CHAPTER

16

Probability

1. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

(a) $\frac{3}{4}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

(2018)

2. A box '*A*' contains 2 white, 3 red and 2 black balls. Another box '*B*' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box '*B*' is :

(a) 7/16 (b) 7/8 (c) 9/16 (d) 9/32

(Online 2018)

3. A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of 'p' is

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

(Online 2018)

4. Let A, B and C be three events, which are pair-wise independent and $\bar{E}$ denotes the complement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P[(\bar{A} \cap \bar{B}) | C]$ is equal to :

(a) $P(A) + P(\bar{B})$ (b) $P(\bar{A}) - P(\bar{B})$

(c) $P(\bar{A}) - P(B)$ (d) $P(\bar{A}) + P(\bar{B})$

(Online 2018)

5. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is :

(a) 3 (b) 5 (c) 4 (d) 6

(Online 2018)

6. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is

(a) 6 (b) 4 (c) $\frac{6}{25}$ (d) $\frac{12}{5}$

(2017)

7. For three events A, B and C,

P(Exactly one of A or B occurs)

$= P$(Exactly one of B or C occurs)

$= P$(Exactly one of C or A occurs) $= \frac{1}{4}$

and P(All the three events occur simultaneously)

$= \frac{1}{16}$. Then the probability that at least one of the events occurs, is

(a) $\frac{7}{16}$ (b) $\frac{7}{64}$ (c) $\frac{3}{16}$ (d) $\frac{7}{32}$

(2017)

8. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10}; then the probability that their sum as well as absolute difference are both multiples of 4, is

(a) $\frac{12}{55}$ (b) $\frac{14}{45}$ (c) $\frac{7}{55}$ (d) $\frac{6}{55}$

(2017)

9. Three persons, P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is

(a) $\frac{39}{64}$ (b) $\frac{21}{64}$ (c) $\frac{15}{64}$ (d) $\frac{9}{64}$

(Online 2017)

10. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is

(a) $\frac{63}{64}$ (b) $\frac{255}{256}$ (c) $\frac{127}{128}$ (d) $\frac{1}{2}$

(Online 2017)

11. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is

(a) $\frac{1}{3}$ (b) $\frac{5}{12}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

(Online 2017)

12. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?

(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_1 and E_3 are independent
(d) E_1, E_2 and E_3 are independent *(2016)*

13. If A and B are any two events such that $P(A) = \frac{2}{5}$ and $P(A \cap B) = \frac{3}{20}$, then the conditional probability, $P(A|(A' \cup B'))$, where A' denotes the complement of A, is equal to

(a) $\frac{11}{20}$ (b) $\frac{5}{17}$ (c) $\frac{8}{17}$ (d) $\frac{1}{4}$

(Online 2016)

14. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is

(a) $\frac{496}{729}$ (b) $\frac{192}{729}$ (c) $\frac{240}{729}$ (d) $\frac{256}{729}$

(Online 2016)

15. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is

(a) $775\left(\frac{6}{8}\right)^{67}$ (b) $77\left(\frac{6}{8}\right)^{66}$

(c) $\frac{::}{8}\left(\frac{7}{8}\right)^{66}$ (d) none of these *(2015)*

16. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to one is

(a) $\frac{6}{6;}$ (b) $\frac{>}{6;}$ (c) $\frac{8}{9}$ (d) $\frac{6:}{6;}$

(Online 2015)

17. Let X be a set containing 10 elements and $P(X)$ be its power set. If A and B are picked up at random from $P(X)$, with replacement, then the probability that A and B have equal number of elements, is

(a) $\frac{^{75}Y_{65}}{7^{65}}$ (b) $\frac{-7^{65}-6.}{7^{75}}$ (c) $\frac{-7^{65}-6.}{7^{65}}$ (d) $\frac{^{75}Y_{65}}{7^{75}}$

(Online 2015)

18. If the lengths of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given that it is an isosceles triangle, is

(a) $\frac{6}{7;}$ (b) $\frac{6}{7<}$ (c) $\frac{6}{76}$ (d) $\frac{6}{6:}$

(Online 2015)

19. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are

(a) equally likely but not independent
(b) independent but not equally likely
(c) independent and equally likely
(d) mutually exclusive and independent

(2014, 2005)

20. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{10}{3^5}$ (b) $\frac{17}{3^5}$ (c) $\frac{13}{3^5}$ (d) $\frac{11}{3^5}$

(2013)

21. Three numbers are chosen at random without replacement from $\{1, 2, 3, ..., 8\}$. The probability that their minimum is 3, given that their maximum is 6, is

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{8}$ (d) $\frac{1}{5}$

(2012)

22. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(a) $P(C|D) < P(C)$ (b) $P(C|D) = \frac{P(D)}{P(C)}$

(c) $P(C|D) = P(C)$ (d) $P(C|D) \geq P(C)$ *(2011)*

23. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(a) $\left[0, \frac{1}{2}\right]$ (b) $\left(\frac{11}{12}, 1\right]$ (c) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (d) $\left(\frac{3}{4}, \frac{11}{12}\right]$

(2011)

24. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(a) 1/3 (b) 2/7 (c) 1/21 (d) 2/23 *(2010)*

25. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an A.P. is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers form an A.P., then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

26. In a binomial distribution $B\left(n,\ p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than

(a) $\frac{1}{\log_{10}4+\log_{10}3}$ (b) $\frac{9}{\log_{10}4-\log_{10}3}$

(c) $\frac{4}{\log_{10}4-\log_{10}3}$ (d) $\frac{1}{\log_{10}4-\log_{10}3}$ *(2009)*

27. One ticket is selected at random from 50 tickets numbered 00, 01, 02, …., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$ (c) $\frac{1}{50}$ (d) $\frac{1}{14}$ *(2009)*

28. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1 *(2008)*

29. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A|B)=\frac{1}{2}$ and $P(B|A)=\frac{2}{3}$. Then $P(B)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ *(2008)*

30. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(a) 8/729 (b) 8/243
(c) 1/729 (d) 8/9 *(2007)*

31. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14 *(2007)*

32. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with a average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$ *(2006)*

33. A random variable X has Poisson distribution with mean 2. The $P(X > 1.5)$ equals

(a) 0 (b) $2/e^2$ (c) $3/e^2$ (d) $1-\frac{3}{e^2}$ *(2005)*

34. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) 1/9 (b) 2/9
(c) 7/9 (d) 8/9 *(2005)*

35. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is:

(a) 128/256 (b) 219/256
(c) 37/256 (d) 28/256 *(2004)*

36. A random variable X has the probability distribution:

X :	1	2	3	4	5	6	7	8
$P(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {$X < 4$}, the probability $P(E \cup F)$ is:

(a) 0.35 (b) 0.77
(c) 0.87 (d) 0.50 *(2004)*

37. The probability that A speaks truth is 4/5, while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact is

(a) 7/20 (b) 1/5
(c) 3/20 (d) 4/5 *(2004)*

38. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is

(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/32 *(2003)*

39. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(a) 3/5 (b) 1/5 (c) 2/5 (d) 4/5 *(2003)*

40. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$. Then set of possible values of x are in the interval

(a) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (b) $\left[\frac{1}{3}, \frac{13}{3}\right]$ (c) $[0, 1]$ (d) $\left[\frac{1}{3}, \frac{1}{2}\right]$ *(2003)*

41. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) 8/3 (b) 3/8
(c) 4/5 (d) 5/4 *(2002)*

42. A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\bar{A}) = 2/3$ then $P(\bar{A} \cap B)$ is

(a) 5/12 (b) 3/8
(c) 5/8 (d) 1/4 *(2002)*

43. A problem in mathematics is given to three students *A, B, C* and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved is

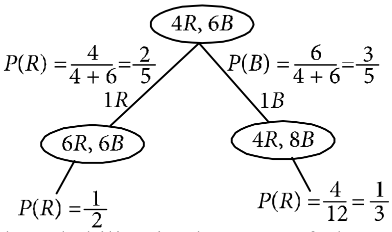
(a) 3/4 (b) 1/2
(c) 2/3 (d) 1/3 *(2002)*

ANSWER KEY

1. (c)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (c)	**11.** (d)	**12.** (d)
13. (b)	**14.** (d)	**15.** (d)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (d)	**21.** (d)	**22.** (d)	**23.** (a)	**24.** (b)
25. (c)	**26.** (d)	**27.** (d)	**28.** (d)	**29.** (c)	**30.** (b)	**31.** (d)	**32.** (d)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (b)
37. (a)	**38.** (d)	**39.** (c)	**40.** (d)	**41.** (d)	**42.** (a)	**43.** (a)					

Explanations

1. (c): Let's draw a state diagram to understand whole condition



The requested probability is the sum of the product of the probabilities along the two possible paths and is equal to $\frac{2}{5}\cdot\frac{1}{2}+\frac{3}{5}\cdot\frac{1}{3} = \frac{1}{5}+\frac{1}{5} = \frac{2}{5}$

2. (a): Let E_1 denotes the event of drawing a white and a red ball and E denotes the event that ball is drawn from box B.

$\therefore \quad P(E) = \frac{1}{2}$

Now, P (ball is drawn from box A) $= P(\bar{E}) = \frac{1}{2}$

$P(E_1/E) = \frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{4\times 2}{36} = \frac{2}{9}$

$P(E_1/\bar{E}) = \frac{{}^2C_1 \times {}^3C_1}{{}^7C_2} = \frac{2\times 3}{21} = \frac{2}{7}$

Using Bayes' Theorem $P(E/E_1) = \frac{P(E)P(E_1/E)}{P(E)P(E_1/E)+P(\bar{E})P(E_1/\bar{E})}$

$$= \frac{\frac{1}{2}\times\frac{2}{9}}{\frac{1}{2}\times\frac{2}{9}+\frac{1}{2}\times\frac{2}{7}} = \frac{1}{9}\times\frac{63}{16} = \frac{7}{16}$$

3. (a): Given : $P(X$ getting head$) = p$

$\therefore \quad P(X$ not getting head$) = 1 - p$

$P(Y$ getting head$) = P(Y$ not getting head$) = \frac{1}{2}$

$P(X \text{ wins}) = p+(1-p)\frac{1}{2}\cdot p+(1-p)\cdot\frac{1}{2}\cdot(1-p)\cdot\frac{1}{2}p+\ldots$

$$= \frac{p}{1-\left(\frac{1-p}{2}\right)} = \frac{2p}{1+p}$$

$P(Y \text{ wins}) = (1-p)\frac{1}{2}+(1-p)\frac{1}{2}\cdot(1-p)\frac{1}{2}+\ldots.$

$$= \left(\frac{1-p}{2}\right)\cdot\frac{1}{\left(1-\frac{1-p}{2}\right)} = \frac{1-p}{1+p}$$

It is given that $P(X$ wins$) = P(Y$ wins$)$

$\therefore \quad \frac{2p}{1+p} = \frac{1-p}{1+p} \Rightarrow 3p = 1 \Rightarrow p = \frac{1}{3}$

4. (c): $P[(\bar{A}\cap\bar{B})\,|\,C] = \frac{P[(\bar{A}\cap\bar{B})\cap C]}{P(C)}$

$$= \frac{P[(\overline{A\cup B})\cap C]}{P(C)} = \frac{P[C-(A\cup B)]}{P(C)}$$

$$= \frac{P(C)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)}{P(C)}$$

$$= \frac{P(C)-P(A\cap C)-P(B\cap C)}{P(C)} \quad [\because P(A\cap B\cap C) = 0]$$

$$= \frac{P(C)-P(A)\cdot P(C)-P(B)\cdot P(C)}{P(C)}$$

[$\because$ A, B and C are independent events]

$= 1 - P(A) - P(B) = P(\bar{A}) - P(B)$ or $P(\bar{B}) - P(A)$

5. (b): Let the number of children in each family be x. So, total number of children in both families are $2x$.

According to question, $\frac{1}{12} = \frac{{}^xC_3\cdot 3!}{{}^{2x}C_3\cdot 3!} \Rightarrow \frac{{}^xC_3}{{}^{2x}C_3} = \frac{1}{12}$

$\Rightarrow \quad \frac{x(x-1)(x-2)\,(x-3)!\,(2x-3)!}{(x-3)!\times(2x)\,(2x-1)\,(2x-2)\,(2x-3)!} = \frac{1}{12}$

$\Rightarrow \quad \frac{x\cdot(x-1)\cdot(x-2)}{(2x)\,(2x-1)\,(2x-2)} = \frac{1}{12}$

$\Rightarrow \quad \frac{(x-2)}{4(2x-1)} = \frac{1}{12} \Rightarrow 3(x-2) = 2x-1 \Rightarrow x = 5$

6. (d): We know that variance $= npq$

P(probability of drawing a green ball) $= \frac{15}{25} = \frac{3}{5}$

Here, $n = 10, p = \frac{3}{5}, q = \frac{2}{5}$ Then, variance $= 10\cdot\frac{3}{5}\cdot\frac{2}{5} = \frac{12}{5}$

7. (a): Given P(exactly one of A or B occurs) $= \frac{1}{4}$

Then, $P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

Similarly, $P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

Also, $P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$

Adding all of them, we have

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$

Now, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3}{8}+\frac{1}{16} = \frac{7}{16}$

8. (d): We have, $4/a - b$ and $4/a + b$

So the possibilities are

a	0	2	4	6	8	10
b	4, 8	6, 10	0, 8	2, 10	0, 4	2, 6

$\therefore \quad$ Required probability $= \frac{6}{{}^{11}C_2} = \frac{6\cdot 2}{11\cdot 10} = \frac{6}{55}$

9. (b): Required probability

$$=\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)+\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)=\frac{21}{64}$$

10. (c): Required probability = 1 – [P(No Head) + P(No Tail)]

$$=1-\left\{\frac{1}{2^8}+\frac{1}{2^8}\right\}=1-\frac{1}{2^7}=1-\frac{1}{128}=\frac{127}{128}$$

11. (d): Let $P(E) = x$ and $P(F) = y$

$P(E \cap F) = P(E) \cdot P(F) = \frac{1}{12} \Rightarrow xy = \frac{1}{12}$

$$P(\bar{E} \cap \bar{F}) = P(\bar{E}) \cdot P(\bar{F}) = \frac{1}{2}$$

$\Rightarrow (1 - P(E))(1 - P(F)) = \frac{1}{2} \Rightarrow (1 - x)(1 - y) = \frac{1}{2}$

$\Rightarrow 1 - x - y + xy = \frac{1}{2} \Rightarrow 1 - x - y + \frac{1}{12} = \frac{1}{2}$

$\Rightarrow 1 - x - y = \frac{1}{2} - \frac{1}{12} = \frac{5}{12} \Rightarrow x + y = \frac{7}{12}$

$\Rightarrow x + \frac{1}{12x} = \frac{7}{12} \Rightarrow \frac{12x^2+1}{12x} = \frac{7}{12}$

$\Rightarrow 12x^2 - 7x + 1 = 0 \Rightarrow 12x^2 - 4x - 3x + 1 = 0$

$\Rightarrow 4x(3x - 1) - 1(3x - 1) = 0$

$\Rightarrow (3x - 1)(4x - 1) = 0 \Rightarrow x = \frac{1}{3}, x = \frac{1}{4}$

$\therefore y = \frac{1}{4}, y = \frac{1}{3} \therefore \frac{P(E)}{P(F)} = \frac{x}{y} = \frac{1/3}{1/4} = \frac{4}{3}$ or $\frac{1/4}{1/3} = \frac{3}{4}$

12. (d): $m\text{-}b_6. = \frac{;}{;\cdot;} = \frac{6}{;}$

$m\text{-}b_7. = \frac{;}{;\cdot;} = \frac{6}{;}\text{B}\, m\text{-}b_8. = \frac{8\cdot 8\cdot 7}{;\cdot;} = \frac{6}{7}$

(For sum to be odd combination as odd + even or even + odd)

$m\text{-}b_6 \cap b_7. = \frac{6}{;\cdot;} = m\text{-}b_6.\cdot m\text{-}b_7.$

$m\text{-}b_6 \cap b_8. = \frac{6\cdot 8}{;\cdot;} = m\text{-}b_6.\cdot m\text{-}b_8.$

$m\text{-}b_7 \cap b_8. = \frac{6\cdot 8}{;\cdot;} = m\text{-}b_7.\cdot m\text{-}b_8.$

As $P(X \cap Y) = P(X) \cdot P(Y)$ the event X and Y are independent. Also, $P(E_1 \cap E_2 \cap E_3) = 0$ as the event cannot happen. So, E_1, E_2, E_3 are pairwise independent, but they together are not independent.

13. (b): We have, $P(A) = \frac{2}{5}; P(A \cap B) = \frac{3}{20}$

$$P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B) = 1 - \frac{3}{20} = \frac{17}{20}$$

Now, $A \cap (A' \cup B') = A \cap (A \cap B)' = A - ((A \cap B)')' = A - (A \cap B)$

$\therefore P(A - (A \cap B)) = \frac{2}{5} - \frac{3}{20} = \frac{1}{4}$

$$\therefore P(A|(A' \cup B')) = \frac{P(A - (A \cap B))}{P(A' \cup B')} = \frac{\frac{1}{4}}{\frac{17}{20}} = \frac{5}{17}$$

14. (d): Let $P(F) = p \Rightarrow P(S) = 2p$

Now, $p + 2p = 1 \Rightarrow p = \frac{1}{3} \therefore P(x \geq 5) = P(x = 5) + P(x = 6)$

$= {}^6C_5 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^5 + {}^6C_6\left(\frac{2}{3}\right)^6 = \frac{256}{729}$

15. (d) : Total number of ways $n(S) = 3^{12}$

$n(E) = {}^{12}C_3 \cdot {}^3C_1 \times (2^9 - {}^9C_3 \cdot 2) + \frac{67! \times 8!}{8!8!;\,!8!}$

Because one of the boxes contains exactly 3 balls, we have to subtract to make the count correct.

$$\text{X\{nmuxµ} = \frac{\text{-}b.}{\text{-}p.} = \frac{{}^{67}Y_8\; {}^8Y_6\text{-}7^{>} - {}^{>}Y_8 \cdot 7. + \frac{67!}{8!8!;\,!}}{8^{67}}$$

16. (d): $np = 2$ ..... (i), $npq = 1$ ..... (ii)

$\Rightarrow \quad = \frac{6}{7}1 \quad = \frac{6}{7}1 \quad = 9$ (from (i) and (ii))

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 6 - {}^9Y_5\left(\frac{6}{7}\right)^5\left(\frac{6}{7}\right)^9 = 6 - \frac{6}{6;} = \frac{6:}{6;}$$

17. (d): Required probability =

$$\frac{\text{-}^{65}Y_5.^7 + \text{-}^{65}Y_6.^7 + \text{-}^{65}Y_7.^7 + 333 + \text{-}^{65}Y_{65}.^7}{\text{-}7^{65}.^7} = \frac{^{75}Y_{65}}{7^{75}}$$

18. (b): Total cases when $a + b > c$ are $\{(1, 1, 1), (2, 2, 1), (2, 2, 2), (2, 2, 3), (3, 3, 1), ..., (3, 3, 5), (4, 4, 1), ..., (4, 4, 6), (5, 5, 1), \ldots, (5, 5, 6), (6, 6, 1), ...,(6, 6, 6)\} = 27$

$\therefore$ Required probability $= \frac{6}{7<}3$

19. (b): $P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - \frac{1}{6} = \frac{5}{6}$

We have $P(\bar{A}) = \frac{1}{4} \Rightarrow P(A) = \frac{3}{4}$

Again, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4} \quad \therefore P(B) = \frac{1}{3}$

Now, $P(A \cap B) = P(A)\,P(B)$ is seem to be true.

Thus A and B are independent.

As $P(A) \neq P(B)$, A and B are not equally likely.

20. (d): P(correct answer) = 1/3

The required probability $= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) + {}^5C_5\left(\frac{1}{3}\right)^5$

$$= \frac{5 \times 2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$$

21. (d) : 3 numbers are chosen from {1, 2, 3, ..., 8} without replacement. Let A be the event that the maximum of chosen numbers is 6.

Let B be the event that the minimum of chosen numbers is 3.

$$P(B/A) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1 \cdot 1 \cdot 2}{{}^8C_3}}{\frac{{}^5C_2}{{}^8C_3}} = \frac{2}{10} = \frac{1}{5}$$

22. (d): $P(C|D)=\dfrac{P(C\cap D)}{P(D)}$ as $C \subset D$, $P(C) \subset P(D)$.

$\therefore$ $P(C \cap D) = P(C)$. We have, $P(C|D)=\dfrac{P(C)}{P(D)}$

As $0 < P(D) \le 1$ we have $P(C|D) \ge P(C)$

23. (a): Probability of at least one failure

$= 1 - P(\text{no failure}) = 1 - p^5$

Now $1-p^5 \ge \dfrac{31}{32} \Rightarrow p^5 \le \dfrac{1}{32}$ thus $p \le \dfrac{1}{2} \therefore p \in [0, 1/2]$

24. (b) : $n(S) = {}^9C_3 = \dfrac{9\times 8\times 7}{6} = 84$

$n(E) = {}^3C_1 \cdot {}^4C_1 \cdot {}^2C_1 = 3 \times 4 \times 2 = 24.$

The desired probability $= \dfrac{24}{84} = \dfrac{2}{7}$.

25. (c) : Number of A.P.'s with common difference 1 = 17

Number of A.P.'s with common difference 2 = 14

Number of A.P.'s with common difference 3 = 11

Number of A.P.'s with common difference 4 = 8

Number of A.P.'s with common difference 5 = 5

Number of A.P.'s with common difference 6 $= \dfrac{2}{57}$

The total number of ways $n(S) = {}^{20}C_4$

The desired probability $= \dfrac{57}{{}^{20}C_4} = \dfrac{57\times 24}{20\times 19\times 18\times 17} = \dfrac{1}{85}$

Now statement-2 is false and statement-1 is true.

26. (d): Probability of at least one success

$= 1 -$ No success $= 1 - {}^nC_n\, q^n$ where $q = 1 - p = 3/4$

we want $1-\left(\dfrac{3}{4}\right)^4 \ge \dfrac{9}{10} \Rightarrow \dfrac{1}{10} \ge \left(\dfrac{3}{4}\right)^4 \Rightarrow \left(\dfrac{3}{4}\right)^4 \le \dfrac{1}{10}$

Taking logarithm on base 10 we have

$n \log_{10}(3/4) \le \log_{10}10^{-1}$

$\Rightarrow n(\log_{10}3 - \log_{10}4) \le -1 \Rightarrow n(\log_{10}4 - \log_{10}3) \ge 1$

$\Rightarrow n \ge \dfrac{1}{\log_{10}4-\log_{10}3}$

27. (d): Any number in the set

$S = \{00, 01, 02, ..., 49\}$ is of the form ab where

$a \in \{0, 1, 2, 3, 4\}$ and $b \in \{0, 1, 2, ..., 9\}$ for the product of digits to be zero, the number must be of the form either $x0$ which are 5 in numbers, because $x \in \{0, 1, 2, 3, 4\}$

or of the form $0x$ which are 10 in numbers because

$x \in \{0, 1, 2, ..., 9\}$

The only number common to both = 00

Thus the number of numbers in S, the product of whose digits is zero = 10 + 5 – 1 = 14

Of these the number whose sum of digits is 8 is just one, *i.e.* 08

The required probability = 1/14.

28. (d) : $A = \{4, 5, 6\}$

Also $B = \{1, 2, 3, 4\}$

We have $A \cup B = \{1, 2, 3, 4, 5, 6\} = S$

Where S is the sample space of the experiment of throwing a die. $P(S) = 1$, for it is a sure event.

Hence $P(A \cup B) = 1$

29. (c) : From the definition of independence of events

$P(A/B)=\dfrac{P(A\cap B)}{P(B)}$

Then $P(B) \cdot P(A/B) = P(A \cap B)$ ...(1)

Interchanging the role of A and B in (1)

$P(A)P(B/A) = P(B \cap A)$ ...(2)

As $A \cap B = B \cap A$, we have from (1) and (2)

$P(A)P(B/A) = P(B)P(A/B)$

$\Rightarrow \dfrac{1}{4}\cdot\dfrac{2}{3} = P(B)\cdot\dfrac{1}{2} \Rightarrow P(B) = \dfrac{1}{4}\cdot\dfrac{2}{3}\cdot 2 = \dfrac{1}{3}$

30. (b) : Possibility of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

Probability of getting score 9 in a single throw $= p = \dfrac{4}{36} = \dfrac{1}{9}$

Required probability = probability of getting score 9 exactly twice

$= {}^3C_2\left(\dfrac{1}{9}\right)^2 \times \left(\dfrac{8}{9}\right) = \dfrac{8}{243}$.

31. (d) : $P(\text{I}) = 0.3, P(\overline{\text{I}}) = 1 - 0.3 = 0.7,$

$P(\text{II}) = 0.2, P(\overline{\text{II}}) = 1 - 0.2 = 0.8$

Required probability $= P(\overline{\text{I}} \cap \text{II}) = P(\overline{\text{I}})P(\text{II}) = (0.7)(0.2) = 0.14$

32. (d) : We know that poission distribution is given by

$P(x=r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$ where $\lambda = 5$

Now $P(x = r \le 1) = P(x = 0) + P(x = 1)$

$= \dfrac{e^{-\lambda}}{0!} + \dfrac{\lambda e^{-\lambda}}{1!} = e^{-5}(1+5) = \dfrac{6}{e^5}$.

33. (d) : $P(X=r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$ $(\lambda = \text{mean})$

$\therefore P(X = r > 1.5) = P(2) + P(3) + ... \infty$

$= 1 - [P(0) + P(1)] = 1 - \left[e^{-2} + \dfrac{e^{-2}\times 2^2}{2}\right] = 1 - \dfrac{3}{e^2}$.

34. (a) : No. of houses = 3 = No. of favourable cases

No. of applicants = 3, $\therefore$ Total number of events $= 3^3$

(because each candidate can apply by 3 ways)

Required probability $= \dfrac{3}{3^3} = \dfrac{1}{9}$.

35. (d) : Given $np = 4$ and $npq = 2$

$q = \dfrac{npq}{np} = \dfrac{2}{4} = \dfrac{1}{2}$ so $p = 1 - 1/2 = \dfrac{1}{2}$

Now $npq = 2 \therefore n = 8$

$\therefore$ BD is given by $P(X = r) = {}^8C_r\, p^r\, q^{n-r}$

$\therefore P(X = r = 2) = {}^8C_2\left(\dfrac{1}{2}\right)^8 = \dfrac{28}{256}$

36. (b) : From the given table prime numbers are 2, 3, 5, 7

'E' denote prime number

'F' denote the number < 4

$\therefore\quad P(E) = P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$

(Events 2, 3, 5, 7 are *M.E*)

$= P(2) + P(3) + P(5) + P(7) = 0.62$

$P(F) = P(1 \text{ or } 2 \text{ or } 3)$ (events 1, 2, 3 are *M.E.*)

$= P(1) + P(2) + P(3) = 0.50$

$P(E \cap F) = P(2 \text{ or } 3) = P(2) + P(3) = 0.35$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.50 - 0.35 = 0.77$

37. (a) : $P(A) = \frac{4}{5} \ \therefore\ P(\bar{A}) = \frac{1}{5}$

$P(B) = \frac{3}{4} \ \therefore\ P(\bar{B}) = \frac{1}{4}$

Now we need $P(A)\,P(\bar{B}) + P(B)\,P(\bar{A}) = \frac{4}{5}\times\frac{1}{4}+\frac{3}{4}\times\frac{1}{5}=\frac{7}{20}$

38. (d) : Given mean $np = 4$, $npq = 2$

$\Rightarrow \frac{npq}{np} = \frac{2}{4} \ \therefore\ q = p = \frac{1}{2}$ and $n = 8$

Now $P(X = r) = {}^8C_r\left(\frac{1}{2}\right)^8$ (Use $P(X = r) = {}^nC_r\, p^r q^{n-r}$)

$\therefore\ P(X = 1) = {}^8C_1\left(\frac{1}{2}\right)^8 = \frac{8}{16\times 16} = \frac{1}{32}$

39. (c) : No. of horses = 5

$\therefore$ Probability that A can't win the race $= \frac{4}{5}\times\frac{3}{4}$

Probability that 'A' must win the race $= 1 - P(\bar{A})P(\bar{B})$

$= 1 - \frac{12}{20} = \frac{2}{5}$

40. (d) : A, B, C are mutually exclusive

$\therefore\quad 0 \le P(A) + P(B) + P(C) \le 1$ ...(i)

$0 \le P(A), P(B), P(C) \le 1$ ...(ii)

Now on solving (i) and (ii) we get $\frac{1}{3} \le x \le \frac{1}{2}$

41. (d) : $n = 5$, $p = q = 1/2$ $\quad P(X = r) = {}^5C_r\left(\frac{1}{2}\right)^5$

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	$\left(\frac{1}{2}\right)^5$	0	0
1	${}^5C_1\left(\frac{1}{2}\right)^5$	$\frac{1\times 5}{32}$	$\frac{5}{32}$
2	${}^5C_2\left(\frac{1}{2}\right)^5$	$\frac{2\times 10}{32}$	$2^2\,\frac{10}{32}$
3	${}^5C_3\left(\frac{1}{2}\right)^5$	$\frac{3\times 10}{32}$	$3^2\,\frac{10}{32}$
4	${}^5C_4\left(\frac{1}{2}\right)^5$	$\frac{4\times 5}{32}$	$4^2\,\frac{5}{32}$
5	${}^5C_5\left(\frac{1}{2}\right)^5$	$5\times\frac{1}{32}$	$5^2\,\frac{1}{32}$
	$\Sigma f_i = 1$	$\Sigma f_i x_i = \frac{80}{32}$	$\Sigma f_i x_i^2 = \frac{240}{32}$

$\bar{x} = \text{mean} = \frac{5}{2}$

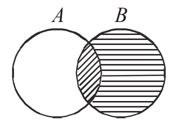
Now variance $= \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2 = \frac{240}{32} - \frac{25}{4} = \frac{40}{32} = \frac{5}{4}$

42. (a) : Given $P(A \cup B) = 3/4$

$P(A \cap B) = 1/4$

$P(\bar{A}) = 2/3$

$\therefore\quad P(B) = \frac{2}{3}$



$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

By using $P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$\therefore\quad P(\bar{A} \cap B) = \frac{2}{3} - \frac{1}{4} = 5/12$

43. (a) : Given $P(A) = 1/2 \quad \therefore\ P(\bar{A}) = 1/2$

$P(B) = 1/3 \ \therefore\ P(\bar{B}) = \frac{2}{3}, \quad P(C) = \frac{1}{4} \ \therefore\ P(\bar{C}) = \frac{3}{4}$

Now problem will be solved if any one of them will solve the problem.

$\therefore\quad P$(at least one of them solve the problem)

= 1 – probability none of them can solve the problem.

or $P(A \cup B \cup C) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C}) = 1 - \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4} = 3/4$