CHAPTER

14 Vector Algebra

1. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to is equal to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to

(a) 336 (b) 315 (c) 256 (d) 84 *(2018)*

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = 0$, then $|\vec{a} \times \vec{c}|$ is equal to :

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{15}}{16}$ (c) $\frac{15}{16}$ (d) $\frac{\sqrt{15}}{4}$ *(Online 2018)*

3. If the position vectors of the vertices *A, B* and *C* of a ΔABC are respectively $4\hat{i} + 7\hat{j} + 8\hat{k}, 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets *BC* is

(a) $\frac{1}{2}(4\hat{i} + 8\hat{j} + 11\hat{k})$ (b) $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

(c) $\frac{1}{4}(8\hat{i} + 14\hat{j} + 19\hat{k})$ (d) $\frac{1}{3}(6\hat{i} + 11\hat{j} + 15\hat{k})$

(Online 2018)

4. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Then $|\vec{b}|$ equals :

(a) $\sqrt{\frac{11}{3}}$ (b) $\frac{11}{\sqrt{3}}$ (c) $\frac{\sqrt{11}}{3}$ (d) $\frac{11}{3}$ *(Online 2018)*

5. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between be 30°. Then $\vec{a} \cdot \vec{c}$ is equal to

(a) 2 (b) 5 (c) $\frac{1}{8}$ (d) $\frac{25}{8}$ *(2017)*

6. The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is

(a) 65 (b) 52 (c) 26 (d) 20 *(Online 2017)*

7. If the vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{b}_2$, perpendicular to then $\vec{b}_1 \times \vec{b}_2$ is equal to

(a) $3\hat{i} - 3\hat{j} + 9\hat{k}$ (b) $-3\hat{i} + 3\hat{j} - 9\hat{k}$

(c) $-6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k}$ (d) $6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

(Online 2017)

8. Let →1→ mzp→ be three unit vectors such that →×-→×→. $= \frac{\sqrt{8}}{7}$-→+→.3 If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between → mzp→ is

(a) $\frac{8\pi}{9}$ (b) $\frac{\pi}{7}$ (c) $\frac{7\pi}{8}$ (d) $\frac{:\pi}{;}$ *(2016)*

9. In a triangle *ABC*, right angled at the vertex *A*, if the position vectors of *A*, *B* and *C* are respectively $3\hat{i} + \hat{j} - \hat{k}, -\hat{i} + 3\hat{j} + p\hat{k}$ and $5\hat{i} + q\hat{j} - 4\hat{k}$, then the point (*p*, *q*) lies on a line

(a) making an obtuse angle with the positive direction of *x*-axis.

(b) parallel to *x*-axis. (c) parallel to *y*-axis

(d) making an acute angle with the positive direction of *x*-axis. *(Online 2016)*

10. Let *ABC* be a triangle whose circumcentre is at *P*. If the position vectors of *A*, *B*, *C* and *P* are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a} + \vec{b} + \vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is

(a) $-\left(\frac{\vec{a} + \vec{b} + \vec{c}}{2}\right)$ (b) $\vec{a} + \vec{b} + \vec{c}$

(c) $\left(\frac{\vec{a} + \vec{b} + \vec{c}}{2}\right)$ (d) $\vec{0}$ *(Online 2016)*

11. Let →1→ mzp→ be three non-zero vectors such that no two of them are collinear and -→×→.×→ $= \frac{6}{8}$|→||→|→. If θ is the angle between vectors → and →, then a value of sin θ is

(a) $\frac{7}{8}$ (b) $\frac{-7\sqrt{8}}{8}$ (c) $\frac{7\sqrt{7}}{8}$ (d) $\frac{-\sqrt{7}}{8}$ *(2015)*

12. Let → mzp→ be two unit vectors such that →+→ $= \sqrt{8}$. If →=→+7→+8-→×→., then 7→ is equal to

(a) $\sqrt{::}$ (b) $\sqrt{:6}$ (c) $\sqrt{98}$ (d) $\sqrt{8<}$ *(Online 2015)*

13. In a parallelogram *ABCD*, $|\overrightarrow{WX}| =$ 1$|\overrightarrow{Wa}| =$ mzp $|\overrightarrow{WY}| =$ 1 µt qz $\overrightarrow{aX} \cdot \overrightarrow{WX}$ has the value

(a) $\frac{6}{7}$- 7 − 7 + 7. (b) $\frac{6}{9}$- 7 + 7 − 7.

(c) $\frac{6}{8}$- 7 + 7 − 7. (d) $\frac{6}{7}$-8 7 + 7 − 7.

(Online 2015)

14. If $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times a] = \lambda[\vec{a}\,\vec{b}\,\vec{c}]^2$, then λ is equal to
(a) 3 (b) 0 (c) 1 (d) 2 *(2014)*

15. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle *ABC*, then the length of the median through *A* is
(a) $\sqrt{45}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$ *(2013)*

16. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} + 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$ *(2012)*

17. Let *ABCD* be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex *B* to the side *AD*, then $\vec{r}$ is given by
(a) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (b) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$
(c) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (d) $\vec{r} = -\vec{q} + \frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ *(2012)*

18. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a} + 2\vec{b}\right)\right]$ is
(a) 5 (b) 3 (c) –5 (d) –3 *(2011)*

19. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d} = 0$. Then the vector $\vec{d}$ is equal to
(a) $\vec{b} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (b) $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$
(c) $\vec{b} - \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (d) $\vec{c} + \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ *(2011)*

20. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b} + \vec{c} = 0$ and $\vec{a}\cdot\vec{b} = 3$ is
(a) $-\hat{i} + \hat{j} - 2\hat{k}$ (b) $2\hat{i} - \hat{j} + 2\hat{k}$
(c) $\hat{i} - \hat{j} - 2\hat{k}$ (d) $\hat{i} + \hat{j} - 2\hat{k}$ *(2010)*

21. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$
(a) (–3, 2) (b) (2, –3)
(c) (–2, 3) (d) (3, –2) *(2010)*

22. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and *p*, *q* are real numbers, then the equality $[3\vec{u}\ \ p\vec{v}\ \ p\vec{w}] - [p\vec{v}\ \ \vec{w}\ \ q\vec{u}] - [2\vec{w}\ \ q\vec{v}\ \ q\vec{u}] = 0$ holds for
(a) exactly two values of (*p*, *q*)
(b) more than two but not all values of (*p*, *q*)
(c) all values of (*p*, *q*)
(d) exactly one value of (*p*, *q*) *(2009)*

23. The non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is
(a) π (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$ *(2008)*

24. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?
(a) $\alpha = 1, \beta = 1$ (b) $\alpha = 2, \beta = 2$
(c) $\alpha = 1, \beta = 2$ (d) $\alpha = 2, \beta = 1$ *(2008)*

25. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for
(a) no value of θ (b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ *(2007)*

26. The values of *a*, for which the points *A*, *B*, *C* with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right-angled triangle at *C* are
(a) 2 and 1 (b) –2 and –1
(c) –2 and 1 (d) 2 and –1 *(2006)*

27. If $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}\cdot\vec{b} \neq 0$, $\vec{b}\cdot\vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are
(a) inclined at an angle of $\pi/3$ between them
(b) inclined at an angle of $\pi/6$ between them
(c) perpendicular (d) parallel *(2006)*

28. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector and λ is a real number then $\left[\lambda(\vec{a} + \vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}\right] = \left[\vec{a}\ \ \vec{b} + \vec{c}\ \ \vec{b}\right]$ for
(a) no value of λ (b) exactly one value of λ
(c) exactly two values of λ (d) exactly three values of λ *(2005)*

29. Let *a*, *b* and *c* be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then *c* is
(a) the arithmetic mean of *a* and *b*
(b) the geometric mean of *a* and *b*
(c) the harmonic mean of *a* and *b*
(d) equal to zero *(2005)*

30. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$. Then $\left[\vec{a}, \vec{b}, \vec{c}\right]$ depends on
(a) only *x* (b) only *y*
(c) neither *x* nor *y* (d) both *x* and *y* *(2005)*

31. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$ is equal to
(a) $\vec{a}^2$ (b) $3\vec{a}^2$ (c) $4\vec{a}^2$ (d) $2\vec{a}^2$ *(2005)*

32. If *C* is the mid point of *AB* and *P* is any point outside *AB*, then
(a) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 0$ (b) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$
(c) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$ (d) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$ *(2005)*

33. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals

(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2\sqrt{2}}{3}$ *(2004)*

34. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals

(a) $\sqrt{14}$ (b) $\sqrt{7}$ (c) 2 (d) 14 *(2004)*

35. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for

(a) all except two values of λ
(b) all except one value of λ
(c) all values of λ (d) no value of λ *(2004)*

36. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by

(a) 25 (b) 30 (c) 40 (d) 15 *(2004)*

37. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals

(a) $\lambda\vec{c}$ (b) $\lambda\vec{b}$ (c) $\lambda\vec{a}$ (d) 0 *(2004)*

38. $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{b}|=2, |\vec{c}|=3$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to

(a) –7 (b) 7 (c) 1 (d) 0 *(2003)*

39. The vectors $\overline{AB} = 3\hat{i}+4\hat{k}$ and $\overline{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(a) $\sqrt{72}$ (b) $\sqrt{33}$ (c) $\sqrt{288}$ (d) $\sqrt{18}$ *(2003)*

40. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$, then $|\vec{w}\cdot\hat{n}|$ is equal to

(a) 1 (b) 2 (c) 3 (d) 0 *(2003)*

41. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w})\cdot(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals

(a) $\vec{u}\cdot\vec{v}\times\vec{w}$ (b) $\vec{u}\cdot\vec{w}\times\vec{v}$
(c) $3u\cdot\vec{u}\times\vec{w}$ (d) 0 *(2003)*

42. Consider A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then $ABCD$ is a

(a) rhombus (b) rectangle
(c) parallelogram but not a rhombus
(d) none of these *(2003)*

43. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$

(a) abc (b) –1 (c) 0 (d) 2 *(2002)*

44. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}| : |\vec{b}| : |\vec{c}| =$

(a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$ (b) $\sqrt{34} : \sqrt{45} : 39$
(c) 34 : 39 : 45 (d) 39 : 35 : 34 *(2002)*

45. $3\lambda\vec{c}+2\mu(\vec{a}\times\vec{b})=0$ then

(a) $3\lambda + 2\mu = 0$ (b) $3\lambda = 2\mu$
(c) $\lambda = \mu$ (d) $\lambda + \mu = 0$ *(2002)*

46. If $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ thus what will be the value of $|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$

(a) 25 (b) 50 (c) –25 (d) –50 *(2002)*

47. If $\vec{a}, \vec{b}, \vec{c}$ are vectors show that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7$, $|\vec{b}|=5$, $|\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is

(a) 60º (b) 30º (c) 45º (d) 90º *(2002)*

48. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}]=4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=$

(a) 16 (b) 64 (c) 4 (d) 8 *(2002)*

49. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then $(\vec{a}\times\vec{b})^2$ is equal to

(a) 48 (b) 16
(c) $\vec{a}$ (d) none of these *(2002)*

ANSWER KEY

1. (a)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (d)	**10.** (c)	**11.** (c)	**12.** (a)
13. (d)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (a)	**21.** (a)	**22.** (d)	**23.** (a)	**24.** (a)
25. (b)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (c)	**31.** (d)	**32.** (d)	**33.** (d)	**34.** (a)	**35.** (a)	**36.** (c)
37. (d)	**38.** (a)	**39.** (b)	**40.** (c)	**41.** (a)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (a)
49. (b)											

Explanations

1. **(a):** Let $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$ gives $2x + 3y - z = 0$ ...(i)

gives $y + z = 24$ ...(ii)

Also, $\vec{u}$ is coplanar with $\vec{a}$ and $\vec{b}$, so $\left[\vec{u}\ \vec{a}\ \vec{b}\right] = 0$

which yields $\begin{vmatrix} x & y & z \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0$

i.e. $4x - 2y + 2z = 0$ ...(iii)

(ii) and (iii) gives $2x + 2z = 24$ *i.e.* $x + z = 12$

From (i), we get $z = 16$ and thus $x = -4$ and $y = 8$.

Hence, $\vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$|\vec{u}| = 4\sqrt{1^2 + 2^2 + 4^2} = 4\sqrt{21}\ \therefore\ |\vec{u}|^2 = 336$

2. **(d):** Given, $\vec{a} + 2\vec{b} + 2\vec{c} = 0 \Rightarrow \vec{a} + 2\vec{c} = -2\vec{b}$

Squaring both sides, we get $|\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}\cdot\vec{c} = 4|\vec{b}|^2$

$\Rightarrow\ 1 + 4 + 4(\vec{a}\cdot\vec{c}) = 4\ \ (\because |\vec{a}| = |\vec{b}| = |\vec{c}| = 1)$

$\Rightarrow\ \vec{a}\cdot\vec{c} = \frac{-1}{4} \Rightarrow |\vec{a}|\cdot|\vec{c}|\cos\theta = \frac{-1}{4} \Rightarrow \cos\theta = \frac{-1}{4}$

Now, $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{16} = \frac{15}{16}$

$\therefore\ |\vec{a}\times\vec{c}| = |\vec{a}||\vec{c}|\sin\theta = \frac{\sqrt{15}}{4}$

3. **(b):** Position vector of A, B and C are respectively $4\hat{i} + 7\hat{j} + 8\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$

$\therefore\ |\overline{AB}| = \sqrt{(2-4)^2 + (3-7)^2 + (4-8)^2} = \sqrt{4+16+16} = 6$

$|\overline{BC}| = \sqrt{(2-2)^2 + (5-3)^2 + (7-4)^2} = \sqrt{4+9} = \sqrt{13}$

$|\overline{CA}| = \sqrt{(2-4)^2 + (5-7)^2 + (7-8)^2} = \sqrt{4+4+1} = 3$

Let D be the bisector of $\angle A$ which meets BC.

$\therefore\ \frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{3} = \frac{2}{1}$

Using section formula, we have

$x = \frac{2\times2 + 2\times1}{3} = \frac{6}{3},\ y = \frac{5\times2 + 3\times1}{3} = \frac{13}{3},\ z = \frac{7\times2 + 4\times1}{3} = \frac{18}{3}$

So, position vector of D is $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

4. **(a):** Here, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$

Now, $\vec{a}\times\vec{b} = \vec{c}$ (Given)

$\Rightarrow\ \vec{a}\times(\vec{a}\times\vec{b}) = \vec{a}\times\vec{c} \Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} = \vec{a}\times\vec{c}$

$\Rightarrow\ 3\vec{a} - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k} \Rightarrow 3(\hat{i} + \hat{j} + \hat{k}) - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$

$\Rightarrow\ 3\hat{i} + 3\hat{j} + 3\hat{k} - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$

$\Rightarrow\ \vec{b} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k}) \Rightarrow |\vec{b}| = \frac{\sqrt{25+4+4}}{3} \Rightarrow |\vec{b}| = \sqrt{\frac{11}{3}}$

5. **(a):** $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k} \Rightarrow |a| = 3$ and $\vec{b} = \hat{i} + \hat{j}$

Now, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}\ \therefore\ |\vec{a}\times\vec{b}| = 3$

We also have, $|(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}||\sin 30°||\hat{n}| = 3|\vec{c}|\cdot\frac{1}{2}n$

$\Rightarrow\ 3 = 3|\vec{c}|\cdot\frac{1}{2}\ \therefore\ |\vec{c}| = 2$

Since, $|\vec{c} - \vec{a}| = 3$ ...(i)

On squaring (i), we get $c^2 + a^2 - 2\vec{c}\cdot\vec{a} = 9$

$\Rightarrow\ 4 + 9 - 2\vec{a}\cdot\vec{c} = 9 \Rightarrow \vec{a}\cdot\vec{c} = 2$

6. **(a):** Let $\vec{a} = 8\hat{i} - 6\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 12\hat{k}$

Area of parallelogram, $A = \frac{1}{2}|\vec{a}\times\vec{b}|$

$\therefore\ \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix} = 72\hat{i} - (-96)\hat{j} + 50\hat{k}$

$\therefore\ |\vec{a}\times\vec{b}| = 5184 + 9216 + 2500 = \sqrt{16900} = 130$

$A = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}\times130 = 65$

7. **(d):** $\vec{b} = 3\hat{j} + 4\hat{k}, \vec{a} = \hat{i} + \hat{j}$

Given that $\vec{b}_1$ is parallel to $\vec{a}$.

$\therefore\ \vec{b}_1 = \frac{(\vec{b}\cdot\vec{a})\hat{a}}{|\vec{a}|} = \left\{\frac{(3\hat{j} + 4\hat{k})\cdot(\hat{i} + \hat{j})}{\sqrt{2}}\right\}\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) = \frac{3(\hat{i} + \hat{j})}{\sqrt{2}\times\sqrt{2}} = \frac{3(\hat{i} + \hat{j})}{2}$

Also, $\vec{b}_1 + \vec{b}_2 = \vec{b} \Rightarrow \vec{b}_2 = \vec{b} - \vec{b}_1 = (3\hat{j} + 4\hat{k}) - \frac{3}{2}(\hat{i} + \hat{j})$

$\Rightarrow\ \vec{b}_2 = -\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 4\hat{k}$ Now, $\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{3}{2} & 0 \\ -\frac{3}{2} & \frac{3}{2} & 4 \end{vmatrix}$

$= \hat{i}(6) - \hat{j}(6) + \hat{k}\left(\frac{9}{4} + \frac{9}{4}\right) = 6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

8. **(d):** We have, $\vec{}\times -\vec{}\times\vec{}. = \frac{\sqrt{8}}{7} -\vec{} + \vec{}.$

$\Rightarrow\ -\vec{}3\vec{}.\vec{} - -\vec{}3\vec{}.\vec{} = \frac{\sqrt{8}}{7} -\vec{} + \vec{}.$

On comparing, $\vec{}\cdot\vec{} = -\frac{\sqrt{8}}{7}$. Then $\theta = \frac{:\pi}{;}$

9. **(d):** We have, $\overrightarrow{AB} = -4\hat{i} + 2\hat{j} + (p+1)\hat{k}$

$\overrightarrow{AC} = 2\hat{i} + (q-1)\hat{j} - 3\hat{k}$

ABC is a right angled triangle, right angle at A.

$\therefore\ \overrightarrow{AB} \perp \overrightarrow{AC} \ \Rightarrow\ \overrightarrow{AB}\cdot\overrightarrow{AC} = 0$

$\Rightarrow\ -8 + 2(q-1) - 3(p+1) = 0 \Rightarrow 3p - 2q + 13 = 0$

$\therefore\ (p, q)$ lies on the line $3x - 2y + 13 = 0$

Now, slope of line $= \frac{3}{2}$

$\therefore$ The point (p, q) lies on a line making acute angle with the positive direction of x-axis.

10. (c): Position vector of centroid $\vec{G} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$

Position vector of circumcentre $\vec{P} = \frac{\vec{a}+\vec{b}+\vec{c}}{4}$

Let $\vec{r}$ be the orthocentre of the triangle.

Now, we know that, $\vec{G} = \frac{2\vec{P}+\vec{r}}{3} \Rightarrow 3\vec{G} = 2\vec{P}+\vec{r}$

$\Rightarrow\ \vec{r} = 3\vec{G} - 2\vec{P} = (\vec{a}+\vec{b}+\vec{c}) - 2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}\right) = \frac{\vec{a}+\vec{b}+\vec{c}}{2}$

11. (c): Expanding -→·→.→ − -→·→.→ = 6/8 → → →

⇒ -→·→.→ − {-→·→. + 6/8 → →}→ = 5

As → mzp → are non-collinear, the coefficients must vanish.

Thus, →·→ = 5 mzp -→·→. = −6/8 → →

Again, o{ −θ = →.→ / (→ →) ⇒ o{ −θ = −6/8 ⇒ −uz θ = $\frac{7\sqrt{7}}{8}$

12. (a): Given that → + → = $\sqrt{8}$ .... (i)

Squaring (i) both sides, we get

→ + → 7 = 8 ⇒ → 7 + → 7 +7→·→ = 8

$\Rightarrow\ 1 + 1 + 2\cos\theta = 3 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

→×→ is perpendicular to plane containing → mzp →

→ = → +7 → +8-→×→. .... (ii)

Squaring (ii) both sides, we get

→ 7 = → 7 +9 → 7 +>-→×→.7 +9-→·→.+;→·-→×→.+67→·-→×→.

$= |1 + 4 + 9\sin^2\theta + 4\cos\theta + 0 + 0|$

= |: +>× 8/9 +9× 6/7| ⇒ → 7 = <+ 7</9 = ::/9 ⇒ 7 → = √::

13. (d): Now, $\overrightarrow{WX} + \overrightarrow{XY} = \overrightarrow{WY}$

$\overrightarrow{WX} + \overrightarrow{Wa} = \overrightarrow{WY}$

$\Rightarrow\ \overrightarrow{WX}^{7} + \overrightarrow{Wa}^{7} + 7\,\overrightarrow{WX}\cdot\overrightarrow{Wa} = \overrightarrow{WY}^{7}$

$\Rightarrow a^2 + b^2 + 7\,\overrightarrow{WX}\cdot\overrightarrow{Wa} = {}^{7} \Rightarrow a^2 + b^2 + 7\overrightarrow{WX}\cdot\text{-}\overrightarrow{WX} - \overrightarrow{Xa}\,. = {}^{7}$

$\Rightarrow a^2 + b^2 + 2a^2 - 7\overrightarrow{WX}\cdot\overrightarrow{Xa} = {}^{7} \Rightarrow \overrightarrow{WX}\cdot\overrightarrow{aX} = \frac{8\ ^{7} +\ ^{7} -\ ^{7}}{7}$

14. (c): $\left[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\right] = (\vec{a}\times\vec{b})\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\}$

$= (\vec{a}\times\vec{b})\{(\vec{b}\times\vec{c}\cdot\vec{a})\vec{c} - (\vec{b}\times\vec{c}\cdot\vec{c})\vec{a}\}$

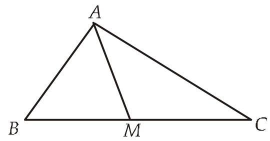
$= (\vec{a}\times\vec{b})\,[\vec{a}\ \vec{b}\ \vec{c}]\vec{c} = [\vec{a}\ \vec{b}\ \vec{c}][\vec{a}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$ $\therefore$ On comparison, $\lambda = 1$

15. (d): $\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

$= \frac{1}{2}\{(3, 0, 4) + (5, -2, 4)\}$

$= \frac{1}{2}(8, -2, 8) = (4, -1, 4)$

$\therefore |\overrightarrow{AM}| = \sqrt{4^2 + 1^2 + 4^2} = \sqrt{33}$



16. (a): $\vec{c} = \hat{a} + 2\hat{b},\ \vec{d} = 5\hat{a} - 4\hat{b}$

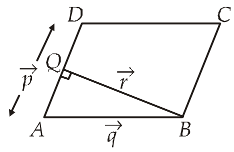
$\therefore \vec{c}\cdot\vec{d} = 0 \Rightarrow (\hat{a}+2\hat{b})\cdot(5\hat{a}-4\hat{b}) = 5 - 4\hat{b}\cdot\hat{a} + 10\hat{b}\cdot\hat{a} - 8$

$\Rightarrow 6\hat{b}\cdot\hat{a} - 3 = 0 \Rightarrow \hat{b}\cdot\hat{a} = \frac{1}{2}\ \therefore \theta = \frac{\pi}{3}$

17. (d): $\vec{r} = \overrightarrow{BA} + \overrightarrow{AQ}$

$= -\vec{q} +$ projection of $\overrightarrow{BA}$ across $\overrightarrow{AD}$

$= -\vec{q} + \frac{(\vec{p}\cdot\vec{q})\vec{p}}{(\vec{p}\cdot\vec{p})}$



18. (c): $(2\vec{a} - \vec{b})\cdot\{(\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b})\}$

$= (2\vec{a} - \vec{b})\cdot\{(\vec{a}\times\vec{b})\times\vec{a} + 2(\vec{a}\times\vec{b})\times\vec{b}\}$

$= (2\vec{a} - \vec{b})\cdot\{(\vec{a}\cdot\vec{a})\vec{b} - (\vec{a}\cdot\vec{b})\vec{a} + 2(\vec{a}\cdot\vec{b})\vec{b} - 2(\vec{b}\cdot\vec{b})\vec{a}\}$

$= (2\vec{a} - \vec{b})\cdot(\vec{b} - 2\vec{a}) = -4\vec{a}\cdot\vec{a} - \vec{b}\cdot\vec{b} = -5$

19. (b): $\vec{a}\cdot\vec{b} \neq 0$ (given) $\vec{a}\cdot\vec{d} = 0$

Now, $\vec{b}\times\vec{c} = \vec{b}\times\vec{d} \Rightarrow \vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times(\vec{b}\times\vec{d})$

$\Rightarrow\ (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})c = (\vec{a}.\vec{d})\vec{b} - (\vec{a}.\vec{b})\vec{d}\ \Rightarrow\ (\vec{a}.\vec{b})\vec{d} = -(\vec{a}.\vec{c})\vec{b} + (\vec{a}.\vec{b})\vec{c}$

$\Rightarrow\ \vec{d} = -\frac{(\vec{a}\cdot\vec{c})\vec{b}}{(\vec{a}.\vec{b})} + \vec{c}$

20. (a) : We have $\vec{a}\times\vec{b} + \vec{c} = 0$

Multiplying vectorially with $\vec{a}$, we have

$\vec{a}\times(\vec{a}\times\vec{b}) + \vec{a}\times\vec{c} = 0 \Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} + \vec{a}\times\vec{c} = 0$

$\vec{a}\times\vec{c} = (\hat{j} - \hat{k})\times(\hat{i} - \hat{j} - \hat{k}) = -2\hat{i} - \hat{j} - \hat{k}$

Thus, $3(\hat{j} - \hat{k}) - 2\vec{b} - 2\hat{i} - \hat{j} - \hat{k} = 0\ \therefore\ \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

21. (a) : $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + 4\hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$

$\vec{a}$ and $\vec{c}$ are orthogonal $\Rightarrow \vec{a}\cdot\vec{c} = 0$ giving $\lambda - 1 + 2\mu = 0$

Also $\vec{b}$ and $\vec{c}$ are orthogonal $\Rightarrow 2\lambda + 4 + 4\mu = 0$

Solving the equation we get $\lambda = -3,\ \mu = 2$.

22. (d): We have $[l\vec{a}\ m\vec{b}\ n\vec{c}] = lmn[\vec{a}\,\vec{b}\,\vec{c}]$ for scalars l, m, n.

Also $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$ (cyclic)

And $[\vec{a}\,\vec{b}\,\vec{c}] = -[\vec{a}\,\vec{c}\,\vec{b}]$ (Interchange of any two vectors)

$[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$

$\Rightarrow\ 3p^2\,[\vec{u}\ \vec{v}\ \vec{w}] - pq[\vec{u}\ \vec{v}\ \vec{w}] + 2q^2[\vec{u}\ \vec{v}\ \vec{w}] = 0$

$\Rightarrow\ (3p^2 - pq + 2q^2)[\vec{u}\ \vec{v}\ \vec{w}] = 0$

As $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar, $[\vec{u}\ \vec{v}\ \vec{w}] \neq 0$

Hence $3p^2 - pq + 2q^2 = 0,\ p, q \in R$

As a quadratic in p, roots are real

$\Rightarrow\ q^2 - 24q^2 \geq 0 \Rightarrow -23q^2 \geq 0 \Rightarrow q^2 \leq 0 \Rightarrow q = 0$

And thus $p = 0$

Thus $(p, q) \equiv (0, 0)$ is the only possibility.

23. **(a) :** $\vec{a} = 8\vec{b}$, $\vec{c} = -7\vec{b}$

$\vec{a}$ and $\vec{b}$ are parallel and $\vec{b}$ and $\vec{c}$ are antiparallel.

Thus $\vec{a}$ and $\vec{c}$ are antiparallel.

Hence the angle between $\vec{a}$ and $\vec{c}$ is π.

24. **(a) :** $\vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$. Also $\vec{a}$ bisects the angle $\vec{b}$ and $\vec{c}$. Thus $\vec{a} = \lambda(\vec{b} + \vec{c})$

$$\alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{j}+\hat{k}}{\sqrt{2}}\right) = \lambda\left(\frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{2}}\right)$$

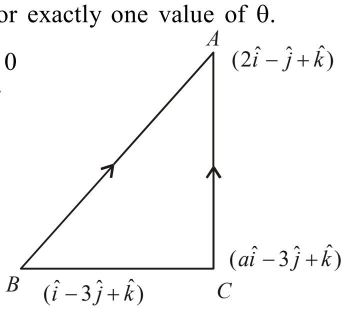
on comparison, $\lambda = \sqrt{2}\alpha, \lambda = \sqrt{2}$ and $\lambda = \sqrt{2}\beta$

Thus $\alpha = 1$ and $\beta = 1$

25. **(b) :** $|2\hat{u} \times 3\hat{v}| = 1 \Rightarrow 6|\hat{u}\|\hat{v}\|\sin\theta| = 1 \Rightarrow \sin\theta = \frac{1}{6}$

$2\hat{u} \times 3\hat{v}$ is a unit vector for exactly one value of θ.

26. **(a) :** Now $\vec{CA} \cdot \vec{CB} = 0$



where $\vec{CA} = (2-a)\hat{i} - 2\hat{j}$

and $\vec{CB} = (1-a)\hat{i} - 6\hat{k}$

$\Rightarrow \quad a^2 - 3a + 2 = 0$

$\Rightarrow (a - 2)(a - 1) = 0$

$\Rightarrow \quad a = 1, 2$

27. **(d) :** Given $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$

$\Rightarrow \quad (\vec{b} \cdot \vec{c})\vec{a} = (\vec{a} \cdot \vec{b})\vec{c} \Rightarrow \lambda_1\vec{a} = \lambda_2\vec{c}$

$(\lambda_1 = \vec{b} \cdot \vec{c}, \lambda_2 = \vec{a} \cdot \vec{b}$ are scalar quantities$) \Rightarrow \vec{a} \parallel \vec{c}$

28. **(a) :** From given $\lambda(\vec{a} + \vec{b}) \cdot (\lambda^2\vec{b} \times \lambda\vec{c}) = \vec{a} \cdot (\vec{b} + \vec{c}) \times \vec{b}$

$\Rightarrow \lambda\vec{a} \cdot (\lambda^2\vec{b} + \lambda\vec{c}) + \lambda\vec{b} \cdot (\lambda^2\vec{b} \times \lambda\vec{c}) = \vec{a} \cdot (\vec{c} \times \vec{b})$

$\Rightarrow \lambda^4[a \ b \ c] = -[a \ b \ c] \Rightarrow \lambda^4 + 1 = 0$

$\Rightarrow (\lambda^2)^2 + 1 = 0 \qquad D < 0$

$\Rightarrow$ No value of λ exist on real axis.

29. **(b) :** We are given that points lies in the same plane. We know that the vector L, M, N are coplanar if

$$L \cdot (\vec{M} \times \vec{N}) = 0 \Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c = \sqrt{ab}$$

$\therefore$ C is G.M. of a and b.

30. **(c) :** $$[a, \ b, \ c] = \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} \ C_3 \to C_3 + C_1 = \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix} = 1(1) = 1$$

which is independent of x and y.

31. **(d) :** Let $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \ \therefore \ \vec{a}^2 = a_1^2 + b_1^2 + c_1^2$

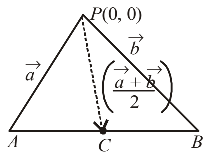
$\therefore \ \vec{a} \times \hat{i} = -b_1\hat{k} + c_1\hat{j} \ \therefore \ (\vec{a} \times \hat{i})^2 = b_1^2 + c_1^2$

Similarly $(\vec{a} \times \hat{j})^2 = a_1^2 + c_1^2$

$(\vec{a} \times \hat{k})^2 = a_1^2 + b_1^2$

$\therefore \ (\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = 2(a_1^2 + b_1^2 + c_1^2) = 2\vec{a}^2$.

32. **(d) :** Let P is origin



Let $\vec{PA} = \vec{a}, \ \vec{PB} = \vec{b}$

$\therefore \ \vec{PC} = \frac{\vec{a} + \vec{b}}{2}$

Now $\vec{PA} + \vec{PB} = \vec{a} + \vec{b} = 2\left(\frac{\vec{a}+\vec{b}}{2}\right) = 2\vec{PC}$.

33. **(d) :** $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}|b||c|\vec{a}$ (As given)

$\Rightarrow \quad \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{b} \cdot \vec{c}\right)\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -\vec{b} \cdot \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}| \Rightarrow \cos\theta = -1/3$

$\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

34. **(a):** Given $\frac{\vec{v} \cdot \vec{u}}{|u|} = \frac{\vec{u} \cdot \vec{w}}{|u|}$ and $\vec{v} \cdot \vec{w} = 0$

Also $|\vec{u}| = 1, \ |\vec{v}| = 2, \ |\vec{w}| = 3$

Now $|\vec{u} - \vec{v} + \vec{w}|^2 = \vec{u}^2 + \vec{v}^2 + \vec{w}^2 - 2\vec{u} \cdot \vec{v} - 2\vec{v} \cdot \vec{w} + 2\vec{u} \cdot \vec{w}$

$= 1 + 4 + 9 + 0 = 14$

35. **(a) :** Using the condition of coplanarity of three vectors

$$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0 \Rightarrow \lambda = 0, \ \frac{1}{2}.$$

36. **(c) :** Total force $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7\hat{i} + 2\hat{j} - 14\hat{k}$ and displacement $\vec{d} = \vec{d}_2 - \vec{d}_1 = (5-1)\hat{i} + (4-2)\hat{j} + (1-3)\hat{k}$

$\therefore$ Work done $= \vec{F} \cdot \vec{d} = 28 + 4 + 8 = 40$

37. **(d):** As $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$

$\therefore \ \vec{a} + 2\vec{b} = P\vec{c}$ ...(i)

and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ $\therefore$ $\vec{b} + 3\vec{c} = Q\vec{a}$ ...(ii)

Now by (i) and (ii) we have $\vec{a} - 6\vec{c} = P\vec{c} - 2Q\vec{a}$

$\Rightarrow \vec{a}(1 + 2Q) + \vec{c}(-6 - P) = 0 \Rightarrow 1 + 2Q = 0$ and $-P - 6 = 0$

$Q = -1/2, \ P = -6$

Putting these values either in (i) or in (ii) we get $\vec{a} + 2\vec{b} + 6\vec{c} = 0$

38. **(a) :** $\vec{a} + \vec{b} + \vec{c} = 0$

Consider $(\vec{a} + \vec{b} + \vec{c})^2 = a^2 + b^2 + c^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$

$$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -\frac{(a^2 + b^2 + c^2)}{2} = -\frac{(1^2 + 2^2 + 3^2)}{2} = -7$$

39. **(b) :** Median through any vertex divide the opposite side into two equal parts $\vec{AB} + \vec{AC} = 2\vec{AD}$

$$\Rightarrow \vec{AD} = \frac{1}{2}[\vec{AB} + \vec{AC}] = \frac{1}{2}[8\hat{i} - 2\hat{j} + 8\hat{k}] \ \therefore \ \left|\vec{AD}\right| = \sqrt{33}$$

40. **(c) :** $\hat{n} \parallel \vec{u}\times\vec{v}$ $\because$ $\vec{u}\cdot\hat{n} = 0 = \vec{v}\cdot\vec{n}$

Now $\hat{n} = \dfrac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{1}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}}(-2\hat{k}) = -\hat{k}$

Now $|\vec{w}\cdot\hat{n}| = \left|(\hat{i}+2\hat{j}+3\hat{k})\cdot(-\hat{k})\right| = |-3| = 3$

41. **(a) :** $(\vec{u}+\vec{v}-\vec{w})\cdot[\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w}]$

$\because \vec{v}\times\vec{v} = 0$

$$\vec{u}\cdot(\vec{u}\times\vec{v})-\vec{u}\cdot(\vec{u}\times\vec{w})+\vec{u}\cdot(\vec{v}\times\vec{w})+\vec{v}\cdot(\vec{u}\times\vec{v})-\vec{v}\cdot(\vec{u}\times\vec{w})+\vec{v}\cdot(\vec{v}\times\vec{w})$$
$$-\vec{w}\cdot(\vec{u}\times\vec{v})+\vec{w}\cdot(\vec{u}\times\vec{w})-\vec{w}\cdot(\vec{v}\times\vec{w})$$

$= \vec{u}\cdot(\vec{v}\times\vec{w})+\vec{v}\cdot(\vec{w}\times\vec{u})-\vec{w}\cdot(\vec{u}\times\vec{v})$

$= \vec{u}\cdot(\vec{v}\times\vec{w})+\vec{u}\cdot(\vec{v}\times\vec{w})-\vec{u}\cdot(\vec{v}\times\vec{w})$

$= \vec{u}\cdot(\vec{v}\times\vec{w})$ $\quad(\because [a\ b\ c] = [b\ c\ a] = [c\ a\ b])$

42. **(d) :** $\overrightarrow{AB} = \overrightarrow{OB}-\overrightarrow{OA} = 6\hat{i}+2\hat{j}-3\hat{k}$ $\therefore$ $|\overrightarrow{AB}| = \sqrt{49} = 7$

Similarly $\overrightarrow{BC} = 2\hat{i}-3\hat{j}+6\hat{k}$ $\therefore |\overrightarrow{BC}| = \sqrt{49} = 7$

$\overrightarrow{CD} = -6\hat{i}-2\hat{j}-\hat{k}$ $\therefore$ $|\overrightarrow{CD}| = \sqrt{41}$

$\overrightarrow{DA} = -2\hat{i}+3\hat{j}-2\hat{k}$ $\therefore$ $|\overrightarrow{DA}| = \sqrt{17}$

43. **(c) :** If possible say $\vec{a}+\vec{b}+\vec{c} = 0 \Rightarrow \vec{b}+\vec{c} = -\vec{a}$

$\vec{a}\times\left(\vec{b}+\vec{c}\right) = -\vec{a}\times\vec{a} \Rightarrow \vec{a}\times\vec{b} = \vec{c}\times\vec{a}$

Similarly $\vec{b}\times\vec{c} = \vec{c}\times\vec{a}$ $\therefore$ $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

44. **(b) :** Given $\vec{c} = \vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -5 & 0\\ 6 & 3 & 0\end{vmatrix}$ $\therefore$ $\vec{c} = 39\hat{k}$

Now $|\vec{a}| = \sqrt{34}, |\vec{b}| = \sqrt{45}$ and $|\vec{c}| = |39\hat{k}| = 39$

$\therefore$ $|\vec{a}|:|\vec{b}|:|\vec{c}| = \sqrt{34}:\sqrt{45}:39$

45. **(b) :** $3\lambda\,\vec{c} = 2\mu\,(\vec{b}\times\vec{a})$

$\Rightarrow$ either $3\lambda = 2\mu$ or $\vec{c}\parallel\vec{b}\times\vec{a}$ but $3\lambda = 2\mu$

46. **(a) :** We have $\vec{a}+\vec{b}+\vec{c} = \vec{0} \Rightarrow (\vec{a}+\vec{b}+\vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = -50$

$\Rightarrow (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = -25 \therefore (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 25$

47. **(a) :** Given $\vec{a}+\vec{b}+\vec{c} = 0$, we need angle between $\vec{b}$ and $\vec{c}$ so consider $\vec{b}+\vec{c} = -\vec{a}$

$\Rightarrow b^2+c^2+2|b|\,|c|\cos\theta = a^2$

$\Rightarrow \cos\theta = \dfrac{a^2-b^2-c^2}{2|b||c|} = \dfrac{49-25-9}{2\times5\times3} = \dfrac{1}{2}$ $\therefore$ $\theta = 60°$

48. **(a) :** Consider $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$

$= (\vec{a}\times\vec{b})\cdot[\vec{k}\times(\vec{c}\times\vec{a})]$ where $k = \vec{b}\times\vec{c}$

$= \vec{a}\times\vec{b}\cdot[(\vec{k}\cdot\vec{a})\vec{c}-(\vec{k}\cdot\vec{c})\vec{a}]$

$= (\vec{a}\times\vec{b})\cdot\left[[(\vec{b}\times\vec{c})\cdot a]\vec{c}-[(\vec{b}\times\vec{c})\cdot\vec{c}]\vec{a}\right]$

$= (\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c})\cdot\vec{a}]\vec{c}-0 = \left((\vec{b}\times\vec{c})\cdot\vec{a}\right)\left[(\vec{a}\times\vec{b})\cdot\vec{c}\right]$

$= \left[\vec{a}\cdot(\vec{b}\times\vec{c})\right]\left[\vec{c}\cdot(\vec{a}\times\vec{b})\right] = \left[\vec{a}\cdot(\vec{b}\times\vec{c})\right]^2 = 16$

49. **(b) :** Using fact: $(\vec{a}\times\vec{b})^2 = a^2b^2-(\vec{a}\cdot\vec{b})^2$

$= a^2b^2-a^2b^2\cos^2\theta = (4\times2)^2-(4\times2)^2\cos^2\dfrac{\pi}{6}$

$= 64\times\sin^2\dfrac{\pi}{6} = 64\times\dfrac{1}{4} = 16$