CHAPTER 13

Three Dimensional Geometry

1. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane containing the lines L_1 and L_2 is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{4\sqrt{2}}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

(2018)

2. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane $x + y + z = 7$ is

(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$ *(2018)*

3. A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel to zx-plane through B, a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is :

(a) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$ (b) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

(c) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (d) $x + y + z = 6$

(Online 2018)

4. An angle between the plane, $x + y + z = 5$ and the line of intersection of the planes, $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$, is

(a) $\sin^{-1}(3/\sqrt{17})$ (b) $\cos^{-1}(\sqrt{3/17})$

(c) $\sin^{-1}(\sqrt{3/17})$ (d) $\cos^{-1}(\sqrt{3}/17)$

(Online 2018)

5. A plane bisects the line segment joining the points (1, 2, 3) and (–3, 4, 5) at right angles. Then this plane also passes through the point

(a) (1, 2, –3) (b) (–1, 2, 3)

(c) (–3, 2, 1) (d) (3, 2, 1) *(Online 2018)*

6. An angle between the lines whose direction cosines are given by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is

(a) $\cos^{-1}\left(\frac{1}{8}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\cos^{-1}\left(\frac{1}{4}\right)$ (d) $\cos^{-1}\left(\frac{1}{6}\right)$ *(Online 2018)*

7. If the angle between the lines, $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to

(a) $-\frac{4}{7}$ (b) $\frac{7}{2}$ (c) $-\frac{7}{4}$ (d) $\frac{7}{2}$

(Online 2018)

8. The sum of the intercepts on the coordinate axes of the plane passing through the point (–2, –2, 2) and containing the line joining the points (1, –1, 2) and (1, 1, 1), is

(a) 4 (b) –4 (c) 12 (d) –8

(Online 2018)

9. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$ is

(a) $\frac{10}{\sqrt{83}}$ (b) $\frac{5}{\sqrt{83}}$ (c) $\frac{10}{\sqrt{74}}$ (d) $\frac{20}{\sqrt{74}}$

(2017)

10. If the image of the point $P(1, -2, 3)$ in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line; $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to

(a) $2\sqrt{42}$ (b) $\sqrt{42}$ (c) $6\sqrt{5}$ (d) $3\sqrt{5}$

(2017)

11. The line of intersection of the planes $\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$, is

(a) $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$ (b) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$

(c) $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$ (d) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

(Online 2017)

12. The coordinates of the foot of the perpendicular from the point (1, –2, 1) on the plane containing the lines, $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$, is

(a) (0, 0, 0) (b) (2, –4, 2)

(c) (–1, 2, –1) (d) (1, 1, 1) *(Online 2017)*

13. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is

(a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$ (b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$

(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$ (d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$

(Online 2017)

14. If $x = a$, $y = b$, $z = c$ is a solution of the system of linear equations $x + 8y + 7z = 0$, $9x + 2y + 3z = 0$, $x + y + z = 0$ such that the point (a, b, c) lies on the plane $x + 2y + z = 6$, then $2a + b + c$ equals

(a) 1 (b) 2 (c) –1 (d) 0

(Online 2017)

15. If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x - 4y + 3z = 2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is

(a) 0 (b) 3 (c) 1 (d) 2

(Online 2017)

16. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is

(a) $8\sqrt{65}$ (b) $65\sqrt{8}$ (c) $\frac{65}{\sqrt{8}}$ (d) $\frac{75}{8}$

(2016)

17. If the line, $\frac{-8}{7}=\frac{+7}{-6}=\frac{+9}{8}$ lies in the plane $lx + my - z = 9$, then $l^2 + m^2$ is equal to

(a) 26 (b) 18 (c) 5 (d) 2 *5GEFL6*

18. The shortest distance between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x+2}{-2}=\frac{y-4}{8}=\frac{z-5}{4}$ lies in the interval

(a) $(3, 4]$ (b) $(2, 3]$ (c) $[1, 2)$ (d) $[0, 1)$

(Online 2016)

19. The distance of the point $(1, -2, 4)$ from the plane passing through the point $(1, 2, 2)$ and perpendicular to the planes $x - y + 2z = 3$ and $2x - 2y + z + 12 = 0$, is

(a) 2 (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

(Online 2016)

20. ABC is a triangle in a plane with vertices $A(2, 3, 5)$, $B(-1, 3, 2)$ and $C(\lambda, 5, \mu)$. If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is

(a) 1130 (b) 1348 (c) 1077 (d) 676

(Online 2016)

21. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is

(a) 2 (b) 4 (c) 3 (d) 1

(Online 2016)

22. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is

(a) $x + 3y + 6z = 7$ (b) $2x + 6y + 12z = -13$

(c) $2x + 6y + 12z = 13$ (d) $x + 3y + 6z = -7$ *(2015)*

23. The distance of the point $(1, 0, 2)$ from the point of intersection of the line $\frac{-7}{8}=\frac{+6}{9}=\frac{-7}{67}$ and the plane $x - y + z = 16$, is

(a) $8\sqrt{76}$ (b) 13 (c) $7\sqrt{69}$ (d) 8 *5GEFK6*

24. If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane, $3x + 4y - 12z + 13 = 0$, then λ satisfies the equation

(a) $3x^2 - 10x + 7 = 0$ (b) $3x^2 + 10x + 7 = 0$

(c) $3x^2 + 10x - 13 = 0$ (d) $3x^2 - 10x + 21 = 0$

(Online 2015)

25. If the shortest distance between the lines

$\frac{-6}{\alpha}=\frac{+6}{-6}=\frac{-}{6}1\text{-}\alpha\neq-6.$ and

$x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{6}{\sqrt{8}}$, then a value of α is

(a) $-\frac{6;}{6>}$ (b) $-\frac{6>}{6;}$ (c) $\frac{87}{6>}$ (d) $\frac{6>}{87}$

(Online 2015)

26. The shortest distance between the z-axis and the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$, is

(a) 1 (b) 2 (c) 3 (d) 4

(Online 2015)

27. A plane containing the point $(3, 2, 0)$ and the line $\frac{-6}{6}=\frac{-7}{:}=\frac{-8}{9}$ also contains the point

(a) $(0, -3, 1)$ (b) $(0, 7, 10)$

(c) $(0, 7, -10)$ (d) $(0, 3, 1)$ *(Online 2015)*

28. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line

(a) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$ (b) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

(c) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$ (d) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(2014)

29. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 + m^2 + n^2$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$ *(2014)*

30. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have

(a) exactly three values (b) any value

(c) exactly one value (d) exactly two values

(2013)

31. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{9}{2}$ (d) $\frac{3}{2}$ *(2013)*

32. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is
(a) $x - 2y + 2z - 1 = 0$ (b) $x - 2y + 2z + 5 = 0$
(c) $x - 2y + 2z - 3 = 0$ (d) $x - 2y + 2z + 1 = 0$
(2012)

33. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to
(a) 9/2 (b) 0 (c) – 1 (d) 2/9
(2012)

34. **Statement-1 :** The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$
Statement-2 : The line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

35. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ then λ equals
(a) 2/5 (b) 5/3 (c) 2/3 (d) 3/2
(2011)

36. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals
(a) 30º (b) 45º
(c) 60º (d) 75º. *(2010)*

37. **Statement-1 :** The point $A(3, 1, 6)$ is the mirror image of the point B $(1, 3, 4)$ in the plane $x - y + z = 5$.
Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining A $(3, 1, 6)$ and $B(1, 3, 4)$.
(a) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement 1.
(b) Statement-1 is true, Statement-2 is true; Statement 2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

38. Let the $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ line lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals
(a) (–6, 7) (b) (5, –15)
(c) (–5, 5) (d) (6, –17) *(2009)*

39. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are
(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$
(c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) 6, –3, 2 *(2009)*

40. If the straight lines
$\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$
intersect at a point, then the integer k is equal to
(a) – 2 (b) – 5
(c) 5 (d) 2 *(2008)*

41. The line passing through the points $(5, 1, a)$ and $(3, b, 1)$ crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then
(a) $a = 8, b = 2$ (b) $a = 2, b = 8$
(c) $a = 4, b = 6$ (d) $a = 6, b = 4$ *(2008)*

42. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals
(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$
(2007)

43. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals
(a) – 4 (b) –2 (c) 0 (d) 1
(2007)

44. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are
(a) (4, 3, 5) (b) (4, 3, –3)
(c) (4, 9, –3) (d) (4, –3, 3) *(2007)*

45. If a line makes an angle of $\pi/4$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$
(2007)

46. The image of the point (–1, 3, 4) in the 3 plane $x - 2y = 0$ is
(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) (15, 11, 4)
(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$ *(2006)*

47. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if
(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$
(c) $\frac{a}{a'}+\frac{c}{c'}=-1$ (d) $\frac{a}{a'}+\frac{c}{c'}=1$ *(2006)*

48. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
(a) 90º (b) 0º (c) 30º (d) 45º
(2005)

49. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius
(a) 1 (b) 3 (c) $\sqrt{2}$ (d) 2 *(2005)*

50. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is

(a) $-\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{3}{4}$ *(2005)*

51. The distance between the line $\vec{r} = 2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+5\hat{j}+\hat{k}) = 5$ is

(a) $\frac{10}{3\sqrt{3}}$ (b) $\frac{10}{9}$ (c) $\frac{10}{3}$ (d) $\frac{3}{10}$ *(2005)*

52. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals

(a) 1 (b) –1 (c) 2 (d) –2 *(2005)*

53. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane

(a) $x - y - 2z = 1$ (b) $x - 2y - z = 1$
(c) $x - y - z = 1$ (d) $2x - y - z = 1$ *(2004)*

54. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by

(a) $(3a, 2a, 3a), (a, a, 2a)$ (b) $(3a, 2a, 3a), (a, a, a)$
(c) $(3a, 3a, 3a), (a, a, a)$ (d) $(2a, 3a, 3a), (2a, a, a)$ *(2004)*

55. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(a) 7/2 (b) 5/2
(c) 3/2 (d) 9/2 *(2004)*

56. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

(a) 3/5 (b) 1/5
(c) 2/3 (d) 2/5 *(2004)*

57. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x=\frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are coplanar, then λ equals

(a) –1/2 (b) –1 (c) –2 (d) 0 *(2004)*

58. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(a) $k = 1$ or -1 (b) $k = 0$ or -3
(c) $k = 3$ or -3 (d) $k = 0$ or -1 *(2003)*

59. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d$ will be perpendicular, if and only if

(a) $aa' + bb' + cc' = 0$
(b) $(a + a')(b + b') + (c + c') = 0$
(c) $aa' + cc' + 1 = 0$
(d) $aa' + bb' + cc' + 1 = 0$ *(2003)*

60. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $(1,\vec{a},\vec{a}^2),(1,\vec{b},\vec{b}^2)$ and $(1,\vec{c},\vec{c}^2)$ are non-coplanar, then the product abc equals

(a) –1 (b) 1 (c) 0 (d) 2 *(2003)*

61. A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, 2, 1)$, $B(2, 1, 3)$ and $C(-1, 1, 2)$. Then the angle between the faces OAB and ABC will be

(a) $\cos^{-1}(17/31)$ (b) 30°
(c) 90° (d) $\cos^{-1}(19/35)$ *(2003)*

62. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

(a) 2 (b) 3 (c) 4 (d) 1 *(2003)*

63. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(a) $11\frac{3}{4}$ (b) 13 (c) 39 (d) 26 *(2003)*

64. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a, b, c and a', b', c' from the origin, then

(a) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(b) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(d) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$ *(2003)*

65. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle π/4 with plane $x + y = 3$ are

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$
(c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$ *(2002)*

ANSWER KEY

1. (c)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (a)	**11.** (d)	**12.** (a)
13. (a)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (b)	**21.** (c)	**22.** (a)	**23.** (b)	**24.** (a)
25. (c)	**26.** (b)	**27.** (b)	**28.** (d)	**29.** (d)	**30.** (d)	**31.** (b)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (c)	**36.** (c)
37. (b)	**38.** (a)	**39.** (b)	**40.** (b)	**41.** (d)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (b)	**46.** (d)	**47.** (a)	**48.** (a)
49. (a)	**50.** (b)	**51.** (a)	**52.** (d)	**53.** (d)	**54.** (b)	**55.** (a)	**56.** (a)	**57.** (c)	**58.** (b)	**59.** (c)	**60.** (a)
61. (d)	**62.** (b)	**63.** (b)	**64.** (c)	**65.** (b)							

Explanations

1. (c): A plane passing through the intersection of the given planes is $(2x-2y+3z-2)+\lambda(x-y+z+1)=0$
i.e. $(\lambda+2)x-(2+\lambda)y+(\lambda+3)z+(\lambda-2)=0$
The plane is having infinite number of solutions with $x+2y-z-3=0$ and $3x-y+2z-1=0$.

$$\therefore \begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}=0$$

$\Rightarrow (\lambda+2)(4-1)+(\lambda+2)(2+3)+(\lambda+3)(-1-6)=0$
$\Rightarrow \lambda=5$
$\therefore$ The equation of the plane becomes $7x-7y+8z+3=0$
The perpendicular distance from origin is

$$\frac{3}{\sqrt{7^2+7^2+8^2}}=\frac{3}{\sqrt{162}}=\frac{3}{9\sqrt{2}}=\frac{1}{3\sqrt{2}}$$

2. (a): The direction ratios of AB, where $A(5,-1,4)$ and $B(4,-1,3)$ are $(1, 0, 1)$
Let the angle between AB and plane is θ, which gives

$$\sin\theta=\frac{2}{\sqrt{6}} \text{ i.e. } \cos\theta=\frac{1}{\sqrt{3}}$$

The projection of AB on the plane $=AB\cos\theta=\sqrt{2}\cdot\frac{1}{\sqrt{3}}=\sqrt{\frac{2}{3}}$

3. (c): Let given plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

It passes through $(3, 2, 1)$ $\therefore \frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$

Now, $A\equiv(a,0,0)$, $B\equiv(0,b,0)$, $C\equiv(0,0,c)$
$\therefore$ Locus of point of intersection of planes

$x=a$, $y=b$, $z=c$ is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

4. (c): Given planes are $3x+4y+z-1=0$ and $5x+8y+2z+14=0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 1 \\ 5 & 8 & 2 \end{vmatrix}=\hat{i}(8-8)-\hat{j}(6-5)+\hat{k}(4)=-\vec{j}+4\vec{k}$$

$\therefore$ Required plane is parallel to $-\vec{j}+4\vec{k}$

So, required angle $=\sin^{-1}\left(\frac{-1+4}{\sqrt{3}\cdot\sqrt{17}}\right)=\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

5. (c): Given points are $(1, 2, 3)$ and $(-3, 4, 5)$
D.r.'s of line are $\langle -3-1,\ 4-2,\ 5-3\rangle=\langle -4,\ 2,\ 2\rangle$

So, equation of normal is $-4\hat{i}+2\hat{j}+2\hat{k}$
As plane bisects the line segment joining the points $(1, 2, 3)$ and $(-3, 4, 5)$ at right angle.
$\therefore$ The point where it bisects is the midpoint of $(1, 2, 3)$ and $(-3, 4, 5)$ *i.e.*, $(-1, 3, 4)$
Now, the required equation of plane is passing through $(-1, 3, 4)$ and having normal $(-4\hat{i}+2\hat{j}+2\hat{k})$
$\therefore$ Equation of plane is $(x+1)(-4)+(y-3)2+(z-4)2=0$
$\Rightarrow -4x-4+2y-6+2z-8=0$
$\Rightarrow 4x-2y-2z+18=0 \Rightarrow 2x-y-z+9=0$
Observing all the points we get point $(-3, 2, 1)$ satisfies the equation of plane.

6. (d): The given equations are
$l+3m+5n=0$ ...(i) and $5lm-2mn+6nl=0$ ...(ii)
From (i), $l=-3m-5n$
Putting this value of l in (ii), we have
$5(-3m-5n)m-2mn+6n(-3m-5n)=0$
$\Rightarrow -15m^2-30n^2-45mn=0 \Rightarrow m^2+2n^2+3mn=0$
$\Rightarrow m^2+3mn+2n^2=0 \Rightarrow m(m+2n)+n(m+2n)=0$
$\Rightarrow (m+n)(m+2n)=0 \Rightarrow$ either $m=-n$ or $m=-2n$
For $m=-n$, $l=-2n$; For $m=-2n$, $l=n$
$\therefore$ Direction ratios of two lines are
$\langle -2n,-n,n\rangle$ and $\langle n,-2n,n\rangle$ *i.e.*, $\langle -2,-1,1\rangle$ and $\langle 1,-2,1\rangle$

$\therefore$ The required angle is $\cos\theta=\frac{-2\cdot1+2\cdot1+1\cdot1}{\sqrt{4+1+1}\sqrt{1+4+1}}$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{6}\cdot\sqrt{6}}=\frac{1}{6} \Rightarrow \theta=\cos^{-1}\left(\frac{1}{6}\right)$

7. (b): Equation of lines are $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ ...(i)

and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ or $\frac{x-5}{2}=\frac{y-2}{p/7}=\frac{z-3}{4}$ ...(ii)

Here, $a_1=2$, $b_1=2$, $c_1=1$, $a_2=2$, $b_2=p/7$, $c_2=4$

Given, angle between lines (i) and (ii) is $\cos^{-1}\left(\frac{2}{3}\right)$.

Angle between two lines $=\cos^{-1}\left(\frac{a_1\times a_2+b_1\times b_2+c_1\times c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$

$$\text{So, } \cos^{-1}\left(\frac{2\times2+2\times\frac{p}{7}+1\times4}{\sqrt{4+4+1}\cdot\sqrt{4+\frac{p^2}{49}+16}}\right)=\cos^{-1}\left(\frac{2}{3}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{8+\frac{2p}{7}}{3\sqrt{20+\frac{p^2}{49}}}\right)=\cos^{-1}\left(\frac{2}{3}\right)$$

$$\Rightarrow 8+\frac{2p}{7}=\frac{2}{3}\left(3\sqrt{20+\frac{p^2}{49}}\right)$$

$$\Rightarrow 4+\frac{p}{7}=\sqrt{20+\frac{p^2}{49}} \Rightarrow \left(4+\frac{p}{7}\right)^2=20+\frac{p^2}{49}$$

$$\Rightarrow \frac{8p}{7}=20-16 \Rightarrow \frac{8p}{7}=4 \Rightarrow p=\frac{7}{2}$$

8. **(b):** Equation of plane is given by

$$\begin{vmatrix} x-(-2) & y-(-2) & z-2 \\ -3 & -1 & 0 \\ -3 & -3 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ -3 & -1 & 0 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $(x + 2)(-1 - 0) - (y + 2)(-3 - 0) + (z - 2)(9 - 3) = 0$

$\Rightarrow$ $-(x + 2) + 3(y + 2) + 6(z - 2) = 0$

$\Rightarrow$ $-x - 2 + 3y + 6 + 6z - 12 = 0$

$\Rightarrow$ $-x + 3y + 6z - 8 = 0 \Rightarrow x - 3y - 6z + 8 = 0$

$\Rightarrow$ $\dfrac{x}{-8}+\dfrac{y}{8/3}+\dfrac{z}{8/6}=1$ $\therefore$ Sum of intercepts $= -8+\dfrac{8}{3}+\dfrac{8}{6}=-4$

9. **(a):** The normal vector to the plane is given by

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i}+7\hat{j}+3\hat{k}$$

The plane is given by $5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

i.e., $5x + 7y + 3z + 5 = 0$

The distance of $(1, 3, -7)$ from the above plane is

$$\left|\frac{5+21-21+5}{\sqrt{5^2+7^2+3^2}}\right| = \frac{10}{\sqrt{83}}$$

10. **(a):** The line PQ is given by $\dfrac{x-1}{1}=\dfrac{y+2}{4}=\dfrac{z-3}{5}=t$

Let a point M on PQ be $(t + 1, 4t - 2, 5t + 3)$.

For this point to lie in the plane

$2x + 3y - 4z + 22 = 0$ we have,

$2(t + 1) + 3(4t - 2) - 4(5t + 3) + 22 = 0$

$\Rightarrow$ $-6t + 6 = 0$ $\Rightarrow$ $t = 1$

Then the point M is $(2, 2, 8)$

$\therefore$ $PQ = 2PM = 2\sqrt{1^2+4^2+5^2} = 2\sqrt{42}$

11. **(d):** We have two equation of planes *i.e.,*

$\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$

The planes have normal vector $\vec{n}_1=(3,-1,1)$ and $\vec{n}_2=(1,4,-2)$

Then $\vec{n}=\vec{n}_1\times\vec{n}_2$, is parallel to line of intersection (L).

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix} = \hat{i}(-2)-\hat{j}(-7)+\hat{k}(13) \quad \therefore \quad \vec{n}=-2\hat{i}+7\hat{j}+13\hat{k}$$

Now to find a point on the line of intersection L, we need to solve the two equations : $3x - y + z = 1$ and $x + 4y - 2z = 2$

We consider the point to be the point on plane $z = 0$.

Put $z = 0$ in systems above, we get $3x - y = 1$ and $x + 4y = 2$

On solving, we get $x = 6/13$ and $y = 5/13$

$\therefore$ Point of intersection is $\left(\dfrac{6}{13},\dfrac{5}{13},0\right)$

Hence, equation of line of intersection to the given planes is

$$\frac{x-6/13}{-2}=\frac{y-5/13}{7}=\frac{z-0}{13} \text{ or } \frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}$$

12. **(a):** We have,

$L_1=\dfrac{x+1}{6}=\dfrac{y-1}{7}=\dfrac{z-3}{8}$; $L_2=\dfrac{x-1}{3}=\dfrac{y-2}{5}=\dfrac{z-3}{7}$

Let $\vec{n}_1, \vec{n}_2$ be the normal vectors of line L_1 and L_2 respectively.

$\therefore$ $\vec{n}_1 = (6, 7, 8)$, $\vec{n}_2 = (3, 5, 7)$

$\therefore$ Normal vector to the plane is,

$$\vec{n}=\vec{n}_1\times\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 9\hat{i}-18\hat{j}+9\hat{k}$$

which is proportional to $\hat{i}-2\hat{j}+\hat{k}$ *i.e.,* $(1, -2, 1)$

$\therefore$ Equation of plane is $1(x + 1) - 2(y - 1) + 1(z - 3) = 0$

$\Rightarrow$ $x - 2y + z = 0$

Now, as $(1, -2, 1)$ is the point on the perpendicular from $(1, -2, 1)$

$\therefore$ Equation of perpendicular line is

$$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-1}{1}=-\frac{(1+4+1)}{6}=-1$$

$\therefore$ $x = 0, y = 0, z = 0$

13. **(a):** Let centroid be (h, k, l).

$\therefore$ x-intercept $= 3h$, y-intercept $= 3k$, z-intercept $= 3l$

Equation of plane is $\dfrac{x}{3h}+\dfrac{y}{3k}+\dfrac{z}{3l}=1$

Distance of plane from $(0, 0, 0)$ is

$$\left|\frac{-1}{\sqrt{\dfrac{1}{9h^2}+\dfrac{1}{9k^2}+\dfrac{1}{9l^2}}}\right| = 3 \Rightarrow 1 = 3\left(\frac{1}{3}\right)\sqrt{\frac{1}{h^2}+\frac{1}{k^2}+\frac{1}{l^2}}$$

Thus locus is $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=1$

14. **(a):** $x + 8y + 7z = 0$ ...(i)

$9x + 2y + 3z = 0$ ...(ii) $x + y + z = 0$ ...(iii)

Subtracting (iii) from (i), we get $7y + 6z = 0$ ...(iv)

Multiplying (iii) by 2 and then subtracting from (ii), we get

$7x + z = 0$ ...(v)

Let $x = \lambda$

Then, from (v), $z = -7\lambda$

From (iv), $y = \dfrac{-6z}{7}=\dfrac{-6}{7}(-7\lambda)=6\lambda$

Given that solution of system lies on the plane $x + 2y + z = 6$

$\therefore$ $\lambda + 2(6\lambda) + (-7\lambda) = 6$

$\Rightarrow$ $\lambda + 12\lambda - 7\lambda = 6 \Rightarrow 6\lambda = 6 \Rightarrow \lambda = 1$ $\therefore x = 1, y = 6, z = -7$

So, $2a + b + c = 2(1) + 6 + (-7) = 1$

15. **(a):** Point $(3, -2, -\lambda)$ lies on plane $2x - 4y + 3z - 2 = 0$

$\therefore$ $6 + 8 - 3\lambda - 2 = 0 \Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$

Now, $\dfrac{x-3}{1}=\dfrac{y+2}{-1}=\dfrac{z+4}{-2}=k_1$ (say) ...(1)

$\dfrac{x-1}{12}=\dfrac{y}{9}=\dfrac{z}{4}=k_2$ (say) ...(2)

Point on first line is $(k_1 + 3, -k_1 - 2, -2k_1 - 4)$

Point on second line is $(12k_2 + 1, 9k_2, 4k_2)$

$\therefore$ $k_1 + 3 = 12k_2 + 1$; $-k_1 - 2 = 9k_2$; $-2k_1 - 4 = 4k_2$

On solving these equations, we ge $k_2 = 0$ and $k_1 = -2$

$\therefore$ Point (1, 0, 0) lies on both lines.

So, given lines intersect each other. $\therefore$ Shortest distance = 0.

16. (b): The equation of line parallel to $x = y = z$ and passing through (1, –5, 9) is $\frac{-6}{6} = \frac{+:}{6} = \frac{->}{6} = \text{--m}.$

Let $A(k + 1, k - 5, k + 9)$ be the point of intersection of line and plane.

We have, $k + 1 - k + 5 + k + 9 = 5 \Rightarrow k = -10$

$\therefore$ The point is (–9, –15, –1)

Required distance $= \sqrt{-6+>.^7 + -\!-: +6:.^7 + ->+6.^7} = 65\sqrt{8}$

17. (d): As the line $\frac{-8}{7} = \frac{+7}{-6} = \frac{+9}{8}$ lies in the plane $lx + my - z = 9$, we have $3l - 2m + 4 = 9$. Also, $2l - m - 3 = 0$

Solving for l and m we get $l = 1$, $m = -1$

So, $l^2 + m^2 = 2$

18. (b): We have, $x_1 = 0, y_1 = 0, z_1 = 0;$

$x_2 = -2, y_2 = 4, z_2 = 5; a_1 = 2, b_1 = 2, c_1 = 1; a_2 = -2, b_2 = 8, c_2 = 4$

$\therefore$ Shortest distance

$$= \left|\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\Sigma(a_1b_2 - a_2b_1)^2}}\right| = \left|\frac{\begin{vmatrix} -2 & 4 & 5 \\ 2 & 2 & 1 \\ -2 & 8 & 4 \end{vmatrix}}{\sqrt{(8-8)^2 + (8+2)^2 + (16+4)^2}}\right|$$

$$= \left|\frac{60}{22.36}\right| = 2.7$$

19. (c): Let the equation of plane passing through the point (1, 2, 2) be $a(x - 1) + b(y - 2) + c(z - 2) = 0$ …(i)

Since, it is perpendicular to the planes

$x - y + 2z = 3$ and $2x - 2y + z + 12 = 0$

$\therefore$ $a - b + 2c = 0$ and $2a - 2b + c = 0$ …(ii)

Solving equations in (ii), we get $c = 0$ and $a = b$

$\therefore$ From (i) equation of plane is $x + y - 3 = 0$

$\therefore$ Distance of point (1, –2, 4) from plane

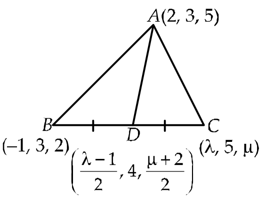
$x + y - 3 = 0$ is $D = \frac{|1 - 2 - 3|}{\sqrt{1+1}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

20. (b): Dr's of AD are $\frac{\lambda - 1}{2} - 2, 4 - 3, \frac{\mu + 2}{2} - 5$

i.e. $\frac{\lambda - 5}{2}, 1, \frac{\mu - 8}{2}$

$\because$ This median is making equal angles with coordinate axes, therefore,

$\frac{\lambda - 5}{2} = 1 = \frac{\mu - 8}{2}$



$\Rightarrow$ $\lambda = 7, \mu = 10$

$\therefore$ $\lambda^3 + \mu^3 + 5 = 1348$

21. (c): $\because$ Lines are coplanar

$$\therefore \begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$$

$\Rightarrow$ $2(4 - \lambda^4) + 4(\lambda^2 - 2) = 0$

$\Rightarrow$ $4 - \lambda^4 + 2\lambda^2 - 4 = 0 \Rightarrow \lambda^2(\lambda^2 - 2) = 0 \Rightarrow \lambda = 0, \sqrt{2}, -\sqrt{2}$

22. (a): 1st solution : Let the equation of line parallel to the plane $x + 3y + 6z = 1$ be $x + 3y + 6z = k$

As a point on line of intersection of planes

$2x - 5y + z = 3$ and $x + y + 4z = 5$ is (4, 1, 0) got by inspection, we have the required plane satisfying this point.

Hence, $k = 4 + 3 \cdot 1 + 0 = 7$

Thus the equation of plane is $x + 3y + 6z = 7$

2nd solution : The equation of plane containing the line $2x - 5y + z = 3$, $x + y + 4z = 5$ is

$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

$\Rightarrow$ $(2 + \lambda)x + (\lambda - 5)y + (4\lambda + 1)z - (5\lambda + 3) = 0$

As this plane is parallel to $x + 3y + 6z - 1 = 0$, the coefficients must be proportional, gives $\frac{7+\lambda}{6} = \frac{\lambda -:}{8} = \frac{9\lambda + 6}{;} = \frac{:\lambda + 8}{6}$

Taking any two of them give, (for example 1st and 2nd)

$6 + 3\lambda = \lambda - 5 \Rightarrow 2\lambda = -11 \Rightarrow \lambda = -\frac{66}{7}$

The equation of plane is $-\frac{<}{6} - \frac{76}{7} - 76 + \frac{9>}{7} = 5$

i.e., $7x + 21y + 42z - 49 = 0$ *i.e.,* $x + 3y + 6z = 7$

23. (b): Let the parameter corresponding to the point of intersection be denoted by t, then $\frac{-7}{8} = \frac{+6}{9} = \frac{-7}{67} =$

Thus $(3t + 2, 4t - 1, 12t + 2)$ is a general point.

Thus point lies on plane $x - y + z = 16$ gives

$(3t + 2) - (4t - 1) + (12t + 2) = 16 \Rightarrow 11t = 11 \quad \therefore \quad t = 1$

Thus the point is (5, 3, 14)

Given point is (1, 0, 2)

The distance between the points is

$\sqrt{-: -6.^7 + -8-5.^7 + -69-7.^7} = \sqrt{6; + > + 699} = \sqrt{6; >} = 68$

24. (a): So, the equation of plane is $3x + 4y - 12z + 13 = 0$ ...(i)

The points $(1, 1, \lambda)$ and (–3, 0, 1) are equidistant from (i)

$\Rightarrow$ $\frac{8 + 9 - 67\lambda + 68}{\sqrt{8^7 + 9^7 + 67^7}} = \frac{-> + 5 - 67 + 68}{\sqrt{8^7 + 9^7 + 67^7}}$

$\Rightarrow$ $|-12\lambda + 20| = |-8| \Rightarrow |-3\lambda + 5| = |-2|$

$\Rightarrow$ $9\lambda^2 + 25 - 30\lambda = 4 \Rightarrow 9\lambda^2 - 30\lambda + 21 = 0$

$\Rightarrow 3\lambda^2 - 10\lambda + 7 = 0$

25. (c): We have, $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ ...(i)

$\therefore$ Point of intersection of above lines are $P(0, 1, -2)$

Given equation of line is $\frac{-6}{\alpha} = \frac{+6}{-6} = \frac{}{6}$ .... (ii)

$\therefore$ Point Q (1, –1, 0) lies on above line

$\therefore$ $\overrightarrow{mn} = -7 + 7$

Also, $\begin{vmatrix} \hat{} & \hat{} & \hat{} \\ 6 & 6 & 6 \\ 7 & -6 & 6 \end{vmatrix} = 7 + -8$ ....(iii) (from (i))

Now $\vec{} = \begin{vmatrix} \hat{} & \hat{} & \hat{} \\ \alpha & -6 & 6 \\ 7 & 6 & -8 \end{vmatrix}$ (from (ii) and (iii))

$$= 7\hat{} + \hat{}(8\alpha + 7) + \hat{}(\alpha + 7)$$

Shortest distance between lines= S. D.= $\overrightarrow{mn}\,.\,\hat{}$

$$= \frac{7 - 7\text{-}8\alpha + 7. + 7\text{-}\alpha + 7.}{\sqrt{9 + \text{-}8\alpha + 7.^7 + \text{-}\alpha + 7.^7}} = \frac{6}{\sqrt{8}} \Rightarrow 3(2 - 4\alpha)^2 = 10\alpha^2 + (16\alpha + 12)$$

$\Rightarrow \quad 19\alpha^2 - 32\alpha = 0 \;\Rightarrow\; \alpha = \frac{87}{6>}$

26. (b): The plane through the given line is
$(x + y + 2z - 3) + \lambda(2x + 3y + 4z - 4) = 0$
or, $(1 + 2\lambda)x + (1 + 3\lambda)y + (2 + 4\lambda)z - (3 + 4\lambda) = 0$
If this plane is || to z-axis whose d.c.'s are $< 0, 0, 1 >$ then normal to this plane must be $\perp$ to z-axis.

$\Rightarrow \quad (1 + 2\lambda) . 0 + (1 + 3\lambda) . 0 + (2 + 4\lambda) . 1 = 0 \Rightarrow \lambda = -\frac{6}{7}$

The equation of the plane through the given line and parallel to z-axis is

$(x + y + 2z - 3) - \frac{6}{7}(2x + 3y + 4z - 4) = 0 \Rightarrow y + 2 = 0$

Required shortest distance = length of $\perp$ from (0, 0, 1) to the plane $= \frac{5+7}{\sqrt{6}} = 7$

27. (b): $A(3, 2, 0)$ and $B(1, 2, 3)$ lie in the plane.

$\Rightarrow \quad \overrightarrow{WX} = 7^j + 5^j + \text{-}{-}8.^j$ mx{ xuq uz µt q | xmz q3

$\therefore$ V{ ~y mx qoµ{ ~{ r | xmz q D $\text{-}7^j - 8^j . \times \text{-}^j + :^j + 9^j .$

$$= 6:^j - 66^j + 65^j$$

$\therefore$ Equation of plane is $\text{-}^{\rightarrow} - \text{-}8^j + 7^j + 5^j \ldots \cdot \text{-}6:^j - 66^j + 65^j . = 5$

$\Rightarrow 15x - 11y + 10z - 23 = 0$

28. (d) : The line is parallel to the plane.
Image of (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is given by

$$\frac{\alpha - 1}{2} = \frac{\beta - 3}{-1} = \frac{\gamma - 4}{1} = \frac{-2(2 - 3 + 4 + 3)}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{-2 \times 6}{6} = -2$$

$\therefore \quad (\alpha, \beta, \gamma) \equiv (-3, 5, 2)$

Thus the required line is $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

29. (d): As $l = -m - n$. We have $l^2 = m^2 + n^2$ gives
$m^2 + n^2 = (m + n)^2 \quad \Rightarrow \quad 2mn = 0 \quad \Rightarrow \quad mn = 0$

So, the d.r.'s is $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ or $\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$\cos\theta = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + 0 + 0 \quad \Rightarrow \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$

30. (d): For the lines to be coplanar $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

Expanding, we get $1(1 + 2k) + 1(1 + k^2) - 1(2 - k) = 0$
$\Rightarrow \quad k^2 + 1 + 2k + 1 - 2 + k = 0$
$\Rightarrow \quad k^2 + 3k = 0 \quad \Rightarrow \quad k(k + 3) = 0 \quad \therefore \quad k = 0, -3$
So there are two values of k.

31. (b): The planes are $4x + 2y + 4z = 16, \; 4x + 2y + 4z = -5$

Distance between planes $= \frac{16 - (-5)}{\sqrt{4^2 + 2^2 + 4^2}} = \frac{21}{6} = \frac{7}{2}$

32. (c) : Equation of a plane parallel to $x - 2y + 2z - 5 = 0$ and at a unit distance from origin is $x - 2y + 2z + k = 0$

$\Rightarrow \quad \frac{|k|}{3} = 1 \Rightarrow |k| = 3$

$\therefore \; x - 2y + 2z - 3 = 0 \quad$ or $\quad x - 2y + 2z + 3 = 0$

33. (a) : $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = r_1$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = r_2$

or $2r_1 + 1 = r_2 + 3, \; 3r_1 - 1 = 2r_2 + k, \; 4r_1 + 1 = r_2$

$\Rightarrow 2r_1 - r_2 = 2$, and $4r_1 - r_2 = -1 - 2r_1 = 3 \Rightarrow r_1 = \frac{-3}{2}$ and $r_2 = -5$

$\therefore \; -\frac{9}{2} - 1 = -10 + k \Rightarrow k = 10 - \frac{11}{2} = \frac{9}{2}$

34. (d): The direction ratios of the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$ is $(0, 6, -4)$.
The direction ratios of the given line is (1, 2, 3).
As $1\cdot0 + 6\cdot2 - 4\cdot3 = 0$ we have the lines as perpendicular
Also the midpoint of AB lies on the given line, so statement 1 and statement 2 are true but statement 2 is not a correct explanation of statement 1.
Statement '2' holds even if the line is not perpendicular. This situation is possible.

35. (c): $\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$

$x + 2y + 3z = 4$

Angle between line and plane (by definition)

$$= \sin^{-1}\left(\frac{1\cdot1 + 2\cdot2 + \lambda\cdot3}{\sqrt{1+4+9}\sqrt{1+4+\lambda^2}}\right) = \sin^{-1}\left(\frac{5 + 3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}}\right)$$

So, $\frac{(5+3\lambda)^2}{14(5+\lambda^2)} + \frac{5}{14} = 1 \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \quad \frac{(5+3\lambda)^2}{5+\lambda^2} + 5 = 14 \quad \Rightarrow \quad (5 + 3\lambda)^2 + 5(5 + \lambda^2) = 14(5 + \lambda^2)$

$\Rightarrow \quad 25 + 30\lambda + 9\lambda^2 + 25 + 5\lambda^2 = 70 + 14\lambda^2 \Rightarrow 30\lambda + 50 = 70$

$\Rightarrow \quad 30\lambda = 20 \quad \therefore \quad \lambda = 2/3$

36. (c) : We have $l = \frac{1}{\sqrt{2}}, m = -\frac{1}{2}$

As $l^2 + m^2 + n^2 = 1$, we have $n^2 = \frac{1}{4} \Rightarrow n = \pm\frac{1}{2}$

We take positive values, so $n = \frac{1}{2} \Rightarrow \cos\theta = \frac{1}{2}. \; \therefore \; \theta = 60°$.

37. (b) : Let the image be (a, b, c)
Thus by image formula, we have

$$\frac{a-1}{1} = \frac{b-3}{-1} = \frac{c-4}{1} = -2\left(\frac{1-3+4-5}{3}\right) \Rightarrow \frac{a-1}{1} = \frac{b-3}{-1} = \frac{c-4}{1} = 2$$

$\therefore \quad (a, b, c) = (3, 1, 6)$

Again, the midpoint of $A(3, 1, 6)$ and $B(1, 3, 4)$ is (2, 2, 5) & the equation of the plane is $x - y + z = 5$.

As the point lies on the plane, so the plane bisects the segment AB. But it does not explain statement-1.

38. (a): The line is $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$

The direction ratios of the line are (3, –5, 2).

As the line lies in the plane $x + 3y - \alpha z + \beta = 0$,

we have $(3)(1) + (-5)(3) + 2(-\alpha) = 0$

$\Rightarrow \quad -12 - 2\alpha = 0. \quad \therefore \quad \alpha = -6$

Again (2, 1, –2) lies on the plane

$\Rightarrow \quad 2 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 5 = 12 - 5 = 7$

Hence (α, β) is (–6, 7).

39. (b): Let the vector $\overrightarrow{PQ}$ be $(x_1 - x_2, y_1 - y_2, z_1 - z_2)$

we have $x_1 - x_2 = 6, y_1 - y_2 = -3, z_1 - z_2 = 2$

Length of $PQ = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

$= \sqrt{6^2+3^2+2^2} = \sqrt{36+9+4} = 7$

The direction cosines of $\overrightarrow{PQ}$ are $\left\langle \frac{x_1-x_2}{PQ}, \frac{y_1-y_2}{PQ}, \frac{z_1-z_2}{PQ} \right\rangle$

i.e., $\left\langle \frac{6}{7}, -\frac{3}{7}, \frac{2}{7} \right\rangle$

40. (b) : As the lines intersect,

we have $\frac{(x-1)}{k}=\frac{(y-2)}{2}=\frac{z-3}{3}=r$

$\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}=t$

which on solving gives $2k^2 + 5k - 25 = 0$

$\Rightarrow \quad 2k^2 + 10k - 5k - 25 = 0 \Rightarrow 2k(k + 5) - 5(k + 5) = 0$

$\Rightarrow \quad (2k - 5)(k + 5) = 0 \therefore \; k = -5, \frac{5}{2}$

41. (d) : The equation of the line passing through (3, b, 1) and (5, 1, a) is $\frac{x-5}{2}=\frac{y-1}{1-b}=\frac{z-a}{a-1}=\mu$ (say)

The line crosses the yz plane where $x = 0$, *i.e.*

$-5 = 2\mu \;\therefore\; \mu = -\frac{5}{2}$

Again , $y = \mu(1-b)+1 = \frac{17}{2}$

$\Rightarrow \; -\frac{5}{2}(1-b)+1=\frac{17}{2} \;\Rightarrow\; -\frac{5}{2}(1-b)=\frac{15}{2}$

$\Rightarrow \; (1-b)=-3 \;\therefore\; b=4$

Again $z=\mu(a-1)+a=-\frac{13}{2}$

$\Rightarrow \; -\frac{5}{2}(a-1)+a=-\frac{13}{2} \;\Rightarrow\; -\frac{3}{2}a+\frac{5}{2}=-\frac{13}{2}$

$\Rightarrow \; -\frac{3}{2}a=-9 \Rightarrow a=6$

42. (c) : Direction of the line, $L = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3\hat{i}-3\hat{j}+3\hat{k}$.

Then $\cos\alpha = \frac{3}{\sqrt{9+9+9}} = \frac{1}{\sqrt{3}}$.

Second method

If direction cosines of L be l, m, n, then

$l + 3m + n = 0, \; l + 3m + 2n = 0$

After solving, we get, $\frac{l}{3}=\frac{m}{-3}=\frac{n}{3}$

$\therefore \; l:m:n = \frac{1}{\sqrt{3}}:-\frac{1}{\sqrt{3}}:\frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$.

43. (b) : $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$

$[\vec{a}\ \vec{b}\ \vec{c}]=0$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0 \Rightarrow 1(1-2x+4)-1(-1-2x)+1(x-2+x)=0$

$\Rightarrow 5 - 2x + 1 + 2x + 2x - 2 = 0 \;\Rightarrow x = -2.$

44. (c) : Centre of sphere $\equiv (3, 6, 1)$

Let the other end of diameter is (α, β, γ)

$3=\frac{\alpha+2}{2} \Rightarrow \alpha=4\ ,\ 6=\frac{\beta+3}{2} \Rightarrow \beta=9\ \ 1=\frac{\gamma+5}{2} \Rightarrow \gamma=-3.$

45. (b) : Let required angle is θ

$\because \; l=\cos\frac{\pi}{4}, m=\cos\frac{\pi}{4}$ then $n=\cos\theta$

We know that $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\theta=1 \;\Rightarrow\; \frac{1}{2}+\frac{1}{2}+\cos^2\theta=1$

$\Rightarrow \quad \cos^2\theta = 0 \Rightarrow \theta = \pi/2$

Thus required angle is $\pi/2$.

46. (d) : Image of point (x', y', z') in $ax + by + cz + d = 0$ is given by

$\frac{x-x'}{a}=\frac{y-y'}{b}=\frac{z-z'}{c}=\frac{-2(ax'+by'+cz'+d)}{a^2+b^2+c^2}$

$\Rightarrow \quad \frac{x+1}{1}=\frac{y-3}{-2}=\frac{z-4}{0}=\frac{-2(-1-6)}{5} \;\therefore\; x=\frac{9}{5}, y=\frac{-13}{5}, z=4$

47. (a) : Two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are $\perp$ if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Given lines can be written as $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ ..(i)

and $\frac{x-b'}{a'}=\frac{y}{1}=\frac{z-d'}{c'}$ ...(ii)

As lines are perpendicular

$\therefore \quad aa' + 1 + cc' = 0 \quad \Rightarrow aa' + cc' = -1$

48. (a) : From given lines $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ and $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$

$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$\cos\theta = \frac{6-24+18}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}} = 0 \therefore \theta = 90^\circ.$

49. (a) : Centre of sphere is (1/2, 0, –1/2)

R = Radius of sphere is $\sqrt{g^2+f^2+w^2-c}$

$= \sqrt{\frac{1}{4}+\frac{1}{4}+2} \;\therefore\; R = \sqrt{\frac{5}{2}}$

$d = \perp$ distance from centre to the plane is equal to

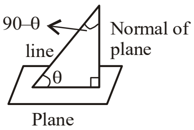
$$d = \left|\frac{\frac{1}{2}+0+\frac{1}{2}-4}{\sqrt{1^2+2^2+1^2}}\right|, \quad d = \frac{3}{\sqrt{6}}.$$

$\therefore$ Radius of the circle

$$= \sqrt{(\text{Radius of sphere})^2 - (\text{perpendicular distance from centre of sphere to plane})^2}$$

$$= \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - \left(\frac{9}{6}\right)} = \sqrt{\frac{15}{6}-\frac{9}{6}} = 1.$$

50. (b) : Angle between the line and plane is same as the angle between the line and normal to the plane



$$\therefore \quad \cos(90-\theta) = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\Rightarrow \frac{1}{3} = \frac{(1\times 2+2\times(-1))+2\sqrt{\lambda}}{\sqrt{1^2+2^2+2^2}\sqrt{2^2+1^2+\lambda}} \quad \Rightarrow \lambda = \frac{5}{3}.$$

51. (a) : $d = \left|\frac{\vec{a}\cdot\vec{n}-d}{\sqrt{n}}\right|$

$$\therefore \ d = \left|\frac{(2i-2j+3k)\cdot(i+5j+k)-(-5)}{\sqrt{1^2+5^2+1^2}}\right|, \quad d = \frac{10}{3\sqrt{3}}.$$

52. (d) : Centre of spheres are (–3, 4, 1) and (5, –2, 1)

$$\underset{C_1(-3,4,1)\qquad\qquad C_2(5,-2,1)}{M(1,1,1)}$$

using mid point in the equation $2ax - 3ay + 4az + 6 = 0$
$\Rightarrow 2a - 3a + 4a + 6 = 0 \Rightarrow a = -2$.

53. (d) : Equation of the plane of intersection of two spheres $S_1 = 0 = S_2$ is given by $S_1 - S_2 = 0$

$\Rightarrow \quad 10x - 5y - 5z = 5 \Rightarrow 2x - y - z = 1$

54. (b) : Given $AB = \frac{x}{1} = \frac{y+a}{1} = \frac{z}{1}$

$CD : \frac{x+a}{2} = \frac{y}{1} = \frac{z}{1}$

Let $P \equiv (r, r-a, r)$ and $Q = (2\lambda - a, \lambda, \lambda)$

Direction ratios of PQ are $r-2\lambda+a,\ r-\lambda-a,\ r-\lambda$

According to question, direction ratios of PQ are (2, 1, 2)

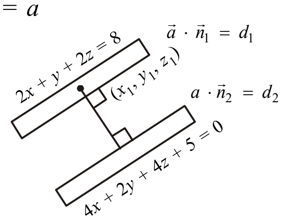
$$\therefore \ \underset{\text{(i)}}{\frac{r-2\lambda+a}{2}} = \underset{\text{(ii)}}{\frac{r-\lambda-a}{1}} = \underset{\text{(iii)}}{\frac{r-\lambda}{2}}$$

(ii) and (iii) $\Rightarrow r - \lambda = 2a$

(i) and (iii) $\Rightarrow \lambda = a \qquad r = 3a,\ \lambda = a$

$\therefore p \equiv (3a, 2a, 3a)$ and $Q \equiv (a, a, a)$.

55. (a) : Let (x_1, y_1, z_1) be any point on the plane



$2x + y + 2z - 8 = 0$

$\therefore \quad 2x_1 + y_1 + 2z_1 - 8 = 0$

$$\therefore \ d = \frac{|2(2x+y+2z-8)+21|}{\sqrt{4^2+2^2+4^2}} = \frac{21}{6} = \frac{7}{2}$$

56. (a) : If a line makes the angle α, β, γ with x, y, z axis respectively then $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 2l^2 + m^2 = 1$ or $2n^2 + m^2 = 1$

$\Rightarrow \quad 2\cos^2\theta = 1 - \cos^2\beta \ (\alpha = \gamma = \theta)$

$\quad 2\cos^2\theta = \sin^2\beta$

$\Rightarrow \quad 2\cos^2\theta = 3\sin^2\theta$ (given $\sin^2\beta = 3\sin^2\theta$) $\Rightarrow 5\cos^2\theta = 3$

57. (c) : From the given lines we have

$$\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s \qquad \ldots (A)$$

and $$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{-2} = t \qquad \ldots (B)$$

As lines (A) and (B) are coplanar $\therefore \begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$

$\Rightarrow \quad (2\lambda - 2\lambda) + 4(-2-\lambda) - 1(2+\lambda) = 0$

$\Rightarrow \quad 5\lambda = -10 \quad \therefore \lambda = -2$

58. (b) : Using fact, two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad k^2 + 3k = 0 \quad \Rightarrow \quad k = 0$ or $k = -3$

59. (c) : Given lines can be written as

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c} \text{ and } \frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d}{c'}$$

$\therefore$ Required condition of perpendicularity is $aa' + cc' + 1 = 0$

60. (a) : As vectors $(1, \vec{a}, \vec{a}^2)$, $(1, \vec{b}, \vec{b}^2)$, $(1, \vec{c}, \vec{c}^2)$ are non coplanar.

$$\therefore \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \ \ldots \text{(A)} \quad \text{Now} \quad \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$$

On solving, we get $\Rightarrow (1 + abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$\Rightarrow (1 + abc) = 0$ by using (A)

61. (d) : Concept : angle between the faces is equal to the angle between their normals.

$\therefore$ Vector $\varnothing$ to the face OAB is $\overrightarrow{OA}\times\overrightarrow{OB}$

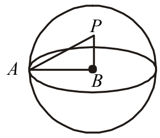
$= 5\hat{i} - \hat{j} - 3\hat{k}$ and vector $\varnothing$ to the face ABC is $\overrightarrow{AB}\times\overrightarrow{AC} = \hat{i} - 5\hat{j} - 3\hat{k}$

$\therefore$ Let θ be the angle between the faces OAB and ABC

$$\therefore \ \cos\theta = \frac{(5\hat{i}-\hat{j}-3\hat{k})\cdot(\hat{i}-5\hat{j}-3\hat{k})}{|5\hat{i}-\hat{j}-3\hat{k}||\hat{i}-5\hat{j}-3\hat{k}|}$$

$$\cos\theta = \frac{19}{35} \quad \therefore \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

62. (b) : The radius and centre of sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is



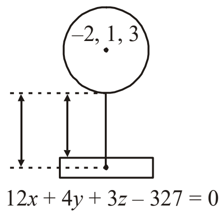
$\sqrt{1^2 + 1^2 + 4 + 19} = 5$ and centre $(-1, 1, 2)$

PB $\perp$ from centre to the plane

$$\frac{|-1 + 2 + 4 + 7|}{\sqrt{1 + 2^2 + 2^2}} = 4$$

Now $(AB)^2 = AP^2 - PB^2 = 25 - 16 = 9 \therefore AB = 3$

63. (b) : In order to determine the shortest distance between the plane and sphere, we find the distance from the centre of sphere to the plane – Radius of sphere



$\therefore$ Centre of sphere is $(-2, 1, 3)$

Required distance is

$$\frac{|-24 + 4 + 9 - 327|}{\sqrt{12^2 + 4^2 + 3^2}} - \sqrt{(2)^2 + 1^2 + 3^2 + 155}$$

$= 26 - 13 = 13$ units.

64. (c) : Now equation of the plane through $(a, 0, 0)$ $(0, b, 0)$ $(0, 0, c)$ is

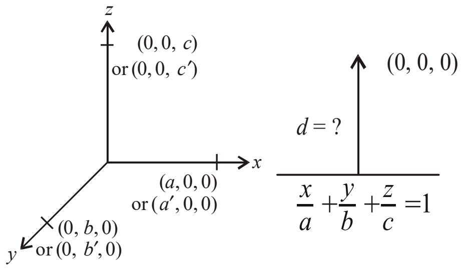
$$\Rightarrow \frac{x}{x\text{-Intercept}} + \frac{y}{y\text{-Intercept}} + \frac{z}{z\text{-Intercept}} = 1 \qquad ...(*)$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

So the distance from (0, 0, 0) to this plane to the plane (*) is given by

$$d_1 = \frac{|0 + 0 + 0 - 1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

Similarly, $d_2 = \dfrac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$



Now $d_1 = d_2$ given (as origin is same)

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$$

65. (b) : Let D.R.'s of normal to plane are a, b, c

$\therefore \quad a(x - 1) + b(y) + c(z) = 0 \qquad ...(*)$

$\Rightarrow \quad a(0 - 1) + b(1) + c(0) = 0$ (by using (0, 1, 0) in (*))

$\Rightarrow \quad -a + b = 0 \Rightarrow a = b$

Also angle between (*) and $x + y + 0z = 3$ is $\pi/4$

$$\therefore \quad \cos\frac{\pi}{4} = \frac{a + a}{\sqrt{1^2 + 1^2}\sqrt{a^2 + b^2 + c^2}} = \frac{2a}{\sqrt{2}\sqrt{2a^2 + c^2}}$$

$\Rightarrow \quad 2a^2 + c^2 = 4a^2 \Rightarrow \quad c = \pm\sqrt{2}\, a$

$\therefore \quad$ D.R.'s a, b, c *i.e.* $a, a, \pm\sqrt{2}a$

$\therefore \quad$ Required D.R.'s are $1, 1, \sqrt{2}$ or $1, 1, -\sqrt{2}$

Hence $1, 1, \sqrt{2}$ match with choice (b)