CHAPTER

12

Two Dimensional Geometry

1. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is

(a) $\frac{3\sqrt{5}}{2}$ (b) $\sqrt{10}$

(c) $2\sqrt{10}$ (d) $3\sqrt{\frac{5}{2}}$ *(2018)*

2. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then the value of c is

(a) 95 (b) 195 (c) 185 (d) 85 *(2018)*

3. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0, 3)$, then the area (in sq. units) of ΔPTQ is

(a) $36\sqrt{5}$ (b) $45\sqrt{5}$ (c) $54\sqrt{3}$ (d) $60\sqrt{3}$ *(2018)*

4. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is

(a) $\frac{4}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 3 *(2018)*

5. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is

(a) $3x + 2y = 6xy$ (b) $3x + 2y = 6$

(c) $2x + 3y = xy$ (d) $3x + 2y = xy$ *(2018)*

6. In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are respectively, $x + y = 5$ and $x = 4$. Then area of ΔABC (in sq. units) is

(a) 12 (b) 9 (c) 4 (d) 5 *(Online 2018)*

7. If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{4}$ *(Online 2018)*

8. If the tangent drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is :

(a) $4x^2 - y^2 - 16x^2y^2 = 0$ (b) $4x^2 - y^2 + 16x^2y^2 = 0$

(c) $x^2 - 4y^2 + 16x^2y^2 = 0$ (d) $x^2 - 4y^2 - 16x^2y^2 = 0$ *(Online 2018)*

9. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to

(a) 1 (b) 2 (c) $\sqrt{5}$ (d) $\sqrt{2}$ *(Online 2018)*

10. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is

(a) $4(x + y) + 3 = 0$ (b) $8(2x + y) + 3 = 0$

(c) $3(x + y) + 4 = 0$ (d) $x + 2y + 3 = 0$ *(Online 2018)*

11. Tangents drawn from the point $(-8, 0)$ to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to

(a) 24 (b) 64 (c) 32 (d) 48 *(Online 2018)*

12. The sides of a rhombus $ABCD$ are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1, 2)$ and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is

(a) 2 (b) $\frac{5}{2}$ (c) $\frac{7}{4}$ (d) $\frac{7}{2}$ *(Online 2018)*

13. A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram $OABP$ (O being the origin) is formed, then the locus of P is

(a) $9x^2 + 4y^2 = 169$ (b) $4x^2 - 9y^2 = 121$

(c) $4x^2 + 9y^2 = 121$ (d) $9x^2 - 4y^2 = 169$ *(Online 2018)*

14. The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose centre is $(3, -2)$. The radius of C_2 is

(a) $\sqrt{2}$ (b) $\sqrt{6}$ (c) 3 (d) 2 *(Online 2018)*

15. The foot of the perpendicular drawn from the origin on the line, $3x + y = \lambda(\lambda \neq 0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio $BP : PA$ is

(a) 9 : 1 (b) 1 : 3 (c) 3 : 1 (d) 1 : 9

(Online 2018)

16. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is

(a) $x + 4y - 2 = 0$ (b) $x + y + 1 = 0$
(c) $x - y + 3 = 0$ (d) $x + 2y = 0$

(Online 2018)

17. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is :

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{1}{2}$

(Online 2018)

18. The number of values of k for which the system of linear equations,

$(k + 2)x + 10y = k$
$kx + (k + 3)y = k - 1$ has no solution is :

(a) infinitely many (b) 1
(c) 2 (d) 3 *(Online 2018)*

19. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle C, on the x-axis is equal to :

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$

(Online 2018)

20. The locus of the point of intersection of the lines,

$\sqrt{2}x - y + 4\sqrt{2}k = 0$ and $\sqrt{2}kx + ky - 4\sqrt{2} = 0$

(k is any non-zero real parameter), is

(a) a hyperbola with length of its transverse axis $8\sqrt{2}$.
(b) a hyperbola whose eccentricity is $\sqrt{3}$.
(c) an ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$.
(d) an ellipse with length of its major axis $8\sqrt{2}$.

(Online 2018)

21. Let k be an integer such that triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point

(a) $\left(1, \frac{3}{4}\right)$ (b) $\left(1, -\frac{3}{4}\right)$ (c) $\left(2, \frac{1}{2}\right)$ (d) $\left(2, -\frac{1}{2}\right)$

(2017)

22. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is

(a) $2(\sqrt{2} - 1)$ (b) $4(\sqrt{2} - 1)$
(c) $4(\sqrt{2} + 1)$ (d) $2(\sqrt{2} + 1)$ *(2017)*

23. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is

(a) $4x - 2y = 1$ (b) $4x + 2y = 7$
(c) $x + 2y = 4$ (d) $2y - x = 2$ *(2017)*

24. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point

(a) $(2\sqrt{2}, 3\sqrt{3})$ (b) $(\sqrt{3}, \sqrt{2})$
(c) $(-\sqrt{2}, -\sqrt{3})$ (d) $(3\sqrt{2}, 2\sqrt{3})$ *(2017)*

25. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is

(a) $\frac{16}{7}$ (b) $\frac{8}{\sqrt{7}}$ (c) $\frac{8}{7}$ (d) $\frac{4}{\sqrt{7}}$

(Online 2017)

26. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is

(a) $2(\sqrt{2} + 1)$ (b) $3 + 2\sqrt{2}$
(c) $2(3 + 2\sqrt{2})$ (d) $\sqrt{2} + 1$ *(Online 2017)*

27. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is

(a) 8 (b) 32 (c) 80 (d) 40

(Online 2017)

28. If a point P has co-ordinates (0, –2) and Q is any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is

(a) $\frac{25 + \sqrt{6}}{2}$ (b) $8 + 5\sqrt{3}$
(c) $14 + 5\sqrt{3}$ (d) $\frac{47 + 10\sqrt{6}}{2}$ *(Online 2017)*

29. The locus of the point of intersection of the straight lines,

$tx - 2y - 3t = 0$
$x - 2ty + 3 = 0$ ($t \in R$), is

(a) a hyperbola with the length of conjugate axis 3
(b) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$
(c) an ellipse with the length of major axis 6
(d) a hyperbola with eccentricity $\sqrt{5}$ *(Online 2017)*

30. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is

(a) $\sqrt{3}-2$ (b) $2\sqrt{3}-1$

(c) $\sqrt{3}-1$ (d) $2\sqrt{3}-2$ *(Online 2017)*

31. A line drawn through the point $P(4, 7)$ cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then $PA \cdot PB$ is equal to

(a) 56 (b) 74 (c) 65 (d) 53

(Online 2017)

32. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, –1) and (–2, 2) is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{2}$

(Online 2017)

33. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to

(a) $8\sqrt{3}$ (b) $2\sqrt{3}$ (c) $16\sqrt{3}$ (d) $10\sqrt{3}$

(Online 2017)

34. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus?

(a) (–3, –9) (b) (–3, –8)

(c) $\left(\frac{6}{8}1-\frac{=}{8}\right)$ (d) $\left(-\frac{65}{8}1-\frac{<}{8}\right)$ *(2016)*

35. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on

(a) a circle

(b) an ellipse which is not a circle

(c) a hyperbola (d) a parabola *(2016)*

36. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is

(a) $:\sqrt{7}$ (b) $:\sqrt{8}$ (c) 5 (d) 10

(2016)

37. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is

(a) $x^2 + y^2 - 4x + 8y + 12 = 0$

(b) $x^2 + y^2 - x + 4y - 12 = 0$

(c) $^7 + {}^7 - \frac{}{9} + 7 \quad -79 = 5$

(d) $x^2 + y^2 - 4x + 9y + 18 = 0$ *(2016)*

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

(a) $\frac{9}{8}$ (b) $\frac{9}{\sqrt{8}}$ (c) $\frac{7}{\sqrt{8}}$ (d) $\sqrt{8}$

(2016)

39. A circle passes through (–2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle?

(a) $2x - 3y + 10 = 0$ (b) $3x + 4y - 3 = 0$

(c) $4x + 5y - 6 = 0$ (d) $5x + 2y + 4 = 0$

(Online 2016)

40. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is

(a) $7xy = 6(x + y)$

(b) $4(x + y)^2 - 28(x + y) + 49 = 0$

(c) $6xy = 7(x + y)$

(d) $14(x + y)^2 - 97(x + y) + 168 = 0$ *(Online 2016)*

41. If the tangent at a point on the ellipse $\frac{x^2}{27}+\frac{y^2}{3}=1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is

(a) $3\sqrt{3}$ (b) $\frac{9}{2}$ (c) 9 (d) $9\sqrt{3}$

(Online 2016)

42. The point (2, 1) is translated parallel to the line $L: x - y = 4$ by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is

(a) $x + y = 2 - \sqrt{6}$ (b) $2x + 2y = 1 - \sqrt{6}$

(c) $x + y = 3 - 3\sqrt{6}$ (d) $x + y = 3 - 2\sqrt{6}$

(Online 2016)

43. The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is

(a) $\frac{\sqrt{15}}{2}$ (b) $\sqrt{\frac{19}{2}}$ (c) $\sqrt{\frac{15}{2}}$ (d) $\frac{\sqrt{19}}{2}$

(Online 2016)

44. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2 - 18e + 5 = 0$. If $S(5, 0)$ is a focus and $5x = 9$ is the corresponding directrix of this hyperbola, then $a^2 - b^2$ is equal to

(a) –7 (b) –5 (c) 5 (d) 7

(Online 2016)

45. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is

(a) 8 (b) 4 (c) 6 (d) 2

(Online 2016)

46. Equation of the tangent to the circle, at the point (1, –1), whose centre is the point of intersection of the straight lines $x - y = 1$ and $2x + y = 3$ is

(a) $x + 4y + 3 = 0$ (b) $3x - y - 4 = 0$

(c) $x - 3y - 4 = 0$ (d) $4x + y - 3 = 0$

(Online 2016)

47. A straight line through origin O meets the lines $3y = 10 - 4x$ and $8x + 6y + 5 = 0$ at points A and B respectively. Then O divides the segment AB in the ratio
(a) 2 : 3 (b) 1 : 2 (c) 4 : 1 (d) 3 : 4
(Online 2016)

48. A ray of light is incident along a line which meets another line, $7x - y + 1 = 0$, at the point (0, 1). The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is
(a) $41x - 25y + 25 = 0$ (b) $41x + 25y - 25 = 0$
(c) $41x - 38y + 38 = 0$ (d) $41x + 38y - 38 = 0$
(Online 2016)

49. A hyperbola whose transverse axis is along the major axis of the conic $\frac{x^2}{3}+\frac{y^2}{4}=4$, and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it?
(a) $(\sqrt{5}, 2\sqrt{2})$ (b) $(0, 2)$
(c) $(5, 2\sqrt{3})$ (d) $(\sqrt{10}, 2\sqrt{3})$ *(Online 2016)*

50. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is
(a) $y^2 = 2x$ (b) $x^2 = 2y$ (c) $x^2 = y$ (d) $y^2 = x$
(2015)

51. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a
(a) circle of radius $\sqrt{7}$.
(b) circle of radius $\sqrt{8}$.
(c) straight line parallel to x-axis.
(d) straight line parallel to y-axis. *(2015)*

52. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is
(a) 3 (b) 4 (c) 1 (d) 2 *(2015)*

53. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{7}{>}+\frac{7}{:}=6$, is
(a) $\frac{7<}{7}$ (b) 27 (c) $\frac{7<}{9}$ (d) 18 *(2015)*

54. The points (0, 8/3), (1, 3) and (82, 30) :
(a) form an obtuse angled triangle
(b) form an acute angled triangle
(c) form a right angled triangle
(d) lie on a straight line *(Online 2015)*

55. Let L be the line passing through the point $P(1, 2)$ such that its intercepted segment between the co-ordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point (–2, 1), then the point of intersection of L and L_1 is
(a) $\left(\frac{9}{:}1\frac{67}{:}\right)$ (b) $\left(\frac{66}{75}1\frac{7>}{65}\right)$
(c) $\left(\frac{8}{65}1\frac{6<}{:}\right)$ (d) $\left(\frac{8}{:}1\frac{78}{65}\right)$ *(Online 2015)*

56. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is
(a) $x^2 + y^2 + 3x + 9y = 0$
(b) $x^2 + y^2 - 3x + 9y = 0$
(c) $x^2 + y^2 - 3x - 9y = 0$
(d) $x^2 + y^2 + 3x - 9y = 0$ *(Online 2015)*

57. If the tangent to the conic, $y - 6 = x^2$ at (2, 10) touches the circle, $x^2 + y^2 + 8x - 2y = k$ (for some fixed k) at a point (α, β); then (α, β) is
(a) $\left(-\frac{;}{6<}1\frac{65}{6<}\right)$ (b) $\left(-\frac{=}{6<}1\frac{7}{6<}\right)$
(c) $\left(-\frac{9}{6<}1\frac{6}{6<}\right)$ (d) $\left(-\frac{<}{6<}1\frac{;}{6<}\right)$ *(Online 2015)*

58. An ellipse passes through the foci of the hyperbola, $9x^2 - 4y^2 = 36$ and its major and minor axes lie along the transverse and conjugate axes of the hyperbola respectively. If the product of eccentricities of the two conics is 1/2, then which of the following points does not lie on the ellipse?
(a) $-\sqrt{68}15.$ (b) $\left(\frac{\sqrt{8>}}{7}1\sqrt{8}\right)$
(c) $\left(\frac{6}{7}\sqrt{68}1\frac{\sqrt{8}}{7}\right)$ (d) $\left(\sqrt{\frac{68}{7}}1\sqrt{;}\right)$
(Online 2015)

59. A straight line L through the point (3, –2) is inclined at an angle of 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is
(a) $+\sqrt{8}\quad +7-8\sqrt{8}=5$ (b) $-\sqrt{8}\quad +7+8\sqrt{8}=5$
(c) $\sqrt{8}\quad -\quad +8+7\sqrt{8}=5$ (d) $\sqrt{8}\quad +\quad -8+7\sqrt{8}=5$
(Online 2015)

60. If the incentre of an equilateral triangle is (1, 1) and the equation of its one side is $3x + 4y + 3 = 0$, then the equation of the circumcircle of this triangle is
(a) $x^2 + y^2 - 2x - 2y - 2 = 0$
(b) $x^2 + y^2 - 2x - 2y - 14 = 0$
(c) $x^2 + y^2 - 2x - 2y + 2 = 0$
(d) $x^2 + y^2 - 2x - 2y - 7 = 0$ *(Online 2015)*

61. If a circle passing through the point (–1, 0) touches y-axis at (0, 2), then the length of the chord of the circle along the x-axis is
(a) $\frac{8}{7}$ (b) $\frac{:}{7}$ (c) 3 (d) 5
(Online 2015)

62. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is

(a) $\frac{6}{7}$ (b) $\frac{7\sqrt{7}-6}{7}$ (c) $\sqrt{7}-6$ (d) $\frac{\sqrt{7}-6}{7}$

(Online 2015)

63. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant. If R divides PQ in the ratio 2 : 1, then the locus of R is

(a) $9y^2 = 4x$ (b) $9y^2 = -4x$
(c) $3y^2 = 2x$ (d) $3y^2 = -2x$

(Online 2015)

64. The locus of the foot of the perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(a) $(x^2 - y^2)^2 = 6x^2 - 2y^2$
(b) $(x^2 + y^2)^2 = 6x^2 + 2y^2$
(c) $(x^2 + y^2)^2 = 6x^2 - 2y^2$
(d) $(x^2 - y^2)^2 = 6x^2 + 2y^2$ *(2014)*

65. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes, then

(a) $2bc + 3ad = 0$ (b) $3bc - 2ad = 0$
(c) $3bc + 2ad = 0$ (d) $2bc - 3ad = 0$ *(2014)*

66. Let PS be the median of the triangle whose vertices $P(2, 2)$, Q (6, –1) and $R(7, 3)$. The equation of the line passing through (1, –1) and parallel to PS is

(a) $2x + 9y + 7 = 0$ (b) $4x + 7y + 3 = 0$
(c) $2x - 9y - 11 = 0$ (d) $4x - 7y - 11 = 0$

(2014)

67. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{8}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$ *(2014)*

68. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally then the radius of T is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

(2014)

69. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point

(a) (2, –5) (b) (5, –2)
(c) (–2, 5) (d) (–5, 2) *(2013)*

70. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola $y^2 = 4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-2 : If the line, $y = mx + \frac{\sqrt{5}}{m} (m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1. *(2013)*

71. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

(a) $\sqrt{3}y = x - \sqrt{3}$ (b) $y = \sqrt{3}x - \sqrt{3}$
(c) $\sqrt{3}y = x - 1$ (d) $y = x + \sqrt{3}$ *(2013)*

72. The equation of the circle passing through the focii of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y + 7 = 0$ (b) $x^2 + y^2 - 6y - 5 = 0$
(c) $x^2 + y^2 - 6y + 5 = 0$ (d) $x^2 + y^2 - 6y - 7 = 0$

(2013)

73. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is

(a) $2 - \sqrt{2}$ (b) $1 + \sqrt{2}$ (c) $1 - \sqrt{2}$ (d) $2 + \sqrt{2}$

(2013)

74. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals

(a) 6 (b) 11/5 (c) 29/5 (d) 5 *(2012)*

75. **Statement 1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$

Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

76. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is

(a) 6/5 (b) 5/3
(c) 10/3 (d) 3/5 *(2012)*

77. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is

(a) $4x^2 + y^2 = 8$ (b) $x^2 + 4y^2 = 16$
(c) $4x^2 + y^2 = 4$ (d) $x^2 + 4y^2 = 8$ *(2012)*

78. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is
(a) -2 (b) $-1/2$ (c) $-1/4$ (d) -4 *(2012)*

79. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if
(a) $a = 2c$ (b) $|a| = 2c$
(c) $2|a| = c$ (d) $|a| = c$ *(2011)*

80. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.
Statement-1 : The ratio $PR : RQ$ equals
Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

81. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is
(a) $\frac{8}{3\sqrt{2}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$ *(2011, 2009)*

82. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{\frac{2}{5}}$ is
(a) $3x^2 + 5y^2 - 15 = 0$ (b) $5x^2 + 3y^2 - 32 = 0$
(c) $3x^2 + 5y^2 - 32 = 0$ (d) $5x^2 + 3y^2 - 48 = 0$ *(2011)*

83. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is
(a) $y = 0$ (b) $y = 1$ (c) $y = 2$ (d) $y = 3$ *(2010)*

84. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is
(a) $x = 1$ (b) $2x + 1 = 0$
(c) $x = -1$ (d) $2x - 1 = 0$ *(2010)*

85. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is
(a) $\frac{23}{\sqrt{15}}$ (b) $\sqrt{17}$ (c) $\frac{17}{\sqrt{15}}$ (d) $\frac{23}{\sqrt{17}}$ *(2010)*

86. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
(a) $-85 < m < -35$ (b) $-35 < m < 15$
(c) $15 < m < 65$ (d) $35 < m < 85$ *(2010)*

87. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point $(-1, 0)$ is equal to 1/3. Then the circumcentre of the triangle ABC is at the point
(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$ (c) $\left(\frac{5}{3}, 0\right)$ (d) (0, 0) *(2009)*

88. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is
(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$
(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$ *(2009)*

89. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for
(a) all except one value of p
(b) all except two values of p
(c) exactly one value of p
(d) all values of p *(2009)*

90. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is
(a) $\frac{5}{3}$ (b) $\frac{8}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$ *(2008)*

91. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is
(a) (3, 4) (b) (3, – 4)
(c) (– 3, 4) (d) (– 3, – 4) *(2008)*

92. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at
(a) (2, 0) (b) (0, 2) (c) (1, 0) (d) (0, 1) *(2008)*

93. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept -4. Then a possible value of k is
(a) -4 (b) 1 (c) 2 (d) -2 *(2008)*

94. The normal to a curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
(a) circle (b) hyperbola
(c) ellipse (d) parabola *(2007)*

95. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval
(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$
(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$ *(2007)*

96. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is
(a) 1 (b) 2 (c) –1/2 (d) –2 *(2007)*

97. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is
(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$ *(2007)*

98. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 square unit, then the set of values which 'k' can take is given by
(a) $\{-1, 3\}$ (b) $\{-3, -2\}$
(c) $\{1, 3\}$ (d) $\{0, 2\}$ *(2007)*

99. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is
(a) (2, 4) (b) (–2, 0)
(c) (–1, 1) (d) (0, 2) *(2007)*

100. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?
(a) abscissae of vertices (b) abscissae of foci
(c) eccentricity (d) directrix *(2007)*

101. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belongs to
(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$
(c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$ *(2006)*

102. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of chord of the circle C that subtend an angle of $2\pi/3$ at its centre is
(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$
(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$ *(2006)*

103. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, then the equation of the circle is
(a) $x^2 + y^2 + 2x - 2y - 47 = 0$
(b) $x^2 + y^2 + 2x - 2y - 62 = 0$
(c) $x^2 + y^2 - 2x + 2y - 62 = 0$
(d) $x^2 + y^2 - 2x + 2y - 47 = 0$ *(2006)*

104. In an ellipse, the distance between its focii is 6 and minor axis is 8. Then its eccentricity is,
(a) 3/5 (b) 1/2
(c) 4/5 (d) $1/\sqrt{5}$ *(2006)*

105. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is
(a) $xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$
(c) $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$ *(2006)*

106. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is
(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$
(c) $4x + 3y = 24$ (d) $3x + 4y = 25$ *(2006)*

107. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then
(a) $3a^2 - 2ab + 3b^2 = 0$ (b) $3a^2 - 10ab + 3b^2 = 0$
(c) $3a^2 + 2ab + 3b^2 = 0$ (d) $3a^2 + 10ab + 3b^2 = 0$
(2005)

108. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
(a) a circle (b) an ellipse
(c) a hyperbola (d) a parabola *(2005)*

109. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{4}$ *(2005)*

110. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is
(a) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(b) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
(c) $2ax + 2by - (a^2 + b^2 + p^2) = 0$
(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ *(2005)*

111. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is
(a) a circle (b) an ellipse
(c) a parabola (d) a hyperbola *(2005)*

112. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for
(a) no value of a
(b) exactly one value of a
(c) exactly two values of a
(d) infinitely many values of a *(2005)*

113. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is

(a) $\left(\frac{-1}{3}, \frac{7}{3}\right)$ (b) $\left(-1, \frac{7}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{3}\right)$ *(2005)*

114. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c} = 0$ always passes through a fixed point. That point is

(a) $(-1, -2)$ (b) $(-1, 2)$

(c) $\left(1, -\frac{1}{2}\right)$ (d) $(1, -2)$ *(2005)*

115. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(a) below the x-axis at a distance of 2/3 from it

(b) below the x-axis at a distance of 3/2 from it

(c) above the x-axis at a distance of 2/3 from it

(d) above the x-axis at a distance of 3/2 from it *(2005)*

116. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(a) $x^2 - 4y + 2 = 0$ (b) $x^2 + 4y + 2 = 0$

(c) $y^2 + 4x + 2 = 0$ (d) $y^2 - 4x + 2 = 0$ *(2005)*

117. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directrices is $x = 4$, then the equation of the ellipse is

(a) $4x^2 + 3y^2 = 12$ (b) $3x^2 + 4y^2 = 12$

(c) $3x^2 + 4y^2 = 1$ (d) $4x^2 + 3y^2 = 1$ *(2004)*

118. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then

(a) $d^2 + (2b - 3c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$

(c) $d^2 + (2b + 3c)^2 = 0$ (d) $d^2 + (3b - 2c)^2 = 0$

(2004)

119. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is

(a) $x^2 + y^2 + x + y = 0$ (b) $x^2 + y^2 - x + y = 0$

(c) $x^2 + y^2 - x - y = 0$ (d) $x^2 + y^2 + x - y = 0$

(2004)

120. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is

(a) $x^2 + y^2 + 2x + 2y - 23 = 0$

(b) $x^2 + y^2 - 2x - 2y - 23 = 0$

(c) $x^2 + y^2 - 2x + 2y - 23 = 0$

(d) $x^2 + y^2 + 2x - 2y - 23 = 0$ *(2004)*

121. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is

(a) $(y - p)^2 = 4qx$ (b) $(x - q)^2 = 4py$

(c) $(x - p)^2 = 4qy$ (d) $(y - q)^2 = 4px$ *(2004)*

122. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is

(a) $2ax - 2by + (a^2 + b^2 + 4) = 0$

(b) $2ax + 2by - (a^2 + b^2 + 4) = 0$

(c) $2ax + 2by + (a^2 + b^2 + 4) = 0$

(d) $2ax - 2by - (a^2 + b^2 + 4) = 0$ *(2004)*

123. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals

(a) 3 (b) -1 (c) 1 (d) -3 *(2004)*

124. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value

(a) 2 (b) -1

(c) 1 (d) -2 *(2004)*

125. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is -1 is

(a) $\frac{x}{2}+\frac{y}{3} = 1$ and $\frac{x}{2}+\frac{y}{1} = 1$

(b) $\frac{x}{2}-\frac{y}{3} = -1$ and $\frac{x}{-2}+\frac{y}{1} = 1$

(c) $\frac{x}{2}+\frac{y}{3} = -1$ and $\frac{x}{-2}+\frac{y}{1} = -1$

(d) $\frac{x}{2}-\frac{y}{3} = 1$ and $\frac{x}{-2}+\frac{y}{1} = 1$ *(2004)*

126. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

(a) $3x + 2y = 5$ (b) $2x - 3y = 7$

(c) $2x + 3y = 9$ (d) $3x - 2y = 3$ *(2004)*

127. The normal to the curve $x = a(1 + \cos\theta)$, $y = a \sin\theta$ at θ always passes through the fixed point

(a) (0, 0) (b) $(0, a)$

(c) $(a, 0)$ (d) (a, a) *(2004)*

128. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$

(c) $(2, 4)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$ *(2004)*

129. If the equation of the locus of point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then $c =$

(a) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (b) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

(c) $\sqrt{(a_1^2 + b_1^2 - a_2^2 - b_2^2)}$ (d) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

(2003)

130. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and (1, 0), where t is a parameter, is

(a) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(b) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

(d) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ *(2003)*

131. If the pairs of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(a) $p = -q$ (b) $pq = 1$

(c) $pq = -1$ (d) $p = q$ *(2003)*

132. A square of side a lies above the x -axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is
(a) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$
(c) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
(d) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ *(2003)*

133. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is
(a) $x^2 + y^2 + 2x - 2y = 47$
(b) $x^2 + y^2 - 2x + 2y = 47$
(c) $x^2 + y^2 - 2x + 2y = 62$
(d) $x^2 + y^2 + 2x - 2y = 62$ *(2003)*

134. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then
(a) $r < 2$ (b) $r = 2$
(c) $r > 2$ (d) $2 < r < 8$ *(2003)*

135. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then
(a) $t_2 = -t_1 + \dfrac{2}{t_1}$ (b) $t_2 = t_1 - \dfrac{2}{t_1}$
(c) $t_2 = t_1 + \dfrac{2}{t_1}$ (d) $t_2 = -t_1 - \dfrac{2}{t_1}$ *(2003)*

136. The foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide. Then the value of b^2 is
(a) 5 (b) 7 (c) 9 (d) 1 *(2003)*

137. A triangle with vertices (4, 0), (–1, –1), (3, 5) is
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles *(2002)*

138. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is
(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$
(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$ *(2002)*

139. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is
(a) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ (b) $\left(\dfrac{1}{2}, -\sqrt{2}\right)$
(c) $\left(\dfrac{3}{2}, \dfrac{1}{2}\right)$ (d) $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$ *(2002)*

140. Locus of mid point of the portion between the axes of $x\cos\alpha + y\sin\alpha = p$ where p is constant is
(a) $x^2 + y^2 = \dfrac{4}{p^2}$ (b) $x^2 + y^2 = 4p^2$
(c) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{2}{p^2}$ (d) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$ *(2002)*

141. The point of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ and $\perp$ to each other for
(a) two values of a (b) $\forall$ a
(c) for one value of a (d) for no values of a *(2002)*

142. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is
(a) $4 \le x^2 + y^2 \le 64$ (b) $x^2 + y^2 \le 25$
(c) $x^2 + y^2 \ge 25$ (d) $3 \le x^2 + y^2 \le 9$ *(2002)*

143. If the pair of lines
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
intersect on the y-axis then
(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \ne ch^2$
(c) $abc = 2fgh$ (d) none of these *(2002)*

144. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is
(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$
(c) $-1 \pm \sqrt{2}$ (d) none of these *(2002)*

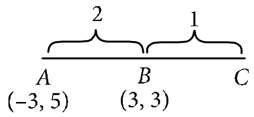
145. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are
(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$ *(2002)*

ANSWER KEY

1. (d)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (a)	**11.** (d)	**12.** (b)
13. (d)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (a)	**21.** (c)	**22.** (b)	**23.** (a)	**24.** (a)
25. (b)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (d)	**31.** (a)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (d)	**36.** (b)
37. (a)	**38.** (c)	**39.** (a)	**40.** (a)	**41.** (c)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (c)
49. (c)	**50.** (b)	**51.** (a)	**52.** (a)	**53.** (b)	**54.** (d)	**55.** (a)	**56.** (b)	**57.** (b)	**58.** (c)	**59.** (b)	**60.** (b)
61. (c)	**62.** (c)	**63.** (b)	**64.** (b)	**65.** (b)	**66.** (a)	**67.** (d)	**68.** (c)	**69.** (b)	**70.** (a)	**71.** (a)	**72.** (d)
73. (a)	**74.** (a)	**75.** (d)	**76.** (c)	**77.** (b)	**78.** (a)	**79.** (d)	**80.** (a)	**81.** (d)	**82.** (c)	**83.** (d)	**84.** (c)
85. (d)	**86.** (b)	**87.** (a)	**88.** (a)	**89.** (a)	**90.** (b)	**91.** (d)	**92.** (c)	**93.** (a)	**94.** (b, c)	**95.** (d)	**96.** (a)
97. (c)	**98.** (a)	**99.** (b)	**100.** (b)	**101.** (c)	**102.** (d)	**103.** (d)	**104.** (a)	**105.** (a)	**106.** (c)	**107.** (c)	**108.** (c)
109. (b)	**110.** (c)	**111.** (c)	**112.** (a)	**113.** (d)	**114.** (d)	**115.** (b)	**116.** (d)	**117.** (b)	**118.** (c)	**119.** (c)	**120.** (c)
121. (c)	**122.** (b)	**123.** (d)	**124.** (a)	**125.** (d)	**126.** (c)	**127.** (c)	**128.** (d)	**129.** (d)	**130.** (a)	**131.** (c)	**132.** (c)
133. (b)	**134.** (d)	**135.** (d)	**136.** (b)	**137.** (a)	**138.** (c)	**139.** (b)	**140.** (d)	**141.** (a)	**142.** (a)	**143.** (a)	**144.** (c)
145. (b)											

Explanations

1. (d) : We know that the centroid divides orthocentre and circumcentre in the ratio 2 : 1.



$$AC = \frac{3}{2}AB = \frac{3}{2}\cdot\sqrt{6^2+2^2} = \frac{3}{2}\cdot 2\sqrt{10} = 3\sqrt{10}$$

Radius of the circle with AC as diameter $= \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$

2. (a) : The equation of tangent at (1, 7) to $x^2 = y - 6$ is $2x - y + 5 = 0$

The perpendicular distance of centre (–8, –6) to the line $2x - y + 5 = 0$ should be equal to the radius of the circle.

$$\therefore \sqrt{64+36-c} = \frac{|-16+6+5|}{\sqrt{5}} \Rightarrow \sqrt{5} = \sqrt{100-c} \Rightarrow c = 95$$

3. (b) : Let the tangent at (α, β) be $4x\alpha - y\beta = 36$

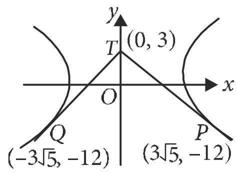
As (0, 3) lies on the tangent, so we have $-3\beta = 36 \Rightarrow \beta = -12$

Now $4\alpha^2 - \beta^2 = 36$ gives $4\alpha^2 - 12^2 = 36$

$$\Rightarrow 4\alpha^2 = 180 \Rightarrow \alpha^2 = 45 \Rightarrow \alpha = \pm 3\sqrt{5}$$

Thus the points P and Q are $P(3\sqrt{5}, -12), Q(-3\sqrt{5}, -12)$

The area of the ΔTQP is given by



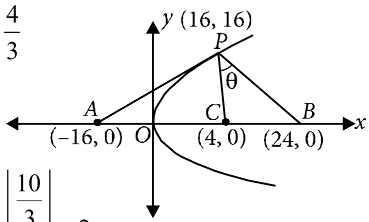
$$\Delta = \left|\frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ 3\sqrt{5} & -12 & 1 \\ -3\sqrt{5} & -12 & 1 \end{vmatrix}\right|$$

$$= \left|\frac{1}{2}[-3(6\sqrt{5}) - 36\sqrt{5} - 3\,6\sqrt{5}]\right| = \frac{1}{2}\,90\sqrt{5} = 45\sqrt{5}$$

4. (c) : The equation of tangent at $P(16, 16)$ is $x - 2y + 16 = 0$

The equation of normal at $P(16, 16)$ is $2x + y - 48 = 0$

The slope of PC : $m_1 = \frac{16}{12} = \frac{4}{3}$



The slope of PB :

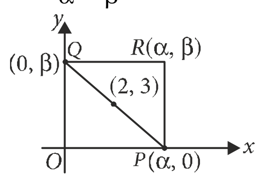
$$m_2 = \frac{-16}{8} = -2$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{4}{3}+2}{1-\frac{4}{3}(2)}\right| = \left|\frac{\frac{10}{3}}{-\frac{5}{3}}\right| = 2$$

5. (d) : The equation of the given line is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ ...(i)

As (2, 3) lies on (i),

$$\therefore \frac{2}{\alpha} + \frac{3}{\beta} = 1$$



$\Rightarrow 2\beta + 3\alpha - \alpha\beta = 0$

changing (α, β) to (x, y) we have the locus of R as $3x + 2y - xy = 0$

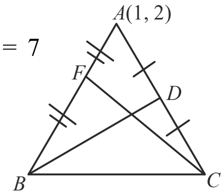
6. (b) : The equation of median BD is $x + y = 5$.

$\because$ B lies on it, therefore co-ordinates of B be $(x_1, 5 - x_1)$

Now, CF is median through C.

$\because$ F is the mid point of AB, where $A = (1, 2)$ and $B = (x_1, 5 - x_1)$

$\therefore$ Co-ordinates of $F = \left(\frac{x_1+1}{2}, \frac{5-x_1+2}{2}\right)$



Also, F lies on $x = 4$. $\therefore \frac{x_1+1}{2} = 4 \Rightarrow x_1 = 7$

$\Rightarrow$ Co-ordinates of $B = (7, -2)$

Similarly, let $C = (4, y_1)$

$\because$ D is the mid point of AC.

$\therefore$ $D = \left(\frac{4+1}{2}, \frac{y_1+2}{2}\right)$

Now, D lies on $x + y = 5 \Rightarrow \frac{5}{2} + \frac{y_1+2}{2} = 5 \Rightarrow y_1 = 3$

$\therefore$ Co-ordinates of $C = (4, 3)$

Now, area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & -2 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}[1(-2-3) - 2(7-4) + 1(21+8)] = \frac{1}{2}[18] = 9$

7. (b) : Given, $\frac{x^2}{9} + \frac{y^2}{3} = 1$

Equation of normal at $P(3\cos\theta, \sqrt{3}\sin\theta)$ is

$3\sec\theta\, x - \sqrt{3}\,\text{cosec}\,\theta\, y = 6$ ...(i)

Slope of (i) is given by $\frac{-3\sec\theta}{-\sqrt{3}\,\text{cosec}\,\theta} = \sqrt{3}\tan\theta$

Equation of normal at $(-3\sin\theta, \sqrt{3}\cos\theta)$

$-3\,\text{cosec}\,\theta x - \sqrt{3}\sec\theta\, y = 6$ ...(ii)

Slope of (ii) is given by $\frac{-(-3\,\text{cosec}\,\theta)}{(-\sqrt{3}\sec\theta)} = -\sqrt{3}\cot\theta$

$\because$ Angle between normals is β $\therefore \tan\beta = \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1-3}\right|$

$$= \left|\frac{-\sqrt{3}}{2}\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\right| = \left|\frac{-\sqrt{3}(\sin^2\theta + \cos^2\theta)}{2\sin\theta\cos\theta}\right| = \frac{\sqrt{3}}{\sin 2\theta}$$

$\Rightarrow \tan\beta = \frac{\sqrt{3}}{\sin 2\theta} \Rightarrow \cot\beta = \frac{\sin 2\theta}{\sqrt{3}} \quad \therefore \frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$

8. (d) : We have, $4y^2 = x^2 + 1$ ...(i)

The tangent to (i) at (x_1, y_1) is given by $4yy_1 = xx_1 + 1$

According to question, $A \equiv \left(\frac{-1}{x_1}, 0\right)$, $B \equiv \left(0, \frac{1}{4y_1}\right)$

Let mid point of AB be $M(h, k)$.

Then, $\frac{-1}{x_1} = 2h \Rightarrow x_1 = \frac{-1}{2h}$ and $\frac{1}{4y_1} = 2k \Rightarrow y_1 = \frac{1}{8k}$

$\because$ (x_1, y_1) lies on (i)

$\therefore$ $4\left(\frac{1}{8k}\right)^2 = \left(\frac{-1}{2h}\right)^2 + 1 \Rightarrow \frac{4\times 1}{64k^2} = \frac{1}{4h^2} + 1$

$\Rightarrow$ $\frac{1}{16k^2} = \frac{1}{4h^2} + 1 \Rightarrow h^2 = 4k^2 + 16h^2k^2$

$\therefore$ Locus of mid point of AB is
$x^2 = 4y^2 + 16x^2y^2$ or $x^2 - 4y^2 - 16x^2y^2 = 0$

9. **(b) :** Let radius be r and centre be (α, β).

$\because$ (α, β) lies on $y - 4x + 3 = 0$

$\therefore$ $\beta - 4\alpha + 3 = 0 \Rightarrow \beta = 4\alpha - 3$ ...(i)

$\therefore$ Centre $\equiv (\alpha, 4\alpha - 3)$ ...(ii)

Now, $OA^2 = OB^2 \Rightarrow (\alpha - 2)^2 + (\beta - 3)^2 = (\alpha - 4)^2 + (\beta - 5)^2$

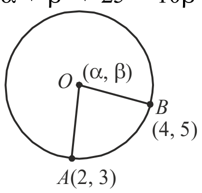
$\Rightarrow$ $\alpha^2 + 4 - 4\alpha + \beta^2 + 9 - 6\beta = \alpha^2 + 16 - 8\alpha + \beta^2 + 25 - 10\beta$

$\Rightarrow$ $4\alpha + 4\beta = 28 \Rightarrow \alpha + \beta = 7$

$\Rightarrow$ $\alpha + 4\alpha - 3 = 7$ (Using (i))

$\Rightarrow$ $5\alpha = 10 \Rightarrow \alpha = 2$

$\therefore$ Centre $(\alpha, \beta) = (2, 5)$



and radius = $\sqrt{OA} = \sqrt{(2-2)^2 + (5-3)^2} = 2$

10. **(a) :** $\because$ Length of latus rectum of each parabola = 3

$\therefore$ Equation of parabolas be $y^2 = 3x$ ...(i)

and $x^2 = 3y$ . ...(ii)

The equation of tangent to (i) is $y = mx + \frac{3}{4m}$ ...(iii)

(iii) is also tangent to $x^2 = 3y$

$\Rightarrow x^2 = 3mx + \frac{9}{4m} \Rightarrow 4mx^2 = 12m^2x + 9 \Rightarrow 4mx^2 - 12m^2x - 9 = 0$

Now, $D = 0 \Rightarrow 144m^4 - 4(-9)(4m) = 0 \Rightarrow m(m^3 + 1) = 0$

$\Rightarrow m = 0$(which is not possible) $\therefore$ $m = -1$

Put $m = -1$ in (iii), we get $y = -x - \frac{3}{4}$

or $4(x + y) + 3 = 0$, which is the required equation of tangent.

11. **(d) :** Let $(2t^2, 4t)$ be the point on the parabola and the focus is $(2, 0)$.

Now, slope of the tangent to the parabola $y^2 = 8x$

is given by $2y\cdot\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{4}{y}$

$\left.\frac{dy}{dx}\right|_{\substack{x=2t^2\\ y=4t}} = \frac{4}{4t} = \frac{1}{t}$ ...(i)

But the slope of the tangent passing through $(2t^2, 4t)$ and $(-8, 0)$ is given by

$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{0 - 4t}{-8 - 2t^2} \Rightarrow m = \frac{4t}{8 + 2t^2}$ ...(ii)

Equating (i) and (ii), we have $\frac{1}{t} = \frac{4t}{8 + 2t^2} \Rightarrow \frac{1}{t} = \frac{2t}{4 + t^2}$

$\Rightarrow$ $4 + t^2 = 2t^2 \Rightarrow 4 = t^2 \Rightarrow t = \pm 2$

For $t = 2$, point is $(8, 8)$ and for $t = -2$, point is $(8, -8)$.

So, the points are $P(8, 8)$, $Q(8, -8)$, $F(2, 0)$

$\therefore$ Area of $\Delta PFQ = \frac{1}{2}\left|8(-8) + 8(-8) + 2(8 + 8)\right|$

$= \frac{1}{2}\left|-128 + 32\right| = \frac{1}{2}\left|-96\right| = 48$ sq. units

12. **(b) :** Let coordinates of A be $(0, a)$

The diagonals intersect at $P(1, 2)$

We know that the diagonals will be parallel to the angle bisectors of the two sides $y = x + 2$ and $y = 7x + 3$

i.e. $\frac{x - y + 2}{\sqrt{2}} = \pm\frac{7x - y + 3}{5\sqrt{2}} \Rightarrow 5x - 5y + 10 = \pm(7x - y + 3)$

$\Rightarrow$ $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$\Rightarrow$ $m_1 = -\frac{1}{2}$ and $m_2 = 2$

(where m_1 and m_2 are the slopes of the given two lines)

Let one diagonal be parallel to $2x + 4y - 7 = 0$ and other be parallel to $12x - 6y + 13 = 0$

The vertex A could be on any of the two diagonals. Hence, slope of AP is either $-1/2$ or 2.

$\Rightarrow$ $\frac{2 - a}{1 - 0} = 2$ or $\frac{2 - a}{1 - 0} = -\frac{1}{2} \Rightarrow a = 0$ or $a = \frac{5}{2}$

But $a \neq 0$ $\therefore$ $a = \frac{5}{2}$. Thus, ordinate of A is $\frac{5}{2}$.

13. **(d) :** Given hyperbola is $4x^2 - 9y^2 = 36$

$\Rightarrow$ $\frac{x^2}{9} - \frac{y^2}{4} = 1$ ... (i)

Equation of normal to (i) is given by

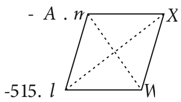
$\frac{3x}{\sec\theta} + \frac{2y}{\tan\theta} = 9 + 4 \Rightarrow \frac{3x}{\sec\theta} + \frac{2y}{\tan\theta} = 13$

$\therefore$ Coordinates of A are $\left(\frac{13}{3}\sec\theta, 0\right)$ and coordinates of B are $\left(0, \frac{13}{2}\tan\theta\right)$

Let the coordinates of P be (h, k).

Since, diagonals of parallelogram bisect each other.

$\therefore$ $\left(\frac{h + \frac{13}{3}\sec\theta}{2}, \frac{0 + k}{2}\right) = \left(0, \frac{13}{4}\tan\theta\right)$



$\Rightarrow$ $h = -\frac{13}{3}\sec\theta,\ k = \frac{13}{2}\tan\theta$

$\Rightarrow$ $\sec\theta = -\frac{3h}{13}$ ...(ii) and $\tan\theta = \frac{2k}{13}$ ...(iii)

Squaring (ii) and (iii) and then subtracting, we get

$\frac{9h^2}{169} - \frac{4k^2}{169} = 1 \Rightarrow 9h^2 - 4k^2 = 169$

Thus, locus of P is $9x^2 - 4y^2 = 169$.

14. **(b) :** Given : $x^2 + y^2 - 2x - 1 = 0$ ...(i)

Differentiating (i) with respect to x, we have

$2x + 2y\frac{dy}{dx} - 2 = 0 \Rightarrow x + y\frac{dy}{dx} - 1 = 0 \Rightarrow \frac{dy}{dx} = \frac{1 - x}{y}$

$\therefore$ $\left.\frac{dy}{dx}\right|_{(2,1)} = -1$

So, equation of tangent is $\frac{y - 1}{x - 2} = -1 \Rightarrow x + y = 3$ ...(ii)

Now, equation of circle with centre $(3, -2)$ is

$(x - 3)^2 + (y + 2)^2 = r^2$ ...(iii)

Putting $x = 3 - y$ from (ii) in (iii), we have

$y^2 + y^2 + 4 + 4y = r^2 \Rightarrow 2y^2 + 4 + 4y = r^2$

$\Rightarrow$ $y^2 + 2y + 2 = \frac{r^2}{2} \Rightarrow (y + 1)^2 = \frac{r^2}{2} - 1 \Rightarrow y = \pm\sqrt{\frac{r^2}{2} - 1} - 1$

So, points are $\left(4-\sqrt{\frac{r^2}{2}-1}, \sqrt{\frac{r^2}{2}-1}-1\right)$

and $\left(4+\sqrt{\frac{r^2}{2}-1}, -\sqrt{\frac{r^2}{2}-1}-1\right)$

Length of chord is given to be 4. So, by distance formula, we have

$$\sqrt{4\left(\frac{r^2}{2}-1\right)+4\left(\frac{r^2}{2}-1\right)}=4 \Rightarrow 8\left(\frac{r^2}{2}-1\right)=16 \Rightarrow \frac{r^2}{2}-1=2$$

$\Rightarrow \frac{r^2}{2}=3 \Rightarrow r=\pm\sqrt{6} \Rightarrow r=\sqrt{6}$ ($\because$ r cannot be negative)

15. (a) : Given, $3x + y = \lambda$ $(\lambda \neq 0) \Rightarrow 3x + y - \lambda = 0$

Foot of perpendicular from (x_1, y_1) to $ax + by + c = 0$ is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$

$\Rightarrow \frac{x-0}{3}=\frac{y-0}{1}=\frac{-(3\times 0+0-\lambda)}{3^2+1^2}$ [$\because$ $(x_1, y_1) = (0, 0)$]

$\Rightarrow \frac{x}{3}=\frac{y}{1}=\frac{\lambda}{10}$

Hence, foot of perpendicular is $P\left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Now, line meets x-axis where $y = 0$, so $3x+0=\lambda \Rightarrow x=\frac{\lambda}{3}$

Hence, coordinates of A are $\left(\frac{\lambda}{3}, 0\right)$.

Similarly, coordinates of B are $(0, \lambda)$.

$$\therefore \frac{BP}{PA}=\frac{\sqrt{\left(\frac{3\lambda}{10}-0\right)^2+\left(\frac{\lambda}{10}-\lambda\right)^2}}{\sqrt{\left(\frac{3\lambda}{10}-\frac{\lambda}{3}\right)^2+\left(\frac{\lambda}{10}-0\right)^2}}$$

$$=\frac{\sqrt{\frac{9\lambda^2}{100}+\frac{81\lambda^2}{100}}}{\sqrt{\frac{\lambda^2}{900}+\frac{\lambda^2}{100}}}=\frac{\sqrt{\frac{90\lambda^2}{100}}}{\sqrt{\frac{10\lambda^2}{900}}}=\frac{\sqrt{\frac{9}{10}}}{\sqrt{\frac{1}{90}}}=\frac{\sqrt{81}}{1}=\frac{9}{1} \Rightarrow BP : PA = 9 : 1$$

16. (b) : Let $P(2t, t^2)$

Given, $x^2 + y^2 + 6x + 8 = 0$ is equation of the circle.

So, coordinates of centre is $(-3, 0)$.

Now, distance of point P from $(-3, 0)=\sqrt{(2t+3)^2+(t^2-0)^2}$

Let $f(t) = (2t + 3)^2 + (t^2 - 0)^2$

$\Rightarrow f(t) = 4t^2 + 9 + 12t + t^4$ ...(i)

Differentiating equation (i) w.r.t. 't', we have

$f'(t) = 8t + 12 + 4t^3 = 4(t^3 + 2t + 3) = 4(t + 1)(t^2 - t + 3)$

Now, $f'(t) = 0 \Rightarrow t = -1$

So, point P becomes $(-2, 1)$

$\therefore$ Equation of tangent to the parabola is given by

$xx_1 = 2(y + y_1)$

$\Rightarrow x(-2) = 2(y + 1)$ [$\because$ $x_1 = -2, y_1 = 1$]

$\Rightarrow x + y + 1 = 0$

17. (a) : Length of latus rectum of ellipse is $\frac{2b^2}{a}$.

$\therefore \frac{2b^2}{a}=4$ [Length of latus rectum = 4 (Given)]

$\Rightarrow b^2 = 2a$ ...(i)

We know that $b^2 = a^2(1 - e^2)$

$\Rightarrow 2a = a^2(1 - e^2)$ [From (i)]

$\Rightarrow 2 = a(1 - e^2) \Rightarrow 2 = a(1 - e)(1 + e)$ ...(ii)

Also, given that distance of focus from vertex is $\frac{3}{2}$.

So, $(-ae+a)^2=\frac{9}{4} \Rightarrow a(1-e)=\frac{3}{2}$ ...(iii)

From (ii) and (iii), we have $2=\frac{3}{2}(1+e) \Rightarrow e=\frac{1}{3}$

18. (b) : Given equations will have no solution if

$\frac{k+2}{k}=\frac{10}{k+3}\neq\frac{k}{k-1}$

On solving first two, we have $(k + 2)(k + 3) = 10k$

$\Rightarrow k^2 - 5k + 6 = 0 \Rightarrow k = 2, 3$

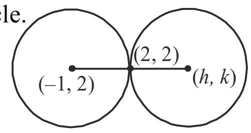
For $k = 2$, these equations are identical.

So, for only $k = 3$ these equations have no solution

$\therefore$ There is only one value of k.

19. (d) : Centre of circle $x^2 + y^2 + 2x - 4y - 4 = 0$ is $(-1, 2)$ and radius = $\sqrt{1+4+4}$ = 3

Let (h, k) be the centre of another circle.



Now, $\frac{h-1}{2}=2$ and $\frac{k+2}{2}=2$

$\Rightarrow h = 4 + 1$ and $k = 4 - 2$

$\Rightarrow h = 5$ and $k = 2$

So, centre of required circle is (5, 2) and radius = 3.

$\therefore$ Equation of circle becomes $(x - 5)^2 + (y - 2)^2 = (3)^2$

$\Rightarrow x^2 + y^2 - 10x - 4y + 20 = 0$ ...(i)

Length of intercept made by (i) on x-axis

$= 2\sqrt{g^2-c}=2\sqrt{25-20}$ ($\because$ $g = -5, c = 20$)

$=2\sqrt{5}$

20. (a) : $\sqrt{2}x-y+4\sqrt{2}k=0$ ...(i), $\sqrt{2}kx+ky-4\sqrt{2}=0$ ...(ii)

On eliminating k from (i) and (ii), we have

$$(\sqrt{2}x+y)\left(\frac{\sqrt{2}x-y}{-4\sqrt{2}}\right)=4\sqrt{2} \Rightarrow 2(\sqrt{2}x)^2-y^2=-(4\sqrt{2})^2$$

$\Rightarrow 2x^2 - y^2 = -32$

Dividing both sides by -32, we have

$\frac{y^2}{32}-\frac{x^2}{16}=1$, which is the equation of hyperbola.

Eccentricity $e=\sqrt{1+\frac{a^2}{b^2}}=\sqrt{1+\frac{16}{32}}=\sqrt{\frac{3}{2}}$

Length of transverse axis = $2b$ = $8\sqrt{2}$

21. (c) : As area is given to be 56, we have $\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix}=\pm 56$

Expanding, we get $k(k - 2) - 5(-3k - 2) - k(-3k - k) = \pm 56$

$\Rightarrow k^2 - 2k + 15k + 10 + 3k^2 + k^2 = \pm 56$

$\Rightarrow\quad 5k^2 + 13k + 10 = \pm 56$

Taking the positive sign $5k^2 + 13k - 46 = 0$

$\Rightarrow\quad (5k + 23)(k - 2) = 0\ \therefore\ k = 2$ is an integer

Taking the negative sign

$5k^2 + 13k + 66 = 0$

$D = 13^2 - 4\cdot5\cdot66 < 0$

Thus there is no solution in this case.

So the vertices are $A(2, -6)$, $B(5, 2)$ and $C(-2, 2)$.

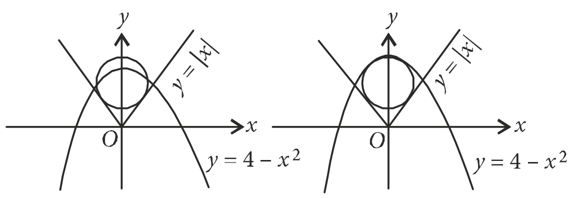
The equation of altitude from A is $x = 2$ and

the equation of altitude from C is $y - 2 = -\dfrac{3}{8}(x+2)$

i.e., $3x + 8y - 10 = 0$

Solving the two we get the orthocentre as $\left(2, \dfrac{1}{2}\right)$.

22. (b) : There are two possibilities



For both of them we get different answers.

For 1st case : $x^2 + (y - b)^2 = r^2$ as $y = x$ is tangent to circle

$\Rightarrow\ \left|\dfrac{0-b}{\sqrt{2}}\right| = r\ \ \therefore\ \ b = r\sqrt{2}.$ Now $x^2 + (y-b)^2 = \dfrac{b^2}{2}$

As $x^2 = 4 - y$

We have, $4 - y + (y-b)^2 = \dfrac{b^2}{2}$

Arranging as a quadratic in y, we have $y^2 - (2b+1)\,y + \dfrac{b^2}{2} + 4 = 0$

The discriminant being zero yields

$$(2b+1)^2 - 4\left(\frac{b^2}{2} + 4\right) = 0$$

$\Rightarrow\quad 4b^2 + 4b + 1 - 2b^2 - 16 = 0$ *i.e.* $2b^2 + 4b - 15 = 0$

$$\therefore\quad b = \frac{-4 \pm \sqrt{16+120}}{4} = \frac{-4 \pm 2\sqrt{34}}{4} = \frac{-2 \pm \sqrt{34}}{2}$$

Taking the positive value $b = \dfrac{\sqrt{34}-2}{2}\ \ \therefore\ r = \dfrac{\sqrt{34}-2}{2\sqrt{2}}$

For 2nd case : Co-ordinates of centre as $(0, 4-r)$, r being radius $y = x$ touch the circle

$\Rightarrow\ \left|\dfrac{0-(4-r)}{\sqrt{2}}\right| = r\ \ \Rightarrow\ \ r - 4 = \pm r\sqrt{2}$

Which gives $r(1 + \sqrt{2}) = 4$

(As r can't be negative) $\therefore\ r = \dfrac{4}{\sqrt{2}+1} = 4(\sqrt{2}-1)$

Remark : The problem, as posed, is ambiguous because of choices. The best choice is (b).

23. (a) : As $x = -\dfrac{a}{e} = -4$

We have, $a = 4e = 4\cdot\dfrac{1}{2} = 2$

Again $b^2 = a^2(1-e^2) = a^2\left(1 - \dfrac{1}{4}\right) = \dfrac{4\cdot3}{4} = 3$

Thus the equation to ellipse is $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$

Differentiating w.r.t x, we get $\dfrac{2x}{4} + \dfrac{2y}{3}\cdot\dfrac{dy}{dx} = 0 \Rightarrow\ \dfrac{dy}{dx} = -\dfrac{3}{4}\dfrac{x}{y}$

At $\left(1, \dfrac{3}{2}\right), \dfrac{dy}{dx} = -\dfrac{3}{4}\cdot\dfrac{1\cdot2}{3} = -\dfrac{1}{2}$

So the slope of normal is 2. The equation is

$y - \dfrac{3}{2} = 2(x-1)$ *i.e.*, $4x - 2y - 1 = 0$

24. (a) : The equation to the hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

We have, $ae = 2\quad \Rightarrow\quad a^2e^2 = 4$

Also, $b^2 = a^2(e^2 - 1)\quad \Rightarrow\quad a^2 + b^2 = a^2e^2 = 4$

The hyperbola passes through $(\sqrt{2}, \sqrt{3})$ means $\dfrac{2}{a^2} - \dfrac{3}{b^2} = 1$

On solving, we get $\dfrac{2}{a^2} - \dfrac{3}{4-a^2} = 1$

$\Rightarrow\quad 2(4 - a^2) - 3a^2 = a^2(4 - a^2)$

$\Rightarrow\quad 8 - 5a^2 = 4a^2 - a^4 = a^4 - 9a^2 + 8 = 0$

$\Rightarrow\quad (a^2 - 1)(a^2 - 8) = 0\quad \therefore\quad a^2 = 1$

As $a^2 = 8$ will give b^2 negative. $\therefore\ a^2 = 1$ and $b^2 = 3$

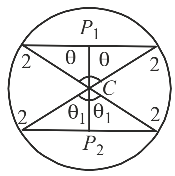
So, equation of the hyperbola is $\dfrac{x^2}{1} - \dfrac{y^2}{3} = 1$

The equation of tangent at P is $\dfrac{x\sqrt{2}}{1} - \dfrac{y\sqrt{3}}{3} = 1$

The point $(2\sqrt{2}, 3\sqrt{3})$ lies on it.

25. (b) : We have, radius of circle = 2 units

Let $\quad 2\theta = \cos^{-1}\left(\dfrac{1}{7}\right) \Rightarrow \cos 2\theta = 1/7\ \Rightarrow\ 2\cos^2\theta - 1 = \dfrac{1}{7}$



$\Rightarrow\quad 2\cos^2\theta = \dfrac{8}{7}\ \Rightarrow \cos^2\theta = \dfrac{4}{7}$

$\Rightarrow\quad \left(\dfrac{CP_1}{2}\right)^2 = \dfrac{4}{7}\quad \Rightarrow\quad CP_1 = \dfrac{4}{\sqrt{7}}$

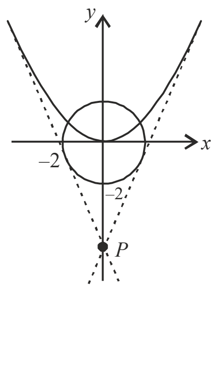
Let $2\theta_1 = \sec^{-1}(7) \Rightarrow \sec 2\theta_1 = 7$

$\Rightarrow\quad \dfrac{1}{2\cos^2\theta_1 - 1} = 7 \Rightarrow 2\left(\dfrac{CP_2}{2}\right)^2 - 1 = \dfrac{1}{7}$

$\Rightarrow\quad 2\left(\dfrac{CP_2}{2}\right)^2 = \dfrac{8}{7} \Rightarrow CP_2 = \dfrac{4}{\sqrt{7}}$

$\therefore\ \ P_1P_2 = CP_1 + CP_2 = \dfrac{4}{\sqrt{7}} + \dfrac{4}{\sqrt{7}} = \dfrac{8}{\sqrt{7}}$ units.

26. (c) : Tangent to $x^2 + y^2 = 4$ is



$y = mx \pm 2\sqrt{1+m^2}$ ...(i)

Given equation of parabola is $x^2 = 4y$

$\therefore\quad x^2 = 4mx + 8\sqrt{1+m^2}$ (from (i))

$\Rightarrow\quad x^2 - 4mx - 8\sqrt{1+m^2} = 0$

As there is only one intersection point

$\therefore\quad$ Discriminant = 0

$\Rightarrow \quad 16m^2 + 4\cdot 8\sqrt{1+m^2} = 0 \Rightarrow m^2 + 2\sqrt{1+m^2} = 0$

or $\quad m^2 = -2\sqrt{1+m^2} \Rightarrow \quad m^4 = 4 + 4m^2 \Rightarrow m^4 - 4m^2 - 4 = 0$

$\Rightarrow \quad m^2 = \dfrac{4 \pm \sqrt{16+16}}{2} = \dfrac{4 \pm 4\sqrt{2}}{2} \Rightarrow \quad m^2 = 2 \pm 2\sqrt{2}$

Put in (i), we get $y = 2\sqrt{1+2+2\sqrt{2}} = 2\sqrt{3+2\sqrt{2}}$ ($\because$ at P, $x = 0$)

27. (d) : We have, $e = 3/5$,

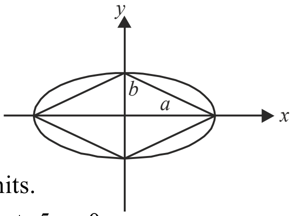
$2ae = 6 \Rightarrow a = 5$

As $b^2 = a^2(1 - e^2)$

$\Rightarrow \quad b^2 = 25(1 - 9/25) \Rightarrow b = 4$

$\therefore$ Area of quadrilateral $= 4(1/2\ ab)$

$= 2ab = 2 \times 5 \times 4 = 40$ sq. units.



28. (c) : We have, $x^2 + y^2 - 5x - y + 5 = 0$

$= \left(x - \dfrac{5}{2}\right)^2 + \left(y - \dfrac{1}{2}\right)^2 = \dfrac{3}{2}$

On circle, $Q \equiv \left(\dfrac{5}{2} + \sqrt{\dfrac{3}{2}}\cos\theta, \dfrac{1}{2} + \sqrt{\dfrac{3}{2}}\sin\theta\right)$

$(PQ)^2 = \left(\dfrac{5}{2} + \sqrt{\dfrac{3}{2}}\cos\theta\right)^2 + \left(\dfrac{5}{2} + \sqrt{\dfrac{3}{2}}\sin\theta\right)^2$

$\Rightarrow \quad PQ^2 = \dfrac{25}{2} + \dfrac{3}{2} + 5\sqrt{3/2}(\cos\theta + \sin\theta)$

$= 14 + 5\sqrt{\dfrac{3}{2}}(\cos\theta + \sin\theta)$ $\quad$ ($\because$ $\text{Max}(\cos\theta + \sin\theta) = \sqrt{2}, \theta = 45°$)

Maximum value of $(PQ)^2 = 14 + 5\sqrt{\dfrac{3}{2}}(\sqrt{2}) = 14 + 5\sqrt{3}$

29. (a) : We have, $tx - 2y - 3t = 0$ ...(i)

and $x - 2ty + 3 = 0$ ...(ii)

Point of intersection of (i) and (ii) is

$y = \left(\dfrac{3t}{t^2-1}\right), x = 3\left(\dfrac{1+t^2}{t^2-1}\right) \quad \Rightarrow \quad \dfrac{x}{3} = \left(\dfrac{1+t^2}{t^2-1}\right), \dfrac{2y}{3} = \dfrac{2t}{t^2-1}$

$\therefore \quad \left(\dfrac{x}{3}\right)^2 - \left(\dfrac{2y}{3}\right)^2 = \dfrac{(1+t^2)^2 - (2t)^2}{(t^2-1)^2} = \dfrac{(t^2-1)^2}{(t^2-1)^2} = 1$

$\Rightarrow \quad \dfrac{x^2}{9} - \dfrac{y^2}{9/4} = 1$ represents a hyperbola with

$a^2 = 9$ and $b^2 = 9/4$

$\therefore$ Length of conjugate axis $= 2b = 3$

and $e = \dfrac{\sqrt{a^2+b^2}}{a} = \dfrac{\sqrt{9+\dfrac{9}{4}}}{3} = \dfrac{\sqrt{45}}{2\times 3} = \dfrac{3\sqrt{5}}{2\times 3} = \dfrac{\sqrt{5}}{2}$

30. (d) : Side of square $= 2$

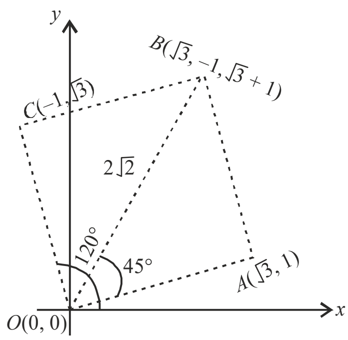
$\dfrac{x}{\cos 30°} = \dfrac{y}{\sin 30°} = 2$

$\Rightarrow \quad x = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$

and $y = 1$

$\dfrac{x}{\cos 120°} = \dfrac{y}{\sin 120°} = 2$

$\Rightarrow \quad x = -1,\ y = \sqrt{3}$



$\dfrac{x}{\cos 75°} = \dfrac{y}{\sin 75°} = 2\sqrt{2}$

$\Rightarrow \quad x = \sqrt{3} - 1$ and $y = \sqrt{3} + 1$

$\therefore \quad$ Required sum $= 0 + \sqrt{3} + \sqrt{3} - 1 + (-1) = 2\sqrt{3} - 2$

31. (a) : We have, $x^2 + y^2 = 9$

Let line through P, A and B make angle θ with x-axis.

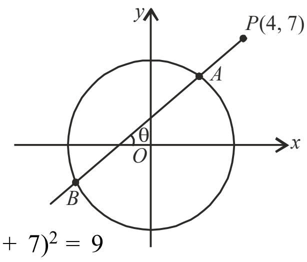
$\therefore \quad$ Equation of line is

$\dfrac{x-4}{\cos\theta} = \dfrac{y-7}{\sin\theta} = k\ (say)$

Any point on this line is

$(k\cos\theta + 4, k\sin\theta + 7)$

This point will also lie on the circle.



$\therefore \quad (k\cos\theta + 4)^2 + (k\sin\theta + 7)^2 = 9$

$\Rightarrow \quad k^2\cos^2\theta + 16 + 8k\cos\theta + k^2\sin^2\theta + 49 + 14k\sin\theta = 9$

$\Rightarrow \quad k^2 + k(8\cos\theta + 14\sin\theta) + 56 = 0$

$\therefore \quad k_1 \times k_2 = PA \cdot PB = \dfrac{56}{1} = 56$

32. (a) : Let equation of the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Ellipse passes through the points $(4, -1)$ and $(-2, 2)$.

$\therefore \quad \dfrac{16}{a^2} + \dfrac{1}{b^2} = 1 \Rightarrow 16b^2 + a^2 = a^2b^2$ ...(1)

And $\dfrac{4}{a^2} + \dfrac{4}{b^2} = 1$

$\Rightarrow \quad 4b^2 + 4a^2 = a^2b^2$ ...(2)

From (1) & (2), we get $16b^2 + a^2 = 4a^2 + 4b^2$

$\Rightarrow \quad 3a^2 = 12b^2 \Rightarrow a^2 = 4b^2$

Now, $e = \sqrt{1 - \dfrac{b^2}{a^2}} = \sqrt{1 - \dfrac{b^2}{4b^2}} = \sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2}$

33. (d) : If $y = mx + c$ is the normal to the parabola $y^2 = 8x$, then $c = at^3 + 2at$ and $m = -t$

Here, $a = 2$

Given, focal distance is 8 units. $\therefore (at^2 - a)^2 + 4a^2t^2 = 64$

$\Rightarrow \quad a^2[(t^2 - 1)^2 + 4t^2] = 64 \Rightarrow a^2(t^2 + 1)^2 = 64$

$\Rightarrow \quad (a(t^2 + 1)) = 8$

$\Rightarrow \quad t^2 + 1 = 4 \Rightarrow t^2 = 3 \quad \Rightarrow \quad t = \sqrt{3}$

Now, $c = at^3 + 2at = 2(\sqrt{3})^3 + 2(2)\sqrt{3} = 6\sqrt{3} + 4\sqrt{3} = 10\sqrt{3}$

$\therefore \quad |c| = 10\sqrt{3}$

34. (c) : Coordinates of $A \equiv (1, 2)$ $\therefore$ Slope of $AE = 2$

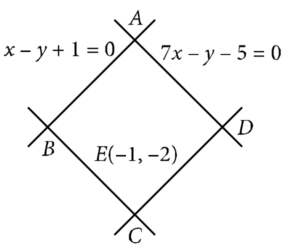
$\Rightarrow$]x{| q{ r $Xa = -\dfrac{6}{7}$

$\Rightarrow$ L}×mµ{ z { r Xa u–

$\dfrac{+7}{+6} = -\dfrac{6}{7}$

$\Rightarrow \quad x + 2y + 5 = 0$

$\therefore \quad$ Co-ordinates of $a = \left(\dfrac{6}{8} 1 \dfrac{-=}{8}\right)$



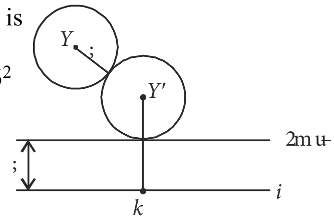
35. (d) : The equation of circle is $x^2 + y^2 - 8x - 8y - 4 = 0$

$\Rightarrow \quad (x-4)^2 + (y-4)^2 = 36 = 6^2$

$\therefore$ Radius = 6.

Consider a line 6 units below the x-axis.

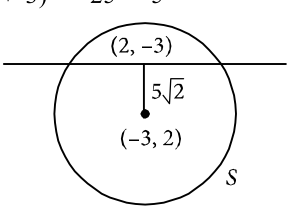
We have $C'C = C'N = r + 6$



Thus, the locus of C' is a parabola with C as focus and L as directrix.

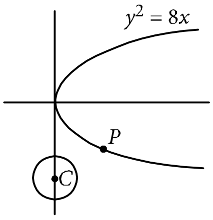
36. (b) : The circle is $x^2 + y^2 - 4x + 6y - 12 = 0$

$(x-2)^2 + (y+3)^2 = 25 = 5^2$



Now, radius of $p = \sqrt{7: +: 5} = \sqrt{<:} =: \sqrt{8}$

37. (a) : The geometry of the situation is as follows.



The point P must lie on the normal common to circle and parabola.

Let the normal be in parametric form.

$y + tx = 4t + 2t^3$

As it has to pass through $(0, -6)$,

we have, $t^3 + 2t + 3 = 0$ gives $(t+1)(t^2 - t + 3) = 0$

The only real value is $t = -1$.

So, point P becomes $P(2, -4)$. We have $Ym = 7\sqrt{7}$

The equation of circle is $(x-2)^2 + (y+4)^2 = -7\sqrt{7}.^7 ==$

i.e. $x^2 + y^2 - 4x + 8y + 12 = 0$

38. (c) : We have $2b = ae$ and $\frac{7^{\;7}}{} ==$

Also, we have $b^2 = a^2(e^2 - 1)$

Now, eliminating a and b from these equations

$\frac{7}{9} = {}^7 - 6 \Rightarrow 9 = 8^{\,7} \;\therefore\; = \frac{7}{\sqrt{8}}$ m E 5

39. (a) : Equation of circle with centre (h, k) and touches y-axis is given by $x^2 + y^2 - 2hx - 2ky + k^2 = 0$

Since, it touches y-axis at $(0, 2)$ $\therefore k = 2$

$\Rightarrow \quad x^2 + y^2 - 2hx - 4y + 4 = 0$

Also, it passes through $(-2, 4)$

$\therefore \quad (-2)^2 + 4^2 - 2h(-2) - 4(4) + 4 = 0$

$\Rightarrow \quad 4 + 16 + 4h - 16 + 4 = 0 \Rightarrow h = -2$

Hence, centre of circle is $(-2, 2)$

$(-2, 2)$ satisfy the equation given in option (a).

So, diameter of circle is $2x - 3y + 10 = 0$.

40. (a) : Given equations of lines can be written as $4x + 3y - 12 = 0$ and $3x + 4y - 12 = 0$

Equation of line passing through the intersection of these two lines is given by $(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$

$\Rightarrow \quad x(4 + 3\lambda) + y(3 + 4\lambda) - 12(1 + \lambda) = 0$

Above line meets the coordinate axes at points A and B.

Now, coordinates of point A are $\left(\frac{12(1+\lambda)}{4+3\lambda}, 0\right)$ and coordinates of point B are $\left(0, \frac{12(1+\lambda)}{3+4\lambda}\right)$

$\therefore$ Coordinates of mid-point of AB are given by

$h = \frac{6(1+\lambda)}{4+3\lambda}$ ...(i) and $k = \frac{6(1+\lambda)}{3+4\lambda}$ ...(ii)

Eliminating λ from (i) and (ii), we get, $6(h + k) = 7hk$

$\therefore$ Locus of the mid-point of AB is, $6(x + y) = 7xy$

41. (c) : Equation of ellipse is $\frac{x^2}{27} + \frac{y^2}{3} = 1$

$a^2 = 27,\ b^2 = 3$

$\therefore$ Equation of tangent to the above ellipse is,

$$\frac{x}{\sqrt{27}}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$$

Since, the tangent meets the coordinate axes at points A and B

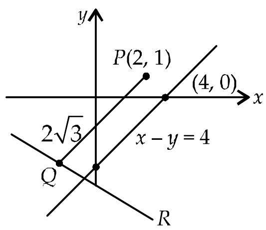
$\therefore$ Coordinates of point A are $\left(\frac{\sqrt{27}}{\cos\theta}, 0\right)$ and coordinates of point B are $\left(0, \frac{\sqrt{3}}{\sin\theta}\right)$

Area of $\Delta OAB = \frac{1}{2}\cdot\frac{\sqrt{27}}{\cos\theta}\cdot\frac{\sqrt{3}}{\sin\theta} = \frac{9}{\sin 2\theta}$ sq. units

Area will be minimum when $\sin 2\theta = 1$

$\therefore$ Minimum area = 9 sq. units

42. (d) : We have, $L : x - y = 4$



Now, slope of $L = 0$ is 1

Since, line L is perpendicular to QR

$\therefore$ Slope of $QR = -1$

Let equation of QR be

$y = mx + c$

$\Rightarrow \quad y = -x + c \Rightarrow x + y - c = 0$

Now, distance of QR from point $(2, 1)$ is $2\sqrt{3}$ units

$\therefore \quad 2\sqrt{3} = \frac{|2 + 1 - c|}{\sqrt{2}} \Rightarrow 2\sqrt{6} = |3 - c|$

$\Rightarrow \quad c - 3 = \pm 2\sqrt{6} \Rightarrow c = 3 \pm 2\sqrt{6}$

$\therefore$ Equation of required line is

$x + y = 3 + 2\sqrt{6}$ or $x + y = 3 - 2\sqrt{6}$

43. (a) : Any point on the curve $y = x^2 - 4$ is of the type $(\alpha, \alpha^2 - 4)$

Distance of point $(\alpha, \alpha^2 - 4)$ from the origin is, $D = \sqrt{\alpha^2 + (\alpha^2 - 4)^2}$

$D^2 = \alpha^4 - 7\alpha^2 + 16$

$\frac{dD^2}{d\alpha} = 4\alpha^3 - 14\alpha$ Now, $\frac{dD^2}{d\alpha} = 0 \Rightarrow 2\alpha(2\alpha^2 - 7) = 0$

$\Rightarrow \quad \alpha = 0$ or $\alpha^2 = \frac{7}{2}$. Again $\frac{d^2D^2}{d\alpha^2} = 12\alpha^2 - 14$

$\left(\frac{d^2D^2}{d\alpha^2}\right)_{\text{at }\alpha=0} = -14 < 0$ and $\left(\frac{d^2D^2}{d\alpha^2}\right)_{\text{at }\alpha^2=\frac{7}{2}} = 28 > 0$

$\therefore$ Distance is minimum at $\alpha^2 = \frac{7}{2}$

$\therefore$ Minimum distance, $D = \sqrt{\frac{49}{4} - \frac{49}{2} + 16} = \frac{\sqrt{15}}{2}$

44. (a) : $S(5, 0)$ is the focus $\therefore$ $ae = 5$ ...(i)

$5x = 9$ *i.e.*, $x = \dfrac{9}{5}$ is the directrix $\therefore$ $\dfrac{a}{e} = \dfrac{9}{5}$ ...(ii)

From (i) and (ii), we get $a = 3$ and $e = \dfrac{5}{3}$

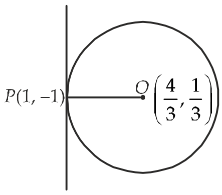
Now, $b^2 = a^2(e^2 - 1) = 9\left(\dfrac{25}{9} - 1\right) = 16$

Hence, $a^2 - b^2 = 9 - 16 = -7$

45. (a) : $t_1 = -t - \dfrac{2}{t}$, $t_1^2 = t^2 + \dfrac{4}{t^2} + 4$

Since, $t^2 + \dfrac{4}{t^2} \ge 2\sqrt{t^2 \cdot \dfrac{4}{t^2}} = 4$ $\therefore$ Minimum value at $t_1^2 = 8$

46. (a) : Point of intersection of lines $x - y = 1$ and $2x + y = 3$ is $O\left(\dfrac{4}{3}, \dfrac{1}{3}\right)$



Slope of $OP = \dfrac{\frac{1}{3} + 1}{\frac{4}{3} - 1} = \dfrac{\frac{4}{3}}{\frac{1}{3}} = 4$

$\therefore$ Slope of tangent $= -\dfrac{1}{4}$

So, equation of tangent at $P(1, -1)$ is $y + 1 = -\dfrac{1}{4}(x - 1)$

$\Rightarrow$ $4y + 4 = -x + 1 \Rightarrow x + 4y + 3 = 0$

47. (c) : Length of $\perp$ from $O(0, 0)$ to $4x + 3y = 10$ is

$$p_1 = \frac{|4(0) + 3(0) - 10|}{\sqrt{4^2 + 3^2}} = \frac{10}{5} = 2$$

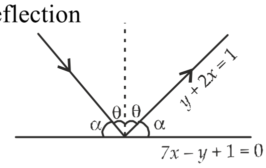
Length of $\perp$ from $O(0, 0)$ to $8x + 6y + 5 = 0$ is

$$p_2 = \frac{|8(0) + 6(0) + 5|}{\sqrt{8^2 + 6^2}} = \frac{5}{10} = \frac{1}{2}$$

Lines are parallel to each other $\Rightarrow$ ratio will be 4 : 1 or 1 : 4.

48. (c) : Let slope of incident ray be m.

Now angle of incidence = angle of reflection



$\therefore$ $\left|\dfrac{m - 7}{1 + 7m}\right| = \left|\dfrac{-2 - 7}{1 - 14}\right| = \dfrac{9}{13}$

$\Rightarrow$ $\dfrac{m - 7}{1 + 7m} = \dfrac{9}{13}$ or $\dfrac{m - 7}{1 + 7m} = -\dfrac{9}{13}$

$\Rightarrow$ $13m - 91 = 9 + 63m$ or $13m - 91 = -9 - 63m$

$\Rightarrow$ $50m = -100$ or $76m = 82$ $\Rightarrow$ $m = -2, m = \dfrac{41}{38}$

$\therefore$ Equations of incident line at (0, 1) are

$$y - 1 = -2(x - 0) \text{ or } y - 1 = \frac{41}{38}(x - 0)$$

i.e., $2x + y - 1 = 0$ or $38y - 38 - 41x = 0$

49. (c) : We have, $\dfrac{x^2}{12} + \dfrac{y^2}{16} = 1$ $\therefore$ $e = \sqrt{1 - \dfrac{12}{16}} = \dfrac{1}{2}$

$\Rightarrow$ Foci $\equiv$ (0, 2) & (0, –2)

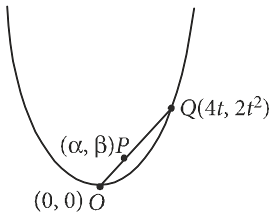
So, transverse axis of hyperbola $= 2b = 4 \Rightarrow b = 2$

& $a^2 = b^2(e^2 - 1)$ $\Rightarrow$ $a^2 = 4\left(\dfrac{9}{4} - 1\right)$ $\Rightarrow a^2 = 5$

$\therefore$ Required equation is $\dfrac{x^2}{5} - \dfrac{y^2}{4} = -1$

50. (b) : $OP : PQ = 1 : 3$

Let the parametric co-ordinates of Q be $(4t, 2t^2)$



We have, by section formula

$$\alpha = \frac{9 \ \ +5}{9} = \ \ \text{mzp}$$

$$\beta = \frac{7 \ ^7 +5}{9} = \frac{7}{7}$$

Eliminating 't', we get the locus of $P(\alpha, \beta)$ as $\alpha^2 = 2\beta$

Thus the locus is $x^2 = 2y$

51. (a) : The line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$ passes through the intersection of line

and $\left.\begin{matrix} 2x - 3y + 4 = 0 \\ x - 2y + 3 = 0 \end{matrix}\right\}$ (A) for different values of k.

Lines given by (A) meet at (1, 2)

Let image of $A(2, 3)$ in the family of lines be $B(\alpha, \beta)$.

Thus (1, 2) is a fixed point for given family of lines, we have

$AP = BP$ $\Rightarrow$ $(\alpha - 1)^2 + (\beta - 1)^2 = 2$

The locus generalises to $(x - 1)^2 + (y - 1)^2 = 2$

Thus it is a circle of radius $\sqrt{7}$.

52. (a) : The circles can be written as

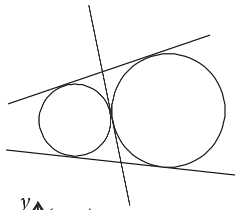
$(x - 2)^2 + (y - 3)^2 = 25$ with centre $O_1(2, 3)$ and $r_1 = 5$

and $(x + 3)^2 + (y + 9)^2 = 64$ with centre $O_2(-3, -9)$ and $r_2 = 8$

O_1O_2 = distance between centres

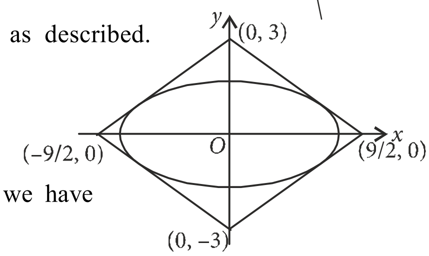
$$= \sqrt{:^7 + 67^7} = 68$$

As $r_1 + r_2 = O_1O_2$



We have that circle touch each other externally, so there are three common tangents as shown.

53. (b) : The ellipse is as described.



$$\frac{7}{>} + \frac{7}{:} = 6$$

$$\text{suq}- \ = 81 \ = \sqrt{:}$$

Using $b^2 = a^2(1 - e^2)$, we have

$$6 - \ ^7 = \frac{:}{>} \ \Rightarrow \ ^7 = \frac{9}{>}$$

$\Rightarrow$ $e = 2/3$

Equation of tangent at (2, 5/3) is

$$\frac{6}{7} + \frac{6}{7} = 6 \ \ 33\!\!1 \ \frac{\cdot 7}{>} + \frac{}{:} \cdot \left(\frac{:}{8}\right) = 6 \ \therefore \ \frac{7}{>} + \frac{}{8} = 6$$

The points of intersection with axes are (0, 3) and (9/2, 0) of the above line.

Using symmetry, picture can be completed.

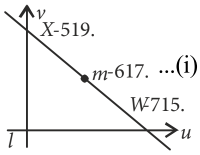
Area required $= \dfrac{6}{7}$-8.->47. $\times 9 = 7<$

54. (d) : The points are $W\left(51\dfrac{=}{8}\right)1$ X-618. mzp Y-=7185.

Slope of $AB = \dfrac{1}{3}$, Slope of $BC = \dfrac{1}{3}$

So, the given points lies on same line.

55. (a) : Equation of line L be



$\frac{}{7}+\frac{}{9}=6 \Rightarrow 2x + y = 4$ ...(i)

Equation of line L_1 perpendicular to (i)

$x - 2y = k$

Since it passes through (–2, 1), so $k = -4$

Equation of L_1, $x - 2y = -4$ ...(ii)

Solving (i) and (ii), we get point of intersection $\left(\frac{9}{:}1\frac{67}{:}\right)$

56. (b) : Given that $y + 3x = 0$

Equation of a chord of the circle is $y = -3x$ ...(i)

$\Rightarrow x^2 + (-3x)^2 - 30x = 0 \Rightarrow 10x^2 - 30x = 0$

$\Rightarrow 10x(x - 3) = 0 \Rightarrow x = 0, y = 0$

and $x = 3, y = -9$ are end points of diameter.

So the equation of the circle is $(x - 3)(x - 0) + (y + 9)(y - 0) = 0$

$\Rightarrow x^2 + y^2 - 3x + 9y = 0$

57. (b) : The equation of the tangent ($T = 0$)

would be $\frac{6}{7}$- +65. –; =7 $\Rightarrow 4x - y + 2 = 0$

The centre of circle is (–4, 1) and point of touch would be the foot of perpendicular from (–4, 1) on $4x - y + 2 = 0$

$\frac{+9}{9}=\frac{-6}{-6}=\frac{-–6;\ -6+7.}{9^7+6^7}$

$\Rightarrow \frac{+9}{9}=\frac{6:}{6<}1\frac{-6}{-6}=\frac{6:}{6<} \Rightarrow =\frac{-=}{6<}1 =\frac{7}{6<}$

58. (c) : The given hyperbola is $\frac{7}{9}-\frac{7}{>}=6$

Foci -$\pm\sqrt{68}$15. and $=\frac{\sqrt{68}}{7}$

Since the product of eccentricities is $\frac{6}{7}$

So, $6\times\frac{\sqrt{68}}{7}=\frac{6}{7} \Rightarrow 6=\frac{6}{\sqrt{68}}$

$\therefore$ Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a^2 = 13$

$=\sqrt{6-\frac{7}{7}} \Rightarrow {}^7=67$

Equation of ellipse is $\frac{7}{68}+\frac{7}{67}=6$ ...(i)

Substituting all the options in (i), we see that only option (c) does not lie on the ellipse.

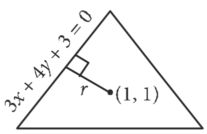
59. (b) : We have, µnz ; 5° $=\left|\frac{-–\sqrt{8}.}{6+\ -–\sqrt{8}.}\right|$

$\Rightarrow$ - $+\sqrt{8}.^7 = 8$ -6– $\sqrt{8}.^7 \Rightarrow$ $=5$ {~ $=\sqrt{8}$

$\therefore$ L}×nµ{z {r~q}×u-qp xz q u- $+7=\sqrt{8}\,(\ -8)$

33 $-\sqrt{8}\ +7+8\sqrt{8}=5$

60. (b) : In an equilateral triangle, incentre and circumcentre are same and $R = 2r$



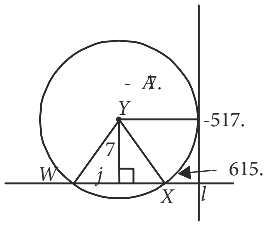
V{ 1 $=\frac{8+9+8}{\sqrt{>+6;}}=7$

$\Rightarrow R = 4$

$\therefore$ Equation of circumcircle is $(x - 1)^2 + (y - 1)^2 = 16$

$\Rightarrow x^2 + y^2 - 2x - 2y - 14 = 0$

61. (c) :



Since $BC = h$ (radius of circle) $\Rightarrow (h + 1)^2 + 2^2 = h^2$

$\Rightarrow 2h + 5 = 0 \Rightarrow h = \frac{-:}{7}$

Also, $AB = 2(AM) = 7\sqrt{\frac{7:\ -6;}{9}} = 3$

62. (c) : Given that 7 $=\frac{6}{7}\left(\frac{7\ ^7}{}\right)$

$\Rightarrow$ 7 $=\frac{7}{}$ $\Rightarrow$ 7 $=\frac{7}{7}$

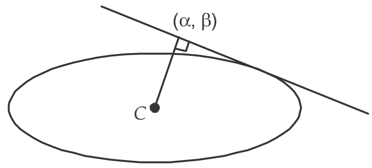
Hx{ 1 $^7=6-\frac{7}{7} \Rightarrow {}^7=6-7 \Rightarrow =\sqrt{7}-6$

63. (b) : Let m– $\frac{7}{6}$17 $_6$.1 n -– $\frac{7}{6}$1–7 $_6$. and $R(h, k)$

$\Rightarrow h = -at_1^2,\ k = \frac{-7\ \ 6}{8}$

$\Rightarrow 9k^2 = -4h \Rightarrow 9y^2 = -4x$ is required equation of locus

64. (b) : 1st solution:



The equation of tangent to the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ is

$$\frac{x\cos\theta}{\sqrt{6}}+\frac{y\sin\theta}{\sqrt{2}}=1 \qquad ...(1)$$

The equation to the tangent is also given by

$$y-\beta=-\frac{\alpha}{\beta}(x-\alpha) \quad i.e., \alpha x+\beta y=\alpha^2+\beta^2 \qquad ...(2)$$

Comparing (1) and (2), we get

$$\frac{\cos\theta}{\sqrt{6}\alpha}=\frac{\sin\theta}{\sqrt{2}\beta}=\frac{1}{\alpha^2+\beta^2}=\frac{\sqrt{\cos^2\theta+\sin^2\theta}}{\sqrt{6\alpha^2+2\beta^2}}$$

$\Rightarrow 6\alpha^2 + 2\beta^2 = (\alpha^2 + \beta^2)^2$

$\therefore$ Locus of (α, β) is $(x^2 + y^2)^2 = 6x^2 + 2y^2$

2nd solution : The equation of the tangent is $y = mx \pm \sqrt{6m^2+2}$

and slope m is given by $m=-\frac{\alpha}{\beta}$

Also, (α, β) lies on the tangent

$\Rightarrow \beta = m\alpha \pm \sqrt{6m^2+2} \Rightarrow \beta=\left(-\frac{\alpha}{\beta}\right)\alpha\pm\sqrt{\frac{6\alpha^2}{\beta^2}+2}$

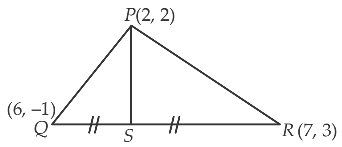
$$\Rightarrow \quad \frac{\alpha^2+\beta^2}{\beta} = \pm\sqrt{\frac{6\alpha^2+2\beta^2}{\beta^2}} \Rightarrow (\alpha^2+\beta^2)^2 = 6\alpha^2+2\beta^2$$

Then the locus is $(x^2 + y^2)^2 = 6x^2 + 2y^2$.

65. (b) : Let the point of intersection be $(\alpha, -\alpha)$. Then
$4a\alpha - 2a\alpha + c = 0$
$5b\alpha - 2b\alpha + d = 0$

We have, $-\frac{c}{2a} = -\frac{d}{3b} \Rightarrow 3bc - 2ad = 0$

66. (a) :



S is $\left(\frac{13}{2}, 1\right)$

Slope of $PS = -2/9$

The equation of line passing through $(1, -1)$ and parallel to PS is $y + 1 = -\frac{2}{9}(x-1) \Rightarrow 2x + 9y + 9 - 2 = 0 \therefore 2x + 9y + 7 = 0$

67. (d) : The equation of tangent at $P(t^2, 2t)$ is $x - yt + t^2 = 0$
$x^2 + 32y = 0$ is the given curve.

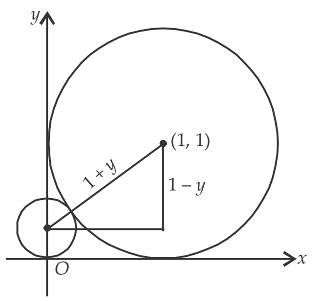
Eliminating y, $x^2 + \frac{32}{t}x + 32t = 0$ For tangent, discriminant $= 0$

$$\Rightarrow \left(\frac{32}{t}\right)^2 - 4(32t) = 0 \Rightarrow \frac{32}{t^2} - 4t = 0 \Rightarrow t^3 = 8 \Rightarrow t = 2$$

$\Rightarrow t^3 = 8 \quad \therefore \quad t = 2$

Slope of tangent is 1/2.

68. (c) : We have by Pythagoras theorem $(1 + y)^2 = (1 - y)^2 + 1$
$\Rightarrow \quad 4y = 1 \therefore \quad y = 1/4$



69. (b) : The system of circles touches the line $y = 0$ at the point $(3, 0)$ is given by $\{(x - 3)^2 + y^2)\} + \lambda y = 0$

As the circle passes through $(1, -2)$, we can determine λ which gives $4 + 4 - 2\lambda = 0 \quad \therefore \quad \lambda = 4$

The circle is $(x - 3)^2 + y^2 + 4y = 0$. A simple calculation shows that $(5, -2)$ lies on the circle.

70. (a) : Let a tangent to the parabola be $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$

As it is a tangent to the circle $x^2 + y^2 = 5/2$, we have

$$\left(\frac{\sqrt{5}}{m}\right) = \frac{\sqrt{5}}{\sqrt{2}}\sqrt{1+m^2} \Rightarrow (1+m^2)m^2 = 2$$

which gives $m^4 + m^2 - 2 = 0 \quad \Rightarrow \quad (m^2 - 1)(m^2 + 2) = 0$

As $m \in R$, $m^2 = 1 \quad \therefore \quad m = \pm 1$

Also $m = \pm 1$ does satisfy $m^4 - 3m^2 + 2 = 0$

Hence common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$

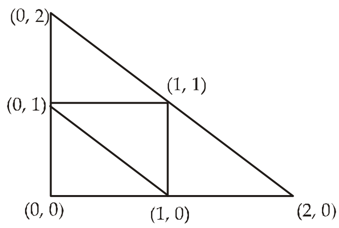
71. (a) : As the slope of incident ray is $-\frac{1}{\sqrt{3}}$. So, the slope of reflected ray has to be $\frac{1}{\sqrt{3}}$.

The point of incidence is $(\sqrt{3}, 0)$. Hence, the equation of reflected ray is $y = \frac{1}{\sqrt{3}}(x - \sqrt{3}) \Rightarrow \sqrt{3}y - x = -\sqrt{3} \Rightarrow x - \sqrt{3}y - \sqrt{3} = 0$

72. (d) : Foci are given by $(\pm ae, 0)$

As $a^2e^2 = a^2 - b^2 = 7$ we have equation of circle as
$(x - 0)^2 + (y - 3)^2 = (\sqrt{7} - 0)^2 + (0 - 3)^2 \therefore x^2 + y^2 - 6y - 7 = 0$

73. (a) : The triangle whose side's midpoints are given to be (0, 1), (1, 0) and (1, 1) happen to be a right angled triangle with vertices as shown.



1st solution : x-coordinate of incentre

$$= \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \frac{2\times 2 + 2\sqrt{2}\times 0 + 2\times 0}{2 + 2 + 2\sqrt{2}}$$

$$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$$

2nd solution : $r = (s-a)\tan\frac{A}{2} = \left(\frac{4+2\sqrt{2}}{2} - 2\sqrt{2}\right)\tan\frac{\pi}{4} = 2 - \sqrt{2}$

74. (a) : $A\ (1, 1)$; $B\ (2, 4)$

$P(x_1, y_1)$ divides line segment AB in the ratio 3 : 2

$$x_1 = \frac{3(2)+2(1)}{5} = \frac{8}{5};\ y_1 = \frac{3(4)+2(1)}{5} = \frac{14}{5}$$

$2x + y = k$ passes through $P(x_1, y_1) \therefore 2\times\frac{8}{5} + \frac{14}{5} = k \Rightarrow k = 6$

75. (d) : Statement 1 : $y^2 = 16\sqrt{3}x$, $y = mx + \frac{4\sqrt{3}}{m}$

$$\frac{x^2}{2} + \frac{y^2}{4} = 1,\ x = m_1 y + \sqrt{4m_1^2 + 2} \Rightarrow y = \frac{x}{m_1} - \sqrt{4 + \frac{2}{m_1^2}} \quad m = \frac{1}{m_1}$$

Now, $\left(\frac{4\sqrt{3}}{m}\right)^2 = \left(-\sqrt{4+\frac{2}{m_1^2}}\right)^2$

$$\Rightarrow \frac{48}{m^2} = 4 + \frac{2}{m_1^2} = 4 + 2m^2$$

$$\Rightarrow \frac{24}{m^2} = 2 + m^2$$

$\Rightarrow \quad m^4 + 2m^2 - 24 = 0$ ...(1)

$\Rightarrow \quad (m^2 + 6)(m^2 - 4) = 0 \Rightarrow m = \pm 2$

Statement 2 : If $y = mx + \frac{4\sqrt{3}}{m}$ is a common tangent to $y^2 = 16\sqrt{3}x$ and ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 - 24 = 0$

From (1), statement 2 is a correct explanation for statement 1.

76. (c) : Let the equation of the circle is $(x-1)^2+(y-k)^2=k^2$. It passes through (2, 3) $\therefore$ $1+9+k^2-6k=k^2$

$\Rightarrow k=\frac{5}{3} \Rightarrow$ diameter $=\frac{10}{3}$

77. (b) : $(x-1)^2+y^2=1,\ r=1 \Rightarrow a=2$
and $x^2+(y-2)^2=4,\ r=2 \Rightarrow b=4$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{4}=1 \Rightarrow x^2+4y^2=16$

78. (a) : $y=mx+c \Rightarrow 2=m+c$
Co-ordinates of P & Q : $P(0, c)$, $Q(-c/m, 0)$

$$\frac{1}{2}\times|c|\times\left|\frac{c}{m}\right|=A \Rightarrow \frac{c^2}{2m}=A$$

$$\Rightarrow \frac{(2-m)^2}{2m}=A \Rightarrow \frac{m^2-4m+4}{2m}=A \Rightarrow \frac{m}{2}-2+\frac{2}{m}=A$$

$\because \frac{dA}{dm}=0$

$$\Rightarrow \frac{1}{2}-\frac{2}{m^2}=0 \Rightarrow \frac{1}{2}=\frac{2}{m^2} \Rightarrow m^2=4 \Rightarrow m=\pm 2$$

79. (d) : The centres and radii are

$$\left(x-\frac{a}{2}\right)^2+y^2=\frac{a^2}{4},\quad x^2+y^2=c^2$$

Centre $\left(\frac{a}{2},0\right)$ and (0, 0) & radius $=\frac{a}{2}$ and c

$$\sqrt{\left(\frac{a}{2}\right)^2+(0-0)}=\left|\left|\frac{a}{2}\right|\pm c\right| \Rightarrow \left|\frac{a}{2}\right|=\left|\left|\frac{a}{2}\right|\pm c\right|$$

$$\Rightarrow \left|\frac{a}{2}\right|=c-\left|\frac{a}{2}\right|.\ \therefore\ |a|=c.$$

80. (a) : In triangle OPQ, O divides PQ in the ratio of $OP:OQ$ which is $2\sqrt{2}:\sqrt{5}$ but it fails to divide triangle into two similar triangles.

81. (d) : Let P be (y^2, y)

Perpendicular distance from P to $x-y+1=0$ is $\frac{|y^2-y+1|}{\sqrt{2}}$

As $|y^2-y+1|=y^2-y+1$ ($\because\ y^2-y+1>0$)

Minimum value $=\frac{1}{\sqrt{2}}\cdot\frac{(4ac-b^2)}{4a}=\frac{1}{\sqrt{2}}\cdot\frac{4-1}{4\cdot 1}=\frac{3}{4\sqrt{2}}$

82. (c) : Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$(-3, 1)$ lies on it $\Rightarrow \frac{9}{a^2}+\frac{1}{b^2}=1$

Also, $b^2=a^2\left(1-\frac{2}{5}\right) \Rightarrow 5b^2=3a^2$

On solving, we get $a^2=\frac{32}{3},\ b^2=\frac{32}{5}$

The equation to ellipse becomes $3x^2+5y^2=32$

83. (d) : On differentiating, the given equation $\frac{dy}{dx}=1-\frac{8}{x^3}$

As the tangent is parallel to x-axis, we have

$$1-\frac{8}{x^3}=0 \Rightarrow x^3=8 \Rightarrow x=2 \text{ So, } y=2+\frac{4}{2^2}=2+1=3$$

Thus (2, 3) is the point of contact and equation of the tangent is $y=3$.

84. (c) : From a property of the parabola, the perpendicular tangents intersect at the directrix.
The equation of directrix is $x=-1$, hence this is the locus of point P.

85. (d) : As the line passes through (13, 32), we have

$$\frac{13}{5}+\frac{32}{b}=1 \Rightarrow \frac{32}{b}=1-\frac{13}{5}=-\frac{8}{5} \Rightarrow b=-20$$

Thus the line is $\frac{x}{5}-\frac{y}{20}=1$, *i.e.*, $4x-y=20$

The equation of line parallel to $4x-y=20$ has slope 4.

Thus $-\frac{3}{c}=4.\ \therefore\ c=-\frac{3}{4}.$

Then the equation to line k is $4x-y=-3$

The distance between lines k and c is $\frac{20+3}{\sqrt{4^2+1^2}}=\frac{23}{\sqrt{17}}$

86. (b) : The circle is $x^2+y^2-4x-8y-5=0$

$\Rightarrow (x-2)^2+(y-4)^2=5^2$

Length of perpendicular from centre (2, 4) on the line $3x-4y-m=0$ should be less than radius.

$$\Rightarrow \frac{|6-16-m|}{5}<5 \Rightarrow (10+m)<25$$

$\Rightarrow -25<10+m<25 \Rightarrow -35<m<15$

87. (a) : Let P be a general point (x, y) such that

$\frac{PM}{PN}=\frac{1}{3}$ where $M\equiv(1, 0)$ and $N\equiv(-1, 0)$

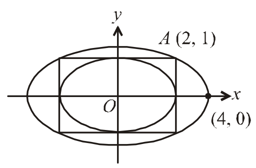
we have $\frac{\sqrt{(x-1)^2+y^2}}{\sqrt{(x+1)^2+y^2}}=\frac{1}{3} \Rightarrow 9[(x-1)^2+y^2]=(x+1)^2+y^2$

which reduces to $8x^2+8y^2-20x+8=0$

$$\Rightarrow x^2+y^2-\frac{10}{4}x+1=0 \Rightarrow x^2+y^2-\frac{5}{2}x+1=0$$

The locus is a circle with centre (5/4, 0)
As points A, B, C lie on this circle, the circumcentre of triangle ABC is (5/4, 0).

88. (a) : The given ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1$
i.e., the point A, the corner of the rectangle in 1st quadrant, is (2, 1). Again the ellipse circumscribing the rectangle passes through the point (4, 0), so its equation is $\frac{x^2}{16}+\frac{y^2}{b^2}=1$



$A(2, 1)$ lies on the above ellipse

$$\Rightarrow \frac{4}{16}+\frac{1}{b^2}=1 \Rightarrow \frac{1}{b^2}=1-\frac{1}{4}=\frac{3}{4} \Rightarrow b^2=4/3$$

Thus, the equation to the desired ellipse is

$$\frac{x^2}{16}+\frac{3}{4}y^2=1 \Rightarrow x^2+12y^2=16$$

89. (a) : The radical axis, which in the case of intersection of the circles is the common chord, of the circles

$S_1 : x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and

$S_2 : x^2 + y^2 + 2x + 2y - p^2 = 0$ is $S_1 - S_2 = 0$

$\Rightarrow \quad x + 5y + 2p - 5 + p^2 = 0$ ...(i)

If there is a circle passing through P, Q and (1, 1) it's necessary and sufficient that (1, 1) doesn't lie on PQ, *i.e.*, $1 + 5 + 2p - 5 + p^2 \neq 0$

$\Rightarrow \quad p^2 + 2p + 1 \neq 0 \Rightarrow \quad (p + 1)^2 \neq 0 \quad \therefore \quad p \neq -1$

Thus for all values of p except '–1' there is a circle passing through P, Q and (1, 1).

90. (b) : Obviously the major axis is along the x-axis

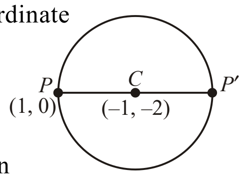
The distance between the focus and the corresponding directrix

$= \left|\frac{a}{e} - ae\right| = 4 \quad \Rightarrow \quad \frac{a}{e} - ae = 4$ (note that $\frac{a}{e} > ae$)

$\Rightarrow \; a\left(\frac{1}{e} - e\right) = 4 \Rightarrow \; a\left(2 - \frac{1}{2}\right) = 4 \Rightarrow \; a \cdot \frac{3}{2} = 4 \therefore \; a = \frac{8}{3}$

Remark : The question should have read "*The corresponding directrix*" in place of "*the directrix*".

91. (d) : The centre C of the circle is seen to be (– 1, – 2). As C is the mid point of P and P', the coordinate of P' is given by



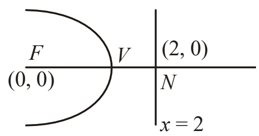
$P' \equiv (2 \times -1 - 1, 2 \times -2 - 0)$

$\equiv (-3, -4)$

Remark : If P be (α, β) and $C(h, k)$ then

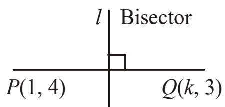
$P' \equiv (2h - \alpha, 2k - \beta)$

92. (c) : The vertex is the mid point of FN, that is, vertex = (1, 0)



93. (a) : The slope of l = $-\dfrac{1}{\text{the slope of the original line } PQ}$

$= -\dfrac{1}{\dfrac{3-4}{k-1}} = (k-1)$



The midpoint = $\left(\frac{k+1}{2}, \frac{7}{2}\right)$

The equation to the bisector l is $\left(y - \frac{7}{2}\right) = (k-1)\left(x - \frac{k+1}{2}\right)$

As $x = 0$, $y = -4$ satisfies it, we have

$\left(-4 - \frac{7}{2}\right) = (k-1)\left(0 - \frac{k+1}{2}\right) \Rightarrow \; -\frac{15}{2} = -\frac{k^2-1}{2}$

$\Rightarrow k^2 - 1 = 15 \quad \Rightarrow \quad k^2 = 16 \quad \therefore \quad k = \pm 4.$

94. (b, c) : Equation of normal at $P(x, y)$ is

$$Y - y = -\frac{dx}{dy}(X - x) \quad \Rightarrow G \equiv \left(x + y \cdot \frac{dy}{dx}, 0\right)$$

$\left|x + y\frac{dy}{dx}\right| = |2x| \;\Rightarrow\; y\frac{dy}{dx} = x \;$ or $\; y\frac{dy}{dx} = -3x$

$ydy = xdx$ or $ydy = -3xdx$

After integrating, we get $\frac{y^2}{2} = \frac{x^2}{2} + c$ or $\frac{y^2}{2} = -\frac{3x^2}{2} + c$

$\Rightarrow x^2 - y^2 = -2c$ or $3x^2 + y^2 = 2c \Rightarrow x^2 - y^2 = c_1$ or $3x^2 + y^2 = c_2$.

95. (d) : Equation of circle $(x - h)^2 + (y - k)^2 = k^2$

It is passing through (–1, 1) then

$(-1 - h)^2 + (1 - k)^2 = k^2 \Rightarrow h^2 + 2h - 2k + 2 = 0, D \geq 0$

$2k - 1 \geq 0 \Rightarrow k \geq \frac{1}{2}.$

96. (a) : Sum of the slopes $= -\dfrac{\text{co-efficient of } xy}{\text{co-efficient of } y^2}$

$\therefore$ Sum of slopes $= -\dfrac{(1-m^2)}{m} = 0 \;\Rightarrow\; m = \pm 1.$

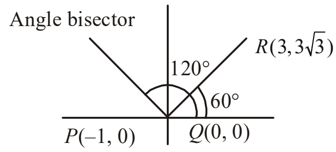
Second method

Equation of bisectors of lines $xy = 0$ are $y = \pm x$

Put $y = \pm x$ in $my^2 + (1 - m^2)xy - mx^2 = 0$, we get

$(1 - m^2)x^2 = 0 \Rightarrow m = \pm 1.$

97. (c) : Slope of the required angle bisector is



$\tan 120° = -\sqrt{3}$

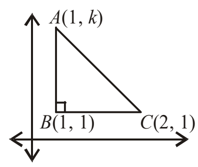
Hence equation of the angle bisector is

$y = -\sqrt{3}(x - 0)$

$\Rightarrow \sqrt{3}x + y = 0$

98. (a) : $\frac{1}{2} \times |k - 1| \times 1 = 1$

$k = -1, 3.$



99. (b) : Let P is the required point, then P lies on directrix $x = -2$ of $y^2 = 8x$

Hence $P \equiv (-2, 0)$.

100. (b) : $\because \; b^2 = a^2(e^2 - 1) \Rightarrow \sin^2\alpha = \cos^2\alpha\,(e^2 - 1)$

$\Rightarrow \tan^2\alpha + 1 = e^2 \Rightarrow e^2 = \sec^2\alpha$

Vertices $\equiv (\pm\cos\alpha, 0)$

Coordinate of focii are $(\pm ae, 0) \equiv (\pm 1, 0)$

$\Rightarrow$ if α varies then the abscissa of foci remain constant.

101. (c) : Given lines are $y = \frac{x}{2}(x > 0)$ and $y = 3x$ $(x > 0)$ using (a, a^2) in these lines

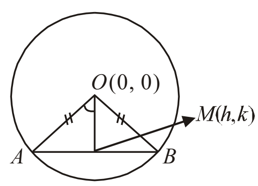
$a^2 - \frac{a}{2} > 0$ ... (i) and $a^2 - 3a < 0$ ... (ii)

Solving (i) and (ii) we get $\frac{1}{2} < a < 3$

102. (d) : Let AB is chord of circle and $M(h, k)$ be mid point of AB & $\angle AOM = 60°$

Now $OA = OB = 3$ and

$OM \perp AB$ (By properties of circle)



Now, $OA = \sqrt{h^2 + k^2}$, $OM = r\cos\theta$

$\sqrt{h^2 + k^2} = 3\cos 60°$

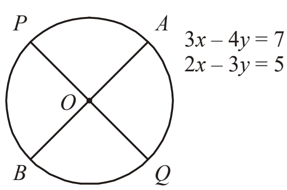
$\sqrt{h^2 + k^2} = \frac{3}{2} \;\Rightarrow\; h^2 + k^2 = \frac{9}{4} \;\Rightarrow\; x^2 + y^2 = \frac{9}{4}$

103. (d) : Let $OA = r$

Given area = 49π

$\Rightarrow \quad \pi r^2 = 49\pi$

$r = 7$



Point of intersection of AB and PQ is (1, –1)

$\therefore$ equation of circle is $(x - 1)^2 + (y + 1)^2 = 7^2$

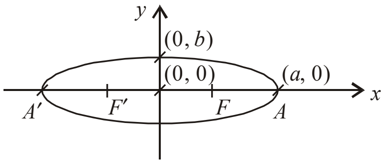
$\Rightarrow \quad x^2 + y^2 - 2x + 2y - 47 = 0$

104. (a) : Let $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\ (a>b)$

Given $2b = 8$ ... (i) and $2ae = 6$ ... (ii)

By (i) and (ii) we have

$\dfrac{b}{ae}=\dfrac{4}{3}$



$\Rightarrow\ \dfrac{b^2}{a^2}=\dfrac{16}{9}e^2$

$\Rightarrow\ 1-e^2=\dfrac{16}{9}e^2$ $\qquad (\because\ b^2 = a^2(1-e^2)$ as $a > b)$

$\Rightarrow\ e=\dfrac{3}{5}$

105. (a) : Let h, k be the locus of the vertex of family of parabola

$$y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$$

$\therefore\ k=\dfrac{a^3h^2}{3}+\dfrac{a^2h}{2}=2a\ \Rightarrow\ \dfrac{3k}{a^3}=h^2+\dfrac{3h}{2a}-\dfrac{6}{a^2}$

$\Rightarrow\ \dfrac{3}{a^3}\left(k+\dfrac{35a}{16}\right)=\left(h+\dfrac{3}{4a}\right)^2$

i.e., $\left\{x^2=\dfrac{3}{a^3}y,\ \text{where } x=h+\dfrac{3}{4a}, y=k+\dfrac{35a}{16}\right\}$

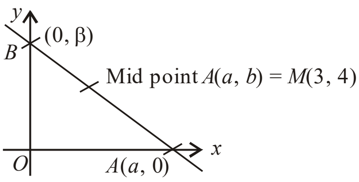
$\Rightarrow$ vertex is $\left(\dfrac{-3}{4a},\dfrac{-35a}{16}\right)\ \therefore\ hk=\left(\dfrac{-3}{4a}\right)\left(\dfrac{-35a}{16}\right)\ \Rightarrow\ hk=\dfrac{105}{64}$

$\Rightarrow\ xy=\dfrac{105}{64}$ $\qquad$ (taking $h = x$, $k = y$)

106. (c) : Now the equation of line which meet the x-axis and y-axis at $A(\alpha, 0)$, $B(0, \beta)$ is given by $\dfrac{x}{\alpha}+\dfrac{y}{\beta}=1$

where $\alpha = 2a = 6$

and $\beta = 2b = 8$



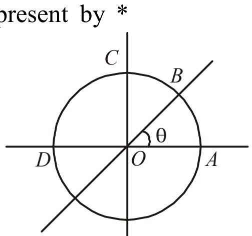
$\therefore$ Required equation is $\dfrac{x}{6}+\dfrac{y}{8}=1\ \Rightarrow\ 4x + 3y = 24$

107. (c) : $ax^2+2(a+b)xy+by^2=0$ ....(*)

Let θ be the angle between the lines represent by *

$\therefore\ \tan\theta=\dfrac{2\sqrt{h^2-ab}}{a+b}$

$\tan\theta=\left|\dfrac{2\sqrt{(a+b)^2-ab}}{a+b}\right|$



$\{3 \text{ area } \overset{\frown}{OAB} = \text{ Area of } \overset{\frown}{DBC}\}$

Now, $\theta = 45°$ ($\because$ area of one sector = 3 time the area of another sector)

$\therefore\ \tan 45° = \left|\dfrac{2\sqrt{(a+b)^2-ab}}{a+b}\right|\ \Rightarrow 3a^2 + 2ab + 3b^2 = 0.$

108. (c) : Given $y = \alpha x + \beta$ and $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

$\therefore\ b^2x^2-a^2y^2=a^2b^2$

$\Rightarrow\ b^2x^2-a^2(\alpha x+\beta)^2=a^2b^2$ (by using $y = \alpha x + \beta$)

$\Rightarrow\ x^2(b^2-a^2\alpha^2)-2a^2\alpha\beta x+(-\beta^2a^2-a^2b^2)=0$ ...(*)

Now the line $y = \alpha x + \beta$ will be tangent to circle if both roots of (*) are equal $\therefore$ keeping $D = 0$ in (*) we have

$4\alpha^2a^4\beta^2=4(b^2-a^2\alpha^2)(-\beta^2a^2-a^2b^2)$

$\Rightarrow\ \alpha^2a^2\beta^2=(b^2-a^2\alpha^2)(-\beta^2-b^2)$

$\Rightarrow\ \alpha^2a^2\beta^2=-b^2\beta^2+\beta^2a^2\alpha^2-b^4+a^2\alpha^2b^2$

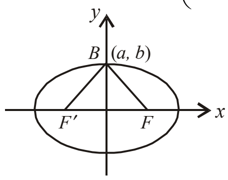
$\Rightarrow\ a^2\alpha^2b^2=b^2(b^2+\beta^2)\Rightarrow a^2\alpha^2=b^2+\beta^2\Rightarrow a^2x^2-y^2=b^2.$

109. (b) : $F'(-ae, 0)$, $F(ae, 0)$

Slope of $BF=\dfrac{b}{ae}=m_1$ (say), Slope of $BF'=\dfrac{b}{-ae}=m_2$ (say)

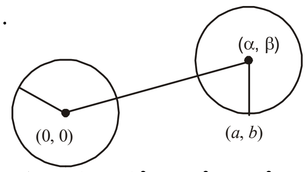
Now $m_1\times m_2=-1\ \Rightarrow\ \dfrac{b}{ae}\times\dfrac{b}{-ae}=-1$

$\Rightarrow\ b^2=a^2e^2\ \Rightarrow\ a^2-a^2e^2=a^2e^2$ $\qquad \left(\because\ e^2=1-\dfrac{b^2}{a^2}\right)$



$\Rightarrow\ 1-e^2=e^2,\ 2e^2=1,\ e=\pm\dfrac{1}{\sqrt{2}}.$

110. (c) : Let locus of the centre of circle be (α, β). If C_1, C_2 are centres of the circles with radii r_1, r_2 respectively then $(C_1C_2)^2=r_1^2+r_2^2$

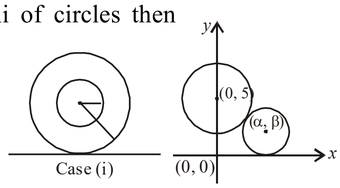


$\Rightarrow\ \alpha^2+\beta^2=p^2+(\alpha-a)^2+(\beta-b)^2$

$\Rightarrow\ p^2+a^2+b^2-2a\alpha-2b\beta=0$

$\Rightarrow\ 2ax+2by-(a^2+b^2+p^2)=0.$

111. (c) : Let locus of centre be α, β then according to given, if r_1, r_2 are radii of circles then



Internal touch. This case does not exist as centre of circle is (0, 3) and radius is 2.

$C_1C_2=r_2\pm r_1\ \Rightarrow\ \sqrt{(\alpha-0)^2+(\beta-3)^2}=|\beta\pm 2|$

$\Rightarrow\ \alpha^2+\beta^2-6\beta+9=\beta^2+4+4\beta$

and $\alpha^2+\beta^2-6\beta+9=\beta^2-4\beta+4$

$\Rightarrow\ \alpha^2-10\beta+5=0$ and $x^2=2\beta+5$

$\Rightarrow\ x^2=10y-5$ and $x^2=2y-5$

Both are parabolas but $x^2 = 2y - 5$ does not exist.

112. (a) : As the line passes through P and Q which are the point of intersection of two circles. It means given line is the equation of common chord and the equation of common chord of two intersecting circle is $S_1 - S_2 = 0 = 5ax + (c-d)y + a + 1 = 0$.

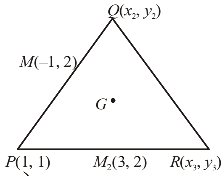
Now $5ax + (c-d)y + a + 1 = 0$ and $5x + by - a = 0$ represent same equation.

$\therefore\ \dfrac{5a}{5}=\dfrac{c-d}{b}=\dfrac{a+1}{-a}\ \Rightarrow a^2+a+1=0$ and $\dfrac{c-d}{b}+1=-\dfrac{1}{a}$

$\Rightarrow\ \left(a+\dfrac{1}{2}\right)^2+\dfrac{3}{4}=0$ and $-(c-d+b)=b/a$

$d-b-c=+b/a$ has no solution. $\therefore$ No value of a exist.

113. (d) : $\because$ $x_2 = 2(-1) - 1 = -3$



$y_2 = 2 \times 2 - 1 = 3$

$x_3 = 3 \times 2 - 1 = 5$

$y_3 = 2 \times 2 - 1 = 3$

$\therefore$ Centroid $G = \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\dfrac{1-3+5}{3}, \dfrac{1+3+3}{3}\right) = \left(1, \dfrac{7}{3}\right).$

114. (d) : Let us take the two set of values of $a = 1$, $b = 1/2$, $c = 1/3$ and $a = 1/2$, $b = 1/3$, $c = 1/4$

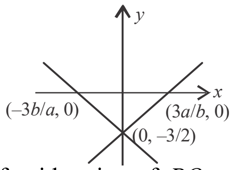
Putting these value in the given equation, we get

$x + 2y + 3 = 0$ and $2x + 3y + 4 = 0$ ...(*)

Solving the equations of (*) we have $x = 1$, $y = -2$

$(1, -2)$ is required point on the line.

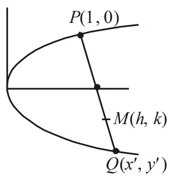
115. (b) : Intercepts made by the lines with co-ordinate axis is



$(-3b/a, 0)$, $(0, -3/2)$ and $(3a/b, 0)$.

Common intercept is $(0, -3/2)$.

116. (d) : Let $M(h, k)$ be point of locus of mid point of PQ.



$\Rightarrow \dfrac{x'+1}{2} = h, \dfrac{y'+0}{2} = k,$

$\therefore$ $x' = 2h - 1$, $y' = 2k$

Now (x', y') lies on $y^2 = 8x$

$\Rightarrow$ $(2k)^2 = 8(2h - 1)$

$\Rightarrow$ $y^2 = 2(2x - 1) \Rightarrow y^2 - 4x + 2 = 0.$

117. (b) : Equation of directrix $x = 4$ which is parallel to y-axis so axis of the ellipse is x-axis. Let equation of ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$

Again $e = 1/2$ and $e^2 = 1 - \dfrac{b^2}{a^2} \Rightarrow \left(\dfrac{b}{a}\right)^2 = 1 - 1/4 = 3/4$ ...(*)

Also the equation of one directrix is $x = 4$

$\therefore$ Equation of directrix $x = \dfrac{a}{e}$ $\therefore$ $4 = \dfrac{a}{e}$

$\Rightarrow$ $a = 2$ $(\because e = 1/2)$

Further, $b^2 = \dfrac{a^2 \times 3}{4}$ (by (*)) $\Rightarrow b^2 = \dfrac{4 \times 3}{4} = 3$

Hence, equation of ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$ or $3x^2 + 4y^2 = 12$

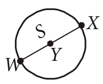
118. (c) : The point of intersection of parabola's $y^2 = 4ax$ and $x^2 = 4ay$ are $A(0, 0)$, $B(4a, 4a)$ as the line $2bx + 3cy + 4d = 0$ passes through these points

$\therefore$ $d = 0$ and $2b(4a) + 3c(4a) = 0$

$\Rightarrow 2b + 3c = 0$ $\Rightarrow (2b + 3c)^2 + d^2 = 0$

119. (c) : Given circle $x^2 + y^2 - 2x = 0$ ... (1)

and line be $y = x$ ... (2)



Solving (1) and (2) we get $x = 0, 1$

$\therefore$ $y = 0, 1$

$\therefore$ $A(0, 0)$, $B(1, 1)$ and equation of circle in the diameter form is $(x - 0)(x - 1) + (y - 0)(y - 1) = 0 \Rightarrow x^2 + y^2 - (x + y) = 0$

120. (c) : As per given condition centre of the circle is the point of intersection of the $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$

$\therefore$ centre is $(1, -1)$

Also circumference of the circle is given $2\pi r = 10\pi$ $\therefore$ $r = 5$

$\therefore$ Required equation of circle is

$(x - 1)^2 + (y + 1)^2 = 5^2$ or $x^2 + y^2 - 2x + 2y - 23 = 0$

121. (c) : Equation of circle with AB as diameter is given by

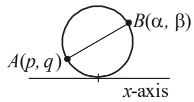
$(x - p)(x - \alpha) + (y - q)(y - \beta) = 0$

$\Rightarrow x^2 + y^2 - x(p + \alpha) - y(q + \beta) + p\alpha + q\beta = 0$ ...(1)

Now (1) touches axis of x so put $y = 0$ in (1) we have

$x^2 - x(p + \alpha) + p\alpha + q\beta = 0$ ...(2)

and $D = 0$ in equation (2)



$\therefore$ $(p + \alpha)^2 = 4[p\alpha + q\beta]$

$\Rightarrow (p - \alpha)^2 = 4q\beta$

Now $\alpha \to x$, $\beta \to y$

$\therefore$ $(p - x)^2 = 4q(y)$ which is required locus of one end point of the diameter.

122. (b) : Let the equation of circle cuts orthogonally the circle $x^2 + y^2 = 4$ is $x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

$\therefore$ $2g_1g_2 + 2f_1f_2 = c_1c_2$ (where $(-g, -f)$ are point of locus)

$\Rightarrow c = -4$

Again circle (i) passes through (a, b), so

$a^2 + b^2 + 2ga + 2fb + 4 = 0$

Now replacing g, f by x, y respectively

$\therefore$ $2ax + 2by - (a^2 + b^2 + 4) = 0$

123. (d) : The equation $ax^2 + 2hxy + by^2 = 0 = (y - m_1x)(y - m_2x)$

$\Rightarrow$ $m_1 + m_2 = -\dfrac{2h}{b} = \dfrac{1}{4c}$ ...(*)

$m_1m_2 = \dfrac{3}{2}c$ and $3x + 4y = 0 \Rightarrow m_1 = -3/4$ $\therefore$ $m_2 = -\dfrac{2}{c}$

Now, by (*) we have $-\left(\dfrac{3}{4} + \dfrac{2}{c}\right) = \dfrac{1}{4c} \Rightarrow -\dfrac{3}{4} = \dfrac{1}{4c} + \dfrac{2}{c}$

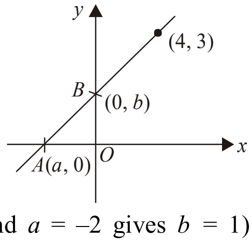
$\Rightarrow -\dfrac{3}{4} = \dfrac{1}{4c} + \dfrac{8}{4c} \Rightarrow -\dfrac{3}{4} = \dfrac{9}{4c}$ $\therefore$ $c = -3$

124. (a) : If m_1 and m_2 are slope of the lines then by given condition $m_1 + m_2 = 4m_1m_2 \Rightarrow -\dfrac{2c}{7} = -\dfrac{4}{7} \Rightarrow c = 2$

By using $ax^2 + 2hxy + by^2 = 0$

$\Rightarrow$ $m_1 + m_2 = \dfrac{-2h}{b}$ and $m_1m_2 = \dfrac{a}{b}$

125. (d) : Given $OA + OB = -1$



i.e. $a + b = -1$

$\therefore$ equation of the line be

$\dfrac{x}{a} - \dfrac{y}{1+a} = 1 \Rightarrow \dfrac{4}{a} - \dfrac{3}{1+a} = 1$

$\Rightarrow$ $a = \pm 2$ (as $a = 2$ gives $b = -3$ and $a = -2$ gives $b = 1$)

So, equation are $\dfrac{x}{2} - \dfrac{y}{3} = 1$ and $\dfrac{x}{-2} + \dfrac{y}{1} = 1$

126. (c) : Let locus of point $C(h, k)$ and centroid (α, β)

As (α, β) lies on $2x + 3y = 1$ $\therefore$ $2\alpha + 3\beta = 1$

Now, centroid of ΔABC is $\left(\dfrac{2+(-2)+h}{3}, \dfrac{-3+1+k}{3}\right)$

or $\left(\frac{h}{3},\frac{k-2}{3}\right)$ $\therefore\ 2\left(\frac{h}{3}\right)+\frac{3(k-2)}{3}=1$

$\Rightarrow\ 2h+3k=9 \Rightarrow\ 2x+3y=9$

127. (c) : The equation of normal at θ is $y-y_1=-\frac{1}{\frac{dy}{dx}}(x-x_1)$

$\Rightarrow\ y-a\sin\theta=\frac{\sin\theta}{\cos\theta}\,(x-a(1-\cos\theta))$

which passes through $(a,\ 0)$

128. (d) : Given $y^2=18x$ and $\frac{dy}{dt}=2\frac{dx}{dt}$ $\therefore\ 2y\frac{dy}{dt}=18\frac{dx}{dt}$

$\Rightarrow 2y\frac{dy}{dt}=\frac{18}{2}\frac{dy}{dt} \Rightarrow y=9/2\ \ \therefore\ x=\frac{y^2}{18}=\frac{81}{72}=\frac{9}{8}$

So, the required point is $\left(x=\frac{9}{8},\ y=9/2\right)$

129. (d) : Let α, β is the point of locus, equidistant from (a_1, b_1) and (a_2, b_2) is given by

$(\alpha-a_1)^2+(\beta-b_1)^2=(\alpha-a_2)^2+(\beta-b_2)^2$

$\Rightarrow a_1^2+b_1^2-2a_1\alpha-2b_1\beta-a_2^2-b_2^2+2a_2\alpha+2b_2\beta=0$

$\Rightarrow 2(a_2-a_1)\alpha+2(b_2-b_1)\beta+a_1^2+b_1^2-b_2^2-a_2^2=0$

$\Rightarrow (a_2-a_1)x+(b_2-b_1)y+\frac{1}{2}(a_1^2+b_1^2-a_2^2-b_2^2)=0$

$\Rightarrow c=-\frac{1}{2}[a_1^2+b_1^2-a_2^2-b_2^2]$

130. (a) : Let (h, k) be the co-ordinate of centroid

$\therefore\ \ h=\frac{a\cos t+b\sin t+1}{3},\ k=\frac{a\sin t-b\cos t+0}{3}$

$\Rightarrow\ 3h-1=a\cos t+b\sin t$ ...(i)

$3k=a\sin t-b\cos t$ ...(ii)

squaring (i) and (ii) then adding, we get

$(3h-1)^2+(3k)^2=a^2(\cos^2 t+\sin^2 t)+b^2(\cos^2 t+\sin^2 t)$

Replacing (h, k) by (x, y) we get choice (a) is correct.

131. (c) : Given equations are

$x^2-2qxy-y^2=0$ ...(1) and $x^2-2pxy-y^2=0$ ...(2)

Joint equation of angle bisector of the line (1) and (2) are same

$\therefore\ \ \frac{x^2-y^2}{1+1}=\frac{xy}{-q} \Rightarrow qx^2+2xy-qy^2=0$ ...(3)

Now, (2) and (3) are same, taking ratio of their coefficients

$\therefore\ \ \frac{1}{q}=\frac{-p}{1}\ \ \Rightarrow\ \ pq=-1$

132. (c) : According to the problem square lies above x-axis Now equation of AB using two point form. We get

$y-y_1=m(x-x_1)$

$(y-a\sin\alpha)=-\frac{(\cos\alpha-\sin\alpha)}{(\cos\alpha+\sin\alpha)}[x-a\cos\alpha]$

$\Rightarrow y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)$

$=a\sin\alpha(\cos\alpha+\sin\alpha)+a\cos\alpha\,(\cos\alpha-\sin\alpha)$

$=a(\sin^2\alpha+\cos^2\alpha)=a(1)$

133. (b): Co-ordinate of centre may be $(1, -1)$ or $(-1, 1)$ but $1, -1$ satisfies the given equations of diameter, so choices (a) and (d) are out of court.

Again $\pi R^2=154,\ R^2=49\ \therefore\ R=7$

$\therefore$ Required equation of circle be $(x-1)^2+(y+1)^2=49$

$\Rightarrow x^2+y^2-2x+2y\ \ =47$

134. (d) : $(x-1)^2+(y-3)^2=r^2\ \therefore\ C_1(1, 3)$ and $r_1=t_1=r$

$(x-4)^2+1(y+1)^2=9\ \therefore\ C_2(4, -1)$ and $r_2=t_2=3$

so $C_1C_2=\sqrt{(4-1)^2+(3+1)^2}=5$

Now, for intersecting circles $r_1+r_2>C_1C_2$ and $|r_1-r_2|<C_1C_2$

$\Rightarrow\ \ r+3>5$ and $|r-3|<5$

$\Rightarrow\ \ r>2$ and $-5<r-3<5$

$\Rightarrow\ \ r>2$ and $-2<r<8 \Rightarrow r\in(2, 8)$

135. (d) : Since the normal at $(bt_1^2, 2bt_1)$, on parabola $y^2=4bx$ meet the parabola again at $(bt_2^2, 2bt_2)$

$\therefore\ \ t_1x+y=2bt_1+bt_1^3$ passes through $(bt_2^2, 2bt_2)$

$\Rightarrow\ \ t_1bt_2^2+2bt_2=2bt_1+bt_1^3 \Rightarrow t_1(t_2^2-t_1^2)=2(t_1-t_2)$

$\Rightarrow\ \ t_1(t_2+t_1)=-2$

$\Rightarrow t_2+t_1=-\frac{2}{t_1}\ \Rightarrow\ t_2=-\frac{2}{t_1}-t_1$

136. (b) : Eccentricity for $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $b^2=a^2(1-e^2)$

and eccentricity for $\frac{x^2}{\frac{144}{25}}-\frac{y^2}{\frac{81}{25}}=1$ is $e_1=\frac{a_1^2+b_1^2}{a_1^2}$

$\therefore\ e_1=\sqrt{1+\frac{81}{144}}=\frac{15}{12}$

Again foci $=a_1e_1=\frac{12}{5}\times\frac{15}{12}=3$

$\therefore$ focus of hyperbola is $(3, 0)=(ae, 0)$

So, focus of ellipse $(ae, 0)=(4e, 0)$

As their foci are same $\therefore\ 4e=3\ \ \ \therefore\ e=3/4$

$\therefore\ e^2=1-\left(\frac{b}{a}\right)^2=1-\frac{b^2}{16}$ or $\frac{b^2}{16}=1-e^2=1-\frac{9}{16}$

$\Rightarrow\ \ b^2=7$

137. (a) : $AB=\sqrt{26}$, $AC=\sqrt{26}$

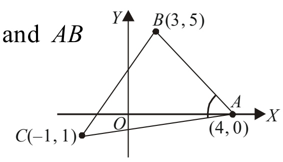
$\therefore\ \ ABC$ is isosceles

Again product of the slope of AC and AB

$=\frac{1}{5}\times(-5)=-1$

$\Rightarrow\ \ AC\perp AB$

$\Rightarrow\ \ $ right angled at A



138. (c) : Given median of the equilateral triangle is $3a$.

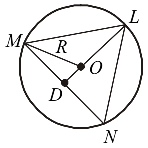
In $\Delta\ LMD$, $(LM)^2=(LD)^2+(MD)^2$

$(LM)^2=9a^2+\left(\frac{LM}{2}\right)^2$

$\Rightarrow \frac{3}{4}(LM)^2=9a^2\ \ \ \ \therefore\ (LM)^2=12a^2$

Again in triangle OMD, $(OM)^2=(OD)^2+(MD)^2$

$$R^2 = (3a - R)^2 + \left(\frac{LM}{2}\right)^2$$



$\Rightarrow R^2 = 9a^2 + R^2 - 6aR + 3a^2 \Rightarrow 6aR = 12a^2$

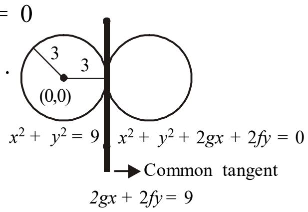
$R = 2a$

So, equation of circle be

$(x - 0)^2 + (y - 0)^2 = R^2 = (2a)^2 \Rightarrow x^2 + y^2 = 4a^2$

139. (b) : Let equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$. As it passes through (0, 0) so, $c = 0$

and as it passes (1, 0) so, $-g = \frac{1}{2}$.



Now $x^2 + y^2 + 2gx + 2fy = 0$

and $x^2 + y^2 = 9$ touches each other.

$\therefore$ Equation of common tangent is

$2gx + 2fy - 9 = 0$

and distance from the centre of circle $x^2 + y^2 = 9$ to the common tangent is equal to the radius of the circle $x^2 + y^2 = 9$

$$\therefore \frac{|0 + 0 - 9|}{\sqrt{4g^2 + 4f^2}} = 3 \Rightarrow 9^2 = 4(g^2 + f^2)\ 9 = 4\left(\frac{1}{4} + f^2\right)$$

$\Rightarrow 9 = 1 + 4f^2$

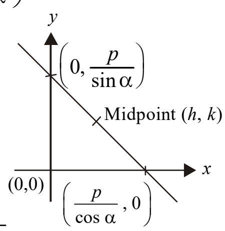
$\therefore f^2 = 2 \therefore f = \pm\sqrt{2}$

$\therefore$ Centre of the required circle be $\left(\frac{1}{2}, \sqrt{2}\right), \left(\frac{1}{2}, -\sqrt{2}\right)$

140. (d) : $\because$ (h, k) is $\left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right)$

$\therefore \cos\alpha = \frac{p}{2h}$,

$\sin\alpha = \frac{p}{2k}$



$\Rightarrow \sin^2\alpha + \cos^2\alpha$

$$= \frac{p^2}{4h^2} + \frac{p^2}{4k^2} \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$$

141. (a) : Pair of lines $Ax^2 + 2hxy + By^2 = 0$ are $\perp$ to each other if $A + B = 0$

$\Rightarrow 3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$

$\Rightarrow$ There exist two value of a as $D > 0$

$$\therefore a = \frac{-3 \pm \sqrt{17}}{2}$$

142. (a) : Let (α, β) is the centre of the circle whose radius is 3.

$\therefore$ Equation of such circle be $(x - \alpha)^2 + (y - \beta)^2 = 3^2$

$\Rightarrow \alpha^2 + \beta^2 - 2\alpha x - 2\beta y + 25 = 9$

$\Rightarrow x^2 + y^2 - 2x^2 - 2y^2 + 25 = 9 \Rightarrow x^2 + y^2 = 25 - 9$

$\Rightarrow x^2 + y^2 = 16$ and $x^2 + y^2 = 25 \Rightarrow 4 \le x^2 + y^2 \le 64$

143. (a) : As $s = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of line

$$\therefore \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

or $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ...(1)

Now say point of intersection on y axis be $(0, y_1)$ and point of intersection of pair of line be obtained by solving the equations

$$\frac{\partial s}{\partial x} = 0 = \frac{\partial s}{\partial y} \quad \therefore \quad \frac{\partial s}{\partial x} = 0 \Rightarrow ax + by + g = 0$$

$$\Rightarrow \frac{\partial s}{\partial y} = 0 \Rightarrow hx + by + f = 0 \Rightarrow \begin{cases} hy_1 + g = 0 \\ by_1 + f = 0 \end{cases} \quad (*)$$

On comparing the equation given in (*) we get

$bg = fh$ and $bg^2 = fgh$ ...(2)

Again $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

meet at y-axis $\quad\therefore x = 0$

$\Rightarrow by^2 + 2fy + c = 0$ whose roots must be equal $\therefore f^2 = bc$

$af^2 = abc$ ...(3)

Now using (2) and (3) in equation (1) we have

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow (abc - af^2) + (fgh - bg^2) + fgh - ch^2 = 0$

$\Rightarrow 0 + 0 + fgh - ch^2 = 0$

$\therefore ch^2 = fgh$ ...(4)

Now, adding (2) and (4) $2fgh = ch^2 + bg^2$

144. (c) : Equation of chord $y = mx + 1$

Equation of circle $x^2 + y^2 = 1$

Joint equation of the curve through the intersection of line and circle be given by $x^2 + y^2 = (y - mx)^2$,

$\Rightarrow x^2(1 - m^2) + 2mxy = 0$

$$\text{Now } \tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b} \text{ where } \begin{cases} a = 1 - m^2 \\ h = m, b = 0 \end{cases}$$

$$\tan 45^\circ = \pm\frac{2\sqrt{m^2 - 0}}{1 - m^2}$$

$\Rightarrow 1(1 - m^2) = \pm 2m \Rightarrow m^2 \pm 2m - 1 = 0 \Rightarrow m = \pm 1 \pm \sqrt{2}$

$\Rightarrow m = 1 \pm \sqrt{2}$ and $-1 \pm \sqrt{2}$

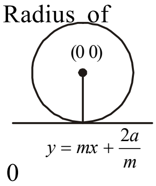
145. (b) : Let common tangent to the curves be $y = mx + c$ ...(1)

$= mx + \frac{a}{m}$ and $y^2 = 8ax = 4(2a)x$

$\therefore$ Equation of tangent to parabola $y = mx + \frac{2a}{m}$ ...(2)

which is also tangent to the circle $x^2 + y^2 = 2a^2 = (\sqrt{2}a)^2$

Now distance from (0, 0) to the tangent line = Radius of circle



$$\therefore \sqrt{2}\,a = \pm\frac{2a}{m} \times \frac{1}{\sqrt{1 + m^2}}$$

$\Rightarrow m^2(1 + m^2) - 2 = 0 \Rightarrow (m^2 - 1)(m^2 + 2) = 0$

$\Rightarrow m = \pm 1$

Required equation of tangent $y = mx + \frac{2a}{m} = \pm(x + 2a)$