# CHAPTER 11 Differential Equations

1. Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(a) $-\frac{4}{9}\pi^2$ (b) $\frac{4}{9\sqrt{3}}\pi^2$ (c) $\frac{-8}{9\sqrt{3}}\pi^2$ (d) $-\frac{8}{9}\pi^2$

*(2018)*

2. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1, & x \in [0, 1] \\ 0, & \text{otherwise} \end{cases}$.
If $y(0) = 0$, then $y(3/2)$ is :

(a) $\frac{e^2 - 1}{2e^3}$ (b) $\frac{e^2 - 1}{e^3}$ (c) $\frac{e^2 + 1}{2e^4}$ (d) $\frac{1}{2e}$

*(Online 2018)*

3. The curve satisfying the differential equation, $(x^2 - y^2)dx + 2xydy = 0$ and passing through the point (1, 1) is

(a) A hyperbola (b) A circle of radius two
(c) A circle of radius one (d) An ellipse

*(Online 2018)*

4. The differential equation representing the family of ellipses having foci either on the $x$-axis or on the $y$-axis, centre at the origin and passing through the point (0, 3) is :

(a) $x + yy'' = 0$ (b) $xyy' - y^2 + 9 = 0$
(c) $xyy'' + x(y')^2 - yy' = 0$ (d) $xyy' + y^2 - 9 = 0$

*(Online 2018)*

5. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$,

then $y\left(\frac{\pi}{2}\right)$ is equal to

(a) $-\frac{2}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{1}{3}$ *(2017)*

6. The curve satisfying the differential equation, $ydx - (x + 3y^2)dy = 0$ and passing through the point (1, 1), also passes through the point

(a) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, -\frac{1}{3}\right)$ (d) $\left(-\frac{1}{3}, \frac{1}{3}\right)$ *(Online 2017)*

7. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)dx = xdy$, then $\left(-\frac{6}{7}\right)$ is equal to

(a) $-\frac{7}{:}$ (b) $-\frac{9}{:}$ (c) $\frac{7}{:}$ (d) $\frac{9}{:}$

*(2016)*

8. The solution of the differential equation
$\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$,
and $y(0) = 1$, is given by

(a) $y^2 = 1 + \frac{x}{\sec x + \tan x}$ (b) $y = 1 + \frac{x}{\sec x + \tan x}$
(c) $y = 1 - \frac{x}{\sec x + \tan x}$ (d) $y^2 = 1 - \frac{x}{\sec x + \tan x}$

*(Online 2016)*

9. Let $y(x)$ be the solution of the differential equation

$(x \log x)$ —+ $=7$ log $x$, $(x \geq 1)$

Then $y(e)$ is equal to

(a) 2 (b) $2e$
(c) $e$ (d) 0 *(2015)*

10. If $y(x)$ is the solution of the differential equation

- +7.—= 7 +9 −>, $x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to

(a) 0 (b) 1
(c) −1 (d) 2 *(Online 2015)*

11. The solution of the differential equation $ydx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to

(a) 4 (b) 3
(c) 2 (d) 1 *(Online 2015)*

12. Let the population of rabbits surviving at a time $t$ be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$.
If $p(0) = 100$ then $p(t)$ equals

(a) $300 - 200\, e^{-t/2}$ (b) $600 - 500\, e^{t/2}$
(c) $400 - 300\, e^{-t/2}$ (d) $400 - 300\, e^{t/2}$ *(2014)*

**13.** Solution of the differential equation

$\cos x dy = y(\sin x - y)dx$, $0 < x < \pi/2$ is

(a) $\sec x = (\tan x + c)y$ (b) $y \sec x = \tan x + c$
(c) $y \tan x = \sec x + c$ (d) $\tan x = (\sec x + c)y$ *(2010)*

**14.** The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where $c_1$ and $c_2$ are arbitrary constants, is

(a) $y'' = y'y$ (b) $yy'' = y'$
(c) $yy'' = (y')^2$ (d) $y' = y^2$ *(2009)*

**15.** The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

(a) $y = x \ln x + x$ (b) $y = \ln x + x$
(c) $y = x \ln x + x^2$ (d) $y = x e^{(x-1)}$ *(2008)*

**16.** The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

(a) $(x-2)^2 y'^2 = 25 - (y-2)^2$
(b) $(x-2) y'^2 = 25 - (y-2)^2$
(c) $(y-2) y'^2 = 25 - (y-2)^2$
(d) $(y-2)^2 y'^2 = 25 - (y-2)^2$ *(2008)*

**17.** The differential equation of all circles passing through the origin and having their centres on the $x$-axis is

(a) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (b) $y^2 = x^2 - 2xy\frac{dy}{dx}$
(c) $x^2 = y^2 + xy\frac{dy}{dx}$ (d) $x^2 = y^2 + 3xy\frac{dy}{dx}$ *(2007)*

**18.** The differential equation whose solution is $Ax^2 + By^2 = 1$, where $A$ and $B$ are arbitrary constants is of

(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree *(2006)*

**19.** If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(a) $x\log\left(\frac{y}{x}\right) = cy$ (b) $y\log\left(\frac{x}{y}\right) = cx$
(c) $\log\left(\frac{x}{y}\right) = cy$ (d) $\log\left(\frac{y}{x}\right) = cx$ *(2005)*

**20.** The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows

(a) order 1, degree 1 (b) order 1, degree 2
(c) order 2, degree 2 (d) order 1, degree 3 *(2005)*

**21.** The solution of the differential equation $ydx + (x + x^2y)dy = 0$ is

(a) $\frac{1}{xy} + \log y = C$ (b) $-\frac{1}{xy} + \log y = C$
(c) $-\frac{1}{xy} = C$ (d) $\log y = Cx$ *(2004)*

**22.** The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where $a$ is an arbitrary constant is

(a) $(x^2 - y^2)y' = 2xy$ (b) $2(x^2 + y^2)y' = xy$
(c) $2(x^2 - y^2)y' = xy$ (d) $(x^2 + y^2)y' = 2xy$ *(2004)*

**23.** If $x = e^{y + e^{y + \ldots \text{to } \infty}}$, $x > 0$ then $\frac{dy}{dx}$ is

(a) $\frac{1-x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{x}{1+x}$ (d) $\frac{1+x}{x}$ *(2004)*

**24.** The solution of the differential equation

$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ is

(a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$ (b) $xe^{\tan^{-1} y} = \tan^{-1} y + k$
(c) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$ (d) $(x-2) = ke^{-\tan^{-1} y}$ *(2003)*

**25.** The degree and order of the differential equation of the family of all parabolas whose axis is $x$-axis, are respectively

(a) 1, 2 (b) 3, 2
(c) 2, 3 (d) 2, 1 (2003)

**26.** The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are

(a) $1, \frac{2}{3}$ (b) 3, 1 (c) 3, 3 (d) 1, 2 *(2002)*

**27.** The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$

(a) $\frac{1}{4}e^{-2x}$ (b) $\frac{1}{4}e^{-2x} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-2x} + c + d$ *(2002)*

**ANSWER KEY**

**1.** (d) **2.** (a) **3.** (c) **4.** (b) **5.** (d) **6.** (d) **7.** (d) **8.** (d) **9.** (a) **10.** (a) **11.** (b) **12.** (d)
**13.** (a) **14.** (c) **15.** (a) **16.** (d) **17.** (a) **18.** (d) **19.** (d) **20.** (d) **21.** (b) **22.** (a) **23.** (a) **24.** (a)
**25.** (a) **26.** (c) **27.** (b)

# Explanations

**1.** **(d) :** $\frac{dy}{dx}+(\cot x)y=4x\text{cosec}x$

$\text{I.F.}=e^{\int\cot x\,dx}=e^{\log(\sin x)}=\sin x$

Then the solution is given by $y\cdot\sin x=\int 4x\,\text{cosec}(x)\sin x\,dx+C$

*i.e.* $y\sin x = 2x^2 + C$

As $y(\pi/2) = 0$, we have $C = -\pi^2/2$

So, $y\sin x = 2x^2 - \pi^2/2$

$\therefore\ y(\pi/6)=2\left\{\frac{2\pi^2}{36}-\frac{\pi^2}{2}\right\}=2\pi^2\left\{\frac{1}{18}-\frac{1}{2}\right\}=-\frac{8}{9}\pi^2$

**2.** **(a) :** We have, $\frac{dy}{dx}+2y=f(x)$. It is a linear differential equation.

$\therefore$ I.F. $=e^{\int 2dx}=e^{2x}$

$\therefore$ The required solution is $y\times(e^{2x})=\int_0^x f(x)\times e^{2x}dx+c$ ...(i)

$\Rightarrow\ y=e^{-2x}\int_0^x f(x)\times e^{2x}dx+ce^{-2x}$

Now, $y(0) = 0 \Rightarrow c = 0$

$\therefore$ Solution becomes $y(x)=e^{-2x}\int_0^x f(x)\times e^{2x}dx$

Now, $y\left(\frac{3}{2}\right)=e^{-3}\int_0^{3/2}f(x)e^{2x}dx=e^{-3}\left[\int_0^1 f(x)e^{2x}dx+\int_1^{3/2}f(x)e^{2x}dx\right]$

$=e^{-3}\left[\int_0^1 e^{2x}dx+0\right]=e^{-3}\left|\frac{e^{2x}}{2}\right|_0^1=\frac{e^{-3}}{2}(e^2-1)=\frac{e^2-1}{2e^3}$

**3.** **(c) :** The given curve is $(x^2-y^2)dx+2xydy=0$

$\Rightarrow\ \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$, Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore\ v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2x^2v}\Rightarrow v+x\cdot\frac{dv}{dx}=\frac{v^2-1}{2v}$

$\Rightarrow x\cdot\frac{dv}{dx}=\frac{v^2-1-2v^2}{2v}=\frac{-(v^2+1)}{2v}\Rightarrow\left(\frac{2v}{v^2+1}\right)dv=-\frac{1}{x}\cdot dx$

$\Rightarrow\ \log(v^2+1)=-\log x+\log C$

$\Rightarrow\ \log(v^2+1)=\log\frac{c}{x}\Rightarrow x(v^2+1)=C$

$\Rightarrow\ x\left(\frac{y^2}{x^2}+1\right)=C$

Now, the curves passes through (1, 1) $\therefore\ 1(1+1)=C\Rightarrow C=2$

$\therefore$ Required equation of curve is $\frac{y^2}{x}+x=2\ \Rightarrow\ y^2+x^2=2x$

$\Rightarrow\ x^2+y^2-2x=0\Rightarrow(x-1)^2+(y-0)^2=(1)^2$

**4.** **(b) :** Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Since ellipse passes through point (0, 3).

So, $0+\frac{9}{b^2}=1\Rightarrow b^2=9\ \therefore\ \frac{x^2}{a^2}+\frac{y^2}{9}=1$ ...(i)

Differentiating (i) w.r.t. '$x$', we have

$\frac{2x}{a^2}+\frac{2y}{9}\cdot\frac{dy}{dx}=0\Rightarrow\frac{x}{a^2}=-\frac{y}{9}\cdot\frac{dy}{dx}\Rightarrow\frac{x}{a^2}=-\frac{y}{9}y'$

$\Rightarrow\ \frac{1}{a^2}=-\frac{y}{9x}y'$ ...(ii)

Using (ii) in (i), we get $x^2\left(-\frac{y}{9x}y'\right)+\frac{y^2}{9}=1$

$\Rightarrow\ -xyy'+y^2=9\Rightarrow xyy'-y^2+9=0$

**5.** **(d) :** We have $\frac{dy}{dx}=-\frac{(y+1)\cos x}{2+\sin x}$

$\int\frac{dy}{y+1}=-\int\frac{\cos x}{2+\sin x}dx$

$\Rightarrow\ \ln(y+1)=-\ln(2+\sin x)+\ln\lambda\Rightarrow(y+1)(2+\sin x)=\lambda$

As $y(0) = 1 \Rightarrow 2\cdot2=\lambda$ or $\lambda = 4$

At $x=\frac{\pi}{2}, y\left(\frac{\pi}{2}\right)=\frac{4}{2+1}-1=\frac{4}{3}-1=\frac{1}{3}.$

**6.** **(d) :** We have, $ydx-(x+3y^2)dy=0$

$\Rightarrow\ ydx=(x+3y^2)dy\Rightarrow\frac{dx}{dy}=\frac{x}{y}+3y\Rightarrow\frac{dx}{dy}-\frac{x}{y}=3y$

This is homogeneous linear differential equation.

$\therefore\ \text{I.F.}=e^{-\int\frac{1}{y}dy}=e^{-\ln y}=\frac{1}{y}$

$\therefore$ Solution is, $\frac{x}{y}=\int 3y\cdot\frac{1}{y}dy\ \Rightarrow\ \frac{x}{y}=3y+c$ ...(i)

Since (i) passes through (1, 1) $\therefore$ $1=3+c\Rightarrow c=-2$

$\therefore$ Required curve is $x=3y^2-2y$

This curve also passes through the point $\left(-\frac{1}{3},\frac{1}{3}\right)$.

**7.** **(d) :** The differential equation can be rewritten as

$xdy=ydx+xy^2dx\Rightarrow\frac{xdy-ydx}{y^2}=xdx$

On integrating, we get, $-\frac{x}{y}=\frac{x^2}{2}+C$

As the curve passes through (1, –1), we have $1=\frac{1}{2}+C\quad\therefore\ C=\frac{1}{2}$

Now the curve $f(x)=x^2+1+\frac{2x}{y}=0$

$\Rightarrow\ y=-\frac{2x}{1+x^2}\ \therefore\ f\left(-\frac{1}{2}\right)=\frac{-2(-1/2)}{1+\frac{1}{4}}=\frac{4}{5}$

**8.** **(d) :** We have, $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$

$\Rightarrow\ 2y\frac{dy}{dx}+y^2\sec x=\tan x$ ...(i)

Put $y^2 = t \Rightarrow 2y\dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\therefore$ Equation (i) becomes, $\dfrac{dt}{dx} + t\sec x = \tan x$

I.F. $= e^{\int \sec x dx} = e^{\ln(\sec x + \tan x)} = \sec x + \tan x$

$\therefore$ Solution is given by

$t(\sec x + \tan x) = \int \tan x(\sec x + \tan x)dx$

$\Rightarrow$ $t(\sec x + \tan x) = \sec x + \tan x - x + c$

$\Rightarrow$ $t = 1 - \dfrac{x}{\sec x + \tan x} + c$ $\Rightarrow$ $y^2 = 1 - \dfrac{x}{\sec x + \tan x} + c$

Now, $y(0) = 1 \Rightarrow$ $1 = 1 - 0 + c \Rightarrow c = 0$

$\therefore$ Particular solution is $^7 = 6 - \dfrac{}{\text{-qo} \;\; +\text{µnz}}$

**9.** **(a) :** The equation can be written as $— + \left(\dfrac{6}{\text{xz}}\right) \;\; = 7$

It is linear in $y$. Thus PMD $\int\frac{}{\text{xz}}$ = $^{\text{xz-xz}}$ $\cdot$ = xz

The solution is $y = 1 - \dfrac{x}{\sec x + \tan x}$

$\cdot$PMD $\int 7 \cdot$ PM $+\lambda$ 331 -xz . $= \int 7$-xz $. + \lambda$

$= 2x(\ln x - 1) + \lambda$

At $x = 1$, we have $\lambda = 2$

The solution become $y \ln x = 2x(\ln x - 1) + 2$

Set $x = e$ in the above to obtain $y = 2e(\ln e - 1) + 2 = 2$

The value of $y$ at $x = e$, *i.e.* $y(e) = 2$

**10.** **(a) :** - $+7.$ — = $^7 + 9$ –>1 $\neq -7$

$\Rightarrow$ $= \dfrac{^7 + 9 \;\; ->}{+7}$ $\Rightarrow$ $\int$ $= \int \dfrac{^7 + 9 \;\; ->}{+7}$

$\Rightarrow$ $= \int\left( +7 - \dfrac{68}{+7}\right)$ $\Rightarrow$ $= \int$- $+7.$ $-68\int\dfrac{6}{+7}$

$\Rightarrow$ $= \dfrac{7}{7} + 7$ $-68$x{s $+7$ +

Given that $y(0) = 0 \Rightarrow 0 = -13\log 2 + c$

$\Rightarrow$ $= \dfrac{7}{7} + 7$ $-68$x{s $+7$ $+68$x{s 7

$\Rightarrow$ $y(-4) = 8 - 8 - 13\log 2 + 13\log 2 = 0$

**11.** **(b) :** We have, –- $+7$ $^7$.— $= 5$

$\Rightarrow$ $=$- $+7$ $^7$.— $\Rightarrow$ — $=$ $+7$ $^7$

$\Rightarrow$ $— + \left(-\dfrac{6}{}\right) = 7$ ...(i)

PM= $\int -\frac{6}{}$ $=$ $^{-\text{xz}}$ $=$ $^{-6} = \dfrac{6}{}$

$\therefore$ ^tq-{xxµ{z {rµtqq}xmµ{z -uu- $\left(\dfrac{6}{}\right) = \int$-7 $. \times \dfrac{6}{}$ +

$\Rightarrow$ $\dfrac{x}{y} = 2y + c$ ...(ii)

When $x = 1$ and $y = -1$ we get, $c = 1$

The equation (ii) becomes — $= 7$ $+6$ …(iii)

Put $y = 1$ in (iii), we get $x = 2 + 1 = 3$

**12.** **(d) :** $\dfrac{dp}{dt} = \dfrac{1}{2}p(t) - 200$

$\Rightarrow$ $\dfrac{dp}{p - 400} = \dfrac{1}{2}dt$

Integrating, we get, $\ln|p - 400| = \dfrac{1}{2}t + c$

$t = 0$, $p = 100$ $\Rightarrow$ $\ln 300 = c$

Again, $\ln\left(\dfrac{p - 400}{300}\right) = \dfrac{t}{2}$ $\Rightarrow$ $|p - 400| = 300e^{t/2}$

$\therefore$ $400 - p = 300e^{t/2}$ $(p < 400)$ $\therefore$ $p = 400 - 300e^{t/2}$

**13.** **(a) : 1st Solution:** $\cos x\, dy = y(\sin x - y)dx$

$\Rightarrow$ $\cos x\, dy = y \sin x\, dx - y^2\, dx$

$\Rightarrow$ $\cos x\, dy - y \sin x\, dx = -y^2\, dx$

$\Rightarrow$ $d(y\cos x) = -y^2\, dx \Rightarrow \dfrac{d(y\cos x)}{(y\cos x)^2} = -\dfrac{dx}{\cos^2 x}$

On integration, we have

$\Rightarrow$ $-\sec x = -y\tan x + yk$

$\Rightarrow$ $\sec x = y(\tan x + c)$ where $c$ is a constant

**2nd Solution:**

$\dfrac{dy}{dx} = \dfrac{y(\sin x - y)}{\cos x} \Rightarrow \dfrac{dy}{dx} = y\tan x - y^2\sec x$

$\Rightarrow$ $\dfrac{dy}{dx} - y\tan x = -y^2\sec x$ $\Rightarrow$ $\dfrac{1}{y^2}\dfrac{dy}{dx} - \dfrac{1}{y}\tan x = -\sec x$

Setting, $-\dfrac{1}{y} = v$, we have

$\dfrac{dv}{dx} + (\tan x)v = -\sec x$, which is linear in $v$.

I.F. $= e^{\int \tan x\, dx} = e^{\ln \sec x} = \sec x$

The solution is $v \times \sec x = \int -\sec^2 x\, dx + k$

$\Rightarrow$ $v\sec x = -\tan x + k$

$\Rightarrow$ $-\dfrac{\sec x}{y} = -\tan x - c$ $\Rightarrow$ $\sec x = y(\tan x + c)$

**14.** **(c) :** $y = c_1 e^{c_2 x}$

Differentiating w.r.t. $x$, we get $y' = c_1 c_2 e^{c_2 x} = c_2 y$ ...(i)

Again differentiating w.r.t. $x$, $y'' = c_2 y'$ ...(ii)

From (i) and (ii) upon division $\dfrac{y'}{y''} = \dfrac{y}{y'} \Rightarrow y''y = (y')^2$

which is the desired differential equation of the family of curves.

**15.** **(a) : 1st Solution (Homogeneous equation):**

Let $y = vx$, so that $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

We have $v + x\dfrac{dv}{dx} = \dfrac{x + vx}{x} = 1 + v$

$\Rightarrow$ $x\dfrac{dv}{dx} = 1 \Rightarrow dv = \dfrac{dx}{x} \Rightarrow v = \ln x + \ln k$

As $v = y/x$ we have $y = x\ln x + (\ln k)x$

At $x = 1$, $y = 1$ giving

$1 = 0 + (\ln k)$ $\therefore$ $\ln k = 1$, Then $y = x\ln x + x$

**2nd Solution (Inspection) :**

Rewriting the equation $\frac{dy}{dx}=\frac{x+y}{x}$ as $xdy - ydx = xdx$

We have $\frac{xdy-ydx}{x^2}=\frac{dx}{x} \Rightarrow d\left(\frac{y}{x}\right)=\frac{dx}{x}$

On integration $\frac{y}{x}=\ln x+k \Rightarrow y = x \ln x + kx$

As before, evaluating constant, $y = x \ln x + x$

**16. (d) :** The equation of circle is

$(x - \alpha)^2 + (y - 2)^2 = 25$ ...(1)

Differentiating w.r.t. $x$

$(x-\alpha)+(y-2)\frac{dy}{dx}=0 \Rightarrow x-\alpha=-(y-2)\frac{dy}{dx}$ ...(2)

From (1) and (2) on eliminating '$\alpha$'

$(y-2)^2\left(\frac{dy}{dx}\right)^2+(y-2)^2=25 \Rightarrow (y-2)^2(y')^2 = 25 - (y-2)^2$

**17. (a) :** General equation of all such circles is

$(x - h)^2 + (y - 0)^2 = h^2$ .... (i) where $h$ is parameter

$\Rightarrow (x - h)^2 + y^2 = h^2$

Differentiating, we get $2(x-h)+2y\frac{dy}{dx}=0$

$h=x+y\frac{dy}{dx}$ to eliminate $h$, putting value of $h$ in equation (i) we get $y^2=x^2+2xy\frac{dy}{dx}$.

**18. (d) :** Given $A x^2 + B y^2 = 1$

As solution having two constants, $\therefore$ order of differential equation is 2 so our choices (b) & (c) are discarded from the list, only choices (a) and (d) are possible

Again $A x^2 + B y^2 = 1$ ...(*)

Differentiating (*) w.r.t. $x$

$-\frac{A}{B}=\frac{y}{x}\frac{dy}{dx}$ ...(i)

Again on differentiating $-\frac{A}{B}=y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2$ ...(ii)

By (i) and (ii) we get $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2=y\left(\frac{dy}{dx}\right)$

$\Rightarrow$ order 2 and degree 1.

**19. (d) :** $x\frac{dy}{dx}=y(\log y-\log x+1)$

$\therefore \frac{dy}{dx}=\frac{y}{x}\left(\log\left(\frac{y}{x}\right)+1\right)$ Now put $\frac{y}{x}=v$

$\therefore v \log v\, dx = x\, dv \Rightarrow \frac{dv}{v\log v}=\frac{dx}{x} \Rightarrow \log\left(\frac{y}{x}\right)=cx.$

**20. (d) :** $y^2=2c\left(x+\sqrt{c}\right)$ ...(i)

$\therefore\ 2yy_1 = 2c \ \therefore\ yy_1 = c$

Now putting $c = yy_1$ in (i) we get

$y^2=2\cdot yy_1\left(x+\sqrt{yy_1}\right) \Rightarrow (y^2-2xyy_1)^2=4(yy_1)^3$

$\Rightarrow (y^2 - 2xyy_1)^2 = 4y^3y_1^3 \Rightarrow$ order 1, degree 3.

**21. (b) :** $y\,dx = -(x^2y + x)\,dy \Rightarrow ydx + xdy = -x^2y\,dy$

$\Rightarrow \frac{ydx+xdy}{(xy)^2}=\frac{-dy}{y} \Rightarrow \frac{d(xy)}{(xy)^2}=-\frac{dy}{y}$

$\Rightarrow d\left(-\frac{1}{xy}\right)=-\frac{dy}{y} \Rightarrow -\frac{1}{xy}=-\log y+C$

$\Rightarrow -\frac{1}{xy}+\log y = C$

**22. (a) :** Given family of curve is $x^2 + y^2 - 2ay = 0$ ...(1)

$\Rightarrow 2a=\frac{x^2+y^2}{y}.$

Also from (1), $2x + 2yy' - 2a\,y' = 0$

$\Rightarrow 2x+2yy'-\left(\frac{x^2+y^2}{y}\right)y'=0$

$\Rightarrow 2xy + y'(2y^2 - x^2 - y^2) = 0 \Rightarrow y'(x^2 - y^2) = 2xy$

**23. (a) :** $x = e^{y+e^{y+e^{y....}}} \Rightarrow x = e^{y+x}$

Differentiate w.r.t. $x$ after taking logarithm both sides

$\therefore \frac{1}{x}=1+\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{1-x}{x}$

**24. (a) :** From the given equation $(1+y^2)\frac{dx}{dy}+1x=e^{\tan^{-1}y}$

$\Rightarrow \frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{e^{\tan^{-1}y}}{1+y^2} \Rightarrow x\cdot \text{I.F.}=\int y\cdot\text{I.F.}\,dy$

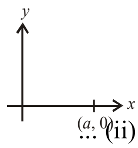
where $\text{I.F.}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y} \Rightarrow x\,e^{\tan^{-1}y}=\frac{e^{2\tan^{-1}y}}{2}+c$

$\Rightarrow 2x\,e^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

**25. (a) :** As axis of parabola is $x$-axis which means focus lies on $x$-axis. Equation of such parabolas is given by

$y^2 = 4a(x - k)$

.. (i)




$\Rightarrow 2yy_1 = 4a$ (by differentiating (i) w.r.t. $x$)

$\Rightarrow y\frac{dy}{dx}=2a$ ... (ii)

$\Rightarrow \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ (by differentiating (ii) w.r. to $x$)

$\Rightarrow$ Order 2 and degree 1 (Concept: Exponent of highest order derivative is called degree and order of that derivative is called order of the differential equation.)

**26. (c) :** $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}}=4\left(\frac{d^3y}{dx^3}\right) \Rightarrow \left(1+3\frac{dy}{dx}\right)^2=\left[4\frac{d^3y}{dx^3}\right]^3$

$\therefore$ Highest order is 3 whose exponent is also 3.

**27. (b) :** Given $\frac{d^2y}{dx^2}=e^{-2x}$

$\therefore \frac{dy}{dx}=\frac{e^{-2x}}{-2}+c \quad \therefore \quad y=\frac{e^{-2x}}{4}+cx+d$