CHAPTER

10 Integral Calculus

1. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{2}$ (d) 4π

(2018)

2. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$ and α, β $(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$ is

(a) $\frac{1}{2}(\sqrt{2}-1)$ (b) $\frac{1}{2}(\sqrt{3}-1)$

(c) $\frac{1}{2}(\sqrt{3}+1)$ (d) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$ *(2018)*

3. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x+\cos^3 x \sin^2 x+\sin^3 x \cos^2 x+\cos^5 x)^2} dx$ is equal to

(a) $\frac{-1}{1+\cot^3 x}+C$ (b) $\frac{1}{3(1+\tan^3 x)}+C$

(c) $\frac{-1}{3(1+\tan^3 x)}+C$ (d) $\frac{1}{1+\cot^3 x}+C$ *(2018)*

4. The value of the integral $\int_{-\pi/2}^{\pi/2} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right) dx$ is

(a) $\frac{3}{8}\pi$ (b) 0 (c) $\frac{3}{16}\pi$ (d) $\frac{3}{4}$

(Online 2018)

5. The area (in sq. units) of the region $\{x \in R : x \ge 0, y \ge 0, y \ge x-2 \text{ and } y \le \sqrt{x}\}$, is

(a) $\frac{10}{3}$ (b) $\frac{13}{3}$ (c) $\frac{5}{3}$ (d) $\frac{8}{3}$

(Online 2018)

6. If $f\left(\frac{x-4}{x+2}\right) = 2x+1$, $(x \in R - \{1, -2\})$, then $\int f(x)dx$ is equal to : (where C is a constant of integration)

(a) $12 \log_e |1 - x| - 3x + C$

(b) $-12 \log_e |1 - x| - 3x + C$

(c) $-12 \log_e |1 - x| + 3x + C$

(d) $12 \log_e |1 - x| + 3x + C$ *(Online 2018)*

7. If $\int \frac{2x+5}{\sqrt{7-6x-x^2}} dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right)+C$

(Where C is a constant of integration), then the ordered pair (A, B) is equal to

(a) $(-2, -1)$ (b) $(2, -1)$

(c) $(-2, 1)$ (d) $(2, 1)$ *(Online 2018)*

8. The value of integral $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} dx$ is

(a) $\pi\sqrt{2}$ (b) $2\pi(\sqrt{2}-1)$

(c) $\frac{\pi}{2}(\sqrt{2}+1)$ (d) $\pi(\sqrt{2}-1)$ *(Online 2018)*

9. If $I_1 = \int_0^1 e^{-x}\cos^2 x\, dx$, $I_2 = \int_0^1 e^{-x^2}\cos^2 x\, dx$ and $I_3 = \int_0^1 e^{-x^3} dx$; then

(a) $I_3 > I_1 > I_2$ (b) $I_2 > I_3 > I_1$

(c) $I_2 > I_1 > I_3$ (d) $I_3 > I_2 > I_1$

(Online 2018)

10. If $f(x) = \int_0^x t(\sin x - \sin t)dt$, then

(a) $f'''(x) + f'(x) = \cos x - 2x\sin x$

(b) $f'''(x) + f''(x) - f'(x) = \cos x$

(c) $f'''(x) - f''(x) = \cos x - 2x\sin x$

(d) $f'''(x) + f''(x) = \sin x$ *(Online 2018)*

11. If $\int \frac{\tan x}{1+\tan x+\tan^2 x} dx = x - \frac{K}{\sqrt{A}}\tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right)+C$, ($C$ is a constant of integration), then the ordered pair (K, A) is equal to :

(a) $(-2, 3)$ (b) $(-2, 1)$ (c) $(2, 1)$ (d) $(2, 3)$

(Online 2018)

12. If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines $y = 0$ and $x = t (t > 1)$ is 1 sq. unit, then t is equal to :

(a) $4/3$ (b) $e^{3/2}$

(c) $3/2$ (d) $e^{2/3}$ *(Online 2018)*

13. Let $I_n = \int \tan^n x\, dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to

(a) $\left(\frac{1}{5}, 0\right)$ (b) $\left(\frac{1}{5}, -1\right)$ (c) $\left(-\frac{1}{5}, 0\right)$ (d) $\left(-\frac{1}{5}, 1\right)$

(2017)

14. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$ is equal to

(a) 2 (b) 4 (c) –1 (d) –2

(2017)

15. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y$ and $y \leq 1 + \sqrt{x}\}$ is

(a) $\frac{3}{2}$ (b) $\frac{7}{3}$ (c) $\frac{5}{2}$ (d) $\frac{59}{12}$

(2017)

16. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x + \cot x)^3} dx$ equals

(a) $\frac{15}{128}$ (b) $\frac{13}{32}$ (c) $\frac{13}{256}$ (d) $\frac{15}{64}$

(Online 2017)

17. The integral $\int \sqrt{1 + 2\cot x\,(\operatorname{cosec} x + \cot x)}\, dx$ $\left(0 < x < \frac{\pi}{2}\right)$ is equal to

(where C is a constant of integration)

(a) $2\log\left(\sin\frac{x}{2}\right) + C$ (b) $2\log\left(\cos\frac{x}{2}\right) + C$

(c) $4\log\left(\cos\frac{x}{2}\right) + C$ (d) $4\log\left(\sin\frac{x}{2}\right) + C$

(Online 2017)

18. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is

(a) $\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$ (b) $\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}$

(c) $\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$ (d) $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$ *(Online 2017)*

19. If $f\left(\frac{3x - 4}{3x + 4}\right) = x + 2, x \neq -\frac{4}{3}$ and $\int f(x)\, dx = A\log|1 - x| + Bx + C$, then the ordered pair (A, B) is equal to

(where C is a constant of integration)

(a) $\left(-\frac{8}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{8}{3}, -\frac{2}{3}\right)$

(c) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (d) $\left(-\frac{8}{3}, -\frac{2}{3}\right)$

(Online 2017)

20. If $\int_1^2 \frac{dx}{(x^2 - 2x + 4)^{3/2}} = \frac{k}{k + 5}$, then k is equal to

(a) 1 (b) 3 (c) 4 (d) 2

(Online 2017)

21. The integral $\int \frac{7\ ^{67} +:\ ^{>}}{-\ ^{:} +\ ^{8} +6.^{8}}$ is equal to

(a) $\frac{-\ :}{-\ ^{:} +\ ^{8} +6.^{7}} + Y$ (b) $\frac{65}{7-\ ^{:} +\ ^{8} +6.^{7}} + Y$

(c) $\frac{:}{7-\ ^{:} +\ ^{8} +6.^{7}} + Y$ (d) $\frac{-\ 65}{7-\ ^{:} +\ ^{8} +6.^{7}} + Y$

where C is an arbitrary constant. *(2016)*

22. $\underset{\to\infty}{\text{xy}} \left(\frac{-\ +6.-\ +7.3338}{7}\right)^{64}$ is equal to

(a) $\frac{6=}{9}$ (b) $\frac{7<}{7}$ (c) $\frac{>}{7}$ (d) 3log3 – 2

(2016)

23. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is

(a) $\pi - \frac{9}{8}$ (b) $\pi - \frac{=}{8}$

(c) $\pi - \frac{9\sqrt{7}}{8}$ (d) $\frac{\pi}{7} - \frac{7\sqrt{7}}{8}$ *(2016)*

24. If $2\int_0^1 \tan^{-1} x\,dx = \int_0^1 \cot^{-1}(1 - x + x^2)\,dx$, then $\int_0^1 \tan^{-1}(1 - x + x^2)\,dx$ is equal to

(a) $\frac{\pi}{2} + \log 2$ (b) log2

(c) $\frac{\pi}{2} - \log 4$ (d) log4 *(Online 2016)*

25. The area (in sq. units) of the region described by $A = \{(x, y) \mid y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0\}$ is

(a) $\frac{19}{6}$ (b) $\frac{17}{6}$ (c) $\frac{7}{2}$ (d) $\frac{13}{6}$

(Online 2016)

26. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then $A + B + C$ equals :

(a) $\frac{16}{5}$ (b) $\frac{27}{10}$ (c) $\frac{7}{10}$ (d) $\frac{21}{5}$

(Online 2016)

27. For $x \in R$, $x \neq 0$, if $y(x)$ is a differentiable function such that

$$x\int_1^x y(t)\,dt = (x+1)\int_1^x t\,y(t)\,dt, \text{ then } y(x) \text{ equals}$$

(where C is a constant.)

(a) $Cx^3 e^{\frac{1}{x}}$ (b) $\frac{C}{x^2}e^{-\frac{1}{x}}$ (c) $\frac{C}{x}e^{-\frac{1}{x}}$ (d) $\frac{C}{x^3}e^{-\frac{1}{x}}$

(Online 2016)

28. The value of the integral $\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$, where $[x]$ denotes the greatest integer less than or equal to x, is

(a) $\frac{1}{3}$ (b) 6 (c) 7 (d) 3

(Online 2016)

29. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to

(where C is a constant of integration.)

(a) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$ (b) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(c) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

(Online 2016)

30. The integral $\int \frac{}{7_{-}\ 9 + 6.^{849}}$ equals

(a) $-\text{-}\ ^9 + 6.^{\frac{6}{9}} +$ (b) $-\left(\frac{^9+6}{9}\right)^{\frac{6}{9}} +$

(c) $\left(\frac{^9+6}{9}\right)^{\frac{6}{9}} +$ (d) $-\ ^9 + 6.^{\frac{6}{9}} +$ *(2015)*

31. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is

(a) $\frac{6:}{;9}$ (b) $\frac{>}{87}$ (c) $\frac{<}{87}$ (d) $\frac{:}{;9}$

(2015)

32. The integral $\int_7^9 \frac{x\{ s\ ^7}{x\{ s\ ^7 + x\{ s\text{-}8;\ -67\ +\ ^7.}$ is equal to

(a) 1 (b) 6 (c) 2 (d) 4 *(2015)*

33. The integral $\int \frac{}{-\ +6.^{849}\ -\ -7.^{:49}}$ u-q} ×mxµ{

(a) $9\left(\frac{+6}{-7}\right)^{649} + Y$ (b) $9\left(\frac{-7}{+6}\right)^{649} + Y$

(c) $-\frac{9}{8}\left(\frac{+6}{-7}\right)^{649} + Y$ (d) $-\frac{9}{8}\left(\frac{-7}{+6}\right)^{649} + Y$

(Online 2015)

34. For $x > 0$, let $-\ . = \int_6 \frac{x\{ s}{6+}\ 3$ Then $-\ . + \left(\frac{6}{-}\right)$ is equal to

(a) $\frac{6}{9}$-x{ s $.^7$ (b) $\frac{6}{7}$-x{ s $.^7$

(c) $\log x$ (d) $\frac{6}{9}$x{ s ^{7} *(Online 2015)*

35. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to

(a) 3/5 (b) 3/4 (c) 1/3 (d) 4/3

(Online 2015)

36. If $\int \frac{x\{ s\left(\ +\sqrt{6+\ ^7}\right)}{\sqrt{6+\ ^7}} = \frac{6}{7}\text{-}\ -..^7 + Y1$

where C is a constant, then $g(2)$ is equal to

(a) 7 x{ s $-7+\sqrt{:}.$ (b) x{ s $-7+\sqrt{:}.$

(c) $\frac{6}{\sqrt{:}}$x{ s $-7+\sqrt{:}.$ (d) $\frac{6}{7}$x{ s $-7+\sqrt{:}.$

(Online 2015)

37. Let $f : R \to R$ be a function such that $f(2 - x) = f(2 + x)$ and $f(4 - x) = f(4 + x)$, for all $x \in R$ and $\int_5^7 -\ .\quad =:3$ Then the value of $\int_{65}^{:5} -\ .$ u-

(a) 80 (b) 100 (c) 125 (d) 200

(Online 2015)

38. Let $f : (-1, 1) \to R$ be a continuous function. If $\int_5^{-u} -\ . = \frac{\sqrt{8}}{7}\ 1$ then $\left(\frac{\sqrt{8}}{7}\right)$ is equal to

(a) $\frac{\sqrt{8}}{7}$ (b) $\sqrt{8}$ (c) $\sqrt{\frac{8}{7}}$ (d) $\frac{6}{7}$

(Online 2015)

39. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(a) $xe^{x+\frac{1}{x}}+c$ (b) $(x+1)e^{x+\frac{1}{x}}+c$

(c) $-xe^{x+\frac{1}{x}}+c$ (d) $(x-1)e^{x+\frac{1}{x}}+c$ *(2014)*

40. The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is

(a) $\frac{\pi}{2}-\frac{4}{3}$ (b) $\frac{\pi}{2}-\frac{2}{3}$ (c) $\frac{\pi}{2}+\frac{2}{3}$ (d) $\frac{\pi}{2}+\frac{4}{3}$

(2014)

41. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals

(a) $\frac{2\pi}{3}-4-4\sqrt{3}$ (b) $4\sqrt{3}-4$

(c) $4\sqrt{3}-4-\frac{\pi}{3}$ (d) $\pi - 4$ *(2014)*

42. If $\int f(x)dx = \psi(x)$ then $\int x^5 f(x^3)dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(b) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(c) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

(d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$ *(2013)*

43. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\, dt, x \in R$, which are parallel to the line $y = 2x$, are equal to

(a) ± 2 (b) ± 3 (c) ± 4 (d) ± 1

(2013)

44. The area (in sq. units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis and lying in the first quadrant is

(a) 36 (b) 18 (c) $\frac{27}{4}$ (d) 9

(2013)

45. **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$.

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

(a) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.
(b) Statement-1 is true, Statement-II is false.
(c) Statement-I is false, Statement-II is true.
(d) Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I. *(2013)*

46. If the integral $\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to

(a) 1 (b) 2 (c) -1 (d) -2

(2012)

47. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi)$ equals

(a) $g(x) - g(\pi)$ (b) $g(x) \cdot g(\pi)$

(c) $\frac{g(x)}{g(\pi)}$ (d) $g(x) + g(\pi)$ *(2012)*

48. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is

(a) $\frac{20\sqrt{2}}{3}$ (b) $10\sqrt{2}$ (c) $20\sqrt{2}$ (d) $\frac{10\sqrt{2}}{3}$

(2012)

49. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is

(a) $\frac{1}{2}\ln 18$ (b) $\ln 18$

(c) $2\ln 18$ (d) $\ln 9$ *(2012)*

50. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$; where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(a) $I - \frac{k(T-t)^2}{2}$ (b) e^{-kT}

(c) $T^2 - \frac{I}{k}$ (d) $I - \frac{kT^2}{2}$ *(2011)*

51. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to

(a) 13 (b) -2 (c) 7 (d) 5 *(2011)*

52. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

(a) $\frac{\pi}{2}\log 2$ (b) $\log 2$ (c) $\pi \log 2$ (d) $\frac{\pi}{8}\log 2$

(2011)

53. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\sin t\, dt$. Then f has

(a) local minimum at π and local maximum at 2π.
(b) local maximum at π and local minimum at 2π.
(c) local maximum at π and 2π.
(d) local minimum at π and 2π. *(2011)*

54. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x-axis is

(a) 3/2 square units (b) 5/2 square units
(c) 1/2 square units (d) 1 square unit *(2011)*

55. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\, dx$ equals

(a) $\sqrt{41}$ (b) 21 (c) 41 (d) 42

(2010)

56. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(a) $4\sqrt{2} - 2$ (b) $4\sqrt{2} + 2$ (c) $4\sqrt{2} - 1$ (d) $4\sqrt{2} + 1$

(2010)

57. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent to the parabola at the point (2, 3) and the x-axis is

(a) 6 (b) 9 (c) 12 (d) 3 *(2009)*

58. $\int_0^{\pi}[\cot x]dx,$ where [.] denotes the greatest integer function, is equal to

(a) 1 (b) –1

(c) $-\pi/2$ (d) $\pi/2$ *(2009)*

59. The value of $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is

(a) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$ (b) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(c) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$ (d) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(2008)

60. The area of the plane region bounded by the curves $x+2y^2=0$ and $x+3y^2=1$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ *(2008)*

61. Let $I=\int_0^1\frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx.$

Then which one of the following is true?

(a) $I>\frac{2}{3}$ and $J<2$ (b) $I>\frac{2}{3}$ and $J>2$

(c) $I<\frac{2}{3}$ and $J<2$ (d) $I<\frac{2}{3}$ and $J>2$ *(2008)*

62. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(a) 1/6 (b) 1/3 (c) 2/3 (d) 1 *(2007)*

63. $\int\frac{dx}{\cos x+\sqrt{3}\sin x}$ equals

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(2007)

64. The solution for x of the equation $\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{2}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$ (c) $-\sqrt{2}$ (d) π

(2007)

65. Let $F(x)=f(x)+f\left(\frac{1}{x}\right),$ where $f(x)=\int_1^x\frac{\log t}{1+t}dt,$

Then $F(e)$ equals

(a) 1 (b) 2 (c) 1/2 (d) 0

(2007)

66. The value of $\int_1^a[x]f'(x)dx,\ a>1,$ where $[x]$ denotes the greatest integer not exceeding x is

(a) $af(a)-\{f(1)+f(2)+...+f([a])\}$

(b) $[a]f(a)-\{f(1)+f(2)+...+f([a])\}$

(c) $[a]f([a])-\{f(1)+f(2)+...+f(a)\}$

(d) $af([a])-\{f(1)+f(2)+...+f(a)\}$ *(2006)*

67. $\int_{-3\pi/2}^{-\pi/2}[(x+\pi)^3+\cos^2(x+3\pi)]dx$ is equal to

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{2}-1$

(2006)

68. $\int_0^{\pi}xf(\sin x)dx$ is equal to

(a) $\pi\int_0^{\pi}f(\cos x)dx$ (b) $\pi\int_0^{\pi}f(\sin x)dx$

(c) $\frac{\pi}{2}\int_0^{\pi/2}f(\sin x)dx$ (d) $\pi\int_0^{\pi/2}f(\cos x)dx$ *(2006)*

69. The value of the integral, $\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(a) 1/2 (b) 3/2 (c) 2 (d) 1

(2006)

70. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+....+\frac{1}{n}\sec^2 1\right]$ equals

(a) $\frac{1}{2}$cosec1 (b) $\frac{1}{2}$sec1 (c) $\frac{1}{2}$tan1 (d) tan1

(2005)

71. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx,\ a>0,$ is

(a) $\pi/2$ (b) $a\pi$ (c) 2π (d) π/a

(2005)

72. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(a) 1 : 2 : 3 (b) 1 : 2 : 1

(c) 1 : 1 : 1 (d) 2 : 1 : 2 *(2005)*

73. The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is

(a) 2 (b) 1 (c) 4 (d) 3 *(2005)*

74. If $I_1=\int_0^1 2^{x^2}dx,\ I_2=\int_0^1 2^{x^3}dx,\ I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then

(a) $I_1>I_2$ (b) $I_2>I_1$ (c) $I_3>I_4$ (d) $I_3=I_4$

(2005)

75. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(a) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$ (b) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$

(c) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$ (d) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ *(2005)*

76. Let $F : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals

(a) 36 (b) 24 (c) 18 (d) 12 *(2005)*

77. $\int \left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{x}{x^2+1} + C$ (b) $\frac{\log x}{(\log x)^2 + 1} + C$

(c) $\frac{x}{(\log x)^2 + 1} + C$ (d) $\frac{xe^x}{1+x^2} + C$ *(2005)*

78. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is

(a) 3 (b) 2

(c) 1 (d) 4 *(2004)*

79. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is

(a) –1 (b) –3 (c) 2 (d) 1 *(2004)*

80. If $\int_0^{\pi} x f(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$, then A is

(a) π/4 (b) π (c) 0 (d) 2π *(2004)*

81. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ is

(a) 2 (b) 1 (c) 0 (d) 3 *(2004)*

82. The value of $\int_{-2}^{3} |1 - x^2| dx$ is

(a) 7/3 (b) 14/3 (c) 28/3 (d) 1/3 *(2004)*

83. $\int \frac{dx}{\cos x - \sin x}$ is equal to

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + C$ (b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right| + C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + C$ (d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$ *(2004)*

84. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log\sin(x-\alpha) + C$, then value of (A, B) is

(a) (–sinα, cosα) (b) (cosα, sinα)

(c) (sinα, cosα) (d) (–cosα, sinα) *(2004)*

85. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n}$ is

(a) $1 - e$ (b) $e - 1$ (c) e (d) $e + 1$ *(2004)*

86. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$, then one of the possible values of k is

(a) 16 (b) 63 (c) 64 (d) 15 *(2003)*

87. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x\sin x}$ is

(a) 2 (b) 1 (c) 0 (d) 3 *(2003)*

88. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is

(a) $\frac{1}{n+2}$ (b) $\frac{1}{n+1} - \frac{1}{n+2}$

(c) $\frac{1}{n+1} + \frac{1}{n+2}$ (d) $\frac{1}{n+1}$ *(2003)*

89. If $f(a + b - x) = f(x)$, then $\int_a^b x f(x)dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(x)dx$ (b) $\frac{b-a}{2}\int_a^b f(x)dx$

(c) $\frac{a+b}{2}\int_a^b f(a+b-x)dx$ (d) $\frac{a+b}{2}\int_a^b f(b-x)dx$ *(2003)*

90. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y)g(y)dy$, then

(a) $F(t) = e^t - (1+t)$ (b) $F(t) = te^t$

(c) $F(t) = te^{-t}$ (d) $F(t) = 1 - e^t(1+t)$ *(2003)*

91. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is

(a) $e + \frac{e^2}{2} - \frac{3}{2}$ (b) $e - \frac{e^2}{2} - \frac{3}{2}$

(c) $e + \frac{e^2}{2} + \frac{5}{2}$ (d) $e - \frac{e^2}{2} - \frac{5}{2}$ *(2003)*

92. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is

(a) 3 sq. units (b) 4 sq. units

(c) 6 sq. units (d) 2 sq. units *(2003)*

93. $\lim_{n\to\infty}\dfrac{1^p+2^p+3^p+...+n^p}{n^{p+1}}$ is

(a) $\dfrac{1}{p+1}$ (b) $\dfrac{1}{1-p}$ (c) $\dfrac{1}{p}-\dfrac{1}{p-1}$ (d) $\dfrac{1}{p+2}$

(2002)

94. The area bounded by the curves $y = \ln x$, $y = \ln |x|$, $y = |\ln x|$ and $y = |\ln|x||$ is

(a) 4 sq. units (b) 6 sq. units
(c) 10 sq. units (d) none of these *(2002)*

95. $\int_{-\pi}^{\pi}\dfrac{2x(1+\sin x)}{1+\cos^2 x}dx$ is

(a) $\pi^2/4$ (b) π^2 (c) 0 (d) $\pi/2$ *(2002)*

96. If $y = f(x)$ makes +ve intercept of 2 and 0 unit with x and y and encloses an area of 3/4 square unit with the axes then $\int_0^2 x\, f'(x)\, dx$ is

(a) 3/2 (b) 1 (c) 5/4 (d) –3/4

(2002)

97. $\int_{\pi}^{10\pi}|\sin x|dx$ is

(a) 20 (b) 8 (c) 10 (d) 18

(2002)

98. $I_n = \int_0^{\pi/4}\tan^n x\, dx$, then $\lim_{n\to\infty} n\left[I_n + I_{n-2}\right]$ equals

(a) 1/2 (b) 1
(c) ∞ (d) 0 *(2002)*

99. $\int_0^{\sqrt{2}}[x^2]dx$ is

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$
(c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$ *(2002)*

ANSWER KEY

1. (a)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (a)	**11.** (d)	**12.** (d)
13. (a)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (a)	**21.** (b)	**22.** (b)	**23.** (b)	**24.** (b)
25. (a)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (c)	**30.** (b)	**31.** (b)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (d)	**36.** (b)
37. (b)	**38.** (b)	**39.** (a)	**40.** (d)	**41.** (c)	**42.** (b)	**43.** (d)	**44.** (d)	**45.** (c)	**46.** (b)	**47.** (a, d)	**48.** (a)
49. (c)	**50.** (d)	**51.** (c)	**52.** (c)	**53.** (b)	**54.** (a)	**55.** (b)	**56.** (a)	**57.** (b)	**58.** (c)	**59.** (d)	**60.** (a)
61. (c)	**62.** (a)	**63.** (c)	**64.** (c)	**65.** (c)	**66.** (b)	**67.** (c)	**68.** (d)	**69.** (b)	**70.** (c)	**71.** (a)	**72.** (c)
73. (b)	**74.** (a)	**75.** (c)	**76.** (c)	**77.** (c)	**78.** (c)	**79.** (c)	**80.** (b)	**81.** (a)	**82.** (c)	**83.** (d)	**84.** (b)
85. (b)	**86.** (c)	**87.** (b)	**88.** (b)	**89.** (a, c)	**90.** (a)	**91.** (b)	**92.** (b)	**93.** (a)	**94.** (a)	**95.** (b)	**96.** (d)
97. (d)	**98.** (b)	**99.** (c)									

Explanations

1. (a) : Let $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ ...(i)

Changing x to $-\pi/2 + \pi/2 - x = -x$, we have

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}} dx \quad \text{...(ii)}$$

Adding (i) & (ii), we get $2I = \int_{-\pi/2}^{\pi/2} \sin^2 x \left\{\frac{1}{1+2^x} + \frac{1}{1+2^{-x}}\right\} dx$

$= \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = 2\int_0^{\pi/2} \sin^2 x\, dx$ (As the integral function is even)

$\therefore \; I = \frac{\pi}{4}$

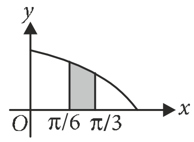
2. (b) : $y = (gof)(x) = \cos\sqrt{x^2} = \cos|x| = \cos x$

$[\because \cos(-x) = \cos x]$

Consider $18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow (3x - \pi)(6x - \pi) = 0$

$\Rightarrow \; x = \frac{\pi}{3}, \frac{\pi}{6}$



$\therefore$ Required area $= \int_{\pi/6}^{\pi/3} \cos x\, dx = [\sin x]_{\pi/6}^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{1}{2}(\sqrt{3} - 1)$

3. (c) : Let $I = \int \frac{\sin^2 x \cos^2 x}{(\sin^2 x + \cos^2 x)^2 (\sin^3 x + \cos^3 x)^2} dx$

$= \int \frac{\sin^2 x \cos^2 x\, dx}{(\sin^3 x + \cos^3 x)^2} = \int \frac{\tan^2 x \sec^2 x}{(1+\tan^3 x)^2} dx$

Put $1 + \tan^3 x = t$ so that $3\tan^2 x \sec^2 x\, dx = dt$

$\therefore \; I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3}\cdot\frac{1}{t} + C = -\frac{1}{3}\cdot\frac{1}{(1+\tan^3 x)} + C$

4. (a) : Let $I = \int_{-\pi/2}^{\pi/2} \sin^4 x \left(1 + \log\left(\frac{2+\sin x}{2-\sin x}\right)\right) dx$

$= \int_{-\pi/2}^{\pi/2} \sin^4 x\, dx + \int_{-\pi/2}^{\pi/2} \sin^4 x \log\left(\frac{2+\sin x}{2-\sin x}\right) dx = I_1 + I_2$

Now, $I_1 = \int_{-\pi/2}^{\pi/2} \sin^4 x\, dx = 2\int_0^{\pi/2} \sin^4 x\, dx$

$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx \text{ if } f(x) = f(-x)\right]$

$I_2 = \int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \log\left(\frac{2+\sin x}{2-\sin x}\right) dx = 0$

$\left[\because \int_{-a}^{a} f(x)dx = 0 \text{ if } f(x) = -f(-x)\right]$

$\therefore \; I = 2\int_0^{\pi/2} \sin^4 x = 2\int_0^{\pi/2} (\sin^2 x)^2 dx = 2\int_0^{\pi/2} \left(\frac{1-\cos 2x}{2}\right)^2 dx$

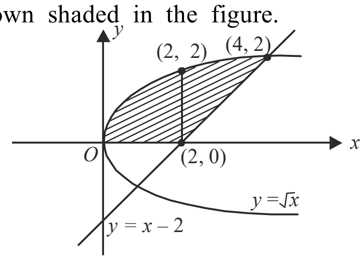
$= \frac{1}{2}\int_0^{\pi/2} [1 + \cos^2 2x - 2\cos 2x]\, dx = \frac{1}{2}\int_0^{\pi/2} \left[1 + \left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right] dx$

$= \frac{1}{2}\int_0^{\pi/2} \left(\frac{3}{2} + \frac{\cos 4x}{2} - 2\cos 2x\right) dx$

$= \frac{1}{2}\left[\frac{3}{2}|x|_0^{\pi/2} + \frac{1}{2}\left|\frac{\sin 4x}{4}\right|_0^{\pi/2} - 2\left|\frac{\sin 2x}{2}\right|_0^{\pi/2}\right]$

$= \frac{1}{2}\left[\left(\frac{3}{2}\times\frac{\pi}{2}\right) + \frac{1}{8}(\sin 2\pi - \sin 0) - \sin\pi + \sin 0\right] = \frac{3\pi}{8}$

5. (a) : Required area is shown shaded in the figure.



On solving $y = \sqrt{x}$

and $y = x - 2$

we get, $(\sqrt{x})^2 = (x-2)^2$

$\Rightarrow \; x = x^2 + 4 - 4x$

$\Rightarrow \; x^2 - 5x + 4 = 0$

$\Rightarrow (x - 4)(x - 1) = 0$

$\Rightarrow \; x = 1, 4$

$\therefore$ Required area

$= \int_0^2 \sqrt{x}\, dx + \int_2^4 (\sqrt{x} - x + 2)\, dx = \int_0^4 \sqrt{x}\, dx + \int_2^4 (2 - x)\, dx$

$= \left|\frac{2}{3}(x)^{3/2}\right|_0^4 + \left|2x - \frac{x^2}{2}\right|_2^4 = \frac{2}{3}(8) + 2(4-2) - \frac{1}{2}(16-4)$

$= \frac{16}{3} + 4 - 6 = \frac{16}{3} - 2 = \frac{10}{3}.$

6. (a) : Given, $f\left(\frac{x-4}{x+2}\right) = 2x + 1$ ...(i)

Put $\frac{x-4}{x+2} = t \Rightarrow x - 4 = 2t + xt$

$\Rightarrow \; x - xt = 2t + 4 \Rightarrow x(1 - t) = 2(t + 2) \Rightarrow x = \frac{2(t+2)}{1-t}$

$\therefore$ (i) becomes $f(t) = 2\left(\frac{2(t+2)}{1-t}\right) + 1 = \frac{-4(t+2)}{t-1} + 1$

or $\; f(x) = \frac{-4(x+2)}{x-1} + 1$ ...(ii)

On integrating (ii), we get $\int f(x) = -4\int \frac{(x+2)}{x-1} dx + \int 1\, dx$

$= -4\int \frac{x+3-1}{x-1} dx + x = -4\int 1\, dx - 12\int \frac{1}{x-1} dx + x = -3x + 12\int \frac{1}{1-x} dx$

$= -3x + 12 \log_e|1 - x| + C$

7. (a) : Let $I = \int \frac{2x+5}{\sqrt{7-6x-x^2}} dx \Rightarrow I = \int \frac{2x+6-1}{\sqrt{7-6x-x^2}} dx$

$$\Rightarrow\ I=\int\frac{2x+6}{\sqrt{7-6x-x^2}}\,dx-\int\frac{1}{\sqrt{(4)^2-(x+3)^2}}\,dx$$

$$\Rightarrow\ I=\int\frac{-dt}{\sqrt{t}}-\int\frac{1}{\sqrt{4^2-(x+3)^2}}\,dx$$

$[\because\ t=7-6x-x^2\ \Rightarrow\ dt=-(2x+6)dx]$

$$\Rightarrow I=-2(7-6x-x^2)^{1/2}-\sin^{-1}\left(\frac{x+3}{4}\right)+C$$

$$\Rightarrow I=-2\sqrt{7-6x-x^2}-\sin^{-1}\left(\frac{x+3}{4}\right)+C\ \ \therefore\ A=-2,\ B=-1$$

8. (d) : Let $I=\int_{\pi/4}^{3\pi/4}\frac{x}{1+\sin x}dx$

$$\Rightarrow\ I=\int_{\pi/4}^{3\pi/4}\left(\frac{x}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}\right)dx\ \Rightarrow\ I=\int_{\pi/4}^{3\pi/4}\frac{x(1-\sin x)}{\cos^2 x}dx$$

$$\Rightarrow I=\int_{\pi/4}^{3\pi/4}x\cdot\sec^2 x\,dx-\int_{\pi/4}^{3\pi/4}x\cdot\frac{\sin x}{\cos^2 x}\,dx$$

$$\Rightarrow I=\int_{\pi/4}^{3\pi/4}x\cdot\sec^2 x dx-\int_{\pi/4}^{3\pi/4}x(\sec x\tan x)dx$$

$$\Rightarrow I=|x\tan x|_{\pi/4}^{3\pi/4}-\log(\sec x)|_{\pi/4}^{3\pi/4}-\left|\frac{x}{\cos x}\right|_{\pi/4}^{3\pi/4}-\int_{\pi/4}^{3\pi/4}\frac{1}{\cos x}dx$$

$$\Rightarrow I=\left|x\tan x\right|_{\pi/4}^{3\pi/4}-\left|\frac{x}{\cos x}\right|_{\pi/4}^{3\pi/4}-\log\left|\sec x+\tan x\right|_{\pi/4}^{3\pi/4}$$

$$\Rightarrow I=-\frac{3\pi}{4}-\frac{\pi}{4}+\frac{3\pi}{4}(\sqrt{2})+\frac{\pi}{4}\sqrt{2}-\log(\sqrt{2}+1)+\log(\sqrt{2}+1)$$

$$\Rightarrow\ I=-\pi+\sqrt{2}\pi\ \Rightarrow\ I=\pi(\sqrt{2}-1)$$

9. (d) : For $0<x<1$

$x^3<x^2<x$ *i.e.*, $-x^3>-x^2>-x$

$e^{-x^3}>e^{-x^2}>e^{-x}\ \therefore\ e^{-x}\cos^2 x<e^{-x^2}\cos^2 x<e^{-x^3}\cos^2 x$

As $\cos^2 x\le 1$, we have $e^{-x^2}\cos^2 x<e^{-x^3}$

Thus, $e^{-x}\cos^2 x<e^{-x^2}\cos^2 x<e^{-x^3}\ \therefore\ I_1<I_2<I_3$

10. (a) : $f(x)=\int_0^x t(\sin x-\sin t)\,dt$

$$\Rightarrow\ f(x)=\int_0^x t\sin x\cdot dt-\int_0^x t\sin t\cdot dt$$

$$\Rightarrow f(x)=\sin x\left[\frac{t^2}{2}\right]_0^x-\left[|t(-\cos t)|_0^x-\int_0^x\frac{d(t)}{dt}\left(\int\sin t\cdot dt\right)dt\right]$$

$$\Rightarrow f(x)=\sin x\cdot\frac{x^2}{2}-\left[-x\cos x-\int_0^x 1\cdot(-\cos t)\cdot dt\right]$$

$$\Rightarrow f(x)=\sin x\cdot\frac{x^2}{2}-\left[-x\cos x+|\sin t|_0^x\right]$$

$$\Rightarrow f(x)=\sin x\cdot\frac{x^2}{2}+x\cos x-\sin x \quad \text{...(i)}$$

Now differentiating (i) w.r.t. 'x', we have

$$f'(x)=\sin x\cdot\left(\frac{2x}{2}\right)+\frac{x^2}{2}\cdot(\cos x)-x\sin x+\cos x-\cos x$$

$$\Rightarrow\ f'(x)=\frac{x^2}{2}\cdot\cos x \quad \text{...(ii)}$$

Differentiating (ii) w.r.t. 'x', we have

$$f''(x)=\frac{x^2}{2}\cdot(-\sin x)+\left(\frac{2x}{2}\right)\cdot\cos x$$

$$\Rightarrow\ f''(x)=x\cos x-\frac{x^2}{2}\cdot\sin x \quad \text{...(iii)}$$

Differentiating (iii) w.r.t. 'x', we have

$$f'''(x)=x(-\sin x)+\cos x-\frac{x^2}{2}(\cos x)-\left(\frac{2x}{2}\right)\sin x$$

$$\Rightarrow\ f'''(x)=\cos x-2x\sin x-\frac{x^2}{2}\cos x \quad \text{...(iv)}$$

Adding (ii) and (iv), we have

$$f'''(x)+f'(x)=\cos x-2x\sin x-\frac{x^2}{2}\cos x+\frac{x^2}{2}\cos x$$

$=\cos x-2x\sin x$.

11. (d) : Let $I=\int\frac{\tan x}{1+\tan x+\tan^2 x}dx=\int\frac{1+\tan x-1}{1+\tan x+\tan^2 x}dx$

$$=\int\frac{1+\tan x+\tan^2 x-\sec^2 x}{1+\tan x+\tan^2 x}dx=\int\left(1-\frac{\sec^2 x}{1+\tan x+\tan^2 x}\right)dx$$

$$=x-\int\frac{\sec^2 x}{1+\tan x+\tan^2 x}dx$$

$$=x-\int\frac{1}{1+t+t^2}dt\ (\text{Putting }\tan x=t\Rightarrow\sec^2 x dx=dt)$$

$$=x-\int\frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=x-\frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\sqrt{3}/2}\right)+C$$

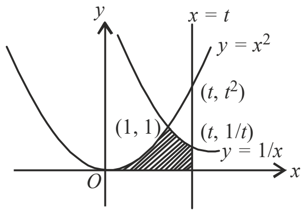
$$=x-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{3}}\right)+C\quad\therefore\quad K=2,\ A=3$$

12. (d) : $y=x^2,\ y=\frac{1}{x}$

$\Rightarrow\ x^3=1$

$\Rightarrow\ x=1$

Since area bounded by the given curves $=1$



$$\therefore\ \int_0^1 y\,dx+\int_1^t y\,dx=1\Rightarrow\int_0^1 x^2\,dx+\int_1^t\frac{1}{x}\,dx=1\Rightarrow\left[\frac{x^3}{3}\right]_0^1+\left[\log x\right]_1^t=1$$

$$\Rightarrow\ \frac{1}{3}+\log t-\log 1=1\ \Rightarrow\ \log t=1-\frac{1}{3}\ \Rightarrow\ \log t=\frac{2}{3}\ \Rightarrow t=e^{2/3}$$

13. (a) : We have $I_n=\int\tan^n x\,dx,(n>1)$

$$=\int\tan^{n-2}x(\sec^2 x-1)dx=\int\tan^{n-2}x\sec^2 x\,dx-\int\tan^{n-2}dx$$

$$=\frac{\tan^{n-1}x}{n-1}-I_{n-2}\ \text{Then,}\ I_n+I_{n-2}=\frac{(\tan x)^{n-1}}{n-1}$$

Now, $I_4+I_6=\frac{\tan x^5}{5}$ ...(i)

And $I_4+I_6=a\tan x^5+bx^5+C$ (Given) ...(ii)

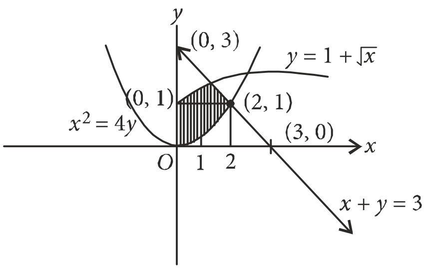
On comparing (i) and (ii), we get $a=\frac{1}{5},b=0,$ and C is a constant of integeration.

14. (a) : Let $I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos(\pi - x)} = \int_{\pi/4}^{3\pi/4} \frac{dx}{1-\cos x}$

On adding, we have, $2I = \int_{\pi/4}^{3\pi/4} \frac{2}{1-\cos^2 x}dx$

$\Rightarrow \; I = \int_{\pi/4}^{3\pi/4} \text{cosec}^2 x\, dx = -\cot x\Big|_{\pi/4}^{3\pi/4} = 2$

15. (c) : The graph of the region is as follows :



Required area $= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$

$= x + \frac{2x^{3/2}}{3}\Big|_0^1 + 3x - \frac{x^2}{2}\Big|_1^2 - \frac{x^3}{12}\Big|_0^2$

$= \left(1+\frac{2}{3}\right) + \left(3\cdot 2 - \frac{2^2}{2} - 3\cdot 1 + \frac{1^2}{2}\right) - \frac{2^3}{12} = \frac{5}{3} + \left(4 - \frac{5}{2}\right) - \frac{2}{3} = \frac{5}{2}.$

16. (a) : We have,

$\int_{\pi/12}^{\pi/4} \frac{8\cos 2x}{(\tan x + \cot x)^3}dx = \int_{\pi/12}^{\pi/4} \frac{8\cos 2x}{8\left(\frac{\sin^2 x + \cos^2 x}{2\sin x \cos x}\right)^3}dx$

$\int_{\pi/12}^{\pi/4} \frac{\cos 2x}{\left(\frac{1}{\sin 2x}\right)^3}dx = \int_{\pi/12}^{\pi/4} \cos 2x \times \sin 2x \times \sin^2(2x)dx$

$= \frac{1}{4}\int_{\pi/12}^{\pi/4} \sin 4x \cdot (1-\cos 4x)dx = \frac{1}{4}\int_{\pi/12}^{\pi/4} \sin 4x\, dx - \frac{1}{8}\int_{\pi/12}^{\pi/4} \sin 8x\, dx$

$= \frac{1}{4}\left[-\frac{\cos 4x}{4}\right]_{\pi/12}^{\pi/4} - \frac{1}{8}\left(-\frac{\cos 8x}{8}\right)_{\pi/12}^{\pi/4}$

$= \frac{-1}{16}\left(\cos \pi - \cos\frac{\pi}{3}\right) + \frac{1}{64}\left(\cos 2\pi - \cos\frac{2\pi}{3}\right)$

$= -\frac{1}{16}\left(-1-\frac{1}{2}\right) + \frac{1}{64}\left(1+\frac{1}{2}\right) = \frac{3}{32} + \frac{3}{128} = \frac{15}{128}$

17. (a) : Let $I = \int \sqrt{1 + 2\cot x(\text{cosec} x + \cot x)}dx$

$= \int \sqrt{1 + 2\cot x \,\text{cosec} x + 2\cot^2 x}\, dx$

$= \int \sqrt{1 + 2\frac{\cos x}{\sin^2 x} + 2\frac{\cos^2 x}{\sin^2 x}}\, dx = \int \sqrt{\frac{\sin^2 x + 2\cos x + 2\cos^2 x}{\sin^2 x}}\, dx$

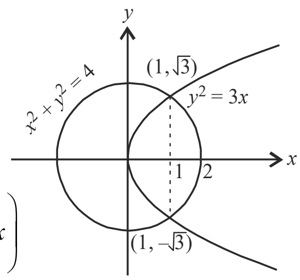
$= \int \sqrt{\frac{\sin^2 x + \cos^2 x + 2\cos x + \cos^2 x}{\sin^2 x}}\, dx$

$= \int \sqrt{\frac{1 + 2\cos x + \cos^2 x}{\sin^2 x}}\, dx = \int \sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}\, dx$

$= \int \frac{(1+\cos x)}{\sin x}dx = \int \frac{2\cos^2(x/2)}{2\sin(x/2)\cos(x/2)}dx$

$= \int \cot(x/2)\, dx = 2\log\left|\sin\frac{x}{2}\right| + C$

18. (d) : We have, $x^2 + y^2 = 4$
and $y^2 = 3x$
$\Rightarrow \; x^2 + 3x - 4 = 0$
$\Rightarrow (x + 4)(x - 1) = 0$
$\Rightarrow \; x = -4, \; x = 1$



Area $= 2\times\left(\int_0^1 \sqrt{3}\cdot\sqrt{x}dx + \int_1^2 \sqrt{4-x^2}\, dx\right)$

$= 2\left(\sqrt{3}\left(\frac{2}{3}\right) + \left\{2\cdot\frac{\pi}{2} - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)\right\}\right)$

$= 2\times\left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right) = 2\times\left(\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}\right) = \frac{1}{\sqrt{3}} + \frac{4\pi}{3}$

19. (a) : $f\left(\frac{3x-4}{3x+4}\right) = x+2, \; x \neq -\frac{4}{3}$

Let $\frac{3x-4}{3x+4} = t \Rightarrow 3x - 4 = 3tx + 4t \;\Rightarrow x = \frac{4t+4}{3-3t}$

Now, $f(t) = \frac{4t+4}{3-3t} + 2 = \frac{10-2t}{3-3t}$ Or, $f(x) = \frac{2x-10}{3x-3}$

Now, $\int f(x)\, dx = \int \frac{2x-10}{3x-3}dx = \int \frac{2x}{3x-3}dx - 10\int \frac{dx}{3x-3}$

$= \frac{2}{3}\int \frac{x-1}{x-1}dx + \frac{2}{3}\int \frac{dx}{x-1} - \frac{10}{3}\int \frac{dx}{x-1}$

$= \frac{2}{3}x + \frac{2}{3}\log(x-1) - \frac{10}{3}\log(x-1) + C = \frac{2x}{3} - \frac{8}{3}\log(x-1) + C$

So, $A = \frac{-8}{3}, B = \frac{2}{3}$

20. (a) : Let $I = \int_1^2 \frac{dx}{(x^2 - 2x + 4)^{3/2}} = \int_1^2 \frac{dx}{((x-1)^2 + 3)^{3/2}}$

Put $x - 1 = \sqrt{3}\tan\theta \Rightarrow dx = \sqrt{3}\sec^2\theta d\theta$

When $x = 1$, $\theta = 0$ and when $x = 2$, $\theta = \frac{\pi}{6}$

$\therefore \; I = \int_0^{\pi/6} \frac{\sqrt{3}\sec^2\theta d\theta}{(3\tan^2\theta + 3)^{3/2}} = \int_0^{\pi/6} \frac{\sqrt{3}\sec^2\theta\, d\theta}{3\sqrt{3}(\sec^2\theta)^{3/2}} = \int_0^{\pi/6} \frac{1}{3\sec\theta}d\theta$

$= \frac{1}{3}\int_0^{\pi/6} \cos\theta\, d\theta = \frac{1}{3}(\sin\theta)_0^{\pi/6} = \frac{1}{3}\left(\frac{1}{2} - 0\right) = \frac{1}{6}$

Now, $\frac{1}{6} = \frac{k}{k+5} \Rightarrow k + 5 = 6k \Rightarrow 5k = 5 \;\Rightarrow\; k = 1$

21. (b) : Let $f = \int \frac{7^{67} + : ^{>}}{- : + ^{8} + 6.^{8}}$

$= \int \frac{\frac{7^{67} + : ^{>}}{6:}}{\left(\frac{: + ^{8} + 6}{:}\right)^{8}} \quad = \int \frac{\frac{7}{8} + \frac{:}{;}}{\left(6 + \frac{6}{7} + \frac{6}{:}\right)^{8}}$

Put, $6 + \frac{6}{7} + \frac{6}{:} =$ 1{ µt mµ — $= -\left(\frac{7}{8} + \frac{:}{;}\right)$

The integral reduces to

$f = -\int \frac{6}{^{8}} = \frac{6}{7\,^{7}} + Y = \frac{65}{7- : + ^{8} + 6.^{7}} + Y$

22. (b) : From the definition of limit as sum

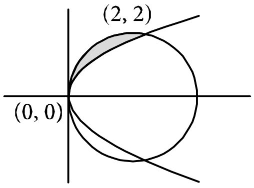
$\underset{\to\infty}{xy} \left(\frac{- +6.- +7.3338}{7}\right)^{64} = \underset{\to\infty}{xy} \frac{6}{} \sum_{=6}^{7} xz\left(6 + -\right) = \int_5^7 xz\,6 + .$

Now, $\int_5^7 xz\,6+ . = g- +6.\, xz- +6. - i_5^7 = 3\ln 3 - 2 = \ln 27 - 2$

$\therefore$ Required limit = $xz\,7< -7 = \frac{7<}{7}$

23. (b) : The area of the required region is shaded.

$\therefore$ Area $= \frac{\pi \cdot 7^7}{9} - \sqrt{7}\int_5^7 \sqrt{\;}$



$= \pi - \sqrt{7} \cdot \frac{847}{847}\Big|_5^7$

$= \pi - \sqrt{7} \cdot \frac{7}{8} \cdot 7\sqrt{7} = \pi - \frac{8}{8}$

24. (b) : We have, $7\int_5^6 \mu m z^{-6} = \int_5^6 o\{\mu^{-6} 6- + ^{7}.$

$= \int_0^1 \left(\frac{\pi}{2} - \tan^{-1}(1 - x + x^2)\right) dx$

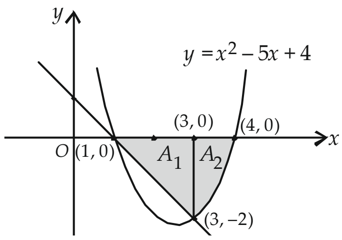
$\Rightarrow 2\int_0^1 \tan^{-1} x\,dx = \int_0^1 \frac{\pi}{2} dx - \int_0^1 \tan^{-1}(1 - x + x^2)\,dx$

$\Rightarrow \int_0^1 \tan^{-1}(1 - x + x^2)\,dx = \frac{\pi}{2} - 2\int_0^1 \tan^{-1} x\,dx$

$= \frac{\pi}{2} - 2\left\{[(\tan^{-1} x)x]_0^1 - \int_0^1 \frac{1}{1+x^2} x\,dx\right\} = \frac{\pi}{2} - 2\left(\frac{\pi}{4}\right) + \int_0^1 \frac{2x}{1+x^2} dx$

$= \frac{\pi}{2} - \frac{\pi}{2} + \left[\log(1+x^2)\right]_0^1 = \log(2) - \log(1) = \log 2$

25. (a) :



Required area $= A_1 + A_2 = \left|\int_1^3 (1-x)dx\right| + \left|\int_3^4 (x^2 - 5x + 4)\,dx\right|$

$= \left|\left[x - \frac{x^2}{2}\right]_1^3\right| + \left|\left[\frac{x^3}{3} - \frac{5}{2}x^2 + 4x\right]_3^4\right| = 2 + \frac{7}{6} = \frac{19}{6}$ sq. units

26. (a) : Consider, $I = \int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$= \int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}} = \int \frac{dx}{2\cos^4 x\sqrt{\tan x}} = \int \frac{\sec^4 x}{2\sqrt{\tan x}} dx$

Put $\tan x = t^2 \Rightarrow \sec^2 x\, dx = 2t\, dt$. Also $\sec^2 x = 1 + t^4$

Now, $I = \int \frac{(1+t^4)\,2t\,dt}{2t} = \int (1+t^4)\,dt = t + \frac{t^5}{5} + k$

$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x + k$

On comparing with the given equation, we get $A = \frac{1}{2}, B = \frac{5}{2}, C = \frac{1}{5}$

Now, $A + B + C = \frac{16}{5}$

27. (d) : $x\int_1^x y(t)dt = x\int_1^x ty(t)dt + \int_1^x ty(t)dt$

Differentiating w.r.t. x, we get $\int_1^x y(t)\,dt + x[y(x) - y(1)]$

$= \int_1^x ty\,(t)dt + x[xy(x) - y(1)] + xy(x) - y(1)$

$\Rightarrow \int_1^x y\,(t)dt = \int_1^x t\,y(t)dt + x^2 y(x) - y(1)$

Again differentiating w.r.t. x, we get

$y(x) - y(1) = xy(x) - y(1) + 2xy(x) + x^2 y'(x)$

$\Rightarrow (1 - 3x)y(x) = x^2 y'(x)$

$\Rightarrow \frac{y'(x)}{y(x)} = \frac{1-3x}{x^2} \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1-3x}{x^2} \Rightarrow \int \frac{1}{y}dy = \int \frac{1}{x^2}dx - \int \frac{3}{x}dx$

$\Rightarrow \ln y = -\frac{1}{x} - 3\ln x + \ln C \Rightarrow \ln\left|\frac{yx^3}{C}\right| = -\frac{1}{x}$

$\Rightarrow \frac{yx^3}{C} = e^{-1/x} \Rightarrow y = \frac{Ce^{-\frac{1}{x}}}{x^3}$

28. (d) : Let $I = \int_4^{10} \frac{[x^2]}{[x^2 - 28x + 196] + [x^2]} dx$ ...(i)

Use $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx$ $\therefore I = \int_4^{10} \frac{[(x-14)^2]}{[x^2] + [(x-14)^2]} dx$ ...(ii)

Adding (i) & (ii), we get

$2I = \int_4^{10} \frac{[(x-14)^2] + [x^2]}{[x^2] + [(x-14)^2]} dx \Rightarrow 2I = \int_4^{10} dx \Rightarrow 2I = 6 \Rightarrow I = 3$

29. (c) : Let $I = \int \frac{dx}{(1+\sqrt{x})\sqrt{x}\sqrt{1-x}}$

Put $1 + \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt \quad \therefore I = \int \frac{2dt}{t\sqrt{2t - t^2}}$

Again put $t = \frac{1}{z} \Rightarrow dt = \frac{-1}{z^2}dz$

$$\therefore \quad I = 2\int \frac{-\frac{1}{z^2}dz}{\frac{1}{z}\sqrt{\frac{2}{z} - \frac{1}{z^2}}} = 2\int \frac{-dz}{\sqrt{2z-1}} = -2\sqrt{2z-1} + C$$

$$= -2\sqrt{\frac{2}{t} - 1} + C = -2\sqrt{\frac{2-t}{t}} + C = -2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$$

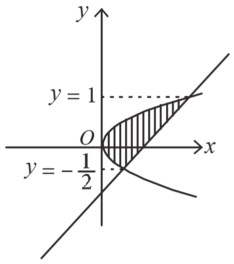
30. (b) : $f = \int \frac{}{7_- \; 9 \; +6.^{849}}$

$$= \int \frac{}{7_- \; 9 \, .^{849} \left\{6 + \frac{6}{9}\right\}^{849}} = \int \frac{}{: \left(6 + \frac{6}{9}\right)^{849}}$$

Set $6 + \frac{6}{9} = \quad \Rightarrow \; -\frac{9}{:} \quad =$

$$\Rightarrow f = -\frac{6}{9}\int \frac{6}{849} \quad = -\frac{6}{9}\cdot\frac{649}{649} = -\;649 = -\left(6 + \frac{6}{9}\right)^{649} + Y$$

31. (b) :



The area is given by $\int - \; 6 - \; 7\cdot$ and in this case

$$\int_{-647}^{6} \left(\frac{+6}{9} - \frac{7}{7}\right) \quad = \frac{- \; +6.^7}{=} - \frac{8}{;}\Bigg|_{-647}^{6}$$

$$= \left[\frac{7^7}{=} - \frac{6}{;}\right] - \left[\frac{\frac{6}{9}}{=} + \frac{6}{=\cdot;}\right] = \left(\frac{6}{7} - \frac{6}{;}\right) - \left(\frac{6}{87} + \frac{6}{9=}\right)$$

$$= \frac{8-6}{;} - \left(\frac{8+7}{>;}\right) = \frac{6}{8} - \frac{:}{>;} = \frac{87-:}{>;} = \frac{7<}{>;} = \frac{>}{87}$$

32. (a) : $\text{Sq}\mu f = \int_7^9 \frac{xz \quad 7}{7\,xz \quad 7 + xz - \; - ;\, .^7}$

$$= \int_7^9 \frac{xz}{xz \quad + xz -; \; - \; .}$$ (Use $\ln x^2 = 2\ln x$, $x > 0$)

Using $\int - \; . \quad = \int \; - \; + \; - \; .$ 1the above rewrite as

$$f = \int_7^9 \frac{xz -; \; - \; .}{xz \quad + xz -; \; - \; .}$$

Adding the two, $7f = \int_7^9 \frac{xz \quad + xz -; \; - \; .}{xz \quad + xz -; \; - \; .} \quad = \int_7^9 \quad = 7 \;\; \therefore \; I = 1$

33. (c) : We have, $\int \frac{}{- \; +6.^{849} - \; -7.^{:\,49}} = \int \frac{}{\left(\frac{+6}{-7}\right)^{849} - \; -7.^7}$

Put $\frac{+6}{-7} = \quad \Rightarrow \quad \frac{-8}{- \; -7.^7} = -$

So, $f = \int \frac{}{-8 \;\; ^{849}} = \frac{-6}{8}\left(\frac{\frac{-8}{9}+6}{\frac{-8}{9}+6}\right) \quad = \frac{-9}{8} \; 649 + Y$

34. (b) : We have, $\left(\frac{6}{\;}\right) = \int_6^{64} \frac{xz}{6+}$ .... (i)

Put $\quad = \frac{6}{\;} \quad \Rightarrow \quad = -\frac{6}{7}$

$f(x) = \int_6 \frac{x\{ \, s}{7\left(6 + \frac{6}{\;}\right)} \quad = \int_6 \frac{x\{ \, s}{\cdot 6 + \; .}$

Now, $- \; . + \quad \left(\frac{6}{\;}\right) = \int_6 x\{ \, s \;\; \left[\frac{6}{6+} + \frac{6}{\cdot 6 + \; .}\right] \quad = \int_6 \frac{x\{ \, s}{}$

$$= \left[\frac{\cdot x\{ \, s \quad .^7}{7}\right]_6 = \frac{\cdot x\{ \, s \quad .^7}{7}$$

35. (d) : We have, $y + 2x^2 = 0$ ....(i)

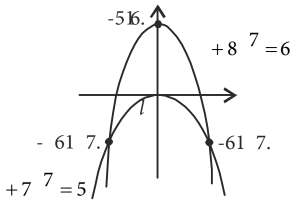
$y + 3x^2 = 1$ ....(ii)

Solving (i) and (ii), we get the point of intersection as (1, –2) and (–1, –2).

Area $= 7\int_5^6 \cdot -6 - 8 \;\; ^7 . - - -7 \;\; ^7 ..$

$$= 7\int_5^6 \cdot 6 - \;\; ^7. \quad = 7\left(\quad - \frac{8}{8}\right)_5^6$$

$= \frac{9}{8}$ sq. units



36. (b) : We have, $\int \frac{x\{ \, s - \; + \sqrt{6 + \;^7}.}{\sqrt{6 + \;^7}} \quad = \frac{6}{7} - \; - \; ..^7 + Y$ ...(i)

Differentiating (i) both sides, we get $\frac{x\{ \, s - \; + \sqrt{6 + \;^7}.}{\sqrt{6 + \;^7}} = g(t)g'(t)$

$\Rightarrow \quad - \; . = x\{ \, s - \; + \sqrt{6 + \;^7}. \quad \therefore \quad \cdot 7. = x\{ \, s \cdot 7 + \sqrt{:}.$

37. (b) : $f(2 - x) = f(2 + x) \Rightarrow$ Function is symmetrical about $x = 2$

and $f(4 - x) = f(4 + x) \Rightarrow$ Function is symmetrical about $x = 4$

$\Rightarrow f(x)$ is periodic with period 2.

$$\int_{65}^{:5} - \; . \quad = \int_{7\cdot:.}^{7\cdot7:.} \; - \; . \quad = \cdot 7: - :.\int_5^7 \; - \; . \quad = 75 \times : = 655$$

38. (b) : $\int_5^{-xz} \quad - \; . \quad = \frac{\sqrt{8}}{7}$ ...(i)

Differentiating (i) both sides w.r.t. x, we get

$-\!\!-\!\!\mathrm{ıɪ}$. o{ $-\ = \dfrac{\sqrt{8}}{7}$ ... (ii)

Putting $x = \dfrac{\pi}{8}$ in (ii), we get

$$\left(\frac{\sqrt{8}}{7}\right)\times\frac{6}{7}=\frac{\sqrt{8}}{7} \quad\Rightarrow\quad \left(\frac{\sqrt{8}}{7}\right)=\sqrt{8}$$

39. (a) : $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx = \int 1\cdot e^{x+\frac{1}{x}}dx+\int\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$

$= \int e^{x+\frac{1}{x}}dx + \int x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$

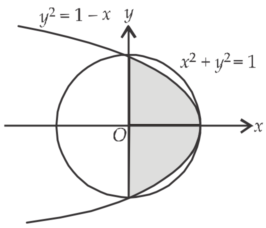
$= \int e^{x+\frac{1}{x}}dx + \left[x\int\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx - \int 1\cdot e^{x+\frac{1}{x}}dx\right]$

(Integration by parts)

$= \int e^{x+\frac{1}{x}}dx + x\,e^{x+\frac{1}{x}} - \int e^{x+\frac{1}{x}}dx = x\,e^{x+\frac{1}{x}}+c$

c being an arbitrary constant.

40. (d) : 1st solution :



$\text{Area} = \frac{\pi}{2}+2\int_0^1\sqrt{1-x}\,dx$

$= \frac{\pi}{2}+2\cdot\left.\frac{(1-x)^{3/2}}{3/2}\right|_0^1$

$= \frac{\pi}{2}+\frac{4}{3}[(1-x)^{3/2}]_0^1 = \frac{\pi}{2}+\frac{4}{3}$

2nd solution: $\text{Area} = \frac{\pi}{2}+2\int_0^1 x\,dy$

$= \frac{\pi}{2}+2\int_0^1(1-y^2)dy = \frac{\pi}{2}+2\left.\left(y-\frac{y^3}{3}\right)\right|_0^1 = \frac{\pi}{2}+2\left(1-\frac{1}{3}\right)=\frac{\pi}{2}+\frac{4}{3}$

41. (c): $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

$= \int_0^{\pi}\sqrt{\left(1-2\sin\frac{x}{2}\right)^2}\,dx = \int_0^{\pi}\left|1-2\sin\frac{x}{2}\right|dx$

$= \int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$

$= \left.\left(x+4\cos\frac{x}{2}\right)\right|_0^{\pi/3}+\left.\left(-4\cos\frac{x}{2}-x\right)\right|_{\pi/3}^{\pi} = -\frac{\pi}{3}+8\cdot\frac{\sqrt{3}}{2}-4 = 4\sqrt{3}-4-\frac{\pi}{3}$

42. (b) : Let $x^3 = u$, then $3x^2dx = du$

Also suppose $\int f(x)dx = \psi(x)$

Now $\int x^5 f(x^3)dx = \frac{1}{3}\int u\,f(u)du = \frac{1}{3}\left[u\int f(u)du - \int(\int f(u))du\right]$

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

43. (d) : $\frac{dy}{dx} = |x| = 2. \quad \therefore\ x = \pm 2$

We can solve for y to get

$y_1 = \int_0^2 |t|\,dt = \int_0^2 t\,dt = \left.\frac{t^2}{2}\right|_0^2 = 2$ and $y_2 = \int_0^{-2}|t|\,dt = -\int_0^{-2}t\,dt = -2$

Tangents are $y - 2 = 2(x - 2)$ and $y + 2 = 2(x + 2)$

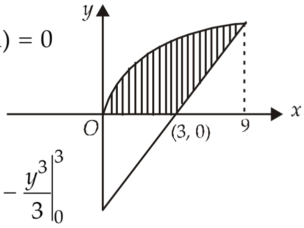
Then the x intercepts are obtained by putting $y = 0$.

We then get $x = \pm 1$

44. (d) : Solving $y = \sqrt{x}$ with $2y - x + 3 = 0$, we have

$2\sqrt{x} - x + 3 = 0 \Rightarrow (\sqrt{x}-3)(\sqrt{x}+1) = 0$

$\therefore\ x = 1, 9$



$\text{Area} = \int_0^3[(2y+3)-y^2]\,dy = \left. y^2+3y-\frac{y^3}{3}\right|_0^3$

$= 9 + 9 - 9 = 9$

45. (c) : $I = \int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\cot x}}$

Adding, $2I = \int_{\pi/6}^{\pi/3}\left(\frac{1}{1+\sqrt{\tan x}}+\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\right)dx$

$= \int_{\pi/6}^{\pi/3}1\cdot dx = \left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{6} \qquad \therefore\ I = \frac{\pi}{12}$

Again Statement-II is true.

46. (b) : $\int\frac{5\tan x}{\tan x-2}dx = x + a\ln|\sin x - 2\cos x| + k$

Differentiating both sides, we get $\frac{5\tan x}{\tan x-2} = 1+\frac{a(\cos x+2\sin x)}{\sin x-2\cos x}$

$\Rightarrow \frac{5\sin x}{\sin x-2\cos x} = \frac{\sin x(1+2a)+\cos x(a-2)}{\sin x-2\cos x} \Rightarrow a = 2$

47. (a, d) : $g(x) = \int_0^x \cos 4t\,dt \Rightarrow g(x) = \left[\frac{\sin 4t}{4}\right]_0^x = \frac{\sin 4x}{4}$

$\Rightarrow g(x+\pi) = \frac{\sin 4(x+\pi)}{4} = \frac{\sin 4x}{4}$

$\Rightarrow g(\pi) = 0 \Rightarrow g(x+\pi) = g(x)+g(\pi)$ or $g(x) - g(\pi)$.

48. (a) : $x^2 = \frac{y}{4}$, $x^2 = 9y$

Area bounded by the parabolas and $y = 2$

$= 2\times\int_0^2\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy = 5\int_0^2\sqrt{y}\,dy = 5\times\frac{(y)^{3/2}}{3/2} = \frac{10}{3}\times 2\sqrt{2} = \frac{20\sqrt{2}}{3}$

49. (c) : $\frac{d(p(t))}{dt} = 0.5p(t) - 450$

$\int_{850}^{p}\frac{2dp}{p-900} = \int_0^t dt \Rightarrow 2\ln\frac{p-900}{-50} = t \Rightarrow p = 900 - 50\cdot e^{t/2}$

If $p = 0$, then $\frac{900}{50} = e^{t/2} \Rightarrow t = 2\ln 18$

50. (d) : $\frac{dV}{dt} = -k(T-t)$

On integration, $V = \frac{k(T-t)^2}{2} + \alpha$

At $t = 0$, $V(t) = I$ $\Rightarrow$ $I = \frac{kT^2}{2} + \alpha$ $\therefore$ $\alpha = I - \frac{kT^2}{2}$

As $t = T$, we have $V(T) = \alpha = I - \frac{kT^2}{2}$

51. (c) : $\frac{dy}{dx} = y + 3$ $\Rightarrow$ $\frac{dy}{y+3} = dx$

As $y(0) = 2$, we have $\ln 5 = C$

Now $\ln(y + 3) = x + \ln 5$

As $x = \ln 2$ we have $\ln(y + 3) = \ln 2 + \ln 5 = \ln 10$

$\Rightarrow$ $y + 3 = 10$ $\Rightarrow$ $y = 7$

52. (c) : $I = \int_0^1 \frac{8\ln(1+x)}{1+x^2}dx$ Let $J = \int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Let $x = \tan\theta \Rightarrow J = \int_0^{\pi/4} \ln(1+\tan\theta)d\theta$

Now $J = \int_0^{\pi/4} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

Adding $2J = \int_0^{\pi/4} \ln(1+\tan\theta) + \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

$= \int_0^{\pi/4} \ln\left\{(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right\}d\theta$

$2J = \int_0^{\pi/4} (\ln 2)d\theta = \frac{\pi}{4}\ln 2$ $\Rightarrow$ $8J = 4\frac{\pi}{4}\ln 2$ $\Rightarrow$ $I = 8J = \pi\ln 2.$

53. (b) : $f(x) = \int_0^x \sqrt{t}\sin t\,dt$

$f'(x) = \sqrt{x}\sin x$

$f''(x) = \sqrt{x}\cos x + \frac{1}{2}x^{-1/2}\sin x$

$f''(\pi) = -\sqrt{\pi} < 0$; $f''(2\pi) = \sqrt{2\pi} > 0$

Thus at π maximum and at 2π minimum.

54. (a) : Area $= \frac{1}{2} + \int_1^e \frac{dx}{x} = \frac{1}{2} + \ln x\,\Big|_1^e = \frac{3}{2}$ sq. units

55. (b) : $p'(x) = p'(1-x)$

On integration, $p(x) = -p(1-x) + k$,

k being the constant of integration.

Set $x = 0$ to obtain $p(0) = -p(1) + k$

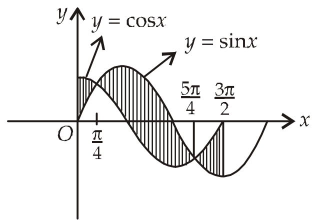
$\Rightarrow$ $1 = -41 + k$. $\therefore$ $k = 42$

Now, $I = \int_0^1 p(x)dx = \int_0^1 p(1-x)dx$

On adding we get $2I = \int_0^1 p(x) + p(1-x)\,dx = \int_0^1 k\,dx = \int_0^1 42\,dx = 42.$

Thus $I = 21$.

56. (a) :



The desired area =

$\int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)dx$

$= 2[\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$

(As the first and third integrals are equal in magnitude)

$= 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right) + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}} - 2 = 4\sqrt{2} - 2$

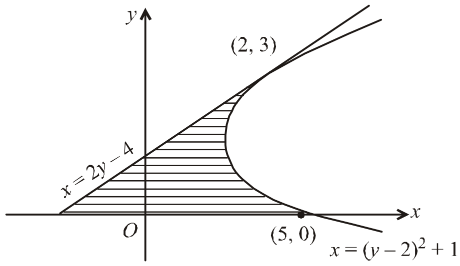
57. (b) : $(y-2)^2 = x - 1$

Differentiating w.r.t. x, we have $2(y-2)y' = 1$

$\Rightarrow$ $y' = \frac{1}{2(y-2)}$ at $(2, 3)$, $y' = 1/2$

The equation of the tangent to the parabola at (2, 3) is

$y - 3 = \frac{1}{2}(x - 2) \Rightarrow x - 2y + 4 = 0$



The area of the bounded region

$= \int_0^3 [(y-2)^2 + 1 - (2y-4)]dy$

$= \int_0^3 (y^2 - 6y + 9)\,dy = \int_0^3 (y-3)^2\,dy$. Put $t = 3 - y \Rightarrow dt = -dy$

$= \int_0^3 t^2 dt = \left[\frac{t^3}{3}\right]_0^3 = \frac{3^3}{3} = 9$

58. (c) : $I = \int_0^\pi [\cot x]dx$

$I = \int_0^\pi [\cot(\pi - x)]dx = \int_0^\pi [-\cot x]dx$

Adding we have $2I = \int_0^\pi \{[\cot x] + [-\cot x]\}dx$

$2I = \int_0^\pi (-1)dx = -\pi$ $\therefore$ $I = -\pi/2$

Note that $[x] + [-x] = 0$, $x \in Z$ and $= -1$, $x \notin Z$.

59. (d) : $\sqrt{2}\int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)}dx = \sqrt{2}\int \frac{\sin\left(x - \frac{\pi}{4} + \frac{\pi}{4}\right)}{\sin\left(x - \frac{\pi}{4}\right)}dx$

$= \sqrt{2}\int\left[\cos\frac{\pi}{4} + \cot\left(x - \frac{\pi}{4}\right)\sin\frac{\pi}{4}\right]dx$

$$= \sqrt{2}\cdot\frac{1}{\sqrt{2}}x + \sqrt{2}\cdot\frac{1}{\sqrt{2}}\ln\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c = x + \ln\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c$$

c being a constant of integration.

60. (a) : Solution $x + 2y^2 = 0$ and $x + 3y^2 = 1$ we have

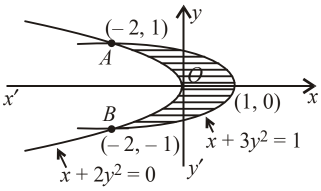
$1 - 3y^2 = -2y^2 \Rightarrow y^2 = 1$

$\therefore\ y = \pm 1$

$y = -1 \Rightarrow x = -2$

$y = 1 \Rightarrow x = -2$

The bounded region is as under



The desired area = $2\int_0^1[(1-3y^2)-(-2y^2)]dy$

$$=2\int_0^1(1-y^2)dy = 2\left[y-\frac{y^3}{3}\right]_0^1 = 2\times\frac{2}{3}=\frac{4}{3} \text{ sq. units}$$

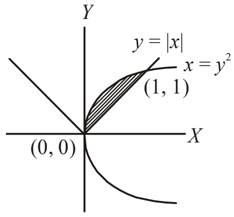
61. (c) : In the interval of integration $\sin x < x$

$$I=\int_0^1\frac{\sin x}{\sqrt{x}}dx<\int_0^1\frac{x}{\sqrt{x}}dx=\int_0^1\sqrt{x}dx=\left[\frac{2}{3}x^{3/2}\right]_0^1=\frac{2}{3}$$

$\therefore\ I<\frac{2}{3}$ Also $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx<\int_0^1\frac{1}{\sqrt{x}}dx=[2\sqrt{x}]_0^1=2$

$\therefore\ J < 2$

62. (a) : Required area



$$=\int_0^1(\sqrt{x}-x)dx$$

$$=\left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^1=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$$

63. (c) : $\frac{1}{2}\int\frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}=\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C.$

64. (c) : $\left[\sec^{-1}t\right]_{\sqrt{2}}^{x}=\frac{\pi}{2} \Rightarrow \sec^{-1}x-\sec^{-1}\sqrt{2}=\frac{\pi}{2}$

$$\Rightarrow \sec^{-1}x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4} \Rightarrow x=-\sqrt{2}.$$

65. (c) : $F(x)=\int_1^x\frac{\ln t}{1+t}dt+\int_1^{1/x}\frac{\ln t}{1+t}dt$

$$F(x)=\int_1^x\left(\frac{\ln t}{1+t}+\frac{\ln t}{(1+t)t}\right)dt=\int_1^x\frac{\ln t}{t}dt=\frac{1}{2}(\ln x)^2$$

$F(e) = 1/2.$

66. (b) : $\int_2^a[x]f'(x)dx$, say $[a] = K$ such that $a > 1$

$$=\int_1^2 1f'(x)dx+\int_2^3 2f'(x)dx+......+\int_{K-1}^{K}(K-1)f'(x)dx+\int_K^a Kf'(x)dx$$

$= f(2) - f(1) + 2[f(3) - f(2)] + 3[f(4) - f(3)] + ...$
$(K - 1)[f(K) - f(K - 1)] + K[f(a) - f(K)]$

$= -[f(1) + f(2) + ... + f(K)] + K f(a)$

$= [a] f(a) - [f(1) + f(2) + ... + f([a])]$

67. (c) : Let $I=\int_{\frac{-3\pi}{2}}^{\frac{-\pi}{2}}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$

Putting $x + \pi = z$

Also $x=\frac{-\pi}{2}\Rightarrow z=\frac{\pi}{2}$ and $x=\frac{-3\pi}{2}\Rightarrow z=\frac{-\pi}{2}$ $\therefore\ dx = dz$

and $x + 3\pi = z + 2\pi$

$$\therefore\ I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}[z^3+\cos^2(2\pi+z)]dz=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}z^3dz+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\cos^2 z\,dz$$

$= 0$ (an odd function) $+2\int_0^{\frac{\pi}{2}}\cos^2 z\,dz=0+2\frac{1}{2}\times\frac{\pi}{2}=\frac{\pi}{2}$

$$\left\{\text{Using fact }\int_0^{\frac{\pi}{2}}\sin^n x\,dx=\begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}......\frac{1}{2}\times\frac{\pi}{2} & \text{if } n=2m\\ \frac{n-1}{n}\frac{n-3}{n-2}......\frac{2}{2} & \text{if } n=2m+1\end{cases}\right\}$$

68. (d) : Let $I=\int_0^{\pi}xf(\sin x)dx$ ..... (i)

$I=\int_0^{\pi}(\pi-x)f(\sin x)dx$ ..... (ii)

Using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

By (i) & (ii) on adding, we get

$$\therefore\ I=\frac{\pi}{2}\int_0^{\pi}f(\sin x)dx=2\frac{\pi}{2}\int_0^{\frac{\pi}{2}}f(\sin x)dx$$

[Using $\int_0^{2a}f(x)dx=2\int_0^a f(x)dx$ if $f(2a - x) = f(x)$]

$$=\pi\int_0^{\frac{\pi}{2}}f\left(\sin\left(\frac{\pi}{2}-x\right)\right)dx=\pi\int_0^{\frac{\pi}{2}}f(\cos x)dx$$

69. (b) : Using fact $\int_a^b\frac{f(x)}{f(a+b+x)+f(x)}=\int_a^b f(x)dx=\frac{b-a}{2}$

$$\therefore\ \int_3^6\frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx=\frac{6-3}{2}=\frac{3}{2}$$

70. (c) : $\lim_{n\to\infty}\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right)+...+\frac{1}{n}\sec^2 1$

$$=\lim_{n\to\infty}\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right)+...+\frac{n}{n^2}\sec^2\left(\frac{n^2}{n^2}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{r=n}\left(\frac{r}{n^2}\right)\sec^2\left(\frac{r}{n}\right)^2=\lim_{n\to\infty}\sum_{r=0}^{r=n}\frac{1}{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2$$

$$=\int_0^1 x\sec^2(x^2)dx=\frac{1}{2}\tan 1.$$

71. (a) : Let $f(x)=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx\quad(a>0)$ ...(1)

$\therefore\ f(x)=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^{-x}}dx$ $\therefore\ \int_a^b f(x)dx=\int_a^b f(a+b-x)$

$\therefore \quad f(x) = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x} dx \quad \ldots (2)$

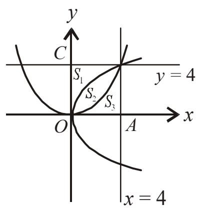
$2f(x) = \int_{-\pi}^{\pi} \cos^2 x \, dx = 2\int_0^{\pi} \cos^2 x \, dx$

$= 2 \times 2\int_0^{\pi/2} \cos^2 x \, dx$, $2f(x) = 4 \times \frac{1}{2} \times \frac{\pi}{2}$

$\left[\begin{array}{l} \text{By using } \int_0^{\pi/2} \sin^n x \, dx \\ = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \ldots\ldots \frac{1}{2} \times \frac{\pi}{2} \text{ if } n \text{ is even} \end{array}\right]$

$\Rightarrow f(x) = \frac{\pi}{2}$

72. (c) : Total area = 4 × 4 = 16 sq. units

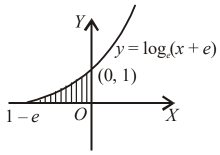


Area of $S_3 = \int_0^4 \frac{x^2}{4} dx = \frac{16}{3} = S_1$

$\therefore \quad S_2 = 16 - \frac{16}{3} \times 2 = \frac{16}{3}.$

$\therefore S_1 : S_2 : S_3$ is 1 : 1 : 1.

73. (b) : Required area = $\int_{1-e}^{0} \log_e (x+e) dx$



$= \int_1^e \log z \, dz$

$= \left[z(\log_e z - 1)\right]_1^e = 1.$

74. (a) : For 0 < x < 1, $\quad x^2 > x^3 \quad \therefore$ $\boxed{2^{x^2} > 2^{x^3}}$

and for 1 < x < 2, $\quad x^3 > x^2 \quad \therefore \; 2^{x^3} > 2^{x^2}$

i.e. $2^{x^2} < 2^{x^3} \Rightarrow I_3 < I_4$ as $2^{x^2} > 2^{x^3}$

$\therefore \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx \quad \therefore \quad I_1 > I_2$

75. (c) : According to question,

$\int_{\pi/4}^{B} f(x) dx = \int_{\pi/4}^{B(>\pi/4)} \left(B \sin B + \frac{\pi}{4} \cos B + B\sqrt{2}\right)$

$f(\beta) = \sin B + B\cos B - \frac{\pi}{4} \sin B + \sqrt{2} \;\therefore\; f\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{4} + \sqrt{2}.$

76. (c) : $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ (0/0) form, $= \lim_{x \to 2} \frac{f'(x) \times 4(f(x))^3}{1}$

$= 4f'(2) \times (f(2))^3 = \frac{1}{48} \times 4 \times 6 \times 6 \times 6 = 18.$

77. (c) : Consider $f(x) = \frac{x}{(\log x)^2 + 1}$

$\therefore \; f'(x) = \frac{1 + (\log x)^2 - \frac{2x\log x}{x}}{(1 + (\log x)^2)^2}$

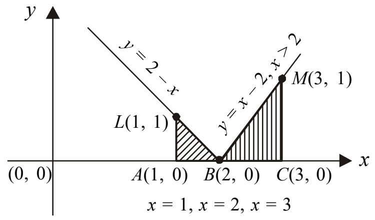
$\therefore \; f'(x) = \frac{1 + (\log x)^2 - 2\log x}{(1 + \log^2 x)^2} = \left(\frac{(1 - \log x)}{1 + (\log x)^2}\right)^2$

$\therefore \int \left(\frac{(1 - \log x)}{1 + (\log x)^2}\right)^2 dx = \int f'(x) \, dx = f(x)$

$\therefore \int \left(\frac{1 - \log x}{1 + (\log x)^2}\right)^2 dx = \frac{x}{1 + (\log x)^2} + C$

78. (c) : $y = \begin{cases} x - 2 & \text{if } x > 2 \\ 0 & \text{if } x = 0 \\ 2 - x & \text{if } x < 2 \end{cases}$

Required area = Area of ΔLAB + Area of ΔMBC



$= \frac{1}{2}[AL \times AB + BC \times CM] = \frac{1}{2}$ [1 × 1 + 1 × 1] = 1

79. (c) : As $f(x) = \frac{e^x}{1 + e^x}$

$\therefore \quad f(a) = \frac{e^a}{1 + e^a}$ and $f(-a) = \frac{e^{-a}}{1 + e^{-a}} \;\therefore\; f(-a) + f(a) = 1$

Now $\int_{f(-a)}^{f(a)} x\, g\{x(1-x)\} dx = \int_{f(-a)}^{f(a)} (1-x) g\{(1-x)(x)\} dx$

using $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$\Rightarrow \; 2\int_{f(-a)}^{f(a)} x\, g\{x(1-x)\} dx = \int_{f(-a)}^{f(a)} g\{(1-x)x\} dx \Rightarrow 2I_1 = I_2$

$\therefore \quad \frac{I_2}{I_1} = \frac{2}{1}$

80. (b) : $\int_0^{\pi} x f(\sin x) dx = A\int_0^{\frac{\pi}{2}} f(\sin x) dx$

or $A\int_0^{\frac{\pi}{2}} f(\sin x) dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x) dx = \int_0^{\pi} x f(\sin x) dx$

$\Rightarrow A\int_0^{\frac{\pi}{2}} f(\sin x) dx = \frac{\pi}{2} \times 2\int_0^{\frac{\pi}{2}} f(\sin x) dx$

$\Rightarrow A\int_0^{\frac{\pi}{2}} f(\sin x) = \pi\int_0^{\frac{\pi}{2}} f(\sin x) dx \Rightarrow A = \pi$

81. (a) : $\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}} dx = \int_0^{\pi/2} (\sin x + \cos x) dx$

$$= \left(\frac{\cos x}{-1} + \sin x\right)_0^{\frac{\pi}{2}} = 1 - (-1) = 2$$

82. (c) : $\int_{-2}^{3} \left|1 - x^2\right| dx = \int_{-2}^{3} \left|(1-x)(1+x)\right| dx$

Putting $1 - x^2 = 0$ $\therefore$ $x = \pm 1$

Points $-2, -1, 1, 3$

$\therefore$ $|1 - x^2| = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ (1-(1-x^2)) & \text{if } x < -1 \text{ and } x \geq 1 \end{cases}$ $\therefore$ $\int_{-2}^{3} \left|(1-x^2)\right| dx$

$$= \int_{-2}^{-1} (x^2 - 1)dx + \int_{-1}^{1} (1 - x^2)dx + \int_{1}^{3} (x^2 - 1)dx$$

$$= \frac{4}{3} + 2\left(\frac{2}{3}\right) + \frac{20}{3} = \frac{28}{3}$$

83. (d) : $\int \frac{1}{a\cos x - b\sin x} dx$ where $a = b = 1$

let $a = r\cos\theta = 1$

$b = r\sin\theta = 1$ $\therefore$ $r = \sqrt{2}$

$\theta = \tan^{-1}(b/a)$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\cos(x + \pi/4)} dx = \frac{1}{\sqrt{2}} \int \frac{1}{\sin\left(\frac{\pi}{2} + x + \frac{\pi}{4}\right)} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{2\sin\left(\frac{x}{2} + \frac{3\pi}{8}\right)\cos\left(\frac{x}{2} + \frac{3\pi}{8}\right)} dx$$

$$= \frac{1}{2\sqrt{2}} \int \frac{\sec^2\left(3\frac{\pi}{8} + \frac{x}{2}\right)}{\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)} dx = \frac{1}{2\sqrt{2}} \times 2\log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$$

$$= \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$$

84. (b) : $\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B\log\sin(x - \alpha) + C$

Differentiating w.r.t. x both sides

$\Rightarrow$ $\frac{\sin x}{\sin(x-\alpha)} = A + \frac{B\cos(x-\alpha)}{\sin(x-\alpha)}$

$\Rightarrow$ $\sin x = A\sin(x - \alpha) + B\cos(x - \alpha)$

$\sin x = A(\sin x\cos\alpha - \cos x\sin\alpha)$

$+ B(\cos x\cos\alpha + \sin x\sin\alpha)$

$\sin x = \sin x(A\cos\alpha + B\sin\alpha) + \cos x(B\cos\alpha - A\sin\alpha)$

Now solving $A\cos\alpha + B\sin\alpha = 1$ and $B\cos\alpha - A\sin\alpha = 0$

$(A, B) = (\cos\alpha, \sin\alpha)$

85. (b) : $\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{r=n} \frac{1}{n} e^{\frac{r}{n}} = \int_0^1 e^x dx = e - 1$

86. (c): Given $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$

$\Rightarrow$ $\int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx = F(k) - F(1)$

$\Rightarrow$ $\int_1^{64} \frac{e^{\sin z}}{z} dz = F(k) - F(1)$ where $(x^3 = z)$

$\Rightarrow$ $\left[F(z)\right]_1^{64} = F(k) - F(1) \Rightarrow F(64) - F(1) = F(k) - F(1)$

$\Rightarrow$ $k = 64$

87. (b) : $\underset{x\to 0}{\text{Lt}} \frac{(\tan t)_0^{x^2}}{x\sin x} = \underset{x\to 0}{\text{Lt}} \frac{\tan x^2}{x\sin x} = \underset{x\to 0}{\text{Lt}} \frac{\tan x^2}{x^2 \frac{\sin x}{x}}$

$$= \underset{x\to 0}{\text{Lt}} \frac{\tan x^2}{x^2} \frac{1}{\underset{x\to 0}{\text{Lt}} \frac{\sin x}{x}} = 1 \times 1 = 1$$

88. (b) : $\int_0^1 x(1-x)^n dx$

Putting $x = \sin^2\theta$

$dx = 2\sin\theta\cos\theta\, d\theta$ and $x = 0, \theta = 0$

$x = 1, \theta = \pi/2$

$\therefore$ $\int_0^1 x(1-x)^n dx = \int_0^{\pi/2} \sin^2\theta\cos^{2n}\theta\,(2\sin\theta\cos\theta)d\theta$

$$= 2\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^{2n+1}\theta\, d\theta$$

$\left[\text{Using } \int_0^{\pi/2} \sin^{2n+1}\theta\cos^{2n+1}\theta\, d\theta = \frac{[(2n)(2n-2)...2][(2n)(2n-2)...2]}{(4n+2)(4n)(4n-2)....2}\right]$

$\therefore$ $2\int_0^{\pi/2} \sin^3\theta\cos^{2n+1}\theta\, d\theta$

$$= \frac{2[2\times(2n)(2n-2)(2n-4)\,...4.2]}{(2n+4)(2n+2)(2n)(2n-2)...\,4.2} = \frac{2\times 2\times 1}{(2n+4)(2n+2)}$$

$$= \frac{1}{(n+2)(n+1)} = \frac{1}{n+1} - \frac{1}{n+2} \quad \text{(By partial fraction)}$$

89. (a, c) : Let $I = \int_a^b x f(x)\, dx$

$$I = \int_a^b (a + b - x)\, f(a + b - x)\, dx$$

$$I = \int_a^b (a+b)\, f(a + b - x)\, dx - \int_a^b x f(a + b - x)\, dx$$

$$I = \int_a^b (a+b) f(x)dx - \int_a^b x f(x)dx$$

$$\therefore\ I = \frac{a+b}{2}\int_a^b f(x)\,dx = \frac{a+b}{2}\int_a^b f(a+b-x)dx$$

90. (a) : From given $F(t) = \int_0^t f(t-y)\,g(y)\,dy$

$= \int_0^t e^{t-y}y\,dy$ (By replacing $y \to t - y$ in $f(y)$)

$F(t) = -\int_t^0 (t-\theta)e^{\theta}d\theta = \int_0^t (t-\theta)e^{\theta}d\theta$

$= [t\ e^{\theta}]_0^t - [(\theta - 1)\ e^{\theta}]_0^t = t(e^t - 1) - (t-1)e^t - 1$

$= e^t(t - t + 1) - t - 1 = e^t - (t+1)$

91. (b) : As $f(x) = f'(x)$ and $f(0) = 1 \Rightarrow \frac{f'(x)}{f(x)} = 1$

$\Rightarrow \log(f(x)) = x \Rightarrow f(x) = e^x + k \Rightarrow f(x) = e^x$ as $f(0) = 1$

Now $g(x) = x^2 - e^x$

$\therefore \int_0^1 f(x)\,g(x)\,dx = \int_0^1 e^x(x^2 - e^x)\,dx = \int_0^1 x^2e^x dx - \int_0^1 e^{2x}dx$

$= [(x^2 - 2x + 2)e^x]_0^1 - \left(\frac{e^{2x}}{2}\right)_0^1$

$= (e-2) - \left(\frac{e^2-1}{2}\right) = e - \frac{e^2}{2} - \frac{3}{2}$

Using $f^n(x)e^x\ dx = e^x[f^n(x) - f_1^n(x) + f_2^n(x) + ... + (-1)^n f_n(x)]$ where $f_1, f_2, ... f_n$ are derivatives of first, second ...n^{th} order.

92. (b) : Required area

$= \int_{-1}^0 (3+x)-(-x+1) + \int_0^1 (3-x)-(-x+1)dx + \int_1^2 (3-x)-(x-1)\,dx$

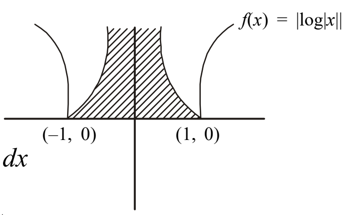
$= \int_{-1}^0 2(1+x)\,dx + \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx = 4$ sq. units

93. (a) : $\underset{n\to\infty}{\text{Lt}} \frac{1^p + 2^p + ... + n^p}{n^p} \times \frac{1}{n}$

$= \underset{n\to\infty}{\text{Lt}} \frac{1}{n}\left[\left(\frac{1}{n}\right)^p + \left(\frac{2}{n}\right)^p + ... + \left(\frac{n}{n}\right)^p\right]$

$= \underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{r=n} \frac{1}{n}\left(\frac{r}{n}\right)^p = \int_0^1 x^p dx = \frac{1}{p+1}$

94. (a) : Required Area



$= 2\int_0^1 \left|\log|x|\right| dx$

$= 2\left[\left(x\left|\log|x|\right|\right)\right]_0^1 - \int_0^1 \left(-\frac{1}{x}\right)\cdot x\,dx$

$= 2[(1-0) + (x)_0^1] = 4$ sq. units.

95. (b) : $2\int_{-\pi}^{\pi} \frac{x}{1+\cos^2 x}dx + 2\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

$= 0 + 2\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = 2 \cdot 2\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\ dx$

$= 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = 4 \times \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$

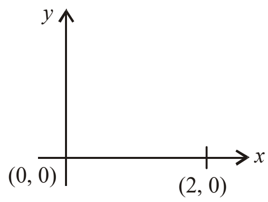
$\left(\text{By using } \int_0^{\pi} xf(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx\right)$

$= 4\frac{\pi}{2} \times 2 \times \int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx = 4\pi\ (\tan^{-1}\cos x)_{\frac{\pi}{2}}^{0}$

(By putting $\cos x = t$)

$= 4\pi \times \left(\frac{\pi}{4} - 0\right) = \pi^2$

96. (d) : Given $\int_0^2 f(x)\,dx = 3/4$



$\therefore \int_0^2 x\,f'(x)\,dx$

$= x\int_0^2 f'(x)\,dx - \int_0^2 f(x)\,dx = [x \cdot f(x)]_0^2 - 3/4 = 2f(2) - \frac{3}{4}$

$= 0 - \frac{3}{4}$ [$\because f(2) = 0$, curve having intercept 2 units on x-axis.]

$= -3/4$

97. (d) : $\int_{\pi}^{10\pi} |\sin x|\,dx = \int_0^{10\pi} |\sin x|\,dx - \int_0^{\pi} |\sin x|\,dx$

$= 10 \times 2 - 1 \times 2 = 18$ (Using period of $|\sin x| = \pi$)

98. (b) : $I_n = \int_0^{\pi/4} \tan^n x\,dx$ $\quad I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x\,dx$

$\therefore I_n + I_{n-2} = \int_0^{\pi/4} \tan^n x\,dx + \int_0^{\pi/4} \tan^{n-2} x\,dx$

$= \int_0^{\pi/4} \tan^{n-2} x \times (\sec^2 x - 1)\,dx + \int_0^{\pi/4} \tan^{n-2} x\,dx$

$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x\,dx$

$I_n + I_{n-2} = \frac{1}{n+1} \therefore n(I_n + I_{n-2}) = \frac{1}{1 + 1/n} \quad \therefore \underset{n\to\infty}{\text{Lt}} n(I_n + I_{n-2}) = 1$

99. (c) : $\int_0^{\sqrt{2}} [x^2]dx = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx$

$= 0 + \int_1^{\sqrt{2}} 1\,dx = \sqrt{2} - 1$