CHAPTER

9 Differential Calculus

1. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

(a) $\frac{9}{2}$ (b) 6 (c) $\frac{7}{2}$ (d) 4

(2018)

2. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+....+\left[\frac{15}{x}\right]\right)$

(a) does not exist in R (b) is equal to 0

(c) is equal to 15 (d) is equal to 120 *(2018)*

3. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is

(a) $2\sqrt{2}$ (b) 3 (c) -3 (d) $-2\sqrt{2}$

(2018)

4. Let $S = \{t \in R : f(x) = |x - \pi|\cdot(e^{|x|} - 1)\sin|x|$ is not differentiable at $t\}$, then the set S is equal to

(a) $\{0, \pi\}$ (b) ϕ (an empty set)

(c) $\{0\}$ (d) $\{\pi\}$ *(2018)*

5. If a right circular cone, having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is :

(a) $6\sqrt{3}\,\pi$ (b) $6\sqrt{2}\,\pi$

(c) $8\sqrt{2}\,\pi$ (d) $8\sqrt{3}\,\pi$ *(Online 2018)*

6. If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim_{x\to 0} \frac{f'(x)}{x}$

(a) exists and is equal to 0

(b) exists and is equal to -2

(c) exists and is equal to 2

(d) does not exist *(Online 2018)*

7. Let $S = \{(\lambda, \mu) \in R \times R : f(t) = (|\lambda|\, e^{|t|} - \mu)\sin(2|t|)$, $t \in R$, is a differentiable function$\}$.Then S is a subset of :

(a) $[0, \infty) \times R$ (b) $R \times (-\infty, 0)$

(c) $R \times [0, \infty)$ (d) $(-\infty, 0) \times R$

(Online 2018)

8. If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point $(-2, 0)$ is

(a) -34 (b) -32 (c) -2 (d) 4

(Online 2018)

9. Let $f(x)$ be a polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$. If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right)=3$ then $f(-1)$is equal to

(a) $\frac{5}{2}$ (b) $\frac{9}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

(Online 2018)

10. $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ equals :

(a) $-\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

(Online 2018)

11. If $f(x) = \sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$, then $f'\left(-\frac{1}{2}\right)$ equals

(a) $\sqrt{3}\log_e\sqrt{3}$ (b) $-\sqrt{3}\log_e 3$

(c) $-\sqrt{3}\log_e\sqrt{3}$ (d) $\sqrt{3}\log_e 3$

(Online 2018)

12. Let $f(x) = \begin{cases} (x-1)^{\frac{1}{2-x}}, & x > 1, x \neq 2 \\ k, & x = 2 \end{cases}$

The value of k for which f is continuous at $x = 2$ is

(a) e^{-1} (b) e (c) e^{-2} (d) 1

(Online 2018)

13. If the function f defined as $f(x) = \frac{1}{x} - \frac{k-1}{e^{2x}-1}$, $x \neq 0$ is continuous at $x = 0$, then the ordered pair $(k, f(0))$ is equal to :

(a) $\left(\frac{1}{3}, 2\right)$ (b) (3, 2) (c) (2, 1) (d) (3, 1)

(Online 2018)

14. If $x = \sqrt{2^{\mathrm{cosec}^{-1} t}}$ and $y = \sqrt{2^{\sec^{-1} t}}$ $(|t| \geq 1)$, then $\frac{dy}{dx}$ is equal to :

(a) $-\frac{y}{x}$ (b) $\frac{x}{y}$ (c) $-\frac{x}{y}$ (d) $\frac{y}{x}$

(Online 2018)

15. Let M and m be respectively the absolute maximum and the absolute minimum value of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0,3]. Then $M - m$ is equal to :
(a) 5 (b) 1 (c) 4 (d) 9
(Online 2018)

16. $\lim\limits_{x \to 0} \dfrac{(27+x)^{1/3} - 3}{9 - (27+x)^{2/3}}$ equals
(a) $-\dfrac{1}{3}$ (b) $\dfrac{1}{6}$ (c) $-\dfrac{1}{6}$ (d) $\dfrac{1}{3}$
(Online 2018)

17. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cot x - \cos x}{(\pi - 2x)^3}$ equals
(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{8}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{24}$ *(2017)*

18. If for $x \in \left(0, \dfrac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals
(a) $\dfrac{3x\sqrt{x}}{1-9x^3}$ (b) $\dfrac{3x}{1-9x^3}$
(c) $\dfrac{3}{1+9x^3}$ (d) $\dfrac{9}{1+9x^3}$ *(2017)*

19. The normal to the curve $y(x-2)(x-3) = x + 6$ at the point where the curve intersects the y-axis passes through the point
(a) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ (b) $\left(\dfrac{1}{2}, -\dfrac{1}{3}\right)$
(c) $\left(\dfrac{1}{2}, \dfrac{1}{3}\right)$ (d) $\left(-\dfrac{1}{2}, -\dfrac{1}{2}\right)$ *(2017)*

20. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is
(a) 10 (b) 25 (c) 30 (d) 12.5 *(2017)*

21. If $y = \left[x + \sqrt{x^2 - 1}\right]^{15} + \left[x - \sqrt{x^2 - 1}\right]^{15}$,
then $(x^2 - 1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is equal to
(a) $225y^2$ (b) $224y^2$
(c) $125y$ (d) $225y$ *(Online 2017)*

22. The tangent at the point (2, –2) to the curve $x^2y^2 - 2x = 4(1 - y)$ does not pass through the point
(a) (–2, –7) (b) (–4, –9)
(c) $\left(4, \dfrac{1}{3}\right)$ (d) (8, 5) *(Online 2017)*

23. $\lim\limits_{x \to 3} \dfrac{\sqrt{3x} - 3}{\sqrt{2x-4} - \sqrt{2}}$ is equal to
(a) $\sqrt{3}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $\dfrac{1}{2\sqrt{2}}$ (d) $\dfrac{1}{\sqrt{2}}$
(Online 2017)

24. If $\lim\limits_{n \to \infty} \dfrac{1^a + 2^a + \dots + n^a}{(n+1)^{a-1}[(na+1) + (na+2) + \dots + (na+n)]} = \dfrac{1}{60}$ for some positive real number a, then a is equal to
(a) $\dfrac{17}{2}$ (b) 8
(c) 7 (d) $\dfrac{15}{2}$ *(Online 2017)*

25. If $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$ and $(x^2 - 1)\dfrac{d^2y}{dx^2} + \lambda x\dfrac{dy}{dx} + ky = 0$, then $\lambda + k$ is equal to
(a) –23 (b) –24
(c) 26 (d) –26 *(Online 2017)*

26. Let f be a polynomial function such that $f(3x) = f'(x) \cdot f''(x)$, for all $x \in R$. Then
(a) $f(2) - f'(2) + f''(2) = 10$ (b) $f''(2) - f(2) = 4$
(c) $f''(2) - f'(2) = 0$ (d) $f(2) + f'(2) = 28$
(Online 2017)

27. A tangent to the curve, $y = f(x)$ at $P(x, y)$ meets x-axis at A and y-axis at B. If $AP : BP = 1 : 3$ and $f(1) = 1$, then the curve also passes through the point
(a) $\left(\dfrac{1}{2}, 4\right)$ (b) $\left(\dfrac{1}{3}, 24\right)$
(c) $\left(2, \dfrac{1}{8}\right)$ (d) $\left(3, \dfrac{1}{28}\right)$ *(Online 2017)*

28. The value of k for which the function
$$f(x) = \begin{cases} \left(\dfrac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0 < x < \dfrac{\pi}{2} \\ k + \dfrac{2}{5}, & x = \dfrac{\pi}{2} \end{cases}$$
is continuous at $x = \dfrac{\pi}{2}$, is
(a) $\dfrac{17}{20}$ (b) $\dfrac{3}{5}$
(c) $-\dfrac{2}{5}$ (d) $\dfrac{2}{5}$ *(Online 2017)*

29. The function f defined by $f(x) = x^3 - 3x^2 + 5x + 7$ is
(a) decreasing in R.
(b) increasing in R.
(c) decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$.
(d) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$.
(Online 2017)

30. Let $= \text{xy}_{\to 5^+}\ \text{-6+}\mu\text{nz}^{7}\sqrt{\ }.^{\frac{6}{7}}$, then log$p$ is equal to
(a) 2 (b) 1 (c) 1/2 (d) 1/4 *(2016)*

31. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then
(a) g is not differentiable at $x = 0$
(b) $g'(0) = \cos(\log 2)$
(c) $g'(0) = -\cos(\log 2)$
(d) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$
(2016)

32. Consider $- \ . = \mu\text{nz}^{-6}\left(\sqrt{\dfrac{6+\text{-uz}}{6-\text{-uz}}}\right)1 \in \left(51\dfrac{\pi}{7}\right)3$

A normal to $y = f(x)$ at $x = \frac{\pi}{2}$ also passes through the point

(a) (0, 0) (b) $\left(51\frac{7\pi}{8}\right)$

(c) $\left(\frac{\pi}{;}1\,5\right)$ (d) $\left(\frac{\pi}{9}1\,5\right)$ *(2016)*

33. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then

(a) $2x = (\pi + 4)r$ (b) $(4 - \pi)x = \pi r$

(c) $x = 2r$ (d) $2x = r$ *(2016)*

34. If m and M are the minimum and the maximum values of $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x, x \in R$, then $M - m$ is equal to

(a) $\frac{9}{4}$ (b) $\frac{15}{4}$

(c) $\frac{7}{4}$ (d) $\frac{1}{4}$ *(Online 2016)*

35. If $f(x)$ is a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$, for each $x > 0$, then $f\left(\frac{3}{2}\right)$ is equal to

(a) $\frac{23}{18}$ (b) $\frac{13}{6}$

(c) $\frac{25}{9}$ (d) $\frac{31}{18}$ *(Online 2016)*

36. If the function $f(x) = \begin{cases} -x, & x < 1 \\ a + \cos^{-1}(x + b), & 1 \le x \le 2 \end{cases}$ is differentiable at $x = 1$, then $\frac{a}{b}$ is equal to

(a) $\frac{\pi + 2}{2}$ (b) $\frac{\pi - 2}{2}$

(c) $\frac{-\pi - 2}{2}$ (d) $-1 - \cos^{-1}(2)$ *(Online 2016)*

37. If $\lim_{x \to \infty}\left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$, then '$a$' is equal to

(a) 2 (b) $\frac{3}{2}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$ *(Online 2016)*

38. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$, $t \in R$, meets the curve again at a point Q, then the coordinates of Q are

(a) $(16t^2 + 3, -64t^3 - 1)$

(b) $(4t^2 + 3, -8t^3 - 1)$

(c) $(t^2 + 3, t^3 - 1)$

(d) $(t^2 + 3, -t^3 - 1)$ *(Online 2016)*

39. Let $a, b \in R$, $(a \neq 0)$. If the function f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^3}, & \sqrt{2} \le x < \infty \end{cases}$$

is continuous in the interval $[0, \infty)$, then an ordered pair (a, b) is

(a) $(-\sqrt{2}, 1 - \sqrt{3})$ (b) $(\sqrt{2}, -1 + \sqrt{3})$

(c) $(\sqrt{2}, 1 - \sqrt{3})$ (d) $(-\sqrt{2}, 1 + \sqrt{3})$

(Online 2016)

40. Let C be a curve given by $y(x) = 1 + \sqrt{4x - 3}, x > \frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is

(a) (1, 7) (b) (3, –4) (c) (4, –3) (d) (2, 3)

(Online 2016)

41. Let $f(x) = \sin^4 x + \cos^4 x$. Then f is an increasing, function in the interval

(a) $\left]\frac{5\pi}{8}, \frac{3\pi}{4}\right[$ (b) $\left]\frac{\pi}{2}, \frac{5\pi}{8}\right[$

(c) $\left]\frac{\pi}{4}, \frac{\pi}{2}\right[$ (d) $\left]0, \frac{\pi}{4}\right[$ *(Online 2016)*

42. $\lim_{x \to 0} \frac{(1 - \cos 2x)^2}{2x \tan x - x \tan 2x}$ is

(a) 2 (b) $\frac{1}{2}$

(c) –2 (d) $\frac{1}{2}$ *(Online 2016)*

43. $\underset{\to 5}{\text{xy}} \frac{\text{-6} - \text{o\{} -7 \quad . \text{-8} + \text{o\{} - \quad .}{\text{µnz } 9}$ is equal to

(a) 2 (b) $\frac{1}{2}$

(c) 4 (d) 3 *(2015, 2013)*

44. If the function $\text{-} \quad . = \begin{cases} \sqrt{\quad +6} \;\; 1 \;\; 5 \le \quad \le 8 \\ +7 \quad 1 \;\; 8 < \quad \le : \end{cases}$ is differentiable, then the value of $k + m$ *is*

(a) $\frac{65}{8}$ (b) 4

(c) 2 (d) $\frac{6;}{:}$ *(2015)*

45. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$. If $\underset{\to 5}{\text{xy}}\left[6 + \frac{\text{-} \quad .}{7}\right] = 8$ then $f(2)$ is equal to

(a) 0 (b) 4 (c) –8 (d) –4

(2015)

46. The normal to the curve, $x^2 + 2xy - 3y^2$ at (1, 1)
(a) meets the curve again in the first quadrant.
(b) meets the curve again in the fourth quadrant.
(c) does not meet the curve again.
(d) meets the curve again in the second quadrant.
(2015)

47. xy →5 (7 −o{− / −uz 7) u-q}×mxµ{
(a) 3 (b) 3/2 (c) 5/4 (d) 2
(Online 2015)

48. The distance, from the origin, of the normal to the curve, $x = 2\cos t + 2t \sin t$, $y = 2\sin t - 2t \cos t$ at $t = \pi/4$ is
(a) 4 (b) $7\sqrt{7}$ (c) 2 (d) $\sqrt{7}$
(Online 2015)

49. If Rolle's theorem holds for the function $f(x) = 2x^3 + bx^2 + cx$, $x \in [-1, 1]$, at the point $x = 1/2$, then $2b + c$ equals
(a) 1 (b) −1 (c) 2 (d) −3
(Online 2015)

50. Let the tangents drawn to the circle, $x^2 + y^2 = 16$ from the point $P(0, h)$ meet the x-axis at points A and B. If the area of ΔAPB is minimum, then h is equal to
(a) $9\sqrt{8}$ (b) $8\sqrt{8}$
(c) $8\sqrt{7}$ (d) $9\sqrt{7}$ *(Online 2015)*

51. Let k be a non-zero real number. If

- . = { (- −6.7) / (−uz(—) x{ s (6 + —/9)) 1 ≠5 ; 67 1 =5 }

is a continuous function, then the value of k is
(a) 1 (b) 2 (c) 3 (d) 4
(Online 2015)

52. The equation of a normal to the curve,
−uz = −uz(π/8 +) mµ =51 u-
(a) 7 $+\sqrt{8}$ =5 (b) 7 $-\sqrt{8}$ =5
(c) 7 $+\sqrt{8}$ =5 (d) 7 $-\sqrt{8}$ =5
(Online 2015)

53. Let k and K be the minimum and the maximum values of the function - . = (-6+ .53) / (6+ 53) in [0, 1] respectively, then the ordered pair (k, K) is equal to
(a) $(1, 2^{0.6})$ (b) $(2^{-0.4}, 2^{0.6})$
(c) $(2^{-0.6}, 1)$ (d) $(2^{-0.4}, 1)$ *(Online 2015)*

54. From the top of a 64 metres high tower, a stone is thrown upwards vertically with the velocity of 48 m/s. The greatest height (in metres) attained by the stone, assuming the value of the gravitational acceleration $g = 32$ m/s^2, is
(a) 100 (b) 88
(c) 128 (d) 112 *(Online 2015)*

55. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha\log|x| + \beta x^2 + x$, then
(a) $\alpha = -6, \beta = -\frac{1}{2}$ (b) $\alpha = 2, \beta = -\frac{1}{2}$
(c) $\alpha = 2, \beta = \frac{1}{2}$ (d) $\alpha = -6, \beta = \frac{1}{2}$ *(2014)*

56. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to
(a) 1 (b) $-\pi$
(c) π (d) $\pi/2$ *(2014)*

57. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$
(a) $2f'(c) = 3g'(c)$ (b) $f'(c) = g'(c)$
(c) $f'(c) = 2g'(c)$ (d) $2f'(c) = g'(c)$ *(2014)*

58. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to
(a) $5x^4$ (b) $\frac{1}{1+\{g(x)\}^5}$
(c) $1 + \{g(x)\}^5$ (d) $1 + x^5$ *(2014)*

59. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is
(a) 3000 (b) 3500 (c) 4500 (d) 2500
(2013)

60. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to
(a) $\frac{1}{2}$ (b) 1
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$ *(2013)*

61. Consider the function, $f(x) = |x - 2| + |x - 5|$, $x \in R$
Statement 1 : $f'(4) = 0$
Statement 2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

62. If $f : R \to R$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

(a) discontinuous only at non-zero integral values of x.
(b) continuous only at $x = 0$.
(c) continuous for every real x.
(d) discontinuous only at $x = 0$. *(2012)*

63. Let $a, b \in R$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement 1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

64. A spherical balloon is filled with 4500 π cubic metres of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic metres per minute, then the rate (in metres per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is

(a) 2/9 (b) 9/2 (c) 9/7 (d) 7/9 *(2012)*

65. $\frac{d^2x}{dy^2}$ equals to

(a) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (b) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(c) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (d) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ *(2011)*

66. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(a) equals $-\sqrt{2}$ (b) equals $\frac{1}{\sqrt{2}}$

(c) does not exist (d) equals $\sqrt{2}$ *(2011)*

67. The values of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all x in R, are

(a) $p = -\frac{3}{2}, q = \frac{1}{2}$ (b) $p = \frac{1}{2}, q = \frac{3}{2}$

(c) $p = \frac{1}{2}, q = -\frac{3}{2}$ (d) $p = \frac{5}{2}, q = \frac{1}{2}$ *(2011)*

68. Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$, $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(a) 4 (b) -4
(c) 0 (d) -2 *(2010)*

69. Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)} =$

(a) 1 (b) 2/3 (c) 3/2 (d) 3 *(2010)*

70. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at $x = -1$, then a possible value of k is

(a) 1 (b) 0 (c) $-1/2$ (d) -1 *(2010)*

71. Let $f : R \to R$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$.

Statement-1 : $f(c) = 1/3$, for some $c \in R$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in R$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

72. Let $f(x) = x|x|$ and $g(x) = \sin x$.

Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : gof is twice differentiable at $x = 0$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1. *(2009)*

73. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$:

(a) $P(-1)$ is not minimum but $P(1)$ is the maximum of P
(b) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P
(c) neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P
(d) $P(-1)$ is the minimum and $P(1)$ is the maximum of P *(2009)*

74. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

(a) 1 (b) log 2 (c) $-\log 2$ (d) -1 *(2009)*

75. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?

(a) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(b) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(c) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(d) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ *(2008)*

76. Let $f(x)=\begin{cases}(x-1)\sin\frac{1}{x-1} & \text{if } x\neq 1\\ 0 & \text{if } x=1\end{cases}$
Then which one of the following is true?
(a) f is differentiable at $x = 1$ but not at $x = 0$
(b) f is neither differentiable at $x = 0$ nor at $x = 1$
(c) f is differentiable at $x = 0$ and at $x = 1$
(d) f is differentiable at $x = 0$ but not at $x = 1$ *(2008)*

77. How many real solutions does the equation
$x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?
(a) 5 (b) 7 (c) 1 (d) 3 *(2008)*

78. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2 *(2007)*

79. The function $f : R - \{0\} \to R$ given by
$$f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$$
can be made continuous at $x = 0$ by defining $f(0)$ as
(a) 0 (b) 1 (c) 2 (d) –1 *(2007)*

80. Let $f : R \to R$ be a function defined by $f(x) = \min \{x + 1, |x| + 1\}$. Then which of the following is true?
(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x) \geq 1$ for all $x \in R$
(d) $f(x)$ is not differentiable at $x = 1$ *(2007)*

81. The function $f(x) = \tan^{-1} (\sin x + \cos x)$ is an increasing function in
(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (c) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{4}\right)$ *(2007)*

82. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is
(a) $\log_3 e$ (b) $\log_e 3$
(c) $2 \log_3 e$ (d) $\frac{1}{2}\log_e 3$ *(2007)*

83. If $x^m \cdot y^n = (x + y)^{m+n}$, then dy/dx is
(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$ (c) xy (d) $\frac{x}{y}$ *(2006)*

84. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is
(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$ (c) $\frac{1}{2}x^2$ (d) πx^2 *(2006)*

85. The set of points where $f(x)=\frac{x}{1+|x|}$ is differentiable, is
(a) $(-\infty, 0) \cup (0, \infty)$ (b) $(-\infty, -1) \cup (-1, \infty)$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$ *(2006)*

86. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (c) $\pi/4$ *(2006)*

87. The function $g(x)=\frac{x}{2}+\frac{2}{x}$ has a local minimum at
(a) $x = 2$ (b) $x = -2$
(c) $x = 0$ (d) $x = 1$ *(2006)*

88. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is
(a) 1/4 (b) 41 (c) 1 (d) 17/7 *(2006)*

89. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is
(a) $\frac{1}{18\pi}$ cm/min (b) $\frac{1}{36\pi}$ cm/min
(c) $\frac{5}{6\pi}$ cm/min (d) $\frac{1}{54\pi}$ cm/min *(2005)*

90. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to
(a) 0 (b) $\frac{a^2}{2}(\alpha-\beta)^2$
(c) $\frac{1}{2}(\alpha-\beta)^2$ (d) $\frac{-a^2}{2}(\alpha-\beta)^2$ *(2005)*

91. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that
(a) it makes angle $\frac{\pi}{2}+\theta$ with x-axis
(b) it passes through the origin
(c) it is at a constant distance from the origin
(d) it passes through $\left(\frac{a\pi}{2},-a\right)$ *(2005)*

92. If f is a real-valued differentiable function satisfying $|f(x)-f(y)| \leq (x-y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals
(a) 1 (b) 2 (c) 0 (d) – 1 *(2005)*

93. Let f be the differentiable for $\forall$ x. If $f(1) = -2$ and $f'(x) \geq 2$ for [1, 6], then
(a) $f(6) < 8$ (b) $f(6) \geq 8$
(c) $f(6) = 5$ (d) $f(6) < 5$ *(2005)*

94. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals

(a) 4 (b) 3 (c) 6 (d) 5 *(2005)*

95. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is

(a) ab (b) $2ab$ (c) a/b (d) $\sqrt{ab}$ *(2005)*

96. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(a) (2, 3) (b) (1, 2) (c) (0, 1) (d) (1, 3) *(2004)*

97. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

(a) $(x + 1)^3$ (b) $(x - 1)^3$
(c) $(x - 1)^2$ (d) $(x + 1)^2$ *(2004)*

98. Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$.

$f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) –1 *(2004)*

99. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are

(a) $a \in R$, $b = 2$ (b) $a = 1$, $b \in R$
(c) $a \in R$, $b \in R$ (d) $a = 1$ and $b = 2$ *(2004)*

100. Let $f(a) = g(a) = k$ and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if

$$\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4,$$

then the value of k is

(a) 2 (b) 1 (c) 0 (d) 4 *(2003)*

101. $\lim_{x\to\pi/2}\frac{[1-\tan(x/2)][1-\sin x]}{[1+\tan(x/2)][\pi-2x]^3}$ is

(a) 0 (b) 1/32 (c) ∞ (d) 1/8 *(2003)*

102. The value of

$$\lim_{n\to\infty}\frac{1+2^4+3^4+....+n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^3+3^3+....+n^3}{n^5}$$ is

(a) zero (b) 1/4 (c) 1/5 (d) 1/30 *(2003)*

103. The real number x when added to its inverse gives the minimum value of the sum at x equal to

(a) 1 (b) –1 (c) –2 (d) 2 *(2003)*

104. If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0, \\ 0, & x=0 \end{cases}$

then $f(x)$ is

(a) continuous for all x, but not differentiable at $x = 0$
(b) neither differentiable not continuous at $x = 0$
(c) discontinuous everywhere
(d) continuous as well as differentiable for all x *(2003)*

105. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(a) 1 (b) 2 (c) 1/2 (d) 3 *(2003)*

106. If $f(x) = x^n$, then the value of

$$f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+....+\frac{(-1)^n f^n(1)}{n!}$$ is

(a) 2^{n-1} (b) 0 (c) 1 (d) 2^n *(2003)*

107. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is

(a) –1/3 (b) 2/3 (c) –2/3 (d) 0 *(2003)*

108. If $2a + 3b + 6c = 0$ ($a, b, c \in R$) then the quadratic equation $ax^2 + bx + c = 0$ has

(a) at least one root in (0, 1)
(b) at least one root in [2, 3]
(c) at least one root in [4, 5]
(d) none of these *(2002)*

109. Let $f(2) = 4$ and $f'(2) = 4$ then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ equals

(a) 2 (b) – 2 (c) – 4 (d) 3 *(2002)*

110. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{\frac{1}{x}} =$

(a) e^4 (b) e^2 (c) e^3 (d) 1 *(2002)*

111. If $f(x + y) = f(x) \cdot f(y)$ $\forall$ x, y and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is

(a) 0 (b) 1 (c) 6 (d) 2 *(2002)*

112. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) –1
(c) 0 (d) does not exist *(2002)*

113. The maximum distance from origin of a point on the curve $x = a\sin t - b\sin\left(\frac{at}{b}\right)$, $y = a\cos t - b\cos\left(\frac{at}{b}\right)$, both $a, b > 0$ is

(a) $a - b$ (b) $a + b$
(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$ *(2002)*

114. If $f(1) = 1$, $f'(1) = 2$, then $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(a) 2 (b) 4 (c) 1 (d) 1/2 *(2002)*

115. $f(x)$ and $g(x)$ are two differentiable function on [0, 2] such that $f''(x)-g''(x)=0$, $f'(1)=2g'(1)=4$, $f(2)=3g(2)=9$ then $f(x)- g(x)$ at $x = 3/2$ is

(a) 0 (b) 2
(c) 10 (d) 5 *(2002)*

116. f is defined in [–5, 5] as

$$f(x)=\begin{cases} x, & \text{if } x \text{ is rational and} \\ -x, & \text{if } x \text{ is irrational. Then} \end{cases}$$

(a) $f(x)$ is continuous at every x, except $x = 0$
(b) $f(x)$ is discontinuous at every x, except $x = 0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere *(2002)*

117. $\lim\limits_{x\to 0}\dfrac{\log x^n-[x]}{[x]}$, $n\in N$, ([x] denotes greatest integer less than or equal to x)
(a) has value – 1 (b) has value 0
(c) has value 1 (d) does not exist *(2002)*

118. If $y=(x+\sqrt{1+x^2})^n$, then $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}$ is
(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$ *(2002)*

ANSWER KEY

1. (a)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (c)	**11.** (a)	**12.** (a)
13. (d)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (b)	**21.** (d)	**22.** (a)	**23.** (d)	**24.** (c)
25. (b)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (c)	**31.** (b)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (a)
37. (b)	**38.** (d)	**39.** (c)	**40.** (a)	**41.** (c)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (c)
49. (b)	**50.** (d)	**51.** (c)	**52.** (a)	**53.** (d)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (c)	**58.** (c)	**59.** (b)	**60.** (d)
61. (a)	**62.** (c)	**63.** (a)	**64.** (a)	**65.** (b)	**66.** (c)	**67.** (a)	**68.** (b)	**69.** (a)	**70.** (d)	**71.** (a)	**72.** (b)
73. (a)	**74.** (d)	**75.** (b)	**76.** (b)	**77.** (c)	**78.** (c)	**79.** (b)	**80.** (a)	**81.** (d)	**82.** (c)	**83.** (a)	**84.** (c)
85. (c)	**86.** (a)	**87.** (a)	**88.** (b)	**89.** (a)	**90.** (b)	**91.** (a, c)	**92.** (c)	**93.** (b)	**94.** (d)	**95.** (b)	**96.** (c)
97. (b)	**98.** (a)	**99.** (b)	**100.** (d)	**101.** (b)	**102.** (c)	**103.** (a)	**104.** (a)	**105.** (b)	**106.** (b)	**107.** (b)	**108.** (a)
109. (c)	**110.** (d)	**111.** (c)	**112.** (a)	**113.** (a)	**114.** (a)	**115.** (d)	**116.** (b)	**117.** (d)	**118.** (a)		

Explanations

1. (a) : Let the curves $y^2 = 6x$ and $9x^2 + by^2 = 16$ intersect at (α, β).

$\beta^2 = 6\alpha$ and $9\alpha^2 + b\beta^2 = 16$

$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$ for curve $y^2 = 6x$ is $\frac{dy}{dx} = \frac{3}{y} = \frac{3}{\beta}$

$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$ for curve $9x^2 + by^2 = 16$ is $\frac{dy}{dx} = -\frac{9x}{by} = -\frac{9\alpha}{b\beta}$

As the curves are orthogonal, we have $\left(\frac{3}{\beta}\right)\left(-\frac{9\alpha}{b\beta}\right) = -1$

As $\beta^2 = 6\alpha$, we get $27\alpha = b(\alpha \cdot 6)$

$\Rightarrow\ b = \frac{27}{6} = \frac{9}{2}$ (as $\alpha \neq 0$)

For $b = 0$ the intersection is non orthogonal. So we can rule out $b = 0$ in the beginning only to conclude $\alpha \neq 0$ in the end.

2. (d) : Observe that $t - 1 < [t] \leq t$

Applying this to numbers $\frac{1}{x}, \frac{2}{x}, \ldots\ldots, \frac{15}{x}$ and summing them, we have $\sum_{k=1}^{15} \frac{k}{x} - 15 < \sum_{k=1}^{15}\left[\frac{k}{x}\right] \leq \sum_{k=1}^{15}\frac{k}{x}$

Multiplying throughout by x, we have

$$\sum_{k=1}^{15} k - 15x < x\sum_{k=1}^{15}\left[\frac{k}{x}\right] \leq \sum_{k=1}^{15} k$$

Putting the limit $x \to 0^+$, we have $120 < L \leq 120$

As the limit from both sides approaches to 120, we have by sandwich principle, the required limit = 120.

3. (a) : We have, $f(x) = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

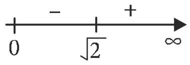
And $g(x) = x - \frac{1}{x}$ for $x \in R - \{-1, 0, 1\}$

$$\therefore\ h(x) = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{x - \frac{1}{x}} = \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}$$

Put $x - \frac{1}{x} = t$

$\therefore\ H(t) = t + \frac{2}{t}$ for $t \in (-\infty, \infty) - \{0\}$

Consider $H(t)$ on the interval $(0, \infty)$



$H(t) = t + \frac{2}{t}$ $H'(t) = 1 - \frac{2}{t^2}$

So, $H(t)$ is decreasing on $(0, \sqrt{2})$ and increasing on $(\sqrt{2}, \infty)$.

Thus $H(t)$ has a local minimum at $t = \sqrt{2}$.

$\therefore\ H(\sqrt{2}) = 2\sqrt{2}$ is the local minimum value of the function at $\sqrt{2}$.

Remark : Observe that $t + \frac{2}{t} \geq 2\sqrt{2}$ thereby again contribute that $2\sqrt{2}$ is a local minimum.

4. (b) : At $x = 0$, we have L.H.D. $= \lim_{h\to 0}\frac{f(0-h) - f(0)}{-h}$

$$= \lim_{h\to 0}\frac{(\pi + h)(e^h - 1)\sin h}{(-h)} = \pi(0)(-1) = 0$$

$$\text{R.H.D.} = \lim_{h\to 0}\frac{f(0+h) - f(0)}{h} = \lim_{h\to 0}\frac{(\pi - h)(e^h - 1)\sin h}{h}$$

$= (\pi)(0)(1) = 0$

Let's check at $x = \pi$

$$\text{L.H.D.} = \lim_{h\to 0}\frac{f(\pi - h) - f(\pi)}{-h}$$

$$= \lim_{h\to 0^+}\frac{h(e^{\pi - h} - 1)\sin h}{-h} = (0)\,(e^{\pi} - 1)(-1) = 0$$

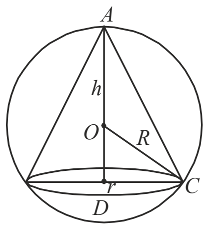
$$\text{R.H.D.} = \lim_{h\to 0}\frac{f(\pi + h) - f(\pi)}{h}$$

$$= \lim_{h\to 0}\frac{h(e^{\pi + h} - 1)(-\sin h)}{h} = (0)\,(e^{\pi} - 1)(-1) = 0$$

Thus f is differentiable at both $x = 0$ and $x = \pi$.

Remark : This happens as $x = 0$ and $x = \pi$ both are repeated roots of the given function.

5. (d) : Let cone of radius r and height h is inscribed in a sphere of radius, $R = 3$



Now, $OD = AD - OA = h - 3$

In ΔODC, $(OC)^2 = (OD)^2 + (CD)^2$

$\Rightarrow\ (3)^2 = (h - 3)^2 + r^2$

$\Rightarrow\ 9 = h^2 + 9 - 6h + r^2$

$\Rightarrow\ r^2 = 6h - h^2$ ...(i)

Volume of cone $(V) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h(6h - h^2)$

$= \frac{1}{3}\pi(6h^2 - h^3)$

Now, $\frac{dV}{dh} = \frac{\pi}{3} \times (12h - 3h^2)$

Put $\frac{dV}{dh} = 0 \Rightarrow 3h^2 - 12h = 0$

$\Rightarrow\ 3h(h - 4) = 0 \Rightarrow h = 4$

Also, $\frac{d^2V}{dh^2} < 0$ at $h = 4$

Put $h = 4$ in (i) we get, $r^2 = 24 - 16 = 8$

$\therefore$ Slant height, $l = \sqrt{h^2 + r^2} = \sqrt{16 + 8} = \sqrt{24}$

Now, curved surface area of cone $= \pi r l$

$= \pi \times 2\sqrt{2} \times 2\sqrt{6} = 8\sqrt{3}\pi$.

6. (b) : Given, $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$f(x) = \cos x(x^2 - 2x^2) - x(2\sin x - 2x\tan x) + 1(2x\sin x - x^2\tan x)$

$= -x^2\cos x + x^2\tan x = x^2(\tan x - \cos x)$

$f'(x) = 2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)$

$$\lim_{x\to 0}\frac{f'(x)}{x} = \lim_{x\to 0}\left[\frac{2x(\tan x - \cos x)}{x} + \frac{x^2(\sec^2 x + \sin x)}{x}\right]$$

$= 2(-1) + 0 = -2$

7. (c) : Given, $f(t) = (|\lambda|\, e^{|t|} - \mu)\,.\,\sin(2\,|t|)$

R.H.D. (at $x = 0$) $= \lim_{h\to 0}\frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0}\frac{(|\lambda|e^h - \mu)\sin 2h}{h} = \lim_{h\to 0}(|\lambda|e^h - \mu)\frac{2\sin h\cos h}{h}$

$= 2(|\lambda| - \mu)$

L.H.D. (at $x = 0$) $= \lim_{h\to 0}\frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0}\frac{(|\lambda|e^h - \mu)\sin 2h}{-h}$

$= -2(|\lambda| - \mu)$

R.H.D. (at $x = 0$) = L.H.D. (at $x = 0$)

$\Rightarrow 4|\lambda| = 4\mu \Rightarrow |\lambda| = \mu \Rightarrow \mu \ge 0$ and $\lambda \in R$

8. (a) : Given, $x^2 + y^2 + \sin y = 4$ ...(i)

On differentiating (i) both sides, we get

$2x + 2y\frac{dy}{dx} + \cos y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2x}{2y + \cos y}$ ...(ii)

On differentiating (ii) both sides, we get

$$\frac{d^2y}{dx^2} = \frac{(2y + \cos y)(-2) - (-2x)(2 - \sin y)\frac{dy}{dx}}{(2y + \cos y)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2(2y + \cos y) + 2x(2 - \sin y)\frac{dy}{dx}}{(2y + \cos y)^2}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{(-2,\,0)} = \frac{-2(0+1) + 2(-2)(2-0)\cdot(4)}{(0+1)^2} \quad \left(\because \left.\frac{dy}{dx}\right|_{(-2,\,0)} = 4\right)$$

$= -2 - 32 = -34$

9. (b) : Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ ...(i)

Given, $\lim_{x\to 0}\left(\frac{f(x)}{x^2} + 1\right) = 3 \Rightarrow \lim_{x\to 0}\left(ax^2 + bx + c + \frac{d}{x} + \frac{e}{x^2} + 1\right) = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$ [$\because$ limit exists finitely, so $d = e = 0$]

$\therefore f(x) = ax^4 + bx^3 + 2x^2 \Rightarrow f'(x) = 4ax^3 + 3bx^2 + 4x$

Given that $f(x)$ has extreme values at $x = 1$ and $x = 2$

$\therefore f'(1) = 0$ and $f'(2) = 0$

$\Rightarrow 4a + 3b + 4 = 0$ ...(ii) and $32a + 12b + 8 = 0$ ...(iii)

From (ii) and (iii), we get $a = \frac{1}{2}$, $b = -2$

Thus, $f(x) = \frac{1}{2}x^4 - 2x^3 + 2x^2 \;\therefore\; f(-1) = \frac{1}{2} + 2 + 2 = \frac{9}{2}$

10. (c) : Let $f(x) = \lim_{x\to 0}\frac{x\tan 2x - 2x\tan x}{(1 - \cos 2x)^2}$

$$= \lim_{x\to 0}\frac{\frac{x\sin 2x}{\cos 2x} - \frac{2x\sin x}{\cos x}}{4\sin^4 x}$$

$$= \lim_{x\to 0}\frac{x\cos x\cdot\sin 2x - 2x\sin x\cos 2x}{4\sin^4 x}\times\lim_{x\to 0}\frac{1}{\cos 2x\cdot\cos x}$$

$$= \lim_{x\to 0}\frac{2x\cos^2 x\sin x - 2x\sin x\cos 2x}{4\sin^4 x}\times 1$$

$$= \lim_{x\to 0}\frac{x(\cos^2 x - \cos 2x)}{2\sin^3 x} = \lim_{x\to 0}\frac{x(\cos^2 x - \cos^2 x + \sin^2 x)}{2\sin^3 x}$$

$$= \lim_{x\to 0}\frac{x}{2\sin x} = \lim_{x\to 0}\frac{1}{\frac{2\sin x}{x}} = \frac{1}{2}\lim_{x\to 0}\frac{1}{\frac{\sin x}{x}} = \frac{1}{2}$$

11. (a) : $f(x) = \sin^{-1}\left(\frac{2\times 3^x}{1 + 9^x}\right)$

Put $3^x = \tan t \Rightarrow t = \tan^{-1} 3^x$

$\therefore f(x) = \sin^{-1}\left(\frac{2\tan t}{1 + \tan^2 t}\right) = \sin^{-1}(\sin 2t) = 2t = 2\tan^{-1}(3^x)$

$\Rightarrow f'(x) = \frac{2}{1 + 9^x}\times 3^x\log 3 \Rightarrow f'\left(\frac{1}{2}\right) = \sqrt{3}\times\frac{1}{2}\log 3 = \sqrt{3}\log\sqrt{3}$

12. (a) : Since the function $f(x)$ is continuous at $x = 2$.

$\therefore \lim_{x\to 2} f(x) = f(2) \Rightarrow \lim_{x\to 2}(x-1)^{\frac{1}{2-x}} = k$

$\Rightarrow e^{\lim_{x\to 2}[(x-1)-1]\frac{1}{2-x}} = k$ [$\because$ L.H.S. is in the form of 1^∞]

$\Rightarrow e^{\lim_{x\to 2} -\left(\frac{x-2}{x-2}\right)} = k \Rightarrow e^{-1} = k \Rightarrow k = \frac{1}{e}$

13. (d) : $f(x) = \frac{1}{x} - \frac{k-1}{e^{2x} - 1}$; $x \ne 0$

Given, $f(x)$ is continuous at $x = 0$

$\Rightarrow f(0) = \lim_{x\to 0}\left(\frac{1}{x} - \frac{k-1}{e^{2x}-1}\right) \Rightarrow f(0) = \lim_{x\to 0}\frac{e^{2x} - 1 - x(k-1)}{x(e^{2x}-1)}$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{\left(1 + (2x) + \frac{1}{2!}(2x)^2 + \dots\right) - 1 - x(k-1)}{x(e^{2x}-1)}$$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{\left(2x + \frac{1}{2!}(2x)^2 + \dots\right) - x(k-1)}{2x^2\left(\frac{e^{2x}-1}{2x}\right)}$$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{\left(2x + \frac{1}{2!}(2x)^2 + \dots\right) - x(k-1)}{2x^2}$$

$$= \lim_{x\to 0}\frac{\left((2 - k + 1)x + 2x^2 + \frac{4}{3}x^3 + \dots\right)}{2x^2} = \lim_{x\to 0}\left[\left(\frac{3-k}{2}\right)\frac{1}{x} + 1 + \frac{2}{3}x + \dots\right]$$

Since, $f(x)$ is continuous at $x = 0$

$\therefore 3 - k = 0 \Rightarrow k = 3 \;\therefore f(0) = 1$

14. (a) : $x = \sqrt{2^{\mathrm{cosec}^{-1}t}}$ ...(i), $y = \sqrt{2^{\sec^{-1}t}}$ ...(ii)

Differentiating (i) and (ii) w.r.t. 't', we get

$$\frac{dx}{dt} = \frac{1}{2\sqrt{2^{\mathrm{cosec}^{-1}t}}}\times(2^{\mathrm{cosec}^{-1}t}\log 2)\times\frac{-1}{t\sqrt{t^2-1}}$$

$$\Rightarrow \quad \frac{dx}{dt}=\frac{-(2^{\operatorname{cosec}^{-1}t})^{1/2}\log 2}{2t\sqrt{t^2-1}}=\frac{-x\log 2}{2t\sqrt{t^2-1}} \quad \text{...(iii)}$$

$$\text{and} \quad \frac{dy}{dt}=\frac{1}{2\sqrt{2^{\sec^{-1}t}}}\times(2^{\sec^{-1}t}\log 2)\times\frac{1}{t\sqrt{t^2-1}}$$

$$=\frac{(2^{\sec^{-1}t})^{1/2}\log 2}{2t\sqrt{t^2-1}}=\frac{y\log 2}{2t\sqrt{t^2-1}} \quad \text{...(iv)}$$

$$\therefore \quad \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{y\log 2/2t\sqrt{t^2-1}}{-x\log 2/2t\sqrt{t^2-1}}=-\frac{y}{x}.$$

15. (d) : $f(x) = 2x^3 - 9x^2 + 12x + 5,\ x \in [0, 3]$

$\Rightarrow f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x - 1)(x - 2)$

For $f'(x) = 0$, $(x - 1)(x - 2) = 0 \ \Rightarrow x = 1$ or $x = 2$

Now, $f(1) = 10$, $f(2) = 9$, $f(0) = 5$, $f(3) = 14$

$\therefore$ $M = 14$ and $m = 5$

So, $M - m = 14 - 5 = 9$

16. (c) : We have, $\displaystyle \lim_{x\to 0}\frac{(27+x)^{1/3}-3}{9-(27+x)^{2/3}}=\lim_{x\to 0}\frac{3\left[\left(1+\frac{x}{27}\right)^{1/3}-1\right]}{9\left[1-\left(1+\frac{x}{27}\right)^{2/3}\right]}$

$$=\lim_{x\to 0}\frac{\left[\left(1+\frac{x}{3\times 27}+...\right)-1\right]}{3\left[1-\left(1+\frac{2x}{3\times 27}+...\right)\right]}=\lim_{x\to 0}\frac{1[x/81]}{3[-2x/81]}=-\frac{1}{6}$$

17. (a) : We have $\displaystyle \lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}=\lim_{h\to 0}\frac{(-\tan h)-(-\sin h)}{(-2h)^3}$

$$=-\frac{1}{8}\lim_{h\to 0}\frac{\sin h-\tan h}{h^3}=\frac{1}{8}\lim_{h\to 0}\frac{\tan h\,(1-\cos h)}{h^3}$$

$$=\frac{1}{8}\lim_{h\to 0}\left(\frac{\tan h}{h}\right)\left(\frac{2\sin^2\frac{h}{2}}{h^2}\right)=\frac{1}{8}\cdot 1\cdot 2\cdot\frac{1}{4}=\frac{1}{16}.$$

18. (d) : Let $u=\tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^3}\right),\ x\in\left(0,\dfrac{1}{4}\right)$

$$=\tan^{-1}\left(\frac{2\cdot(3x^{3/2})}{1-(3x^{3/2})^2}\right)=2\tan^{-1}(3x^{3/2})$$

This holds as $3x^{3/2} \in (0, 3/8)$

Differentiating with respect to x, we obtain

$$\frac{du}{dx}=2\cdot\frac{1}{1+9x^3}\cdot 3\cdot\frac{3}{2}\cdot x^{\frac{1}{2}}=\frac{9\sqrt{x}}{1+9x^3}$$

$$\Rightarrow \quad \sqrt{x}\cdot g(x)=\frac{9\sqrt{x}}{1+9x^3} \Rightarrow \quad g(x)=\frac{9}{1+9x^3}.$$

19. (a) : We have, $y(x - 2)(x - 3) = x + 6$

It meets the y-axis where $x = 0$, *i.e.* $y(6) = 6 \ \therefore\ y = 1$

The point of intersection is (0,1).

$$\text{Now,} \quad y=\frac{x+6}{x^2-5x+6} \quad \text{...(i)}$$

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx}=\frac{(x^2-5x+6)\cdot 1-(x+6)\cdot(2x-5)}{(x^2-5x+6)^2}$$

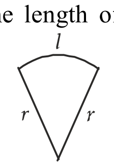
$$\text{Now,} \quad \left.\frac{dy}{dx}\right|_{x=0}=\frac{6-(6)(-5)}{36}=1$$

$\therefore$ Slope of normal = –1

Then the equation to curve is $y - 1 = -1(x - 0)$

i.e., $x + y - 1 = 0$.

20. (b) : Let r be the radius of circle and l the length of arc of the circle.



Now $l + 2r = 20$ (given)

Also $l = r\theta \quad \Rightarrow \theta r + 2r = 20$

$$\therefore \quad \theta=\frac{20-2r}{r}$$

$$\text{Now,} \quad A=\frac{\pi r^2\theta}{2\pi}=\frac{r^2}{2}\cdot\frac{20-2r}{r}=r(10-r)$$

We have $\dfrac{dA}{dr} = 10 - 2r$

$$\frac{dA}{dr}=0 \ \Rightarrow\ r=5 \quad \text{Also,} \quad \frac{d^2A}{dr^2}=-2<0$$

$\therefore$ $A\ (r)$ is maximum at $r = 5$.

Area = 5(10 – 5) = 25

Alternative solution : We have, $A = r(10 - r)$

Applying A.M. & G.M. inequality, we get

$$\sqrt{r(10-r)}\le\frac{r+10-r}{2} \quad i.e., \ \sqrt{r(10-r)}\le 5 \quad \therefore\ r(10-r)\le 25$$

Then the maximum area is 25 and is achieved at $r = 10 - r$ *i.e.,* $r = 5$.

21. (d) : $\displaystyle \frac{dy}{dx}=15\left(x+\sqrt{x^2-1}\right)^{14}\left(1+\frac{x}{\sqrt{x^2-1}}\right)$

$$+15\left(x-\sqrt{x^2-1}\right)^{14}\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

$$=15\frac{\left(x+\sqrt{x^2-1}\right)^{15}}{\sqrt{x^2-1}}-\frac{15\left(x-\sqrt{x^2-1}\right)^{15}}{\sqrt{x^2-1}}$$

$$=\frac{15}{\sqrt{x^2-1}}\left[\left(x+\sqrt{x^2-1}\right)^{15}-\left(x-\sqrt{x^2-1}\right)^{15}\right]$$

$$\Rightarrow \sqrt{x^2-1}\frac{dy}{dx}=15\left[\left(x+\sqrt{x^2-1}\right)^{15}-\left(x-\sqrt{x^2-1}\right)^{15}\right]$$

$$\Rightarrow \quad \sqrt{x^2-1}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

$$=15\left(x+\sqrt{x^2-1}\right)^{14}\left(1+\frac{x}{\sqrt{x^2-1}}\right)+15\left(x-\sqrt{x^2-1}\right)^{14}\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

$$=\frac{15}{\sqrt{x^2-1}}\left(15\left(x+\sqrt{x^2-1}\right)^{15}+15\left(x+\sqrt{x^2-1}\right)^{15}\right)=\frac{225y}{\sqrt{x^2-1}}$$

$$\Rightarrow \quad \left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=225y$$

22. (a) : We have, $x^2y^2 - 2x = 4(1 - y)$

$\Rightarrow$ $x^2y^2 - 2x = 4 - 4y$

Differentiating both sides w.r.t. x, we get

$$2xy^2 + 2y \cdot x^2 \frac{dy}{dx} - 2 = -4\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2yx^2 + 4) = 2 - 2xy^2 \Rightarrow \frac{dy}{dx} = \frac{2 - 2xy^2}{2yx^2 + 4}$$

$$\left.\frac{dy}{dx}\right|_{(2,-2)} = \frac{2 - 2 \times 2 \times (-2)^2}{2(-2) \times (2)^2 + 4} = \frac{-14}{-12} = \frac{7}{6}$$

$\therefore$ Slope of tangent to the curve $= \frac{7}{6}$

Equation of tangent passes through (2, –2) is

$$y + 2 = \frac{7}{6}(x - 2) \Rightarrow 7x - 6y = 26$$

$\therefore$ Equation of tangent does not pass through (–2, –7).

23. (d) : We have $\lim\limits_{x \to 3} \frac{\sqrt{3x} - 3}{\sqrt{2x - 4} - \sqrt{2}}$

$$= \lim_{x \to 3} \frac{\sqrt{3x} - 3}{\sqrt{2x - 4} - \sqrt{2}} \times \frac{\sqrt{2x - 4} + \sqrt{2}}{\sqrt{2x - 4} + \sqrt{2}}$$

$$= \lim_{x \to 3} \frac{(\sqrt{3x} - 3)(\sqrt{2x - 4} + \sqrt{2})}{(2x - 4 - 2)}$$

$$= \lim_{x \to 3} \frac{(\sqrt{3x} - 3)(\sqrt{3x} + 3)(\sqrt{2x - 4} + \sqrt{2})}{(2x - 6)(\sqrt{3x} + 3)}$$

$$= \lim_{x \to 3} \frac{(3x - 9)(\sqrt{2x - 4} + \sqrt{2})}{2(x - 3)(\sqrt{3x} + 3)} = \lim_{x \to 3} \frac{3(x - 3)(\sqrt{2x - 4} + \sqrt{2})}{2(x - 3)(\sqrt{3x} + 3)}$$

$$= \frac{3}{2} \times \frac{(\sqrt{2} + \sqrt{2})}{(3 + 3)} = \frac{1}{\sqrt{2}}$$

24. (c)

25. (b) : $y^{1/5} + y^{-1/5} = 2x \Rightarrow \left(\frac{1}{5}y^{-4/5} - \frac{1}{5}y^{-6/5}\right)\frac{dy}{dx} = 2$

$\Rightarrow y'\,(y^{1/5} - y^{-1/5}) = 10\,y \Rightarrow y'\left(2\sqrt{x^2 - 1}\right) = 10y$

$[\because 2x = y^{1/5} + y^{-1/5} \Rightarrow y^{1/5} - y^{-1/5} = 2\sqrt{x^2 - 1}]$

$$\Rightarrow y''\left(2\sqrt{x^2 - 1}\right) + y'\left(2\frac{2x}{2\sqrt{x^2 - 1}}\right) = 10y'$$

$\Rightarrow y''\,(x^2 - 1) + xy' = 5\sqrt{x^2 - 1}(y') \Rightarrow y''\,(x^2 - 1) + xy' - 25y = 0$

$\therefore \lambda = 1,\ k = -25$ So, $\lambda + k = 1 - 25 = -24$

26. (c) : Let $f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow f(3x) = 27ax^3 + 9bx^2 + 3cx + d$

$f'(x) = 3ax^2 + 2bx + c,\ f''(x) = 6ax + 2b$

Now, $f(3x) = f'(x)\,f''(x)$

$\Rightarrow 27ax^3 + 9bx^2 + 3cx + d = 18a^2x^3 + (6ab + 12ab)\,x^2 + (4b^2 + 6ac)x + 2bc$

Comparing coeff. of like powers, we get

$27a = 18a^2,\ 9b = 18ab,\ 3c = 4b^2 + 6ac,\ d = 2bc$

$\Rightarrow a = \frac{3}{2}, b = 0, c = 0, d = 0 \quad \therefore f(x) = \frac{3}{2}x^3$

$f'(x) = \frac{9}{2}x^2,\ f''(x) = 9x$

$\therefore f(2) = 12,\ f'(2) = 18,\ f''(2) = 18$

So, $f''(2) - f'(2) = 0$

27. (c)

28. (b) : $\because f(x)$ is continuous at $x = \frac{\pi}{2}$.

$$\therefore \lim_{x \to \frac{\pi}{2}} \left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}} = f\left(\frac{\pi}{2}\right) \Rightarrow \left(\frac{4}{5}\right)^0 = k + \frac{2}{5}$$

$$\Rightarrow k + \frac{2}{5} = 1 \Rightarrow k = 1 - \frac{2}{5} = \frac{3}{5}$$

29. (b) : $f(x) = x^3 - 3x^2 + 5x + 7$

$\Rightarrow f'(x) = 3x^2 - 6x + 5 = 3(x - 1)^2 + 2 > 0,\ \forall\ x \in R$

So, $f(x)$ is increasing in R.

30. (c) : We have, $= \underset{\to 5^+}{\text{xy}}\ \text{-6+µnz}^7\sqrt{\ }.^{\frac{6}{7}}$

It is of the form 1^∞ hence the limit is given by

$$= \underset{\to 5}{\text{xy}} \frac{\text{µnz}^7\sqrt{\ }}{7} = \underset{\to 5}{\text{xy}} \frac{6}{7}\left(\frac{\text{µnz}\sqrt{\ }}{\sqrt{\ }}\right)^7 = \frac{6}{7} = \sqrt{\ } \quad \therefore \log p = \frac{1}{2}$$

31. (b) : As we are concerned about differentiability at '0' in the vicinity of $\sin x$

$f(x) = \log 2 - \sin x$

$g(x) = f(f(x)) = \log 2 - \sin(\log 2 - \sin x)$

As g is sum of two differentiable functions, so g is differentiable.

$g'(x) = \cos(\log 2 - \sin x)\cdot\cos x$

Then $g'(0) = \cos(\log 2)$.

32. (b) : - . = µnz$^{-6}\left(\sqrt{\frac{6+\text{-uz}}{6-\text{-uz}}}\right)$

$= \text{µnz}^{-6}\left(\frac{6+\text{-uz}}{\sqrt{6-\text{-uz}^7}}\right) = \text{µnz}^{-6}\left(\frac{6+\text{-uz}}{\sqrt{\text{o\{}-^7}}\right)$

$= \text{µnz}^{-6}\left(\frac{6+\text{-uz}}{\text{o\{}-}\right)$ H– $\in\left(51\frac{\pi}{7}\right)$

$= \text{µnz}^{-6}\left[\text{µnz}\left(\frac{\pi}{9} + \frac{\ }{7}\right)\right] = \frac{\pi}{9} + \frac{\ }{7} \therefore$ ′- . $= \frac{6}{7} \quad \therefore$ ′$\left(\frac{\pi}{;}\right) = \frac{6}{7}$

Equation of normal is $-\frac{\pi}{8} = -7\left(\ -\frac{\pi}{;}\right)$ 33 7 + $= \frac{7\pi}{8}$

It passes through $\left(51\frac{7\pi}{8}\right)$

33. (c) : We have from hypothesis, $4x + 2\pi r = 2$

$\therefore \quad = \frac{6-7}{\pi}$

Area, $W = \ ^7 + \pi\ ^7 = \ ^7 + \frac{\pi}{\pi^7}\text{-7}\ -6.^7 = \ ^7 + \frac{6}{\pi}\text{-7}\ -6.^7$

For maximum/minimum

$\frac{W}{\ } = 5 \Rightarrow 7\ + \frac{9}{\pi}\text{-7}\ -6. = 5 \quad \therefore \quad = \frac{7}{\pi + 9}$

Also, $\frac{^7W}{7} > 5$ at this value. Thus there is a minimum.

Again, on comparing, $x = 2r$

34. (a) : We have, $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$

$= 4 + 2(1 - \cos^2 x)\cos^2 x - 2\cos^4 x$

$= 4 + 2\cos^2 x - 4\cos^4 x$

$$= -4\left\{\cos^4 x - \frac{\cos^2 x}{2} - 1\right\} = -4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\}$$

Now, $0 \le \cos^2 x \le 1$

$$\Rightarrow -\frac{1}{4} \le \cos^2 x - \frac{1}{4} \le \frac{3}{4} \Rightarrow 0 \le \left(\cos^2 x - \frac{1}{4}\right)^2 \le \frac{9}{16}$$

$$\Rightarrow -\frac{17}{16} \le \left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16} \le -\frac{1}{2}$$

$$\Rightarrow 2 \le -4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\} \le \frac{17}{4}$$

$\therefore$ Maximum value, $M = \frac{17}{4}$ and minimum value, $m = 2$

$\therefore \; M - m = \frac{17}{4} - 2 = \frac{9}{4}$

35. (d) : Let $L = \lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Applying L' Hospital rule, $L = \lim_{t \to x} \frac{2t\,f(x) - x^2 f'(t)}{1} = 1$

or $2xf(x) - x^2 f'(x) = 1 \Rightarrow f'(x) - \frac{2}{x} f(x) = \frac{-1}{x^2}$

I.F. $= e^{\int \frac{-2}{x} dx} = e^{-2\log x} = \frac{1}{x^2}$

Solution is $[f(x)]\frac{1}{x^2} = \int \frac{1}{x^2}\left(-\frac{1}{x^2}\right)dx \Rightarrow \frac{f(x)}{x^2} = \frac{1}{3x^3} + C$

We have, $f(1) = 1$

$\Rightarrow 1 = \frac{1}{3} + C \Rightarrow C = \frac{2}{3} \quad \therefore \; f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$

$\therefore \; f\left(\frac{3}{2}\right) = \frac{2}{3} \times \left(\frac{3}{2}\right)^2 + \frac{1}{3} \times \frac{2}{3} = \frac{31}{18}$

36. (a) : We have, $f(x) = \begin{cases} -x, & x < 1 \\ a + \cos^{-1}(x + b), & 1 \le x \le 2 \end{cases}$

Since, $f(x)$ is differentiable at $x = 0$, therefore continuous.

$\therefore \; \lim_{x \to 1^-} (-x) = \lim_{x \to 1^+} (a + \cos^{-1}(x + b)) = f(1)$

$\Rightarrow -1 = a + \cos^{-1}(1 + b) \Rightarrow \cos^{-1}(1 + b) = -1 - a$ ...(i)

Since, $f(x)$ is differentiable. $\therefore$ L.H.D. = R.H.D.

L.H.D. $= \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{-(1-h) - (-1)}{-h}$

$= \lim_{h \to 0} \frac{-1 + h + 1}{-h} = \lim_{h \to 0} \frac{h}{-h} = -1$

R.H.D.$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \frac{a + \cos^{-1}(1 + h + b) - [a + \cos^{-1}(1 + b)]}{h}$

$= \lim_{h \to 0} \frac{\cos^{-1}(1 + h + b) - \cos^{-1}(1 + b)}{h} = \lim_{h \to 0} \frac{-1}{\sqrt{1 - (1 + h + b)^2}}$

(Using L'Hospital rule)

$= \frac{-1}{\sqrt{1 - (1 + b)^2}}$

Hence, $-1 = \frac{-1}{\sqrt{1 - (1 + b)^2}}$

$\Rightarrow 1 - (1 + b)^2 = 1 \Rightarrow (1 + b)^2 = 0 \Rightarrow b = -1$

$\therefore$ From (i), we have $-1 = a + \cos^{-1}(0)$

$\Rightarrow a = -1 - \frac{\pi}{2} \Rightarrow a = \frac{-\pi - 2}{2} \quad \therefore \; \frac{a}{b} = \frac{\pi + 2}{2}$

37. (b) : We have, $\lim_{x \to \infty}\left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$

Now, $\lim_{x \to \infty}\left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x}$ (1^∞ form)

$$= e^{\lim_{x \to \infty}\left[\left(1 + \frac{a}{x} - \frac{4}{x^2} - 1\right)2x\right]} = e^{\lim_{x \to \infty}\left(2a - \frac{8}{x}\right)} = e^{2a}$$

Hence, $e^{2a} = e^3 \;\therefore\; 2a = 3 \Rightarrow a = \frac{3}{2}$

38. (d) : We have, $x = 4t^2 + 3$, $y = 8t^3 - 1$

$\therefore \; P \equiv (4t^2 + 3, 8t^3 - 1)$ Now, $\frac{dx}{dt} = 8t$ and $\frac{dy}{dt} = 24t^2$

Slope of tangent at $P = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 3t$

Let $Q \equiv (4\lambda^2 + 3, 8\lambda^3 - 1)$

Slope of $PQ = 3t$

$$\Rightarrow \frac{8t^3 - 8\lambda^3}{4t^2 - 4\lambda^2} = 3t \Rightarrow \frac{8(t - \lambda)(t^2 + \lambda^2 + t\lambda)}{4(t - \lambda)(t + \lambda)} = 3t$$

$\Rightarrow t^2 + t\lambda - 2\lambda^2 = 0 \Rightarrow (t - \lambda)(t + 2\lambda) = 0 \Rightarrow t = \lambda$ or $\lambda = \frac{-t}{2}$

$\therefore \; Q \equiv [t^2 + 3, -t^3 - 1]$

39. (c) : Since $f(x)$ is continuous at $x = 1 \;\therefore\; \frac{2}{a} = a \Rightarrow a = \pm\sqrt{2}$

Also $f(x)$ is continuous at $x = \sqrt{2} \;\therefore\; a = \frac{2b^2 - 4b}{2\sqrt{2}}$

When $a = \sqrt{2}$, we get $2 = b^2 - 2b \Rightarrow b^2 - 2b - 2 = 0$

$\Rightarrow b = \frac{2 \pm \sqrt{4 + 4 \cdot 2}}{2} = 1 \pm \sqrt{3} \qquad \therefore \; (a, b) \equiv (\sqrt{2}, 1 \pm \sqrt{3})$

When $a = -\sqrt{2}$, we get $-2 = b^2 - 2b \Rightarrow b^2 - 2b + 2 = 0$

$\Rightarrow b = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i$ (Neglected)

40. (a) : We have, $y = 1 + \sqrt{4x - 3}$

$\therefore \; \frac{dy}{dx} = \frac{1}{2\sqrt{4x - 3}} \times 4 = \frac{2}{3} \Rightarrow 4x - 3 = 9 \Rightarrow x = 3$

So, $y = 4 \;\therefore$ Equation of normal at $P(3, 4)$ is

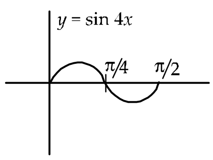
$y - 4 = -\frac{3}{2}(x - 3) \Rightarrow 2y - 8 = -3x + 9 \Rightarrow 3x + 2y - 17 = 0$

41. (c) : $f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x\cos x + 4\cos^3 x(-\sin x)$

$= 4\sin x\cos x(\sin^2 x - \cos^2 x)$

$= -2\sin 2x\cos 2x = -\sin 4x$



Since, $f(x)$ is increasing when $f'(x) > 0$

$\Rightarrow -\sin 4x > 0 \Rightarrow \sin 4x < 0 \Rightarrow x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

42. (c) : $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$

$$=\lim_{x\to 0}\frac{(2\sin^2 x)^2}{2x\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+...\right)-x\left(2x+\frac{2^3x^3}{3}+2\frac{2^5x^5}{15}+...\right)}$$

$$=\lim_{x\to 0}\frac{4\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-...\right)^4}{x^4\left(\frac{2}{3}-\frac{8}{3}\right)+x^6\left(\frac{4}{15}-\frac{64}{15}\right)+...}=\lim_{x\to 0}\frac{4\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-...\right)^4}{-2+x^2\left(-\frac{60}{15}\right)+...}=-2$$

43. (a) : $\text{xy}_{\to 5}\frac{\text{-6}-\text{o\{}-7\ \ .\text{-8}+\text{o\{}-\ .}{\text{µnz }9} = \text{xy}_{\to 5}\frac{7\text{--uz}^7\ \ .\text{-8}+\text{o\{}-\ .}{\text{µnz }9}$

$$=\text{xy}_{\to 5}\,7\left(\frac{\text{-uz}}{}\right)^7\frac{\text{-8}+\text{o\{}-\ .}{\left(\text{µnz }9\right)}=7\cdot 6\cdot\frac{\text{-8}+6.}{9}=7$$

44. (c) : Ist solution : As $\ -\ .=\begin{cases}\sqrt{\ \ +61} & 5\le\ \ \le 8\\ \ \ +71 & 8<\ \ \le :\end{cases}$ is differentiable at $x = 3$, it must be first continuous at $x = 3$.

Hence, $\text{xy}_{\to 8^+}\ -\ .=\ \text{-8.}\ \Rightarrow\ 8\ \ +7=7$

Again, $\ '^{+}\text{-8.}=\text{xy}_{\to 5}\frac{\text{-8+}\ \ .-\ \ \text{-8.}}{}$

$$=\text{xy}_{\to 5}\frac{\text{-8+}\ \ .+7-7}{}=\text{xy}_{\to 5}\frac{}{}=\ \ \ \text{-m}\ 8\ \ +7=7\ \ .$$

Also, $\ '^{-}\text{-8.}=\text{xy}_{\substack{\to 5\\ -\ >5.}}\frac{\text{-8}-\ \ .-\ \ \text{-8.}}{-}$

$$=\text{xy}_{\to 5}\frac{\sqrt{9-\ \ }-7}{-}=\text{xy}_{\to 5}\frac{\text{-}\sqrt{9-\ \ }-7.}{-}$$

$$=\text{xy}_{\to 5}\frac{\text{-9}-\ \ .-9}{-}\cdot\frac{6}{\sqrt{9-\ \ }+7}=\text{xy}_{\to 5}\frac{}{\sqrt{9-\ \ }+7}=\frac{-}{9}$$

Hence set $m = k/4$

Now, $3m + 2 = 2k$ yields $m = 2/5$, $k = 8/5$

$\therefore\ \ k + m = 2$

2nd solution : Since $g(x)$ is differentiable

$$\Rightarrow\ \ '\text{-}\ .\ _{=8}=\left.\frac{}{7\sqrt{\ \ +6}}\right|_{=8}=\frac{-}{9}=$$

(we earlier did it by ‘ab initio’ - first principle)

2nd solution can then be completed as before.

45. (a) : 1st solution : Let $f(x) = ax^4 + bx^3 + cx^2 + dx + \lambda$

As $\text{xy}_{\to 5}\left(6+\frac{\ ^9+\ ^8+\ ^7+\ \ +\lambda}{7}\right)=8$

We have $d = \lambda = 0$, the coefficient of exponents lower than 2 must vanish.

$\Rightarrow\ \text{xy}_{\to 5}\ \text{-6+}\ \ ^7+\ \ +\ .=8\ \Rightarrow\ c = 2$

$f(x) = ax^4 + bx^3 + 2x^2$

$f'(x) = 4ax^3 + 3bx^2 + 4x = x(4ax^2 + 3bx + 4)$

$x = 1$ and 2 are roots of $4ax^2 + 3bx + 4 = 0$

Thus, $-\frac{8}{9}=8\ \text{nzp}\ 7=\frac{9}{9}$ (Using sum and product of roots)

Solving, we get $\ =\frac{6}{7}1\ \ =-7\ \ \ \ -\ .=\frac{9}{7}-7\ ^8+7\ ^7$

Put $x = 2$ to get $f(2) = 8 - 16 + 8 = 0$.

2nd solution : As f has extreme values at $x = 1$ and $x = 2$, we build f from f'.

$f'(x) = k(x-1)(x-2)(x-\alpha)$

As f' is a polynomial of degree 3.

As $\text{xy}_{\to 5}\left(6+\frac{-\ .}{7}\right)=8\ \Rightarrow\ \text{xy}_{\to 5}\frac{-\ .}{7}=7$

Thus, $f(x) = x^2 g(x)$

Hence $x = 0$ is a repeated root of $f(x)$.

Here $f'(x) = k(x-1)(x-2)x = k(x^3 - 3x^2 + 2x)$

$$\Rightarrow\ \ -\ .=\left(\frac{9}{9}-\ ^8+\ ^7\right)=\ \ ^7\left(\frac{7}{9}-\ \ +6\right)$$

$$\Rightarrow\ \text{xy}_{\to 5}\frac{-\ .}{7}=\ \ =7$$

Thus, $\ -\ .=\frac{9}{7}-7\ ^8+7\ ^7$

46. (b) : The given curve is $x^2 + 2xy - 3y^2 = 0$

Factorizing it becomes $(x-y)(x+3y) = 0$

Normal at (1, 1) is $x + y = \lambda$ *i.e.* $1 + 1 = \lambda\ \ \therefore\ \lambda = 2$

Thus the equation is $x + y = 2$

Obviously $x + 3y = 0$ doesn’t have the point (1, 1) on it.

Now, $x + y = 2$ meets $x + 3y = 0$ in the point $(3, -1)$ obtained by solving the system of linear equations. Hence the point is in 4th quadrant.

47. (b) : $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{\sin^2 x}=\text{xy}_{\to 5}\frac{7\ \ ^7\ +\text{-uz}}{7\text{-uz}\ \ \text{o\{}-}$

(Using L’ Hospital Rule)

$$=\text{xy}_{\to 5}\left(\frac{}{\text{-uz}}\ \ ^7+\frac{6}{7}\right)\frac{6}{\text{o\{}-}=6+\frac{6}{7}=\frac{8}{7}$$

48. (c) : Given that $x = 2\cos t + 2t\sin t$

$\Rightarrow\ \ \frac{}{}=-7\text{-uz}\ \ +7\text{g o\{}-\ +\text{-uz}\ \ \text{i}=2t\cos t$ ...(i)

Also, $y = 2\sin t - 2t\cos t$

$\Rightarrow\ \ \frac{}{}=7\text{o\{}-\ -7\text{g}-\ \text{-uz}\ \ +\text{o\{}-\ \text{i}=7\ \ \text{-uz}$ ...(ii)

So, $\frac{}{}=\frac{7\ \ \text{-uz}}{7\ \ \text{o\{}-}\ \{\sim\frac{}{}=\text{µnz}$ (from (i) & (ii))

$$\left(\frac{}{}\right)_{=\pi 49}=6$$

So the slope of the normal is -1.

and at $t = \pi/4 \Rightarrow\ \ =\sqrt{7}+\frac{\pi}{7\sqrt{7}}$ and $\ =\sqrt{7}-\frac{\pi}{7\sqrt{7}}$

∴ The equation of normal is

$$\left[\quad -\left(\sqrt{7}-\frac{\pi}{7\sqrt{7}}\right)\right]=-6\left[\quad -\left(\sqrt{7}+\frac{\pi}{7\sqrt{7}}\right)\right]$$

$$\Rightarrow \quad -\sqrt{7}+\frac{\pi}{7\sqrt{7}}=- \quad +\sqrt{7}+\frac{\pi}{7\sqrt{7}}$$

$\Rightarrow \quad + \quad =7\sqrt{7}$. So the distance from the origin is 2.

49. (b) : We have, $f(x) = 2x^3 + bx^2 + cx$

Now, $f(1) = f(-1)$ and $f'\left(\frac{1}{2}\right)=0$ So, $f(1) = 2 + b + c$

$f(-1) = -2 + b - c$

Now, $f(1) = f(-1) \Rightarrow c = -2$ ...(i)

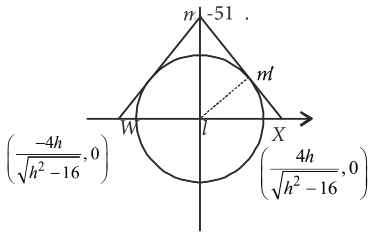
Also, $f'(x) = 6x^2 + 2bx + c$

$\Rightarrow \quad '\left(\frac{6}{7}\right)=\frac{8}{7}+ \quad + \quad =5 \quad \Rightarrow \frac{8}{7}+ \quad -7=5$ ...(ii)

So, $2b + c = \left(7\times\frac{6}{7}\right)+\text{-}-7.=-6$ (using (i) and (ii))

50. (d) : Let the equation of the tangent be $(y - h) = m(x - 0)$

$\Rightarrow mx - y + h = 0$ ...(i)



Since $OP' = \perp$ distance of origin from the tangent of circle = radius

$\Rightarrow \left|\frac{}{\sqrt{\ ^7+6}}\right|=9 \quad \Rightarrow h^2 = 16\,(m^2 + 1)$

$\Rightarrow h^2 = 16m^2 + 16 \Rightarrow \quad =\frac{\sqrt{\ ^7-6;}}{9}$

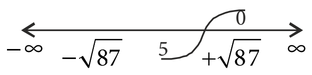
∴ x co-ordinates of A and B are $=\frac{-}{}=\mp\frac{9}{\sqrt{\ ^7-6;}}$ respectively (from (i))

Area of $\Delta = \frac{6}{7}$ × base × height

$$\Rightarrow \Delta = \frac{6}{7}\times\frac{=}{\sqrt{\ ^7-6;}}\times \quad =\frac{9\ ^7}{\sqrt{\ ^7-6;}}$$

$$\frac{\Delta}{}=9\left[\frac{7\sqrt{\ ^7-6;}-\frac{7\ \cdot\ ^7}{7\sqrt{\ ^7-6;}}}{^7-6;}\right]$$

$$=9\left[\frac{9\text{-}\ ^7-6;.-7\ ^7}{7\left(\sqrt{\ ^7-6;}\right)\text{-}\ ^7-6;.}\right]=\frac{9\ g7\ ^7-;9i}{7\sqrt{\ ^7-6;}\text{-}\ ^7-6;.}$$



For area to be minimum, $h=\sqrt{87} \Rightarrow \quad =9\sqrt{7}$

51. (c) : Since $f(x)$ is a continuous function ∴ $\underset{\to 5}{\text{xy}}\ \ \text{-}\ .=\ \text{-}5.$

$$\Rightarrow \underset{\to 5}{\text{xy}}\frac{\frac{\text{-}\quad -6.^7}{7}}{\frac{\text{-uz-}\ 4\ .\cdot\text{x\{s-6+}\ 49.}{\cdot-9\cdot\frac{-}{9}}}=67 \quad \Rightarrow 4k = 12 \Rightarrow k = 3$$

52. (a) : We have, $\sin y = x \sin\left(\frac{\pi}{8}+\quad\right)$ ...(i)

Differentiating (i) w.r.t. x, we get

$\text{o\{}-\ \ \frac{}{}=\ \text{o\{}-\left(\frac{\pi}{8}+\quad\right)\frac{}{}+\text{-uz}\left(\frac{\pi}{8}+\quad\right)$ ...(ii)

Put $x = 0$ and $y = 0$ in (ii), we get $\frac{}{}=\frac{\sqrt{8}}{7} \Rightarrow \quad \frac{-}{}=\frac{-7}{\sqrt{8}}$

∴ Equation of normal passing through (0, 0) is $=\frac{-7}{\sqrt{8}}$

$33\ \sqrt{8}\ =-7 \quad \Rightarrow 7\ +\sqrt{8}\ =5$

53. (d) : Let $\text{-}\ .=\frac{\text{-6+}\ .^{84}}{6+\ ^{84}}$

When $f'(x) = 0 \Rightarrow x = 1$

$\text{Hx\{}1 \quad \text{-}5.=6\ \text{mzp}\ \ \text{-}6.=\frac{7^{53}}{7}=7^{-53}$ ∴ $f(x) \in (2^{-0.4}, 1)$

54. (a) : We know that, $v^2 - u^2 = 2gh$

$\Rightarrow 0 - (48)^2 = 2(-32)h$

$\Rightarrow \quad =\frac{7859}{;9}=8;\ y$

∴ The greatest height = 64 + 36 = 100 metres

55. (b) : We have $f(x) = \alpha \log|x| + \beta x^2 + x$

$$f'(x)=\frac{\alpha}{x}+2\beta x+1=\frac{2\beta x^2+x+\alpha}{x}$$

-1 and 2 are the roots of $2\beta x^2 + x + \alpha = 0$

Hence $-\frac{1}{2\beta}=-1+2 \Rightarrow -\frac{1}{2\beta}=1$ ∴ $\beta=-\frac{1}{2}$

Also, $\frac{\alpha}{2\beta}=(-1)(2) \Rightarrow \frac{\alpha}{2\beta}=-2$ ∴ $\alpha=2$

56. (c) : $\lim\limits_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}=\lim\limits_{x\to 0}\frac{\sin\{\pi-\pi(\cos^2 x)\}}{x^2}$

$$=\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{x^2}=\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{(\pi\sin^2 x)}\cdot\frac{\pi\sin^2 x}{x^2}$$

$$=\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\cdot\pi\cdot\left(\frac{\sin x}{x}\right)^2=1\cdot\pi\cdot 1=\pi$$

57. (c) : $\frac{f'(c)}{g'(c)}=\frac{f(1)-f(0)}{g(1)-g(0)}=\frac{6-2}{2-0}=2 \Rightarrow f'(c) = 2g'(c)$

58. (c) : $f(g(x)) = x \Rightarrow f'(g(x))g'(x) = 1$

$\Rightarrow g'(x)=\frac{1}{f'(g(x))} \Rightarrow g'(x) = 1 + \{g(x)\}^5$

59. (b) : $\frac{dP}{dx}=100-12\sqrt{x}$

Integrating, we have, $dP=(100-12\sqrt{x})dx$

$P = 100x - 12 \cdot \frac{2}{3} \cdot x^{3/2} + \lambda \Rightarrow P = 100x - 8x^{3/2} + \lambda$

$P(0) = 2000 = \lambda \quad \therefore \quad \lambda = 2000$

$P(25) = 100 \times 25 - 8 \times 25^{3/2} + 2000 = 3500.$

60. (d) : We have, $y = \sec(\tan^{-1}x)$

$\frac{dy}{dx} = \sec(\tan^{-1}x) \cdot \tan(\tan^{-1}x) \cdot \frac{1}{1+x^2}$

$\left.\frac{dy}{dx}\right|_{x=1} = \sqrt{2} \cdot 1 \cdot \frac{1}{2} = \frac{1}{\sqrt{2}}$

61. (a) : $f(x) = |x-2| + |x-5| \Rightarrow f(x) = \begin{cases} 7-2x, & x<2 \\ 3, & 2 \le x \le 5 \\ 2x-7, & x>5 \end{cases}$

Statement-1 : $f'(4) = 0$. True

Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$. True

But Statement 2 is not a correct explanation for statement 1.

62. (c) : $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi = [x]\cos\left(\pi x - \frac{\pi}{2}\right) = [x]\sin \pi x$

Let n be an integer.

$\lim_{x \to n^+} f(x) = 0, \lim_{x \to n^-} f(x) = 0 \quad \therefore f(n) = 0$

$\Rightarrow f(x)$ is continuous for every real x.

63. (a) : $f(x) = \ln|x| + bx^2 + ax,\ x \ne 0$ has extreme values at $x = -1,\ x = 2$.

$\Rightarrow f'(x) = \frac{1}{x} + 2bx + a$

$f'(-1) = 0$ and $f'(2) = 0$ [Given]

$\Rightarrow -1 - 2b + a = 0 \Rightarrow b = -\frac{1}{4}$ and $\frac{1}{2} + 4b + a = 0 \Rightarrow a = \frac{1}{2}$

$f''(x) = -\frac{1}{x^2} + 2b = -\frac{1}{x^2} - \frac{1}{2} = -\left(\frac{1}{x^2} + \frac{1}{2}\right) < 0$ for all $x \in R - \{0\}$

$\Rightarrow f$ has a local maximum at $x = -1,\ x = 2$

$\therefore$ **Statement 1 :** f has local maxima at $x = -1, x = 2$

$\therefore$ **Statement 2 :** $a = \frac{1}{2}, b = -\frac{1}{4}$

64. (a) : $\frac{dv}{dt} = -72\pi \text{ m}^3/\text{min},\ v_0 = 4500\pi$

$v = \frac{4}{3}\pi r^3 \ \therefore \ \frac{dv}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$

After 49 min, $v = v_0 + 49 \cdot \frac{dv}{dt} = 4500\pi - 49 \times 72\pi$

$= 4500\pi - 3528\pi = 972\pi$

$\Rightarrow 972\pi = \frac{4}{3}\pi r^3 \Rightarrow r^3 = 243 \times 3 = 3^6 \Rightarrow r = 9$

$\therefore -72\pi = 4\pi \times 81 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{18}{81} = -\frac{2}{9}$

Thus, radius decreases at a rate of $\frac{2}{9}$ m/min

65. (b) : $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$

$= \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\} \cdot \frac{dx}{dy} = -\left(\frac{dy}{dx}\right)^{-2} \frac{d^2y}{dx^2} \cdot \left(\frac{dy}{dx}\right)^{-1} = -\left(\frac{dy}{dx}\right)^{-3} \frac{d^2y}{dx^2}$

66. (c) : Let $x = 2 + h$

$\lim_{h \to 0} \frac{\sqrt{1-\cos 2h}}{h} = \lim_{h \to 0} \frac{|\sin h|}{h}$

RHL = 1, LHL = –1. Thus limit doesn't exist.

67. (a) : $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x<0 \\ q, & x=0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x>0 \end{cases}$

$\lim_{x \to 0^+} f(x) = \frac{1}{2}$

Again, $\lim_{x \to 0^-} f(x) = \frac{\sin(p+1) + \sin x}{x} = p + 2$

Now, $p + 2 = q = 1/2 \ \therefore \ p = -3/2,\ q = 1/2$.

68. (b) : $g(x) = \{f(2f(x) + 2\}^2$

We have on differentiation with respect to x,

$g'(x) = 2f(2f(x) + 2) \cdot f'(2f(x) + 2) \cdot 2f'(x)$

Let $x = 0$

$g'(0) = 2f(2f(0) + 2) \cdot f'(2f(0) + 2) \cdot 2f'(0)$

$= 2f(0) \cdot f'(0) \cdot 2f'(0) = (-2)(1)(2) = -4.$

69. (a) : As f is a positive increasing function, we have $f(x) < f(2x) < f(3x)$

Dividing by $f(x)$ leads to $1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$

As $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$, we have by Squeez theorem or Sandwich theorem, $\lim_{x \to \infty} \frac{f(2x)}{f(x)} = 1.$

70. (d) : $\lim_{x \to 1^+} f(x) = 1$

As $f(-1) = k + 2$

As f has a local minimum at $x = -1$

$f(-1^+) \ge f(-1) \ge f(-1^-) \Rightarrow 1 \ge k + 2 \Rightarrow k + 2 \le 1. \quad \therefore \quad k \le -1$

Thus $k = -1$ is a possible value.

71. (a) : Using A.M.-G.M. inequality, $\frac{e^x + 2e^{-x}}{2} \ge \sqrt{e^x \cdot 2e^{-x}}$.

Thus, $e^x + 2e^{-x} \ge 2\sqrt{2}$. Then $\frac{1}{e^x + 2e^{-x}} \le \frac{1}{2\sqrt{2}}$

As $\frac{1}{e^x + 2e^{-x}}$ is always positive, we have $0 < \frac{1}{e^x + 2e^{-x}} \le \frac{1}{2\sqrt{2}}$

Observe that $f(0) = 1/3$. Thus such that $f(c) = 1/3$.

Using extreme-value theorem, we can say that as f is continuous, f will attain a value 1/3 at some point. Here we are able to identify the point as well.

72. (b) : $gof(x) = g(f(x)) = \sin(x|x|) = \begin{cases} -\sin x^2, & x<0 \\ \sin x^2, & x \ge 0 \end{cases}$

Let the composite function $gof(x)$ be denoted by $H(x)$.

Then $H(x) = \begin{cases} -\sin x^2, & x<0 \\ \sin x^2, & x \ge 0 \end{cases}$

$LH'(0) = \lim_{h\to 0^-} \frac{H(0-h)-H(0)}{-h}$

$= \lim_{h\to 0^-} \frac{-\sin h^2}{-h} = \lim_{h\to 0^-} \cdot \frac{\sin h^2}{h^2} \cdot h = 1 \cdot 0 = 0$

$RH'(0) = \lim_{h\to 0^+} \frac{H(0+h)-H(0)}{h} = \lim_{h\to 0^+} \frac{\sin h^2 - 0}{h} = \lim_{h\to 0^+} \left(\frac{\sin h^2}{h^2}\right) h$

$= 1 \cdot 0 = 0$

Thus $H(x)$ is differentiable at $x = 0$

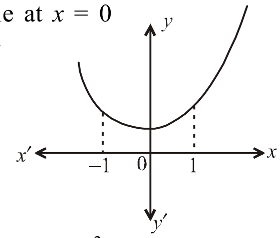
Also $H'(x) = \begin{cases} -2x\cos x^2 & , x < 0 \\ 0 & , x = 0 \\ 2x\cos x^2 & , x > 0 \end{cases}$

$H'(x)$ is continuous at $x = 0$ for $H'(0) = LH'(0) = RH'(0)$

Again $H''(x) = \begin{cases} -2\cos x^2 + 4x^2 \sin x^2 & , x < 0 \\ 2\cos x^2 - 4x^2 \sin x^2 & , x \ge 0 \end{cases}$

$LH''(0) = -2$ and $RH''(0) = 2$

Thus $H(x)$ is NOT twice differentiable at $x = 0$



73. (a) : $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$P'(0) = 0 \Rightarrow c = 0$

Also $P'(x) = x(4x^2 + 3ax + 2b)$

As $P'(x) = 0$ has no real roots except $x = 0$, we have

D of $4x^2 + 3ax + 2b$ is less than zero *i.e.*, $(3a)^2 - 4 \cdot 4 \cdot 2b < 0$ then $4x^2 + 3ax + 2b > 0 \ \ \forall x \in R$

(If $a > 0$, $b^2 - 4ac < 0$ then $ax^2 + bx + c > 0 \ \forall x \in R$)

So $P'(x) < 0$ if $x \in [-1, 0)$ *i.e.*, decreasing

and $P'(x) > 0$ if $x \in (0, 1]$ *i.e.*, increasing

Max. of $P(x) = P(1)$

But minimum of $P(x)$ doesn't occur at $x = -1$, *i.e.*, $P(-1)$ is not the minimum.

74. (d) : $x^{2x} - 2x^x \cot y - 1 = 0$ .....(i)

At $x = 1$ we have

$1 - 2\cot y - 1 = 0 \Rightarrow \cot y = 0 \quad \therefore \quad y = \pi/2$

Differentiating (i) w.r.t. x, we have

$2x^{2x}(1+\ln x) - 2[x^x(-\text{cosec}^2 y)\frac{dy}{dx} + \cot y \cdot x^x(1+\ln x)] = 0$

At $P(1, \pi/2)$ we have $2(1+\ln 1) - 2[1(-1)\left(\frac{dy}{dx}\right)_P + 0] = 0$

$\Rightarrow 2 + 2\left(\frac{dy}{dx}\right)_P = 0 \quad \therefore \quad \left(\frac{dy}{dx}\right)_P = -1$

75. (b) : Let $f(x) = x^3 - px + q$

Now $f'(x) = 0$, *i.e.* $3x^2 - p = 0 \Rightarrow x = -\sqrt{\frac{p}{3}}, \sqrt{\frac{p}{3}}$

Also, $f''(x) = 6x \Rightarrow f''\left(-\sqrt{\frac{p}{3}}\right) < 0 \Rightarrow f''\left(\sqrt{\frac{p}{3}}\right) > 0$

Thus maxima at $-\sqrt{\frac{p}{3}}$ and minima at $\sqrt{\frac{p}{3}}$.

76. (b) : By definition $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$, if the limit exists.

$\therefore \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{(1+h-1)\sin\frac{1}{(1+h-1)} - 0}{h} = \lim_{h\to 0} \sin\frac{1}{h}$

As the limit dosen't exist, $\therefore$ it is not differentiable at $x = 1$

Again $f'(0) = \lim_{h\to 0} \frac{f(h)-f(0)}{h}$, if the limit exists

$\therefore \lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0} \frac{(h-1)\sin\frac{1}{h-1} - \sin 1}{h}$

But this limit dosen't exist. Hence it is not differentiable at $x = 0$.

77. (c) : Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$\therefore f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 \Rightarrow f'(x) > 0 \ \forall \ x \in R$

i.e. $f(x)$ is an strictly increasing function.

so it can have at the most one solution. It can be shown that it has exactly one solution.

78. (c) : I[st] solution : Let $p = \cos\theta$, $q = \sin\theta$, $0 \le \theta \le \pi/2$

$p + q = \cos\theta + \sin\theta$

$\Rightarrow$ maximum value of $(p + q) = \sqrt{2}$

II[nd] solution : By using A.M ≥ G.M., $\frac{p^2+q^2}{2} \ge pq \Rightarrow pq \le \frac{1}{2}$

$(p+q)^2 = p^2 + q^2 + 2pq \Rightarrow (p+q) \le \sqrt{2}$.

79. (b) : $f(0) = \lim_{x\to 0}\left[\frac{1}{x} - \frac{2}{e^{2x}-1}\right] = \lim_{x\to 0} \frac{e^{2x}-1-2x}{x(e^{2x}-1)} \left(\frac{0}{0}\text{form}\right)$

By using L' Hospital rule $f(0) = \lim_{x\to 0} \frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}} \left(\frac{0}{0}\text{ form}\right)$

Again use L' Hospital rule $f(0) = \lim_{x\to 0} \frac{4e^{2x}}{4e^{2x}+4xe^{2x}} = 1$.

80. (a) : $f(x) = \min\{x+1, |x|+1\} \Rightarrow f(x) = x + 1, x \in R$

Hence $f(x)$ is differentiable for all $x \in R$.

81. (d) : $f'(x) = \frac{1}{1+(\sin x + \cos x)^2} . (\cos x - \sin x)$

$f'(x) = \frac{\cos x - \sin x}{2 + \sin 2x}$

If $f'(x) > 0$ then $f(x)$ is increasing function

For $-\frac{\pi}{2} < x < \frac{\pi}{4}, \cos x > \sin x$

Hence $y = f(x)$ is increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

82. (c) : By LMVT, $f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(1)}{3-1}$

$f'(c) = \frac{\log_e 3 - \log_e 1}{2} = \frac{1}{2}\log_e 3 \Rightarrow \frac{1}{c} = \frac{1}{2}\log_e 3 = \frac{1}{2\log_3 e}$

$\therefore c = 2\log_3 e$.

83. (a) : $x^m \times y^n = (x + y)^{m+n}$

Taking log both sides we get

$m\log x + n\log y = (m+n)\log(x+y)$

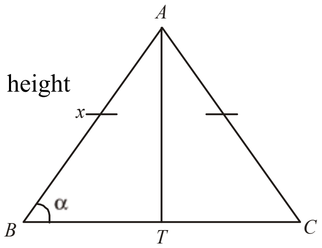
Differentiating w.r.t. x we get $\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{nx+ny-my-ny}{y(x+y)}\right) = \frac{mx+nx-mx-my}{x(x+y)}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{nx-my}{nx-my}\right)\frac{y}{x} = \frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}.$

84. (c) : $AT = x \sin \alpha$
$BT = x \cos \alpha$



Area of triangle $ABC = \frac{1}{2}$ base × height

$= \frac{1}{2}(2BT)(AT)$

$= \frac{1}{2}(2x^2 \cos\alpha \sin\alpha)$

$= \frac{1}{2}x^2 \sin 2\alpha \leq \frac{1}{2}x^2$ as $-1 \leq \sin 2\alpha \leq 1$

$\therefore$ Maximum area of $\Delta ABC = \frac{1}{2}x^2$

85. (c) : Given $f(x) = \frac{x}{1+|x|} \Rightarrow f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \geq 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} \frac{1}{(1-x)^2}, & x < 0 \\ \frac{1}{(1+x)^2}, & x \geq 0 \end{cases}$

$f'(x)$ is finite quantity $\forall\, x \in R$

$\therefore f'(x)$ is differentiable $\forall\, x \in (-\infty, \infty)$

86. (a) : Given equation $y = x^2 - 5x + 6$, given points (2, 0), (3, 0)

$\therefore \frac{dy}{dx} = 2x - 5$

say $m_1 = \left(\frac{dy}{dx}\right)_{\substack{x=2\\y=0}} = 4-5 = -1$ and $m_2 = \left(\frac{dy}{dx}\right) at_{\substack{x=3\\y=0}} = 6-5 = 1$

since $m_1 m_2 = -1 \Rightarrow$ tangents are at right angle *i.e.,* $\frac{\pi}{2}$

87. (a) : Let $g(x) = \frac{x}{2} + \frac{2}{x}$

$\therefore \quad g'(x) = \frac{1}{2} - \frac{2}{x^2}$

for maxima and minima $g'(x) = 0 \Rightarrow x = \pm 2$

Again $g''(x) = \frac{4}{x^3} > 0$ for $x = 2$

< 0 for $x = -2 \quad \therefore x = 2$ is point of minima

88. (b) : For the range of the expression

$$\frac{3x^2+9x+17}{3x^2+9x+7} = y = \frac{ax^2+bx+c}{px^2+qx+r},$$

[find the solution of the inequality $A\, y^2 + B\, y + K \geq 0$]

where $A = q^2 - 4pr = -3$, $B = 4ar + 4PC - 2bq = 126$

$K = b^2 - 4ac = -123$ *i.e.,* solve $-3y^2 + 126 + y - 123 \geq 0$

$\Rightarrow \; 3y^2 - 126y + 123 \leq 0$

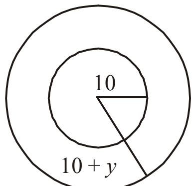
$\Rightarrow \; y^2 - 42y + 41 \leq 0$

$\Rightarrow \; (y-1)(y-42) \leq 0$

$\Rightarrow \; 1 \leq y \leq 42$

$\Rightarrow$ maximum value of y is 42

89. (a) : $v = \frac{4}{3}\pi(y+10)^3$ where y is thickness of ice



$\Rightarrow \frac{dv}{dt} = 4\pi(10+y)^2 \frac{dy}{dt}$

$\left(\frac{dy}{dt}\right)_{\text{at } y=5} = \frac{50}{4\pi(15)^2} \quad \left(\text{as } \frac{dv}{dt} = 50 \text{ cm}^3/\text{min.}\right)$

$= \frac{1}{18\pi}$ cm/min.

90. (b) : As α is root of $ax^2 + bx + c = 0$

$\therefore a\alpha^2 + b\alpha + c = 0$. Now

$$\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2} = \lim_{x\to\alpha} \frac{2\sin^2\left(\frac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha} \frac{2\sin^2\left[\frac{a(x-\alpha)(x-\beta)}{2}\right]}{a^2\left[\frac{(x-\alpha)^2(x-\beta)^2}{4}\right]} \times \frac{a^2(x-\beta)^2}{4}$$

$$= \lim_{x\to\alpha}\left[\frac{\sin\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)}{\frac{a(x-\alpha)(x-\beta)}{2}}\right]^2 \times \frac{a^2(x-\beta)^2}{2} = 1 \times \frac{a^2}{2}(\alpha-\beta)^2.$$

91. (a, c) : $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \tan\theta$ = slope of tangent

$\therefore$ Slope of normal to the curve $= -\cot\theta = \tan(90 + \theta)$.

Now equation of normal to the curve

$[y - a(\sin\theta - \theta\cos\theta)] = -\frac{\cos\theta}{\sin\theta}(x - a(\cos\theta + a\sin\theta))$

$\Rightarrow x\cos\theta + y\sin\theta = a(1)$

Now distance from (0, 0) to $x \cos\theta + y \sin\theta = a$ is

distance $(d) = \frac{(0+0-a)}{1} \quad \therefore$ distance is constant $= a$.

92. (c) : Given $|f(x) - f(y)| \leq (x-y)^2$

$$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \leq \lim_{x\to y}|x-y|$$

$\Rightarrow |f'(x)| \leq 0$, $f(x) = 0$ $\left(|f'(x)| < 0, \text{ not possible}\right)$

$\Rightarrow f(x) = k$ (by integration)

$\Rightarrow f(x) = 0$ $(\because f(0) = 0)$

$\Rightarrow f(x)\ (\forall\, x \in R) = 0 \;\therefore\; f(1) = 0$.

93. (b) : Let if possible $f'(x) = 2$ for

$\Rightarrow \; f(x) = 2x + c$ (Integrating both side w.r.t. x)

$\therefore \; f(1) = 2 + c, -2 = 2 + c \Rightarrow c = -4 \;\therefore f(x) = 2x - 4$

$\therefore \; f(6) = 2 \times 6 - 4 = 8 \;\therefore\; f(6) \geq 8$.

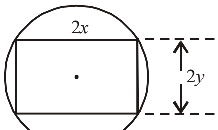
94. (d) : As $f(x)$ is differentiable at $x = 1$

$5 = \lim_{h\to 0} \frac{f(1+h)}{h}$ assumes 0/0 form

$5 = \lim_{h\to 0} \frac{f'(1)}{1} \;\therefore f'(1) = 5.$

95. (b) : Any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(a\cos\theta, b\sin\theta)$ so the area of rectangle inscribed in the ellipse is given by

$A = (2a\cos\theta)(2b\sin\theta)$



$\therefore\ A = 2ab\sin 2\theta \Rightarrow \frac{dA}{d\theta} = 4ab\cos 2\theta$

Now for maximum area

$\frac{dA}{d\theta} = 0 \Rightarrow \theta = \frac{\pi}{4}$ and $\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\pi/4} = -8ab\sin 2\theta$

as $\frac{d^2A}{d\theta^2} < 0$. $\therefore$ Area is maximum for $\theta = \pi/4$.

$\therefore$ sides of rectangle are $\frac{2a}{\sqrt{2}}, \frac{2b}{\sqrt{2}}$

Required area $= 2ab$.

96. (c) : Let $f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx$

Note : In such type of problems we always consider $f(x)$ as the integration of L.H.S of the given equation without constant. Here integration of $ax^2 + bx + c$ is $\frac{ax^3}{3}+\frac{bx^2}{2}+cx$ called it by $f(x)$. Now use the intervals in $f(x)$ if $f(x)$ satisfies the given condition then at least one root of the equation $ax^2 + bx + c = 0$ must lies in that interval.

Now $f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx$

$f(0) = 0$ and $f(1) = \frac{a}{3}+\frac{b}{2}+c = \frac{2a+3b+6c}{6} = 0$

given $2a + 3b + 6c = 0$ $\therefore$ $x = 0$ and $x = 1$ are roots of

$f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx = 0$

$\therefore$ at least one root of the equation $ax^2 + bx + c = 0$ lies in (0, 1)

97. (b) : Given $f''(x) = 6(x-1)$

$\Rightarrow f'(x) = \frac{6(x-1)^2}{2} + c$

$\Rightarrow 3 = 3 + c$ $\left[\because f(x) = y = 3x+5,\ f'(x) = 3\ \forall\ x \in R\right]$

$\Rightarrow c = 0$

so $f'(x) = 3(x-1)^2$

$\Rightarrow f(x) = (x-1)^3 + c_1$ as curve passes through (2, 1)

$\Rightarrow 1 = (2-1)^3 + c_1 \Rightarrow c_1 = 0$ $\therefore$ $f(x) = (x-1)^3$

98. (a) : $\underset{x\to\pi/4}{\text{Lt}} \frac{1-\tan x}{4x-\pi}$ putting $4x - \pi = t$

$\therefore \underset{x\to\pi/4}{\text{Lt}} \frac{(1-\tan x)\times(1+\tan x)}{(1+\tan x)\left[-4\left(\frac{\pi}{4}-x\right)\right]}$

$\underset{x\to\pi/4}{\text{Lt}} -\frac{\tan\left(\frac{\pi}{4}-x\right)\times(1+\tan x)}{4\left(\frac{\pi}{4}-x\right)} = -1/2$

99. (b) : $e^2 = \left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}$ (1^∞ form)

$e^2 = e^{\underset{x\to\infty}{\text{Lt}}\left[1+\frac{a}{x}+\frac{b}{x^2}-1\right](2x)}$

$e^2 = e^{2a} \Rightarrow 2a = 2$ $\therefore$ $a = 1$ and $b \in R$

100. (d) : $\underset{x\to a}{\text{Lt}} \frac{f(a)\,g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4$

$\Rightarrow \underset{x\to a}{\text{Lt}} \frac{f(a)[g(x)-f(x)]}{g(x)-f(x)} = 4 \Rightarrow \underset{x\to a}{\text{Lt}} f(a) = 4 \Rightarrow k = 4$

101. (b) : $\underset{x\to\pi/2}{\text{Lt}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{4.\left(\frac{\pi-2x}{4}\right)(\pi-2x)^2}$

$\underset{x\to\pi/2}{\text{Lt}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4.\left(\frac{\pi}{4}-\frac{x}{2}\right)} \frac{1-\cos\left(\frac{\pi}{2}-x\right)}{(\pi-2x)^2}$

$\underset{x\to\pi/2}{\text{Lt}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4.\left(\frac{\pi}{4}-\frac{x}{2}\right)} \frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4^2\left(\frac{\pi-2x}{4}\right)^2} = \frac{1}{4}\times\frac{2}{16} = \frac{1}{32}$

102. (c) : $\underset{n\to\infty}{\text{Lt}} \frac{1^4+2^4+3^4+...+n^4}{n^5} - \underset{n\to\infty}{\text{Lt}} \frac{1^3+2^3+...+n^3}{n^5}$

$= \underset{n\to\infty}{\text{Lt}} \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30\cdot n^5} - \underset{n\to\infty}{\text{Lt}} \frac{n^2(n+1)^2}{4\times n^5}$

$\left[\text{Using } 1^4 + 2^4 + ... + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}\right]$

$= \frac{6}{30} - 0 = \frac{1}{5}$

103. (a) : $f(x) = x + 1/x$

$f'(x) = 1 - 1/x^2$ and $f''(x) = \frac{2}{x^3}$, now $f'(x) = 0$

$\Rightarrow x = \pm 1$ $\therefore$ $f''(1) > 0 \Rightarrow x = 1$ is point of minima.

104. (a) : Given $f(x) = \begin{cases} x\,e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\underset{x\to0^+}{\text{Lt}} f(x) = \underset{x\to0^+}{\text{Lt}} xe^{-2/x} = 0$ ...(A)

and $\underset{x\to0^-}{\text{Lt}} f(x) = \underset{x\to0^-}{\text{Lt}} e^{\left[-\frac{1}{x}+\frac{1}{x}\right]} = 0$ ...(B)

As LHL = RHL $\therefore$ $f(x)$ is continuous at $x = 0$

Again RHD at $x = 0$ is $\underset{x\to0^+}{\text{Lt}} \frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-0}{h} = 0$

also we have LHD at $x = 0$ is $\frac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}-0}{-h} = 1$

so L.H.D $\neq$ R.H.D at $x = 0$

$\therefore f(x)$ is non differentiable at $x = 0$

105. (b) : For maximum and minima $f'(x) = 0$

$\Rightarrow 6x^2 - 18ax + 12a^2 = 0$ and $f''(x) = 12x - 18a$

$f'(x) = 0$

$\Rightarrow x = a, 2a$ and $f''(a) < 0$ and $f''(2a) > 0$

Now $p = a$ and $q = 2a$ and $p^2 = q$

$\Rightarrow a^2 = 2a \Rightarrow a^2 - 2a = 0$

$\Rightarrow a(a-2) = 0 \Rightarrow a = 0, a = 2$

106. (b) : $f(x) = x^n \therefore f(1) = 1 = {}^nC_0$

$\therefore f'(x) = nx^{n-1}$ so $-f'(1) = -n = -{}^nC_1$

$f''(x) = n(n-1)x^{n-2}$ so $\dfrac{f''(1)}{2!} = \dfrac{n(n-1)}{2!} = {}^nC_2$

$f^n(x) = n(n-1)\ ...1 \therefore \dfrac{f^n(1)(-1)^n}{n!} = (-1)^n\ {}^nC_n$

$\therefore f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + ... + \dfrac{(-1)^n f^n(1)}{n!}$

$= {}^nC_0 - {}^nC_1 + {}^nC_2\ ....+ (-1)^n C_n$

Now $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ ...(i)

Putting $x = -1$ in both sides of (i) we get

$0 = C_0 - C_1 + C_2 - C_3 + ...$

107. (b) : $\underset{x\to 0}{\text{Lt}} \dfrac{\log(3+x) - \log(3-x)}{x} = k$

$\therefore k = \underset{x\to 0}{\text{Lt}} \dfrac{\log\left(1+\frac{x}{3}\right) - \log\left(1-\frac{x}{3}\right)}{x}$

$k = \underset{x\to 0}{\text{Lt}} \dfrac{\log\left(1+\frac{x}{3}\right)}{\frac{x}{3}\times 3} + \underset{x\to 0}{\text{Lt}} \dfrac{\log\left(1-\frac{x}{3}\right)}{-\frac{x}{3}\times 3} \Rightarrow k = \dfrac{1}{3}+\dfrac{1}{3} = \dfrac{2}{3}$

108. (a)

109. (c) : $\underset{x\to 2}{\text{Lt}} \dfrac{xf(2) - 2f(x) + 2f(2) - 2f(2)}{x-2}$

$= \underset{x\to 2}{\text{Lt}} \dfrac{(x-2)f(2) - 2[f(x) - f(2)]}{x-2}$

$= \lim\limits_{x\to 2} [f(2) - 2f'(x)] = 4 - 2 \times 4 = -4$

110. (d) : We have $\lim\limits_{x\to\infty}\left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^{1/x} = \lim\limits_{x\to\infty}\left(\dfrac{1+\frac{5}{x}+\frac{3}{x^2}}{1+\frac{1}{x}+\frac{3}{x^2}}\right)^{1/x} = 1^0 = 1$

111. (c) : Given $f(x+y) = f(x) f(y) \therefore f(0+0) = (f(0))^2$

$\Rightarrow f(0) = 0$ or $f(0) = 1$ but $f(0) \neq 0$

Now $f'(x) = \underset{h\to 0}{\text{Lt}} \dfrac{f(x+h) - f(x)}{h} = \underset{h\to 0}{\text{Lt}} \dfrac{f(x)f(h) - f(x)}{h}$

$f'(x) = f(x) \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h} \quad \therefore f'(0) = f(0) \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$3 = \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$ $(\because f(0) = 1)$

Now $f'(x) = f(x) \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$\therefore f'(5) = f(5) \times 3 = 2 \times 3 = 6$

112. (a) : $\underset{x\to 0}{\text{Lt}} \dfrac{\sqrt{2}\sqrt{\sin^2 x}}{x\sqrt{2}} = \underset{x\to 0}{\text{Lt}} \dfrac{\sin x}{x} = 1.$

113. (a) : Let $A(0,0)$, $B(x, y) = \begin{cases} a\sin t - b\sin\dfrac{at}{b} = x \\ a\cos t - b\cos\dfrac{at}{b} = y \end{cases}$

$\therefore \sqrt{x^2+y^2} = AB = \sqrt{a^2(\sin^2 t + \cos^2 t) + b^2\left(\sin^2\left(\dfrac{at}{b}\right) + \cos^2\left(\dfrac{at}{b}\right)\right) - 2ab\cos\left(t - \dfrac{at}{b}\right)}$

$= \sqrt{a^2 + b^2 - 2ab\cos\alpha}$ (since $|\cos\alpha| \le 1$)

$\le \sqrt{a^2+b^2-2ab} = a - b.$

114. (a) : $\underset{x\to 1}{\text{Lt}} \dfrac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$ (0/0 form)

$= \underset{x\to 1}{\text{Lt}} \dfrac{1}{2\sqrt{f(x)}} \times \dfrac{2\sqrt{x}}{1} \times f'(x) = \dfrac{2\times 1\times 2}{2} = 2$

115. (d) : As $f''(x) - g''(x) = 0 \Rightarrow f'(x) - g'(x) = k$

$f'(1) - g'(1) = k \quad \therefore k = 2$

So $f'(x) - g'(x) = 2 \Rightarrow f(x) - g(x) = 2x + k_1$

$f(2) - g(2) = 4 + k_1$

$k_1 = 2$

So $f(x) - g(x) = 2x + 2$

$\therefore [f(x) - g(x)]_{x=\frac{3}{2}} = \dfrac{2\times 3}{2} + 2 = 5$

116. (b)

117. (d) : $\underset{x\to 0}{\text{Lt}} \dfrac{\log x^n - [x]}{[x]} = \underset{x\to 0}{\text{Lt}} \dfrac{n\log x}{[x]} - 1$

which does not exist as $\underset{x\to 0}{\text{Lt}} \dfrac{\log x}{[x]}$ does not exist

118. (a) : $y_1 = n\left[x + \sqrt{1+x^2}\right]^{n-1}\left[1 + \dfrac{x}{\sqrt{1+x^2}}\right]$

$y_1 = n\left[x+\sqrt{1+x^2}\right]^n \cdot \dfrac{1}{\sqrt{1+x^2}}$

$y_1 = \dfrac{ny}{\sqrt{1+x^2}} \quad \left(y_1 = \dfrac{dy}{dx}\right)$

$\Rightarrow y_1^2(1+x^2) = n^2y^2 \Rightarrow y_1^2(2x) + (1+x^2)(2y_1y_2) = 2yy_1n^2$

$\Rightarrow y_2(1+x^2) + xy_1 = n^2y$