CHAPTER

8 Sequences and Series

1. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series
$1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 + 5^2 + 2\cdot 6^2 + ...$
If $B - 2A = 100\lambda$, then λ is equal to
(a) 496 (b) 232 (c) 248 (d) 464 *(2018)*

2. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that
$\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.
$a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to
(a) 33 (b) 66 (c) 68 (d) 34 *(2018)*

3. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval :
(a) $(-\infty, -10]$ (b) $(-10, 0)$
(c) $(0,10)$ (d) $[10, \infty)$ *(Online 2018)*

4. If $x_1, x_2, ..., x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, ..., \frac{1}{h_n}$ are two A.P.s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 . h_{10}$ equals
(a) 2560 (b) 2650 (c) 3200 (d) 1600 *(Online 2018)*

5. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 .... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$
and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \geq p$, is
(a) 11 (b) 9 (c) 5 (d) 7 *(Online 2018)*

6. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is
(a) $\frac{1}{4} - \frac{1}{\sqrt{2}}$ (b) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$ (c) $\frac{1}{4} - \frac{1}{4\sqrt{2}}$ (d) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$ *(Online 2018)*

7. The sum of the first 20 terms of the series
$1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + ...,$ is
(a) $39 + \frac{1}{2^{19}}$ (b) $39 + \frac{1}{2^{20}}$ (c) $38 + \frac{1}{2^{20}}$ (d) $38 + \frac{1}{2^{19}}$ *(Online 2018)*

8. Let $\frac{1}{x_1}, \frac{1}{x_2}, ..., \frac{1}{x_n}$ ($x_i \neq 0$ for $i = 1, 2, ..., n$) be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to :
(a) 1/8 (b) 3 (c) 13/8 (d) 13/4 *(Online 2018)*

9. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then
(a) b, c and a are in A.P. (b) a, b and c are in A.P.
(c) a, b and c are in G.P. (d) b, c and a are in G.P. *(2017)*

10. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy$, $\forall\ x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to
(a) 165 (b) 190 (c) 255 (d) 330 *(2017)*

11. If the sum of the first n terms of the series $\sqrt{3} + \sqrt{75} + \sqrt{243} + \sqrt{507} + ...$ is $435\sqrt{3}$, then n equals
(a) 29 (b) 18 (c) 15 (d) 13 *(Online 2017)*

12. If the arithmetic mean of two numbers a and b, $a > b > 0$, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to
(a) $\frac{\sqrt{6}}{2}$ (b) $\frac{3\sqrt{2}}{4}$ (c) $\frac{5\sqrt{6}}{12}$ (d) $\frac{7\sqrt{3}}{12}$ *(Online 2017)*

13. Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + ... + \frac{1+2+...+n}{1^3+2^3+...+n^3}$.
If $100\ S_n = n$, then n is equal to
(a) 99 (b) 19
(c) 200 (d) 199 *(Online 2017)*

14. If three positive numbers a, b and c are in A.P. such that $abc = 8$, then the minimum possible value of b is
(a) $4^{2/3}$ (b) $4^{1/3}$
(c) 4 (d) 2 *(Online 2017)*

15. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is
(a) 8/5 (b) 4/3 (c) 1 (d) 7/4 *5GEFL6*

16. If the sum of the first ten terms of the series
$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...,$ is $\frac{16}{5}m$,
then m is equal

(a) 102 (b) 101
(c) 100 (d) 99 *(2016)*

17. Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to
(a) 342 (b) 216 (c) 258 (d) 270 *(Online 2016)*

18. Let $a_1, a_2, a_3, \ldots a_n, \ldots$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to
(a) 306 (b) 204 (c) 153 (d) 612 *(Online 2016)*

19. The sum of first 9 terms of the series
$$\frac{6^8}{6}+\frac{6^8+7^8}{6+8}+\frac{6^8+7^8+8^8}{6+8+:}+3333u$$
(a) 142 (b) 192 (c) 71 (d) 96 *(2015)*

20. If m is A. M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $d_6^9 + 7d_7^9 + d_8^9$ equals
(a) 4 lmn^2 (b) $4l^2m^2n^2$ (c) 4 l^2mn (d) 4 lm^2n *(2015)*

21. Let the sum of the first three terms of an A.P. be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is
(a) 26.5 (b) 28 (c) 29.5 (d) 31 *(Online 2015)*

22. The value of $\sum_{=6;}^{85} - +7.- -8.$ is equal to
(a) 7785 (b) 7780 (c) 7775 (d) 7770 *(Online 2015)*

23. The sum of the 3rd and the 4th terms of a G.P. is 60 and the product of its first three terms is 1000. If the first term of this G.P. is positive, then its 7th term is
(a) 7290 (b) 320 (c) 640 (d) 2430 *(Online 2015)*

24. If $\sum_{=6}^{:} \frac{6}{- +6.- +7.- +8.} = \frac{-1}{8}$ then k is equal to
(a) $\frac{::}{88;}$ (b) $\frac{6<}{65:}$ (c) $\frac{6}{;}$ (d) $\frac{6>}{667}$ *(Online 2015)*

25. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is
(a) $3+\sqrt{2}$ (b) $2-\sqrt{3}$ (c) $2+\sqrt{3}$ (d) $\sqrt{2}+\sqrt{3}$ *(2014)*

26. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + \ldots + 10(11)^9 = k(10)^9$, then k is equal to
(a) $\frac{441}{100}$ (b) 100 (c) 110 (d) $\frac{121}{100}$ *(2014)*

27. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is
(a) $\frac{7}{9}(99-10^{-20})$ (b) $\frac{7}{81}(179+10^{-20})$
(c) $\frac{7}{9}(99+10^{-20})$ (d) $\frac{7}{81}(179-10^{-20})$ *(2013)*

28. **Statement 1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.
Statement 2 : $\sum_{k=1}^{n}(k^3-(k-1)^3)=n^3$ for any natural number n.
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement- 2 is a correct explanation for Statement 1. *(2012)*

29. If 100 times the 100th term of an A.P. with non-zero common difference equals the 50 times its 50th term, then the 150th term of this A.P. is
(a) 150 (b) zero
(c) – 150 (d) 150 times its 50th term *(2012)*

30. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after
(a) 20 months (b) 21 months
(c) 18 months (d) 19 months *(2011)*

31. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an A.P. with common difference –2, then the time taken by him to count all notes is
(a) 24 minutes (b) 34 minutes
(c) 125 minutes (d) 135 minutes *(2010)*

32. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots\ldots$ is
(a) 3 (b) 4 (c) 6 (d) 2 *(2009)*

33. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
(a) 4 (b) – 4 (c) – 12 (d) 12 *(2008)*

34. The sum of the series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots.$ upto infinity is
(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$ (c) e^{-2} (d) e^{-1} *(2007)*

35. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression is equals
(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5}-1)$
(c) $\frac{1}{2}(1-\sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$ *(2007)*

36. Let $a_1, a_2, a_3, \ldots$ be terms of an A.P. If $\frac{a_1+a_2+\ldots+a_p}{a_1+a_2+\ldots+a_q}=\frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals
(a) 41/11 (b) 7/2 (c) 2/7 (d) 11/41 *(2006)*

37. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to
(a) $n(a_1 - a_n)$ (b) $(n - 1)(a_1 - a_n)$
(c) na_1a_n (d) $(n - 1)a_1a_n$ *(2006)*

38. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation
(a) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$
(b) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$
(c) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$
(d) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$ *(2005)*

39. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in
(a) H.P.
(b) Arithmetic-Geometric progression
(c) A.P. (d) G.P. *(2005)*

40. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4 *(2005)*

41. The sum of the series $1 + \frac{1}{4 \cdot 2!} + \frac{1}{16 \cdot 4!} + \frac{1}{64 \cdot 6!} + ...\ \infty$ is
(a) $\frac{e+1}{\sqrt{e}}$ (b) $\frac{e-1}{\sqrt{e}}$ (c) $\frac{e+1}{2\sqrt{e}}$ (d) $\frac{e-1}{2\sqrt{e}}$ *(2005)*

42. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$, and $T_n = \frac{1}{m}$, then $a - d$ equals
(a) $1/mn$ (b) 1 (c) 0 (d) $\frac{1}{m} + \frac{1}{n}$ *(2004)*

43. The sum of first n terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd, the sum is
(a) $\frac{n(n+1)^2}{4}$ (b) $\frac{n^2(n+1)}{2}$
(c) $\frac{3n(n+1)}{2}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$ *(2004)*

44. The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + ...$ is
(a) $\frac{(e-1)^2}{2e}$ (b) $\frac{(e^2-1)}{2e}$ (c) $\frac{(e^2-1)}{2}$ (d) $\frac{(e^2-2)}{e}$ *(2004)*

45. If the system of linear equations $x + 2ay + az = 0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c
(a) are in G.P. (b) are in H.P.
(c) satisfy $a + 2b + 3c = 0$ (d) are in A.P. *(2003)*

46. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A.P., then $f'(a), f'(b)$ and $f'(c)$ are in
(a) G.P. (b) H.P.
(c) Arithmetic-Geometric Progression
(d) A.P. *(2003)*

47. The sum of the series $\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} - ...$ upto ∞ is equal to
(a) $\log_e 2 - 1$ (b) $\log_e 2$
(c) $\log_e (4/e)$ (d) $2\log_e 2$ *(2003)*

48. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)
(a) lie on an ellipse (b) lie on a circle
(c) are vertices of a triangle
(d) lie on a straight line *(2003)*

49. Let R_1 and R_2 respectively be the maximum ranges up and down on an inclined plane and R be the maximum range on the horizontal plane. Then, R_1, R, R_2 are in
(a) A.P. (b) G.P.
(c) H.P.
(d) Arithmetic-Geometric Progression (A.G.P.) *(2003)*

50. If 1, $\log_9(3^{1-x} + 2)$, $\log_3[4 \cdot 3^x - 1]$ are in A.P. then x equals
(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$. *(2002)*

51. $1^3 - 2^3 + 3^3 - 4^3 + ... + 9^3 =$
(a) 425 (b) -425 (c) 475 (d) -475 *(2002)*

52. Sum of infinite number of terms in GP is 20 and sum of their square is 100. The common ratio of GP is
(a) 5 (b) 3/5 (c) 8/5 (d) 1/5 *(2002)*

53. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} ...\infty$ is
(a) 1 (b) 2 (c) 3/2 (d) 4 *(2002)*

54. Fifth term of a GP is 2, then the product of its 9 terms is
(a) 256 (b) 512
(c) 1024 (d) none of these *(2002)*

ANSWER KEY

1. (c)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (d)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (d)	**11.** (c)	**12.** (c)
13. (d)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (d)	**21.** (c)	**22.** (b)	**23.** (b)	**24.** (a)
25. (c)	**26.** (b)	**27.** (b)	**28.** (d)	**29.** (b)	**30.** (b)	**31.** (b)	**32.** (a)	**33.** (c)	**34.** (d)	**35.** (b)	**36.** (d)
37. (d)	**38.** (b)	**39.** (a)	**40.** (a)	**41.** (c)	**42.** (c)	**43.** (b)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (c)	**48.** (d)
49. (c)	**50.** (c)	**51.** (a)	**52.** (b)	**53.** (b)	**54.** (b)						

Explanations

1. **(c) :** Let $S = 1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$
The sum of first 20 terms is
$A = (1^2 + 2^2 + ... + 20^2) + (2^2 + 4^2 + ... + 20^2)$

$$= \frac{20 \cdot 21 \cdot 41}{6} + 4 \cdot \frac{10 \cdot 11 \cdot 21}{6} = \frac{20 \cdot 21}{6}(41 + 22) = 4410$$

$B = 1^2 + 2.2^2 + ... + 2.40^2$
$= (1^2 + 2^2 + ... + 40^2) + (2^2 + 4^2 + ... + 40^2)$

$$= \frac{40 \cdot 41 \cdot 81}{6} + \frac{4 \cdot 20 \cdot 21 \cdot 41}{6} = \frac{40 \times 41}{6}(81 + 42)$$

$$= \frac{40 \cdot 41}{6} \times 123 = 33620$$

$B - 2A = 33620 - 8820 = 24800 \therefore 100\lambda = 24800 \Rightarrow \lambda = 248$

2. **(d) :** $\sum_{k=0}^{12} a_{4k+1} = 416$

$\Rightarrow a_1 + a_5 + ... + a_{49} = 416 \Rightarrow \frac{13}{2}(a_1 + a_{49}) = 416$

As $a_{49} = a_1 + 48d$, so we have $\frac{13}{2}(2a_1 + 48d) = 416$,

$\Rightarrow a_1 + 24d = 32$ ...(i)
Given, $a_9 + a_{43} = 66 \Rightarrow a_1 + 8d + a_1 + 42d = 66$
$\Rightarrow a_1 + 25d = 33$ ...(ii)
The equation (i) and (ii) gives $a_1 = 8$, $d = 1$
Now $a_1^2 + a_2^2 + ... + a_{17}^2 = 8^2 + 9^2 + ... + 24^2$
$= (1^2 + 2^2 + ... + 24^2) - (1^2 + 2^2 + ... + 7^2)$

$$= \frac{24 \cdot 25 \cdot 49}{6} - \frac{7 \cdot 8 \cdot 15}{6}$$

$= 4.25.49 - 7.20 = 4900 - 140 = 4760 = 34(140) \therefore m = 34$

3. **(c) :** Let b, br, br^2, ... be an infinite G.P. with first term b, common ratio r and sum = 5.

$\therefore 5 = \frac{b}{1-r}; |r| < 1 \Rightarrow \frac{b}{5} = 1 - r \Rightarrow r = 1 - \frac{b}{5}$

Now, $|r| < 1 \Rightarrow \left|1 - \frac{b}{5}\right| < 1 \Rightarrow -1 < 1 - \frac{b}{5} < 1$

$\Rightarrow 0 < -\frac{b}{5} + 2 < 2 \Rightarrow 0 < \frac{10-b}{5} < 2$

$\Rightarrow 0 < -b + 10 < 10 \Rightarrow -b + 10 > 0$ and $-b + 10 < 10$
$\Rightarrow b < 10$ and $b > 0$ or $b \in (0, 10)$

4. **(a) :** Let d_1 and $1/d_2$ be the common difference of A.P.

$x_1, x_2, ..., x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}, \dots, \frac{1}{h_n}$ respectively.

Given, $x_3 = 8$; $x_8 = 20$
$\Rightarrow x_1 + 2d_1 = 8$ ...(i) and $x_1 + 7d_1 = 20$ ...(ii)

On solving (i) and (ii), we get $d_1 = \frac{12}{5}$ and $x_1 = \frac{16}{5}$

Similarly; $h_2 = 8$ and $h_7 = 20$

$\Rightarrow \frac{1}{h_2} = \frac{1}{8}$ and $\frac{1}{h_7} = \frac{1}{20} \Rightarrow \frac{1}{h_1} + \frac{1}{d_2} = \frac{1}{8}$ ...(iii)

and $\frac{1}{h_1} + \frac{6}{d_2} = \frac{1}{20}$ ...(vi)

On solving (iii) and (iv), we get $\frac{1}{d_2} = \frac{-3}{200}$, $\frac{1}{h_1} = \frac{28}{200}$

Now, $x_5 = x_1 + 4d_1 = \frac{16}{5} + \frac{48}{5} = \frac{64}{5}$

Also, $\frac{1}{h_{10}} = \frac{1}{h_1} + \frac{9}{d_2} = \frac{28}{200} - \frac{27}{200} = \frac{1}{200}$

$\therefore x_5 \cdot h_{10} = \frac{64}{5} \times 200 = 2560$

5. **(d) :** We have, $A_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 ...... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$

which is a G.P. with $a = \frac{3}{4}$, $r = -\frac{3}{4}$

$$\therefore A_n = \frac{3}{4}\left(\frac{1 - \left(-\frac{3}{4}\right)^n}{1 + \frac{3}{4}}\right) = \frac{3}{7}\left(1 - \left(\frac{-3}{4}\right)^n\right)$$

Also, $B_n = 1 - A_n$
Consider $B_n > A_n \Rightarrow 1 - A_n > A_n$

$\Rightarrow 2A_n < 1 \Rightarrow \frac{6}{7}\left(1 - \left(\frac{-3}{4}\right)^n\right) < 1$

$\Rightarrow 1 - \left(\frac{-3}{4}\right)^n < \frac{7}{6} \Rightarrow (-1)^{n+1}\left(\frac{3}{4}\right)^n < \frac{1}{6} = \frac{1}{2 \times 3}$

$\Rightarrow (-1)^{n+1} 3^{n+1} < 2^{2n-1} \Rightarrow (-3)^{n+1} < 2^{2n-1}$
which is true for all even natural numbers and for all odd natural numbers ≥ 7.
$\therefore$ Least odd natural number p is 7.

6. **(d) :** Given that a, b, c are in A.P.
$\Rightarrow 2b = a + c$ ...(i) Again, a^2, b^2, c^2 are in G.P.
$\Rightarrow b^4 = (ac)^2 \Rightarrow ((b^2)^2 - (ac)^2) = 0$
$\Rightarrow (b^2 - ac)(b^2 + ac) = 0$
$\Rightarrow b^2 = ac$ or $b^2 = -ac$ ...(ii)
But $b^2 = ac$ is not possible. ($\because$ This gives $a = b = c = 1/4$)

Also, $a + b + c = \frac{3}{4} \Rightarrow 3b = \frac{3}{4}$ (From (i))

$\Rightarrow b = 1/4$
Putting this value of b in (i) and (ii), we have

$\frac{1}{2} = a + c$ ... (iii) and $\frac{1}{16} = -ac$ ...(iv)

From (iii), we have $\frac{1}{2} - a = c$

Putting the value of c in (iv), we have $16a^2 - 8a - 1 = 0$

$\Rightarrow a = \frac{8 \pm \sqrt{64+64}}{2\times 16} = \frac{8 \pm 8\sqrt{2}}{2 \times 16} \Rightarrow a = \frac{1 \pm \sqrt{2}}{4} = \frac{1}{4} \pm \frac{1}{2\sqrt{2}}$

$\Rightarrow a = \frac{1}{4} - \frac{1}{2\sqrt{2}}$

7. **(d) :** $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + ...$

$= (2-1) + \left(2 - \frac{1}{2}\right) + \left(2 - \frac{1}{4}\right) + \left(2 - \frac{1}{8}\right) + ... + 20 \text{ terms}$

$= (2 + 2 + ... + 20 \text{ terms}) - \left(1 + \frac{1}{2} + \frac{1}{4} + ... + 20 \text{ terms}\right)$

$= 2 \times 20 - \left(\frac{1 - \left(\frac{1}{2}\right)^{20}}{1 - \frac{1}{2}}\right) = 40 - \left(\frac{1 - \left(\frac{1}{2}\right)^{20}}{\frac{1}{2}}\right)$

$= 40 - 2 + 2\left(\frac{1}{2}\right)^{20} = 38 + \frac{1}{2^{19}}$

8. **(d) :** $x_1 = 4$ and $x_{21} = 20 \Rightarrow \frac{1}{x_1} = \frac{1}{4}$ and $\frac{1}{x_{21}} = \frac{1}{20}$

Now, $a_{21} = a + 20d = \frac{1}{20}$ $\left(\because a_{21} = \frac{1}{20}\right)$

$\Rightarrow \frac{1}{4} + 20d = \frac{1}{20} \Rightarrow d = -\frac{1}{100}$

Now, $x_n > 50$ (Given) $\Rightarrow \frac{1}{x_n} < \frac{1}{50} \Rightarrow \frac{1}{4} - \frac{(n-1)}{100} < \frac{1}{50}$

$\Rightarrow n > 24$

Now, $\sum_{i=1}^{n} \left(\frac{1}{x_i}\right) = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \sum_{i=1}^{25} \left(\frac{1}{x_i}\right) = \frac{25}{2}\left[2 \times \frac{1}{4} - \frac{1}{100} \times 24\right] = \frac{13}{4}$

9. **(a) :** $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$

$\Rightarrow \frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

$\Rightarrow (15a - 3b)^2 = 0, (3b - 5c)^2 = 0, (5c - 15a)^2 = 0$

$\Rightarrow 15a = 3b = 5c \Rightarrow \frac{a}{1} = \frac{b}{5} = \frac{c}{3} \Rightarrow b, c, a$ are A.P.

10. **(d) :** Given, $f(x) = ax^2 + bx + c$

and $f(x + y) = f(x) + f(y) + xy$

$\Rightarrow a(x + y)^2 + b(x + y) + c = ax^2 + bx + c + ay^2 + by + c + xy$

$\Rightarrow 2axy = c + xy$ *i.e.*, $(2a - 1)xy - c = 0 \ \forall \ x, y \in R$

Then, $a = \frac{1}{2}, c = 0$

Also, $a + b + c = 3 \quad \therefore \quad b = 5/2$

Now, $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x$

$\sum_{n=1}^{10} f(x) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n$

$= \frac{1}{2}\frac{10 \cdot 11 \cdot 21}{6} + \frac{5}{2}\frac{10 \cdot 11}{2} = \frac{10 \cdot 11}{12}[21 + 15] = 330$

Alternative solution :

Let $x = m$, $y = 1$ in $f(x + y) = f(x) + f(y) + xy$ to obtain

$f(m + 1) = f(m) + f(1) + m = f(m) + 3 + m$

$\Rightarrow f(m + 1) - f(m) = 3 + m$

Changing m to m–1, we get

$f(m) - f(m - 1) = 3 + (m - 1)$ ... (i)

$f(2) - f(1) = 3 + 1$ ... (ii)

Adding (i) and (ii), we get $f(m + 1) - 3 = 3 + \frac{m(m+1)}{2}$

$f(m+1) = 3m + \frac{m(m+1)}{2} + 3 \ \therefore \ f(m) = 3(m-1) + \frac{(m-1)m}{2} + 3$

$= 3m + \frac{m^2 - m}{2} = \frac{m^2 + 5m}{2} = \frac{m^2}{2} + \frac{5}{2}m$

from here calculation is same as in previous solution.

11. **(c) :** We have, $\sqrt{3} + \sqrt{75} + \sqrt{243} + ... = 435\sqrt{3}$

$\Rightarrow \sqrt{3}[1 + \sqrt{25} + \sqrt{81} + \sqrt{169} + ...] = 435\sqrt{3}$

$\Rightarrow \sqrt{3}[1 + 5 + 9 + 13 + ...] = 435\sqrt{3}$

$\Rightarrow \sqrt{3} \times \frac{n}{2}[2 + (n-1)4] = 435\sqrt{3}$ [as 1 + 5 + 9 + 13 ..... is A.P.]

$\Rightarrow n + \frac{4n^2}{2} - \frac{4n}{2} = 435 \Rightarrow 2n + 4n^2 - 4n = 870$

$\Rightarrow 4n^2 - 2n - 870 = 0 \Rightarrow 2n^2 - n - 435 = 0$

$\Rightarrow n = \frac{1 \pm \sqrt{1 + 4 \times 2 \times 435}}{4} = \frac{1 \pm 59}{4}$

$\Rightarrow n = \frac{1 + 59}{4} = 15 \quad \left[\because \frac{1-59}{4} \text{ is neglected}\right]$

12. **(c) :** Arithmetic mean of a and $b = \frac{a+b}{2}$ and geometric mean of a and $b = \sqrt{ab}$

According to question, $\frac{a+b}{2} = 5\sqrt{ab} \Rightarrow (a + b)^2 = 100ab$

Now, $(a + b)^2 - (a - b)^2 = 4ab$

$\Rightarrow 100ab - (a - b)^2 = 4ab \Rightarrow 96ab = (a - b)^2$

$\therefore \frac{(a+b)^2}{(a-b)^2} = \frac{100}{96} \Rightarrow \frac{a+b}{a-b} = \frac{10}{4\sqrt{6}}$

$\Rightarrow \frac{a+b}{a-b} = \frac{5}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$

13. **(d) :** $T_r = \frac{\frac{r(r+1)}{2}}{\left(\frac{r(r+1)}{2}\right)^2} \Rightarrow T_r = \frac{2}{r(r+1)} = 2\left[\frac{1}{r} - \frac{1}{r+1}\right]$

$\therefore S_n = 2\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 2\left(1 - \frac{1}{n+1}\right) \Rightarrow S_n = \frac{2n}{n+1}$

Now, $100S_n = n \Rightarrow 100 \times \frac{2n}{n+1} = n$

$\Rightarrow n + 1 = 200 \Rightarrow n = 199$

14. (d) : a, b, c are in A.P. $\therefore\ a + c = 2b$
Now, $abc = 8$

$\Rightarrow\ a\cdot c\cdot\left(\frac{a+c}{2}\right) = 8 \Rightarrow ac(a + c) = 16 = 4 \times 4$

$\Rightarrow\ ac = 4$ and $a + c = 4$

So, $b = \frac{4}{2} = 2$

15. (b) : Let d be the common difference and a the first term of the A.P. , then we have
$(a + 4d)^2 = (a + d)(a + 8d)$

Now, $\frac{+9}{+} = \frac{+=}{+9} = \frac{9}{8} = \frac{9}{8}$ mzp $\neq 5$

[Using properties of ratios.]

16. (b) : Let us denote the expression as A

$\therefore\ W = \left(6\frac{8}{:}\right)^7 + \left(7\frac{7}{:}\right)^7 + \left(8\frac{6}{:}\right)^7 +$ 333×| µ{ 65 µq~y –

$= \frac{=^7}{:^7} + \frac{67^7}{:^7} +$ 333×| µ{ 65 µq~y –

$= \frac{9^7}{:^7}\text{-}7^7 + 8^7 + 9^7$ 333×| µ{ 65 µq~y –.

$= \frac{6;}{7:}\left(\frac{66\cdot 67\cdot 78}{;} - 6\right) = \frac{6;}{7:}\cdot{:}\ 5{:} = \frac{6;}{:}\cdot 656$

As the sum is given to be $\frac{6;}{:}$ $\quad\therefore\ m = 101$

17. (b) : $x + y + z = 12$
Now, A.M. $\geq$ G.M.

$$\Rightarrow\ \frac{3\left(\frac{x}{3}\right)+4\left(\frac{y}{4}\right)+5\left(\frac{z}{5}\right)}{12} \geq \left[\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^4\left(\frac{z}{5}\right)^5\right]^{\frac{1}{12}}$$

$\Rightarrow\ \frac{x^3y^4z^5}{3^34^45^5} \leq 1 \ \Rightarrow x^3y^4z^5 \leq 3^3 \cdot 4^4 \cdot 5^5$

But, given $x^3y^4z^5 = (0.1)\ (600)^3$

$\therefore$ All the numbers are equal.

$\therefore\ \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k$ (say) $\Rightarrow\ x = 3k,\ y = 4k,\ z = 5k$

But, $x + y + z = 12 \Rightarrow\ 3k + 4k + 5k = 12 \Rightarrow k = 1$

$\therefore\ x = 3\ ;\ y = 4\ ;\ z = 5$ So, $x^3 + y^3 + z^3 = 216$

18. (a) : We have, $a_3 + a_7 + a_{11} + a_{15} = 72$

$\Rightarrow\ (a_3 + a_{15}) + (a_7 + a_{11}) = 72$

Now, $a_3 + a_{15} = a_7 + a_{11} = a_1 + a_{17}\ \therefore\ a_1 + a_{17} = 36$

Now, $S_{17} = \frac{17}{2}[a_1 + a_{17}] = 17 \times 18 = 306$

19. (d) : The n^{th} term, t_n is

$$\frac{6^8 + 7^8 + \text{333}+\ \ ^8}{6 + 8 + \text{333}+\text{-}7\ \ -6.} = \frac{\frac{^7\text{-}\ \ +6.^7}{9}}{7} = \frac{\text{-}\ \ +6.^7}{9}$$

$$\sum_{=6}^{>} \ = \sum_{=6}^{>} \frac{\text{-}\ \ +6.^7}{9} = \frac{6}{9}\left\{\sum_{=6}^{65}\ ^7 - 6\right\}$$

$$= \frac{6}{9}\left\{\frac{65\cdot 66\cdot 76}{;} - 6\right\} = \frac{6}{9}\ 8{=}\!: -6 = \frac{6}{9} \times 8{=}9 = >;$$

20. (d) : Given that l, G_1, G_2, G_3, n are in G.P.
$G_1 = lr,\ G_2 = lr^2,\ G_3 = lr^3,\ n = lr^4$

Then $G_1{}^4 + 2G_2{}^4 + G_3{}^4 = (lr)^4 + 2(lr^2)^4 + (lr^3)^4$

$= (l^3)(lr^4) + 2l^2(lr^4)^2 + l\cdot(lr^4)^3$

$= l^3\cdot n + 2l^2\cdot n^2 + ln^3 = ln(l^2 + 2nl + n^2) = ln(n + l)^2 = 4m^2nl$

21. (c) : Let $a,\ a + d,\ a + 2d$ be first three terms of an A.P.
So, $a + a + d + a + 2d = 39$

$\Rightarrow\ 3a + 3d = 39 \Rightarrow d = 3\ (\because\ a = 10)$

Sum of last four terms = 178

$\Rightarrow\ l - 3d + l - 2d + l - d + l = 178$

$\Rightarrow\ 4l - 6d = 178\ \Rightarrow 4l = 196 \Rightarrow l = 49$

A. P. is 10, 13, 16, 19, ..., 46, 49

Median $= \frac{65 + 9>}{7} = 7{>}3$

22. (b) : $\sum_{=6;}^{85}\text{-}\ \ +7.\text{-}\ \ -8. = \sum_{=6}^{85}\text{-}\ ^7 - \ \ -;. - \sum_{=6}^{6:}\text{-}\ ^7 - \ \ -;.$ ... (i)

Put $r = 30$ in $\left(\frac{r(r+1)(2r+1)}{6} - \frac{r(r+1)}{2} - 6r\right)$

$= \frac{85\cdot\text{-}86.\cdot;\ 6}{;} - 6{:}\ \text{-}86. - ;\ \text{-}85.$

$= 9455 - 465 - 180 = 8810$ .... (ii)

and putting $r = 15$, we get $\frac{6{:}\ \cdot 6;\ \cdot 86}{;} - \frac{6{:}\ \cdot 6;}{7} - ;\ \cdot 6{:}.$

$= {:}\ \times{=}\times 86 - \frac{6{:}\ \times 6;}{7} - ;\ \times 6{:}$

$= 1240 - 120 - 90 = 1030$ .... (iii)

Substituting the value of (ii) and (iii) in (i), we get the value = 8810 – 1030 = 7780

23. (b) : $ar^2 + ar^3 = 60$ and $a \times ar \times ar^2 = 1000$

$\Rightarrow\ ar(r + r^2) = 60$ and $a^3r^3 = 1000\ \Rightarrow\ ar = 10$

$\Rightarrow\ r + r^2 = 6\ \Rightarrow\ r = -3, 2$

$\Rightarrow\ r = 2\ \Rightarrow\ a = 5\ \therefore\ T_7 = ar^6 = 5 \times 2^6 = 320$

24. (a) : $\sum_{=6}^{:} \frac{6}{\text{-}\ \ +6.\text{-}\ \ +7.\text{-}\ \ +8.} = \frac{}{8}$

$\Rightarrow\ \frac{6}{6\cdot 7\cdot 8\cdot 9} + \frac{6}{7\cdot 8\cdot 9\cdot{:}} + \text{3333333}+ \frac{6}{{:}\ \cdot;\ \cdot<\cdot=} = \frac{}{8}$

$\Rightarrow\ \frac{6}{8}\left[\left(\frac{6}{6\cdot 7\cdot 8} - \frac{6}{7\cdot 8\cdot 9}\right) + \left(\frac{6}{7\cdot 8\cdot 9} - \frac{6}{8\cdot 9\cdot{:}}\right) + \text{333}\right.$

$\left.+\left(\frac{6}{{:}\ \cdot;\ \cdot<} - \frac{6}{;\ \cdot<\cdot=}\right)\right] = \frac{}{8}$

$\Rightarrow\ \frac{6}{8}\left[\frac{6}{6\cdot 7\cdot 8} - \frac{6}{;\ \cdot<\cdot=}\right] = \frac{}{8} \Rightarrow\ \frac{6}{8}\left[\frac{6}{;} - \frac{6}{88;}\right] = \frac{}{8}\ \Rightarrow\ \ = \frac{{:}\,{:}}{88;}$

25. (c) : Let the numbers be a, ar, ar^2, we have
$2|2ar| = a + ar^2\ \Rightarrow\ 4r = r^2 + 1 \Rightarrow r^2 - 4r + 1 = 0$

$\Rightarrow\ r = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

$\therefore\ r = 2 + \sqrt{3}$, as number is positive.

26. (b) : Let $P = 10^9 + 2 \cdot 11 \cdot 10^8 + ... + 10 \cdot 11^9$

we have $\frac{11}{10}P = 11\cdot 10^8 + ... + 9\cdot 11^9 + 11^{10}$ on subtracting, we get

$\Rightarrow \ \frac{1}{10}P = 11^{10} - [10^9 + 11^1 \cdot 10^8 + 11^2 \cdot 10^7 + ... + 11^9]$

$$= 11^{10} - 10^9 \left\{ \frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1} \right\} = 11^{10} - 11^{10} + 10^{10}$$

$\therefore \quad P = 10^{11} = (100) \cdot 10^9$

On comparison, $k = 100$

27. (b) : $t_r = \underbrace{0.77777....7}_{r \text{ terms}} = \frac{7}{10} + \frac{7}{10^2} + ... + \frac{7}{10^r} = \frac{7}{9}(1 - 10^{-r})$

$$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9}\left(20 - \sum_{r=1}^{20} 10^{-r}\right) = \frac{7}{9}\left\{20 - \frac{1}{9}(1 - 10^{-20})\right\}$$

$$= \frac{7}{81}(179 + 10^{-20})$$

28. (d) : Statement 1 :

$1 + (1 + 2 + 4) + (4 + 6 + 9) + ... + (361 + 380 + 400)$ is 8000

Statement 2 : $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$

Statement 1 : $T_1 = 1$, $T_2 = 7 = 8 - 1$,

$T_3 = 19 = 27 - 8 \ \Rightarrow T_n = n^3 - (n - 1)^3$

$\therefore$ Statement 2 is a correct explanation of statement 1.

29. (b) : $100\ (a + 99d) = 50\ (a + 49d)$

$\Rightarrow \ a + 149d = 0$ *i.e.*, $T_{150} = 0$

30. (b) : Let it happens after n months.

$$3 \times 200 + \frac{n-3}{2}\{2 \times 240 + (n-4)40\} = 11040$$

$$\Rightarrow \ \left(\frac{n-3}{2}\right)(480 + 40n - 160) = 11040 - 600 = 10440$$

$\Rightarrow \ n^2 + 5n - 546 = 0 \ \Rightarrow \ (n + 26)(n - 21) = 0 \ \therefore \ n = 21.$

31. (b) : We have $a_1 + a_2 + ... + a_n = 4500$

$\Rightarrow \ a_{11} + a_{12} + ... + a_n = 4500 - 10 \times 150 = 3000$

$\Rightarrow \ 148 + 146 + ... = 3000$

$$\Rightarrow \ \frac{n-10}{2} \cdot \left(2 \times 148 + (n - 10 - 1)(-21)\right) = 3000$$

Let $n - 10 = m \ \Rightarrow m \times 148 - m(m - 1) = 3000$

$\Rightarrow \ m^2 - 149m + 3000 = 0 \ \Rightarrow (m - 24)(m - 125) = 0$

$\therefore \ \ m = 24, 125$, giving $n = 34, 135$

But for $n = 135$, we have $a_{135} = 148 + (135 - 1)(-2) = 148 - 268 < 0$

But a_{34} is positive.

Hence, $n = 34$ is the only answer.

32. (a) : Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ ...(1)

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...$ ...(2)

Subtracting (2) from (1), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...$$

$$= \frac{4}{3}\left[1 + \frac{1}{3} + ...\text{to } \infty\right] = \frac{4}{3} \cdot \frac{1}{1 - \frac{1}{3}} = \frac{4}{3} \cdot \frac{1}{2/3} = 2$$

$\Rightarrow \ \frac{2}{3}S = 2 \ \therefore \ S = 3$

33. (c) : Let the G.P. be $a, ar, ar^2, ar^3, ...$

we have $a + ar = 12$ ...(i)

$ar^2 + ar^3 = 48$ ...(ii)

on division we have $\frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \ \Rightarrow \ r^2 = 4$

$\therefore \quad r = \pm 2$

But the terms are alternately positive and negative, $\therefore$ $r = -2$

Now $a = \frac{12}{1+r} = \frac{12}{1-2} = \frac{12}{-1} = -12$ (From (i))

34. (d) : $\because \ e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - ....$ upto infinity

Then put $x = 1$, we get

$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ....$ upto infinity.

35. (b) : Given, $a = ar + ar^2 \ \Rightarrow r^2 + r - 1 = 0 \Rightarrow r = \frac{-1+\sqrt{5}}{2}$.

36. (d) : Given $a_1, a_2, a_3, ...$ be terms of A.P.

$$\frac{a_1 + a_2 + ...a_p}{a_1 + a_2 + ...a_q} = \frac{p^2}{q^2} \ \Rightarrow \ \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \ \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$\Rightarrow \ [2a_1 + (p - 1)d]q = p[2a_1 + (q - 1)d]$

$\Rightarrow \ 2a_1(q - p) = d[(q - 1)p - (p - 1)q]$

$\Rightarrow \ 2a_1(q - p) = d(q - p) \Rightarrow 2a_1 = d$

$$\Rightarrow \ \frac{a_6}{a_{21}} = \frac{a_1 + 5d}{a_1 + 20d} = \frac{a_1 + 10a_1}{a_1 + 40a_1} \ \Rightarrow \ \frac{a_6}{a_{21}} = \frac{11}{41}$$

37. (d) : Given $a_1, a_2, ... a_n$ are in H.P.

$\Rightarrow \ \frac{1}{a_1}, \frac{1}{a_2} ..... \frac{1}{a_n} \in$ A.P. $\ \Rightarrow \ \frac{1}{a_2} - \frac{1}{a_1} = d$

$\Rightarrow \ a_1 a_2 = \frac{a_1 - a_2}{d} = \frac{a_1}{d} - \frac{a_2}{d}$ ... (i)

$a_2 a_3 = \frac{a_2}{d} - \frac{a_3}{d}$ ... (ii)

$\vdots$

$a_{n-1}a_n = \frac{a_{n-1}}{d} - \frac{a_n}{d}$ ... (n)

Adding (i), (ii) .... (n) equations we get

$a_1a_2 + a_2a_3 + a_3a_4 + ... a_{n-1}\, a_n = \frac{a_1}{d} - \frac{a_n}{d}$

Also $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d \ \Rightarrow \ \frac{a_1 - a_n}{d} = (n-1)a_1 a_n$

$\therefore \ \ a_1a_2 + a_2a_3 + ... a_{n-1}\, a_n = (n - 1)a_1a_n.$

38. (b) : $T_{r+1} = {}^mC_r\, y^r. \quad \therefore \ {}^mC_{r-1} + {}^mC_{r+1} = 2 \times {}^mC_r$

$$\Rightarrow \frac{m!}{(r-1)!(m-r+1)!} + \frac{m!}{(r+1)!(m-r-1)!} = 2\frac{m!}{r!(m-r)!}$$

$$\Rightarrow \frac{r(r+1)}{(r+1)!(m-r+1)!} + \frac{(m-r+1)(m-r)}{(r+1)!(m-r+1)!} = \frac{2(r+1)(m-r+1)}{(r+1)!(m-r+1)!}$$

$\Rightarrow$ $r(r+1) + (m-r+1)(m-r) = 2(r+1)(m-r+1)$

$\Rightarrow$ $r(r+1) + (m-r)^2 + m - r - 2(r+1)(m-(r-1)) = 0$

$\Rightarrow$ $r(r+1) + m^2 + r^2 - 2mr + m - r + 2(r^2-1)\ 2m(r+1) = 0$

$\Rightarrow$ $m^2 - m(4r+1) + 4r^2 - 2 = 0.$

39. (a) : Given $|a| < 1, |b| < 1, |c| < 1,\ a, b, c \in$ A.P.

and $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a},\ \sum_{n=0}^{\infty} b^n = \frac{1}{1-b},\ \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$

$\therefore\ x = \frac{1}{1-a},\ y = \frac{1}{1-b},\ z = \frac{1}{1-c}$

$\Rightarrow a = \frac{x-1}{x},\ b = \frac{y-1}{y},\ c = \frac{z-1}{z}$

as $a, b, c \in$ A.P. $\therefore\ 2b = a + c$

$2\left(\frac{y-1}{y}\right) = \frac{x-1}{x} + \frac{z-1}{z} \Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z} \Rightarrow x, y, z \in$ H.P.

40. (a) : Let t_r denote the r^{th} term of G.P. with first term b and common ratio R

$\therefore\ t_r = bR^{r-1}. \therefore\ \log t_r = \log b + (r-1)\log R$

Now from given determinant we have

$$\begin{vmatrix} \log b+(r-1)\log R & \log b + r\log R & \log b+(r+1)\log R \\ \log b+(r+2)\log R & \log b+(r+3)\log R & \log b+(r+4)\log R \\ \log b+(r+5)\log R & \log b+(r+6)\log R & \log b+(r+7)\log R \end{vmatrix}$$

(applying $C_2 \to 2C_2 - (C_1 + C_3)$)

$$= \frac{1}{2}\begin{vmatrix} \log b+(r-1)\log R & 0 & \log b+\log R(r+1) \\ \log b+(r+2)\log R & 0 & \log b+(r+4)\log R \\ \log b+(r+5)\log R & 0 & \log b+(r+7)\log R \end{vmatrix} = \frac{1}{2}\times 0 = 0.$$

41. (c) : $1 + \frac{1}{4(2!)} + \frac{1}{16(4!)} + \frac{1}{64(6!)} + \ldots.\infty$

$= 1 + \frac{1}{2^2\,2!} + \frac{1}{2^4(4!)} + \frac{1}{2^6(6!)} + \ldots.\infty$

$= \frac{1}{2}\left[2\left(1 + \frac{1}{2^2\,2!} + \frac{1}{2^4(4!)} + \frac{1}{2^6(6!)} + \ldots.\infty\right)\right]$

$= \frac{1}{2}\left[2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots\infty\right)\right]$

$= \frac{1}{2}\left[e^x + e^{-x}\right]$ where $x = 1/2$

$= \frac{1}{2}\left[e^{1/2} + e^{-1/2}\right] = \frac{e+1}{2\sqrt{e}}$

42. (c) : $T_m = a + (m-1)d = \frac{1}{n}$ ...(i)

$T_n = a + (n-1)d = \frac{1}{m}$ ...(ii)

Now $T_m - T_n = \frac{1}{n} - \frac{1}{m} = (m-n)d \Rightarrow d = \frac{1}{mn}$ and $a = \frac{1}{mn}$

$\therefore\ a - d = 0$

43. (b): As S_n is needed for n is odd let $n = 2k+1$

$\therefore\ S_n = S_{2k+1}$ = Sum up to $2k$ terms + $(2k+1)^{th}$ term

$= \frac{2k(2k+1)^2}{2}$ + last term

$= \frac{(n-1)n^2}{2} + n^2$ as $n = 2k+1 = \frac{n^2(n+1)}{2}$

44. (a) : $e^x + e^{-x} = 2\left[1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\infty\right]$

$\frac{e+e^{-1}}{2} - 1 = \frac{1}{2!} + \frac{1}{4!} + \ldots\infty$

$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\infty$

45. (b) : For non trivial solution the determinant of the coefficient of various term vanish

i.e. $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$\Rightarrow (3bc - 4bc) - 2a(c-b) + a(4c-3b) = 0$

$\Rightarrow \frac{2ac}{a+c} = b$

$\Rightarrow a, b, c \in$ H.P.

46. (d) : Let the polynomial be $f(x) = ax^2 + bx + c$

given $f(1) = f(-1) \Rightarrow b = 0$

$\therefore f(x) = ax^2 + c$

now $f'(x) = 2ax$

$\therefore f'(a) = 2a^2, f'(b) = 2ab, f'(c) = 2ac$ as $a, b, c \in$ A.P.

$\Rightarrow$ $a^2, ab, ac \in$ A.P. $\Rightarrow$ $2a^2, 2ab, 2ac \in$ A.P.

$\Rightarrow$ $f'(a), f'(b), f'(c) \in$ A.P.

47. (c): $s = \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots.\infty$

Let $s_1 = \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \ldots.\infty$

$\therefore\ t_n = \frac{1}{(2n-1)(2n)} = \frac{1}{2n-1} - \frac{1}{2n}$

$\therefore\ s_n = \Sigma t_n = \sum\left(\frac{1}{2n-1} - \frac{1}{2n}\right)$

$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}\ \ldots..\infty = \log_e 2$ ...(A)

Again $s_2 = \frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + \ldots.\infty$

$t'_n = \frac{1}{(2n)(2n+1)}$

$s_2 = \Sigma t_n' = \sum\frac{1}{(2n)(2n+1)} = \sum\left(\frac{1}{2n} - \frac{1}{2n+1}\right)$

$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots.\infty = -\left[-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \ldots\infty\right]$

$= -[\log_e 2 - 1] = 1 - \log_e 2$ ...(B)

Now $s = s_1 - s_2 = (A) - (B)$

$= \log_e 2 - 1 + \log_e 2 = \log(4/e)$

48. **(d) :** Let $x_1 = a \therefore x_2 = ar, x_3 = ar^2$
and $y_1 = b \therefore y_2 = br, y_3 = br^2$
Now $A(a, b)$, $B(ar, br)$, $C(ar^2, br^2)$

Now slope of $AB = \dfrac{b(1-r)}{a(1-r)} = \dfrac{b}{a}$ and

slope of $BC = \dfrac{br(1-r)}{ar(1-r)} = \dfrac{b}{a}$ as slope of AB = slope of BC

$\therefore AB \parallel BC$, but point B is common so A, B, C are collinear.

49. **(c) :** Let θ be the angle of inclination of plane to horizontal and u be the velocity of projection of the projectile

$\therefore \quad R_1 = \dfrac{u^2}{g(1+\sin\theta)}, R_2 = \dfrac{u^2}{g(1-\sin\theta)}$

$\therefore \quad \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2g}{u^2} = \dfrac{2}{R} \Rightarrow R_1, R, R_2 \in$ H.P.

50. **(c) :** As 1, $\dfrac{1}{2}\log_3(3^{1-x}+2)$, $\log_3(4\cdot 3^x - 1) \in$ A.P.

$\Rightarrow \quad \log_3(3^{1-x}+2) = \log_3(4\cdot 3^x - 1) + 1$
$\Rightarrow \quad 3^{1-x} + 2 = (4\cdot 3^x - 1) \times 3 \because \log_3 3 = 1.$
$\Rightarrow \quad 3^{1-x} + 2 = 12\cdot 3^x - 3$
$\Rightarrow \quad 3^x[(3^{1-x}) + 2] = 12\cdot 3^{2x} - 3\cdot 3^x$ (multiplying 3^x both side)
$\Rightarrow \quad 12t^2 - 5t - 3 = 0$ where $t = 3^x$
$\Rightarrow \quad (3t+1)(4t-3) = 0 \Rightarrow t = -1/3, t = 3/4$
$\Rightarrow \quad 3^x = -1/3$ which is not possible and $t = \dfrac{3}{4} \Rightarrow 3^x = \dfrac{3}{4}$
$\Rightarrow \quad x\log_3 3 = \log_3 3 - \log_3 4$
(By taking logarithm at the base 3 both sides)
$\Rightarrow \quad x = 1 - \log_3 4$

51. **(a) :** $(1^3 + 3^3 + 5^3 + ... + 9^3) - (2^3 + 4^3 + 6^3 + 8^3)$
$= (1^3 + 3^3 + 5^3 + ... + 9^3) - 2^3(1^3 + 2^3 + 3^3 + 4^3)$
$= [1^3 + 3^3 + ... + (2n-1)^3]_{n = \text{odd} = 5} - 2^3[1^3 + 2^3 + ... + n^3]_{n = \text{even} = 4}$
$= [2n(n+1)(n+2)(n+3) - 12n(n+1)(n+2) + 13n(n+1) - n]_{n = 5\ (\text{odd})} - 2^3\left[\dfrac{n^2(n+1)^2}{4}\right]_{\substack{n=4 \\ (\text{even})}}$
$= [2 \times 5 \times 6 \times 7 \times 8 - 12 \times 5 \times 6 \times 7 + 13 \times 5 \times 6 - 5] - 2^3\left(\dfrac{16 \times 25}{4}\right)$
$= [3750 - 5(505)] - 2 \times 16 \times 25 = 1225 - 800 = 425$

52. **(b) :** Let terms of G.P. are $a, ar, ar^2, ...$
$\therefore S_\infty = \dfrac{a}{1-r}$ where a = first term, r = common ratio
$S_\infty = 20$
According to question $\dfrac{a}{1-r} = 20$
$\Rightarrow a = 20(1-r)$ ...(i)
Also $\dfrac{a^2}{1-r^2} = 100 \Rightarrow \dfrac{a}{1-r}\cdot\dfrac{a}{1+r} = 100$
$\Rightarrow a = 5(1+r)$ ...(ii)
Solving (i) and (ii) we have $r = 3/5$

53. **(b) :** $S_\infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+...\infty} = 2^\lambda$(say) ...(*)

Where $\lambda = \dfrac{1}{4} + \dfrac{2}{8} + \dfrac{3}{16} + \dfrac{4}{32} + ...\infty$ ... (*A*)

$\dfrac{\lambda}{2} = 0 + \dfrac{1}{8} + \dfrac{2}{16} + \dfrac{3}{32} + \dfrac{4}{64} + ...\infty$ ... (*B*)

Now $(B) - (A) \Rightarrow \dfrac{\lambda}{2} = \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \dfrac{1}{32} + \dfrac{1}{64} + ...$

$\dfrac{\lambda}{2} = \dfrac{a}{1-r} = \dfrac{1}{4} \times \dfrac{2}{1} \therefore \lambda = 1$

so $S_\infty = 2^1$

54. **(b) :** Let first term of a G.P is a and common ratio r
$\therefore \quad t_5 = ar^4 = 2$

$\therefore \quad \prod_{i=1}^{9} a_i = a \cdot ar\ ar^2\ .....ar^8$

$= a^9 r^{\frac{8\times 9}{2}}$
$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$