CHAPTER

7 Binomial Theorem and its Simple Applications

1. The sum of the co-efficients of all odd degree terms in the expansion of $(x+\sqrt{x^3-1})^5+(x-\sqrt{x^3-1})^5$, $(x > 1)$ is

(a) 2 (b) –1 (c) 0 (d) 1 *(2018)*

2. If n is the degree of the polynomial,

$$\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$

and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to :

(a) $(24, (10)^8)$ (b) $(12, (20)^4)$

(c) $(12, 8(10)^4)$ (d) $(8, 5(10)^4)$ *(Online 2018)*

3. The coefficient of x^{10} in the expansion of $(1 + x)^2 (1 + x^2)^3 (1 + x^3)^4$ is equal to

(a) 50 (b) 52 (c) 44 (d) 56

(Online 2018)

4. The coefficient of x^2 in the expansion of the product $(2 - x^2)\cdot((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ is :

(a) 155 (b) 106 (c) 108 (d) 107

(Online 2018)

5. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots\ldots\ldots + ({}^{21}C_{10} - {}^{10}C_{10})$ is

(a) $2^{21} - 2^{10}$ (b) $2^{20} - 2^9$

(c) $2^{20} - 2^{10}$ (d) $2^{21} - 2^{11}$ *(2017)*

6. If $(27)^{999}$ is divided by 7, then the remainder is

(a) 6 (b) 1

(c) 2 (d) 3 *(Online 2017)*

7. The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$, is

(a) 1 (b) –4

(c) –1 (d) 4 *(Online 2017)*

8. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$ is 28, then the sum of the coefficients of all the terms in this expansion is

(a) 64 (b) 2187 (c) 243 (d) 729

(2016)

9. For $x \in R$, $x \neq -1$, if $(1 + x)^{2016} + x(1 + x)^{2015} + x^2 (1 + x)^{2014} + \ldots. + x^{2106} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to

(a) $\frac{2017!}{17!\,2000!}$ (b) $\frac{2016!}{17!\,1999!}$

(c) $\frac{2016!}{16!}$ (d) $\frac{2017!}{2000!}$ *(Online 2016)*

10. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, $(x > 0)$, are m and n respectively, then $\frac{m}{n}$ is equal to

(a) 27 (b) 182 (c) $\frac{5}{4}$ (d) $\frac{4}{5}$

(Online 2016)

11. The sum of coefficients of integral powers of x in the binomial expansion of $\left(6-7\sqrt{\ }\right)^{:5}$ is

(a) $\frac{6}{7}\left(8^{:5}-6\right)$ (b) $\frac{6}{7}\left(7^{:5}+6\right)$

(c) $\frac{6}{7}\left(8^{:5}+6\right)$ (d) $\frac{6}{7}\left(8^{:5}\right)$ *(2015)*

12. If the coefficients of the three successive terms in the binomial expansion of $(1 + x)^n$ are in the ratio 1 : 7 : 42, then the first of these terms in the expansion is

(a) 6th (b) 7th (c) 8th (d) 9th

(Online 2015)

13. The term independent of x in the binomial expansion of

$$\left(6-\frac{6}{\ }+8\ :\right)\left(7\ ^7-\frac{6}{\ }\right)^{=}\ u\!\!-$$

(a) 400 (b) 496 (c) –400 (d) –496

(Online 2015)

14. If $X = \{4^n - 3n - 1, n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to

(a) $Y - X$ (b) X (c) Y (d) N *(2014)*

15. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(a) $\left(14, \frac{251}{3}\right)$ (b) $\left(14, \frac{272}{3}\right)$

(c) $\left(16, \frac{272}{3}\right)$ (d) $\left(16, \frac{251}{3}\right)$ *(2014)*

16. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

(a) 120 (b) 210 (c) 310 (d) 4 *(2013)*

17. If n is a positive integer, then $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}$ is
(a) an even positive integer.
(b) a rational number other than positive integer
(c) an irrational number.
(d) an odd positive integer. *(2012)*

18. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is
(a) –144 (b) 132 (c) 144 (d) –132 *(2011)*

19. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is
(a) 2 (b) 7 (c) 8 (d) 0 *(2009)*

20. **Statement-1 :** $\sum_{r=0}^{n}(r+1)\,{}^{n}C_r = (n+2)2^{n-1}$

Statement-2 : $\sum_{r=0}^{n}(r+1)\,{}^{n}C_r\,x^r = (1+x)^n + nx(1+x)^{n-1}$

(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

21. In the binomial expansion of $(a - b)^n, n \geq 5$, the sum of 5th and 6th terms is zero, then a/b equals
(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$ *(2007)*

22. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then a_n is
(a) $\frac{b^n - a^n}{b-a}$ (b) $\frac{a^n - b^n}{b-a}$
(c) $\frac{a^{n+1} - b^{n+1}}{b-a}$ (d) $\frac{b^{n+1} - a^{n+1}}{b-a}$ *(2006)*

23. For natural numbers m, n if
$(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ...,$
and $a_1 = a_2 = 10$, then (m, n) is
(a) (20, 45) (b) (35, 20)
(c) (45, 35) (d) (35, 45) *(2006)*

24. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
(a) $a + b = 1$ (b) $a - b = 1$
(c) $ab = 1$ (d) $\frac{a}{b} = 1$ *(2005)*

25. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as
(a) $3x + \frac{3}{8}x^2$ (b) $1 - \frac{3}{8}x^2$
(c) $\frac{x}{2} - \frac{3}{8}x^2$ (d) $-\frac{3}{8}x^2$ *(2005)*

26. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals
(a) –3/10 (b) 10/3 (c) –5/3 (d) 3/5 *(2004)*

27. The coefficient of x^n in expansion of $(1 + x)(1 - x)^n$ is
(a) $(-1)^{n-1}(n - 1)^2$ (b) $(-1)^n(1 - n)$
(c) $(n - 1)$ (d) $(-1)^{n-1}\, n$ *(2004)*

28. If $s_n = \sum_{r=0}^{n} \frac{1}{{}^{n}C_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^{n}C_r}$, then $\frac{t_n}{s_n}$ is equal to
(a) $n - 1$ (b) $\frac{1}{2}n - 1$ (c) $\frac{1}{2}n$ (d) $\frac{2n-1}{2}$ *(2004)*

29. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is
(a) 5th term (b) 8th term (c) 6th term (d) 7th term *(2003)*

30. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is
(a) 33 (b) 34 (c) 35 (d) 32 *(2003)*

31. The positive integer just greater than $(1 + .0001)^{1000}$ is
(a) 4 (b) 5 (c) 2 (d) 3 *(2002)*

32. r and n are positive integers $r > 1$, $n > 2$ and coefficient of $(r + 2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1 + x)^{2n}$ are equal, then n equals
(a) $3r$ (b) $3r + 1$ (c) $2r$ (d) $2r + 1$ *(2002)*

33. The coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ are
(a) equal (b) equal with opposite signs
(c) reciprocals of each other (d) none of these *(2002)*

34. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is
(a) 1594 (b) 792 (c) 924 (d) 2924 *(2002)*

ANSWER KEY

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (b)	**11.** (c)	**12.** (b)
13. (a)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (a)	**20.** (c)	**21.** (b)	**22.** (d)	**23.** (d)	**24.** (c)
25. (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (a)	**31.** (c)	**32.** (c)	**33.** (a)	**34.** (c)		

Explanations

1. (a) : We have, $(a + b)^5 + (a - b)^5 = 2\{a^5 + {}^5C_2 . a^3b^2 + {}^5C_4\, ab^4\}$

with $a = x$, $b = \sqrt{x^3 - 1}$ $\therefore$ $(x + \sqrt{x^3-1})^5 + (x - \sqrt{x^3-1})^5$

$= 2\{x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2\}$

Sum of the coeff. of odd degree terms is

$2\{1 - 10 + 5 + 5\} = 2$

2. (b) : On rationalizing given polynomial, we get

$$\left[\frac{2\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)}{2}\right]^8 + \left[\frac{2\left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)}{2}\right]^8$$

$$= 2[{}^8C_0(\sqrt{5x^3+1})^8 + {}^8C_2(\sqrt{5x^3+1})^6(5x^3-1) + {}^8C_4(\sqrt{5x^3+1})^4(5x^3-1)^2 + {}^8C_6(\sqrt{5x^3+1})^2(5x^3-1)^3 + {}^8C_8(5x^3-1)^4]$$

$= 2[(5x^3 + 1)^4 + 28(5x^3 + 1)^3(5x^3 - 1) + 70(5x^3 + 1)^2 (5x^3 - 1)^2 + 28(5x^3 + 1)(5x^3 - 1)^3 + (5x^3 - 1)^4]$

$\therefore$ $n = 12$

and $m = 2(5^4 + 140 \cdot 5^3 + 70 \cdot 5^4 + 140 \cdot 5^3 + 5^4) = 1{,}60{,}000 = (20)^4$

3. (b) : $(1 + x)^2(1 + x^2)^3(1 + x^3)^4 = (1 + x^2 + 2x) \times (1 + x^6 + 3x^2 + 3x^4) \times (1 + 4x^3 + 6x^6 + 4x^9 + x^{12})$

So, coefficient of $x^{10} = 36 + 8 + 8 = 52$

4. (b) : $(1 + 2x + 3x^2)^6 = {}^6C_0 + {}^6C_1(2x + 3x^2) + {}^6C_2(2x + 3x^2)^2 + \ldots + (2x + 3x^2)^6$

$(1 - 4x^2)^6 = {}^6C_0 - {}^6C_1(4x^2) + {}^6C_2(4x^2)^2 - \ldots + (4x^2)^6$

So, coefficient of x^2 = 2 Coefficient of x^2 in $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ – constant term in $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$= 2(18 + 60 - 24) - 2 = 108 - 2 = 106$

5. (c) : $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_{10} - {}^{10}C_{10})$

$= ({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10})$

$$= \frac{2^{21}}{2} - 2^{10} = 2^{20} - 2^{10}$$

6. (a) : We have, $\dfrac{(27)^{999}}{7} = \dfrac{(28-1)^{999}}{7} = \dfrac{28\lambda - 1}{7}$

$$= \frac{28\lambda - 7 + 7 - 1}{7} = \frac{7(4\lambda - 1) + 6}{7}$$

$\therefore$ Remainder = 6

7. (a) : $\left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}}\right)^{10}$

$$= \left[\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)} - \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$= (x^{1/3} + 1 - 1 - 1/x^{1/2})^{10} = (x^{1/3} - 1/x^{1/2})^{10}$

General term in given expansion

$$= {}^{10}C_r (x^{\frac{1}{3}})^{10-r}\left(\frac{-1}{x^{1/2}}\right)^r = {}^{10}C_r\, x^{\frac{10}{3}-\frac{r}{3}-\frac{r}{2}}(-1)^r$$

Now for x^{-5}, $\dfrac{10}{3} - \dfrac{r}{3} - \dfrac{r}{2} = -5$

$$\Rightarrow \frac{10}{3} + 5 = r\left(\frac{1}{3} + \frac{1}{2}\right) \Rightarrow \frac{25}{3} = \frac{5}{6}r \Rightarrow r = \frac{25}{3} \times \frac{6}{5} = 10$$

$\therefore$ Coeff. of $x^{-5} = {}^{10}C_{10}(1)(-1)^{10} = 1$

8. (d) : The number of terms in the expansion of

$\left(6 - \dfrac{7}{\ } + \dfrac{9}{7}\right)$ u $^{n+2}C_2$

We have $^{n+2}C_2 = 28$ giving $(n + 1)(n + 2) = 56$

Then $n = 6$ $\therefore$ Sum of coefficients $= (1 - 2 + 4)^6 = 3^6 = 729$

9. (a) : Let $S = (1 + x)^{2016} + x(1 + x)^{2015} + x^2(1 + x)^{2014} + \ldots + x^{2015}(1 + x) + x^{2016}$ ...(i)

$$\left(\frac{x}{1+x}\right)S = x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots + x^{2016} + \frac{x^{2017}}{1+x} \quad \ldots(ii)$$

Subtracting (ii) from (i), we get

$$\frac{S}{1+x} = (1+x)^{2016} - \frac{x^{2017}}{1+x} \quad \therefore\ S = (1 + x)^{2017} - x^{2017}$$

a_{17} = coefficient of $x^{17} = {}^{2017}C_{17} = \dfrac{2017!}{17!2000!}$

10. (b) : $T_{r+1} = {}^{18}C_r\left(x^{\frac{1}{3}}\right)^{18-r}\left(\dfrac{1}{2x^{\frac{1}{3}}}\right)^r = {}^{18}C_r x^{6-\frac{2r}{3}}\dfrac{1}{2^r}$

Put $6 - \dfrac{2r}{3} = -2 \Rightarrow r = 12$ and $6 - \dfrac{2r}{3} = -4 \Rightarrow r = 15$

$$\therefore \frac{\text{Coefficient of } x^{-2}}{\text{Coefficient of } x^{-4}} = \frac{{}^{18}C_{12}\frac{1}{2^{12}}}{{}^{18}C_{15}\frac{1}{2^{15}}} = 182$$

11. (c) : By Binomial theorem

$$-6-7\sqrt{\ }.^{:5} = {}^{:5}Y_5 - {}^{:5}Y_6 -7\sqrt{\ }. + {}^{:5}Y_7 -7\sqrt{\ }.^7 + 3333333333 + {}^{:5}Y_{:5} -7\sqrt{\ }.^{:5}$$

$$-6+7\sqrt{\ }.^{:5} = {}^{:5}Y_5 + {}^{:5}Y_6 -7\sqrt{\ }. + {}^{:5}Y_7 -7\sqrt{\ }.^7 + 3333333333 + {}^{:5}Y_{:5} -7\sqrt{\ }.^{:5}$$

On addition

$$-6+7\sqrt{\ }.^{:5} + -6-7\sqrt{\ }.^{:5} = 7 - {}^{:5}Y_5 + {}^{:5}Y_7 -7\sqrt{\ }.^7 + {}^{:5}Y_8 -7\sqrt{\ }.^8 + 333333 + {}^{:5}Y_{:5} -7\sqrt{\ }.^{:5}.$$

Set $x = 1$ to obtain

$3^{50} + 1 = 2$ (sum of coefficients of integral powers of x)

$\therefore$ Sum of coeff. of integral powers of $= \dfrac{6}{7} -8^{:5} + 6.$

12. **(b) :** We have $\frac{Y}{6} = \frac{Y_{+6}}{<} = \frac{Y_{+7}}{97}$

Solving, we get $r = 6$

13. **(a) :** The general term in second bracket is

$^{=}Y\ \text{-}7\ \ ^{7}.^{=-}\ \left(-\frac{6}{\ }\right)$

$\Rightarrow\quad {}^8C_r\,(2x^2)^{8-r}\left(-\frac{6}{\ }\right)\ -\frac{6}{\ }\ ^{=}Y\ \text{-}7\ \ ^{7}.^{=-}\ \cdot\left(\frac{-6}{\ }\right)$

$+8\ :\ ^{=}Y\ \text{-}7\ \ ^{7}.^{=-}\ \left(\frac{-6}{\ }\right)$

$= {}^8C_r\,2^{8-r}\,(-1)^r\,x^{16-3r} - {}^8C_r\,2^{8-r}\,(-1)^r\,x^{15-3r} + 3\,.\,{}^8C_r\,2^{8-r}\,x^{21-3r}$

For independent term,

$16 - 3r = 0,\ 15 - 3r = 0 \Rightarrow r = 5,\quad 21 - 3r = 0 \Rightarrow r = 7$

$r = 5,\ r = 7$ is in 2nd term and 3rd term resp.

$\therefore$ term independent of $x = -\,{}^8C_5\,2^3\,(-1)^5 - 3\,.\,{}^8C_7\,.\,2$

$= 448 - (6 \times 8) = 400$

14. **(c) :** $X = \{4^n - 3n - 1\},\ Y = \{9(n-1) : n \in N\}$

$4^n - 3n - 1 = (1+3)^n - 3n - 1$

$= (1 + {}^nC_1\,3 + {}^nC_2\,3^2 + ... + 3^n) - 3n - 1$

$= (1 + 3n) + 9({}^nC_2 + ... + 3^{n-2}\} - 3n - 1$

$= 9({}^nC_2 + ... + 3^{n-2}),\ n \ge 2 = 9I$

Also for $n = 1$, $4^n - 3n - 1 = 0$, a multiple of 9

Every element in X is a multiple of 9. But Y contains all multiples of 9. Hence $X \in Y = Y$.

15. **(c) :** The given expansion is

$(1-2x)^{18} + ax(1-2x)^{18} + bx^2(1-2x)^{18}$

The coefficient of x^3 is

$(-2)^3 \cdot {}^{18}C_3 + a(-2)^2 \cdot {}^{18}C_2 + b(-2) \cdot {}^{18}C_1 = 0$ ...(1)

The coefficient of x^4 is

$(-2)^4 \cdot {}^{18}C_4 + a(-2)^3 \cdot {}^{18}C_3 + b(-2)^2 \cdot {}^{18}C_2 = 0$ ...(2)

From (1) and (2), we get

$153a - 9b = 1632$ ...(3)

$3b - 32a = -240$ ...(4)

Solving (3) and (4), we get $a = 16,\ b = \frac{272}{3}$

16. **(b) :** $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$

$= \left\{\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1} - \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)}\right\}^{10}$

$= (x^{1/3} - x^{-1/2})^{10}\ \therefore\qquad T_{r+1} = (-1)^r\,{}^{10}C_r\,x^{\frac{20-5r}{6}}$

Thus $\frac{20-5r}{6} = 0 \quad \Rightarrow \quad r = 4 \quad \therefore$ Term $= {}^{10}C_4 = 210$.

17. **(c) :** $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$

$= 2\left[{}^{2n}C_1 \cdot (\sqrt{3})^{2n-1} + {}^{2n}C_3 \cdot (\sqrt{3})^{2n-3} + ...\right],$

is an irrational number.

18. **(a) :** $(1 - x - x^2 + x^3)^6 = ((1-x)(1-x^2))^6 = (1-x)^6\,(1-x^2)^6$

$= (1 - {}^6C_1x + {}^6C_2x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6)$

$(1 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 - {}^6C_5x^{10} + {}^6C_6x^{12})$

Coeff. of $x^7 = (-{}^6C_1)(-{}^6C_3) + (-{}^6C_3)({}^6C_2) + (-{}^6C_5)(-{}^6C_1)$

$= 6 \cdot 20 - 20 \cdot 15 + 6 \cdot 6 = 120 - 300 + 36 = -144$

19. **(a) :** Using Modulo Arithmetic

$8 = -1$ (modulo 9) Also $62 = -1$ (modulo 9)

$\Rightarrow 8^{2n} - (62)^{2n+1} = [(-1)^{2n} - (-1)^{2n+1}]\text{mod } 9$

$= (1 + 1)\text{mod } 9 = 2 \text{ mod } 9 \Rightarrow$ Remainder $= 2$

20. **(c) :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n} r \cdot {}^nC_r + \sum_{r=0}^{n} {}^nC_r$

$= \sum_{r=0}^{n} r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} + \sum_{r=0}^{n} {}^nC_r = n \cdot 2^{n-1} + 2^n = 2^{n-1}(n+2)$

Thus Statement-1 is true.

Again $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = \sum_{r=0}^{n} r \cdot {}^nC_r x^r + \sum {}^nC_r x^r$

$= n\sum_{r=0}^{n} {}^{n-1}C_{r-1}x^r + \sum_{r=0}^{n} {}^nC_r x^r = nx(1+x)^{n-1} + (1+x)^n$

Substitute $x = 1$ in the above identity to get

$\sum(r+1)\,{}^nC_r = n \cdot 2^{n-1} + 2^n$

Statement-2 is also true & explains Statement-1 also.

21. **(b) :** ${}^nC_4\,a^{n-4}(-b)^4 = -\left({}^nC_5\,a^{n-5}(-b)^5\right) \Rightarrow \frac{a}{b} = \frac{n-4}{5}$.

22. **(d) :** From given $\frac{1}{(1-ax)(1-bx)} = (1-ax)^{-1}(1-bx)^{-1}$

$= (a_0 + a_1x + ... + a_nx^n + ...)$

$= (1 + ax + a^2x^2 + ... + a^{n-1}x^{n-1} + a^nx^n + ...)$

$(1 + bx + b^2x^2\ .... + b^nx^n + ...)$

$\Rightarrow\quad (a_0 + a_1x + ... + a_nx^n + ...)$

$= 1 + x\,(a+b) + x^2\,(a^2 + ab + b^2) + x^3\,(a^3 + a^2b + ab^2 + b^3) + ... + ... + ... + x^n\,(a^n + a^{n-1}\,b + a^{n-2}\,b^2 + ... + ab^{n-1} + b^n) + ...$

On comparing the coefficient of x^n both sides, we have

$a^n = a^n + a^{n-1}\,b + a^{n-2}\,b^2 + ... + a\,b^{n-1} + b^n$

$= \frac{(a^n + a^{n-1}b + a^{n-2}b^2 + ... + ab^{n-1} + b^n)(b-a)}{b-a}$

(Multiplying and dividing by $b - a$)

$= \frac{b^{n+1} - a^{n+1}}{b-a}$.

23. **(d) :** $(1-y)^m\,(1+y)^n = 1 + a_1y + a_2y^2 + a_3y^3 + ... + ...$ (*)

Differentiating w.r.t. y both sides of (*) we have

$-m(1-y)^{m-1}(1+y)^n + (1-y)^m\,n(1+y)^{n-1}$

$= a_1 + 2a_2y + 3a_3y^2 + 4a_4y^3 + ...$

$\Rightarrow\quad n(1+y)^{n-1}(1-y)^m - m(1-y)^{m-1}(1+y)^n$

$= a_1 + 2a_2y + 3a_2y^2 + 4a_4y^3 + ...$ .....(**)

Again differentiating (**) with respect to y we have

$[n(n-1)(1+y)^{n-2}(1-y)^m + n(1+y)^{n-1}(-m)\,(1-y)^{m-1}]$

$-[m(1+y)^n(m-1)(1-y)^{m-2}(1-y)^{m-1}n(1+y)+n-1]$

$= 2a_2 + 6a_3y + ...$ .... (***)

Now putting $y = 0$ in (**) and (***) we get

$n - m = a_1 = 10$ ...(A)

and $m^2 + n^2 - (m+n) - 2\,mn = 2a_2 = 20$ ...(B)

Solving (A) and (B), we get $n = 45,\ m = 35$

$\therefore\quad (m, n) = (35, 45)$

24. (c) : T_{r+1} of $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_r(ax^2)^r\left(\frac{1}{bx}\right)^{11-r}$

T_{r+1} of $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_r(ax)^r\left(-\frac{1}{bx^2}\right)^{11-r}$

$\therefore$ Coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_5\frac{a^6}{b^5}$

and coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_6\frac{a^5}{b^6}$

Now ${}^{11}C_5\frac{a^6}{b^5} = {}^{11}C_6\frac{a^5}{b^6}$ $\quad\therefore\quad ab = 1.$

25. (d) : $\dfrac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$$=\frac{\left(1+\frac{3}{2}x+\frac{3}{2}\frac{1}{2}\frac{1}{2!}x^2+...\right)-\left(1+3\cdot\frac{1}{2}x+\frac{3\cdot 2}{2!}\cdot\frac{1}{4}x^2+....\right)}{(1-x)^{1/2}}$$

$$=-\frac{3}{8}x^2(1-x)^{-1/2}=-\frac{3}{8}x^2\left[1+\frac{1}{2}x+\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{1}{2!}x^2+...\right]$$

$= -\frac{3}{8}x^2 +$ higher powers of x^2.

26. (a) : Coefficient of middle term in $(1+\alpha x)^4$ = coefficient of middle term in $(1-\alpha x)^6$

$\therefore\ {}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3 \quad\Rightarrow \alpha = -\frac{3}{10}$

27. (b): $(1+x)(1-x)^n = (1-x)^n + x(1-x)^n$

$\therefore$ Coefficient of x^n is $= (-1)^n + (-1)^{n-1}\,{}^nC_1 = (-1)^n\,[1-n]$

28. (c): $t_n = \sum_{r=0}^{n}\frac{r}{{}^nC_r}$

$t_n = \sum_{r=0}^{n}\frac{n-(n-r)}{{}^nC_{n-r}} \quad\Rightarrow\quad t_n = n\sum_{r=0}^{n}\frac{1}{{}^nC_r}-\sum_{r=0}^{n}\frac{n-r}{{}^nC_{n-r}}$

$t_n = n\sum\frac{1}{{}^nC_r}-\sum_{r=0}^{n}\frac{r}{{}^nC_r}$ replacing $n-r$ by r

$t_n = ns_n - t_n \ \therefore\ \frac{t_n}{s_n} = \frac{n}{2}$

29. (d) : General term in the expansion of $(1+x)^{\frac{27}{5}}$

$T_{r+1} = \dfrac{n(n-1)\,.....\,(n-r+1)}{r!}x^r$

$\therefore\ n-r+1<0 \ \Rightarrow\ \frac{27}{5}+1<r \ \Rightarrow\ r>\frac{32}{5} \ \Rightarrow r>6$

30. (a) : $T_{r+1} = {}^{256}C_r\ (\sqrt{3})^{256-r}5^{\frac{r}{8}}$

For integral terms $\frac{256-r}{2},\frac{r}{8}$ are both positive integers

$\therefore\ r = 0, 8, 16, ..., 256$

$\therefore\ 256 = 0 + (n-1)8$ using $t_n = a + (n-1)d$

$\therefore\ \frac{256}{8} = n-1 \ \therefore\ n = \frac{256}{8}+1$

$n = 32 + 1 \ \Rightarrow\ n = 33$

31. (c) : Let $R = \left(1+\frac{1}{10^4}\right)^{1000}$

$= 1 + 1000\left(\frac{1}{10^4}\right)^1 + 1000\frac{999}{2}\left(\frac{1}{10^4}\right)^2 + ... + \left(\frac{1}{10^4}\right)^{10^3}$

$< 1 + \frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+...\infty = \frac{10}{9}$

$\therefore\quad R < \frac{10}{9}$

$\therefore$ The positive integer just greater than $\frac{10}{9}$ is 2.

32. (c) : Given $r > 1$ and $n > 2$

Coefficient of T_{r+2} = Coefficient of T_{3r} in $(1+x)^{2n}$

$\Rightarrow {}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$

$\Rightarrow 3r-1 = r+1$ and $\begin{cases}2n-3r+1=r+1\\ \Rightarrow 2n=4r\\ n=2r\end{cases}$

$2r = 2$

$r = 1 \qquad \{\because {}^nC_x = {}^nC_y \Rightarrow x+y=n$ or $x=y$

33. (a) : In the expansion of $(1+x)^{p+q}$

$T_{r+1} = {}^{p+q}C_rx^r$

$\therefore$ Coefficient of $x^p = {}^{p+q}C_p$

$$=\frac{(p+q)!}{p!(p+q-p)!}=\frac{(p+q)!}{p!q!} \qquad ...(i)$$

Also coefficient of x^q in $(1+x)^{p+q}$ is

$= {}^{p+q}C_q$

$$=\frac{(p+q)!}{q!(p+q-q)!}=\frac{(p+q)!}{q!p!} \qquad ....(ii)$$

$\therefore$ By (i) and (ii), we get

Coefficient of x^p in $(1+x)^{p+q}$ = Coefficient of x^q in $(1+x)^{p+q}$

34. (c) : Consider $(a+b)^n = C_0a^n + C_1a^{n-1}b + C_2a^{n-2}b^2 + ... + C_nb^n$

Putting $a = b = 1 \ \therefore\ 2^n = C_0 + C_1 + C_2 + ... + C_n$

$2^n = 4096 = 2^{12} \Rightarrow\ n = 12$ (even)

Now $(a+b)^n = (a+b)^{12}$

as $n = 12$ is even so coefficient of greatest term is

${}^nC_{\frac{n}{2}} = {}^{12}C_{\frac{12}{2}} = {}^{12}C_6$

$= \frac{12}{6}\times\frac{11}{5}\cdot\frac{10}{4}\frac{9}{3}\cdot\frac{8}{2}\cdot\frac{7}{1} = \frac{11\times 9.8.7}{3.2.1}$

$= 11 \times 3\cdot4\cdot7 = 924$