CHAPTER

6

Mathematical Induction

1. **Statement-1 :** For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\ldots+\frac{1}{\sqrt{n}}>\sqrt{n}.$$

Statement-2 : For every natural number $n \geq 2$,

$$\sqrt{n(n+1)}<n+1.$$

(a) Statement-1 is true, Statement-2 is false

(b) Statement-1 is false, Statement-2 is true

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(2008)

2. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true?

(a) $S(k) \Rightarrow S(k-1)$ (b) $S(k) \Rightarrow S(k+1)$

(c) $S(1)$ is correct

(d) principle of mathematical induction can be used to prove the formula *(2004)*

3. If $a_n=\sqrt{7+\sqrt{7+\sqrt{7+\ldots.}}}$ having n radical signs then by methods of mathematical induction which is true

(a) $a_n > 7,\ \forall\ n \geq 1$ (b) $a_n > 3,\ \forall\ n \geq 1$

(c) $a_n < 4,\ \forall\ n \geq 1$ (d) $a_n < 3,\ \forall\ n \geq 1.$

(2002)

ANSWER KEY

1. (d) **2.** (b) **3.** (b)

Explanations

1. (d) : Statement-1

Let $P(n): \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{n}} > \sqrt{n}$

Step 1 : For $n = 2$, $P(2): \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}} > \sqrt{2}$ is true

Step 2 : Assume $P(n)$ is true for $n = k$, i.e.

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}} > \sqrt{k} \quad ...(i)$$

Step 3 : For $n = k + 1$, we have to show that

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad ...(ii)$$

By Assumption step, we get

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}} > \sqrt{k}$$

Adding $\frac{1}{\sqrt{k+1}}$ on both sides, we get

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}} > \sqrt{k}+\frac{1}{\sqrt{k+1}} \quad ...(iii)$$

Statement-2

For $n = k$

$\sqrt{k(k+1)} < k+1$

$\Rightarrow \sqrt{k}\sqrt{k+1} < \sqrt{k+1}\sqrt{k+1} \Rightarrow \sqrt{k} < \sqrt{k+1}$

$\because \sqrt{k+1} > \sqrt{k}$ For $k \geq 2$

$\Rightarrow 1 > \frac{\sqrt{k}}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \frac{k}{\sqrt{k+1}}$

Multiplying by $\sqrt{k}$

$\Rightarrow \sqrt{k} > \frac{(k+1)-1}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \sqrt{k+1} - \frac{1}{\sqrt{k+1}}$

$$\Rightarrow \sqrt{k}+\frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad ...(iv)$$

From (iii) & (iv)

$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}} > \sqrt{k+1}$

hence (ii) is true for $n = k + 1$

hence $P(n)$ is true for $n \geq 2$

So, Statement-1 and Statement-2 are correct but Statement-2 is not a correct explanation of Statement-1

2. (b) : $S(k) = 1 + 3 + ... + (2k - 1) = 3 + k^2$ ...(i)

When $k = 1$, L.H.S of $S(k) \neq$ R.H.S of $S(k)$

So $S(1)$ is not true.

Now $S(k + 1)$; $1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= 3 + (k + 1)^2$ ...(ii)

Let $S(k)$ is true $\therefore 1 + 3 + 5 + ... + (2k - 1) = k^2 + 3$

$\Rightarrow 1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= 3 + k^2 + 2k + 1 = (k + 1)^2 + 3$

$\Rightarrow S(k + 1)$ true $\therefore S(k) \Rightarrow S(k + 1)$

3. (b) : $a_n = \sqrt{7 + a_n} \Rightarrow a_n^2 - a_n - 7 = 0$

$\therefore a_n = \frac{1 \pm \sqrt{1 + 28}}{2} = \frac{1 \pm \sqrt{29}}{2} > 3$