CHAPTER 5

Permutations and Combinations

1. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is
(a) at least 750 but less than 1000
(b) at least 1000
(c) less than 500
(d) at least 500 but less than 750 *(2018, 2009)*

2. n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is
(a) 9 (b) 6 (c) 8 (d) 7 *(Online 2018)*

3. The number of four letter words that can be formed using the letters of the word BARRACK is
(a) 270 (b) 120 (c) 264 (d) 144 *(Online 2018)*

4. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0,1,2,3,4 (repetition of digits is not allowed) and are multiple of 3 is :
(a) 36 (b) 48 (c) 24 (d) 30 *(Online 2018)*

5. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is
(a) 468 (b) 469
(c) 484 (d) 485 *(2017)*

6. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is
(a) 47th (b) 44th
(c) 45th (d) 46th *(Online 2017)*

7. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is
(a) 7! (b) $5 \times 6!$
(c) $6 \times 6!$ (d) $5 \times 7!$ *(Online 2017)*

8. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is
(a) $\frac{1}{11}$ (b) $\frac{21}{220}$ (c) $\frac{2}{23}$ (d) $\frac{3}{11}$ *(Online 2017)*

9. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is
(a) 46th (b) 59th (c) 52nd (d) 58th *(2016)*

10. The value of $\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$ is equal to
(a) 1240 (b) 560 (c) 1085 (d) 680 *(Online 2016)*

11. If the four letter words (need not be meaningful) are to be formed using the letters from the word “MEDITERRANEAN” such that the first letter is R and the fourth letters is E, then the total number of all such words is
(a) 110 (b) 59 (c) $\frac{11!}{(2!)^3}$ (d) 56 *(Online 2016)*

12. The sum $\sum_{r=1}^{10}(r^2+1)\times(r!)$ is equal to
(a) $11 \times (11!)$ (b) $10 \times (11!)$
(c) $(11!)$ (d) $101 \times (10!)$ *(Online 2016)*

13. If $\frac{{}^{n+2}C_6}{{}^{n-2}P_2}=11$ then n satisfies the equation
(a) $n^2 + n - 110 = 0$ (b) $n^2 + 2n - 80 = 0$
(c) $n^2 + 3n - 108 = 0$ (d) $n^2 + 5n - 84 = 0$ *(Online 2016)*

14. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is
(a) 820 (b) 780
(c) 901 (d) 861

5CEFK6

15. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is

(a) 275 (b) 510 (c) 219 (d) 256

(2015)

16. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8 without repetition, is

(a) 120 (b) 72 (c) 216 (d) 192

(2015)

17. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is

(a) 1120 (b) 1240 (c) 1880 (d) 1960

(Online 2015)

18. Let $A = \{x_1, x_2, ..., x_7\}$ and $B = \{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f : A \to B$ that are onto, if there exist exactly three elements x in A such that $f(x) = y_2$, is equal to

(a) $14.{}^7C_2$ (b) $16.{}^7C_3$ (c) $12.{}^7C_2$ (d) $14.{}^7C_3$

(Online 2015)

19. If in a regular polygon, the number of diagonals is 54, then the number of sides of the polygon is

(a) 10 (b) 12 (c) 9 (d) 6

(Online 2015)

20. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is

(a) 5 (b) 10 (c) 8 (d) 7

(2013)

21. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

(a) 630 (b) 879 (c) 880 (d) 629

(2012)

22. **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

23. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$

and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement-1 : $S_3 = 55 \times 2^9$.

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation of statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true. *(2010)*

24. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(a) 3 (b) 36 (c) 66 (d) 108 *(2010)*

25. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1 : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.

Statement-2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2008)*

26. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(a) $7 \cdot {}^6C_4 \cdot {}^8C_4$ (b) $8 \cdot {}^6C_4 \cdot {}^7C_4$

(c) $6 \cdot 7 \cdot {}^8C_4$ (d) $6 \cdot 8 \cdot {}^7C_4$ *(2008)*

27. The sum of the series

${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - ...... + {}^{20}C_{10}$ is

(a) 0 (b) ${}^{20}C_{10}$ (c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$

(2007)

28. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

(a) 5040 (b) 6210 (c) 385 (d) 1110

(2006)

29. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(a) 602 (b) 603 (c) 600 (d) 601 *(2005)*

30. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(a) ${}^{56}C_4$ (b) ${}^{56}C_3$ (c) ${}^{55}C_3$ (d) ${}^{55}C_4$ *(2005)*

31. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?

(a) 360 (b) 240
(c) 120 (d) 480 *(2004)*

32. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is
(a) 3^8 (b) 21
(c) 5 (d) ${}^8C_3$ *(2004)*

33. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by
(a) 30 (b) $5! \times 4!$
(c) $7! \times 5!$ (d) $6! \times 5!$ *(2003)*

34. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is
(a) 196 (b) 280
(c) 346 (d) 140 *(2003)*

35. If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals
(a) ${}^{n+2}C_{r+1}$ (b) ${}^{n+1}C_r$
(c) ${}^{n+1}C_{r+1}$ (d) ${}^{n+2}C_r$. *(2003)*

36. Number greater than 1000 but less than 4000 is formed using the digits 0, 2, 3, 4 repetition allowed is
(a) 125 (b) 105
(c) 128 (d) 625 *(2002)*

37. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are
(a) 312 (b) 3125
(c) 120 (d) 216 *(2002)*

38. The sum of integers from 1 to 100 that are divisible by 2 or 5 is
(a) 3000 (b) 3050
(c) 3600 (d) 3250 *(2002)*

39. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are
(a) 216 (b) 375
(c) 400 (d) 720 *(2002)*

ANSWER KEY

1. (b)	**2.** (d)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (d)	**11.** (b)	**12.** (b)
13. (c)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (d)	**19.** (b)	**20.** (a)	**21.** (b)	**22.** (c)	**23.** (c)	**24.** (d)
25. (b)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (d)	**30.** (a)	**31.** (a)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (a)	**36.** (c)
37. (d)	**38.** (b)	**39.** (d)									

Explanations

1. **(b) :** The number of ways to choose 4 novels out of 6 is ${}^6C_4$. The number of ways to choose 1 dictionary out of 3 is ${}^3C_1$. As the place of dictionary is fixed, so total number of ways $= {}^6C_4\ {}^3C_1.\ 4! = 15.3.24 = 1080$

2. **(d) :** Since, n digit numbers are formed using 2, 5 and 7 digits.
$\therefore$ The number of ways in which it can be done $= 3^n$
Now, $3^6 < 900$ and $3^7 > 900$
$\therefore$ $n = 7$

3. **(a) :** There are three ways to make four letter words from the letters of the word BARRACK.
(i) When two letters are same *i.e.*, A, A, R, R

So, number of words $= \dfrac{{}^2C_2 \cdot 4!}{2!\,2!} = 6$

(ii) When one letter is same and other is different

So, number of words $= \dfrac{{}^2C_1 \times {}^4C_2 \times 4!}{2!} = 144$

(iii) When all letters are different
So, number of words $= {}^5C_4 \times 4! = 120$
$\therefore$ Total number of words $= 6 + 144 + 120 = 270$

4. **(d) :** Number of 4 digit numbers between 2000 and 5000 which are multiple of $3 = 2 \times 3! + 3 \times 3! = 30$
[$\because$ Numbers which are multiple of 3 can be formed by using (0, 1, 2, 3) or (0, 2, 3, 4)]

5. **(d) :** We can do casework on number of ladies and men to be invited.
X, Y can satisfy the condition in 4 ways
(i) X invites 3 ladies and Y invites 3 men.
(ii) X invites 2 ladies, 1 man and Y invites 1 lady 2 men.
(iii) X invites 1 lady, 2 men and Y invites 2 ladies, 1 man.
(iv) X invites 3 men and Y invites 3 ladies.
The number of ways
$= {}^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3$
$= 16 + 324 + 144 + 1 = 485.$

6. **(d) :** We have E, E, N, Q, U
According to the English dictionary,
(i) Words starts with E
E ___ ___ ___ ___ $= 4! = 24$
(ii) Words starts with N
N ___ ___ ___ ___ $= \dfrac{4!}{2!} = 12$
(iii) Words starts with QE
QE ___ ___ ___ $= 3! = 6$
(iv) Words starts with QN
QN ___ ___ ___ $= \dfrac{3!}{2!} = 3$
(v) Word QUEEN = 1
So, according to the English dictionary, the word QUEEN will be at $(24 + 12 + 6 + 3 + 1)^{\text{th}}$ position $= 46^{\text{th}}$ position.

7. **(b) :** Required number of ways $= 5! \times {}^6C_2 \times 2!$
$= 5! \times \dfrac{6!}{4!2!} \times 2! = 5 \times 6!$

8. **(a)**

9. **(d) :** The number of words in all formed by using the letters of the word SMALL $= \dfrac{:\,!}{7!} = ;\,5$
Let's count backwards.
The 59^{th} word is SMALL $\therefore$ 58^{th} word is SMALL.

10. **(d) :** We have,

$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}} = \frac{15!}{r!(15-r)!} \times \frac{(r-1)!(15-r+1)!}{15!} = \frac{16-r}{r}$$

$$\therefore \sum_{r=1}^{15} r^2 \left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right) = \sum_{r=1}^{15} r^2 \left(\frac{16-r}{r}\right) = \sum_{r=1}^{15} (16r - r^2)$$

$$= 16 \times \frac{15 \times 16}{2} - \frac{15 \times 16 \times 31}{6} = 680$$

11. **(b) :** M, EEE, D, I, T, RR, AA, NN
<u>R</u> – – <u>E</u>
Two empty places can be filled with identical letters [EE, AA, NN] in 3 ways.
Two empty places can be filled with distinct letters [M, E, D, I, T, R, A, N] in ${}^8P_2$ ways.
$\therefore$ Number of words formed $= 3 + {}^8P_2 = 59$

12. **(b) :** We have, $T_r = (r^2 + 1 + r - r)\underline{|r} = (r^2 + r)\underline{|r} - (r-1)\underline{|r}$

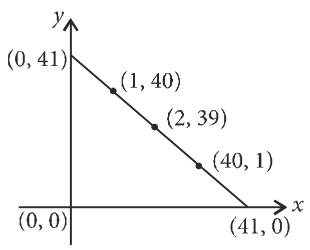
$\Rightarrow T_r = r\underline{|r+1} - (r-1)\underline{|r}$ $\quad \therefore T_1 = 1\underline{|2} - 0$,
$T_2 = 2\underline{|3} - 1 \cdot \underline{|2}$
$T_3 = 3\underline{|4} - 2 \cdot \underline{|3}$
$\vdots \qquad \vdots$
$T_{10} = 10\underline{|11} - 9\underline{|10}$

$$\therefore \sum_{r=1}^{10} (r^2 + 1)\underline{|r} = 10\underline{|11}$$

13. **(c) :** We have, $\dfrac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11 \Rightarrow \dfrac{(n+2)(n+1)n(n-1)}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 11$

$\Rightarrow (n + 2)(n + 1)n(n - 1) = 11 \cdot 10 \cdot 9 \cdot 8 \Rightarrow n = 9$

14. **(b) : 1st solution :**



We count the number of points on line $x = n$, $1 \le n < 40$, that lie in the interior of the triangle.

At line $x = 40$, we have 1 point.

$x = 39$, we have 2 points

......

$x = n$, we have $(40 - n)$ points

The total number of points = 1 + 2 + 3 + ... + 39

$$= \frac{6}{7} \times 8> \times 95 = 8> \times 75 = <=5$$

2nd solution : The points $P(\alpha, \beta)$ satisfying the requirements of the problem are given by

$\alpha \geq 1$

$\beta \geq 1$

$\alpha + \beta < 41$ *i.e.* $\alpha + \beta \leq 40$

The number of integral solution to the equation

$\alpha + \beta + \gamma = 40$, ($\gamma \geq 0$ is a slack variable) is the answer to the problem. Hence the number of points is equal to the non-negative integral solution of $x + y + z = 40$, $x, y, z \geq 0$ which is given by $^{38 + 3 - 1}C_{3 - 1} = {}^{40}C_2 = 780$

15. (c) : We have, $n(A) = 4$ and $n(B) = 2$

Thus the number of elements in $A \times B$ is 8

Number of subsets having at least 3 elements

$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$

$= ({}^8C_0 + {}^8C_1 + {}^8C_2 + ... + {}^8C_8) - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 2^8 - (1 + 8 + 28) = 256 - 37 = 219$

16. (d) : Numbers having 5 digits = $\lfloor\!:$ = 120

Numbers having 4 digits = (3)(4)(3)(2) = 72

As the first digit can be filled in 3 ways, viz 6, 7, and 8, and as repetition is not allowed, the other choices are 4, 3 and 2 in that order.

17. (b) : Number of ways of selecting 1st team from 15 men and 15 women = $^{6:}Y_6 \times {}^{6:}Y_6 = 6:^7$

2nd team = $^{69}Y_6 \times {}^{69}Y_6 = 69^7$ and so on.

So, total number of ways = $1^2 + 2^2 + ... + 15^2$

$$= \frac{6: \times 6; \times 86}{;} = 6795$$

18. (d) : Number of ways of selection of three elements in A such that $f(x) = y_2$ is $^7C_3$

Now for remaining 4 elements in A, we have 2 elements in B

$\therefore$ Total number of onto functions

$= {}^7C_3 \times (2^4 - {}^2C_1(2 - 1)^4) = {}^7C_3 \times 14$

19. (b) : $^nC_2 - n = 54 \Rightarrow \frac{- \;-6.}{7} - \;=:9$

$\Rightarrow n^2 - 3n - 108 = 0 \Rightarrow n = 12$

20. (a) : 1st solution : $^{n+1}C_3 - {}^nC_3 = 10$

$$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 10$$

$\Rightarrow 3n(n - 1) = 60 \Rightarrow n(n - 1) = 20 \Rightarrow n^2 - n - 20 = 0$

$\Rightarrow (n - 5)(n + 4) = 0 \quad \therefore n = 5$

2nd solution : $^{n+1}C_3 - {}^nC_3 = 10$

$$\Rightarrow {}^nC_2 = 10 \Rightarrow \frac{n(n-1)}{2} = 10$$

$\Rightarrow n^2 - n - 20 = 0. \quad \therefore n = 5$

Here we have used $^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

21. (b) : Number of ways in which one or more balls can be selected from 10 white, 9 green, 7 black balls is

= (10 + 1) (9 + 1) (7 + 1) – 1

= 880 – 1 = 879 ways

22. (c) : $x_1 + x_2 + x_3 + x_4 = 10$

The number of positive integral solution is $^{6+4-1}C_{4-1} = {}^9C_3$

It is the same as the number of ways of choosing any 3 places from 9 different places.

23. (c) : $S_1 = \sum j(j-1)\,{}^{10}C_j = \sum j(j-1) \cdot \frac{10(10-1)}{j(j-1)} \cdot {}^8C_{j-2}$

$$= 9 \times 10 \sum_{j=2}^{10} {}^8C_{j-2} = 90 \times 2^8$$

$$S_2 = \sum_{j=1}^{10} j \cdot {}^{10}C_j = 10 \sum_{j=1}^{10} {}^9C_{j-1} = 10 \times 2^9$$

$$S_3 = \sum_{j=1}^{10} j^2 \cdot {}^{10}C_j = \sum_{j=1}^{10} (j(j-1) + j) \cdot {}^{10}C_j$$

$$= \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j + \sum_{j=1}^{10} j \cdot {}^{10}C_j$$

$= 90 \cdot 2^8 + 10 \cdot 2^9 = (45 + 10)2^9 = 55 \cdot 2^9.$

Thus statement-1 is true and statement-2 is false.

24. (d) : Thus number of ways = $({}^3C_2) \times ({}^9C_2) = 3 \times \frac{9 \times 8}{2} = 108$

25. (b) : We have to find the number of integral solutions if $x_1 + x_2 + x_3 + x_4 + x_5 = 6$ and that equals $^{5+6-1}C_{5-1} = {}^{10}C_4$

Thus Statement-1 is false.

Number of different ways of arranging $6A$'s and $4B$'s in a row

$= \frac{\lfloor 10}{\lfloor 6 \times \lfloor 4} = {}^{10}C_4 =$ Number of different ways the child can buy the six ice-creams. $\therefore$ Statement-2 is true.

So, Statement-1 is false, Statement-2 is true.

26. (a) : Leaving S, we have 7 letters M, I, I, I, P, P, I.

ways of arranging them = $\frac{\lfloor 7}{\lfloor 2 \lfloor 4} = 7 \cdot 5 \cdot 3$

And four S can be put in 8 places in $^8C_4$ ways.

The required number of ways = $7 \cdot 5 \cdot 3 \cdot {}^8C_4 = 7 \cdot {}^6C_4 \cdot {}^8C_4$.

27. (d) : $\because$ $^{20}C_0 + {}^{20}C_1 x + ... + {}^{20}C_{10} x^{10} +$ ....

$$....... + {}^{20}C_{20} x^{20} = (1 + x)^{20}$$

After putting $x = -1$, we get $^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ...$

$$..... + {}^{20}C_{10} - {}^{20}C_{11} - {}^{20}C_{12} + ... + {}^{20}C_{20} = 0$$

$$2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9) + {}^{20}C_{10} = 0$$

$$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$$

28. (a) : A voter can vote one candidate or two or three or four candidates

$\therefore$ Required number of ways = $^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$

Fixed

29. (d) : $\boxed{S}$ A C H I N

No. of words start with A = 5!
No. of words start with C = 5!
No. of words start with H = 5!
No. of words start with I = 5!
No. of words start with N = 5!
Total words = 5! + 5! + 5! + 5! + 5! = 5(5 !) = 600
Now add the rank of SACHIN so required rank of SACHIN = 600 + 1 = 601.

30. (a) $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

Putting r = 6, 5, 4, 3, 2, 1 we get

$$^{50}C_4 + {}^{50}C_3 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$\left(\because\ {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}\right)$$

$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 = {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$

31. (a) : Number of letters = 6
Number of vowels = 2 namely A & E these alphabets can be arrange themselves by 2! ways

$\therefore$ Number of words = $\dfrac{6!}{2!}$ = 360

32. (b) : (i) Each box must contain at least one ball since no box remains empty so we have the following cases

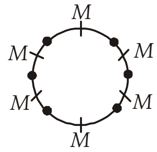
Box	Number of balls				
I	1,	1,	1,	2,	2,
I	1,	2,	3,	3,	2,
III	6,	5,	4,	3,	4,

$\therefore$ Number of ways = 3 × $\dfrac{1 \times 3!}{2!}$ + 3! × 2 = 9 + 6 × 2 = 21

As $\boxed{1, 1, 6}$ $\boxed{2, 3, 4}$ $\boxed{2, 2, 4}$ have case ways and $\boxed{1, 2, 5}$ $\boxed{1, 3, 4}$ have equal number of ways of arranging the balls in the different boxes.

(ii): Let the number of balls in the boxes are x, y, z respectively then $x + y + z = 8$ and no box is empty so each $x, y, z \geq 1$
$\Rightarrow l + m + n + 3 = 8$ where $l = x - 1$,
$m = y - 1$, $n = z - 1$
i.e. $(l + 1) + (m + 1) + (n + 1) = 8$ are non negative integers
$\therefore$ Required number of ways = ${}^{n+r-1}C_r$
$= {}^{3+5-1}C_5 = {}^7C_5 = {}^7C_2$

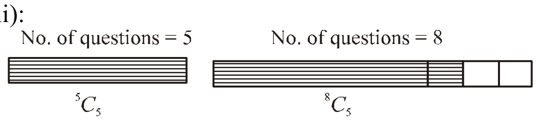
33. (d) : Number of women = 5



Number of men = 6
Number of ways of 6 men at a round table is $n - 1! = (6 - 1)! = 5!$
Now we left with six places between the men and there are 5 women, these 5 women can be arranged themselves by ${}^6P_5$ ways.
$\therefore$ Required number of ways = $5! \times {}^6P_5 = 5! \times 6!$

34. (a) : Case (i) :

Required ways for first case = ${}^5C_4 \times {}^8C_6$ = 140
Case (ii):



$\therefore$ Required ways for case (ii) = ${}^5C_5 \times {}^8C_5 = \dfrac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

Total number of ways = 140 + 56 = 196

35. (a) : Consider ${}^nC_{r-1} + 2{}^nC_r + {}^nC_{r+1}$

$= ({}^nC_{r-1} + {}^nC_r) + ({}^nC_r + {}^nC_{r+1})$

$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

36. (c) : Let number of digits formed be x.
$\therefore$ $1000 < x < 4000$, which means left extreme digit will be either 2 or 3.
$\therefore$ Required numbers = ${}^2C_1$ × H T U
where H = Hundred's place, T = Ten's place and
U = Unit's place = ${}^2C_1 \times 4 \times 4 \times 4 = 128$

37. (d)

38. (b) : Set of numbers divisible by 2 are 2, 4, 6, ..., 100
Set of numbers divisible by 5 are 5, 10, 15, ..., 100
Set of numbers divisible by 10 are 10, 20, 30, ..., 100
Now sum of numbers divisible by 2 is given by

$S_{50} = \dfrac{50}{2}$ [2 + 100] using $S_n = \dfrac{n}{2}[a + l]$

$S_{50} = 25[102]$

Similarly, $S_{20} = \dfrac{20}{2}[5 + 100] = 10 \times 105 = 1050$

and $S_{10} = \dfrac{10}{2}[10 + 100] = 5 \times 110$

$\therefore$ Required sum = 25 × 102 + 1050 – 550 = 3050

39. (d) : Odd numbers are 1, 3, 5, 7
We have to fill up four places like TH H T U
(Case: If repetition is allowed)

${}^5C_1\ 6^2\ {}^4C_1 = 5 \times 6^2 \times 4 = 5 \times 36 \times 4 = 720$