CHAPTER

4 Quadratic Equations

1. If α, $\beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to
(a) 2 (b) –1 (c) 0 (d) 1 *(2018)*

2. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S
(a) contains exactly four elements
(b) is an empty set
(c) contains exactly one element
(d) contains exactly two elements *(2018)*

3. If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is :
(a) $4\sqrt{2}$ (b) 20 (c) $2\sqrt{7}$ (d) $2\sqrt{5}$ *(Online 2018)*

4. If $f(x)$ is a quadratic expression such that $f(1) + f(2) = 0$, and –1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is
(a) $-\frac{5}{8}$ (b) $\frac{5}{8}$ (c) $-\frac{8}{5}$ (d) $\frac{8}{5}$ *(Online 2018)*

5. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to :
(a) $p^2 + q^2$ (b) $2(p^2 + q^2)$
(c) $p^2 + q^2 + r^2$ (d) $\frac{p^2 + q^2}{2}$ *(Online 2018)*

6. If for a positive integer n, the quadratic equation $x(x + 1) + (x + 1)(x + 2) + ... + (x + \overline{n - 1})(x + n) = 10n$ has two consecutive integral solutions, then n is equal to
(a) 9 (b) 10
(c) 11 (d) 12 *(2017)*

7. Let $p(x)$ be a quadratic polynomial such that $p(0) = 1$. If $p(x)$ leaves remainder 4 when divided by $x - 1$ and it leaves remainder 6 when divided by $x + 1$; then
(a) $p(-2) = 11$ (b) $p(2) = 11$
(c) $p(2) = 19$ (d) $p(-2) = 19$ *(Online 2017)*

8. The sum of all the real values of x satisfying the equation $2^{(x - 1)(x^2 + 5x - 50)} = 1$ is
(a) –5 (b) 14 (c) –4 (d) 16 *(Online 2017)*

9. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is
(a) 3 (b) –4
(c) 6 (d) 5 *(2016)*

10. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from –1, then $|b|$ is equal to
(a) 2 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$ *(Online 2016)*

11. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) 2 *(Online 2016)*

12. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{65\ -7\ =}{7\ \ >}$ is equal to
(a) 3 (b) –3 (c) 6 (d) –6 *(2015)*

13. If $2 + 3i$ is one of the root of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation
(a) does not exist
(b) exists and is equal to 1/2
(c) exists and is equal to –1/2
(d) exists and is equal to 1 *(Online 2015)*

14. If the two roots of the equation, $(a - 1)(x^4 + x^2 + 1) + (a + 1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is
(a) $\left(-\frac{6}{7} 15\right)$ (b) $(-\infty, -2) \cup (2, \infty)$
(c) $\left(-\frac{6}{7} 15\right) \cup \left(51\frac{6}{7}\right)$ (d) $\left(51\frac{6}{7}\right)$ *(Online 2015)*

15. Let α and β be the roots of the equation $px^2 + qx + r = 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha - \beta|$ is

(a) $\frac{2\sqrt{17}}{9}$ (b) $\frac{\sqrt{34}}{9}$

(c) $\frac{2\sqrt{13}}{9}$ (d) $\frac{\sqrt{61}}{9}$ *(2014)*

16. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval
(a) $(1, 2)$ (b) $(-2, -1)$
(c) $(-\infty, -2) \cup (2, \infty)$ (d) $(-1, 0) \cup (0, 1)$ *(2014)*

17. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in $[0, 1]$
(a) lies between 2 and 3
(b) lies between –1 and 0
(c) does not exist
(d) lies between 1 and 2 *(2013)*

18. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$ have a common root, then $a : b : c$ is
(a) 3 : 2 : 1 (b) 1 : 3 : 2
(c) 3 : 1 : 2 (d) 1 : 2 : 3 *(2013)*

19. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has
(a) exactly one real root
(b) exactly four real roots
(c) infinite number of real roots
(d) no real roots *(2012)*

20. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that
(a) $|\beta| = 1$ (b) $\beta \in (1, \infty)$
(c) $\beta \in (0, 1)$ (d) $\beta \in (-1, 0)$ *(2011)*

21. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
(a) –2 (b) –1 (c) 1 (d) 2 *(2010)*

22. If the roots of the equation $bx^2 + cx + a = 0$ are imaginary, then for all real values of x. The expression $3b^2x^2 + 6bcx + 2c^2$ is
(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$ *(2009)*

23. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is
(a) 2 (b) 1 (c) 4 (d) 3 *(2008)*

24. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is
(a) $(3, \infty)$ (b) $(-\infty, -3)$
(c) $(-3, 3)$ (d) $(-3, \infty)$ *(2007)*

25. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan30° and tan15°, respectively then the value of $2 + q - p$ is
(a) 2 (b) 3 (c) 0 (d) 1 *(2006)*

26. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval
(a) $-2 < m < 0$ (b) $m > 3$
(c) $-1 < m < 3$ (d) $1 < m < 4$ *(2006)*

27. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is
(a) 0 (b) 1 (c) 2 (d) 3 *(2005)*

28. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals
(a) 3 (b) –2 (c) 1 (d) 2 *(2005)*

29. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval
(a) $(6, \infty)$ (b) $(5, 6]$
(c) $[4, 5]$ (d) $(-\infty, 4)$ *(2005)*

30. If the equation $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x = 0$, $a_1 \ne 0$, $n \ge 2$, has a positive root $x = \alpha$, then the equation $na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_1 = 0$ has a positive root, which is
(a) smaller than α (b) greater than α
(c) equal to α (d) greater than or equal to α *(2005)*

31. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation
(a) $x^2 + 18x - 16 = 0$ (b) $x^2 - 18x + 16 = 0$
(c) $x^2 + 18x + 16 = 0$ (d) $x^2 - 18x - 16 = 0$ *(2004)*

32. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are
(a) 0, –1 (b) –1, 1 (c) 0, 1 (d) –1, 2 *(2004)*

33. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is
(a) 3 (b) 12 (c) 49/4 (d) 4 *(2004)*

34. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is
(a) –2/3 (b) 1/3 (c) –1/3 (d) 2/3 *(2003)*

35. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in
(a) geometric progression
(b) harmonic progression
(c) arithmetic-geometric progression
(d) arithmetic progression *(2003)*

36. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(a) 4 (b) 1
(c) 3 (d) 2 *(2003)*

37. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation whose roots are α/β and β/α is
(a) $3x^2 - 25x + 3 = 0$ (b) $x^2 + 5x - 3 = 0$
(c) $x^2 - 5x + 3 = 0$ (d) $3x^2 - 19x + 3 = 0$ *(2002)*

38. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then
(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$ *(2002)*

39. Product of real roots of the equation $x^2 + |x| + 9 = 0$
(a) is always positive (b) is always negative
(c) does not exist (d) none of these
(2002)

40. If p and q are the roots of the equation $x^2 + px + q = 0$, then
(a) $p = 1, q = -2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$ *(2002)*

41. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is
(a) less than 1 (b) equal to 1
(c) greater than 1 (d) any real number *(2002)*

ANSWER KEY

1. (d)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (a)	**12.** (a)
13. (b)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (b)	**21.** (b)	**22.** (b)	**23.** (a)	**24.** (c)
25. (b)	**26.** (c)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (a)	**31.** (b)	**32.** (a)	**33.** (c)	**34.** (d)	**35.** (b)	**36.** (a)
37. (d)	**38.** (a)	**39.** (c)	**40.** (a)	**41.** (a)							

Explanations

1. **(d) :** $x^2 - x + 1 = 0$ has its roots $-\omega, -\omega^2$.
Now $(-\omega)^{101} + (-\omega^2)^{107} = -\{\omega^2 + \omega^4\} = -(\omega^2 + \omega) = 1$

2. **(d) :** Given $2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0$

$\Rightarrow\ 2|\sqrt{x}-3| + (\sqrt{x}-3)^2 - 3 = 0$

set $|\sqrt{x}-3| = t$, which gives $t^2 + 2t - 3 = 0$

$\Rightarrow\ (t+3)(t-1) = 0 \Rightarrow t = -3, 1$

As $t \geq 0$ we have $t = 1$ Now $\left|\sqrt{x}-3\right| = 1$

$\Rightarrow\ \sqrt{x} - 3 = 1$ or $-1 \Rightarrow \sqrt{x} = 4, 2$ So, $x = 16, 4$
Thus, there are two solutions.

3. **(d) :** Let α, β be the roots of
$x^2 + (2-\lambda)x + (10-\lambda) = 0$ ...(i)
$\therefore\ \alpha + \beta = \lambda - 2$ and $\alpha\beta = 10 - \lambda$ ...(ii)
Now, A
$= \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$
$= (\lambda-2)^3 - 3(10-\lambda)(\lambda-2)$
$= \lambda^3 - 8 - 6\lambda^2 + 12\lambda + 3(\lambda^2 - 12\lambda + 20)$
$= \lambda^3 - 3\lambda^2 - 24\lambda + 52$

Now, $\dfrac{dA}{d\lambda} = 3\lambda^2 - 6\lambda - 24$ Put $\dfrac{dA}{d\lambda} = 0 \Rightarrow \lambda^2 - 2\lambda - 8 = 0$

$\Rightarrow\ (\lambda+2)(\lambda-4) = 0 \Rightarrow \lambda = -2, 4$

$\dfrac{d^2A}{d\lambda^2} = 2\lambda - 2$

For $\lambda = -2$, $2\lambda - 2 = 2(-2) - 2 = -6 < 0$ (max.)
For $\lambda = 4$, $2\lambda - 2 = 2(4) - 2 = 6 > 0$ (min.)
$\therefore$ Eqn. (i) becomes $x^2 - 2x + 6 = 0$
Thus, $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (2)^2 - 4(6) = 4 - 24 = -20$

$\Rightarrow\ \alpha - \beta = 2\sqrt{5}i \Rightarrow |\alpha - \beta| = 2\sqrt{5}$

4. **(d) :** Let $f(x) = ax^2 + bx + c$ such that $f(1) + f(2) = 0$

$\Rightarrow\ a + b + c + 4a + 2b + c = 0$

$\Rightarrow\ 5a + 3b + 2c = 0$ ...(i)

Since -1 is a root of $f(x) = 0$

$\therefore\ a(-1)^2 + b(-1) + c = 0 \Rightarrow a - b + c = 0$ ...(ii)

Eliminating c from (i) and (ii), we get $3a + 5b = 0$

$\Rightarrow\ \dfrac{b}{a} = -\dfrac{3}{5}$ ...(iii)

If another root is α, then $\alpha + (-1) = -\dfrac{b}{a}$

$\Rightarrow\ \alpha + (-1) = \dfrac{3}{5}$ (From (iii))

$\Rightarrow\ \alpha = \dfrac{3}{5} + 1 = \dfrac{8}{5}$

5. **(a) :** $\dfrac{1}{x+p} + \dfrac{1}{x+q} = \dfrac{1}{r} \Rightarrow \dfrac{x+q+x+p}{(x+p)(x+q)} = \dfrac{1}{r}$

$\Rightarrow\ r(2x+p+q) = (x+p)(x+q)$
$\Rightarrow\ x^2 + (p+q-2r)x + pq - pr - qr = 0$
Let α, β be the roots of the given equation.
We have, $\alpha + (-\alpha) = -(p+q-2r)$ $(\because \beta = -\alpha)$
$\Rightarrow\ p + q = 2r$ ...(i) and $\alpha\beta = pq - pr - qr$
Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= 0 - 2(pq - pr - qr)$ $(\because \alpha + \beta = 0)$
$= -2pq + 2r(p+q) = -2pq + (p+q)^2$ [From (i)]
$= p^2 + q^2$

6. **(c) :** We have, $\sum_{k=1}^{n}(x+k-1)(x+k) = 10n$

$\Rightarrow\ \sum_{k=1}^{n}[x^2 + (2k-1)x + k(k-1)] = 10n$

$\Rightarrow\ nx^2 + n^2x + \dfrac{1}{3}n(n^2-1) = 10n \Rightarrow x^2 + nx + \dfrac{1}{3}(n^2-1) - 10 = 0$

$\Rightarrow\ 3x^2 + 3nx + n^2 - 31 = 0$
Let consecutive roots be n and $n+1$, then
$(n+1-n)^2 = (n+1+n)^2 - 4n(n+1)$

$\Rightarrow\ 1 = n^2 - 4\left(\dfrac{n^2-31}{3}\right) \Rightarrow n^2 = 121 \ \therefore\ n = 11$

7. **(d) :** We have, $p(x) = ax^2 + bx + c$
As, $p(0) = 1 \Rightarrow c = 1$
If $p(x)$ is divided by $x - 1$, remainder $= 4$
$\Rightarrow\ p(1) = 4$ ...(i)
If $p(x)$ is divided by $x + 1$, remainder $= 6$
$\Rightarrow\ p(-1) = 6$ ...(ii)
(i) $\Rightarrow a + b + c = 4$ (ii) $\Rightarrow a - b + c = 6$
On solving above two equations, we get
$a = 4$ and $b = -1$, $c = 1$ $\therefore\ p(x) = 4x^2 - x + 1$
$p(-2) = 4(-2)^2 - (-2) + 1 = 16 + 2 + 1 = 19$
$p(2) = 4(2)^2 - 2 + 1 = 16 - 1 = 15$

8. **(c) :** $2^{(x-1)(x^2+5x-50)} = 1 = 2^0 \ \therefore\ (x-1)(x^2+5x-50) = 0$
$\Rightarrow\ (x-1)(x+10)(x-5) = 0 \Rightarrow x = 1, 5, -10$
$\therefore$ Required sum $= 1 + 5 - 10 = -4$.

9. **(a) :** $a^b = 1$ holds iff
1) $a = 1, b \in R$ 2) or $b = 0, a > 0$
The first possibility yields, $x^2 - 5x + 5 = 1$
$\Rightarrow\ x^2 - 5x + 4 = 0 \Rightarrow (x-1)(x-4) = 0 \ \therefore\ x = 1, 4$
The 2nd possibility yields
$x^2 + 4x - 60 = 0 \Rightarrow (x+10)(x-6) = 0 \ \therefore\ x = -10, 6$
At these values the base is positive.
The sum of all values $= 1 + 4 + 6 - 10 = 1$

But none of it matches.
Allow the base to be –1. Then
$x^2 - 5x + 5 = -1 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$
At $x = 2$, $x^2 + 4x - 60$ = even
$x = 3$, $x^2 + 4x - 60$ = odd
So, $x = 2$ is selected.
Sum of value of $x = -10 + 6 + 4 + 1 + 2 = 3$.
This is the best answer out of choices.

10. (c) : We have, $x^2 + bx - 1 = 0$ …(i)
and $x^2 + x + b = 0$ …(ii)
On subtracting (ii) from (i), we get
$x(1 - b) + 1 + b = 0 \Rightarrow x = \frac{b+1}{b-1}$

On putting value of x in (ii), we get $\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$
$\Rightarrow (b + 1)^2 + (b + 1)(b - 1) + b(b - 1)^2 = 0$
$\Rightarrow b^3 + 3b = 0 \Rightarrow b(b^2 + 3) = 0$ But $b \neq 0$, $\therefore b^2 = -3$
$\Rightarrow b = \pm\sqrt{3}\,i \Rightarrow |b| = \sqrt{3}$

11. (a) : We have, $\sqrt{2x+1} - \sqrt{2x-1} = 1$ ...(i)
$\Rightarrow 2x+1+2x-1-2\sqrt{4x^2-1} = 1 \Rightarrow 4x-1 = 2\sqrt{4x^2-1}$
$\Rightarrow 16x^2 - 8x + 1 = 16x^2 - 4 \Rightarrow 8x = 5$
$\Rightarrow x = \frac{5}{8}$ which satisfies equation (i) So, $\sqrt{4x^2-1} = \frac{3}{4}$

12. (a) : α is a root of $x^2 - 6x - 2 = 0$
Then $\alpha^2 - 6\alpha - 2 = 0$
Multiplying by α^n it become, $\alpha^{n+2} - 6\alpha^{n+1} - 2\alpha^n = 0$
Similarly, $\beta^{n+2} - 6\beta^{n+1} - 2\beta^n = 0$
Subtracting, we get
$(\alpha^{n+2} - \beta^{n+2}) - 6(\alpha^{n+1} - \beta^{n+1}) - 2(\alpha^n - \beta^n) = 0$
i.e., $a_{n+2} - 6a_{n+1} - 2a_n = 0$

Thus, $\frac{+7 - 7}{7 \; +6} = 8$

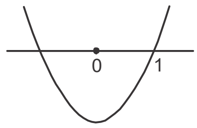
Set $n = 8$ to obtain the desired value $\frac{65 - 7}{7 \; >} = 8$

13. (b) : We have, $\alpha = 2 + 3i$; $\beta = 2 - 3i$ be the roots of the equation.
Let γ be the third root, so product of roots, $\alpha\beta\gamma = \frac{68}{7}$
Now, $-9 + >.\gamma = \frac{68}{7} \Rightarrow \gamma = \frac{6}{7}$. Putting the value of x, and then solving the equation, we can prove that the equation exists.

14. (c) : $(a - 1)(x^2 + x + 1)(x^2 - x + 1) + (a + 1)(x^2 + x + 1)^2 = 0$
$\Rightarrow x^2 + x + 1 = 0$ or $(a - 1)(x^2 - x + 1) + (a + 1)(x^2 + x + 1) = 0$
$\Rightarrow ax^2 + x + a = 0$
For real and unequal roots, $D > 0 \Rightarrow 1 - 4a^2 > 0$
$\Rightarrow \in \left(-\frac{6}{7} 1 \frac{6}{7}\right) - 5 \quad \because \neq 5$

15. (c) : We have $\frac{1}{\alpha} + \frac{1}{\beta} = 4$ and $2q = p + r$
Also, $-2(\alpha + \beta) = \alpha\beta + 1 \Rightarrow -2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = 1 + \frac{1}{\alpha\beta} \Rightarrow \frac{1}{\alpha\beta} = -9$
The equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is $x^2 - 4x - 9 = 0$
The equation having roots α and β is $9x^2 + 4x - 1 = 0$
$\alpha, \beta = \frac{-4 \pm \sqrt{16+36}}{2\times 9} = \frac{-4 \pm 2\sqrt{13}}{2\times 9} = \frac{-2 \pm \sqrt{13}}{9} \therefore |\alpha - \beta| = \frac{2\sqrt{13}}{9}$

16. (d) : Let $\{x\} = t$, so $0 \le t < 1$, we have
$3t^2 - 2t - a^2 = 0$
$D = 4 + 12a^2 > 0$
Then the equation has discriminant as positive. Assume that the roots are between 0 and 1, we have to ensure that there is no integral root, i.e., $t \neq 0$.
When $t = 0 \Rightarrow a^2 = 0 \quad \therefore \quad a = 0$
Product of roots $= -\frac{a^2}{3} < 0$
Thus one root is positive and the other is negative.
The condition that the root is strictly between 0 and 1 is



$f(0)f(1) < 0 \Rightarrow -a^2(1 - a^2) < 0$
$\Rightarrow a^2 - 1 < 0 \Rightarrow a^2 < 1. \quad \therefore \quad a \in (-1, 1)$
For integral roots, $a = 0$
$\therefore$ The set of all possible values of a is $(-1, 0) \cup (0, 1)$.
Remark : The question assumes that the equation does have a solution. Otherwise no answer is correct.

17. (c) : Let $f(x) = 2x^3 + 3x + k \Rightarrow f'(x) = 6x^2 + 3 > 0$
Thus f is strictly increasing. Hence it has atmost one real root. But a polynomial equation of odd degree has atleast one root. Thus the equation has exactly one root. Then the two distinct roots, in any interval whatsoever is an impossibility. No such k exists.

18. (d) : In the equation $x^2 + 2x + 3 = 0$, both the roots are imaginary.
Since $a, b, c \in R$, we have $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$
Hence $a : b : c : : 1 : 2 : 3$

19. (d) : $e^{\sin x} - e^{-\sin x} - 4 = 0$
$\Rightarrow (e^{\sin x})^2 - 4e^{\sin x} - 1 = 0 \Rightarrow t^2 - 4t - 1 = 0$
$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$
i.e., $e^{\sin x} = 2 + \sqrt{5}$ or $\underbrace{2 - \sqrt{5}}_{-\text{ve}}$ (neglected)
$\sin x = \ln(2 + \sqrt{5}) > 1 \therefore$ No real roots.

20. (b) : Let roots be $1 + ai$, $1 + bi$, then we have, $(a \in R)$
$(1 + ai) + (1 + bi) = -\alpha \Rightarrow 2 + (a + b)i = -\alpha$
$(1 + ai)(1 + bi) = \beta$
Comparing, we have $\alpha = -2$ and $a = -b$
Now $(1 + ai)(1 - ai) = \beta \Rightarrow 1 + a^2 = \beta \Rightarrow \beta = 1 + a^2$
As $a^2 \ge 0$ we have $\beta \in (1, \infty)$

21. (b) : We have $x^2 - x + 1 = 0$ giving $x = \frac{1 \pm i\sqrt{3}}{2}$.
Identifying these roots as ω and ω^2, we have $\alpha = \omega$, $\beta = \omega^2$.

We can also take the other way round that would not affect the result.

Now $\alpha^{2009} + \beta^{2009} = \omega^{2009} + \omega^{4018}$

$= \omega^{3k+2} + \omega^{3m+1}$ $(k, m \in N)$

$= \omega^2 + \omega = -1.$ $(\because \omega^{3k} = 1)$

22. (b) : The roots of $bx^2 + cx + a = 0$ are imaginary means $c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$

Again the coefficient of x^2 in $3b^2x^2 + 6bcx + 2c^2$ is positive, so the minimum value of the expression

$$= -\frac{36b^2c^2 - 4(3b^2)(2c^2)}{(3b^2)} = \frac{12b^2c^2}{12b^2} = -c^2$$

As $c^2 < 4ab$ we have $-c^2 > -4ab$

Thus, the minimum value is $-4ab$.

23. (a) : Let α and 4β be the roots of $x^2 - 6x + a = 0$ and α and 3β be those of the equation $x^2 - cx + 6 = 0$

From the relation between roots and coefficients

$\alpha + 4\beta = 6$ and $4\alpha\beta = a$

$\alpha + 3\beta = c$ and $3\alpha\beta = 6$

we obtain $\alpha\beta = 2$ giving $a = 8$

The first equation is $x^2 - 6x + 8 = 0 \Rightarrow x = 2, 4$

For $\alpha = 2$, $4\beta = 4 \Rightarrow 3\beta = 3$

For $\alpha = 4$, $4\beta = 2 \Rightarrow 3\beta = 3/2$ (not an integer)

So the common root is $\alpha = 2$.

24. (c) : $x^2 + ax + 1 = 0$

Let roots be α and β, then $\alpha + \beta = -a$ and $\alpha\beta = 1$

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} \Rightarrow |\alpha - \beta| = \sqrt{a^2 - 4}$

Since, $|\alpha - \beta| < \sqrt{5} \Rightarrow \sqrt{a^2 - 4} < \sqrt{5}$

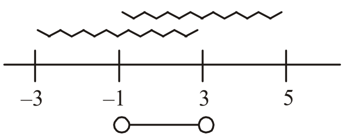
$\Rightarrow a^2 - 4 < 5 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3.$

25. (b) : $\alpha = \tan 30°$, $\beta = \tan 15°$ are roots of the equation $x^2 + px + q = 0 \therefore \alpha + \beta = -p$ and $\alpha \cdot \beta = q$

using $\tan A + \tan B = \tan(A + B) \cdot (1 - \tan A \tan B)$

$\Rightarrow -p = 1 - q \Rightarrow q - p = 1 \Rightarrow 2 + q - p = 3$

26. (c) : Let α, β are roots of the equation $(x^2 - 2mx + m^2) = 1$



$\Rightarrow x = m \pm 1 = m + 1, m - 1$

Now $-2 < m + 1 < 4$ ...(i)

and $-2 < m - 1 < 4$ ...(ii)

$$\begin{cases} \Rightarrow \quad -3 < m < 3 & \text{...(A)} \\ \text{and} \quad -1 < m < 5 & \text{...(B)} \end{cases}$$

By (A) & (B) we get $-1 < m < 3$ as shown by the number line.

27. (b) : Let $f(a) = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a - 2)^2 + 2(a + 1)$

$\therefore f'(a) = 2(a - 2) + 2$

For Maxima/Minima $f'(a) = 0 \Rightarrow 2[a - 2 + 1] = 0 \Rightarrow a = 1$

Again $f''(a) = 2$, $f''(1) = 2 > 0 \Rightarrow$ at $a = 1$, $f(a)$ will be least.

28. (c) : Let α, $\alpha + 1$ are consecutive integers.

$\therefore (x + \alpha)(x + \alpha + 1) = x^2 - bx + c$

Comparing both sides, we get $-b = 2\alpha + 1$

$c = \alpha^2 + \alpha \quad \therefore b^2 - 4c = (2\alpha + 1)^2 - 4(\alpha^2 + \alpha) = 1.$

29. (d) : Given $x^2 - 2kx + k^2 + k - 5 = 0$

Roots are less than $5 \Rightarrow D \geq 0$

$\Rightarrow (-2k)^2 \geq 4(k^2 + k - 5) \Rightarrow k \leq 5$ ...(A)

Again $f(5) > 0 \Rightarrow 25 - 10k + k^2 + k - 5 > 0$

$\Rightarrow k^2 - 9k + 20 > 0 \Rightarrow (k - 4)(k - 5) > 0$

$\Rightarrow k < 4 \cup k > 5$ ...(B)

Also $\frac{\text{sum of roots}}{2} < 5 \Rightarrow k < 5$ ...(C)

From (A), (B), (C), we have

$k \in (-\infty, 4)$ as the choice gives number $k < 5$ is (d).

30. (a) : If possible say

$f(x) = a_0x^n + a_1x^{n-1} + ... + a_nx \quad \therefore f(0) = 0$

Now $f(\alpha) = 0$ $(\because x = \alpha$ is root of given equation)

$\therefore f'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has at least one root in $]0, \alpha[$

$\Rightarrow na_n x^{n-1} + (n - 1) a_{n-1} x^{n-2} + ... + a_1 = 0$

has a +ve root smaller than α.

31. (b) : Let the two numbers be α, β.

$\therefore \quad \frac{\alpha + \beta}{2} = 9$ and $\sqrt{\alpha\beta} = 4$

$\therefore$ Required equation is

$x^2 - 2(\text{Average value of } \alpha, \beta)x + (\sqrt{\text{G.M.}})^2 = 0$

$\therefore \quad x^2 - 2(9)x + 16 = 0$

32. (a) : As $1 - p$ is root of $x^2 + px + 1 - p = 0$

$\Rightarrow (1 - p)^2 + p(1 - p) + (1 - p) = 0$

$(1 - p)[1 - p + p + 1] = 0 \Rightarrow p = 1$

$\therefore$ Given equation becomes $x^2 + x = 0 \Rightarrow x = 0, -1$

33. (c) : As $x^2 + px + q = 0$ has equal roots $\therefore p^2 = 4q$ and one root of $x^2 + px + 12 = 0$ is 4.

$\therefore 16 + 4p + 12 = 0 \Rightarrow p = -7 \Rightarrow p^2 = 4q \Rightarrow q = \frac{49}{4}$

34. (d) : Let α, 2α are roots of the given equation.

$\therefore$ Sum of the roots, $\alpha + 2\alpha = 3\alpha = \frac{1 - 3a}{a^2 - 5a + 3}$ ...(i)

and product of roots, $\alpha(2\alpha) = 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$ ...(ii)

By (i) and (ii), we have

$$\frac{9\alpha^2}{2\alpha^2} = \frac{(1 - 3a)^2}{(a^2 - 5a + 3)^2} \times \frac{a^2 - 5a + 3}{2}$$

$\Rightarrow 9(a^2 - 5a + 3) = (1 - 3a)^2 \Rightarrow a = \frac{2}{3}$

35. (b) : Given $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

$\Rightarrow 2a^2c = bc^2 + ab^2$

$\Rightarrow \dfrac{2a}{b} = \dfrac{c}{a}+\dfrac{b}{c} \Rightarrow \dfrac{c}{a},\dfrac{a}{b},\dfrac{b}{c} \in$ A.P.

$\Rightarrow$ reciprocals are in H.P.

36. **(a) :** Given $x^2 - 3|x| + 2 = 0$

If $x \geq 0$ *i.e.* $|x| = x$

$\therefore$ The given equation can be written as

$x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow x = 1, 2$

Similarly for $x < 0$, $x^2 - 3|x| + 2 = 0$

$\Rightarrow x^2 + 3x + 2 = 0 \Rightarrow x = -1, -2$

Hence 1, –1, 2, –2 are four solutions of the given equation.

37. **(d) :** We need the equation whose roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ which are reciprocal of each other, which means product of roots is $\dfrac{\alpha}{\beta}\dfrac{\beta}{\alpha} = 1$. In our choice (a) and (d) have product of roots 1, so choices (b) and (c) are out of court. In the problem choice, None of these is not given. If out of four choices only one choice satisfies that product of root is 1 then you select that choice for correct answer. Now for proper choice we proceed as, $\alpha \neq \beta$, but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$,

Changing α, β by x

$\therefore$ α, β are roots of $x^2 - 5x + 3 = 0$

$\Rightarrow \alpha + \beta = 5, \alpha\beta = 3$

now, $S = \dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha} = \dfrac{\alpha^2+\beta^2}{\alpha\beta} = \dfrac{19}{3}$ and product $\dfrac{\alpha}{\beta}\cdot\dfrac{\beta}{\alpha} = 1$

$\therefore$ Required equation,

x^2 – (sum of roots) x + product of roots = 0

$\Rightarrow x^2 - \dfrac{19}{3}x + 1 = 0 \Rightarrow 3x^2 - 19x + 3 = 0$

38. **(a) :** Let α, β are roots of $x^2 + bx + a = 0$

$\therefore$ $\alpha + \beta = -b$ and $\alpha\beta = a$

again let γ, δ are roots of $x^2 + ax + b = 0$

$\therefore$ $\gamma + \delta = -a$ and $\gamma\delta = b$

Now given $\alpha - \beta = \gamma - \delta \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow b^2 - 4a = a^2 - 4b \Rightarrow b^2 - a^2 = -4(b - a)$

$\Rightarrow (b - a)(b + a + 4) = 0 \Rightarrow b + a + 4 = 0$ as $(a \neq b)$

39. **(c) :** $x^2 + |x| + 9 = 0$

$\Rightarrow |x|^2 + |x| + 9 = 0$

$\Rightarrow \exists$ no real roots $\qquad (\because D < 0)$

40. **(a) :** Given $S = p + q = -p$ and product $pq = q$

$\Rightarrow q(p - 1) = 0 \Rightarrow q = 0, p = 1$

Now if $q = 0$ then $p = 0 \Rightarrow p = q$

If $p = 1$, then $p + q = -p$

$\Rightarrow q = -2p \Rightarrow q = -2(1) \Rightarrow q = -2 \Rightarrow p = 1$ and $q = -2$

41. **(a) :** In such type of problem if sum of the squares of number is known and we need product of numbers taken two at a time or need range of the product of numbers taken two at a time. We start square of the sum of the numbers like

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow 2(ab + bc + ca) = (a + b + c)^2 - (a^2 + b^2 + c^2)$

$\Rightarrow ab + bc + ca = \dfrac{(a+b+c)^2 - 1}{2} < 1$