CHAPTER

3 Matrices and Determinants

1. If the system of linear equations
$x + ky + 3z = 0$
$3x + ky - 2z = 0$
$2x + 4y - 3z = 0$
has a non-zero solutions (x, y, z), then $\frac{xz}{y^2}$ is equal to
(a) 30 (b) –10 (c) 10 (d) –30 *(2018)*

2. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to
(a) (4, 5) (b) (–4, –5)
(c) (–4, 3) (d) (–4, 5) *(2018)*

3. Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals :
(a) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$
(c) $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$ (d) $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$ *(Online 2018)*

4. Let S be the set of all real values of k for which the system of linear equations $x + y + z = 2$; $2x + y - z = 3$; $3x + 2y + kz = 4$ has a unique solution.Then S is
(a) an empty set (b) equal to R
(c) equal to {0} (d) equal to R – {0}
(Online 2018)

5. If the system of linear equations :
$x + ay + z = 3$, $x + 2y + 2z = 6$, $x + 5y + 3z = b$
has no solution, then
(a) $a = -1, b = 9$ (b) $a \neq -1, b = 9$
(c) $a = 1, b \neq 9$ (d) $a = -1, b \neq 9$ *(Online 2018)*

6 Suppose A is any 3 × 3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to
(a) 13 (b) 7 (c) 12 (d) 8 *(Online 2018)*

7. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is :
(a) 211 (b) 251 (c) 231 (d) 210
(Online 2018)

8. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to
(a) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (b) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$
(c) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (d) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ *(2017)*

9. If S is the set of distinct values of 'b' for which the following system of linear equations
$x + y + z = 1$, $x + ay + z = 1$, $ax + by + z = 0$
has no solution then S is
(a) an infinite set
(b) a finite set containing two or more elements
(c) a singleton (d) an empty set *(2017)*

10. If $S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\}$,
then $\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to
(a) $-2+\sqrt{3}$ (b) $4+2\sqrt{3}$
(c) $-4-2\sqrt{3}$ (d) $-2-\sqrt{3}$ *(Online 2017)*

11. The number of real values of λ for which the system of linear equations
$2x + 4y - \lambda z = 0$, $4x + \lambda y + 2z = 0$, $\lambda x + 2y + 2z = 0$
has infinitely many solutions, is
(a) 0 (b) 1
(c) 2 (d) 3 *(Online 2017)*

12. Let A be any 3 × 3 invertible matrix. Then which one of the following is not always true?
(a) adj (adj (A)) = $|A| \cdot (\text{adj}(A))^{-1}$
(b) adj (adj (A)) = $|A|^2 \cdot (\text{adj}(A))^{-1}$
(c) adj $(A) = |A| \cdot A^{-1}$
(d) adj (adj (A)) = $|A| \cdot A$ *(Online 2017)*

13. For two 3 × 3 matrices A and B, let $A + B = 2B'$ and $3A + 2B = I_3$, where B' is the transpose of B and I_3 is 3 × 3 identity matrix. Then
(a) $10A + 5B = 3I_3$ (b) $5A + 10B = 2I_3$
(c) $3A + 6B = 2I_3$ (d) $B + 2A = I_3$
(Online 2017)

14. If $W=\begin{bmatrix} : & - \\ 8 & 7 \end{bmatrix}$ and A adj $A = AA^T$, then $5a + b$ is equal to
(a) –1 (b) 5 (c) 4 (d) 13 *(2016)*

15. The system of linear equations
$x + \lambda y - z = 0, \lambda x - y - z = 0, x + y - \lambda z = 0$
has a non-trivial solution for
(a) infinitely many values of λ
(b) exactly one value of λ
(c) exactly two values of λ
(d) exactly three values of λ *(2016)*

16. The number of distinct real roots of the equation,

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0 \text{ in the interval } \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \text{ is}$$

(a) 1 (b) 4 (c) 2 (d) 3
(Online 2016)

17. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is:

(a) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$
(Online 2016)

18. Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = O$.
Statement - I : $A^{-1} = \frac{1}{7}(5I - A)$.
Statement - II : The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$. Then :
(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement-I is true, but Statement-II is false.
(d) Statement-I is false, but Statement-II is true.
(Online 2016)

19. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is
(a) –175 (b) 2014 (c) 2016 (d) –25
(Online 2016)

20. The set of all values of λ for which the system of linear equations
$2x_1 - 2x_2 + x_3 = \lambda x_1, 2x_1 - 3x_2 + 2x_3 = \lambda x_2,$
$-x_1 + 2x_2 = \lambda x_3$
has a non-trivial solution,
(a) contains two elements.
(b) contains more than two elements.
(c) is an empty set. (d) is a singleton. *(2015)*

21. If $W=\begin{bmatrix} 6 & 7 & 7 \\ 7 & 6 & -7 \\ & 7 & \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is a 3 × 3 identity matrix, then the ordered pair (a, b) is equal to
(a) (2, 1) (b) (–2, –1)
(c) (2, –1) (d) (–2, 1) *(2015)*

22. The least value of the product xyz for which the determinant

$$\begin{vmatrix} & 6 & 6 \\ 6 & & 6 \\ 6 & 6 & \end{vmatrix} \text{ is non-negative is}$$

(a) $-7\sqrt{7}$ (b) $-6; \sqrt{7}$
(c) –8 (d) –1 *(Online 2015)*

23. If $W=\begin{bmatrix} 5 & -6 \\ 6 & 5 \end{bmatrix}$, then which one of the following statements is not correct?
(a) $A^4 - I = A^2 + I$ (b) $A^3 - I = A(A - I)$
(c) $A^2 + I = A(A^2 - I)$ (d) $A^3 + I = A(A^3 - I)$
(Online 2015)

24. If A is a 3 × 3 matrix such that $|5 \,.\, \text{adj}A| = 5$, then $|A|$ is equal to
(a) $\pm\frac{6}{:}$ (b) ±5 (c) ±1 (d) $\pm\frac{6}{7:}$
(Online 2015)

25. If $\begin{vmatrix} {}^{7}+ & +6 & -7 \\ 7\;{}^{7}+8 & -6 \quad 8 & 8\;-8 \\ {}^{7}+7 & +8 \quad 7\;-6 & 7\;-6 \end{vmatrix} = \quad -671$ then 'a' is equal to
(a) 12 (b) 24 (c) –12 (d) –24
(Online 2015)

26. If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals
(a) I (b) B^{-1}
(c) $(B^{-1})'$ (d) $I + B$ *(2014)*

27. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2 (1-\beta)^2 (\alpha-\beta)^2,$$

then K is equal to
(a) $\frac{1}{\alpha\beta}$ (b) 1 (c) –1 (d) $\alpha\beta$ *(2014)*

28. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to
(a) 11 (b) 5 (c) 0 (d) 4 *(2013)*

29. The number of values of k, for which the system of equations $(k+1)x + 8y = 4k$, $kx + (k+3)y = 3k - 1$ has no solution, is

(a) 1 (b) 2 (c) 3 (d) infinite *(2013)*

30. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to

(a) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ *(2012)*

31. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to

(a) 0 (b) – 1 (c) – 2 (d) 1 *(2012)*

32. Let A and B be two symmetric matrices of order 3.

Statement-1 : $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

33. The number of values of k for which the linear equations

$4x + ky + 2z = 0$
$kx + 4y + z = 0$
$2x + 2y + z = 0$

possess a non-zero solution is

(a) 1 (b) zero (c) 3 (d) 2 *(2011)*

34. Consider the system of linear equations

$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(a) infinite number of solutions
(b) exactly 3 solutions
(c) a unique solution
(d) no solution *(2010)*

35. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(a) less than 4 (b) 5
(c) 6 (d) at least 7 *(2010)*

36. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define $Tr(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement-1 : $Tr(A) = 0$.

Statement-2 : $|A| = 1$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

37. Let A be a 2×2 matrix

Statement-1: adj (adj A) = A

Statement-2: |adj A| = $|A|$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. *(2009)*

38. Let a, b, c be such that $b(a + c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of n is

(a) any even integer (b) any odd integer
(c) any integer (d) zero *(2009)*

39. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by $tr(A)$, the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement-1 : If $A \neq I$ and $A \neq -I$, then det $A = -1$.

Statement-2 : If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2008)*

40. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) 1 (b) 2 (c) – 1 (d) 0 *(2008)*

41. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

(a) If det $A = \pm 1$, then A^{-1} need not exist
(b) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(c) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non-integers
(d) If det $A = \pm 1$, then A^{-1} exists and all its entries are integers *(2008)*

42. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is

(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y. *(2007)*

43. Let $A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

(a) 1/5 (b) 5 (c) 5^2 (d) 1. *(2007)*

44. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will always be true?

(a) $A = B$
(b) $AB = BA$
(c) either A or B is a zero matrix
(d) either A or B is an identity matrix *(2006)*

45. Let $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}$, $a, b \in N$. Then

(a) there cannot exist any B such that $AB = BA$
(b) there exist more than one but finite number B's such that $AB = BA$
(c) there exists exactly one B such that $AB = BA$
(d) there exist infinitely many B's such that $AB = BA$ *(2006)*

46. If $A^2 - A + I = 0$, then the inverse of A is

(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$ *(2005)*

47. If $A = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction

(a) $A^n = 2^{n-1}A - (n - 1)I$
(b) $A^n = nA - (n - 1)I$
(c) $A^n = 2^{n-1}A + (n - 1)I$
(d) $A^n = nA + (n - 1)I$. *(2005)*

48. If $a^2 + b^2 + c^2 = -2$ and

$$f(x)=\begin{vmatrix}1+a^2x & (1+b^2)x & (1+c^2)x\\ (1+a^2)x & 1+b^2x & (1+c^2)x\\ (1+a^2)x & (1+b^2)x & 1+c^2x\end{vmatrix}$$

then $f(x)$ is a polynomial of degree

(a) 0 (b) 1 (c) 2 (d) 3 *(2005)*

49. The system of equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solutions, if α is

(a) either –2 or 1 (b) –2
(c) 1 (d) not –2 *(2005)*

50. Let $A=\begin{pmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{pmatrix}$. The only correct statement about the matrix A is

(a) A^{-1} does not exist
(b) $A = (-1)I$, where I is a unit matrix
(c) A is a zero matrix (d) $A^2 = I$ *(2004)*

51. Let $A=\begin{pmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{pmatrix}$ and $10(B)=\begin{pmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{pmatrix}$. If B is the inverse of matrix A, then α is

(a) 2 (b) –1 (c) –2 (d) 5 *(2004)*

52. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are G.P., then the value of the determinant $\begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}$ is

(a) 2 (b) 1 (c) 0 (d) –2 *(2004)*

53. If 1, ω, ω^2 are the cube roots of unity,

then $\Delta=\begin{vmatrix}1 & \omega^n & \omega^{2n}\\ \omega^n & \omega^{2n} & 1\\ \omega^{2n} & 1 & \omega^n\end{vmatrix}$ is equal to

(a) 1 (b) ω (c) ω^2 (d) 0 *(2003)*

54. If $A=\begin{bmatrix}a & b\\ b & a\end{bmatrix}$ and $A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}$, then

(a) $\alpha = a^2 + b^2$, $\beta = 2ab$
(b) $\alpha = a^2 + b^2$, $\beta = a^2 - b^2$
(c) $\alpha = 2ab$, $\beta = a^2 + b^2$
(d) $\alpha = a^2 + b^2$, $\beta = ab$ *(2003)*

55. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is –ve, then $\begin{vmatrix}a & b & ax+b\\ b & c & bx+c\\ ax+b & bx+c & 0\end{vmatrix}$ is

(a) +ve (b) $(ac - b^2)(ax^2 + 2bx + c)$
(c) –ve (d) 0. *(2002)*

56. If l, m, n are the p^{th}, q^{th} and r^{th} term of a G.P., all positive,

then $\begin{vmatrix}\log l & p & 1\\ \log m & q & 1\\ \log n & r & 1\end{vmatrix}$ equals

(a) –1 (b) 2 (c) 1 (d) 0 *(2002)*

ANSWER KEY

1. (c)	**2.** (d)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (d)	**11.** (b)	**12.** (a)
13. (a)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (a)	**21.** (b)	**22.** (c)	**23.** (c)	**24.** (a)
25. (b)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (b)	**31.** (a)	**32.** (d)	**33.** (d)	**34.** (d)	**35.** (d)	**36.** (c)
37. (a)	**38.** (b)	**39.** (a)	**40.** (a)	**41.** (d)	**42.** (d)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (b)	**48.** (c)
49. (b)	**50.** (d)	**51.** (d)	**52.** (c)	**53.** (d)	**54.** (a)	**55.** (c)	**56.** (d)				

Explanations

1. **(c) :** For non-zero solutions, we have $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\Rightarrow$ $1(-3k + 8) - k(-9 + 4) + 3(12 - 2k) = 0$ which gives $k = 11$

Now, the system of equations become

$x + 11y + 3z = 0$ ...(i)

$3x + 11y - 2z = 0$ ...(ii)

$2x + 4y - 3z = 0$ ...(iii)

The equation (i) and (iii) gives

$3x + 15y = 0$ *i.e.* $x = -5y$

Putting $x = -5y$ in (i), we have $-5y + 11y + 3z = 0$

$\Rightarrow$ $z = -2y$ Now $\frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$

2. **(d) :** As both sides are polynomial in x, let's set $x = 0$ to obtain

$$\begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3$$

which gives $A^3 = -64$ $\therefore$ $A = -4$

Taking x common from all rows of given determinant, we get

$$\begin{vmatrix} 1-\frac{4}{x} & 2 & 2 \\ 2 & 1-\frac{4}{x} & 2 \\ 2 & 2 & 1-\frac{4}{x} \end{vmatrix} = \left(B - \frac{4}{x}\right)\left(1 + \frac{4}{x}\right)^2$$

Take the limit as $x \to \infty$ to obtain $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

Alternative solution :

The most efficient way to obtain the result is to use this result

$$\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = (a + 2b)(a - b)^2$$

This result gives $A = -4$, $B = 5$

Note that the determinant, when $a = b$, vanish and all the three rows become identical hence $(a - b)^2$ is a factor.

3. **(a) :** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $a, b, c, d \in R$ ...(i)

According to given condition, we have

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$ for some scalar λ.

$\Rightarrow$ $a = \lambda$ ...(ii); $2c + 3d = \lambda$ ...(iii)

and $c = 0$ ...(iv); $2a + 3b = 0$ ...(v)

Using (ii), (iii), (iv) and (v), we get

$a = \lambda$, $b = \frac{-2\lambda}{3}$, $c = 0$, $d = \frac{\lambda}{3}$

Also, $|3A| = 108 \Rightarrow 3^2|A| = 108$

$\Rightarrow |A| = 12$ ...(vi)

Putting the values of a, b, c and d in (i), we get

$A = \begin{bmatrix} \lambda & \frac{-2\lambda}{3} \\ 0 & \frac{\lambda}{3} \end{bmatrix} \Rightarrow |A| = \frac{\lambda^2}{3}$ ...(vii)

From (vi) and (vii), we have $\frac{\lambda^2}{3} = 12 \Rightarrow \lambda^2 = 36 \Rightarrow \lambda = \pm 6$

Put $\lambda = 6$, we get $A = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 36 & -24-8 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

4. **(d) :** For unique solution $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

$\Rightarrow$ $1(k + 2) - 1(2k + 3) + 1(4 - 3) \neq 0$

$\Rightarrow$ $-k + 2 - 3 + 1 \neq 0 \Rightarrow k \neq 0$ $\therefore S = R - \{0\}$

5. **(d) :** The given system of equations has no solution

$\therefore$ $\begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$\Rightarrow$ $\begin{vmatrix} 0 & a-2 & -1 \\ 0 & -3 & -1 \\ 1 & 5 & 3 \end{vmatrix} = 0 \Rightarrow a = -1$

Now, for no solution, (adj A) $B \neq O$

$\therefore$ adj$(A) = \begin{bmatrix} -4 & 8 & -4 \\ -1 & 2 & -1 \\ 3 & -6 & 3 \end{bmatrix}$

$\Rightarrow$ (adj A)(B) $= \begin{bmatrix} -4 & 8 & -4 \\ -1 & 2 & -1 \\ 3 & -6 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \\ b \end{bmatrix} \neq O$

$\Rightarrow$ $-12 + 48 - 4b \neq 0 \Rightarrow b \neq 9$

6. **(d) :** Given, $(A - 3I)(A - 5I) = O$ $\therefore A^2 - 8A + 15I = O$

Post multiplying by A^{-1} on both sides, we have

$A \cdot AA^{-1} - 8A \cdot A^{-1} + 15I \cdot A^{-1} = O$

$\Rightarrow$ $A - 8I + 15A^{-1} = O \Rightarrow A + 15A^{-1} = 8I$

$\Rightarrow$ $\frac{1}{2}A + \frac{15}{2}A^{-1} = 4I$ ...(i)

Comparing (i) with $\alpha A + \beta A^{-1} = 4I$, we get $\alpha = \frac{1}{2}$ and $\beta = \frac{15}{2}$

$\therefore \quad \alpha + \beta = \frac{1}{2} + \frac{15}{2} = 8$

7. (c) : $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}; A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}; A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 15 & 5 & 1 \end{bmatrix}, ... A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 20 & 1 & 0 \\ 210 & 20 & 1 \end{bmatrix}$$

Given, $B = A^{20} \left\{ \begin{array}{l} \because \quad a_{31} \text{ in } A^n = \sum_{i=1}^{3} a_{i1} \text{ of } A^{n-1} \\ \text{and } a_{21} \text{ in } A^n = a_{32} \text{ in } A^n = n \end{array} \right\}$

$\therefore$ Sum of the elements of the first column of $B = 1 + 20 + 210 = 231$

8. (a) : Given, $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

Then, A satisfies the characteristic equation
$A^2 - 3A - 10I = 0$
Now $3A^2 + 12A = 3(3A + 10I) + 12A = 21A + 30I$

$$= \begin{bmatrix} 42 & -63 \\ -84 & 21 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$\therefore \quad \text{adj}\,(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

9. (c) : The equation can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & 0 \\ a & b & 1 \end{vmatrix} = -(1-a)^2$

The necessary condition is $\Delta = 0 \Rightarrow a = 1$
But for $a = 1$ the equation becomes
$x + y + z = 1$
$x + y + z = 1$
$x + by + z = 0$
For no solution $b = 1$. Then S is a singleton set.

10. (d) : Let $A = \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix}$

$\therefore \quad |A| = 0(0 - \cos x \sin x) - \cos x(0 - \cos^2 x) - \sin x(\sin^2 x - 0) = 0$
$\Rightarrow \quad \cos^3 x - \sin^3 x = 0$
$\Rightarrow \quad \tan^3 x = 1 \Rightarrow \tan x = 1$

$$\sum_{x \in s} \frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \quad (\because \tan x = 1)$$

$$\Rightarrow \quad \frac{1 + 3 + 2\sqrt{3}}{-2} = \frac{4}{-2} - \frac{2\sqrt{3}}{2} = -2 - \sqrt{3}$$

11. (b) : Let $A = \begin{bmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{bmatrix}$

For the system of linear equations to have infinitely many solutions, $|A| = 0$

$$\Rightarrow \quad \begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2) = 0$
$\Rightarrow \quad \lambda^3 + 4\lambda - 40 = 0$
$\therefore$ Number of solutions = 1 (lies between 3 and 4)

12. (a)

13. (a) : $A + B = 2B' \Rightarrow (A + B)' = (2B')' \Rightarrow A' + B' = 2B$

$\Rightarrow \quad B = \frac{A' + B'}{2}$

Now, $A + \left(\frac{B' + A'}{2}\right) = 2B'$ [$\because \ A + B = 2B'$]

$\Rightarrow \quad 2A + B' + A' = 4B' \quad \Rightarrow 2A + A' = 3B' \Rightarrow A = \frac{3B' - A'}{2}$

Also, $3A + 2B = I_3$ ...(1)

$\Rightarrow \quad 3\left(\frac{3B' - A'}{2}\right) + 2\left(\frac{A' + B'}{2}\right) = I_3$

$\Rightarrow \quad \left(\frac{9B' + 2B'}{2}\right) + \left(\frac{2A' - 3A'}{2}\right) = I_3$

$\Rightarrow \quad 11B' - A' = 2I_3 \Rightarrow (11B' - A')' = (2I_3)'$
$\Rightarrow \quad 11B - A = 2I_3$ ...(2)

Multiplying (2) by 3 and then adding (1) and (2), we get

$35B = 7I_3 \Rightarrow B = \frac{I_3}{5}$

From (2), $11\frac{I_3}{5} - A = 2I_3 \Rightarrow 11\frac{I_3}{5} - 2I_3 = A \Rightarrow A = \frac{I_3}{5}$

$\therefore \quad 5A = 5B = I_3 \Rightarrow 10A + 5B = 3I_3$

14. (b) : We have $WW^q = \begin{bmatrix} : & - \\ 8 & 7 \end{bmatrix} \begin{bmatrix} : & 8 \\ - & 7 \end{bmatrix}$

$$= \begin{bmatrix} 7:\ ^7 +\ ^7 & 6: \ -7 \\ 6: \ -7 & 68 \end{bmatrix}$$

A (adj A)= AA^T is known, so equating the two expressions,

$$\begin{bmatrix} 7:\ ^7 +\ ^7 & 6: \ -7 \\ 6: \ -7 & 68 \end{bmatrix} = \begin{bmatrix} 65 \ +8 & 5 \\ 5 & 65 \ +8 \end{bmatrix}$$

We have, $10a + 3b = 13$ and $15a - 2b = 0$
On solving, we get $a = 2/5$, $b = 3$
Then, $5a + b = 2 + 3 = 5$

15. (d) : The system $AX = 0$ has non-trivial solution iff det$A = 0$

$$331 \begin{vmatrix} 6 & \lambda & -6 \\ \lambda & -6 & -6 \\ 6 & 6 & -\lambda \end{vmatrix} = 5$$

$\Rightarrow \quad (\lambda + 1) - \lambda(-\lambda^2 + 1) - (\lambda + 1) = 0$

$\Rightarrow \quad \lambda^3 - \lambda = 0 \quad \Rightarrow \quad \lambda(\lambda^2 - 1) = 0 \therefore \lambda = 0, 1, -1$

16. (c) : We have, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} \cos x - \sin x & 0 & \sin x - \cos x \\ 0 & \cos x - \sin x & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Applying $C_3 \to C_3 + C_2$

$$\begin{vmatrix} \cos x - \sin x & 0 & \sin x - \cos x \\ 0 & \cos x - \sin x & 0 \\ \sin x & \sin x & \sin x + \cos x \end{vmatrix} = 0$$

Expanding along first column, we get

$(\sin x - \cos x)^2 (2 \sin x + \cos x) = 0$

$\Rightarrow \quad \cos x = -2 \sin x$ or $\cos x = \sin x$

$\Rightarrow \quad \tan x = \frac{-1}{2}$ or $\tan x = 1 \quad \Rightarrow \quad x = -\tan^{-1}\left(\frac{1}{2}\right), \frac{\pi}{4}$

$\therefore \quad$ Two solutions.

17. (c) : $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$PP^T = P^TP = I$

$Q^{2015} = (PAP^T)\ (PAP^T)$ …… (2015 terms) $= PA^{2015}\ P^T$

$P^TQ^{2015}P = A^{2015}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad A^{2015} = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ So, $P^TQ^{2015}P = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

18. (a) : We have, $A^2 - 5A = -7I$

$\Rightarrow AAA^{-1} - 5AA^{-1} = -7IA^{-1} \Rightarrow AI - 5I = -7A^{-1}$

$\Rightarrow \quad A^{-1} = \frac{1}{7}(5I - A)$

Also, $A^3 - 2A^2 - 3A + I = A(5A - 7I) - 2A^2 - 3A + I$

$= 5A^2 - 7A - 2A^2 - 3A + I = 3A^2 - 10A + I$

$= 3(5A - 7I) - 10A + I = 5A - 20I = 5(A - 4I)$

19. (d) : We have,

$A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix}$

Also, $|A| = -1$

Now, $A^{2016} - 2A^{2015} - A^{2014} = A^{2014}\ (A^2 - 2A - I)$

$\therefore \quad |A^{2016} - 2A^{2015} - A^{2014}| = |A^{2014}||A^2 - 2A - I|$

$$= |A|^{2014} \begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = -25$$

20. (a) : The system is $(2 - \lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3 + \lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

For non-trivial solution, the determinant of the coefficient matrix must vanish. Then

$$\begin{vmatrix} 7-\lambda & -7 & 6 \\ 7 & -8-\lambda & 7 \\ -6 & 7 & -\lambda \end{vmatrix} = 5$$

$\Rightarrow \quad (2 - \lambda)\{\lambda(3 + \lambda) - 4\} + 2\ \{-2\lambda - 3\} + 1\ \{4 - (3 + \lambda)\} = 0$

$\Rightarrow \quad (2 - \lambda)(\lambda^2 + 3\lambda - 4) - 4\lambda - 6 + 1 - \lambda = 0$

$\Rightarrow \quad (2 - \lambda)(\lambda^2 + 3\lambda - 4) - 5\lambda - 5 = 0$

$\Rightarrow \quad (2 - \lambda)(\lambda - 1)(\lambda + 4) - 5(\lambda + 1) = 0$

$\Rightarrow \quad (\lambda - 1)\ (-\lambda^2 - 2\lambda + 8 - 5) = 0$

$\Rightarrow \quad (\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0 \Rightarrow \ (\lambda - 1)^2(\lambda + 3) = 0$

Thus, $\lambda = 1, 1, -3 \ \therefore$ Set of all λ's contain 2 elements.

21. (b) : As $AA^T = 9I$, we have $\left(\frac{W}{8}\right)\left(\frac{W}{8}\right)^q = f$

Hence, $\frac{6}{8} W$ is an orthogonal matrix. V{ $\quad \frac{6}{8} W = \begin{bmatrix} \frac{6}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{6}{8} & -\frac{7}{8} \\ \frac{}{8} & \frac{7}{8} & \frac{}{8} \end{bmatrix}$

We know that row (column) form mutually orthogonal unit vectors.

Then $\left(\frac{}{8} 1\frac{7}{8} 1 \frac{}{8}\right)$ is a unit vector, gives $a^2 + 4 + b^2 = 9$

Also, $a + 2b + 4 = 0$ and $2a - 2b + 2 = 0$

The solution is (–2, –1), which is consistent with all the equations.

22. (c) : $\begin{vmatrix} & 6 & 6 \\ 6 & & 6 \\ 6 & 6 & \end{vmatrix} = \quad -- \ + \ + \ . +7$

Since A.M. ≥ G.M.

$\Rightarrow \quad \frac{+ \ +}{8} \geq - \quad .^{648} \Rightarrow x + y + z \geq 3(xyz)^{1/3}$

For least value of xyz, $xyz - 3\ (xyz)^{1/3} + 2 \geq 0$

$\Rightarrow \quad t^3 - 3t + 2 \geq 0$ (Put $t = (xyz)^{1/3}$)

$\Rightarrow \quad (t + 2)\ (t^2 - 2t + 1) \geq 0 \Rightarrow \quad t = -2, 1$

So, least value of $t^3 = xyz$ is -8

23. (c) : Given that $W = \begin{bmatrix} 5 & -6 \\ 6 & 5 \end{bmatrix}$

So, $W^7 = \begin{bmatrix} -6 & 5 \\ 5 & -6 \end{bmatrix} \Rightarrow W^7 = -f$

$W^8 = \begin{bmatrix} 5 & 6 \\ -6 & 5 \end{bmatrix}$ B $W^9 = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} = f$

If we check, then options (a), (b), and (d) are correct.

Now, for option (c), $A^2 + I = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$, $A\ (A^2 - I) = \begin{bmatrix} 5 & 7 \\ -7 & 5 \end{bmatrix}$

24. (a) : Given that $|5 \text{ adj}A| = 5$

$\Rightarrow \quad 5^3|\text{adj}A| = 5 \quad \Rightarrow \quad \text{npv}W = \frac{6}{:^7}$

$\Rightarrow \quad W^{8-6} = \frac{6}{:^7}$ $\qquad\qquad (\because\ |\text{adj } A| = |A|^{n-1})$

$\Rightarrow \quad W = \pm\frac{6}{:}$

25. (b) : Put $x = 1$ on both sides, we get $\begin{vmatrix} 7 & 7 & -6 \\ 9 & 8 & 5 \\ ; & 6 & 6 \end{vmatrix} = -67$

$\Rightarrow \ a = 24$

26. (a) : $BB^T = (A^{-1}A^T)\ (A^{-1}A^T)^T$

$= (A^{-1}A^T)\ ((A^T)^T\ (A^{-1})^T) = (A^{-1}A^T)(A(A^{-1})^T)$

$= A^{-1}(A^TA)(A^T)^{-1} = A^{-1}(AA^T)\ (A^T)^{-1} = (A^{-1}A)\ (A^T)(A^T)^{-1} = I \cdot I = I$

Recall that $(AB)^T = B^TA^T$ and that the matrix multiplication is associative.

27. (b) : $\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$

(By multiplication of determinants)

$= [(1-\alpha)(1-\beta)(\beta-\alpha)]^2$

On comparison, $K = 1$

28. (a) : $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

$\det P = 1(12-12) - \alpha(4-6) + 3(4-6) = 2\alpha - 6$

Also, $\det(\text{adj } P) = (\det P)^2$

$\Rightarrow \quad 2\alpha - 6 = 16 \ \Rightarrow \ 2\alpha = 22. \qquad \therefore \ \alpha = 11$

Remark : $\det(\text{adj } A) = (\det A)^{n-1}$, where A is a matrix of order n.

29. (a) : The equation is $\begin{bmatrix} k+1 & 8 \\ k & k+3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4k \\ 3k-1 \end{bmatrix}$

For no solution of $AX = B$ a necessary condition is $\det A = 0$.

$\Rightarrow \quad \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = 0$

$\Rightarrow \quad (k+1)(k+3) - 8k = 0 \Rightarrow k^2 + 4k + 3 - 8k = 0$

$\Rightarrow \quad k^2 - 4k + 3 = 0 \Rightarrow (k-1)(k-3) = 0 \ \therefore \quad k = 1, 3$

For $k = 1$, the equation becomes $2x + 8y = 4$, $x + 4y = 2$ which is just a single equation in two variables.

$x + 4y = 2$ has infinite solutions.

For $k = 3$, the equation becomes $4x + 8y = 12$, $3x + 6y = 8$ which are parallel lines. So no solution in this case.

30. (b) : $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Let $u_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\Rightarrow a = 1,\ 2a + b = 0$ $\Rightarrow b = -2,\ 3a + 2b + c = 0 \Rightarrow c = 1$

Let $u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\Rightarrow \ p = 0,\ 2p + q = 1 \ \Rightarrow \ q = 1,$ $3p + 2q + r = 0 \ \Rightarrow \ r = -2$

$u_1 + u_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

31. (a) : $P^3 = Q^3$, $\ P^2Q = Q^2P$, $\ PQ^2 = P^2Q$

$\Rightarrow \quad P(P^2 + Q^2) = (Q^2 + P^2) \Rightarrow P(P^2 + Q^2) = (P^2 + Q^2)Q$

$P \neq Q \Rightarrow \ P^2 + Q^2$ is singular. Hence, $|P^2 + Q^2| = 0$

32. (d) : Let $A(BA) = P$

Then $P^T = (ABA)^T = A^TB^TA^T$ (Transversal rule)

$= ABA = P$

Thus P is symmetric.

Again, $A(BA) = (AB)A$ by associativity.

Also $(AB)^T = B^TA^T = BA = AB$ ($\because$ A and B are commutative)

$\Rightarrow \quad AB$ is also symmetric.

33. (d) : For the system to possess non-zero solution,

we have $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

which on expansion gives $k^2 - 6k + 8 = 0$

$\Rightarrow \quad (k-2)(k-4) = 0. \qquad \therefore \ k = 2, 4$

34. (d) : $x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

A quick observation tells us that the sum of first two equations yields

$(x_1 + 2x_2 + x_3) + (2x_1 + 3x_2 + x_3) = 3 + 3$

$\Rightarrow \quad 3x_1 + 5x_2 + 2x_3 = 6$

But this contradicts the third equation, *i.e.*, $3x_1 + 5x_2 + 2x_3 = 1$

As such the system is inconsistent and hence it has no solution.

35. (d) : $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\det A = (a_1b_2c_3 + a_2c_1b_3 + a_3b_1c_2) - (a_1c_2b_3 + a_2b_1c_3 + a_3c_1b_2)$

If any of the terms be non-zero, then det A will be non-zero and all the elements of that term will be 1 each.

Number of non-singular matrices $= {}^6C_1 \times {}^6C_1 = 36$

We can also exhibit more than 6 matrices to pick the right choice.

36. (c) : Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Which gives $\alpha + \delta = 0$ and $\alpha^2 + \beta\gamma = 1$

So we have $Tr(A) = 0$

$\det A = \alpha\delta - \beta\gamma = -\alpha^2 - \beta\gamma = -(\alpha^2 + \beta\gamma) = -1$

Thus statement-1 is true but statement-2 is false.

37. (a) : We have $\text{adj}(\text{adj } A) = |A|^{n-2}A$

Here $n = 2$, which gives $\text{adj}(\text{adj } A) = A$

The statement-1 is true.

Again $|\text{adj } A| = |A|^{n-1}$

Here $n = 2$, which gives $|\text{adj } A| = |A|$

Thus statement-2 is also true. But statement-2 doesn't explain statement-1.

38. (b) : $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ a & -b & c \end{vmatrix} = 0$$

$$\Rightarrow D + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c+1 & c-1 & c \end{vmatrix} = 0$$

(Changing rows to columns)

$$\Rightarrow D + (-1)^n \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c+1 & c-1 \end{vmatrix} = 0$$

(Changing columns in cyclic order doesn't change the determinant)

$\Rightarrow D + (-1)^n D = 0 \Rightarrow \{1 + (-1)^n\}D = 0$

Now $D = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = \begin{vmatrix} a & 2 & a-1 \\ -b & 2 & b-1 \\ c & -2 & c+1 \end{vmatrix}$ $(C_2 \to C_2 - C_3)$

$= \begin{vmatrix} a+c & 0 & a+c \\ -b+c & 0 & b+c \\ c & -2 & c+1 \end{vmatrix}$ $(R_1 \to R_1 + R_3,\ R_2 \to R_2 + R_3)$

Expanding along 2nd column

$D = 2\{(a + c)(b + c) - (a + c)(c - b)\}$

$= 2(a + c)2b = 4b(a + c) \neq 0$ (By hypothesis)

Now $\{1 + (-1)^n\} D = 0 \Rightarrow 1 + (-1)^n = 0$

Which means n = odd integer.

39. (a) : Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. We have

$$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2 + \beta\gamma \end{bmatrix}$$

$$A^2 = I = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

giving $\alpha^2 + \beta\gamma = 1 = \delta^2 + \beta\gamma$ and $\gamma(\alpha + \delta) = \beta(\alpha + \delta) = 0$

As $A \neq I$, $A \neq -I$, we have $\alpha = -\delta$

$$\det A = \begin{vmatrix} \sqrt{1-\beta\gamma} & \beta \\ \gamma & -\sqrt{1-\beta\gamma} \end{vmatrix} = -1 + \beta\gamma - \beta\gamma = -1$$

Statement-1 is therefore true.

$\text{tr}(A) = \alpha + \delta = 0 \quad \{\alpha = -\delta\}$

Statement-2 is false because $\text{tr}(A) = 0$

40. (a) : System of equations

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

has non trivial solution if the determinant of coefficient matrix is zero

$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow 1(1 - a^2) + c(-c - ab) - b(ca + b) = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

41. (d) : Each entry of A is an integer, so the cofactor of every entry is an integer. And then each entry of adjoint is integer.

Also $\det A = \pm 1$ and we know that $A^{-1} = \frac{1}{\det A}(adj\ A)$

This means all entries in A^{-1} are integers.

42. (d) : $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ (Apply $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$)

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = 1(xy - 0) = xy$$

Hence D is divisible by both x and y.

43. (a) : $A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 5\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 25\alpha + 5\alpha^2 \\ 0 & 0 & 25 \end{bmatrix}$$

Given $|A^2| = 25$, $625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$.

44. (b) : Give $A^2 - B^2 = (A + B)(A - B)$

$\Rightarrow 0 = BA - AB \Rightarrow BA = AB$

45. (d) : $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

Now $AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$ ... (i)

and $BA = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a & 2a \\ 3b & 4b \end{pmatrix}$ ... (ii)

As $AB = BA \Rightarrow 2a = 2b \Rightarrow a = b$

$\therefore\ B = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = aI_2 \Rightarrow \exists$ infinite value of $a = b \in N$

46. (c) : $A^2 - A + I = 0 \Rightarrow I = A - A \cdot A$

$IA^{-1} = AA^{-1} - A(AA^{-1})$, $A^{-1} = I - A$.

47. (b) : $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$\therefore\ A^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, A^3 = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ so $A^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$

and $nA - (n-1)I = \begin{pmatrix} n & 0 \\ n & n \end{pmatrix} - \begin{pmatrix} n-1 & 0 \\ 0 & n-1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} = A^n.$

48. (c) : Applying $C_2 \to C_2 + C_3 + C_1$

$$f(x) = 1 + 2x + x(a^2 + b^2 + c^2)\begin{vmatrix} 1+a^2x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ and using $a^2 + b^2 + c^2 = -2$ we have

$$(1+2x-2x)\begin{vmatrix} 1-x & 0 & 0 \\ 0 & 0 & x-1 \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix} = (1-x)^2$$

$= x^2 - 2x + 1$ $\therefore$ degree of $f(x)$ is 2.

49. (b) : For no solution $|A| = 0$ and $(\text{adj } A)(B) \neq 0$

Now $|A| = 0 \Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$\Rightarrow \alpha^3 - 3\alpha + 2 = 0 \Rightarrow (\alpha - 1)^2(\alpha + 2) = 0$

$\Rightarrow \alpha = 1, -2.$

But for $\alpha = 1$, $|A| = 0$ and $(\text{adj } A)(B) = 0$

$\Rightarrow$ for $\alpha = 1$ there exist infintiely many solution.

Also each equation becomes

$x + y + z = 0$ again for $\alpha = -2$

$|A| = 0$ but $(\text{adj } A)(B) \neq 0 \Rightarrow \exists$ no solution.

50. (d) : (i) $|A| = 1$ $\therefore A^{-1}$ does not exist is wrong statement

(ii) $(-1)\, I = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \neq A \Rightarrow$ (b) is false

(iii) A is clearly a non zero matrix $\therefore$ (c) is false.

We are left with (d) only.

51. (d) : Given $A^{-1} = B = 10\, A^{-1} = 10\, B$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix} = 10\, A^{-1} \Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix} (A) = 10I$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}$ ...(*)

$\Rightarrow -5 + \alpha = 0$

(equating A_{21} entry both sides of (*))

$\Rightarrow \alpha = 5$

52. (c) : $\dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = \ldots = \dfrac{a_n}{a_{n-1}} = r$

which means $a_n, a_{n+1}, a_{n+2} \in$ G.P.

$\Rightarrow a_{n+1}^{\ 2} = a_n\, a_{n+2}$

$\Rightarrow 2 \log a_{n+1} - \log a_n - \log a_{n+2} = 0$ ...(i)

Similarly $2 \log a_{n+4} - \log a_{n+3} - \log a_{n+5} = 0$ ...(ii)

and $2 \log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0$ ...(iii)

Using $C_1 \to C_1 + C_3 - 2C_2$

we get $\Delta = 0$

53. (d) : As ω is cube root of unity $\therefore \omega^3 = \omega^{3n} = 1$

$$\therefore \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} = (\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^n) = 0$$

54. (a) : $A^2 = AA = \begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$= \begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$

55. (c) : $C_1 \to xC_1 + C_2 - C_3$

$$= \frac{1}{x}\begin{vmatrix} 0 & b & ax+b \\ 0 & c & bx+c \\ ax^2+2bx+c & bx+c & 0 \end{vmatrix}$$

$= \dfrac{(ax^2+2bx+c)}{x}\,[b^2x + bc - acx - bc]$

$= (b^2 - ac)\,(ax^2 + 2bx + c)$

$= (+\text{ve})\,(-\text{ve}) < 0$

56. (d) : Let A be the first term and R be the common ratio of G.P.

$\therefore\ l = t_p = AR^{p-1}$

$\Rightarrow\ \log l = \log A + (p-1)\log R$

Similarly, $\log m = \log A + (q-1)\log R$

and $\log n = \log A + (r-1)\log R$

$$\therefore \begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A - \log R & p & 1 \\ \log A - \log R & q & 1 \\ \log A - \log R & r & 1 \end{vmatrix} + \begin{vmatrix} p\log R & p & 1 \\ q\log R & q & 1 \\ r\log R & r & 1 \end{vmatrix}$$

$C_1 \approx C_3$ $\qquad$ $C_1 \approx C_2$

$= 0 + 0 = 0$