CHAPTER 1

Sets, Relations and Functions

1. Two sets A and B as under :
$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;
$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$.
Then
(a) neither $A \subset B$ nor $B \subset A$
(b) $B \subset A$
(c) $A \subset B$
(d) $A \cap B = \phi$(an empty set) *(2018)*

2. Consider the following two binary relations on the set $A = \{a, b, c\}$:
$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$ and
$R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)\}$.
Then
(a) R_1 is not symmetric but it is transitive
(b) both R_1 and R_2 are transitive
(c) R_2 is symmetric but it is not transitive
(d) both R_1 and R_2 are not symmetric *(Online 2018)*

3. Let $f : A \to B$ be a function defined as $f(x) = \frac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$. Then f is
(a) Invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$
(b) Not invertible
(c) Invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$
(d) Invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$ *(Online 2018)*

4. Let N denote the set of all natural numbers. Define two binary relations on N as
$R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$
and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then :
(a) Both R_1 and R_2 are symmetric relations
(b) Range of R_1 is $\{2, 4, 8\}$
(c) Both R_1 and R_2 are transitive relations
(d) Range of R_2 is $\{1, 2, 3, 4\}$ *(Online 2018)*

5. The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is
(a) injective but not surjective
(b) surjective but not injective
(c) neither injective nor surjective
(d) invertible *(2017)*

6. Let $f(x) = 2^{10} \cdot x + 1$ and $g(x) = 3^{10} \cdot x - 1$. If $(fog)(x) = x$, then x is equal to
(a) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (b) $\frac{2^{10}-1}{2^{10}-3^{-10}}$
(c) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$ (d) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$ *(Online 2017)*

7. The function $f : N \to N$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$ where N is the set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x, is
(a) one-one and onto.
(b) onto but not one-one.
(c) neither one-one nor onto.
(d) one-one but not onto. *(Online 2017)*

8. If - . +7 $\left(\frac{6}{\ }\right)$ =8 1 ≠51 mzp
$S = \{x \in R : f(x) = f(-x)\}$; then S
(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements *(2016)*

9. For $x \in R, x \ne 0, x \ne 1$, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2$ .... Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to
(a) $\frac{8}{3}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{1}{3}$
(Online 2016)

10. In a certain town, 25% of the families own a phone and 15% own a car, 65% families own neither a phone nor a car and 2000 families own both a car and a phone. Consider the following three statements :
(1) 5% families own both a car and a phone.
(2) 35% families own either a car or a phone.
(3) 40,000 families live in the town.

Then
(a) only (1) and (2) are correct
(b) only (1) and (3) are correct
(c) only (2) and (3) are correct
(d) all (1), (2) and (3) are correct *(Online 2015)*

11. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is
(a) 211 (b) 256 (c) 220 (d) 219 *(2013)*

12. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is
(a) 2^5 (b) 5^3 (c) 5^2 (d) 3^5 *(2012)*

13. Let R be the set of real numbers.
Statement-1 : $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.
Statement-2 : $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

14. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is
(a) $(-\infty, 0)$ (b) $(-\infty, \infty) - \{0\}$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$ *(2011)*

15. Consider the following relations:
$R = \{(x, y) | x, y$ are real numbers and $x = wy$ for some rational number $w\}$;
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that}\right.$
n, q ≠ 0 and qm = pn}.
Then
(a) R is an equivalence relation but S is not an equivalence relation
(b) neither R nor S is an equivalence relation
(c) S is an equivalence relation but R is not an equivalence relation
(d) R and S both are equivalence relations *(2010)*

16. Let $f(x) = (x + 1)^2 - 1, x \geq -1$.
Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.
Statement-2 : f is bijection.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2009)*

17. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then
(a) $A = C$ (b) $B = C$
(c) $A \cap B = \phi$ (d) $A = B$ *(2009)*

18. For real x, let $f(x) = x^3 + 5x + 1$, then
(a) f is onto R but not one-one
(b) f is one-one and onto R
(c) f is neither one-one nor onto R
(d) f is one-one but not onto R *(2009)*

19. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y) : x - y \text{ is an integer}\}$.
Which one of the following is true?
(a) T is an equivalence relation on R but S is not
(b) Neither S nor T is an equivalence relation on R
(c) Both S and T are equivalence relations on R
(d) S is an equivalence relation on R but T is not *(2008)*

20. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where
$Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Show that f is invertible and its inverse is
(a) $g(y) = \dfrac{y-3}{4}$ (b) $g(y) = \dfrac{3y+4}{3}$
(c) $g(y) = 4 + \dfrac{y+3}{4}$ (d) $g(y) = \dfrac{y+3}{4}$ *(2008)*

21. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is
(a) $\dfrac{12!}{(4!)^3}$ (b) $\dfrac{12!}{(4!)^4}$ (c) $\dfrac{12!}{3!(4!)^3}$ (d) $\dfrac{12!}{3!(4!)^4}$ *(2007)*

22. Let W denote the words in the English dictionary. Define the relation R by :
$R = \{(x, y) \in W \times W |$ the words x and y have at least one letter in common$\}$.
Then R is
(a) not reflexive, symmetric and transitive
(b) reflexive, symmetric and not transitive
(c) reflexive, symmetric and transitive
(d) reflexive, not symmetric and transitive. *(2006)*

23. Let $R = \{(3, 3) (6, 6) (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is
(a) reflexive and symmetric only
(b) an equivalence relation
(c) reflexive only
(d) reflexive and transitive only *(2005)*

24. Let $f : (-1, 1) \to B$, be a function defined by

$$f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right),$$

then f is both one-one and onto when B is the interval

(a) $\left[0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ *(2005)*

25. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

	Interval	**Function**
(a)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(b)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(c)	$(-\infty, -4]$	$x^3 + 6x^2 + 6$
(d)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$

(2005)

26. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where a is a given constant and $f(0) = 1$. $f(2a - x)$ is equal to

(a) $f(x)$ (b) $-f(x)$

(c) $f(-x)$ (d) $f(a) + f(a - x)$ *(2005)*

27. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

(a) not symmetric (b) transitive

(c) a function (d) reflexive *(2004)*

28. The range of the function $F(x) = {}^{7-x}P_{x-3}$ is

(a) $\{1, 2, 3, 4\}$ (b) $\{1, 2, 3, 4, 5, 6\}$

(c) $\{1, 2, 3\}$ (d) $\{1, 2, 3, 4, 5\}$. *(2004)*

29. If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

(a) $[0, 1]$ (b) $[-1, 1]$

(c) $[0, 3]$ (d) $[-1, 3]$. *(2004)*

30. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

(a) $f(x) = f(-x)$

(b) $f(2 + x) = f(2 - x)$

(c) $f(x + 2) = f(x - 2)$

(d) $f(x) = -f(-x)$ *(2004)*

31. The domain of the function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) $[1, 2]$ (b) $[2, 3)$

(c) $[2, 3]$ (d) $[1, 2)$ *(2004)*

32. The function $f(x) = \log(x + \sqrt{x^2 + 1})$ is

(a) an odd function

(b) a periodic function

(c) neither an even nor an odd function

(d) an even function *(2003)*

33. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$ is

(a) onto but not one-one

(b) one-one and onto both

(c) neither one-one nor onto

(d) one-one but not onto *(2003)*

34. Domain of definition of the function

$$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x), \text{ is}$$

(a) $(-1, 0) \cup (1, 2)$

(b) $(1, 2) \cup (2, \infty)$

(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (d) $(1, 2)$ *(2003)*

35. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7(n+1)}{2}$ (b) $7n(n + 1)$

(c) $\frac{7n(n+1)}{2}$ (d) $\frac{7n}{2}$. *(2003)*

36. Which one is not periodic?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$. *(2002)*

37. The period of $\sin^2\theta$ is

(a) π^2 (b) π (c) π^3 (d) $\pi/2$. *(2002)*

38. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

(a) $[1, 9]$ (b) $[-1, 9]$

(c) $[-9, 1]$ (d) $[-9, -1]$. *(2002)*

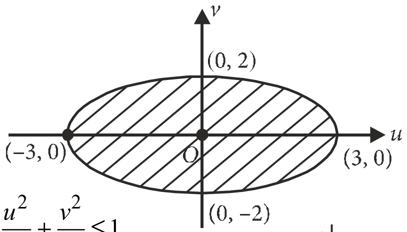
ANSWER KEY

1. (c)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (d)	**11.** (d)	**12.** (d)
13. (a)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (a)	**21.** (a)	**22.** (b)	**23.** (d)	**24.** (c)
25. (d)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (d)	**30.** (c)	**31.** (b)	**32.** (a)	**33.** (b)	**34.** (c)	**35.** (c)	**36.** (b)
37. (b)	**38.** (a)										

Explanations

1. **(c) :** Let's effect a change of origin $a - 6 = u$ and $b - 5 = v$

The set B becomes which is the interior of the ellipse.

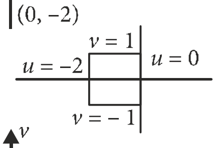


Now the set A becomes $\dfrac{u^2}{9} + \dfrac{v^2}{4} \le 1$

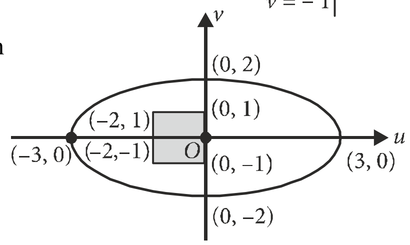
$|u + 1| < 1$ *i.e.* $-2 < u < 0$

$|v| < 1$ *i.e.* $-1 < v < 1$

which is a square.



Drawing the two diagram together, we have



So $A \subset B$ as all the four points $(0, 1)$, $(0, -1)$, $(-2, -1)$ and $(-2, 1)$ lie inside the ellipse.

2. **(c) :** R_2 is symmetric but R_2 is not transitive as $(c, a), (a, b) \in R_2$ but $(c, b) \notin R_2$.

3. **(a) :** Here $f(x) : A \to B$ such that $f(x) = \dfrac{x-1}{x-2}$ where $A = R - \{2\}$ and $B = R - \{1\}$ (Given)

Consider any $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$\Rightarrow \dfrac{x_1 - 1}{x_1 - 2} = \dfrac{x_2 - 1}{x_2 - 2} \Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$

$\Rightarrow x_1 = x_2 \quad \therefore f$ is one-one.

Consider any $y \in B \;\therefore\; y = \dfrac{x-1}{x-2} \;\Rightarrow\; x = \dfrac{2y-1}{y-1} \in R - \{1\}$

Also, $\dfrac{2y-1}{y-1} \ne 2$. Since if $\dfrac{2y-1}{y-1} = 2$, then $1 = 2$, which is not possible

$\Rightarrow$ Every element in the co-domain of f has its pre-image in the domain of f $\Rightarrow$ Function f is onto.

Hence, function is bijective and invertible.

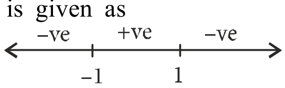
So, $f^{-1}(y) = x \;\Rightarrow\; f^{-1}(y) = \dfrac{2y-1}{y-1}$

4. **(d) :** Here, $R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$ and $R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$ $\therefore$ Range of $R_2 = \{1, 2, 3, 4\}$

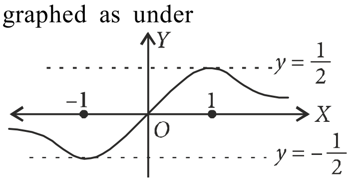
5. **(b) :** We have $f(x) = \dfrac{x}{1+x^2}$

$\therefore \quad f'(x) = \dfrac{(1+x^2)\cdot 1 - x\cdot 2x}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2} = \dfrac{(1-x)(1+x)}{(1+x^2)^2}$

The sign of $f'(x)$ is given as



Now f can be graphed as under



Clearly function is surjective but not injective, as a horizontal line meet the curve in two points.

6. **(d) :** We have, $f(g(x)) = x$

$f(3^{10}x - 1) = 2^{10}(3^{10}\cdot x - 1) + 1 = x$

$\Rightarrow \; 2^{10}\cdot 3^{10}x - 2^{10} + 1 = x \Rightarrow \; x(2^{10}3^{10} - 1) = 2^{10} - 1$

$\Rightarrow \quad x = \dfrac{2^{10} - 1}{2^{10}3^{10} - 1} \Rightarrow x = \dfrac{2^{10}(1 - 2^{-10})}{2^{10}(3^{10} - 2^{-10})} \quad \Rightarrow \quad x = \dfrac{1 - 2^{-10}}{3^{10} - 2^{-10}}$

7. **(c)**

8. **(c) :** Change x to $1/x$ in the equation

- . +7 $\left(\dfrac{6}{\;}\right)$ = 8 1 ≠ 5 to obtain $\left(\dfrac{6}{\;}\right)$ + 7 - . = $\dfrac{8}{\;}$

Eliminating $\left(\dfrac{6}{\;}\right)$ between these two equations, we get

8 - . = $\dfrac{;}{\;}$ − 8 33 - . = $\dfrac{7}{\;}$ −

Now to get the elements of S, we solve $f(x) = f(-x)$

$\Rightarrow$ $\dfrac{7}{\;}$ − = − $\dfrac{7}{\;}$ + $\Rightarrow$ $\dfrac{7}{\;}$ − = 5 $\Rightarrow$ ^{7} = 7 $\therefore$ = $\pm\sqrt{7}$

9. **(c) :** We have,

$f_1(x) = f_{0+1}(x) = f_0(f_0(x)) = \dfrac{1}{1 - \dfrac{1}{1-x}} = \dfrac{x-1}{x}$

Similarly, $f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = \dfrac{1}{1 - \dfrac{x-1}{x}} = x$

and $f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = f_0(x) = \dfrac{1}{1-x}$

and $f_4(x) = f_{3+1}(x) = f_0(f_3(x)) = f_0(f_0(x)) = \dfrac{x-1}{x}$

$\therefore \quad f_0 = f_3 = f_6 = \ldots.. = \dfrac{1}{1-x}$

and $f_1 = f_4 = f_7 = f_{10} = \ldots.. = \dfrac{x-1}{x}$ and $f_2 = f_5 = f_8 = \ldots.. = x$

So, $f_{100}(3) = \dfrac{3-1}{3} = \dfrac{2}{3}$, $f_1\left(\dfrac{2}{3}\right) = \dfrac{\dfrac{2}{3} - 1}{\dfrac{2}{3}} = -\dfrac{1}{2}$ and $f_2\left(\dfrac{3}{2}\right) = \dfrac{3}{2}$

$\therefore \quad f_{100}(3) + f_1\left(\dfrac{2}{3}\right) + f_2\left(\dfrac{3}{2}\right) = \dfrac{5}{3}$

10. **(d) :** Let set P be the families who own a phone and set C be the families who own a car.

$n(P) = 25\%$, $n(C) = 15\%$,

$n(P' \cup C') = 65\%$ and $n(P \cup C) = 35\%$

Now, $n(P \cap C) = n(P) + n(C) - n(P \cup C) = 25 + 15 - 35 = 5\%$

$\Rightarrow \quad x \times 5\% = 2000 \quad \Rightarrow \quad x = 40{,}000$

11. **(d) :** $A \times B$ will have $2 \times 4 = 8$ elements.

The number of subsets having atleast 3 elements

$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$

$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2) = 256 - 1 - 8 - 28 = 219$

12. (d) : $X = \{1, 2, 3, 4, 5\}$; $Y \subseteq X$, $Z \subseteq X$, $Y \cap Z = \phi$
Number of ways $= 3^5$.

13. (a) : $y - x$ = integer and $z - y$ = integer $\Rightarrow$ $z - x$ = integer
$\therefore$ $(x, y) \in A$ and $(y, z) \in A \Rightarrow (x, z) \Rightarrow$ Transitive
Also $(x, x) \in A$ is true $\Rightarrow$ Reflexive
As $(x, y) \in A \Rightarrow (y, x) \Rightarrow$ Symmetric
Hence A is an equivalence relation but B is not.
$(0, y)$ is in B but $(y, 0)$ is not in B.

14. (a) : $f(x) = \dfrac{1}{\sqrt{|x| - x}}$
$|x| - x > 0 \Rightarrow |x| > x$ Thus x must be –ve. $\therefore$ $x \in (-\infty, 0)$.

15. (c) : We have $(x, x) \in R$ for $w = 1$ implying that R is reflexive. For $a \neq 0$, $(a, 0) \notin R$ for any w but $(0, a) \in R$. Thus R is not symmetric. Hence R is not an equivalence relation.
As $\left(\dfrac{m}{n}, \dfrac{m}{n}\right) \in S$ since $mn = mn$, S is reflexive.
$\left(\dfrac{m}{n}, \dfrac{p}{q}\right) \in S \Rightarrow qm = pn$
But this can be written as $np = mq$, giving $\left(\dfrac{p}{q}, \dfrac{m}{n}\right) \in S$. Thus S is symmetric.
Again, $\left(\dfrac{m}{n}, \dfrac{p}{q}\right) \in S$ and $\left(\dfrac{p}{q}, \dfrac{a}{b}\right) \in S$
means $qm = pn$ and $bp = aq$.
i.e. $\dfrac{m}{n} = \dfrac{p}{q}$ and $\dfrac{p}{q} = \dfrac{a}{b}$. *i.e.* $\dfrac{m}{n} = \dfrac{a}{b}$ Thus $\left(\dfrac{m}{n}, \dfrac{a}{b}\right) \in S$
This means S is transitive.

16. (b) : The solution of $f(x) = f^{-1}(x)$ are given by
$f(x) = x$, which gives $(x + 1)^2 - 1 = x$
$\Rightarrow$ $(x + 1)^2 - (x + 1) = 0 \Rightarrow (x + 1)x = 0 \therefore x = -1, 0$
But as no co-domain of f is specified, nothing can be said about f being ONTO or not.

17. (b) : Let $x \in C$
Suppose $x \in A \Rightarrow x \in A \cap C$
$\Rightarrow$ $x \in A \cap B$ $(\because A \cap C = A \cap B)$ Thus $x \in B$
Again suppose $x \notin A \Rightarrow x \in C \cup A$
$\Rightarrow$ $x \in B \cup A \Rightarrow x \in B$
Thus in both cases $x \in C \Rightarrow x \in B$
Hence $C \subseteq B$ ....(1)
Similarly we can show that $B \subseteq C$ ....(2)
Combining (1) and (2) we get $B = C$.

18. (b) : The function is $f : R \rightarrow R$
$f(x) = x^3 + 5x + 1$
Let $y \in R$ then $y = x^3 + 5x + 1 \Rightarrow x^3 + 5x + 1 - y = 0$
As a polynomial of odd degree has always at least one real root, corresponding to any $y \in$ co-domain there $\exists$ some $x \in$ domain such that $f(x) = y$. Hence f is ONTO.
Also f is continuous on R, because it is a polynomial function $f'(x) = 3x^2 + 5 > 0$
$\therefore$ f is strictly increasing. Hence f is one-one also.

19. (a) : To be an equivalence relation the relation must be all – reflexive, symmetric and transitive.
$T = \{(x, y) : x - y \in Z\}$ is
reflexive – for $(x, x) \in Z$ i.e. $x - x = 0 \in Z$
symmetric – for $(x, y) \in Z \Rightarrow x - y \in Z$
$\Rightarrow$ $y - x \in Z$ i.e. $(y, x) \in Z$
transitive – for $(x, y) \in Z$ and $(y, w) \in Z$
$\Rightarrow$ $x - y \in Z$ and $y - w \in Z$, giving $x - w \in Z$ i.e. $(x, w) \in Z$.
$\therefore$ T is an equivalence relation on R.
$S = \{(x, y) : y = x + 1, 0 < x < 2\}$ is not reflexive for $(x, x) \in S$ would imply $x = x + 1$
$\Rightarrow$ $0 = 1$ (impossible). Thus S is not an equivalence relation

20. (a) : Let $f(x_1) = f(x_2)$, $x_1, x_2 \in N$
$\Rightarrow$ $4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$
Thus $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence the function is one-one.
Let $y \in Y$ be a number of the form $y = 4k + 3$, for some $k \in N$, then $y = f(x) \Rightarrow 4k + 3 = 4x + 3 \Rightarrow x = k \in N$
Thus corresponding to any $y \in Y$ we have $x \in N$. The function then is onto.
The function, being both one-one and onto is invertible.
$y = 4x + 3 \Rightarrow x = \dfrac{y-3}{4}$ $\therefore$ $f^{-1}(x) = \dfrac{x-3}{4}$
or $g(y) = \dfrac{y-3}{4}$ is the inverse of the function.

21. (a) : Number of ways $= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4 = \dfrac{12!}{(4!)^3}$.

22. (b) : Given relation R such that
$R = \{(x, y) \in W \times W \mid$ the word x and y have atleast one letter in common$\}$
where W denotes set of words in English dictionary
Clearly $(x, x) \in R \ \forall \ x \in W$
$\therefore$ (x, x) has every letter common $\therefore$ R is reflexive
Let $(x, y) \in R$ then $(y, x) \in R$ as x and y have atleast one letter in common. $\Rightarrow R$ is symmetric.
But R is not transitive
$\therefore$ Let x = DON, y = NEST, z = SHE
then $(x, y) \in R$ and $(y, z) \in R$. But $(x, z) \notin R$.
$\therefore$ R is reflexive, symmetric but not transitive.

23. (d) : For $(3, 9) \in R$, $(9, 3) \notin R$
$\therefore$ relation is not symmetric which means our choice (a) and (b) are out of court. We need to prove reflexivity and transitivity.
For reflexivity $a \in R$, $(a, a) \in R$ which is hold *i.e.* R is reflexive.
Again, for transitivity of $(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R$ which is also true in $R = \{(3, 3)(6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$.

24. (c) : For $x \in (-1, 1)$, we have $f(x) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$
$\therefore$ $f(\tan\theta) = \tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)$ (By $x = \tan\theta$)
$= \tan^{-1}(\tan 2\theta) \Rightarrow f(x) = 2\tan^{-1}x$
$\Rightarrow -\dfrac{\pi}{2} < \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) < \dfrac{\pi}{2}$.

25. (d) : $f(x) = x^3 + 6x^2 + 6 \Rightarrow f'(x) = 3x^2 + 12x = 3x(x+4)$
$\Rightarrow$ $f'(x) > 0 \Rightarrow x < -4 \cup x > 0$
the interval $x < -4$ *i.e* $(-\infty, -4]$ is matched correctly
and after checking others we find that $f(x) = 3x^2 - 2x + 1$
$\Rightarrow f'(x) > 0$ for $x > 1/3$ which is not given in the choice.

26. (b) : Given $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ ...(*)
let $x = 0 = y$
$f(0) = (f(0))^2 - (f(a))^2$
$1 = 1 - (f(a))^2 \Rightarrow f(a) = 0$
$\therefore$ $f(2a - x) = f(a - (x - a)) = f(a)f(x - a) - f(a + x - a)f(0)$
By using (*), we get
$f(2a - x) = 0 - f(x)(1) = -f(x)$ $(\because f(a) = 0, f(0) = 1)$

27. (a) : R is a function as $A = \{1, 2, 3, 4\}$ and $(2, 4) \in R$ and $(2, 3) \in R$
R is not reflexive as $(1, 1) \notin R$
R is not symmetric as $(2, 3) \in R$ but $(3, 2) \notin R$
R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$.

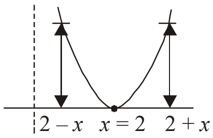
28. (c) : $F(x)$ to be defined for $x \in N$.
(i) $\therefore\ 7 - x > 0 \Rightarrow x < 7$ (ii) $x - 3 \geq 0 \Rightarrow x \geq 3$
(iii) $x - 3 \leq 7 - x \Rightarrow x \leq 5$
$\therefore$ from (i), (ii), (iii) $x = 3, 4, 5$
$\therefore$ $F(3) = {}^4P_0,\ F(4) = {}^3P_1,\ F(5) = {}^2P_2$
$\therefore$ $\{1, 2, 3\}$ is required range

29. (d): Let $f(x) = g(x) + 1$

where $g(x) = 2\left[\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right] = 2\sin(x - 60°)$

$\therefore$ $-2 \leq 2\sin(x - 60°) \leq 2 - 1 \leq 2\sin(x - 60°) + 1 \leq 3$

30. (c): If $y = f(x)$ is symmetrical about the line $x = \alpha$ then
$f(x + \alpha) = f(x - \alpha)$
$\therefore$ $f(x + 2) = f(x - 2)$



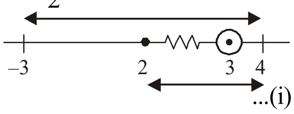
31. (b) : $f(x) = \frac{p(x)}{q(x)}$ (say)

then Domain of $f(x)$ is $D_f\, p(x) \cap D_f\, q(x),\ q(x) \neq 0$

now D_f of $p(x)$ is $-\frac{\pi}{2} \leq \sin^{-1}(x - 3) \leq \frac{\pi}{2}$

$\Rightarrow -\sin\frac{\pi}{2} \leq x - 3 \leq \sin\frac{\pi}{2}$

$\Rightarrow\ 2 \leq x \leq 4$



...(i)

Again $9 - x^2 > 0 \Rightarrow x^2 < 9$
$|x| < 3$
i.e. $-3 < x < 3$ ...(ii)
From (i) and (ii), we have $\therefore\ 2 \leq x < 3$

32. (a) : $f(x) = \log\left[\sqrt{x^2 + 1} + x\right]$

$\therefore\ f(-x) = \log\left[\sqrt{1 + x^2} - x\right]$

$= -\log\left[\frac{1}{\sqrt{1 + x^2} - x}\right] = -\log\left[\frac{\sqrt{1 + x^2} + x}{1}\right]$

$= -f(x) \Rightarrow f(x) + f(-x) = 0 \Rightarrow f(x)$ is an odd function.

33. (b) : If n is odd, let $n = 2k - 1$
Let $f(2k_1 - 1) = f(2k_2 - 1)$

$\Rightarrow \frac{2k_1 - 1 - 1}{2} = \frac{2k_2 - 1 - 1}{2} \Rightarrow k_1 = k_2$

$\Rightarrow f(n)$ is one-one functions if n is odd
Again, If $n = 2k$ (i.e. n is even)
Let $f(2k_1) = f(2k_2)$

$\Rightarrow -\frac{2k_1}{2} = -\frac{2k_2}{2} \Rightarrow k_1 = k_2 \Rightarrow f(n)$ is one-one if n is even

Again $f(n) = \frac{n - 1}{2}$; $f'(n) = \frac{1}{2} > 0\ n \in N$ if n is odd

and $f'(n) = \frac{-1}{2} < 0\ n \in N$ if n is even

Now all such functions which are either increasing or decreasing in the stated domain are said to be onto function. Finally $f(n)$ is one-one onto function.

34. (c) : Let $g(x) = \frac{3}{4 - x^2}$ $\therefore\ x \neq \pm 2$

$\therefore$ $D_f\, g(x) = R - \{-2, 2\}$
$h(x) = \log_{10}(x^3 - x)$ $\therefore\ x^3 - x > 0$
$x(x + 1)(x - 1) > 0$

$-\quad +\quad -\quad +$
$-1\quad 0\quad 1$ $\therefore\ x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Domain of $f(x)$ is $(-1, 0) \cup (1, 2) \cup (2, \infty)$

35. (c) : Let $x = 0 = y \Rightarrow f(0) = 0$
and $x = 1, y = 0 \Rightarrow f(1 + 0) = f(1) + f(0) = 7$ (given)
$x = 1, y = 1 \Rightarrow f(1 + 1) = 2f(1) = 2(7)$
$\Rightarrow f(2) = 2(7)$
$x = 1, y = 2$ $\therefore$ $f(3) = f(1) + f(2) = 7 + 2(7) = 3(7)$
and so on.

$\therefore \sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + ... + f(n)$

$= 7(1 + 2 + 3 + ...+ n) = \frac{7n(n+1)}{2}$

36. (b) : Period of $|\sin 3x|$ is $\frac{\pi}{3}$ and period of $\sin^2 x$ is π

(a) Same as the period of $|\sin x|$ or $\frac{1 - \cos 2x}{2}$ whose period is π

Now period of $|\sin 3x| + \sin^2 x$ is the L.C.M of their periods

$\therefore$ L.C.M of $\left(\frac{\pi}{3}, \pi\right) = \frac{\text{LCM }(\pi,\pi)}{\text{HCF }(3,1)} = \pi$

(c, d) Similarly we can say that $\cos 4x + \tan^2 x$ and $\cos 2x + \sin x$ are periodic function.

(b) Now $\cos^2 x$ is periodic with period π and for period of $\cos\sqrt{x}$ let us take.
$f(x) = \cos\sqrt{x}$ Let $f(x + T) = f(x)$
$\Rightarrow \cos\sqrt{T + x} = \cos\sqrt{x} \Rightarrow \sqrt{T + x} = 2n\pi \pm \sqrt{x}$
which gives no value of T independent of x
$\therefore f(x)$ cannot be periodic
Now say $g(x) = \cos^2 x + \cos\sqrt{x}$ which is sum of a periodic and non periodic function and such function have no period.
So, $\cos\sqrt{x} + \cos^2 x$ is non periodic function.

37. (b) : Let $f(\theta) = \sin^2\theta = |\sin\theta|$ Period of $|\sin\theta|$ is π

38. (a) : If $y = \sin^{-1}a$, then $-1 \leq a \leq 1$

$\therefore -1 \leq \log_3\left(\frac{x}{3}\right) \leq 1$ $\left[\text{as } y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]\right]$

$\Rightarrow \frac{1}{3} \leq \frac{x}{3} \leq 3^1 \Rightarrow 1 \leq x \leq 9$