CHAPTER 10

Oscillations and Waves

1. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} s^{-1}. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avogadro number = 6.02×10^{23} gm mole^{-1})
(a) 6.4 N m^{-1} (b) 7.1 N m^{-1}
(c) 2.2 N m^{-1} (d) 5.5 N m^{-1} *(2018)*

2. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg m^{-3} and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?
(a) 5 kHz (b) 2.5 kHz (c) 10 kHz (d) 7.5 kHz *(2018)*

3. A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe ?
(Speed of sound in air is 340 m s^{-1})
(a) 190 cm (b) 180 cm (c) 200 cm (d) 220 cm *(Online 2018)*

4. 5 beats/second are heard when a tuning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95 m or 1 m. The frequency of the fork will be
(a) 251 Hz (b) 300 Hz (c) 195 Hz (d) 150 Hz *(Online 2018)*

5. Two simple harmonic motions, as shown here, are at right angles. They are combined to form Lissajous figures.
$x(t) = A \sin (at + \delta)$
$y(t) = B \sin (bt)$
Identify the correct match below.

Parameters	**Curve**
(a) $A = B, a = b; \delta = \pi/2$	Line
(b) $A \neq B, a = b; \delta = 0$	Parabola
(c) $A = B, a = 2b; \delta = \pi/2$	Circle
(d) $A \neq B, a = b; \delta = \pi/2$	Ellipse

(Online 2018)

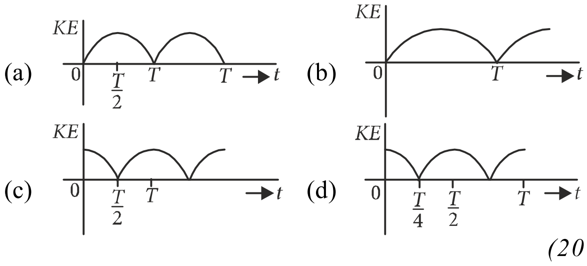
6. An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2}k(x-X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is ($M = 10, m = 5, u = 1, k = 1$)
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{3}$ (c) $\sqrt{\frac{3}{5}}$ (d) $\frac{1}{2}$ *(Online 2018)*

7. Two sitar strings, A and B, playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease by 3 Hz. If the frequency of A is 425 Hz, the original frequency of B is
(a) 428 Hz (b) 430 Hz (c) 420 Hz (d) 422 Hz *(Online 2018)*

8. A particle executes simple harmonic motion and is located at $x = a$, b and c at times t_0, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is
(a) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$ (b) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$
(c) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+2b}{3c}\right)$ (d) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$ *(Online 2018)*

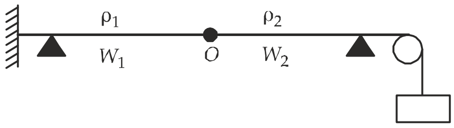
9. The end correction of a resonance column is 1 cm. If the shortest length resonating with the tuning fork is 10 cm, the next resonating length should be
(a) 36 cm (b) 40 cm (c) 28 cm (d) 32 cm *(Online 2018)*

10. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like



(2017)

11. Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary wave is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is



(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 4 : 1

(Online 2017)

12. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is

(a) $\frac{1}{2\sqrt{2}}$ Hz (b) $\frac{1}{2}$ Hz
(c) 2 Hz (d) $\frac{1}{4}$ Hz

(Online 2017)

13. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is 10 s^{-1}. At, $t = 0$ the displacement is 5 m. What is the maximum acceleration? The initial phase is $\frac{\pi}{4}$.

(a) $500\sqrt{2}$ m/s^2 (b) 500 m/s^2
(c) $750\sqrt{2}$ m/s^2 (d) 750 m/s^2

(Online 2017)

14. A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by

$$y(x, t) = 0.5 \sin\left(\frac{5\pi}{4}x\right)\cos(200\,\pi t).$$

What is the speed of the travelling wave moving in the positive x direction?
(x and t are in meter and second, respectively.)

(a) 180 m/s (b) 160 m/s
(c) 120 m/s (d) 90 m/s

(Online 2017)

15. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 N m^{-1} and oscillates in a damping medium of damping constant 10^{-2} kg s^{-1}. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to

(a) 2 s (b) 3.5 s
(c) 7 s (d) 5 s *(Online 2017)*

16. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{7W}{8}$ from equilibrium position. The new amplitude of the motion is

(a) $\frac{W}{8}\sqrt{96}$ (b) $3A$ (c) $W\sqrt{8}$ (d) $\frac{<W}{8}$

(2016)

17. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (take g = 10 m s^{-2})

(a) $7\pi\sqrt{7}$ – (b) 2 s (c) $7\sqrt{7}$ – (d) $\sqrt{7}$ –

(2016)

18. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

(a) $\frac{}{7}$ (b) $\frac{8}{9}$ (c) $2f$ (d) f

(2016, 2012)

19. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time $t = 0$ one particle has displacement A while the other one has displacement $-\frac{A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is

(a) $\frac{5T}{6}$ (b) $\frac{T}{3}$ (c) $\frac{T}{4}$ (d) $\frac{T}{6}$

(Online 2016)

20. Two engines pass each other moving in opposite directions with uniform speed of 30 m s^{-1}. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 m s^{-1}.

(a) 450 Hz (b) 540 Hz
(c) 270 Hz (d) 648 Hz *(Online 2016)*

21. In an engine the piston undergoes vertical simple harmonic motion with amplitude 7 cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close to

(a) 0.7 Hz (b) 1.9 Hz
(c) 1.2 Hz (d) 0.1 Hz *(Online 2016)*

22. A toy-car, blowing its horn, is moving with a steady speed of 5 m/s, away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is 340 m/s, the frequency of the horn of the toy car is close to

(a) 680 Hz (b) 510 Hz
(c) 340 Hz (d) 170 Hz

(Online 2016)

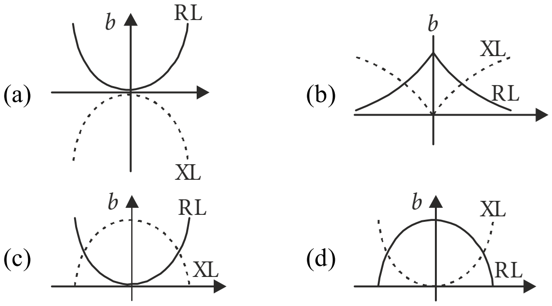
23. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{6}{v}$ is equal to
(g = gravitational acceleration)

(a) $\left[6-\left(\frac{q_j}{q}\right)^7\right]\frac{W}{j}$ (b) $\left[6-\left(\frac{q}{q_j}\right)^7\right]\frac{W}{j}$

(c) $\left[\left(\frac{q_j}{q}\right)^7-6\right]\frac{W}{j}$ (d) $\left[\left(\frac{q_j}{q}\right)^7-6\right]\frac{j}{W}$

(2015)

24. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale)

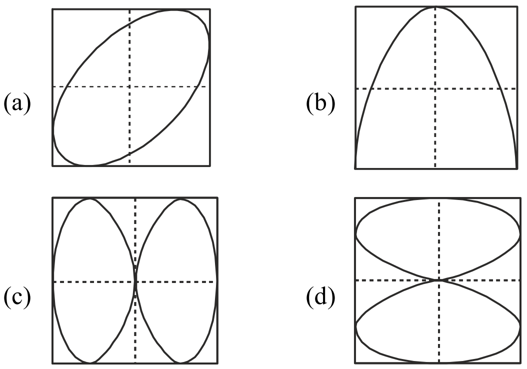


(2015)

25. A train is moving on a straight track with speed 20 m s^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 m s^{-1}) close to

(a) 18% (b) 24% (c) 6% (d) 12% *(2015)*

26. x and y displacements of a particle are given as $x(t) = a\sin\omega t$ and $y(t) = a\sin 2\omega t$. Its trajectory will look like



(Online 2015)

27. A simple harmonic oscillator of angular frequency 2 rad s^{-1} is acted upon by an external force $F = \sin t$ N. If the oscillator is at rest in its equilibrium position at $t = 0$, its position at later times is proportional to

(a) -uz $+\frac{6}{7}$-uz 7 (b) -uz $+\frac{6}{7}$o{ −7

(c) o{ − $-\frac{6}{7}$-uz 7 (d) -uz $-\frac{6}{7}$-uz 7

(Online 2015)

28. A bat moving at 10 m s^{-1} towards a wall sends a sound signal of 8000 Hz towards it. On reflection it hears a sound of frequency f. The value of f in Hz is close to (speed of sound = 320 m s^{-1})

(a) 8258 (b) 8516 (c) 8000 (d) 8424

(Online 2015)

29. A cylindrical block of wood (density = 650 kg m^{-3}), of base area 30 cm^2 and height 54 cm, floats in a liquid of density 900 kg m^{-3}. The block is depressed slightly and then released. The time period of the resulting oscillations of the block would be equal to that of a simple pendulum of length (nearly)

(a) 65 cm (b) 52 cm (c) 39 cm (d) 26 cm

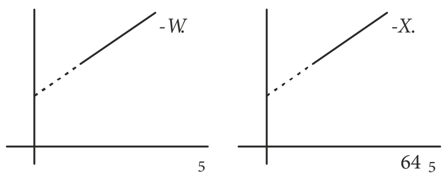
(Online 2015)

30. A pendulum with time period of 1 s is losing energy due to damping. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s^{-1}) is

(a) $\frac{6}{85}$ xz 8 (b) $\frac{6}{6:}$ xz 8 (c) 2 (d) $\frac{6}{7}$

(Online 2015)

31. A source of sound emits sound waves at frequency f_0. It is moving towards an observer with fixed speed v_s($v_s < v$, where v is the speed of sound in air). If the observer were to move towards the source with speed v_0, one of the following two graphs (A and B) will give the correct variation of the frequency f heard by the observer as v_0 is changed.



The variation of f with v_0 is given correctly by

(a) s~m|t W ψt -x{ | qD $\frac{5}{- \; - \; .}$

(b) s~m|t W ψt -x{ | qD $\frac{5}{- \; + \; .}$

(c) s~m|t X ψt -x{ | qD $\frac{5}{- \; - \; .}$

(d) s~m|t X ψt -x{ | qD $\frac{5}{- \; + \; .}$ *(Online 2015)*

32. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$, in same direction, then

(a) time period of oscillations is 6τ
(b) amplitude of motion is $3a$
(c) time period of oscillations is 8τ
(d) amplitude of motion is $4a$ *(2014)*

33. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m s^{-1}.
(a) 4 (b) 12 (c) 8 (d) 6 *(2014)*

34. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5 s. In another 10 s it will decrease to α times its original magnitude where α equals
(a) 0.6 (b) 0.7 (c) 0.81 (d) 0.729 *(2013)*

35. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency
(a) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (b) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$
(c) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (d) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ *(2013)*

36. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?
(a) 770 Hz (b) 188.5 Hz
(c) 178.2 Hz (d) 200.5 Hz *(2013)*

37. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds
(a) b (b) $\frac{1}{b}$ (c) $\frac{2}{b}$ (d) $\frac{0.693}{b}$ *(2012)*

38. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is ($X_0 + A$), the phase difference between their motion is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$ *(2011)*

39. A mass M, attached to a horizontal spring, executes SHM with an amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is
(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$
(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$ *(2011)*

40. The transverse displacement $y(x,t)$ of a wave on a string is given by $y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}$
This represents a
(a) wave moving in $+x$-direction with speed $\sqrt{\frac{a}{b}}$
(b) wave moving in $-x$-direction with speed $\sqrt{\frac{b}{a}}$
(c) standing wave of frequency $\sqrt{b}$
(d) standing wave of frequency $\frac{1}{\sqrt{b}}$ *(2011)*

41. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02\,(\text{m}) \sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right].$$

The tension in the string is
(a) 6.25 N (b) 4.0 N (c) 12.5 N (d) 0.5 N *(2010)*

42. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?
(a) $a^2T^2 + 4\pi^2v^2$ (b) aT/x
(c) $aT + 2\pi v$ (d) aT/v *(2009)*

43. Three sound waves of equal amplitudes have frequencies ($\upsilon - 1$), υ, ($\upsilon + 1$). They superpose to give beats. The number of beats produced per second will be
(a) 4 (b) 3 (c) 2 (d) 1 *(2009)*

44. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest?
(Speed of sound = 330 ms^{-1}).
(a) 49 m (b) 98 m (c) 147 m (d) 196 m *(2009)*

45. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are
(a) $\alpha = 12.50\pi,\ \beta = \frac{\pi}{2.0}$ (b) $\alpha = 25.00\pi,\ \beta = \pi$
(c) $\alpha = \frac{0.08}{\pi},\ \beta = \frac{2.0}{\pi}$ (d) $\alpha = \frac{0.04}{\pi},\ \beta = \frac{1.0}{\pi}$ *(2008)*

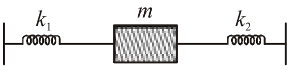
46. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)
(a) 330 m s^{-1} (b) 460 m s^{-1}
(c) 500 m s^{-1} (d) 650 m s^{-1}. *(2008)*

47. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then
(a) $A = x_0\omega^2$, $\delta = 3\pi/4$ (b) $A = x_0$, $\delta = -\pi/4$
(c) $A = x_0\omega^2$, $\delta = \pi/4$ (d) $A = x_0\omega^2$, $\delta = -\pi/4$
(2007)

48. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is
(a) 0.25 s (b) 0.5 s (c) 0.75 s (d) 0.125 s
(2007)

49. A particle of mass m executes simple harmonic motion with amplitude a and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end is
(a) $2\pi^2 m a^2 \upsilon^2$ (b) $\pi^2 m a^2 \upsilon^2$
(c) $\frac{1}{4} m a^2 \upsilon^2$ (d) $4\pi^2 m a^2 \upsilon^2$ *(2007)*

50. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $2f$ (b) $f/2$ (c) $f/4$ (d) $4f$
(2007)

51. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of
(a) 100 (b) 1000 (c) 10000 (d) 10
(2007)

52. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time
(a) at the highest position of the platform
(b) at the mean position of the platform
(c) for an amplitude of $\frac{g}{\omega^2}$
(d) for an amplitude of $\frac{g^2}{\omega^2}$. *(2006)*

53. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is
(a) 100 s (b) 0.01 s (c) 10 s (d) 0.1 s.
(2006)

54. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy become 75% of the total energy?
(a) $\frac{1}{12}$ s (b) $\frac{1}{6}$ s (c) $\frac{1}{4}$ s (d) $\frac{1}{3}$ s.
(2006)

55. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is
(a) 10.5 Hz (b) 105 Hz
(c) 1.05 Hz (d) 1050 Hz. *(2006)*

56. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v m s^{-1}. The velocity of sound in air is 300 m s^{-1}. If the person can hear frequencies upto a maximum of 10000 Hz, the maximum value of v upto which he can hear the whistle is
(a) 30 m s^{-1} (b) $15\sqrt{2}$ m s^{-1}
(c) $15/\sqrt{2}$ m s^{-1} (d) 15 m s^{-1}. *(2006)*

57. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
(a) remain unchanged
(b) increase towards a saturation value
(c) first increase and then decrease to the original value
(d) first decrease and then increase to the original value
(2005)

58. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is
(a) $2\pi\alpha$ (b) $2\pi\sqrt{\alpha}$ (c) $2\pi/\alpha$ (d) $2\pi/\sqrt{\alpha}$
(2005)

59. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is
(a) $-\pi/3$ (b) $\pi/6$ (c) $-\pi/6$ (d) $\pi/3$.
(2005)

60. The function $\sin^2(\omega t)$ represents
(a) a simple harmonic motion with a period $2\pi/\omega$
(b) a simple harmonic motion with a period π/ω
(c) a periodic, but not simple harmonic motion with a period $2\pi/\omega$
(d) a periodic, but not simple harmonic motion with a period π/ω *(2005)*

61. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
(a) 5% (b) 20% (c) zero (d) 0.5%
(2005)

62. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the

tuning forks are sounded again, 6 beats per second are heard. if the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?
(a) 196 Hz (b) 204 Hz (c) 200 Hz (d) 202 Hz
(2005)

63. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then
(a) $\omega_1 = \omega_2$
(b) $\omega_1 > \omega_2$
(c) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(d) $\omega_1 < \omega_2$ *(2004)*

64. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t$ ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to
(a) $\dfrac{m}{\omega_0^2 - \omega^2}$
(b) $\dfrac{1}{m(\omega_0^2 - \omega^2)}$
(c) $\dfrac{1}{m(\omega_0^2 + \omega^2)}$
(d) $\dfrac{m}{\omega_0^2 + \omega^2}$. *(2004)*

65. The total energy of a particle, executing simple harmonic motion is (where x is the displacement from the mean position)
(a) $\propto x$
(b) $\propto x^2$
(c) independent of x
(d) $\propto x^{1/2}$ *(2004)*

66. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then
(a) $T = t_1 + t_2$
(b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$
(d) $T^{-2} = t_1^{-2} + t_2^{-2}$. *(2004)*

67. The displacement y of a particle in a medium can be expressed as: $y = 10^{-6}\sin(100t + 20x + \pi/4)$m
where t is in second and x in meter. The speed of the wave is
(a) 2000 m/s
(b) 5 m/s
(c) 20 m/s
(d) 5π m/s. *(2004)*

68. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000$ kg/m^3. What relationship between t and t_0 is true?
(a) $t = t_0$ (b) $t = t_0/2$ (c) $t = 2t_0$ (d) $t = 4t_0$.
(2003)

69. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statement is true?
(a) K.E. is maximum when $x = 0$
(b) T.E. is zero when $x = 0$
(c) K.E. is maximum when x is maximum
(d) P.E. is maximum when $x = 0$. *(2003)*

70. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(a) 11% (b) 21% (c) 42% (d) 10%.
(2003)

71. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations, are equal, the ratio of amplitudes of A and B is
(a) $\sqrt{k_1/k_2}$
(b) k_2/k_1
(c) $\sqrt{k_2/k_1}$
(d) k_1/k_2. *(2003)*

72. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$. Then the ratio of m/M is
(a) 3/5 (b) 25/9 (c) 16/9 (d) 5/3.
(2003)

73. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was
(a) (256 + 2) Hz
(b) (256 – 2) Hz
(c) (256 – 5) Hz
(d) (256 + 5) Hz. *(2003)*

74. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency υ. The frequency υ of the alternating source is
(a) 50 Hz (b) 100 Hz (c) 200 Hz (d) 25 Hz.
(2003)

75. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is
(a) – 4 (b) 4 (c) $4\sqrt{2}$ (d) 8.
(2003)

76. The displacement y of a wave travelling in the x-direction is given by

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right) \text{ metre,}$$

where x is expressed in metre and t in second. The speed of the wave-motion, in ms^{-1} is
(a) 300 (b) 600 (c) 1200 (d) 200.
(2003)

77. A child swinging on a swing in sitting position, stands up, then the time period of the swing will
(a) increase
(b) decrease
(c) reamains same
(d) increases if the child is long and decreases if the child is short. *(2002)*

78. In a simple harmonic oscillator, at the mean position
(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum. *(2002)*

79. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be
(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$ (c) nT (d) T. *(2002)*

80. When temperature increases, the frequency of a tuning fork
(a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material. *(2002)*

81. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is
(a) 20 (b) 80 (c) 40 (d) 120. *(2002)*

82. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is
(a) $y = a\sin(\omega t + kx)$ (b) $y = -a\sin(\omega t + kx)$
(c) $y = a\sin(\omega t - kx)$ (d) $y = -a\sin(\omega t - kx)$. *(2002)*

83. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is
(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps. *(2002)*

84. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is
(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1. *(2002)*

ANSWER KEY

1. (b)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (d)	**10.** (d)	**11.** (b)	**12.** (b)
13. (a)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (d)	**21.** (b)	**22.** (d)	**23.** (c)	**24.** (d)
25. (d)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (a)	**31.** (a)	**32.** (a)	**33.** (d)	**34.** (d)	**35.** (d)	**36.** (c)
37. (c)	**38.** (b)	**39.** (d)	**40.** (b)	**41.** (a)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (*)	**47.** (a)	**48.** (b)
49. (b)	**50.** (a)	**51.** (a)	**52.** (c)	**53.** (b)	**54.** (b)	**55.** (b)	**56.** (d)	**57.** (c)	**58.** (d)	**59.** (c)	**60.** (d)
61. (b)	**62.** (a)	**63.** (a)	**64.** (b)	**65.** (c)	**66.** (b)	**67.** (b)	**68.** (c)	**69.** (a)	**70.** (d)	**71.** (c)	**72.** (c)
73. (c)	**74.** (a)	**75.** (c)	**76.** (a)	**77.** (b)	**78.** (c)	**79.** (b)	**80.** (b)	**81.** (b)	**82.** (b)	**83.** (b)	**84.** (c)

Explanations

1. **(b) :** Frequency of a particle executing SHM,

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}};\ k = 4\pi^2 \times \upsilon^2 \times m$$

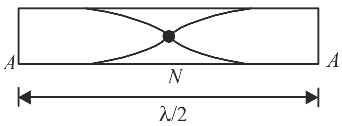
Here, $\upsilon = 10^{12}\ \text{s}^{-1}$, $m = \frac{108}{6.02\times10^{23}}\times10^{-3}$ kg , k = ?

$$\therefore\ k = 4\times(3.14)^2\times(10^{12})^2\times\frac{108\times10^{-3}}{6.02\times10^{23}} = 7.1\ \text{N m}^{-1}$$

2. **(a) :** Velocity of wave, $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}}$

$= \sqrt{3.433\times10^7} = 10^3\times\sqrt{34.33} = 5.86\times10^3\ \text{m s}^{-1}$

Since rod is clamped at the middle, shape of fundamental wave is as follows



As, $\frac{\lambda}{2} = L\ ;\ \lambda = 2L$

$L = 60\ \text{cm} = 0.6\ \text{m}$

$\therefore\ \lambda = 1.2\ \text{m}$

So fundamental frequency,

$$\upsilon = \frac{v}{\lambda} = \frac{5.86\times10^3}{1.2} = 4.88\times10^3\ \text{Hz} = 5\ \text{kHz}$$

3. **(c) :** Organ pipe will have frequency either 255 or 257 Hz. For frequency of tuning fork, 255 Hz

$$255 = \frac{3v}{2l},\ l = \frac{3\times340}{2\times255}\ \text{m} = 2\ \text{m}$$

$l = 200$ cm

4. **(c) :** Frequency of a sonometer wire $\upsilon = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}\ \therefore\ \upsilon \propto \frac{1}{L}$

As per question, $\frac{\upsilon_1}{\upsilon_2} = \frac{100}{95}$ ... (i)

Also, $\upsilon_1 - \upsilon_2 = 10$ .... (ii)

From equations (i) and (ii), $\upsilon_1 = 200$ Hz, $\upsilon_2 = 190$ Hz

Frequency of tuning fork, $\upsilon = \upsilon_1 - 5 = \upsilon_2 + 5 = 195$ Hz

5. **(d) :** For $A = B$, $a = b$ and $\delta = \frac{\pi}{2}$

$x^2 + y^2 = A^2$; Circle

For $A \neq B$, $a = b$ and $\delta = 0$

$$- = \frac{W}{X}\ \Rightarrow\ = \left(\frac{W}{X}\right)$$ B]µ-mst µxuz q

For $A = B$, $a = 2b$ and $\delta = \frac{\pi}{2}$

$x^2 + y^2 = A^2\,[\cos^2(2bt) + \sin^2 bt]$

For $A \neq B$, $a = b$ and $\delta = \frac{\pi}{2}$

$$x = A\sin\left(at + \frac{\pi}{2}\right);\ y = B\sin(at)$$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1\ ;\ \text{Ellipse}$$

6. **(a) :** In first collision momentum mu will be imparted to system. In second collision when momentum of $(M + m)$ is in opposite direction with momentum of particle mu will make its momentum zero.

On 13th collision, $\boxed{M+12m} \xleftarrow{u} \boxed{m}\quad v \longleftarrow \boxed{M+13m}$

Using momentum conservation principle, $mu = (M + 13\,m)v$

$$v = \frac{mu}{M+13m} = \frac{u}{15}$$

Also, $v = \omega A$

$$\Rightarrow\ \frac{u}{15} = \sqrt{\frac{k}{M+13m}}\times A\ \Rightarrow\ A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$$

7. **(c) :** Frequency of sitar string B is either 420 Hz or 430 Hz. As tension in string B is increased, its frequency will increase. If the frequency is 430 Hz, beat frequency will increase.
If the frequency is 420 Hz, beat frequency will decrease, hence correct answer is 420 Hz.

8. **(d) :** Different positions of a particle excuting simple harmonic motion is given by

$a = A\sin\omega t_0$, $b = A\sin2\omega t_0$, $c = A\sin3\omega t_0$

Now, $a + c = A[\sin\omega t_0 + \sin3\omega t_0] = 2A\sin2\omega t_0\cos\omega t_0$

$$\frac{a+c}{b} = 2\cos\omega t_0$$

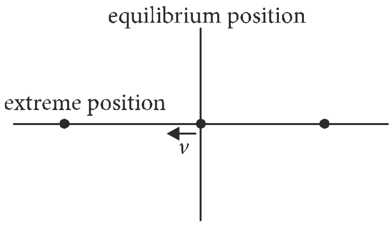
$$\omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)\ \Rightarrow\ \upsilon = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$$

9. **(d) :** Given : $e = 1$ cm

For first resonance, $\frac{\lambda}{4} = l_1 + e = 11\text{cm}$

For second resonance, $\frac{3\lambda}{4} = l_2 + e \Rightarrow l_2 = 3\times11 - 1 = 32\text{cm}$

10. **(d) :** In a simple harmonic motion, velocity of the body is maximum at the equilibrium position.



Now, time taken by a particle executing simple harmonic motion to reach extreme position (where velocity of the body is zero) from equilibrium position is $T/4$. Hence, option (d) is correct.

11. **(b) :** The fundamental frequency of a stretched string is given by, $\upsilon = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$.

Here, n = number of antinodes, μ is the mass per unit length.

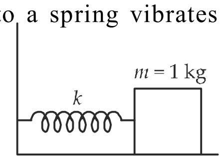
Since the O is the midpoint of two bridges and node of the stationary wave lies here, hence, length of two wires is equal, *i.e.*, $L_1 = L_2 = L$.

$\because$ Frequency remains same for both wires, *i.e.*, $\upsilon_1 = \upsilon_2$

$$\Rightarrow \frac{n_1}{2L}\sqrt{\frac{T}{\pi r^2 \rho_1}} = \frac{n_2}{2L}\sqrt{\frac{T}{\pi r^2 \rho_2}} \text{ or } \frac{n_1}{\sqrt{\rho_1}} = \frac{n_2}{\sqrt{\rho_2}}$$

$$\therefore \frac{n_1}{n_2} = \sqrt{\frac{\rho_1}{4\rho_1}} = \frac{1}{2} \qquad [\because \rho_2 = 4\rho_1]$$

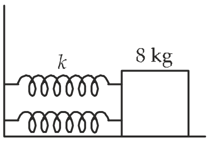
12. (b) : If 1 kg block attached to a spring vibrates with a frequency,



$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1\text{ Hz}$$

$\Rightarrow k = 4\pi^2$ N m^{-1}

When two springs are attached in parallel to an 8 kg block, then



$k_{eq} = k + k = 2k$

$$\text{Frequency, } \upsilon' = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m'}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2k}{8}} = \frac{1}{2\pi}\sqrt{\frac{2\times 4\pi^2}{8}} = \frac{1}{2}\text{ Hz}$$

13. (a) : For simple harmonic motion,

$$\frac{\text{Maximum acceleration}}{\text{Maximum velocity}} = 10 \Rightarrow \frac{\omega^2 a}{\omega a} = 10 \text{ or } \omega = 10$$

At $t = 0$; displacement, $x = 5$ m

$x = a\sin(\omega t + \phi)$

$$5 = a\sin\left(0 + \frac{\pi}{4}\right) \text{ or } 5 = a\sin\frac{\pi}{4} \text{ or } a = 5\sqrt{2}\text{ m}$$

Maximum acceleration $= \omega^2 a = 10^2 \times 5\sqrt{2} = 500\sqrt{2}$ m s^{-2}

14. (b) : Here, $y(x, t) = 0.5\left(\frac{5\pi}{4}x\right)$ sincos$(200\pi t)$

Comparing this equation with standard equation of standing wave, $y(x, t) = 2a\sin kx\cos\omega t$, we get, $k = \frac{5\pi}{4}$ rad/m, $\omega = 200\,\pi$ rad/s

$$\text{Speed of the travelling wave, } v = \frac{\omega}{k} = \frac{200\pi}{\frac{5\pi}{4}} = 160\text{ m/s}$$

15. (c) : Here, $m = 0.1$ kg, $k = 640$ N m^{-1},

$b = 10^{-2}$ kg s^{-1}, $E = \frac{E_0}{2}$, $t = ?$

Amplitude of damped oscillation, $A = A_0\,e^{-bt/2m}$

Total energy of the system, $E = E_0\,e^{-bt/m}$

$$\frac{bt}{m} = \ln\left(\frac{E_0}{E}\right) \Rightarrow t = \frac{m}{b}\ln\left(\frac{E_0}{E}\right) = \frac{0.1}{10^{-2}}\ln(2)$$

$= 10 \times 0.693 = 6.93$ s ≈ 7 s

16. (d) : Speed of a particle performing SHM is given by

At $= \frac{7W}{8}1$ initial speed of the particle, $= \omega\sqrt{W^7 - \quad ^7}$

$$= \omega\sqrt{W^7 - \left(\frac{7W}{8}\right)^7} = \omega\sqrt{\frac{:}{>}W^7} = \frac{\omega W\!\!\sqrt{:}}{8}$$

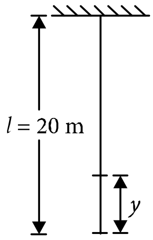
Now, its speed is trebled at the instant $= \frac{7W}{8}$ from equilibrium position, then new amplitude of the SHM is A' (say).

$$\text{Hence, } \ ' = 8 \ = \omega\sqrt{W'^7 - \left(\frac{7W}{8}\right)^7}$$

or, $\omega W\!\sqrt{:} = \omega\sqrt{W'^7 - \frac{9}{>}W^7}$ or, $: W^7 = W'^7 - \frac{9}{>}W^7$

or, $W'^7 = \frac{9>}{>}W^7 \ \therefore \ W' = \frac{<}{8}W3$

17. (c) : Speed of the wave pulse (wave) in the string,



$$= \sqrt{\frac{q}{\mu}}$$

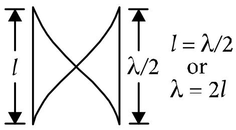
Here, $q = \text{—} \times \ \times$ and $\mu = \text{—}$

$$\therefore \ = \sqrt{\frac{\text{—}\times\ \times}{4}} = \sqrt{\ }$$

Also, $= \text{——} = \sqrt{\ }$ or, $\int_5^{75} \frac{}{\sqrt{\ }} = \int_5 \sqrt{\ }$

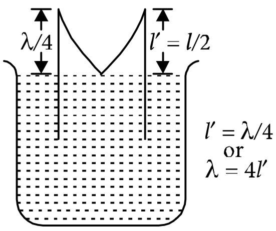
or, $\left[\frac{647}{647}\right]_5^{75} = \sqrt{\ }\ [\ ]_5 \Rightarrow 7\text{-}\sqrt{75} - 5. = \sqrt{65}\times \quad \therefore \ = 7\sqrt{7} -$

18. (d) : Fundamental frequency produced in an open pipe



$$= \frac{}{\lambda} = \frac{}{7}$$

Now, half portion of pipe is dipped vertically in water as shown in the figure, then it behaves as a closed pipe of length $l/2$. So fundamental frequency produced by it,



$$' = \frac{}{\lambda} = \frac{}{9\,'} = \frac{}{7} =$$

19. (d) : Angular displacement (θ_1) of particle 1 from equilibrium point is given by $y_1 = A\sin\theta_1$,

$A = A\sin\theta_1$ or $\sin\theta_1 = 1$ or $\theta_1 = \frac{\pi}{2}$

Similarly, for particle 2, $\theta_2 = \frac{-\pi}{6}$

Relative angular displacement between the two particles,

$$\theta = \theta_1 - \theta_2 = \frac{\pi}{2} - \left(\frac{-\pi}{6}\right) = \frac{2\pi}{3}$$

Relative angular velocity between the two particles

$\omega' = \omega_1 - \omega_2 = \omega - (-\omega) = 2\omega$

$$t = \frac{\theta}{\omega'} = \frac{2\pi}{3\times 2\omega} = \frac{2\pi}{3\times 2\times\frac{2\pi}{T}} = \frac{T}{6}$$

20. (d) : Frequency heard by the driver of second engine

$$\upsilon' = \frac{v+v_0}{v-v_s}\upsilon$$

Here, $v = 330 \text{ m s}^{-1}$, $v_0 = v_s = 30 \text{ m s}^{-1}$, $\upsilon = 540$ Hz

$$\therefore \quad \upsilon' = \frac{330+30}{330-30}\times 540 = \frac{360}{300}\times 540 = 648 \text{ Hz}$$

21. (b) : Amplitude of S.H.M., $A = 7$ cm $= 0.07$ m

As the washer does not stay in contact with the piston, at some particular frequency *i.e.* normal force on the washer = 0

$\Rightarrow$ Maximum acceleration of washer $= A\omega^2 = g$

$$\omega = \sqrt{\frac{g}{A}} = \sqrt{\frac{10}{0.07}} = \sqrt{\frac{1000}{7}}$$

Frequency of the piston, $\upsilon = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{1000}{7}} = 1.9$ Hz

22. (d) : $v = 340 \text{ m s}^{-1}$, $v_s = 5 \text{ m s}^{-1}$

υ = Frequency of horn = ?

$\upsilon_1 - \upsilon_2 = 5$ beats per second

Apparent frequency heard by the observer, directly.

$$\upsilon_1 = \left(\frac{v}{v-v_s}\right)\upsilon = \left(\frac{340}{340-5}\right)\upsilon = \frac{340}{335}\upsilon$$

Apparent frequency heard by the observer on reflection from the wall,

$$\upsilon_2 = \left(\frac{v}{v+v_s}\right)\upsilon = \left(\frac{340}{340+5}\right)\upsilon = \frac{340}{345}\upsilon$$

$\upsilon_1 - \upsilon_2 = 5$

$$\left(\frac{340}{335} - \frac{340}{345}\right)\upsilon = 5 \Rightarrow \upsilon = \frac{5}{340}\times\frac{335\times 345}{10} = 169.96 \text{ Hz} \approx 170 \text{ Hz}$$

23. (c) : Let length of the pendulum wire be l.

$\therefore$ Time period, $q = 7\pi\sqrt{—}$ ...(i)

When an additional mass M is added to bob, let Δl be the extension produced in wire.

^t qz $q_j = 7\pi\sqrt{\dfrac{+\Delta}{}}$ ...(ii)

Now, $v = \dfrac{\text{-µ-q-}}{\text{-µ-mz}} = \dfrac{j \quad 4W}{\Delta \quad 4} \Rightarrow \dfrac{\Delta}{} = \dfrac{j}{W}$ ...(iii)

From eqns. (i) and (ii), we get $\dfrac{q_j}{q} = \sqrt{\dfrac{+\Delta}{}}$

$$\{\sim \quad \left(\frac{q_j}{q}\right)^7 = \frac{+\Delta}{} = 6 + \frac{\Delta}{} = 6 + \frac{j}{W} \quad \text{(Using (iii))}$$

$$\{\sim \quad \frac{j}{W} = \left(\frac{q_j}{q}\right)^7 - 6 \quad \{\sim \quad \frac{6}{v} = \frac{W}{j}\left[\left(\frac{q_j}{q}\right)^7 - 6\right]$$

24. (d) : For a simple pendulum, variation of kinetic energy and potential energy with displacement d is

$$\text{R3L3} = \frac{6}{7} \quad \omega^7\text{-}W^7 - {}^7.\ \text{mz p X3L3} = \frac{6}{7} \quad \omega^7\ {}^7$$

where A is amplitude of oscillation.

When $d = 0$, $\text{R3L3} = \dfrac{6}{7} \quad \omega^7 W^7 \text{1X3L3} = 5$

When $d = \pm A$, K.E. = 0, P.E. $= \dfrac{1}{2}m\omega^2 A^2$

Therefore, graph (d) represents the variation correctly.

25. (d) : Frequency of sound emitted by train, $\upsilon = 1000$ Hz

Speed of train (source), $v_s = 20 \text{ m s}^{-1}$

Speed of sound, $v = 320 \text{ m s}^{-1}$

Observer is stationary.

Frequency heard by person as train approaches him

$$\upsilon_6 = \left(\frac{}{-}\right)\upsilon = \left(\frac{875}{875-75}\right)\times 6555 = \frac{8755}{8}\ \text{O}$$

Frequency heard by person as train moves away from him

$$\upsilon_7 = \left(\frac{}{+}\right)\upsilon = \left(\frac{875}{875+75}\right)\times 6555 = \frac{87555}{89}\ \text{O}$$

$\therefore$ Percentage change in frequency $= \left(\dfrac{\upsilon_7 - \upsilon_6}{\upsilon_6}\right)\times 655$

$$= \left(\frac{\dfrac{87555}{89} - \dfrac{8755}{8}}{\dfrac{8755}{8}}\right)\times 655 \approx -12\%$$

Negative sign implies that the frequency heard by person decreases as the train passes him.

26. (c) : Here, $x = a \sin \omega t$

$y = a \sin 2\omega t$

$y = 2\, a \sin \omega t \cos \omega t$

$y = 7 \quad \sqrt{6 - \dfrac{7}{7}}$ or $y = \dfrac{7}{} \quad \sqrt{- \ - \ .- \ + \ .}$

$y = 0$, at $x = 0, \pm a$

Hence, option (c) is correct.

27. (d) : It is given that oscillator is at rest at $t = 0$ *i.e.,* at $t = 0$, $v = 0$.

So, we can check options for $\quad = — = 5$ by putting $t = 0$.

(a) If $\quad \propto \text{-uz} \ + \dfrac{1}{2}\text{-uz}\ 7$

$\quad \propto \text{o\{} - \ + \dfrac{1}{2} \times 7\text{o\{} - 7$ at $t = 0$, $v \propto 1 + 1 = 2 \neq 0$

(b) If $\quad \propto \text{-uz} \ + \dfrac{6}{7}\text{o\{} - 7$

$\quad \propto \text{o\{} - \ + \dfrac{1}{2} \times 7\text{-—-uz}\ 7\ .$ at $t = 0$, $v \propto 1 - 0 \neq 0$

(c) If $\quad \propto \text{o\{} - \ - \dfrac{1}{2}\text{-uz}\ 7$

$\quad \propto -\text{-uz} \ - \dfrac{1}{2} \times 7\text{o\{} - 7$ at $t = 0$, $v \propto -1 \neq 0$

(d) If $\quad \propto \text{-uz} \ - \dfrac{1}{2}\text{-uz}\ 7$

$\quad \propto \text{o\{} - \ - \dfrac{1}{2} \times 7\text{o\{} - 7$ at $t = 0$, $v \propto 1 - 1 = 0$

So, in option (d) $v = 0$, at $t = 0$

28. (b) : Apparent frequency heard by bat

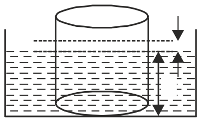
$\upsilon' = \left(\frac{+}{-}\right)\times \upsilon$

Here, $\upsilon' = f = ?$

$v = 320\ \text{m s}^{-1}$, $u = 10\ \text{m s}^{-1}$, $\upsilon = 8000$ Hz

$f = \left(\frac{320+10}{320-10}\right)\times 8000 = \frac{330}{310}\times 8000 = 8516$ Hz

29. (c) : Let block is floating with depth h inside the liquid. Then at equilibrium



$M_{\text{Block}}\,g = F_{\text{up}}$

$(AH\rho_B)\,g = (Ah)\rho_L\,g$ ...(i)

(H = height of the block)

When block depressed slightly by distance x then

$F_{\text{Net}} = Mg - F'_{\text{up}} = AH\rho_B\,g - A(h+x)\rho_L\,g$

$\therefore F_{\text{Net}} = -Ax\rho_L\,g$ [Using eqn. (i)]

$e\ \rho_X \frac{7}{7} = -\ \rho_i$

$\frac{7}{7} = -\frac{\rho_i}{e\ \rho_X} = -\omega^7 \quad \therefore \quad \omega^7 = \frac{\rho_i}{e\ \rho_X}$

For simple pendulum $\omega^7 = -\quad \therefore \quad = \frac{e\ \rho_X}{\rho_i} = \frac{;:5\times:9}{>55} = 8>\text{oy}$

30. (a) : Amplitude in a damped oscillation is given by $A = A_0 e^{-\beta t}$

Energy, $E \propto A^2$ $\therefore$ $\sqrt{b} = \sqrt{b_5}\ ^{-\beta}$ t q~q b_5 u- uz ųµmxqz q~s

Here, $E_0 = 45$ J, $T = 1$ s, $E = 15$ J

$t = nT = 15 \times 1 = 15$ s

Then, $\sqrt{6:} = \sqrt{9:}\ ^{-\beta\times 6:}$

$8^{-\frac{6}{7}} = {}^{-6:\beta}$

Taking log on both sides $-\frac{6}{7}$ xz -8. $= -6:\beta$

$\beta = \frac{\text{xz}\ 8}{85}$

31. (a) : For the given situation, frequency heard by an observer is given by

$D\ _5\left[\frac{0\quad 5}{-\quad -}\right]$

$= \frac{5}{-} + \frac{5\ 5}{-}$

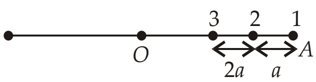
Comparing the equation with the straight line equation

$y = mx + C$

So, slope of the graph $= \frac{5}{-}$

32. (a) : As the particle starts from rest so we choose

$x = A\cos\omega t$



At $t = 0, x = A$

When $t = \tau$, $x = A - a$

When $t = 2\tau$, $x = A - 3a$

$\therefore$ $(A - a) = A\cos\omega\tau$ ...(i)

and $(A - 3a) = A\cos 2\omega\tau = A(2\cos^2\omega\tau - 1)$

$\Rightarrow\ (A-3a) = A\left[2\left(\frac{A-a}{A}\right)^2 - 1\right] \Rightarrow \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$

On solving, $A = 2a$

Now, $A - a = A\cos\omega\tau$

$\Rightarrow\ \cos\omega\tau = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \omega\tau = \frac{\pi}{3} \Rightarrow \frac{2\pi}{T}\tau = \frac{\pi}{3} \Rightarrow T = 6\tau.$

33. (d) : Here, $l = 85$ cm $= 0.85$ m, $v = 340\ \text{m s}^{-1}$

Pipe is closed from one end so it behaves as a closed organ pipe. Frequencies in the closed organ pipe is given by,

$\upsilon = \frac{(2n-1)v}{4l}$ where, $n = 1, 2, 3, 4, \ldots\ldots\ldots$

According to question, $\upsilon < 1250$ Hz

$\left(\frac{2n-1}{4l}\right)v < 1250 \Rightarrow \frac{(2n-1)\times 340}{4\times 0.85} < 1250$

$\Rightarrow (2n-1) < 12.5$

Possible value of $n = 1, 2, 3, 4, 5, 6$

So, number of possible natural frequencies lie below 1250 Hz is 6.

34. (d) : The amplitude of a damped oscillator at a given instant of time t is given by $A = A_0 e^{-bt/2m}$

where A_0 is its amplitude in the absence of damping, b is the damping constant.

As per question

After 5 s (*i.e.* $t = 5$ s) its amplitude becomes

$0.9A_0 = A_0 e^{-b(5)/2m} = A_0 e^{-5b/2m}$

$0.9 = e^{-5b/2m}$ ...(i)

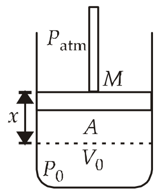
After 10 more second (*i.e.* $t = 15$ s), its amplitude becomes

$\alpha A_0 = A_0 e^{-b(15)/2m} = A_0 e^{-15b/2m}$

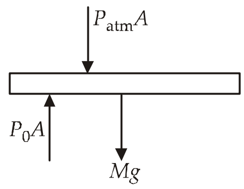
$\alpha = (e^{-5b/2m})^3 = (0.9)^3$ (Using (i))

$= 0.729$

35. (d) :

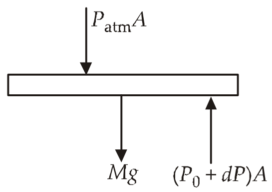


FBD of piston at equilibrium



$P_{\text{atm}}A + Mg = P_0A$ ...(i)

FBD of piston when piston is pushed down a distance x



$$(P_0 + dP)A - (P_{\text{atm}}A + Mg) = M\frac{d^2x}{dt^2} \quad \ldots\text{(ii)}$$

As the system is completely isolated from its surrounding therefore the change is adiabatic.

For an adiabatic process PV^γ = constant

$\therefore\ V^\gamma dP + V^{\gamma-1}PdV = 0$ or $dP = -\frac{\gamma PdV}{V}$

$\therefore\ dP = -\frac{\gamma P_0(Ax)}{V_0}$ $(\because dV = Ax)$ ...(iii)

Using (i) and (iii) in (ii), we get

$$M\frac{d^2x}{dt^2} = -\frac{\gamma P_0 A^2}{V_0}x \text{ or } \frac{d^2x}{dt^2} = -\frac{\gamma P_0 A^2}{MV_0}$$

Comparing it with standard equation of SHM, $\frac{d^2x}{dt^2} = -\omega^2 x$

We get $\omega^2 = \frac{\gamma P_0 A^2}{MV_0}$ or $\omega = \sqrt{\frac{\gamma P_0 A^2}{MV_0}}$

Frequency, $\upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{MV_0}}$

36. (c) : Fundamental frequency of vibration of wire is

$$\upsilon = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where L is the length of the wire, T is the tension in the wire and μ is the mass per length of the wire

As $\mu = \rho A$

where ρ is the density of the material of the wire and A is the area of cross-section of the wire. $\therefore \quad \upsilon = \frac{1}{2L}\sqrt{\frac{T}{\rho A}}$

Here tension is due to elasticity of wire

$$\therefore \quad T = YA\left[\frac{\Delta L}{L}\right] \qquad \left[\text{As } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{TL}{A\Delta L}\right]$$

Hence, $\upsilon = \frac{1}{2L}\sqrt{\frac{Y\Delta L}{\rho L}}$

Here, $Y = 2.2 \times 10^{11}$ N/m^2, $\rho = 7.7 \times 10^3$ kg/m^3

$$\frac{\Delta L}{L} = 0.01, \; L = 1.5 \text{ m}$$

Substituting the given values, we get

$$\upsilon = \frac{1}{2 \times 1.5}\sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}} = \frac{10^3}{3}\sqrt{\frac{2}{7}} \text{ Hz} = 178.2 \text{ Hz}$$

37. (c) **38. (b)**

39. (d) : $T_1 = 2\pi\sqrt{\frac{M}{k}}$ ...(i)

When a mass m is placed on mass M, the new system is of mass $= (M + m)$ attached to the spring. New time period of oscillation

$$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}} \qquad \text{...(ii)}$$

Consider v_1 is the velocity of mass M passing through mean position and v_2 velocity of mass $(m + M)$ passing through mean position.

Using, law of conservation of linear momentum

$$Mv_1 = (m + M)v_2$$

$$M(A_1\omega_1) = (m + M)(A_2\omega_2) \quad (\because \; v_1 = A_1\omega_1 \text{ and } v_2 = A_2\omega_2)$$

or $\frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1} = \left(\frac{m+M}{M}\right) \times \frac{T_1}{T_2}\left(\because \; \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$

$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$ (Using (i) and (ii))

40. (b) : $y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$

$$y(x, t) = e^{-(\sqrt{a}x + \sqrt{b}t)^2} \qquad \text{...(i)}$$

Comparing equation (i) with standard equation

$$y(x, t) = f(ax + bt)$$

As there is positive sign between x and t terms, hence wave travel in $-x$ direction.

$$\text{Wave speed} = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \sqrt{\frac{b}{a}}$$

41. (a) : Here, linear mass density $\mu = 0.04$ kg m^{-1}

The given equation of a wave is

$$y = 0.02\sin\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$$

Compare it with the standard wave equation

$$y = A\sin(\omega t - kx)$$

we get, $\omega = \frac{2\pi}{0.04}$ rad s^{-1}; $k = \frac{2\pi}{0.5}$ rad m^{-1}

Wave velocity, $v = \frac{\omega}{k} = \frac{(2\pi/0.04)}{(2\pi/0.5)}$ m s^{-1} ...(i)

Also $v = \sqrt{\frac{T}{\mu}}$ ...(ii)

where T is the tension in the string and μ is the linear mass density

Equating equations (i) and (ii), we get $\frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$ or $T = \frac{\mu\omega^2}{k^2}$

$$T = \frac{0.04 \times \left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.05}\right)^2} = 6.25 \text{ N}$$

42. (b) : For a simple harmonic motion,

acceleration, $a = -\omega^2 x$ where ω is a constant $= \frac{2\pi}{T}$.

$$a = -\frac{4\pi^2}{T^2} \cdot x \;\Rightarrow\; \frac{aT}{x} = -\frac{4\pi^2}{T}.$$

The period of oscillation T is a constant. $\therefore \; \frac{aT}{x}$ is a constant.

43. (a) : The given sources of sound produce frequencies, $(\upsilon - 1)$, υ and $(\upsilon + 1)$.

For two sources of frequencies υ_1 and υ_2,

$y_1 = A\cos 2\pi\upsilon_1 t$

$y_2 = A\cos 2\pi\upsilon_2 t$

Superposing, one gets

$$y = 2A\cos 2\pi\left(\frac{\upsilon_1 - \upsilon_2}{2}\right)t\cos 2\pi\left(\frac{\upsilon_1 + \upsilon_2}{2}\right)t.$$

The resultant frequency obtained is $\frac{\upsilon_1 + \upsilon_2}{2}$ and this wave is modulated by a wave of frequency $\frac{\upsilon_1 - \upsilon_2}{2}$ (rather the difference of frequencies/2).

The intensity waxes and wanes. For a cosine curve (or sine curve), the number of beats $= \upsilon_1 \sim \upsilon_2$.

Frequencies	Mean	Beats
$\upsilon + 1$ and υ	$(\upsilon + 0.5)$ Hz	1
υ and $\upsilon - 1$	$\upsilon - 0.5$	1
$(\upsilon + 1)$ and $(\upsilon - 1)$	υ	2

Total number of beats = 4.

One should detect three frequencies, υ, $\upsilon + 0.5$ and $\upsilon - 0.5$ and each frequency will show 2 beats, 1 beat and 1 beat per second, respectively.

Total number of beats = 4

44. (b) : The source is at rest, the observer is moving away from the source.

$\therefore \quad f' = f\dfrac{(v_{\text{sound}} - v_{\text{obs}})}{v_{\text{sound}}}$

$\Rightarrow \quad \dfrac{f'}{f} \times v_{\text{sound}} = v_{\text{sound}} - v_{\text{obs}} \quad \Rightarrow \quad \dfrac{f'}{f} \times v_{\text{sound}} - v_{\text{sound}} = -v_{\text{obs}}$

$v_{\text{sound}}\left(\dfrac{f'}{f} - 1\right) = -v_{\text{obs}}$

$330(0.94 - 1) = -v_{\text{obs}} \;\Rightarrow\; v_{\text{obs}} = 330 \times 0.06 = 19.80 \text{ ms}^{-1}$.

$\therefore \quad s = \dfrac{v^2 - u^2}{2a} = \dfrac{(19.80)^2}{2 \times 2} = 98 \text{ m}.$

45. (b) : The wave travelling along the x-axis is given by $y(x, t) = 0.005 \cos(\alpha x - \beta t)$.

Therefore $\alpha = k = \dfrac{2\pi}{\lambda}$. As $\lambda = 0.08$ m.

$\therefore \quad \alpha = \dfrac{2\pi}{0.08} = \dfrac{\pi}{0.04} \;\Rightarrow\; \alpha = \dfrac{\pi}{4} \times 100.00 = 25.00\pi.$

$\omega = \beta \Rightarrow \dfrac{2\pi}{2.0} = \pi \quad \therefore \; \alpha = 25.00\,\pi,\ \beta = \pi$

46. (*) : $v = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M}}$

γ for $O_2 = 1 + 2/5 = 1.4$; γ for He $= 1 + 2/3 = 5/3$

$v_2 = \left(\sqrt{\dfrac{\gamma_{\text{He}}}{4} \times \dfrac{32}{\gamma_{O_2}}}\right) \times 460 = 460 \times \sqrt{\dfrac{5}{3} \times \dfrac{1}{4} \times \dfrac{32 \times 5}{7}} = 1420 \text{ m/s}.$

* The value of the speed of sound in He should have been 965 m/s and that of O_2, about 320 m/s. The value of the velocity given for O_2 is quite high. Option not given.

47. (a) : Given : $x = x_0 \cos\left(\omega t - \dfrac{\pi}{4}\right)$ ...(i)

Acceleration $a = A \cos(\omega t + \delta)$ ...(ii)

Velocity $v = \dfrac{dx}{dt}$ or $v = -x_0\omega \sin\left(\omega t - \dfrac{\pi}{4}\right)$ ...(iii)

Acceleration $a = \dfrac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \dfrac{\pi}{4}\right) = x_0\omega^2 \cos\left[\pi + (\omega t - \dfrac{\pi}{4})\right]$

$= x_0\omega^2 \cos\left[\omega t + \dfrac{3\pi}{4}\right]$ ...(iv)

Compare (iv) with (ii), we get $A = x_0\omega^2,\ \delta = \dfrac{3\pi}{4}$.

48. (b) : Given : displacement $x = 2 \times 10^{-2} \cos \pi t$

Velocity $v = \dfrac{dx}{dt} = -2 \times 10^{-2}\,\pi \sin \pi t$

For the first time when $v = v_{\max}$, $\sin \pi t = 1$

or $\sin \pi t = \sin\dfrac{\pi}{2}$ or $\pi t = \dfrac{\pi}{2}$ or $t = \dfrac{1}{2}\text{ s} = 0.5\text{ s}.$

49. (b) : For a particle to execute simple harmonic motion its displacement at any time t is given by $x(t) = a(\cos \omega t + \phi)$ where, a = amplitude, ω = angular frequency, ϕ = phase constant. Let us choose $\phi = 0$ $\therefore$ $x(t) = a\cos\omega t$

Velocity of a particle $v = \dfrac{dx}{dt} = -a\,\omega \sin \omega t$

Kinetic energy of a particle is $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}ma^2\omega^2 \sin^2 \omega t$

Average kinetic energy $\langle K \rangle = \left\langle \dfrac{1}{2}ma^2\omega^2 \sin^2 \omega t \right\rangle$

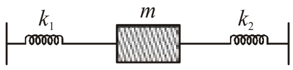
$= \dfrac{1}{2}m\omega^2 a^2 \langle \sin^2 \omega t \rangle$

$= \dfrac{1}{2}m\,\omega^2 a^2 \left(\dfrac{1}{2}\right) \left[\because \langle \sin^2 \theta \rangle = \dfrac{1}{2}\right]$

$= \dfrac{1}{4}ma^2(2\pi\upsilon)^2 \qquad [\because \omega = 2\pi\upsilon]$

$= \pi^2 ma^2 \upsilon^2.$

50. (a) : In the given figure two springs are connected in parallel. Therefore the effective spring constant is given by $k_{eff} = k_1 + k_2$



Frequency of oscillation,

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k_{eff}}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1 + k_2}{m}}$ ...(i)

As k_1 and k_2 are increased four times

New frequency, $f' = \dfrac{1}{2\pi}\sqrt{\dfrac{4(k_1 + k_2)}{m}} = 2f$ (using (i))

51. (a) : $L_1 = 10 \log\left(\dfrac{I_1}{I_0}\right)$; $L_2 = 10 \log\left(\dfrac{I_2}{I_0}\right)$

$\therefore \quad L_1 - L_2 = 10 \log\left(\dfrac{I_1}{I_0}\right) - 10 \log\left(\dfrac{I_2}{I_0}\right)$ or $\Delta L = 10 \log\left(\dfrac{I_1}{I_2}\right)$

or $20 \text{ dB} = 10 \log\left(\dfrac{I_1}{I_2}\right)$ or $10^2 = \dfrac{I_1}{I_2}$ or $I_2 = \dfrac{I_1}{100}$.

52. (c) : In vertical simple harmonic motion, maximum acceleration ($a\omega^2$) and so the maximum force ($ma\omega^2$) will be at extreme positions. At highest position, force will be towards mean position and so it will be downwards. At lowest position, force will be towards mean position and so it will be upwards. This is opposite to weight direction of the coin. The coin will leave contact will the platform for the first time when $m(a\omega^2) \geq mg$ at the lowest position of the platform.

53. (b) : Maximum velocity $v_m = a\omega = a\left(\dfrac{2\pi}{T}\right)$

$\therefore \quad T = \dfrac{2\pi a}{v_m} = 2 \times \dfrac{22}{7} \times \dfrac{(7 \times 10^{-3})}{4.4} = 10^{-2}$ sec $= 0.01$ sec.

54. (b) : During simple harmonic motion,

Kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\,(a\,\omega\cos\omega t)^2$

Total energy $E = \dfrac{1}{2}ma^2\omega^2 \because$ (Kinetic energy) $= \dfrac{75}{100}(E)$

or $\dfrac{1}{2}ma^2\omega^2\cos^2\omega t = \dfrac{75}{100} \times \dfrac{1}{2}ma^2\omega^2$

or $\cos^2 \omega t = \dfrac{3}{4} \Rightarrow \cos\omega t = \dfrac{\sqrt{3}}{2} = \cos\dfrac{\pi}{6} \quad \therefore \; \omega t = \dfrac{\pi}{6}$

or $t = \dfrac{\pi}{6\omega} = \dfrac{\pi}{6(2\pi/T)} = \dfrac{2\pi}{6 \times 2\pi} = \dfrac{1}{6}$ sec.

55. (b) : Let the successive loops formed be p and $(p + 1)$ for frequencies 315 Hz and 420 Hz

$\therefore \quad \upsilon = \dfrac{p}{2l}\sqrt{\dfrac{T}{\mu}} = \dfrac{pv}{2l} \quad \therefore \quad \dfrac{pv}{2l} = 315 \text{ Hz and } \dfrac{(p+1)v}{2l} = 420 \text{ Hz}$

or $\dfrac{(p+1)v}{2l} - \dfrac{pv}{2l} = 420 - 315$ or $\dfrac{v}{2l} = 105 \Rightarrow \dfrac{1 \times v}{2l} = 105$ Hz

$p = 1$ for fundamental mode of vibration of string.

$\therefore$ Lowest resonant frequency = 105 Hz.

56. (d) : $\frac{\upsilon'}{\upsilon} = \frac{v_s}{v_s - v}$ where v_s is the velocity of sound in air.

$\frac{10000}{9500} = \frac{300}{300-v} \Rightarrow (300-v) = 285 \Rightarrow v = 15 \text{ m/s}$.

57. (c) : For a pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$ where l is measured upto centre of gravity. The centre of gravity of system is at centre of sphere when hole is plugged. When unplugged, water drains out. Centre of gravity goes on descending. When the bob becomes empty, centre of gravity is restored to centre.

$\therefore$ Length of pendulum first increases, then decreases to original value.

$\therefore$ T would first increase and then decrease to the original value.

58. (d) : Standard differential equation of SHM is $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Given equation is $\frac{d^2x}{dt^2} + \alpha x = 0$

$\therefore$ $\omega^2 = \alpha$ or $\omega = \sqrt{\alpha}$ $\therefore$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$.

59. (c) : $v_1 = \frac{d}{dt}(y_1) = (0.1 \times 100\pi)\cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{d}{dt}(y_2) = (-0.1 \times \pi)\sin \pi t$

$= (0.1 \times \pi)\cos\left(\pi t + \frac{\pi}{2}\right)$ $\therefore$ $\Delta\phi = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$.

60. (d) : $y = \sin^2 \omega t = \frac{1-\cos 2\omega t}{2} = \frac{1}{2} - \frac{\cos 2\omega t}{2}$

It is a periodic motion but it is not SHM

$\therefore$ Angular speed $= 2\omega$

$\therefore$ Period $T = \frac{2\pi}{\text{angular speed}} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Hence option (d) represents the answer.

61. (b) : By Doppler's effect

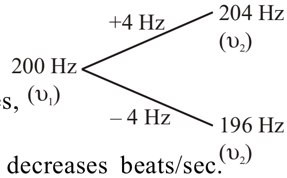
$\frac{\upsilon'}{\upsilon} = \frac{v_s + v_O}{v_s}$ (where v_s is the velocity of sound)

$= \frac{v + (v/5)}{v} = \frac{6}{5}$

$\therefore$ Fractional increase $= \frac{\upsilon' - \upsilon}{\upsilon} = \left(\frac{\upsilon'}{\upsilon} - 1\right) = \left(\frac{6}{5} - 1\right) = \frac{1}{5}$

$\therefore$ Percentage increase $= \frac{100}{5} = 20\%$.

62. (a) : Let the two frequencies be υ_1 and υ_2.



υ_2 may be either 204 Hz or 196 Hz.

As mass of second fork increases, υ_2 decreases.

If υ_2 = 204 Hz, a decrease in υ_2 decreases beats/sec. But this is not given in question

If υ_2 = 196 Hz, a decrease in υ_2 increases beats/sec. This is given in the question when beats increase to 6

$\therefore$ Original frequency of second fork = 196 Hz.

63. (a) : In case of forced oscillations

(i) The amplitude is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_1

(ii) The energy is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_2

$\therefore$ From (i) and (ii), $\omega_1 = \omega_2$.

64. (b) : In case of forced oscillations,

$x = a\sin(\omega t + \phi)$ where $a = \frac{F_0/m}{\omega_0^2 - \omega^2}$

$\therefore$ x is proportional to $\frac{1}{m(\omega_0^2 - \omega^2)}$.

65. (c) : Under simple harmonic motion, total energy

$$= \frac{1}{2}ma^2\omega^2$$

Total energy is independent of x.

66. (b) : When springs are in series, $k = \frac{k_1 k_2}{k_1 + k_2}$

For first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$

For second spring $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\therefore$ $t_1^2 + t_2^2 = \frac{4\pi^2 m}{k_1} + \frac{4\pi^2 m}{k_2} = 4\pi^2 m\left(\frac{k_1 + k_2}{k_1 k_2}\right)$

or $t_1^2 + t_2^2 = \left[2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}\right]^2$ or $t_1^2 + t_2^2 = T^2$.

67. (b) : Given wave equation :

$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)\text{m}$

Standard equation : $y = a \sin(\omega t + kx + \phi)$

Compare the two

$\therefore$ $\omega = 100$ and $k = 20$ $\therefore$ $\frac{\omega}{k} = \frac{100}{20} \Rightarrow \frac{2\pi n}{2\pi/\lambda} = n\lambda = v = 5$

$\therefore$ $v = 5$ m/s.

68. (c) : $t_0 = 2\pi\sqrt{l/g}$ ...(i)

Due to upthrust of water on the top, its apparent weight decreases

upthrust = weight of liquid displaced

$\therefore$ Effective weight $= mg - (V\sigma g) = V\rho g - V\sigma g$

$V\rho g' = Vg(\rho - \sigma)$, where σ is density of water

or $g' = g\left(\frac{\rho - \sigma}{\rho}\right)$ $\therefore$ $t = 2\pi\sqrt{l/g'} = 2\pi\sqrt{\frac{l\rho}{g(\rho-\sigma)}}$ ...(ii)

$\therefore$ $\frac{t}{t_0} = \sqrt{\frac{l\rho}{g(\rho-\sigma)} \times \frac{g}{l}} = \sqrt{\frac{\rho}{\rho - \sigma}} = \sqrt{\frac{4 \times 1000/3}{\left(\frac{4000}{3} - 1000\right)}} = 2$

or $t = t_0 \times 2 = 2t_0$.

69. (a) : Kinetic energy is maximum at $x = 0$.

70. (d) : Let the lengths of pendulum be $(100l)$ and $(121l)$

$\therefore$ $\frac{T'}{T} = \sqrt{\frac{121}{100}} = \frac{11}{10}$

$\therefore$ Fractional change $= \frac{T' - T}{T} = \frac{11-10}{10} = \frac{1}{10}$

$\therefore$ Percentage change = 10%.

71. (c) : Maximum velocity under simple harmonic motion $v_m = a\omega$

$\therefore \quad v_m = \frac{2\pi a}{T} = (2\pi a)\left(\frac{1}{T}\right) = (2\pi a)\left(\frac{1}{2\pi}\sqrt{\frac{k}{m}}\right)$ or $v_m = a\sqrt{\frac{k}{m}}$

$\because \quad (v_m)_A = (v_m)_B$

$\therefore \quad a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$.

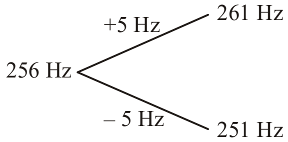
72. (c) : Initially, $T = 2\pi\sqrt{M/k}$

Finally, $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

$\therefore \quad \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M+m}{k}}$ or $\frac{25}{9}\frac{M}{k} = \frac{M+m}{k}$

or $9m + 9M = 25M$ or $\frac{m}{M} = \frac{16}{9}$.

73. (c) : The possible frequencies of piano are (256 + 5) Hz and (256 – 5) Hz.



For piano string, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

When tension T increases, υ increases

(i) If 261 Hz increases, beats/sec increase. This is not given in the question.

(ii) If 251 Hz increases due to tension, beats per second decrease. This is given in the question.

Hence frequency of piano = (256 – 5) Hz.

74. (a) : At resonance, frequency of vibration of wire become equal to frequency of a.c.

For vibration of wire, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$\therefore \quad \upsilon = \frac{1}{2\times 1}\sqrt{\frac{10\times 9.8}{9.8\times 10^{-3}}} = \frac{100}{2} = 50$ Hz.

75. (c) : $x = 4(\cos\pi t + \sin\pi t) = 4\times\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\pi t + \frac{1}{\sqrt{2}}\sin\pi t\right]$

or $x = 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos\pi t + \cos\frac{\pi}{4}\sin\pi t\right] = 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$

Hence amplitude $= 4\sqrt{2}$.

76. (a) : Given wave equation : $y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$m

Standard wave equation : $y = a\sin(\omega t - kx + \phi)$

Compare them

Angular speed = ω = 600 sec^{-1}

Propagation constant = k = 2 m^{-1}

$\frac{\omega}{k} = \frac{2\pi\upsilon}{2\pi/\lambda} = \upsilon\lambda$ = velocity $\therefore$ velocity $= \frac{\omega}{k} = \frac{600}{2} = 300$ m/sec .

77. (b) : Time period will decrease.

When the child stands up, the centre of gravity is shifted upwards and so length of swing decreases. $T = 2\pi\sqrt{l/g}$.

78. (c) : In a simple harmonic oscillator, kinetic energy is maximum and potential energy is minimum at mean position.

79. (b) : For a spring, $T = 2\pi\sqrt{\frac{m}{k}}$

For each piece, spring constant = nk

$\therefore \quad T' = 2\pi\sqrt{\frac{m}{nk}}$

or $T' = 2\pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{n}} = \frac{T}{\sqrt{n}}$.

80. (b) : When temperature increases, l increases

Hence frequency decreases.

81. (b) : $\frac{\lambda_{\max}}{2} = 40 \Rightarrow \lambda_{\max} = 80$ cm.

82. (b) : Consider option (a)

Stationary wave :

$Y = a\sin(\omega t + kx) + a\sin(\omega t - kx)$

when $x = 0$, Y is not zero. The option is not acceptable.

Consider option (b)

Stationary wave : $Y = a\sin(\omega t - kx) - a\sin(\omega t + kx)$

At $x = 0$, $Y = a\sin\omega t - a\sin\omega t$ = zero

This option holds good

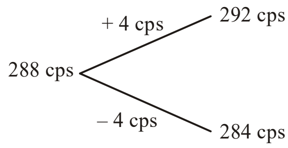
Option (c) gives $Y = 2a\sin(\omega t - kx)$

At $x = 0$, Y is not zero

Option (d) gives $Y = 0$

Hence only option (b) holds good.

83. (b) : The wax decreases the frequency of unknown fork. The possible unknown frequencies are (288 + 4)cps and (288 – 4) cps.



Wax reduces 284 cps and so beats should increases. It is not given in the question. This frequency is ruled out. Wax reduced 292 cps and so beats should decrease. It is given that the beats decrease to 2 from 4.

Hence unknown fork has frequency 292 cps.

84. (c) : In tube A, $\lambda_A = 2l$

In tube B, $\lambda_B = 4l$

$\therefore \quad \upsilon_A = \frac{v}{\lambda_A} = \frac{v}{2l}$

$\upsilon_B = \frac{v}{\lambda_B} = \frac{v}{4l}$

$\therefore \quad \frac{\upsilon_A}{\upsilon_B} = \frac{2}{1}$.