CHAPTER

9

Kinetic Theory of Gases

1. The mass of a hydrogen molecule is 3.32×10^{-27}/ kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m s^{-1}, then the pressure on the wall is nearly

(a) 2.35×10^3 N m^{-2} (b) 4.70×10^3 N m^{-2}
(c) 2.35×10^2 N m^{-2} (d) 4.70×10^2 N m^{-2}

(2018)

2. The value closest to the thermal velocity of a Helium atom at room temperature (300 K) in m s^{-1} is
[$k_B = 1.4 \times 10^{-23}$ J/K; $m_{He} = 7 \times 10^{-27}$ kg]

(a) 1.3×10^3 (b) 1.3×10^5
(c) 1.3×10^2 (d) 1.3×10^4

(Online 2018)

3. Two moles of helium are mixed with n moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value of n is

(a) 1 (b) 2 (c) 3 (d) $\frac{3}{2}$

(Online 2018)

4. C_P and C_V are specific heats at constant pressure and constant volume respectively. It is observed that
$C_P - C_V = a$ for hydrogen gas
$C_P - C_V = b$ for nitrogen gas
The correct relation between a and b is

(a) $a = \frac{1}{14}b$ (b) $a = b$
(c) $a = 14b$ (d) $a = 28b$

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5. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_P) and at constant volume (C_V) is

(a) 6 (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{5}$

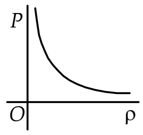
(Online 2017)

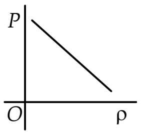
6. N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas?

(a) 0 (b) $\frac{5}{2}nRT$ (c) $\frac{1}{2}nRT$ (d) $\frac{3}{2}nRT$

(Online 2017)

7. Which of the following shows the correct relationship between the pressure 'P' and density ρ of an ideal gas at constant temperature?

(a)


(b)


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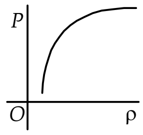

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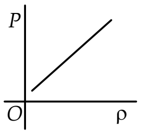


(Online 2016)

8. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is $\left(\gamma = \frac{Y}{Y}\right)$

(a) $\frac{\gamma + 6}{7}$ (b) $\frac{\gamma - 6}{7}$ (c) $\frac{8\gamma + :}{;}$ (d) $\frac{8\gamma - :}{;}$

(2015)

9. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is

(a) 2 (b) 1 (c) $\frac{6}{7}$ (d) $\frac{6}{9}$

(Online 2015)

10. Using equipartition of energy, the specific heat (in J kg^{-1} K^{-1}) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27)

(a) 25 (b) 410 (c) 925 (d) 1850

(Online 2015)

11. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(a) $\frac{(\gamma - 1)}{2(\gamma + 1)R}Mv^2$ K (b) $\frac{(\gamma - 1)}{2\gamma R}Mv^2$ K
(c) $\frac{\gamma M v^2}{2R}$ K (d) $\frac{(\gamma - 1)}{2R}Mv^2$ K *(2011)*

12. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J *(2009)*

13. If C_P and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $C_P - C_V = 28R$ (b) $C_P - C_V = R/28$
(c) $C_P - C_V = R/14$ (d) $C_P - C_V = R$ *(2007)*

14. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is

(a) $T_f = \frac{5}{2}T_0$ (b) $T_f = \frac{3}{7}T_0$
(c) $T_f = \frac{7}{3}T_0$ (d) $T_f = \frac{3}{2}T_0$. *(2006)*

15. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_P/C_V of the mixture is

(a) 1.4 (b) 1.54 (c) 1.59 (d) 1.62
(2005)

16. One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume.

(a) 3/2 (b) 23/15 (c) 35/23 (d) 4/3.
(2004)

17. 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of a gas with $\gamma = 5/3$, then the value of g for the resulting mixture is

(a) 7/5 (b) 2/5 (c) 24/16 (d) 12/7.
(2002)

18. At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(a) 80 K (b) –73 K (c) 3 K (d) 20 K.
(2002)

19. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will

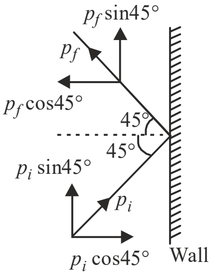
(a) increase
(b) decrease
(c) remain same
(d) decrease for some, while increase for others.
(2002)

ANSWER KEY

1. (a) **2.** (a) **3.** (c) **4.** (c) **5.** (d) **6.** (c) **7.** (d) **8.** (a) **9.** (b) **10.** (c) **11.** (d) **12.** (b)
13. (b) **14.** (d) **15.** (d) **16.** (a) **17.** (c) **18.** (d) **19.** (c)

Explanations

1. **(a) :** As $p_i = p_f$



Net force on the wall,

$$F = \frac{dp}{dt} = 2np_f \cos 45° = 2nmv\cos 45°$$

Here, n is the number of hydrogen molecules striking per second.

$$\text{Pressure} = \frac{F}{A} = \frac{2nmv\cos 45°}{\text{Area}}$$

$$= \frac{2\times10^{23}\times3.32\times10^{-27}\times10^3\times(1/\sqrt{2})}{2\times10^{-4}} = 2.35\times\ 10^3\ \text{N m}^{-2}$$

2. **(a) :** $\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3\times1.4\times10^{-23}\times300}{7\times10^{-27}}}$$

$$= \sqrt{1.8}\times10^3\ \text{m s}^{-1} \approx 1.3\times10^3\ \text{m s}^{-1}$$

3. **(c) :** As we know, $\frac{C_P}{C_V} = \frac{f_{\text{mix}}+2}{f_{\text{mix}}} \Rightarrow \frac{3}{2} = \frac{f_{\text{mix}}+2}{f_{\text{mix}}}$

$\Rightarrow\ f_{\text{mix}} = 4$

$$f_{\text{mix}} = \frac{n_1f_1+n_2f_2}{n_1+n_2} \Rightarrow 4 = \frac{2\times3+n_2\times5}{2+n_2} \Rightarrow n_2 = 2\ \text{moles}$$

4. **(c) :** For an ideal gas, $c_P - c_V = R$
where c_P and c_V are the molar heat capacities.
$MC_P - MC_V = R$
($c_P = MC_P$ and $c_V = MC_V$)
Here, C_P and C_V are specific heats and M is the molar mass.

$$\therefore\ C_P - C_V = \frac{R}{M}$$

For hydrogen gas, $C_P - C_V = \frac{R}{2} = a$ … (i)

For nitrogen gas, $C_P - C_V = \frac{R}{28} = b$ … (ii)

Dividing eqn. (i) by (ii), we get $\frac{a}{b} = 14$ or $a = 14b$

5. **(d) :** An ideal gas has molecules with 5 degrees of freedom, then

$$C_V = \frac{5}{2}R \text{ and } C_P = \frac{7}{2}R\ \therefore\ \frac{C_P}{C_V} = \frac{7/2R}{5/2R} = \frac{7}{5}$$

6. **(c) :** Initial kinetic energy of the system $K_i = \frac{5}{2}RTN$
Final kinetic energy of the system,

$$K_f = \frac{5}{2}RT(N-n) + \frac{3}{2}RT(2n)$$

$$\Delta K = K_f - K_i = nRT\left(3-\frac{5}{2}\right) = \frac{1}{2}nRT$$

7. **(d) :** Ideal gas equation, $PV = nRT$
As temperature is constant.

$$PV = \text{constant} \Rightarrow P\frac{m}{\rho} = \text{constant}$$

$P \propto \rho$ (for given m)

8. **(a) :** Average time of collision between molecules,

$$\tau = \frac{\text{Mean free path}(\lambda)}{\text{Mean speed}(\bar{v})} = \frac{1}{\left(\sqrt{2}\pi d^2\frac{N}{V}\right)\left(\sqrt{\frac{8k_BT}{m\pi}}\right)}$$

$$\Rightarrow\quad \tau \propto \frac{s}{\sqrt{q}} \text{ or } T \propto \frac{s^{\ 7}}{\tau^7} \qquad \text{…(i)}$$

For adiabatic expansion, $TV^{\gamma-1} = \text{constant}$

$$\text{or}\quad \frac{s^{\ 7}}{\tau^7}V^{\gamma-1} = \text{constant} \Rightarrow \tau \propto s^{\ \frac{(\gamma+6)}{7}}$$

Comparing it with $\tau \propto V^q$, we get $q = \frac{\gamma+6}{7}$

9. **(b) :** Average force applied on the walls by a molecule,

$$c = \frac{7}{\quad}$$

$$\because\quad = \frac{7}{\ } \text{ or } \propto - \quad \therefore\ F \propto v^2 \qquad \text{...(i)}$$

$$\text{K.E.} \propto T;\ \frac{6}{7}\ \ ^7 \propto q \text{ or, } v^2 \propto T \qquad \text{...(ii)}$$

From (i) and (ii), we get $F \propto T$

10. **(c) :** For metals, there is no free motion but rather oscillation about mean position.
Thus these have kinetic energy and potential energy, which are almost equal.

$$33\quad \text{X3L}_{\text{ms}} = \text{R3L}_{\text{ms}} = \frac{8}{7}hq$$

$\therefore$ Total energy, E = K.E. + P.E = $3KT$ per mole
Also, $E = m\ CT\ \therefore\ 3KT = m\ CT$

$$Y = \frac{8h}{\ } = \frac{8o}{j} \text{ or } Y = \frac{8\times=3869}{7<\times65^{-8}} \approx 925\ \text{J kg}^{-1}\ \text{K}^{-1}$$

11. **(d) :** Kinetic energy of vessel $= \frac{1}{2}mv^2$
Increase in internal energy $\Delta U = nC_V\Delta T$
where n is the number of moles of the gas in vessel.
As the vessel is stopped suddenly, its kinetic energy is used to increase the temperature of the gas $\therefore\ \frac{1}{2}mv^2 = \Delta U$

$$\frac{1}{2}mv^2 = nC_V\Delta T$$

$$\frac{1}{2}mv^2 = \frac{m}{M}C_V\Delta T \qquad \left(\because\ n = \frac{m}{M}\right)$$

$$\Delta T=\frac{Mv^2}{2C_V} \quad \text{or} \quad \Delta T=\frac{Mv^2(\gamma-1)}{2R}\text{ K} \qquad \left(\because\ C_V=\frac{R}{(\gamma-1)}\right)$$

12. (b) : The thermal energy or internal energy is $U=\frac{5}{2}\mu RT$ for diatomic gases. (5 is the degrees of freedom as the gas is diatomic)

But $PV=\mu RT$

$$V=\frac{\text{mass}}{\text{density}}=\frac{1\text{ kg}}{4\text{ kg/m}^3}=\frac{1}{4}\text{ m}^3$$

$P = 8\times10^4\text{ N/m}^2 \ \therefore\ U=\frac{5}{2}\times8\times10^4\times\frac{1}{4}=5\times10^4\text{ J}$

13. (b) : Molar heat capacity = Molar mass × specific heat capacity

So, the molar heat capacities at constant pressure and constant volume will be $28C_P$ and $28C_V$ respectively

$\therefore \quad 28C_P-28C_V=R \quad \text{or} \quad C_P-C_V=\frac{R}{28}.$

14. (d) : $\Delta U=0\ \therefore\ 1\times\left(\frac{5}{2}R\right)(T_f-T_0)+1\times\frac{3}{2}R\left(T_f-\frac{7}{3}T_0\right)=0$

or $\quad 5T_f-5T_0+3T_f-7T_0=0$ or $8T_f=12T_0$

or $\quad T_f=\frac{3}{2}T_0.$

15. (d) : For 16 g of helium, $n_1=\frac{16}{4}=4$

For 16 g of oxygen, $n_2=\frac{16}{32}=\frac{1}{2}$

For mixture of gases,

$$C_V=\frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2} \quad \text{where} \quad C_V=\frac{f}{2}R$$

$$C_P=\frac{n_1C_{P_1}+n_2C_{P_2}}{n_1+n_2} \quad \text{where} \quad C_P=\left(\frac{f}{2}+1\right)R$$

For helium, $f=3$, $n_1=4$

For oxygen, $f=5$, $n_2=1/2$

$$\therefore \quad \frac{C_P}{C_V}=\frac{\left(4\times\frac{5}{2}R\right)+\left(\frac{1}{2}\times\frac{7}{2}R\right)}{\left(4\times\frac{3}{2}R\right)+\left(\frac{1}{2}\times\frac{5}{2}R\right)}=\frac{47}{29}=1.62.$$

16. (a) : For mixture of gases,

$$\frac{n_1+n_2}{\gamma_m-1}=\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1} \quad \text{or} \quad \frac{1+1}{\gamma_m-1}=\frac{1}{\frac{5}{3}-1}+\frac{1}{\frac{7}{5}-1}$$

or $\quad \frac{2}{\gamma_m-1}=\frac{3}{2}+\frac{5}{2}=4\Rightarrow\gamma_m-1=0.5 \ \therefore\ \gamma_m=1.5=3/2.$

17. (c) : For mixture of gases $\frac{n_1+n_2}{\gamma_m-1}=\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}$

$$\frac{1+1}{\gamma_m-1}=\frac{1}{\left(\frac{7}{5}-1\right)}+\frac{1}{\left(\frac{5}{3}-1\right)}$$

$$\frac{2}{\gamma_m-1}=\frac{5}{2}+\frac{3}{2} \quad \text{or} \quad \frac{2}{\gamma_m-1}=\frac{8}{2}$$

or $\quad 8\gamma_m-8=4 \quad$ or $\quad 8\gamma_m=12 \quad$ or $\quad \gamma_m=\frac{12}{8}=\frac{24}{16}$

18. (d) : $v_{\text{rms}}=\sqrt{\frac{RT}{M}}\ \therefore\ (v_{\text{rms}})_{O_2}=(v_{\text{rms}})_{H_2}$

or $\quad \sqrt{\frac{273+47}{32}}=\sqrt{\frac{T}{2}}\Rightarrow T=20\text{ K}.$

19. (c) : It is the relative velocities between molecules that is important. Root mean square velocities are different from lateral translation.