CHAPTER 6

Gravitation

1. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$ (c) Zero (d) $-\frac{3}{2}\frac{k}{a^2}$

(2018)

2. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then

(a) $T \propto R^{3/2}$ for any n (b) $T \propto R^{\frac{n}{2}+1}$

(c) $T \propto R^{(n+1)/2}$ (d) $T \propto R^{n/2}$ *(2018)*

3. A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is

(a) $+\frac{GMm}{6R}$ (b) $\frac{GMm}{2R}$ (c) $-\frac{GMm}{2R}$ (d) $-\frac{GMm}{6R}$

(Online 2018)

4. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is

(a) 2 (b) 0.6 (c) 6 (d) 10^{-2}

(Online 2018)

5. Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence

(a) there will be no change in weight anywhere on the earth

(b) weight of the object, everywhere on the earth, will increase

(c) except at poles, weight of the object on the earth will decrease

(d) weight of the object, everywhere on the earth, will decrease.

(Online 2018)

6. Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$

The first particle is at a distance $r = 1$, and the second, at $r = 4$. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to

(a) 6×10^{-2} (b) 10^{-1} (c) 3×10^{-3} (d) 6×10^{2}

(Online 2018)

7. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius)

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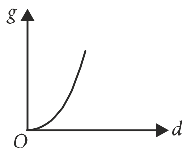

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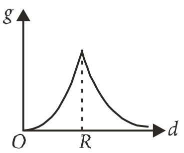

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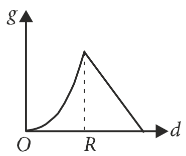

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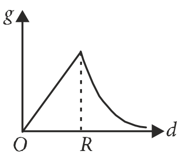


(2017)

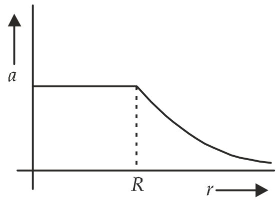
8. If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weigh $\frac{3}{4}W$. Radius of the Earth is 6400 km and $g = 10$ m/s^2.

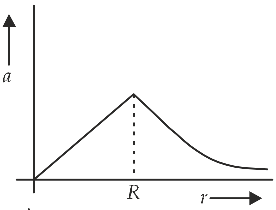
(a) 0.63×10^{-3} rad/s (b) 0.83×10^{-3} rad/s

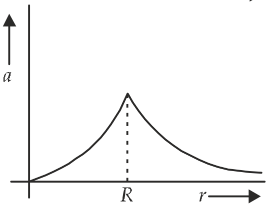
(c) 0.28×10^{-3} rad/s (d) 1.1×10^{-3} rad/s

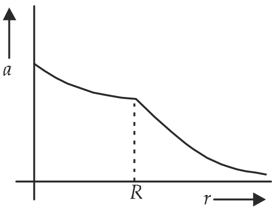
(Online 2017)

9. The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \le R$ and $\rho(r) = 0$ for $r > R$, where r is the distance from the centre. The correct graph that describes qualitatively the acceleration, a of a test particle as a function of r is

(a) 

(b) 

(c) 

(d) 

(Online 2017)

10. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; $h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

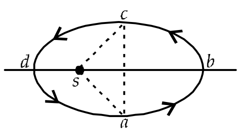
(a) $\sqrt{7\ o}$ (b) $\sqrt{gR}$

(c) $\sqrt{gR/2}$ (d) $\sqrt{\ o}\left(\sqrt{7}-6\right)$

(2016)

11. Figure shows elliptical path $abcd$ of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then

(a) $t_1 = 4t_2$
(b) $t_1 = 2t_2$
(c) $t_1 = 3t_2$
(d) $t_1 = t_2$



(Online 2016)

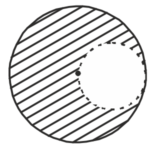
12. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is

(a) Zero since astronaut feels weightless

(b) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$ (c) $F_G = \frac{GMm}{(R+h)^2}$

(d) $0 < F_G < \frac{GMm}{R^2}$ *(Online 2016)*

13. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{o}{7}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is
(G = gravitational constant)



(a) $\frac{-7dj}{8o}$ (b) $\frac{-7dj}{o}$

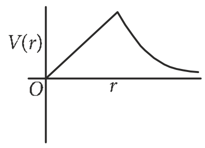
(c) $\frac{-dj}{7o}$ (d) $\frac{-dj}{o}$ *(2015)*

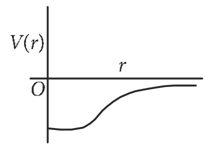
14. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R ($R << L$). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then

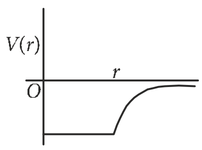
(a) $T^2 \propto r^3$ (b) $T \propto r^2$

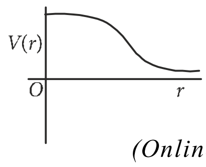
(c) $T \propto r$ (d) $q \propto \sqrt{\ }$ *(Online 2015)*

15. Which of the following most closely depicts the correct variation of the gravitational potential $V(r)$ due to a large planet of radius R and uniform mass density?
(figures are not drawn to scale)

(a) 

(b) 

(c) 

(d) 

(Online 2015)

16. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (b) $\sqrt{\frac{GM}{R}}$

(c) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ (d) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ *(2014)*

17. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?

(a) $\frac{GmM}{3R}$ (b) $\frac{5GmM}{6R}$ (c) $\frac{2GmM}{3R}$ (d) $\frac{GmM}{2R}$

(2013)

18. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be
(a) 6.4×10^8 Joules (b) 6.4×10^9 Joules
(c) 6.4×10^{10} Joules (d) 6.4×10^{11} Joules *(2012)*

19. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is
(a) zero (b) $-\frac{4Gm}{r}$ (c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$
(2011)

20. The height at which the acceleration due to gravity becomes $g/9$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is
(a) $2R$ (b) $\frac{R}{\sqrt{2}}$ (c) $R/2$ (d) $\sqrt{2}R$
(2009)

21. **Directions :** The following question contains statement-1 and statement-2. Of the four choices given, choose the one that best describes the two statements.
Statement-1 : For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.
Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.
(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is false, statement-2 is true.
(c) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(d) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1. *(2008)*

22. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be
(a) 0.11 km s^{-1} (b) 1.1 km s^{-1}
(c) 11 km s^{-1} (d) 110 km s^{-1} *(2008)*

23. Average density of the earth
(a) is directly proportional to g
(b) is inversely proportional to g
(c) does not depend on g
(d) is a complex function of g *(2005)*

24. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere.
(you may take $G = 6.67 \times 10^{-11}$ Nm^2/kg^2)
(a) 6.67×10^{-9} J (b) 6.67×10^{-10} J
(c) 13.34×10^{-10} J (d) 3.33×10^{-10} J *(2005)*

25. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which of the following is correct?
(a) $d = 2h$ (b) $d = h$
(c) $d = h/2$ (d) $d = 3h/2$ *(2005)*

26. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to
(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$.
(2004)

27. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is
(a) $2\,mgR$ (b) $\frac{1}{2}mgR$ (c) $\frac{1}{4}mgR$ (d) mgR.
(2004)

28. The time period of an earth satellite in circular orbit is independent of
(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit.
(2004)

29. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is
(a) gx (b) $\frac{gR}{R-x}$
(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$. *(2004)*

30. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be
(a) $11\sqrt{2}$ km/s (b) 22 km/s
(c) 11 km/s (d) $11/\sqrt{2}$ m/s. *(2003)*

31. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is
(a) $2.5R$ (b) $4.5R$ (c) $7.5R$ (d) $1.5R$.
(2003)

32. The time period of a satellite of earth is 5 hour. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 hour (b) 80 hour
(c) 40 hour (d) 20 hour. *(2003)*

33. The escape velocity of a body depends upon mass as

(a) m^0 (b) m^1 (c) m^2 (d) m^3. *(2002)*

34. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is

(a) $mgR/2$ (b) $2mgR$
(c) mgR (d) $mgR/4$. *(2002)*

35. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R$. *(2002)*

36. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will

(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit in the same velocity
(c) become stationary in its orbit
(d) move towards the earth. *(2002)*

ANSWER KEY

1. (c)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (d)	**11.** (c)	**12.** (c)
13. (d)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (a)	**21.** (c)	**22.** (d)	**23.** (a)	**24.** (b)
25. (a)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (d)	**30.** (c)	**31.** (c)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (d)	**36.** (b)

Explanations

1. **(c) :** Here, $U = -\frac{k}{2r^2}$

Force acting on the particle, $F = -\frac{dU}{dr} = \frac{k}{r^3}$

This force provides necessary centripetal force.

So, $\frac{mv^2}{r} = \frac{k}{r^3}$; $mv^2 = \frac{k}{r^2}$

Kinetic energy of particle, $K = \frac{1}{2}mv^2 = \frac{k}{2r^2}$

Total energy of the particle = $U + K$ $= -\frac{k}{2r^2} + \frac{k}{2r^2} = 0$

2. **(c) :** According to the question, central force is given by

$F_c \propto \frac{1}{R^n}$; $F_c = k\frac{1}{R^n}$

$m\omega^2 R = k\frac{1}{R^n}$; $m\frac{(2\pi)^2}{T^2} = k\frac{1}{R^{n+1}}$ or $T^2 \propto R^{n+1}$; $\therefore$ $T \propto R^{(n+1)/2}$

3. **(d) :** Initially, total energy $E_i = -\frac{GMm}{2R}$

Final total energy, $E_f = -\frac{GM(m/2)}{2(R/2)} - \frac{GM(m/2)}{2(3R/2)} = -\frac{2GMm}{3R}$

Required difference in energies $= E_f - E_i$

$= -\frac{GMm}{R}\left(\frac{2}{3} - \frac{1}{2}\right) = -\frac{GMm}{6R}$

4. **(a) :** $F_1 = \frac{GM_eM_m}{r_1^2}$, $F_2 = \frac{GM_eM_s}{r_2^2}$

$\Delta F_1 = -\frac{2GM_eM_m}{r_1^3}\Delta r_1$, $\Delta F_2 = -\frac{2GM_eM_s}{r_2^3}\Delta r_2$

$\frac{\Delta F_1}{\Delta F_2} = \frac{M_m \Delta r_1}{r_1^3}\ \frac{r_2^3}{M_s \Delta r_2} = \left(\frac{M_m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$

Using $\Delta r_1 = \Delta r_2 = 2R_{\text{earth}}$

$M_m = 8 \times 10^{22}$ kg, $M_s = 2 \times 10^{30}$ kg

$r_1 = 0.4 \times 10^6$ km, $r_2 = 150 \times 10^6$ km. We get $\frac{\Delta F_1}{\Delta F_2} = 2$

5. **(c) :** Effect of rotation of earth on acceleration due to gravity is given by $g' = g - \omega^2 R\cos^2\phi$

Where ϕ is latitude. There will be no change in gravity at poles as $\phi = 90°$, at all other points as ω increases, g' will decrease.

6. **(a) :** Given force provides the centripetal force on the particle moving in circular orbits, $\frac{mv^2}{r} = \frac{16}{r} + r^3$

Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{1}{2}[16 + r^4]$

$\frac{K_1}{K_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} = 0.0625$ or $\frac{K_1}{K_2} \simeq 6 \times 10^{-2}$

7. **(d) :** Variation of g inside the earth's surface at depth h is given by $g' = g\left(1 - \frac{h}{R}\right) = g\left(\frac{R-h}{R}\right) = \frac{gd}{R}$

where d is the distance from the centre of the Earth.

i.e., $g \propto d$ (inside the earth's surface)

Acceleration due to gravity outside the Earth's surface at height h is $' = \frac{}{\left(6 + \frac{}{o}\right)^7} = \frac{o^7}{7}$ *i.e.,* $g' \propto \frac{1}{d^2}$

Hence, option (d) is correct.

8. **(a) :** Here, the weight of person on the equator = W. If earth rotate about its axis, then weight = $\frac{3W}{4}$

Radius of the earth = 6400 km

The acceleration due to gravity at the equator,

$g_e = g - \omega^2 R \cos^2\theta$

$\frac{3}{4}g = g - \omega^2 R \cos^2 0°$ or $\omega^2 R = \frac{g}{4}$

$\omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}} = 0.63 \times 10^{-3}$ rad/s

9. **(a)**

10. **(d) :** Orbital velocity of the satellite,

$_5 = \sqrt{\frac{dj}{o+}}$ also $_5 \approx \sqrt{\frac{dj}{o}}$ $(\because h << R)$

Let v_e be the minimum velocity required by the satellite to escape from its orbit.

$\therefore$ $\frac{6}{7}$ $_7 = \frac{d\ \ j}{o+}$ $\Rightarrow$ $= \sqrt{\frac{7dj}{o+}} \approx \sqrt{\frac{7dj}{o}}$ $(\because h << R)$

so, required increment in the orbital velocity

$= \ - \ _5 = \sqrt{\frac{7dj}{o}} - \sqrt{\frac{dj}{o}} = \sqrt{\frac{dj}{o}}\text{-}\sqrt{7} - 6. = \sqrt{\ o}\text{-}\sqrt{7} - 6.$

11. **(c) :** Let the area of the ellipse be A.

As per Kepler's 2nd law, areal velocity of a planet around the sun is constant, *i.e.,* $\frac{dA}{dt} =$ constant.

$\therefore$ $\frac{t_1}{t_2} = \frac{\text{Area of } abcsa}{\text{Area of } adcsa} = \frac{\frac{A}{2} + \frac{A}{4}}{\frac{A}{2} - \frac{A}{4}} = \frac{\frac{3A}{4}}{\frac{A}{4}} = 3$ $\Rightarrow$ $t_1 = 3t_2$

Note : Here db is the major axis of the ellipse, not semi-major axis and ca is the minor axis of the ellipse, not semi-minor axis.

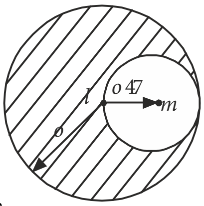
12. **(c) :** Gravitational pull on the astronaut $F_G = \frac{GMm}{(R+h)^2}$

Net force on the astronaunt is zero.

13. **(d) :** Potential at point P (centre of cavity) before removing the spherical portion,

$$s_6 = \frac{-dj}{7o^8}\left(8o^7 - \left(\frac{o}{7}\right)^7\right)$$

$$= \frac{-dj}{7o^8}\left(8o^7 - \frac{o^7}{9}\right) = \frac{-66dj}{=o}$$



Mass of spherical portion to be removed,

$$M' = \frac{j\ s\ '}{s} = \frac{j\ \frac{9\pi}{8}\left(\frac{o}{7}\right)^8}{\frac{9\pi}{8}o^8} = \frac{j}{=}$$

Potential at point P due to spherical portion to be removed

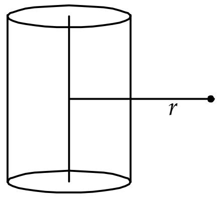
$$V_2 = \frac{-8dj\ '}{7o'} = \frac{-8d\text{-}j\ \ 4=.}{7\text{-}o\ 47.} = \frac{-8dj}{=o}$$

$\therefore$ Potential at the centre of cavity formed $V_P = V_1 - V_2$

$$= \frac{-66dj}{=o} - \left(\frac{-8dj}{=o}\right) = \frac{-dj}{o}$$

14. (c) : Centripetal force is provided by gravitational force $c = \frac{7dj}{i}$ Bc $= \left(-\right)$

k is some constant.



So, $\frac{7}{\quad} = \frac{m}{\quad} \Rightarrow v = \text{constant}$

$T = \frac{7\pi}{\quad} \quad \therefore T \propto r$

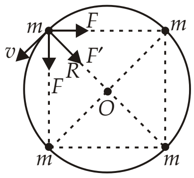
15. (b) : Potential $V(r)$ due to a large planet of radius R is given by

$$s\ \text{-}\ .= -\frac{dj}{\quad};\ r > R$$

$$s\ \text{-}\ .= \frac{-dj}{o}\text{B}\ r = R$$

$$s_{\text{ız}} = -\frac{8}{7}\frac{dj}{o}\left[6 - \frac{7}{8o^7}\right];\ r < R$$

16. (a) : As shown in figure, each mass experiences three forces namely F, F and F'.



Here, F = force between two masses at R separation.

F' = force between two masses at $2R$ separation.

As the all particle move in the circular path of radius R due to their mutual gravitational attraction.

Then net force on mass m = mass × centripetal acceleration.

$$\Rightarrow \frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} + F' = m\frac{v^2}{R} \quad \text{(from figure)}$$

$$\Rightarrow F\sqrt{2} + F' = m\frac{v^2}{R} \Rightarrow \frac{\sqrt{2}Gm^2}{(R\sqrt{2})^2} + \frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \frac{Gm}{R}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = v^2 \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}(1 + 2\sqrt{2})}$$

17. (b): Energy of the satellite on the surface of the planet is $E_i = \text{K.E.} + \text{P.E.} = 0 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{R}$

If v is the velocity of the satellite at a distance $2R$ from the surface of the planet, then total energy of the satellite is

$$E_f = \frac{1}{2}mv^2 + \left(-\frac{GMm}{(R+2R)}\right) = \frac{1}{2}m\left(\sqrt{\frac{GM}{(R+2R)}}\right)^2 - \frac{GMm}{3R}$$

$$= \frac{1}{2}\frac{GMm}{3R} - \frac{GMm}{3R} = -\frac{GMm}{6R}$$

$\therefore$ Minimum energy required to launch the satellite is

$$\Delta E = E_f - E_i = -\frac{GMm}{6R} - \left(-\frac{GMm}{R}\right) = -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5GMm}{6R}$$

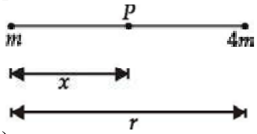
18. (c): Energy required $= \frac{GMm}{R}$

$$= gR^2 \times \frac{m}{R} \qquad \left(\because\ g = \frac{GM}{R^2}\right)$$

$= mgR$

$= 1000 \times 10 \times 6400 \times 10^3 = 64 \times 10^9\ \text{J} = 6.4 \times 10^{10}\ \text{J}$

19. (d) : Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\therefore \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

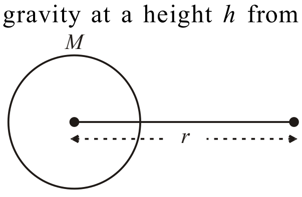
or $\left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4}$ or $x = \frac{r}{3}$ ...(i)

Gravitational potential at a point P is

$$= -\frac{Gm}{x} - \frac{G(4m)}{(r-x)} = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r - \frac{r}{3}\right)} \text{ (Using (i))}$$

$$= -\frac{3Gm}{r} - \frac{3G(4m)}{2r} = -9\frac{Gm}{r}$$

20. (a) : The acceleration due to gravity at a height h from the ground is given as $g/9$.

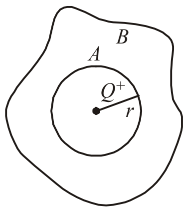


$$\frac{GM}{r^2} = \frac{GM}{R^2}\cdot\frac{1}{9}$$

$\therefore\ r = 3R$

The height above the ground is $2R$.

21. (c) : Let A be the Gaussian surface enclosing a spherical charge Q.

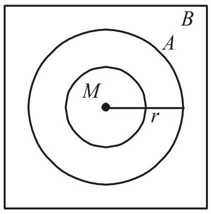


$$\vec{E}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 . r^2}$$

$$\text{Flux } \phi = \vec{E}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

Every line passing through A, has to pass through B, whether B is a cube or any surface. It is only for Gaussian surface, the lines of field should be normal. Assuming the mass is a point mass.



$\vec{g}$, gravitational field $= -\frac{GM}{r^2}$

$$\text{Flux } \phi_g = |\vec{g}\cdot 4\pi r^2| = \frac{4\pi r^2 \cdot GM}{r^2} = 4\pi GM.$$

Here B is a cube. As explained earlier, whatever be the shape, all the lines passing through A are passing through B, although all the lines are not normal.

Statement-2 is correct because when the shape of the earth is spherical, area of the Gaussian surface is $4\pi r^2$. This ensures inverse square law.

22. (d) : $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}}$ for the earth

$v_e = 11 \text{ km s}^{-1}$

Mass of the planet = 10 M_e, Radius of the planet = $R/10$.

$\therefore \quad v_e = \sqrt{\dfrac{2GM \times 10}{R/10}} = 10 \times 11 = 110 \text{ km s}^{-1}$

23. (a) : $\text{Density}(\rho) = \dfrac{\text{Mass of earth}}{\text{Volume of earth}} = \dfrac{M}{(4/3)\pi R^3} = \dfrac{3M}{4\pi R^3}$ ...(i)

$g = \dfrac{GM}{R^2}$ ...(ii)

$\therefore \quad \dfrac{\rho}{g} = \dfrac{3M}{4\pi R^3} \times \dfrac{R^2}{GM} = \dfrac{3}{4\pi RG}$ or $\rho = \dfrac{3}{4\pi RG} g$

$\therefore$ Average density is directly proportional to g.

24. (b) : Gravitational force $F = \dfrac{Gm_1m_2}{R^2}$

$\therefore \quad dW = FdR = \dfrac{Gm_1m_2}{R^2}dR$

$\therefore \quad \int_0^W dW = Gm_1m_2 \int_R^\infty \dfrac{dR}{R^2} = Gm_1m_2\left[-\dfrac{1}{R}\right]_R^\infty = \dfrac{Gm_1m_2}{R}$

$\therefore$ Work done = $\dfrac{(6.67 \times 10^{-11}) \times (100) \times (10 \times 10^{-3})}{10 \times 10^{-2}}$

$= 6.67 \times 10^{-10}$ J.

25. (a) : At height h, $g_h = g\left(1 - \dfrac{2h}{R}\right)$ where $h << R$

or $\quad g - g_h = \dfrac{2hg}{R}$ or $\Delta g_h = \dfrac{2hg}{R}$ ...(i)

At depth d, $g_d = g\left(1 - \dfrac{d}{R}\right)$ where $d << R$

or $\quad g - g_d = \dfrac{dg}{R}$ or $\Delta g_d = \dfrac{dg}{R}$ ...(ii)

From (i) and (ii), when $\Delta g_h = \Delta g_d$

$\dfrac{2hg}{R} = \dfrac{dg}{R}$ or $\quad d = 2h$.

26. (a) : For motion of a planet in circular orbit,
Centripetal force = Gravitational force

$\therefore \quad mR\omega^2 = \dfrac{GMm}{R^n}$ or $\omega = \sqrt{\dfrac{GM}{R^{n+1}}}$

$\therefore \quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R^{n+1}}{GM}} = \dfrac{2\pi}{\sqrt{GM}} R^{\left(\frac{n+1}{2}\right)}$

$\therefore \quad T$ is proportional to $R^{\left(\frac{n+1}{2}\right)}$.

27. (b) : Force on object $= \dfrac{GMm}{x^2}$ at x from centre of earth.

$\therefore$ Work done $= \dfrac{GMm}{x^2}dx \quad \therefore \quad \int \text{Work done} = GMm \int_R^{2R} \dfrac{dx}{x^2}$

$\therefore$ Potential energy gained $= GMm\left[-\dfrac{1}{x}\right]_R^{2R} = \dfrac{GMm \times 1}{2R}$

$\therefore$ Gain in P.E. $= \dfrac{1}{2}mR\left(\dfrac{GM}{R^2}\right) = \dfrac{1}{2}mgR \quad \left[\because g = \dfrac{GM}{R^2}\right]$.

28. (a) : For a satellite
Centripetal force = Gravitational force

$\therefore \quad mR\omega^2 = \dfrac{GMM_e}{R^2}$ where $R = r_e + h$

or $\quad \omega = \sqrt{\dfrac{GM_e}{R^3}} = \sqrt{\dfrac{GM_e}{(r_e+h)^3}} \quad \therefore \quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{(r_e+h)^3}{GM_e}}$

$\therefore \quad T$ is independent of mass (m) of satellite.

29. (d) : For a satellite, centripetal force = gravitational force

$\therefore \quad \dfrac{mv_0^2}{(R+x)} = \dfrac{GMm}{(R+x)^2}$

or $\quad v_0^2 = \dfrac{GM}{(R+x)} = \dfrac{gR^2}{(R+x)} \quad \left[\because g = \dfrac{GM}{R^2}\right]$ or $v_0 = \sqrt{\dfrac{gR^2}{R+x}}$.

30. (c) : The escape velocity of a body does not depend on the angle of projection from earth. It is 11 km/sec.

31. (c) : Let the spheres collide after time t, when the smaller sphere covered distance x_1 and bigger sphere covered distance x_2.

The gravitational force acting between two spheres depends on the distance which is a variable quantity.

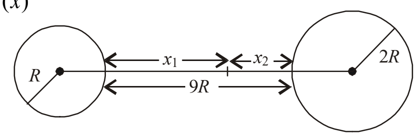
The gravitational force, $F(x) = \dfrac{GM \times 5M}{(12R - x)^2}$

Acceleration of smaller body, $a_1(x) = \dfrac{G \times 5M}{(12R - x)^2}$

Acceleration of bigger body, $a_2(x) = \dfrac{GM}{(12R - x)^2}$

From equation of motion, $x_1 = \dfrac{1}{2}a_1(x)t^2$ and $x_2 = \dfrac{1}{2}a_2(x)t^2$

$\Rightarrow \dfrac{x_1}{x_2} = \dfrac{a_1(x)}{a_2(x)} = 5 \Rightarrow x_1 = 5x_2$



We know that $x_1 + x_2 = 9R$

$x_1 + \dfrac{x_1}{5} = 9R \qquad \therefore \; x_1 = \dfrac{45R}{6} = 7.5R$

Therefore the two spheres collide when the smaller sphere covered the distance of $7.5R$.

32. (c) : According to Kepler's law, $T^2 \propto r^3$

$\therefore \quad \left(\dfrac{T_1}{T_2}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^3 = \left(\dfrac{1}{4}\right)^3 = \dfrac{1}{64}$ or $\dfrac{T_1}{T_2} = \dfrac{1}{8}$

or $\quad T_2 = 8T_1 = 8 \times 5 = 40$ hour.

33. (a) : Escape velocity $= \sqrt{2gR} = \sqrt{\dfrac{2GM_e}{R}}$

Escape velocity does not depend on mass of body which escapes or it depends on m^0.

34. (c) : Escape velocity, $v_e = \sqrt{2gR}$

$\therefore$ Kinetic energy, K.E. $= \dfrac{1}{2}mv_e^2 = \dfrac{1}{2}m \times 2gR = mgR$.

35. (d) : Energy = $(\text{P.E.})_{3R} - (\text{P.E.})_{2R}$

$= -\dfrac{GmM}{3R} - \left(-\dfrac{GmM}{2R}\right) = +\dfrac{GmM}{6R}$.

36. (b) : The centripetal and centrifugal forces disappear, the satellite has the tangential velocity and it will move in a straight line.

Compare Lorentzian force on charges in the cyclotron.