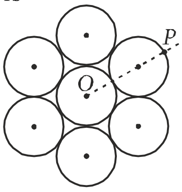
CHAPTER

5 Rotational Motion

1. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is



(a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$

(c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$

(2018)

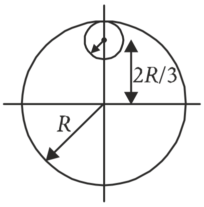
2. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

(a) $4MR^2$

(b) $\frac{40}{9}MR^2$

(c) $10MR^2$

(d) $\frac{37}{9}MR^2$



(2018)

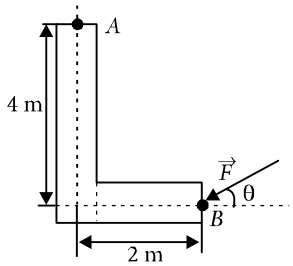
3. A force of 40 N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by

(a) $\tan\theta = \frac{1}{2}$

(b) $\tan\theta = 4$

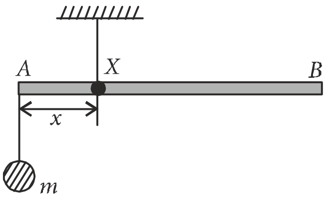
(c) $\tan\theta = 2$

(d) $\tan\theta = \frac{1}{4}$



(Online 2018)

4. A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variables that give a straight line, when plotted, are



(a) $m, \frac{1}{x^2}$ (b) m, x^2 (c) m, x (d) $m, \frac{1}{x}$

(Online 2018)

5. A disc rotates about its axis of symmetry in a horizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of 1.25 cm from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is (g = 10 m/s^2)

(a) 0.7 (b) 0.5 (c) 0.3 (d) 0.6

(Online 2018)

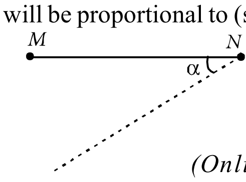
6. A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to (see figure)

(a) $\cos\alpha$

(b) $\sin\alpha$

(c) $\sqrt{\cos\alpha}$

(d) $\sqrt{\sin\alpha}$

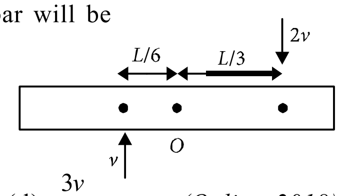


(Online 2018)

7. A thin uniform bar of length L and mass 8 m lies on a smooth horizontal table. Two point masses m and $2m$ are moving in the same horizontal plane from opposite sides of the bar with speeds $2v$ and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be

(a) $\frac{6v}{5L}$

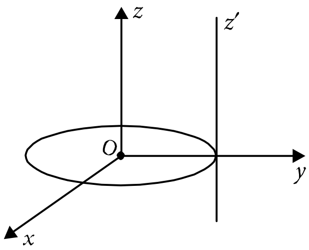
(b) $\frac{v}{6L}$



(c) $\frac{v}{5L}$ (d) $\frac{3v}{5L}$

(Online 2018)

8. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z' axes will be



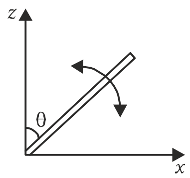
(a) 1 : 4 (b) 1 : 3
(c) 1 : 2 (d) 1 : 5 *(Online 2018)*

9. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum ?

(a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{3}{\sqrt{2}}$

(2017)

10. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is

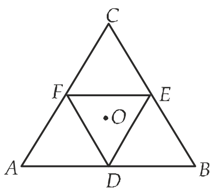


(a) $\frac{3g}{2l}\sin\theta$

(b) $\frac{2g}{3l}\sin\theta$

(c) $\frac{3g}{2l}\cos\theta$ (d) $\frac{2g}{3l}\cos\theta$ *(2017)*

11. Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is I_0 as shown in the figure. A cavity DEF is cut out from the lamina, where D, E, F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is

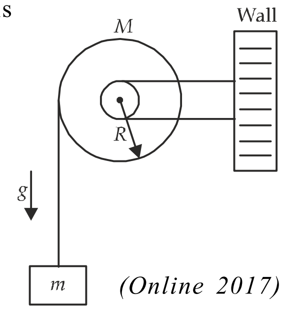


(a) $\frac{7I_0}{8}$ (b) $\frac{31I_0}{32}$

(c) $\frac{3I_0}{4}$ (d) $\frac{15I_0}{16}$

(Online 2017)

12. A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in the figure. The body is released from rest. Then the acceleration of the body is



(a) $\frac{2mg}{2m+M}$

(b) $\frac{2Mg}{2m+M}$

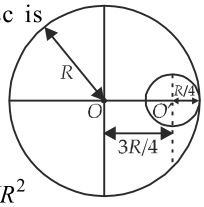
(c) $\frac{2Mg}{2M+m}$

(d) $\frac{2mg}{2M+m}$ *(Online 2017)*

13. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?

(a) Left arm is shorter than the right arm
(b) Left arm is longer than the right arm
(c) Every object that is weighed using this balance appears lighter than its actual weight
(d) Both the arms are of same length *(Online 2017)*

14. A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is



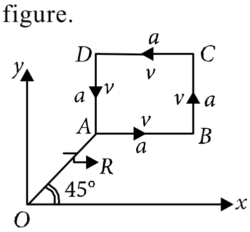
(a) $\frac{219MR^2}{256}$

(b) $\frac{197MR^2}{256}$

(c) $\frac{19MR^2}{512}$ (d) $\frac{237MR^2}{512}$

(Online 2017)

15. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure.



Which of the following statements is false for the angular momentum $\vec{i}$ about the origin?

(a) $\vec{i} = -\frac{}{\sqrt{7}} o\ ^{\mathrm{j}}$ when the particle is moving from A to B.

(b) $\vec{i} = \left[\frac{o}{\sqrt{7}} - \quad\right]^{\mathrm{j}}$ when the particle is moving from C to D.

(c) $\vec{i} = \left[\frac{o}{\sqrt{7}} + \quad\right]^{\mathrm{j}}$ when the particle is moving from B to C.

(d) $\vec{i} = \frac{}{\sqrt{7}} o\ ^{\mathrm{j}}$ when the particle is moving from D to A.

(2016)

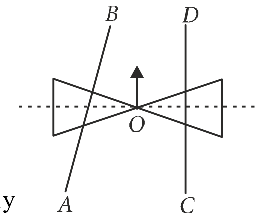
16. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to

(a) turn left

(b) turn right

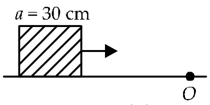
(c) go straight

(d) turn left and right alternately



(2016)

17. A cubical block of side 30 cm is moving with velocity 2 m s^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is



(a) 13.3 (b) 5.0 (c) 9.4 (d) 6.7

(Online 2016)

18. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to (Take the radius of the drum to be 1.25 m and its axle to be horizontal)

(a) 27.0 (b) 0.4 (c) 1.3 (d) 8.0

(Online 2016)

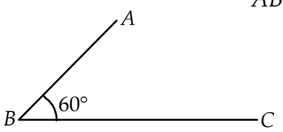
19. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to

(a) 1.85

(b) 1.5

(c) 1.37

(d) 3



(Online 2016)

20. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to

(a) $\frac{:}{=}$ (b) $\frac{8^{7}}{=o}$ (c) $\frac{7}{9o}$ (d) $\frac{8}{9}$

(2015)

21. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is

(a) $\frac{9j\ o^7}{>\sqrt{8}\pi}$ (b) $\frac{9j\ o^7}{8\sqrt{8}\pi}$ (c) $\frac{j\ o^7}{87\sqrt{7}\pi}$ (d) $\frac{j\ o^7}{6;\sqrt{7}\pi}$

(2015)

22. A uniform solid cylindrical roller of mass m is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is a and it is rolling without slipping then the value of F is

(a) ma (b) $2ma$ (c) $\frac{8}{7}$ (d) $\frac{:}{8}$

(Online 2015)

23. Consider a thin uniform square sheet made of a rigid material. If its side is a, mass m and moment of inertia I about one of its diagonals, then

(a) $f > \frac{7}{67}$ (b) $\frac{7}{79} < f < \frac{7}{67}$

(c) $f = \frac{7}{67}$ (d) $f = \frac{7}{79}$ *(Online 2015)*

24. A uniform thin rod AB of length L has linear mass density $\mu- \ . = \ + \frac{\ }{i} 1$ where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{<}{67} i\right)$ from A, then a and b are related as

(a) $a = b$ (b) $a = 2b$

(c) $2a = b$ (d) $3a = 2b$ *(Online 2015)*

25. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is

(a) 8.64 kg m^2 s^{-1} (b) 11.52 kg m^2 s^{-1}

(c) 14.4 kg m^2 s^{-1} (d) 20.16 kg m^2 s^{-1}

(Online 2015)

26. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad s^{-1} about the vertical. About the point of suspension

(a) angular momentum changes both in direction and magnitude.

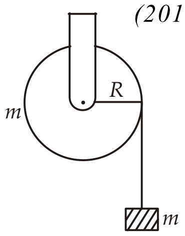
(b) angular momentum is conserved.

(c) angular momentum changes in magnitude but not in direction.

(d) angular momentum changes in direction but not in magnitude.

(2014)

27. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



(a) g (b) $\frac{2g}{3}$ (c) $\frac{g}{2}$ (d) $\frac{5g}{6}$

(2014)

28. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(a) $r\omega_0$ (b) $\frac{r\omega_0}{4}$ (c) $\frac{r\omega_0}{3}$ (d) $\frac{r\omega_0}{2}$

(2013)

29. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is
(a) less than 3
(b) more than 3 but less than 6
(c) more than 6 but less than 9
(d) more than 9 *(2011)*

30. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

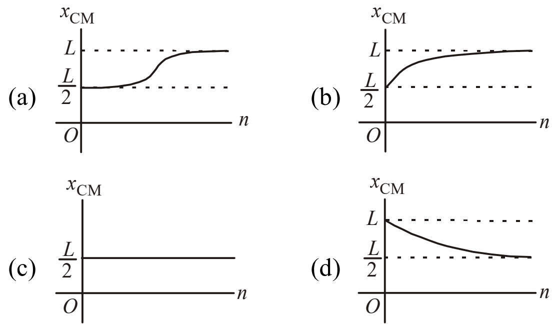
(a) $\frac{3}{2}g$ (b) g (c) $\frac{2}{3}g$ (d) $\frac{g}{3}$ *(2011)*

31. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc
(a) remains unchanged
(b) continuously decreases
(c) continuously increases
(d) first increases and then decreases *(2011)*

32. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$ (c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$ *(2009)*

33. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k(x/L)^n$ where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n?

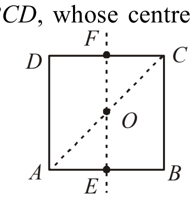


(2008)

34. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\frac{2}{3}ma^2$ (b) $\frac{5}{6}ma^2$ (c) $\frac{1}{12}ma^2$ (d) $\frac{7}{12}ma^2$ *(2008)*

35. For the given uniform square lamina $ABCD$, whose centre is O,



(a) $I_{AC} = \sqrt{2}I_{EF}$
(b) $\sqrt{2}I_{AC} = I_{EF}$
(c) $I_{AD} = 3I_{EF}$
(d) $I_{AC} = I_{EF}$ *(2007)*

36. Angular momentum of the particle rotating with a central force is constant due to
(a) constant torque (b) constant force
(c) constant linear momentum
(d) zero torque *(2007)*

37. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(a) $\frac{g\sin\theta}{1 - MR^2/I}$ (b) $\frac{g\sin\theta}{1 + I/MR^2}$

(c) $\frac{g\sin\theta}{1 + MR^2/I}$ (d) $\frac{g\sin\theta}{1 - I/MR^2}$ *(2007)*

38. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R from the centre of the bigger disc. The value of α is
(a) 1/4 (b) 1/3 (c) 1/2 (d) 1/6. *(2007)*

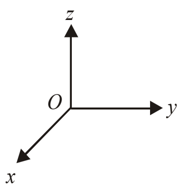
39. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis through A and parallel to BD is
(a) ml^2 (b) $2ml^2$ (c) $\sqrt{3}\,ml^2$ (d) $3ml^2$. *(2006)*

40. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$

(a) $\frac{\omega m}{(m+2M)}$ (b) $\frac{\omega(m+2M)}{m}$

(c) $\frac{\omega(m-2M)}{(m+2M)}$ (d) $\frac{\omega m}{(m+M)}$ *(2006)*

41. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, –1) is



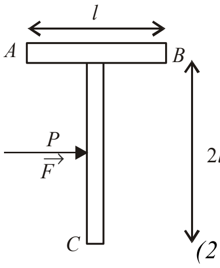
(a) $-F(\hat{i} - \hat{j})$
(b) $F(\hat{i} - \hat{j})$
(c) $-F(\hat{i} + \hat{j})$
(d) $F(\hat{i} + \hat{j})$. *(2006)*

42. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) d (b) $\frac{m_2}{m_1}d$ (c) $\frac{m_1}{m_1+m_2}d$ (d) $\frac{m_1}{m_2}d$.

(2006)

43. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.



(a) $\frac{4}{3}l$

(b) l

(c) $\frac{3}{4}l$

(d) $\frac{3}{2}l$

(2005)

44. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The center of mass of bodies B and C taken together shifts compared to that of body A towards

(a) body C (b) body B

(c) depends on height of breaking

(d) does not shift *(2005)*

45. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the center is

(a) Mr^2 (b) $\frac{1}{2}Mr^2$ (c) $\frac{1}{4}Mr^2$ (d) $\frac{2}{5}Mr^2$

(2005)

46. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $I_A/I_B = d_A/d_B$

where d_A and d_B are their densities. *(2004)*

47. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?

(a) Moment of inertia (b) Angular momentum

(c) Angular velocity (d) Rotational kinetic energy.

(2004)

48. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then

(a) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}\neq 0$ (b) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}=0$

(c) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}\neq 0$ (d) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}=0$

(2003)

49. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

(a) $L/4$ (b) $2L$ (c) $4L$ (d) $L/2$.

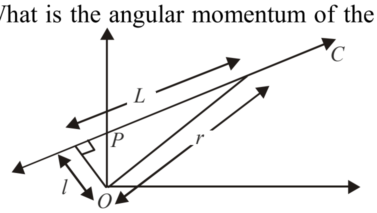
(2003)

50. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$

(c) $I_Y = I_X$ (d) $I_Y = 64I_X$. *(2003)*

51. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P?



(a) mvL (b) mvl (c) mvr (d) zero.

(2002)

52. Moment of inertia of a circular wire of mass M and radius R about its diameter is

(a) $MR^2/2$ (b) MR^2

(c) $2MR^2$ (d) $MR^2/4$. *(2002)*

53. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)

(a) solid sphere (b) hollow sphere

(c) ring (d) all same. *(2002)*

54. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$. *(2002)*

55. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is

(a) v (b) $v/3$ (c) $v/2$ (d) zero.

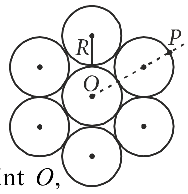
(2002)

ANSWER KEY

1. (d)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (a)	**11.** (d)	**12.** (a)
13. (a)	**14.** (d)	**15.** (b, d)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (d)	**21.** (a)	**22.** (c)	**23.** (c)	**24.** (c)
25. (c)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (b)	**30.** (c)	**31.** (d)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (d)	**36.** (d)
37. (b)	**38.** (*)	**39.** (d)	**40.** (a)	**41.** (d)	**42.** (d)	**43.** (a)	**44.** (d)	**45.** (b)	**46.** (c)	**47.** (b)	**48.** (d)
49. (a)	**50.** (d)	**51.** (d)	**52.** (a)	**53.** (d)	**54.** (c)	**55.** (c)					

Explanations

1. (d) : Moment of inertia of one of the outer disc about an axis passing through point O and perpendicular to the plane



$$I_1 = \frac{1}{2}MR^2 + M(2R)^2 = \frac{9}{2}MR^2$$

Moment of inertia of the system about point O,

$$I_O = \frac{1}{2}MR^2 + 6I_1 = \frac{1}{2}MR^2 + 6\times\frac{9}{2}MR^2 = \frac{55}{2}MR^2$$

Required moment of inertia of the system about point P,

$$I_P = I_O + 7M(3R)^2 = \frac{55}{2}MR^2 + 63MR^2 = \frac{181}{2}MR^2$$

2. (a) : Mass per unit area of disc $= \frac{9M}{\pi R^2}$

$\therefore$ Mass of removed portion $= \frac{9M}{\pi R^2}\times\pi\left(\frac{R}{3}\right)^2 = M$

Let moment of inertia of removed portion $= I_1$

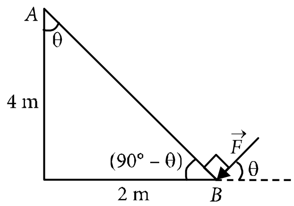
$$\therefore\ I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{MR^2}{2}$$

Let I_2 = Moment of inertia of the whole disc $= \frac{9MR^2}{2}$

Moment of inertia of remaining disc, $I = I_2 - I_1$

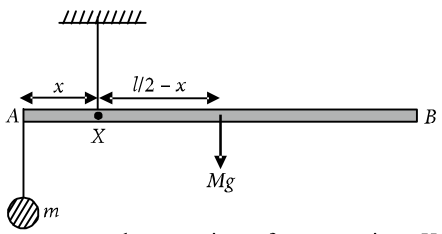
$$\text{or}\quad I = \frac{9MR^2}{2} - \frac{MR^2}{2} = \frac{8MR^2}{2} = 4MR^2$$

3. (a) : Moment of force will be maximum when line of action of force is perpendicular to line AB.



$$\tan\theta = \frac{2}{4} = \frac{1}{2}$$

4. (d) :



Balancing torque about point of suspension X,

$$mgx = Mg\left(\frac{l}{2} - x\right) \Rightarrow mx = M\frac{l}{2} - Mx \Rightarrow m = \left(M\frac{l}{2}\right)\frac{1}{x} - M$$

This is equation of straight line with variables m and $1/x$.

5. (d) : As coin is at rest on rotating disc, centripetal force is provided by the friction force between the coin and disc.

$f = m\omega^2 R$

$$\text{or}\quad \mu mg = m\omega^2 r \quad\text{or}\quad \mu = \frac{\omega^2 r}{g} = \frac{(2\pi\upsilon)^2 r}{g}$$

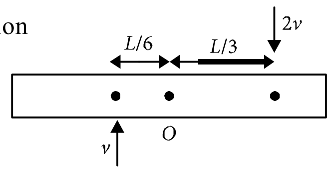
$$\mu = \frac{4\pi^2(3.5)^2\times 1.25\times 10^{-2}}{10} = 604\times 10^{-3} \approx 0.6$$

6. (d) : Using energy conservation principle, loss in potential energy = gain in kinetic energy

$$mgl\sin\alpha = \frac{1}{2}\frac{ml^2}{3}\omega^2 \Rightarrow 6gl\sin\alpha = v^2$$

$$\Rightarrow v = \sqrt{6gl\sin\alpha} \quad\text{or}\quad v\propto\sqrt{\sin\alpha}$$

7. (a) : Using law of conservation of angular momentum, $L_i = L_f$



$$m(2v)\times\frac{L}{3} + 2m(v)\times\frac{L}{6} = I\omega$$

$$mvL = \left[\frac{1}{12}(8m)L^2 + m\left(\frac{L}{3}\right)^2 + 2m\left(\frac{L}{6}\right)^2\right]\omega$$

$$v = L\left(\frac{2}{3}+\frac{1}{9}+\frac{1}{18}\right)\omega = \frac{5}{6}\omega L \quad\text{or}\quad \omega = \frac{6v}{5L}$$

8. (b) : Moment of inertia about z-axis, $I_z = \frac{mR^2}{2}$

Moment of inertia about z'-axis,

$$I_{z'} = I_z + mR^2 = \frac{3}{2}mR^2 \quad\therefore\quad I_z : I_{z'} = 1:3$$

9. (a) : Moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is given by

$$I = \frac{1}{12}ml^2 + \frac{mR^2}{4} \quad\text{or}\quad I = \frac{m}{4}\left(\frac{1}{3}l^2 + R^2\right) \quad \text{... (i)}$$

Also, $m = \rho V = \rho\pi R^2 l$ or $R^2 = \frac{m}{\rho\pi l}$

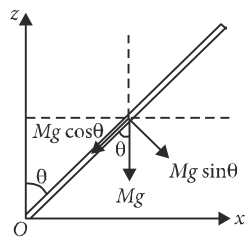
Substitute R^2 in eqn. (i), we get $I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\rho\pi l}\right)$

For moment of inertia to be maximum or minimum,

$$\frac{dI}{dl} = 0 \Rightarrow \frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\rho\pi l^2}\right) = 0$$

$$\Rightarrow \frac{2l}{3} = \frac{R^2}{l} \Rightarrow \frac{l}{R} = \sqrt{\frac{3}{2}} \qquad \left(\text{Using } \frac{R^2}{l} = \frac{m}{\rho\pi l^2}\right)$$

10. (a) : The torque of the weight Mg of the rod about the pivot O is given by



$$\tau = Mg\sin\theta\times\left(\frac{l}{2}\right) \quad \text{...(i)}$$

($Mg\cos\theta$ is passing through the pivot O. Hence, its contribution to the torque will be zero.)

Also,

$$\tau = I\alpha \quad \text{...(ii)}$$

$$\therefore\quad I\alpha = Mg\sin\theta\times\left(\frac{l}{2}\right) \qquad \text{(Using (i) and (ii))}$$

Now, moment of inertia of the rod about the pivot O is

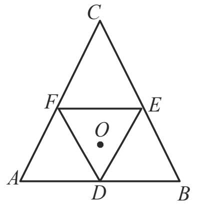
$$I = \frac{1}{3}Ml^2 \quad \therefore \quad \frac{1}{3}Ml^2\alpha = Mg\sin\theta\left(\frac{l}{2}\right) \Rightarrow \quad \alpha = \frac{3}{2}\frac{g}{l}\sin\theta$$

11. (d) : Given $AB = BC = AC = l$

Moment of inertia of a triangular lamina ABC,

$I_0 = kml^2$

$\therefore\ DE = EF = DF = \frac{1}{2}AB = \frac{1}{2}l$



Moment of inertia of ΔDEF,

$$I_{DEF} = k \times \frac{m}{4}\left(\frac{l}{2}\right)^2 = \frac{k}{16}ml^2 = \frac{I_0}{16}$$

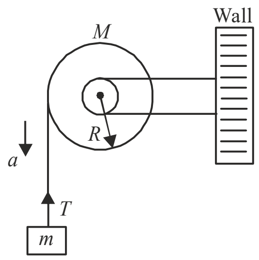
Moment of inertia of the remaining part,

$$I_{\text{remain}} = I_0 - I_{DEF} = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

12. (a) : From figure, we conclude

$mg - T = ma$ ...(i)

Moment of inertia of a uniform disc,



$I = \frac{MR^2}{2}$ and an acceleration is,

$a = \alpha R$

$\because\ RT = I\,\alpha$

$\therefore\ RT = \frac{MR^2}{2} \times \frac{a}{R} \Rightarrow T = \frac{Ma}{2}$

Putting this value in equation (i),

$$mg - \frac{Ma}{2} = ma \text{ or } mg = a\left(m + \frac{M}{2}\right) \Rightarrow a = \frac{2mg}{M + 2m}$$

13. (a)

14. (d) : Moment of inertia of the disc about the given axis,

$$I_D = \frac{MR^2}{2}$$

$$\text{Mass of removed portion} = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{4}\right)^2 = \frac{M}{16}$$

Moment of inertia of removed portion about the given axis (Using parallel axes theorem)

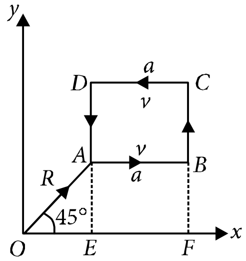
$$I_R = \frac{1}{2}\frac{M}{16}\frac{R^2}{16} + \frac{M}{16} \times \frac{9R^2}{16} = \frac{19MR^2}{512}$$

Required moment of inertia,

$$I = I_D - I_R = \frac{1}{2}MR^2 - \frac{19MR^2}{512} = \frac{237}{512}MR^2$$

15. (b, d) : Here v = speed of the particle

a = side of square



$AE = R \sin 45° = \frac{o}{\sqrt{7}}$

$OE = R \cos 45° = \frac{o}{\sqrt{7}}$

We know,

$\vec{i} = \vec{\ } \times \vec{\ } = \ -uz\,\theta\ \hat{}$

$|\vec{i}| = -\ -uz\,\theta. \ = \perp$

When the particle is moving along AB,

$\vec{i} = \text{-}Wb.\text{-}\ \ .\text{-}\text{–}\hat{}. = -\frac{}{\sqrt{7}}o\,\hat{}$

When the particle is moving along BC,

$\vec{i} = \text{-}l\ c.\text{-}\ \ .\text{-}\hat{}. = \left(\frac{o}{\sqrt{7}} + \right)\hat{}$

When the particle is moving along CD,

$\vec{i} = \text{-}a\,b.\text{-}\ \ .\text{-}\hat{}. = \left(\frac{o}{\sqrt{7}} + \right)\hat{}$

When the particle is moving along DA,

$\vec{i} = \text{-}l\ b.\text{-}\ \ .\text{-}\text{–}\hat{}. = \frac{-}{\sqrt{7}}o\,\hat{}$

Hence, options (b) and (d) are incorrect.

16. (a)

17. (b) : Since no external torque acts on the system, therefore total angular momentum of the system about point O remains constant.

Before hitting, $L_i = mv\frac{a}{2}$

After hitting, $L_f = I\omega \quad \therefore\ mv\frac{a}{2} = I\omega$ or $\omega = \frac{mva}{2I}$

Here I = moment of inertia of cube about its edge

$$= m\frac{a^2}{6} + m\left(\frac{\sqrt{2}a}{2}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

$$\therefore\ \omega = \frac{mva \times 3}{2 \times 2ma^2} = \frac{3v}{4a} = \frac{3 \times 2}{4 \times 0.3} = 5 \text{ rad s}^{-1}$$

18. (a) : Radius of the drum, $R = 1.25$ m

For just one complete rotation, speed of the drum at top position,

$v = \sqrt{Rg}$

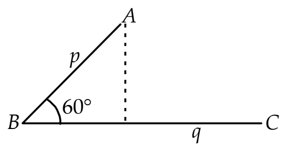
Angular velocity of the drum, $\omega = \frac{v}{R} = \sqrt{\frac{g}{R}}$

$$\omega = \sqrt{\frac{10}{1.25}} \text{ rad s}^{-1} = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} \text{ rpm} = 27 \text{ rpm}$$

19. (c) : Let $AB = p$

$BC = q$

λ = linear mass density of the rod



According to question, centre of mass of the rod lies vertically below point A.

$$\therefore\ X_{CM} = p\cos 60° = \frac{(\lambda q)\left(\frac{q}{2}\right) + (\lambda p)\left(\frac{p}{2}\right)\cos 60°}{\lambda(p+q)}$$

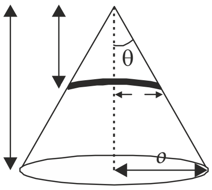
$$\Rightarrow\ \frac{p}{2} = \frac{\frac{q^2}{2} + \frac{p^2}{4}}{(p+q)} \Rightarrow\ p^2 + pq = q^2 + \frac{p^2}{2}$$

$$\Rightarrow\ 1 + \frac{q}{p} = \frac{q^2}{p^2} + \frac{1}{2} \Rightarrow\ \left(\frac{q}{p}\right)^2 - \frac{q}{p} - \frac{1}{2} = 0$$

$$\frac{q}{p} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)\left(-\frac{1}{2}\right)}}{2 \times 1} = \frac{1 \pm \sqrt{3}}{2}$$

$\therefore$ Possible value of $\frac{q}{p} = \frac{1+\sqrt{3}}{2} = 1.366 \approx 1.37$

20. (d) : Let ρ be the density of solid cone. Consider a disc of radius r, thickness dy at a distance of y from its vertex. Then mass of this disc is



$dm = \rho\pi r^2\, dy$

$$\therefore \quad _{oy} = \frac{\int_5}{\int_5} = \frac{\int_5 \rho\pi\ ^7}{\int_5 \rho\pi\ ^7} \qquad ...(i)$$

From figure, $\mu\!\!\!\text{nz}\ \theta = - = \frac{o}{-}\ \{\sim\ = \frac{o}{-}$ ...(ii)

Putting eqn. (ii) in (i), we get

$$_{oy} = \frac{\int_5 \frac{\rho\pi o^7}{7}\ ^8}{\int_5 \frac{\rho\pi o^7}{7}\ ^7} = \frac{\int_5\ ^8}{\int_5\ ^7} = \frac{\left[\frac{9}{9}\right]_5}{\left[\frac{8}{8}\right]_5} = \frac{9}{9} \times \frac{8}{8} = \frac{8}{9}$$

∴ Distance of the centre of mass of a solid uniform cone from its vertex, $_5 = \ _{oy} = \frac{8}{9}3$

21. (a) : A cube of maximum possible volume is cut from a solid sphere of radius R, it implies that the diagonal of the cube is equal to the diameter of sphere, $331\sqrt{8} = 7o$

$$\{\sim \quad = \frac{7o}{\sqrt{8}}$$

Density of solid sphere, $\rho' = \frac{j}{s} = \frac{j}{\frac{9\pi}{8}o^8}$

∴ Mass of cube, $M' = \rho V' = \rho a^3$ (ρ = ρ′)

$$= \frac{j}{\frac{9\pi}{8}o^8}\left(\frac{7o}{\sqrt{8}}\right)^8 = \frac{7j}{\sqrt{8}\pi}$$

Moment of inertia of the cube about an axis passing through its center and perpendicular to one of its faces is

$$f = \frac{j\ '\ ^7}{;} = \frac{6}{;}\frac{7j}{\sqrt{8}\pi}\left(\frac{7o}{\sqrt{8}}\right)^7 = \frac{9j\ o^7}{>\sqrt{8}\pi}$$

22. (c) : Equation of motion for solid cylinder,

$F - f = ma$ …(i) and $fR = I\alpha$

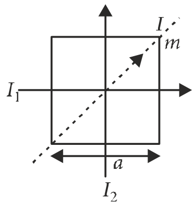
For pure rolling $a = \alpha R$ ∴ $fR = \frac{mR^2}{2}.\frac{a}{R}$

$f = \frac{ma}{2}$ …(ii)

From eqns. (i) and (ii), we get $F - \frac{ma}{2} = ma$

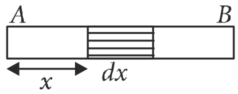
∴ $F = \frac{3}{2}ma$

23. (c) : For a thin uniform square sheet,



$I_1 = I_2 = I = \frac{ma^2}{12}$

24. (c) : Consider a small segment dx of the rod at a distance x from A. Mass of this small segment,



$dm = \mu\, dx = \left(\ + \frac{}{i}\right)$

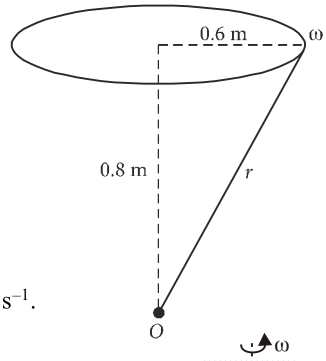
Then CM of the rod AB is given by

$$_{JT} = \frac{\int_5^i -\mu\ .}{\int_5^i \mu}$$

$$\frac{<}{67}i = \frac{\int_5^i \left(\ + \frac{7}{i}\right)}{\int_5^i \left(\ + \frac{}{i}\right)} = \frac{\left(\frac{i^7}{7} + \frac{i^7}{8}\right)}{\left(i + \frac{i}{7}\right)}$$

$$\frac{<}{67} = \frac{\frac{-}{7} + \frac{-}{8}}{+\frac{-}{7}} \quad \therefore \quad = 7$$

25. (c) : Here, $m = 2$ kg, $\omega = 12$ rad s^{-1}



$= \sqrt{-53{=}.^7 + -53.^7} = 6\text{y}$

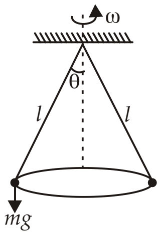
Angular momentum of the particle about point O,

$L = mvr \sin 90°$

$= m \times (0.6\ \omega)r$

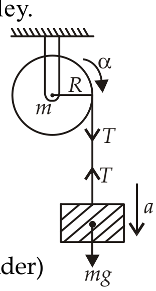
$= 2 \times 0.6 \times 12 \times 1 = 14.4$ kg m^2 s^{-1}.

26. (d) : $\tau(mg) = mg \times l\sin\theta$



Direction of torque by weight is parallel to the plane of rotation of the particle. As $\vec{\tau}$ is perpendicular to the angular momentum of the bob so the magnitude of angular momentum remains same but direction changes.

27. (c) : Here, string is not slipping over pulley.



$a = R\alpha$ ...(i)

Applying Newton's second law on hanging block

$mg - T = ma$ ...(ii)

Torque on cylinder due to tension in string about the fixed point

$T \times R = I\alpha$

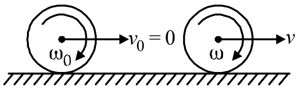
$T \times R = mR^2\alpha$ (∵ $I = mR^2$ for hollow cylinder)

⇒ $T = mR\alpha$

⇒ $T = ma$ [Using eqn. (i)] ...(iii)

From eqns (ii) and (iii)

$$mg = 2ma \Rightarrow a = \frac{g}{2}$$

28. (d) : 

According to law of conservation at point of contact,

$$mr^2\omega_0 = mvr + mr^2\omega = mvr + mr^2\left(\frac{v}{r}\right)$$

$$mr^2\omega_0 = mvr + mvr$$

$$mr^2\omega_0 = 2mvr \quad \text{or} \quad v = \frac{r\omega_0}{2}$$

29. (b) : Torque exerted on pulley $\tau = FR$

or $\alpha = \frac{FR}{I}$ $\qquad\left(\because \alpha = \frac{\tau}{I}\right)$

Here, $F = (20t - 5t^2)$, $R = 2$ m, $I = 10$ kg m^2

$$\therefore \quad \alpha = \frac{(20t - 5t^2) \times 2}{10}$$

$\alpha = (4t - t^2)$ or $\frac{d\omega}{dt} = (4t - t^2)dt$

$d\omega = (4t - t^2)dt$

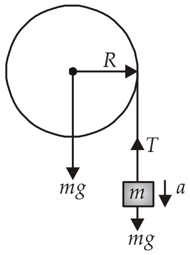
On integrating, $\omega = 2t^2 - \frac{t^3}{3}$. At $t = 6$ s, $\omega = 0$

$$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \text{ or } d\theta = \left(2t^2 - \frac{t^3}{3}\right)dt$$

On integration, $\theta = \frac{2t^3}{3} - \frac{t^4}{12}$. At, $t = 6$ s, $\theta = 36$ rad

$$2\pi n = 36 \Rightarrow n = \frac{36}{2\pi} < 6$$

30. (c) : The free body diagram of pulley and mass



$mg - T = ma$

$\therefore \quad a = \frac{mg - T}{m}$ $\qquad$ ...(i)

As per question, pulley to be consider as a circular disc.

$\therefore$ Angular acceleration of disc

$\alpha = \frac{\tau}{I}$ $\qquad$ ...(ii)

Here, $\tau = T \times R$

and $I = \frac{1}{2}mR^2$ $\qquad$ (For circular disc)

$\therefore \quad T = \frac{mR\alpha}{2}$ $\qquad$ (Using (ii))

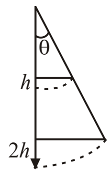
Therefore, $a = \frac{mg - \frac{mR\alpha}{2}}{m}$ $\qquad$ (Using (i))

$ma = mg - \frac{ma}{2}$ $\qquad \left(\because \alpha = \frac{a}{R}\right)$

$\therefore \quad a = \frac{2g}{3}$

31. (d)

32. (d) : The uniform rod of length l and mass m is swinging about an axis passing through the end.



When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the kinetic energy $= \frac{1}{2}I\omega^2$.

$$\Rightarrow mgh = \frac{1}{2}\left(m\frac{l^2}{3}\right)\cdot\omega^2 \quad \therefore \quad h = \frac{l^2\omega^2}{6g}.$$

33. (b) : $x_{\text{C.M.}} = \frac{\int_0^L \left(\frac{k}{L^n}\cdot x^n \cdot dx\right)x}{\int_0^L \frac{k}{L^n}\cdot x^n \cdot dx}$

$$\Rightarrow x_{\text{C.M.}} = \frac{\int_0^L x^{n+1}dx}{\int_0^L x^n dx} = \frac{L^{n+2}}{n+2}\cdot\frac{(n+1)}{L^{n+1}} \Rightarrow x_{\text{C.M}} = \frac{L(n+1)}{(n+2)}$$

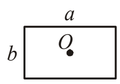
The variation of the centre of mass with x is given by

$$\frac{dx}{dn} = L\left\{\frac{(n+2)1 - (n+1)}{(n+2)^2}\right\} = \frac{L}{(n+2)^2}$$

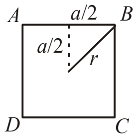
If the rod has the same density as at $x = 0$ *i.e.*, $n = 0$, therefore uniform, the centre of mass would have been at $L/2$. As the density increases with length, the centre of mass shifts towards the right. Therefore it can only be (b).

34. (a) : For a rectangular sheet moment of inertia passing through O, perpendicular to the plate is

$$I_0 = m\left(\frac{a^2 + b^2}{12}\right)$$



for square plate it is $\frac{ma^2}{6}$.



$$r = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}. \quad \therefore r^2 = \frac{a^2}{2}$$

$\therefore \quad I$ about B parallel to the axis through O is

$$I_o + md^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{4ma^2}{6} \text{ or } I = \frac{2}{3}ma^2$$

35. (d) : By perpendicular axes theorem,

$$I_{EF} = M\frac{a^2 + b^2}{12} = \frac{M(a^2 + a^2)}{12} = M\frac{2a^2}{12}$$

$$I_z = \frac{M(2a^2)}{12} + \frac{M(2a^2)}{12} = \frac{Ma^2}{3}.$$

By perpendicular axes theorem,

$$I_{AC} + I_{BD} = I_z \Rightarrow I_{AC} = \frac{I_z}{2} = \frac{Ma^2}{6}$$

By the same theorem $I_{EF} = \frac{I_z}{2} = \frac{Ma^2}{6} \therefore I_{AC} = I_{EF}$.

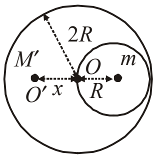
36. (d) : Central forces passes through axis of rotation so torque is zero.

If no external torque is acting on a particle, the angular momentum of a particle is constant.

37. (b) : Acceleration of a uniform body of radius R and mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal is given by

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}.$$

38. (*) : $(M' + m) = M = \pi\,(2R)^2\cdot\sigma$



where σ = mass per unit area

$m = \pi R^2\cdot\sigma$, $M' = 3\pi R^2\sigma$

$$\frac{3\pi R^2\sigma\cdot x+\pi R^2\sigma\cdot R}{M}=0$$

Because for the full disc, the centre of mass is at the centre O.

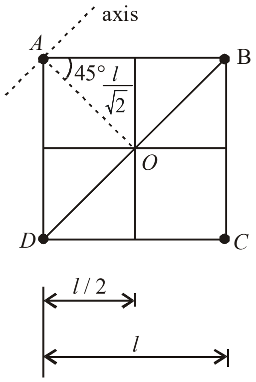
$\Rightarrow \quad x=-\frac{R}{3}=\alpha R \quad \therefore\ |\alpha| = \left|\frac{-1}{3}\right|$

The centre of mass is at $R/3$ to the left on the diameter of the original disc.

The question should be at a distance αR and not α/R.

* None of the option is correct.

39. (d) : $AO\cos 45° = \frac{l}{2}$



$\therefore \quad AO\times\frac{1}{\sqrt{2}}=\frac{l}{2}$

or $\quad AO = \frac{l}{\sqrt{2}}$

$I = I_D + I_B + I_C$

or $\quad I=\frac{2ml^2}{2}+m\left(\frac{2l}{\sqrt{2}}\right)^2$

$I=\frac{2ml^2}{2}+\frac{4ml^2}{2}$ or $I=\frac{6ml^2}{2}=3ml^2.$

40. (a) : Angular momentum is conserved $\therefore L_1 = L_2$

$\therefore \quad mR^2\,\omega = (mR^2 + 2\,MR^2)\,\omega' = R^2\,(m + 2M)\,\omega'$

or $\quad \omega'=\frac{m\omega}{m+2M}.$

41. (d) : Torque $\vec{\tau}=\vec{r}\times\vec{F}$

$\vec{F}=-F\hat{k},\ \vec{r}=\hat{i}-\hat{j} \quad \therefore\ \vec{r}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 0\\ 0 & 0 & -F\end{vmatrix}$

$=\hat{i}F-\hat{j}(-F)=F(\hat{i}+\hat{j}).$

42. (d) : Let m_2 be moved by x so as to keep the centre of mass at the same position $\therefore\ m_1d + m_2\,(-x) = 0$

or $\quad m_1d = m_2x \quad$ or $\quad x=\frac{m_1}{m_2}d.$

43. (a) : It is a case of translation motion without rotation. The force should act at the centre of mass

$$Y_{cm}=\frac{(m\times 2l)+(2m\times l)}{m+2m}=\frac{4l}{3}.$$

44. (d) : The centre of mass of bodies B and C taken together does not shift as no external force is applied horizontally.

45. (b) : $I=\frac{(\text{Mass of semicircular disc})\times r^2}{2}$ or $I=\frac{Mr^2}{2}.$

46. (c) : For solid sphere, $I_A=\frac{2}{5}MR^2$

For hollow sphere, $I_B=\frac{2}{3}MR^2$

$\therefore \quad \frac{I_A}{I_B}=\frac{2MR^2}{5}\times\frac{3}{2MR^2}=\frac{3}{5}$ or $I_A < I_B$.

47. (b) : Free space implies that no external torque is operating on the sphere. Internal changes are responsible for increase in radius of sphere. Here the law of conservation of angular momentum applies to the system.

48. (d) : $\because\ \vec{T}=\vec{r}\times\vec{F} \ \therefore\ \vec{r}\cdot\vec{T}=\vec{r}\cdot(\vec{r}\times\vec{F})=0$

Also $\vec{F}\cdot\vec{T}=\vec{F}\cdot(\vec{r}\times\vec{F})=0$.

49. (a) : Angular momentum $L = I\omega$

Rotational kinetic energy $(K)=\frac{1}{2}I\omega^2$

$\therefore \quad \frac{L}{K}=\frac{I\omega\times 2}{I\omega^2}=\frac{2}{\omega}\Rightarrow L=\frac{2K}{\omega}$

or $\quad \frac{L_1}{L_2}=\frac{K_1}{K_2}\times\frac{\omega_2}{\omega_1}=2\times 2=4 \quad \therefore\ L_2=\frac{L_1}{4}=\frac{L}{4}$

50. (d) : Mass of disc $X=(\pi R^2t)\sigma$ where σ = density

$\therefore \quad I_X=\frac{MR^2}{2}=\frac{(\pi R^2t\sigma)R^2}{2}=\frac{\pi R^4\sigma t}{2}$

Similarly, $I_Y=\frac{(\text{Mass})(4R)^2}{2}=\frac{\pi(4R)^2}{2}\frac{t}{4}\sigma\times 16R^2$

or $\quad I_Y = 32\pi R^4t\sigma \quad \therefore\ \frac{I_X}{I_Y}=\frac{\pi R^4\sigma t}{2}\times\frac{1}{32\pi R^4\sigma t}=\frac{1}{64}$

$\therefore \quad I_Y = 64\ I_X$

51. (d) : The particle moves with linear velocity v along line PC. The line of motion is through P.

Hence angular momentum is zero.

52. (a) : A circular wire behaves like a ring

M.I. about its diameter $=\frac{MR^2}{2}.$

53. (d) : The bodies slide along inclined plane. They do not roll. Acceleration for each body down the plane = $g\sin\theta$. It is the same for each body.

54. (c) : Angular momentum of the system is conserved

$\therefore \quad \frac{1}{2}MR^2\ \omega_1=2mR^2\omega+\frac{1}{2}MR^2\omega$

or $\quad M\omega_1 = (4m + M)\omega \quad$ or $\quad \omega=\frac{M\omega_1}{M+4m}.$

55. (c) : $v_c=\frac{m_1v_1+m_2v_2}{m_1+m_2}$ or $v_c=\frac{m(2v)+m(-v)}{m+m}=\frac{v}{2}.$