CHAPTER

4

Work, Energy and Power

1. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

(2018)

2. A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90° with respect to each other. The mass of unknown particle is

(a) m (b) $2m$ (c) $\frac{m}{\sqrt{3}}$ (d) $\frac{m}{2}$

(Online 2018)

3. A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t seconds after the start of the motion is

(a) $\frac{1}{4}ma^4t^2$ (b) $4ma^4t^2$

(c) $\frac{1}{8}ma^4t^2$ (d) $8ma^4t^2$ *(Online 2018)*

4. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -k v^2$. Its initial speed is $v_0 = 10$ m s^{-1}. If, after 10 s, its energy is $1/8\, mv_0^2$, the value of k will be

(a) 10^{-3} kg m^{-1} (b) 10^{-3} kg s^{-1}

(c) 10^{-4} kg m^{-1} (d) 10^{-1} kg m^{-1} s^{-1} *(2017)*

5. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec will be

(a) 4.5 J (b) 22 J

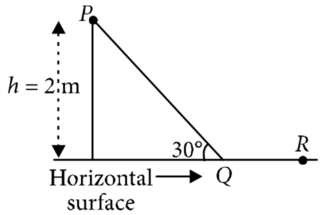
(c) 9 J (d) 18 J *(2017)*

6. An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered at $t \to \infty$ is

(a) $2h$ (b) $\frac{8}{3}h$ (c) $\frac{5}{3}h$ (d) ∞

(Online 2017)

7. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.



The values of the coefficient of friction μ and the distance x (= QR), are respectively close to :

(a) 0.2 and 6.5 m

(b) 0.2 and 3.5 m

(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m *(2016)*

8. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8$ m s^{-2}.

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg

(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

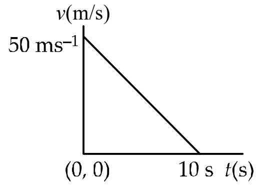
(2016)

9. A car of weight W is on an inclined road that rises by 100 m over a distance of 1 km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of 10 m s^{-1}, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed v then value of v is

(a) 20 m s^{-1} (b) 5 m s^{-1}

(c) 15 m s^{-1} (d) 10 m s^{-1} *(Online 2016)*

10. Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two seconds of the motion is



(a) – 9300 J

(b) 12000 J

(c) – 4500 J

(d) – 12000 J

(Online 2016)

11. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n^2Rt^2 where n is a constant. The power delivered to the particle by the force acting on it, is

(a) $\frac{1}{2}Mn^2R^2t^2$ (b) Mn^2R^2t

(c) MnR^2t^2 (d) MnR^2t

(Online 2016)

12. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(a) 56% (b) 62% (c) 44% (d) 50%

(2015)

13. A block of mass $m = 0.1$ kg is connected to a spring of unknown spring constant k. It is compressed to a distance x from its equilibrium position and released from rest. After approaching half the distance $\left(\frac{}{7}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity 3 m s^{-1}. The total initial energy of the spring is

(a) 1.5 J (b) 0.6 J (c) 0.3 J (d) 0.8 J

(Online 2015)

14. A particle is moving in a circle of radius r under the action of a force $F = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for $r = 0$)

(a) αr^3 (b) $\frac{6}{7}\alpha$ 8 (c) $\frac{9}{8}\alpha$ 8 (d) $\frac{:}{;}\alpha$ 8

(Online 2015)

15. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is

(a) $\frac{1}{2}\left(\frac{aL^2}{2}+\frac{bL^3}{3}\right)$ (b) $aL^2 + bL^3$

(c) $\frac{1}{2}(aL^2+bL^3)$ (d) $\frac{aL^2}{2}+\frac{bL^3}{3}$ *(2014)*

16. This question has Statement-I and Statement-II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement-1 : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $h\left(\frac{1}{2}mv^2\right)$ then $h=\left(\frac{m}{M+m}\right)$

Statement-2 : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement-I is false, Statement-II is true.

(b) Statement-I is true, Statement-II is true, Statement-II is a correct explanation of Statement-I.

(c) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation of statement-I.

(d) Statement-I is true, Statement-II is false. *(2013)*

17. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement 2 : $k_1 < k_2$.

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(d) Statement 1 is false, Statement 2 is true. *(2012)*

18. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

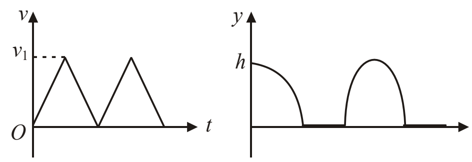
(a) Statement-1 is true, Statement-2 is false.

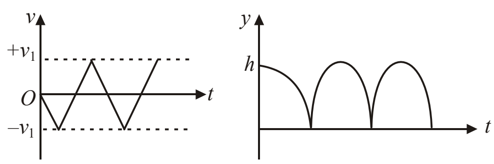
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.

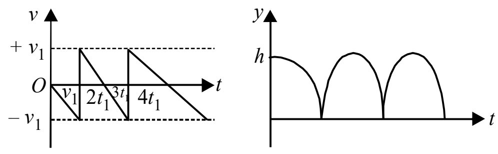
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

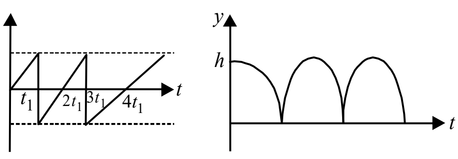
(d) Statement-1 is false, Statement-2 is true. *(2010)*

19. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as function of time will be

(a)


(b)


(c)


(d)


(2009)

20. A block of mass 0.50 kg is moving with a speed of 2.00 m s^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is
(a) 0.34 J (b) 0.16 J (c) 1.00 J (d) 0.67 J.
(2008)

21. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range
(a) 2,000 J - 5,000 J (b) 200 J - 500 J
(c) 2×10^5 J - 3×10^5 J (d) 20,000 J - 50,000 J.
(2008)

22. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is
(a) $K/2$ (b) K (c) zero (d) $K/4$
(2007)

23. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by
(a) 8.5 cm (b) 5.5 cm (c) 2.5 cm (d) 11.0 cm
(2007)

24. The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) \text{J}.$$

The total mechanical energy of the particle 2 J. Then, the maximum speed (in m/s) is
(a) 2 (b) $3/\sqrt{2}$ (c) $\sqrt{2}$ (d) $1/\sqrt{2}$.
(2006)

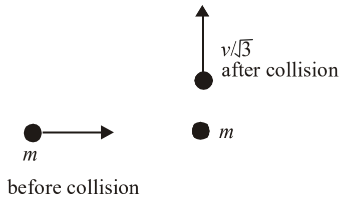
25. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is
(a) 0.5 J (b) –0.5 J (c) –1.25 J (d) 1.25 J.
(2006)

26. A bomb of mass 16 kg at rest explodes into two pieces of masses of 4 kg and 12 kg. The velocity of the 12 kg mass is 4 m s^{-1}. The kinetic energy of the other mass is
(a) 96 J (b) 144 J (c) 288 J (d) 192 J.
(2006)

27. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string making an angle of 45° with the initial vertical direction is
(a) $Mg(\sqrt{2}-1)$ (b) $Mg(\sqrt{2}+1)$
(c) $Mg\sqrt{2}$ (d) $\frac{Mg}{\sqrt{2}}$.
(2006)

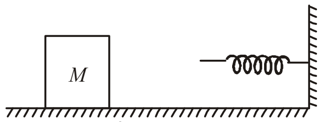
28. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by
(a) $\frac{1}{2}\frac{mv^2}{T^2}t$ (b) $\frac{1}{2}\frac{mv^2}{T^2}t^2$
(c) $\frac{mv^2}{T^2}\cdot t$ (d) $\frac{mv^2}{T^2}\cdot t^2$
(2005, 2004)

29. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the first mass moves with velocity in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



(a) $\frac{2}{\sqrt{3}}v$
(b) $\frac{v}{\sqrt{3}}$
(c) v
(d) $\sqrt{3}v$
(2005)

30. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is



(a) zero (b) $\frac{ML^2}{K}$ (c) $\sqrt{MK}\,L$ (d) $\frac{KL^2}{2M}$
(2005)

31. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is
(a) 10 m/s (b) 34 m/s (c) 40 m/s (d) 20 m/s
(2005)

32. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle in joule is
(a) –7 (b) +7 (c) +10 (d) +13.
(2004)

33. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?
(a) 7.2 J (b) 3.6 J (c) 120 J (d) 1200 J.
(2004)

34. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that
(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line.
(2004)

35. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(a) x^2 (b) e^x (c) x (d) $\log_e x$. *(2004)*

36. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(a) 12.50 N m (b) 18.75 N m
(c) 25.00 N m (d) 6.25 N m. *(2003)*

37. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$. *(2003)*

38. Consider the following two statements.
A. Linear momentum of a system of particles is zero.
B. Kinetic energy of a system of particles is zero.
Then

(a) A does not imply B and B does not imply A
(b) A implies B but B does not imply A
(c) A does not imply B but B implies A
(d) A implies B and B implies A. *(2003)*

39. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 16 J (b) 8 J (c) 32 J (d) 24 J. *(2002)*

40. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should

(a) increase
(b) remain unchanged
(c) decrease
(d) first increase then decrease. *(2002)*

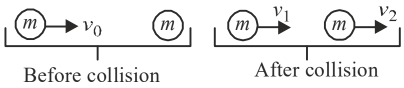
41. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be

(a) E (b) $E/\sqrt{2}$
(c) $E/2$ (d) zero *(2002)*

ANSWER KEY

1. (b)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (*)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (c)	**11.** (b)	**12.** (a)
13. (b)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (d)	**21.** (a)	**22.** (d)	**23.** (b)	**24.** (b)
25. (c)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (c)	**31.** (b)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (b)
37. (b)	**38.** (c)	**39.** (b)	**40.** (c)	**41.** (c)							

Explanations

1. (b) : 

Using conservation of linear momentum, $p_i = p_f$

$mv_0 = mv_1 + mv_2$ or $v_1 + v_2 = v_0$ ...(i)

According to the question,

$$K_f = \frac{3}{2}K_i \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\times\frac{1}{2}mv_0^2$$

$$v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \quad \text{...(ii)}$$

From eqn (i), $(v_1 + v_2)^2 = v_0^2$ or $v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

$$2v_1v_2 = v_0^2 - \frac{3}{2}v_0^2 \qquad \text{(Using equation (ii))}$$

$$2v_1v_2 = -\frac{v_0^2}{2} \text{ or } 4v_1v_2 = -v_0^2$$

Now, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$

$$= v_0^2 - (-v_0^2) = 2v_0^2; \therefore\ v_1 - v_2 = \sqrt{2}\,v_0$$

2. (a) : Before collision net momentum $\vec{p}_i = m\vec{u}$

After collision, $\vec{p}_f = m\vec{v}_1 + m'\vec{v}_2$

Using momentum conservation, $m\vec{u} = m\vec{v}_1 + m'\vec{v}_2$ ...(i)

Using kinetic energy conservation, $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}m'v_2^2$

$$mu^2 = mv_1^2 + m'v_2^2 \quad \text{...(ii)}$$

From equation (i); $m^2u^2 = m^2v_1^2 + m'^2v_2^2$ ...(iii)

Dividing equation (iii) by equation (ii) we get,

$$\frac{m^2(u^2 - v_1^2)}{m(u^2 - v_1^2)} = \frac{m'^2v_2^2}{m'v_2^2} \quad \therefore\ m' = m$$

3. (c) : $v = a\sqrt{s}$ or $\frac{ds}{dt} = a\sqrt{s}$ or $\int_0^s \frac{ds}{\sqrt{s}} = \int_0^t a\,dt$

$$2\sqrt{s} = at \Rightarrow s = \frac{a^2t^2}{4}$$

Velocity at any time t, $v = \frac{ds}{dt} = \frac{a^2t}{2}$

Using work energy theorem,

Work done by all forces in first t seconds = Change in kinetic energy $= \frac{1}{2}mv^2 - 0 = \frac{1}{2}m\times\frac{a^4t^2}{4} = \frac{1}{8}ma^4t^2$

4. (c) : Initial K.E. of the body, $K_i = \frac{1}{2}mv_0^2$

Final K.E. of the body, $K_f = \frac{1}{8}mv_0^2$. Now, $\frac{K_i}{K_f} = 4$

Let initial velocity = v_i, Final velocity = v_f

$$\frac{v_i^2}{v_f^2} = \frac{4}{1} \text{ or } v_f = \frac{v_i}{2} \Rightarrow v_f = \frac{v_0}{2} = \frac{10}{2} = 5\text{ m s}^{-1}$$

(Given v_0 = 10 m s^{-1})

Also, $F = -kv^2 \Rightarrow m\frac{dv}{dt} = -kv^2$; $\frac{-m}{k}\frac{dv}{v^2} = dt$

Integrating both sides

$$\Rightarrow \frac{-m}{k}\int_{10}^{5}\frac{dv}{v^2} = \int_0^{10} dt;\ \frac{-m}{k}\left[\frac{-1}{v}\right]_{10}^{5} = [t]_0^{10}$$

$$\Rightarrow \frac{-10^{-2}}{k}\left(\frac{-1}{5} + \frac{1}{10}\right) = (10 - 0);\ \frac{10^{-3}}{k} = 10$$

$\therefore$ $k = 10^{-4}$ kg m^{-1}

5. (a) : We have been given, $F = 6t$ or $m\frac{dv}{dt} = 6t$

Rearranging and integrating both sides

$$\Rightarrow \int_0^v dv = 6\int_0^1 t\,dt \qquad (\because\ m = 1\text{ kg})$$

$$\Rightarrow v = 6\left[\frac{t^2}{2}\right]_0^1 \Rightarrow v = \frac{6}{2} = 3\text{ m s}^{-1}$$

Work done by the force during the first 1 s is given by the change in the kinetic energy of the object.

$$W = \Delta K.E. = \frac{1}{2}mv^2 \Rightarrow W = \frac{1}{2}\times 1\times(3)^2 = 4.5\text{ J}$$

6. (*) : The kinetic energy of an object just after it hits the ground = 50% of K.E. of the object

$$\frac{1}{2}mv'^2 = \frac{1}{2}\times\frac{1}{2}mv^2 \Rightarrow v' = \frac{v}{\sqrt{2}}$$

$$e = \frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}} = \frac{v'}{v} = \frac{1}{\sqrt{2}}$$

When a ball dropped from a height h, total distance covered at $t \to \infty$, $S = h\left[\frac{1+e^2}{1-e^2}\right]$

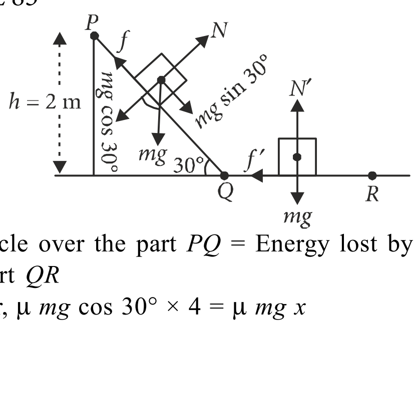
$$S = h\left[\frac{1+1/2}{1-1/2}\right] = 3h$$

* None of the given options is correct.

7. (c) : Here, $m = \frac{}{-\mathbf{u}\ 85°} = 7\ = 9$ y

$QR = x = ?$, $\mu = ?$

Energy of the particle is lost only due to friction between the track and the particle.



According to the question,

Energy lost by the particle over the part PQ = Energy lost by the particle over the part QR

or, $f \times PQ = f' \times QR$ or, $\mu\, mg \cos 30° \times 4 = \mu\, mg\, x$

or, $x = 4 \cos 30° = 9\frac{\sqrt{8}}{7} = 7\sqrt{8}$ y ≈ 83 y

Using work energy theorem for the motion of the particle,
$mgh - (f \times PQ) - f'(QR) = 0 - 0$
or $mgh - 2f'(QR) = 0$ or, $mgh - 2\mu mg\,x = 0$

$\therefore\ \mu = \frac{}{7} = \frac{7}{7\times 7\sqrt{8}} = 537 = \approx 537>$

8. (d) : Here, $m = 10$ kg, $h = 1$ m, $g = 9.8$ m s^{-2}
$n = 1000$
Energy of fat $= 3.8 \times 10^7$ J kg^{-1}

Efficiency, $\eta = 20\% = \frac{1}{5}$

Net work done by the man in lifting the mass
$= n \times$ (Gain in potential energy of the mass)
$= n(mgh) = 1000 \times 10 \times 9.8 \times 1 = 98000$ J

$$\eta = \frac{\text{Net work done by the man}}{\text{Energy in the fat}}$$

$$\frac{6}{:} = \frac{>=555}{\times 83 = \times 65^{<}} \ \{\sim 1 \quad = \frac{>=555\times :}{83 = \times 65^{<}} \quad \therefore\ m = 12.89 \times 10^{-3} \text{ kg}$$

9. (c) : Inclination of road, $\theta = \tan^{-1}\left(\frac{100}{1000}\right) = \tan^{-1}\left(\frac{1}{10}\right)$

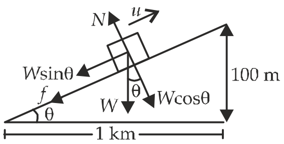
$\Rightarrow\ \tan\theta = \frac{1}{10} \Rightarrow \tan\theta \approx \sin\theta = \frac{1}{10}$ (for very small value of θ)

When car is moving uphill

$P = Fu = (W\sin\theta + f)u$

$= \left(\frac{W}{10} + \frac{W}{20}\right)\times 10$

$= \frac{3W}{20}\times 10 = \frac{3W}{2}$

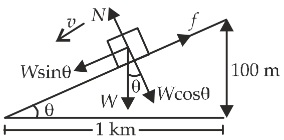


When car is moving downhill

$\frac{P}{2} = F'v = (W\sin\theta - f)v$

$\Rightarrow\ \frac{3W}{4} = \left(\frac{W}{10} - \frac{W}{20}\right)v$

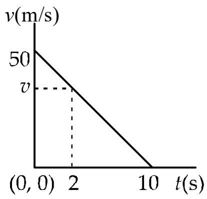
$\Rightarrow\ \frac{3W}{4} = \frac{W}{20}v$ or $v = 15$ m s^{-1}



10. (c) : Here, $m = 10$ kg, $t = 2$ s
$u = 50$ m s^{-1} at $t = 0$ s

$a = \frac{\Delta v}{\Delta t} = \frac{50-0}{0-10} = -5$ m s^{-2}

Speed of the body at $t = 2$ s
$v = u + at = 50 + (-5) \times 2 = 40$ m s^{-1}



Using work energy theorem,
Work done on the body = change in kinetic energy of the body

$$\Delta W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}\times 10\times 40^2 - \frac{1}{2}\times 10 \times 50^2$$

$= 5 \times (40 - 50)(40 + 50) = -4500$ J

11. (b) : Centripetal acceleration $= n^2Rt^2 = \frac{v_t^2}{R}$

$v_t^2 = n^2R^2t^2 \quad \Rightarrow\ v_t = nRt$

Tangential force on the particle, $F_t = M\frac{dv_t}{dt} = MnR$

Power delivered to the particle $= F_tv_t = (MnR)(nRt) = Mn^2R^2t$

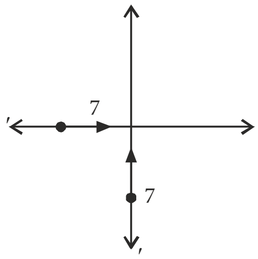
12. (a) : Applying the principle of momentum conservation

$-7\ \hat{}\ .+7\ \ -\ \hat{}\ .=-\ \ +7\ .\ \vec{}$

$7\ \ \hat{}+7\ \ \hat{}=8\ \vec{}$

$\Rightarrow\ \vec{} = \frac{7}{8}(\hat{} + \hat{})$

$\{\sim\ \ ' = |\vec{}| = \frac{7\sqrt{7}}{8}$



Initial energy of the system,

$E_i = \frac{6}{7}m(2v)^2 + \frac{6}{7}(2m)v^2 = 2mv^2 + mv^2 = 3mv^2$

Final energy of the system, $E_f = \frac{6}{7}(3m)v'^2$

$= \frac{8}{7}\left(\frac{7\sqrt{7}}{8}\right)^7 = \frac{9}{8}\ \ 7$

$\therefore$ Percentage loss in the energy $= \frac{b\ - b}{b} \times 100$

$= \frac{8\ \ 7 - \frac{9}{8}\ \ 7}{8\ \ 7} \times 100 = \frac{:}{>} \times 100 \simeq 56\%$

13. (b) : The block comes to rest means its velocity at that point was 3 m s^{-1}.
So at that point, kinetic energy of the spring block system is,

K.E. $= \frac{6}{7}\times\ \ ^7 = \frac{6}{7}\times 536\times -8.^7 = \frac{53>}{7} = 539:$ Ç$[\because\ m = 0.1$ kg$]$

At displacement $\frac{}{7}1$ P.E. $= \frac{6}{9}$^3L3B K.E. $= \frac{8}{9}$^3L3

So T.E. $= \frac{9}{8}\times 539: = 0.6$ J

14. (d) : Centripetal force $F = \alpha r^2$

$\frac{7}{} = \alpha\ ^7$

$\therefore$ K.E. of the particle, $K = \frac{6}{7}\ \ ^7 = \frac{\alpha\ ^8}{7}$

P.E. of the particle, $U = \int_5 c\ \ = \int_5 \alpha\ ^7\ \ = \frac{\alpha\ ^8}{8}$

Total energy of the particle, $E = K + U = \frac{\alpha\ ^8}{7} + \frac{\alpha\ ^8}{8}$

$= \frac{:}{;}\alpha\ ^8$

15. (d) : Restoring force, $F = ax + bx^2$
Work done in stretching the rubber-band by a small amount dx is given by $dW = Fdx$
Net work done in stretching the rubber-band by L is

$$W = \int dW = \int_0^L Fdx \Rightarrow \quad W = \int_0^L (ax + bx^2)dx = \left[a\frac{x^2}{2} + b\frac{x^3}{3}\right]_0^L$$

$$\Rightarrow\quad W = \frac{aL^2}{2} + \frac{bL^3}{3}$$

16. (a) : Loss of energy is maximum when collision is inelastic.

Maximum energy loss $= \frac{1}{2}\frac{mM}{(M+m)}u^2 \quad \therefore \quad f = \frac{mM}{(M+m)}$

Hence, Statement-1 is false, Statement-2 is true.

17. (d) : For the same force, $F = k_1x_1 = k_2x_2$ ...(i)

Work done on spring S_1 is

$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1}$ (Using (i))

Work done on spring S_2 is

$W_2 = \frac{1}{2}k_2x_2^2 = \frac{(k_2x_2)^2}{2k_2} = \frac{F^2}{2k_2}$ (Using (i))

$\therefore \frac{W_1}{W_2} = \frac{k_2}{k_1}$

As $W_1 > W_2 \quad \therefore \quad k_2 > k_1$ or $k_1 < k_2$

Statement 2 is true.

For the same extension, $x_1 = x_2 = x$ ...(ii)

Work done on spring S_1 is $W_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}k_1x^2$ (Using (ii))

Work done on spring S_2 is $W_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}k_2x^2$ (Using (ii))

$\therefore \frac{W_1}{W_2} = \frac{k_1}{k_2}$ As $k_1 < k_2 \quad \therefore \quad W_1 < W_2$

Statement 1 is false.

18. (b)

19. (c) : $v = u + gt$. As the ball is dropped, $v = gt$ when coming down. v increases, makes collision, the value of v becomes +ve, decreases, comes to zero and increases. The change from $+v$ to $-v$ is almost instantaneous. Using –ve signs when coming down, (c) is correct.

Further $h = \frac{1}{2}gt^2$ is a parabola. Therefore (c).

20. (d) : By the law of conservation of momentum

$mu = (M + m)v$

$0.50 \times 2.00 = (1 + 0.50)\,v, \quad \frac{1.00}{1.50} = v$

Initial K.E. $= (1/2) \times 0.50 \times (2.00)^2 = 1.00$ J.

Final K.E. $= \frac{1}{2} \times 1.50 \times \frac{1.00^2}{(1.50)^2} = \frac{1.00}{3.00} = 0.33$

$\therefore$ Loss of energy $= 1.00 - 0.33 = 0.67$ J.

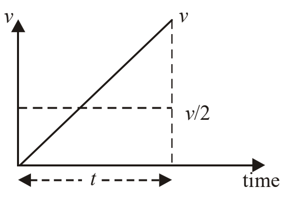
21. (a) : $\bar{v} = v/2$ is average velocity

$s = 100$ m, $t = 10$ s.

$\therefore \ (v/2) = 10$ m/s.

$v_{average} = (v/2) = 10$ m/s.

Assuming an athelete has about 50 to 100 kg, his kinetic energy would have been $\frac{1}{2}mv_{av}^2$.



For 50 kg, $(1/2) \times 50 \times 100 = 2500$ J.

For 100 kg, $(1/2) \times 100 \times 100 = 5000$ J.

It could be in the range of 2000 to 5000 J.

22. (d) : The kinetic energy of a particle is K.

At highest point velocity has its horizontal component. Therefore kinetic energy of a particle at highest point is

$K_H = K\cos^2\theta = K\cos^2 60° = \frac{K}{4}$.

23. (b) : Let the spring be compressed by x

Initial kinetic energy of the mass = potential energy of the spring + work done due to friction

$\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 10000 \times x^2 + 15x$

or $5000x^2 + 15x - 16 = 0$ or $x = 0.055$ m $= 5.5$ cm.

24. (b) : Total energy $E_T = 2$ J. It is fixed.

For maximum speed, kinetic energy is maximum.

The potential energy should therefore be minimum.

$\because \ V(x) = \frac{x^4}{4} - \frac{x^2}{2}$ or $\frac{dV}{dx} = \frac{4x^3}{4} - \frac{2x}{2} = x^3 - x = x(x^2 - 1)$

For V to be minimum, $\frac{dV}{dx} = 0$

$\therefore \ x(x^2 - 1) = 0$, or $x = 0, \pm 1$

At $x = 0$, $V(x) = 0$. At $x = \pm 1$, $V(x) = -\frac{1}{4}$ J

$\therefore$ (Kinetic energy)$_{max} = E_T - V_{min}$

or (Kinetic energy)$_{max} = 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$ J or $\frac{1}{2}m\,v_m^2 = \frac{9}{4}$

or $v_m^2 = \frac{9 \times 2}{m \times 4} = \frac{9 \times 2}{1 \times 4} = \frac{9}{2} \ \therefore \ v_m = \frac{3}{\sqrt{2}}$ m/s.

25. (c) : Kinetic energy at projection point is converted into potential energy of the particle during rise. Potential energy measures the workdone against the force of gravity during rise.

$\therefore$ (– work done) = Kinetic energy $= \frac{1}{2}mv^2$

or (– work done) $= \frac{1}{2} \times \left(\frac{100}{1000}\right)(5)^2 = \frac{5 \times 5}{2 \times 10} = 1.25$ J

$\therefore$ Work done by force of gravity $= -1.25$ J

26. (c) : Linear momentum is conserved

$\therefore \ 0 = m_1v_1 + m_2v_2 = (12 \times 4) + (4 \times v_2)$

or $4v_2 = -48 \Rightarrow v_2 = -12$ m/s

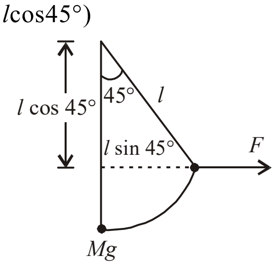
$\therefore$ Kinetic energy of mass $m_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2} \times 4 \times (-12)^2 = 288$ J.

27. (a) : Work done in displacement is equal to gain in potential energy of mass

Work done $= F \times l\sin 45° = \frac{Fl}{\sqrt{2}}$

Gain in potential energy $= Mg(l - l\cos 45°)$

$= Mgl\left(1 - \frac{1}{\sqrt{2}}\right)$



$\therefore \ \frac{Fl}{\sqrt{2}} = \frac{Mgl(\sqrt{2} - 1)}{\sqrt{2}}$

or $F = Mg(\sqrt{2} - 1)$.

28. (c) : Power = Force × velocity = $(ma)(v) = (ma)(at) = ma^2t$

or Power $= m\left(\frac{v}{T}\right)^2(t) = \frac{mv^2}{T^2}t$

29. (a) : Let v_1 = speed of second mass

$\because$ Momentum is conserved

Along X-axis, $mv_1\cos\theta = mv$ ...(i)

Along Y-axis, $mv_1 \sin\theta = \frac{mv}{\sqrt{3}}$ ...(ii)

From (i) and (ii)

$\therefore \quad (mv_1\cos\theta)^2 + (mv_1\sin\theta)^2 = (mv)^2 + \left(\frac{mv}{\sqrt{3}}\right)^2$

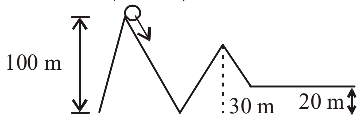
or $\quad m^2v_1^2 = \frac{4m^2v^2}{3}$ or $\quad v_1 = \frac{2}{\sqrt{3}}v$.

30. (c) : Elastic energy stored in spring $= \frac{1}{2}KL^2$

$\therefore$ kinetic energy of block $E = \frac{1}{2}KL^2$

Since $p^2 = 2ME$ $\therefore$ $p = \sqrt{2ME} = \sqrt{\frac{2M \times KL^2}{2}} = \sqrt{MK}\,L$.

31. (b) : $mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right) = \frac{1}{2}mv^2 \cdot \frac{7}{5}$



$\therefore \frac{1}{2}mv^2\left(\frac{7}{5}\right) = mg \times 80$

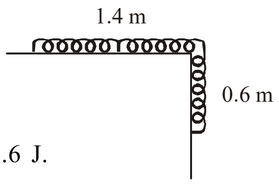
or $\quad v^2 = 2 \times 10 \times 80 \times \frac{5}{7} = 1600 \times \frac{5}{7}$ or $v = 34$ m/s.

32. (b) : Work done $= \vec{F} \cdot \vec{r}$

or $\quad$ work done $= (5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})$

or $\quad$ work done $= 10 - 3 = 7$ J.

33. (b) : The centre of mass of the hanging part is at 0.3 m from table



mass of hanging part

$= \frac{4 \times 0.6}{2} = 1.2$ kg

$\therefore \quad W = mgh = 1.2 \times 10 \times 0.3 = 3.6$ J.

34. (c) : No work is done when a force of constant magnitude always acts at right angles to the velocity of a particle when the motion of the particle takes place in a plane.
Hence kinetic energy of the particle remains constant.

35. (a) : Given : Retardation $\propto$ displacement

or $\quad \frac{dv}{dt} = kx$

or $\quad \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = kx$ or $\quad dv\,(v) = kx\,dx$

or $\quad \int_{v_1}^{v_2} v\,dv = k\int_0^x x\,dx$ or $\frac{v_2^2}{2} - \frac{v_1^2}{2} = \frac{kx^2}{2}$

or $\quad \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \frac{mkx^2}{2}$ or $(K_2 - K_1) = \frac{mk}{2}x^2$

or $\quad$ Loss of kinetic energy is proportional to x^2.

36. (b) : Force constant of spring $(k) = F/x$ or $F = kx$

$\therefore \quad dW = kxdx$ or $\int dW = \int_{0.05}^{0.1} kx\,dx = \frac{k}{2}\left[(0.1)^2 - (0.05)^2\right]$

$= \frac{k}{2} \times [0.01 - 0.0025]$

or $\quad$ Work done $= \frac{(5\times 10^3)}{2} \times (0.0075) = 18.75$ N m.

37. (b) : Power $= \frac{\text{Work}}{\text{Time}} = \frac{\text{Force}\times\text{distance}}{\text{Time}} = \text{Force}\times\text{velocity}$

$\therefore \quad$ Force $\times$ velocity = constant (K)

or $\quad (ma)\,(at) = K$ $\quad$ or $\quad a = \left(\frac{K}{mt}\right)^{1/2}$ $\quad \because \quad s = \frac{1}{2}at^2$

$\therefore \quad s = \frac{1}{2}\left(\frac{K}{mt}\right)^{1/2} t^2 = \frac{1}{2}\left(\frac{K}{m}\right)^{1/2} t^{3/2}$ or s is proportional to $t^{3/2}$.

38. (c) : Kinetic energy of a system of particles is zero. It means that each particle has zero velocity. Hence linear momentum of the system is zero. So, B implies A. Linear momentum of system of particles is zero. It means that velocity of particles may have different directions hence kinetic energy of the system cannot be zero. So, A does not implies B.

39. (b) : $W = \int_{x_1}^{x_2} F dx = \int_{0.05}^{0.15} kx\,dx$

$\therefore \quad W = \int_{0.05}^{0.15} 800x dx = \frac{800}{2}\left[x^2\right]_{0.05}^{0.15} = 400\left[(0.15)^2 - (0.05)^2\right]$

or $\quad W = 8$ J.

40. (c) : When water is cooled to form ice, its thermal energy decreases. By mass energy equivalent, mass should decrease.

41. (c) : Kinetic energy point of projection $(E) = \frac{1}{2}mu^2$

At highest point velocity $= u\cos\theta$

$\therefore \quad$ Kinetic energy at highest point

$= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2 45° = \frac{E}{2}$.