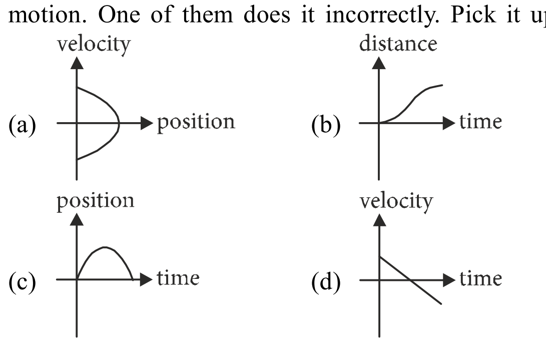
CHAPTER 2

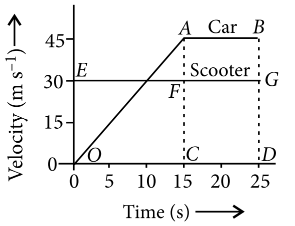
Kinematics

1. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



(2018)

2. The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively.



(a) 112.5 m and 15 s (b) 337.5 m and 25 s
(c) 225.5 m and 10 s (d) 112.5 m and 22.5 s

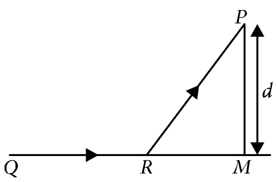
(Online 2018)

3. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding)

(a) 100 m (b) 75 m (c) 160 m (d) 150 m

(Online 2018)

4. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum?



(a) $\frac{d}{2}$ (b) $\frac{d}{\sqrt{3}}$ (c) $\frac{d}{\sqrt{2}}$ (d) d

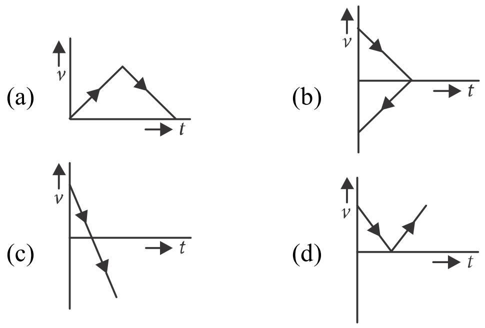
(Online 2018)

5. Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (2\hat{i} - \hat{j})$. The magnitude of a coplanar vector such that $\vec{A}\cdot\vec{C} = \vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{B}$, is given by

(a) $\sqrt{\frac{20}{9}}$ (b) $\sqrt{\frac{5}{9}}$ (c) $\sqrt{\frac{9}{12}}$ (d) $\sqrt{\frac{10}{9}}$

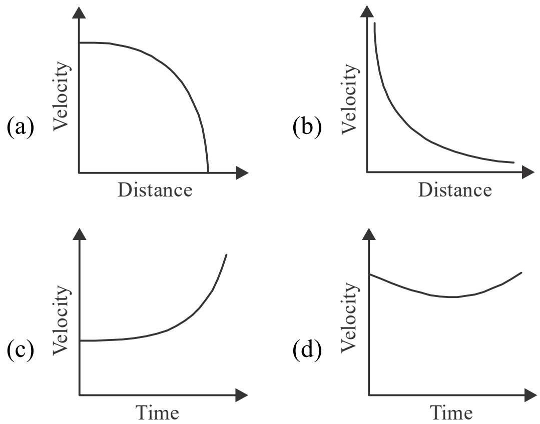
(Online 2018)

6. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity versus time?



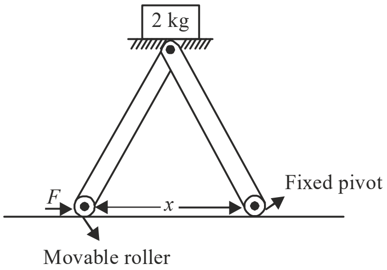
(2017)

7. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity?



(Online 2017)

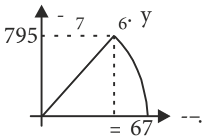
8. The machine as shown has 2 rods of length 1 m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot. As the roller goes back and forth, a 2 kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a

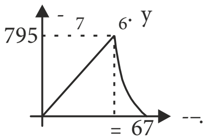


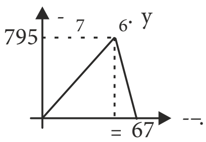
(a) speed which is $\frac{3}{4}$th of that of the roller when the weight is 0.4 m above the ground
(b) constant speed (c) decreasing speed
(d) increasing speed *(Online 2017)*

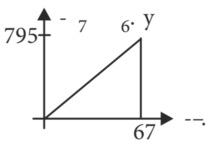
9. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2m/s^2 and the car has acceleration 4m/s^2.The car will catch up with the bus after a time of
(a) $\sqrt{120}$ s (b) 15 s
(c) $10\sqrt{2}$ s (d) $\sqrt{110}$ s *(Online 2017)*

10. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)
(The figures are schematic and not drawn to scale)

(a) 

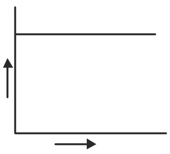
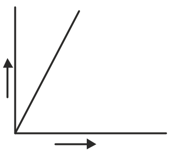
(b) 

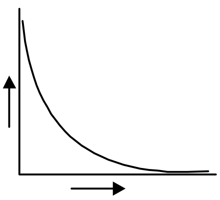
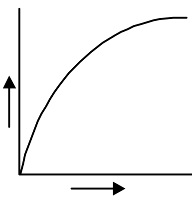
(c) 

(d) 

(2015)

11. If a body moving in a circular path maintains constant speed of 10 m s^{-1}, then which of the following correctly describes relation between acceleration and radius ?

(a) 
(b) 

(c) 
(d) 

(Online 2015)

12. A vector $\vec{W}$ is rotated by a small angle $\Delta\theta$ radians ($\Delta\theta$ < < 1) to get a new vector $\vec{X}$. In that case $|\vec{B}-\vec{A}|$ is
(a) 0 (b) $\vec{W}\left(6-\frac{\Delta\theta^7}{7}\right)$
(c) $\vec{W}\,\Delta\theta$ (d) $\vec{X}\,\Delta\theta-\vec{W}$
(Online 2015)

13. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is
(a) $gH = (n-2)u^2$ (b) $2\,gH = n^2u^2$
(c) $gH = (n-2)^2u^2$ (d) $2\,gH = nu^2(n-2)$ *(2014)*

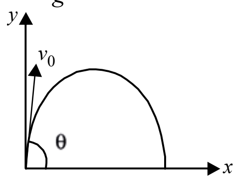
14. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is
(a) $4y = 2x - 25x^2$ (b) $y = x - 5x^2$
(c) $y = 2x - 5x^2$ (d) $4y = 2x - 5x^2$ *(2013)*

15. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be
(a) 10 m (b) $10\sqrt{2}$ m (c) 20 m (d) $20\sqrt{2}$ m
(2012)

16. An object moving with a speed of 6.25 m s^{-1}, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object, to come to rest, would be
(a) 1 s (b) 2 s (c) 4 s (d) 8 s
(2011)

17. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is
(a) $\pi\frac{v^2}{g}$ (b) $\pi\frac{v^4}{g^2}$ (c) $\frac{\pi}{2}\frac{v^4}{g^2}$ (d) $\pi\frac{v^2}{g^2}$
(2011)

18. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0\sin\theta}{g}$, the angular momentum of the particle is



(a) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$
(b) $-mgv_0t^2\cos\theta\,\hat{j}$
(c) $mgv_0t\cos\theta\,\hat{k}$ (d) $-\frac{1}{2}mg\,v_0t^2\cos\theta\,\hat{k}$
where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. *(2010)*

19. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is
(a) $y^2 = x^2$ + constant (b) $y = x^2$ + constant
(c) $y^2 = x$ + constant (d) xy = constant *(2010)*

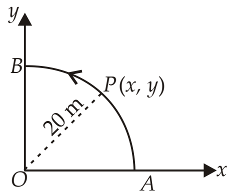
20. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

(a) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$ (b) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$

(c) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$ (d) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$

(2010)

21. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly



(a) 14 m s^{-2}

(b) 13 m s^{-2}

(c) 12 m s^{-2}

(d) 7.2 m s^{-2} *(2010)*

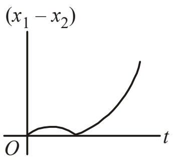
22. A particle has an initial velocity $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is

(a) 10 units (b) $7\sqrt{2}$ units

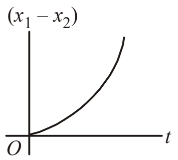
(c) 7 units (d) 8.5 units *(2009)*

23. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

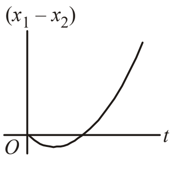
(a)



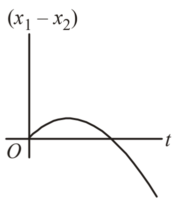
(b)



(c)



(d)



(2008)

24. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$

(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$ *(2007)*

25. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

(a) t^3 (b) t^2 (c) t (d) $t^{1/2}$.

(2006)

26. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?

(a) 293 m (b) 111 m

(c) 91 m (d) 182 m *(2005)*

27. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed in 15 s, then

(a) $s = \frac{1}{2}ft^2$ (b) $s = \frac{1}{4}ft^2$

(c) $s = ft$ (d) $s = \frac{1}{6}ft^2$ *(2005)*

28. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is

(a) $-2av^3$ (b) $2av^2$ (c) $-2av^2$ (d) $2bv^3$

(2005)

29. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is

(a) zero

(b) $\frac{1}{\sqrt{2}}$ m s^{-2} towards north-west

(c) $\frac{1}{\sqrt{2}}$ m s^{-2} towards north-east

(d) $\frac{1}{2}$ m s^{-2} towards north *(2005)*

30. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flights in the two cases, then the product of the two time of flights is proportional to

(a) $1/R$ (b) R

(c) R^2 (d) $1/R^2$. *(2005, 04)*

31. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e.* 120 km/h, the stopping distance will be

(a) 20 m (b) 40 m (c) 60 m (d) 80 m.

(2004)

32. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $T/3$ second?

(a) $h/9$ metre from the ground

(b) $7h/9$ metre from the ground

(c) $8h/9$ metre from the ground

(d) $17h/18$ metre from the ground. *(2004)*

33. If $\vec{A}\times\vec{B} = \vec{B}\times\vec{A}$, then the angle between A and B is

(a) π (b) $\pi/3$ (c) $\pi/2$ (d) $\pi/4$.

(2004)

34. Which of the following statements is false for a particle moving in a circle with a constant angular speed?

(a)` The velocity vector is tangent to the circle.
(b) The acceleration vector is tangent to the circle.
(c) The acceleration vector points to the centre of the circle.
(d) The velocity and acceleration vectors are perpendicular to each other. *(2004)*

35. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) yes, 60°
(b) yes, 30°
(c) no
(d) yes, 45°. *(2004)*

36. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is
(a) 12 m (b) 18 m (c) 24 m (d) 6 m. *(2003)*

37. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ?
[g = 10 m/s^2, sin30° = 1/2, cos30° = $\sqrt{3}/2$]
(a) 5.20 m
(b) 4.33 m
(c) 2.60 m
(d) 8.66 m. *(2003)*

38. The co-ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by
(a) $3t\sqrt{\alpha^2+\beta^2}$
(b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$
(d) $\sqrt{\alpha^2+\beta^2}$. *(2003)*

39. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then
(a) $v_B > v_A$
(b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses. *(2002)*

40. Speeds of two identical cars are u and $4u$ at a specific instant. If the same deceleration is applied on both the cars, the ratio of the respective distances in which the two cars are stopped from that instant is
(a) 1 : 1 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16. *(2002)*

41. If a body looses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?
(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm. *(2002)*

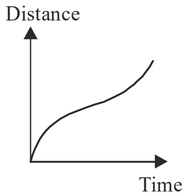
42. Two forces are such that the sum of their magnitudes is 18 N and their resultant is 12 N which is perpendicular to the smaller force. Then the magnitudes of the forces are
(a) 12 N, 6 N
(b) 13 N, 5 N
(c) 10 N, 8 N
(d) 16 N, 2 N. *(2002)*

ANSWER KEY

1. (b)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)	**11.** (c)	**12.** (c)
13. (d)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (d)	**21.** (a)	**22.** (b)	**23.** (c)	**24.** (c)
25. (b)	**26.** (a)	**27.** (*)	**28.** (a)	**29.** (b)	**30.** (b)	**31.** (d)	**32.** (c)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (c)
37. (d)	**38.** (b)	**39.** (b)	**40.** (d)	**41.** (a)	**42.** (b)						

Explanations

1. **(b) :** In options (a), (c) and (d), given graphs represent uniformly decelerated motion of a particle in a straight line with positive initial velocity. Distance-time graph of such a motion is shown here.



2. **(d) :** Distance travelled by car in 15 s

$= \frac{1}{2} \times AC \times OC = \frac{1}{2}(45)(15) = \frac{675}{2}$ m

Distance travelled by scooter in 15 s

$= v \times t = 30 \times 15 = 450$ m

Required difference in distance

$= 450 - \frac{675}{2} = \frac{225}{2} = 112.5$ cm

Let car catches scooter in time t,

$\frac{675}{2} + 45(t - 15) = 30t$

$337.5 + 45t - 675 = 30t$

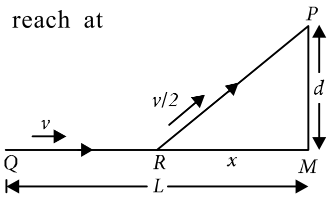
$\Rightarrow \; 15\,t = 337.5 \;\Rightarrow t = 22.5$ s

3. **(c) :** Using, $v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$s = \frac{u^2}{2a} \; \therefore \; \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2} \Rightarrow s_2 = \left(\frac{u_2}{u_1}\right)^2 s_1 = (2)^2(40) = 160$ m

4. **(b) :** Time taken by car to reach at location P from location Q,



$t = \frac{QR}{v} + \frac{RP}{(v/2)}$

$t = \frac{(L - x)}{v} + \frac{2\sqrt{d^2 + x^2}}{v}$

$\frac{dt}{dx} = \frac{1}{v}(0 - 1) + 2 \times \left(\frac{1}{2}\right)\frac{1}{v} \times \frac{2x}{\sqrt{d^2 + x^2}} = \frac{-1}{v} + \frac{2x}{v\sqrt{d^2 + x^2}}$

For minimum value of t, $\frac{dt}{dx} = 0 \; \therefore \; -\frac{1}{v} + \frac{2x}{v\sqrt{d^2 + x^2}} = 0$

$1 = \frac{2x}{\sqrt{d^2 + x^2}}$ or $4x^2 = d^2 + x^2$ or $3x^2 = d^2 \; \therefore \; x = \frac{d}{\sqrt{3}}$

5. **(b) :** If $\vec{C} = a\hat{i} + b\hat{j}$ then

$\vec{A}.\vec{C} = \vec{A}.\vec{B}$, $\quad a + b = 1 \quad$ ...(i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$, $\quad 2a - b = 1 \quad$ ...(ii)

Solving equations (i) and (ii), we get $a = \frac{1}{3}, b = \frac{2}{3}$

$|\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$

6. **(c) :** Velocity of the body going upwards is given by

$v = v_0 - gt \quad (v_0 = \text{initial velocity})$

Hence, the graph between velocity and time should be a straight line with negative slope (g) and intercept v_0.

Also, during the whole motion, acceleration of the body is constant *i.e.,* slope should be constant. Hence option (c) is correct.

7. **(a) :** Here, acceleration is given by, $a = -c$

$\frac{dv}{dt} = -c$ or $\frac{dx}{dt} \cdot \frac{dv}{dx} = -c$

$vdv = -cdx$

$\frac{v^2}{2} = -cx + k$ or $x = -\frac{v^2}{2c} + \frac{k}{c}$

From this equation, we conclude option (a) is correct.

8. **(c)**

9. **(c) :** Acceleration of car, $a_C = 4$ m s^{-2}

Acceleration of bus, $a_B = 2$ m s^{-2}

Initial separation between the bus and car, $s_{CB} = 200$ m

Acceleration of car with respect to bus, $a_{CB} = a_C - a_B = 2$ m s^{-2}

Initial velocity $u_{CB} = 0$, $t = ?$

As, $s_{CB} = u_{CB} \times t + \frac{1}{2} a_{CB} t^2$

$200 = 0 \times t + \frac{1}{2} \times 2 \times t^2$ *i.e.,* $t^2 = 200$; $\therefore \; t = 10\sqrt{2}$ s

10. **(a) :** Using $h = ut + \frac{6}{7}at^2$

For stone 1, $y_1 = 10t - \frac{6}{7}gt^2$; For stone 2, $y_2 = 40t - \frac{6}{7}gt^2$

Relative position of the second stone with respect to the first,

$\Delta y = y_2 - y_1 = 40t - \frac{6}{7}gt^2 - 10t + \frac{6}{7}gt^2$

$\Delta y = 30\,t$

After 8 seconds, stone 1 reaches ground, *i.e.,* $y_1 = -240$ m

$\therefore \; \Delta y = y_2 - y_1 = 40t - \frac{6}{7}gt^2 + 240$

Therefore, it will be a parabolic curve till other stone reaches ground.

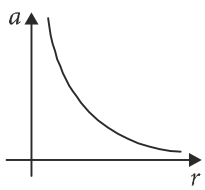
11. **(c) :** Speed $v = 10$ m s^{-1}

We know, centripetal acceleration is given by,

$= \frac{7}{\;}$

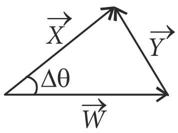
$\because \; \vec{\;}$ = constant

so $a \propto \frac{6}{\;}$ or, ar = constant



This represents a rectangular hyperbola.

12. (c) : By triangle rule



$\vec{W}+\vec{Y}=\vec{X}$B$\vec{X}-\vec{W}=\vec{Y}$

$\vec{X}-\vec{W}= \vec{Y} = \vec{X}$ -uz $\Delta\theta$ -∵ $\Delta\theta << 6.$

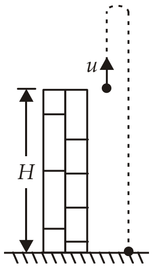
$\vec{X}-\vec{W}= \vec{X}\ \Delta\theta$ -∵ -uz $\Delta\theta \simeq \Delta\theta.$

Hs mz $\vec{X}$ o{ $-\Delta\theta = \vec{W}$

$\therefore\ \vec{X} = \vec{W}$ -∵ o{ $-\Delta\theta \simeq 6.$

]{1 $\vec{X}-\vec{W}= \vec{X}\ \Delta\theta = \vec{W}\Delta\theta$

13. (d) : Time taken by the particle to reach the top most point is,



$$t=\frac{u}{g} \quad \text{... (i)}$$

Time taken by the particle to reach the ground = nt

Using, $s=ut+\frac{1}{2}at^2$

$$\Rightarrow \quad -H=u(nt)-\frac{1}{2}g(nt)^2$$

$$\Rightarrow \quad -H=u\times n\left(\frac{u}{g}\right)-\frac{1}{2}gn^2\left(\frac{u}{g}\right)^2 \quad \text{[using (i)]}$$

$\Rightarrow \quad -2gH = 2nu^2 - n^2u^2 \Rightarrow 2gH = nu^2(n-2)$

14. (c) : Given : $u=\hat{i}+2\hat{j}$

As $\vec{u}=u_x\hat{i}+u_y\hat{j}$ ∵ $u_x = 1$ and $u_y = 2$

Also $x = u_xt$ and $y=u_yt-\frac{1}{2}gt^2$

$\therefore \quad x = t$

and $y=2t-\frac{1}{2}\times 10\times t^2=2t-5t^2$

Equation of trajectory is $y = 2x - 5x^2$

15. (c) : Let u be the velocity of projection of the stone.

The maximum height a boy can throw a stone is

$$H_{\max}=\frac{u^2}{2g}=10\text{ m} \quad \text{...(i)}$$

The maximum horizontal distance the boy can throw the same stone is

$$R_{\max}=\frac{u^2}{g}=20\text{ m} \quad \text{(Using (i))}$$

16. (b) : $\frac{dv}{dt}=-2.5\sqrt{v}$ or $\frac{1}{\sqrt{v}}dv=-2.5\,dt$

On integrating, within limit ($v_1 = 6.25$ m s^{-1} to $v_2 = 0$)

$$\therefore \quad \int_{v_1=6.25\text{ ms}^{-1}}^{v_2=0} v^{-1/2}dv=-2.5\int_0^t dt$$

$$2\times[v^{1/2}]_{6.25}^{0}=-(2.5)t \Rightarrow t=\frac{-2\times(6.25)^{1/2}}{-2.5}=2\text{ s}$$

17. (b) : $R_{\max}=\frac{v^2\sin 90^\circ}{g}=\frac{v^2}{g}$

Area $=\pi(R_{\max})^2=\frac{\pi v^4}{g^2}$

18. (d) : The position vector of the particle from the origin at any time t is

$\vec{r}=v_0t\cos\theta\hat{i}+(v_0t\sin\theta-\frac{1}{2}gt^2)\hat{j}$ $\therefore$ Velocity vector, $\vec{v}=\frac{d\vec{r}}{dt}$

$$\vec{v}=\frac{d}{dt}(v_0t\cos\theta\,\hat{i}+(v_0t\sin\theta-\frac{1}{2}gt^2)\hat{j})$$

$$=v_0\cos\theta\hat{i}+(v_0\sin\theta-gt)\hat{j}$$

The angular momentum of the particle about the origin is

$\vec{L}=\vec{r}\times m\vec{v}$ or $\vec{L}=m(\vec{r}\times\vec{v})$

$$=m\Big[(v_0t\cos\theta\hat{i}+(v_0t\sin\theta-\frac{1}{2}gt^2)\hat{j}) \times(v_0\cos\theta\,\hat{i}+(v_0\sin\theta-gt)\hat{j})\Big]$$

$$=m\Big[(v_0^2t\cos\theta\sin\theta-v_0gt^2\cos\theta)\hat{k} +(v_0^2t\sin\theta\cos\theta-\frac{1}{2}gt^2v_0\cos\theta)(-\hat{k})\Big]$$

$$=m\Big[v_0^2t\sin\theta\cos\theta\hat{k}-v_0gt^2\cos\theta\hat{k} -v_0^2t\sin\theta\cos\theta\hat{k}+\frac{1}{2}v_0gt^2\cos\theta\hat{k}\Big]$$

$$=m\Big[-\frac{1}{2}v_0gt^2\cos\theta\hat{k}\Big]=-\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$$

19. (a) : Here, $\vec{v}=K(y\hat{i}+x\hat{j})$

$\vec{v}=Ky\hat{i}+Kx\hat{j}$ ...(i) ∵ $\vec{v}=\frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}$ ...(ii)

Equating equations (i) and (ii), we get $\frac{dx}{dt}=Ky;\frac{dy}{dt}=Kx$

$$\therefore \quad \frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$$

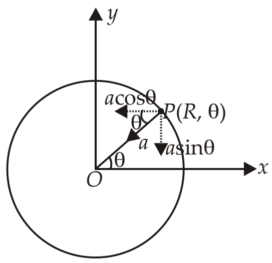
$$\frac{dy}{dx}=\frac{Kx}{Ky}=\frac{x}{y} \quad \text{...(iii)}$$

Integrating both sides of the above equation, we get

$\int y\,dy=\int x\,dx$

$y^2 = x^2 +$ constant

20. (d) :



For a particle in uniform circular motion,

Acceleration, $a=\frac{v^2}{R}$ (towards the centre)

From figure, $\vec{a}=-a\cos\theta\hat{i}-a\sin\theta\hat{j}=-\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$

21. (a) : $s = t^3 + 3$ $\therefore$ $v=\frac{ds}{dt}=\frac{d}{dt}(t^3+3)=3t^2$

Tangential acceleration, $a_t=\frac{dv}{dt}=\frac{d}{dt}(3t^2)=6t$

At $t = 2$ s,

$v = 3(2)^2 = 12$ m/s, $a_t = 6(2) = 12$ m/s^2

Centripetal acceleration, $a_c=\frac{v^2}{R}=\frac{(12)^2}{20}=\frac{144}{20}=7.2\text{ m/s}^2$

Net acceleration, $a=\sqrt{(a_c)^2+(a_t)^2}=\sqrt{(7.2)^2+(12)^2}\approx 14\text{ ms}^{-2}$

22. (b) : $v = u + at$

$\vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$

$\vec{v} = (3+4)\hat{i} + (4+3)\hat{j} \Rightarrow |\vec{v}| = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2}$ units

(This value is about 9.9 units close to 10 units. If (a) is given that is also not wrong).

23. (c) : As $u = 0$, $v_1 = at$, v_2 = constant for the other particle. Initially both are zero. Relative velocity of particle 1 w.r.t. 2 is velocity of 1 – velocity of 2. At first the velocity of first particle is less than that of 2. Then the distance travelled by particle 1 increases as $x_1 = (1/2)\, a t_1^2$. For the second it is proportional to t. Therefore it is a parabola after crossing x-axis again. Curve (c) satisfies this.

24. (c) : Given : velocity $v = v_0 + gt + ft^2$

$\therefore \quad v = \frac{dx}{dt}$ or $\int_0^x dx = \int_0^t v dt$ or $x = \int_0^t (v_0 + gt + ft^2) dt$

$x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} + C$ where C is the constant of integration

Given : $x = 0$, $t = 0$. $\therefore$ $C = 0$ or $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$ sec $\therefore$ $x = v_0 + \frac{g}{2} + \frac{f}{3}$

25. (b) : $v = \alpha\sqrt{x}$ or $\frac{dx}{dt} = \alpha\sqrt{x}$ or $\frac{dx}{\sqrt{x}} = \alpha\, dt$

or $\int \frac{dx}{\sqrt{x}} = \alpha \int dt$ or $2x^{1/2} = \alpha t + C \{\because$ at $t = 0, x = 0, C = 0\}$

or $x = \left(\frac{\alpha}{2}\right)^2 t^2$ or displacement is proportional to t^2.

26. (a) : Initially, the parachutist falls under gravity

$\therefore$ $u^2 = 2ah = 2 \times 9.8 \times 50 = 980$ m^{2}s^{-2}

He reaches the ground with speed = 3 m/s, $a = -2$ m s^{-2}

$\therefore$ $(3)^2 = u^2 - 2 \times 2 \times h_1$ or $9 = 980 - 4h_1$

or $h_1 = \frac{971}{4}$ or $h_1 = 242.75$ m

$\therefore$ Total height = 50 + 242.75 = 292.75 = 293 m.

27. (*) : For first part of journey, $s = s_1$,

$s_1 = \frac{1}{2} f t_1^2 = s$ ...(i) $\quad v = f t_1$ ...(ii)

For second part of journey, $s_2 = vt$ or $s_2 = f t_1 t$ ...(iii)

For the third part of journey, $s_3 = \frac{1}{2}\left(\frac{f}{2}\right)(2t_1)^2$ or $s_3 = \frac{1}{2} \times \frac{4 f t_1^2}{2}$

or $s_3 = 2s_1 = 2s$ ...(iv)

$s_1 + s_2 + s_3 = 15s$

or $s + f t_1 t + 2s = 15s$ or $f t_1 t = 12s$ ...(v)

From (i) and (v), $\frac{s}{12s} = \frac{ft_1^2}{2 \times ft_1 t}$

or $t_1 = \frac{t}{6}$ or $s = \frac{1}{2} ft_1^2 = \frac{1}{2} f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$ or $s = \frac{ft^2}{72}$

* None of the given options provide this answer.

28. (a) : $t = ax^2 + bx$

Differentiate the equation with respect to t

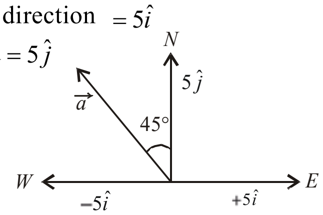
$\therefore \quad 1 = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$ or $1 = 2axv + bv$ as $\frac{dx}{dt} = v$

or $v = \frac{1}{2ax+b}$ or $\frac{dv}{dt} = \frac{-2a(dx/dt)}{(2ax+b)^2} = -2av \times v^2$

or Acceleration = $-2av^3$

29. (b) : Velocity in eastward direction $= 5\hat{i}$

velocity in northward direction $= 5\hat{j}$



$\therefore$ Acceleration $\vec{a} = \frac{5\hat{j} - 5\hat{i}}{10}$

or $\vec{a} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$

or $|\vec{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$

or $|\vec{a}| = \frac{1}{\sqrt{2}}$ ms^{-2} towards north-west.

30. (b) : Range is same for angles of projection θ and $(90 - \theta)$

$\therefore \quad t_1 = \frac{2u\sin\theta}{g}$ and $t_2 = \frac{2u\sin(90-\theta)}{g}$

$\therefore \quad t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g} \times \left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$

$\therefore$ $t_1 t_2$ is proportional to R.

31. (d) : Let a be the retardation for both the vehicles

For automobile, $v^2 = u^2 - 2as$

$\therefore$ $u_1^2 - 2as_1 = 0 \Rightarrow u_1^2 = 2as_1$

Similarly for car, $u_2^2 = 2as_2$

$\therefore \quad \left(\frac{u_2}{u_1}\right)^2 = \frac{s_2}{s_1} \Rightarrow \left(\frac{120}{60}\right)^2 = \frac{s_2}{20}$ or $s_2 = 80$ m

32. (c) : Equation of motion : $s = ut + \frac{1}{2}gt^2$

$\therefore \quad h = 0 + \frac{1}{2}gT^2$ or $2h = gT^2$ ...(i)

After $T/3$ sec, $s = 0 + \frac{1}{2} \times g\left(\frac{T}{3}\right)^2 = \frac{gT^2}{18}$

or $18s = gT^2$ ...(ii)

From (i) and (ii), $18s = 2h$ or $s = \frac{h}{9}$ m from top.

$\therefore$ Height from ground = $h - \frac{h}{9} = \frac{8h}{9}$ m.

33. (a) : $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ or $AB\sin\theta\,\hat{n} = AB\sin(-\theta)\,\hat{n}$

or $\sin\theta = -\sin\theta$ or $2\sin\theta = 0$

or $\theta = 0, \pi, 2\pi.....$ $\therefore$ $\theta = \pi$

34. (b) : The acceleration vector acts along the radius of the circle. The given statement is false.

35. (a) : The person will catch the ball if his speed and horizontal speed of the ball are same

$= v_0\cos\theta = \frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} = \cos 60° \therefore \theta = 60°$

36. (c) : For first case, $u_1 = 50\frac{\text{km}}{\text{hour}} = \frac{50\times1000}{60\times60} = \frac{125}{9}\frac{\text{m}}{\text{s}}$

$\therefore$ Acceleration $a = -\frac{u_1^2}{2s_1} = -\left(\frac{125}{9}\right)^2 \times \frac{1}{2\times6} = -16\ \text{m/s}^2$

For second case, $u_2 = 100\frac{\text{km}}{\text{hour}} = \frac{100\times1000}{60\times60} = \frac{250}{9}\frac{\text{m}}{\text{s}}$

$\therefore$ $s_2 = \frac{-u_2^2}{2a} = \frac{-1}{2}\left(\frac{250}{9}\right)^2 \times \left(-\frac{1}{16}\right) = 24$ m or s_2 = 24 m

37. (d) : Height of building = 10 m
The ball projected from the roof of building will be back to roof, height of 10 m after covering the maximum horizontal range.

Maximum horizontal range $(R) = \frac{u^2 \sin 2\theta}{g}$

or $R = \frac{(10)^2 \times \sin 60°}{10} = 10 \times 0.866$ or R = 8.66 m.

38. (b) : $\because$ $x = \alpha t^3$ $\therefore$ $\frac{dx}{dt} = 3\alpha t^2 \Rightarrow v_x = 3\alpha t^2$

Again $y = \beta t^3$ $\therefore$ $\frac{dy}{dt} \Rightarrow v_y = 3\beta t^2$ $\therefore$ $v^2 = v_x^2 + v_y^2$

or $v^2 = (3\alpha t^2)^2 + (3\beta t^2)^2 = (3t^2)^2 (\alpha^2 + \beta^2)$

or $v = 3t^2\sqrt{\alpha^2 + \beta^2}$

39. (b) : Ball A projected upwards with velocity u, falls back with velocity u downwards. It completes its journey to ground under gravity.
$\therefore$ $v_A^2 = u^2 + 2gh$ ...(i)
Ball B starts with downwards velocity u and reaches ground after travelling a vertical distance h
$\therefore$ $v_B^2 = u^2 + 2gh$ ...(ii)
From (i) and (ii), $v_A = v_B$

40. (d) : Both are given the same deceleration simultaneously and both finally stop.
Formula relevant to motion : $u^2 = 2as$

$\therefore$ For first car, $s_1 = \frac{u^2}{2a}$

For second car, $s_2 = \frac{(4u)^2}{2a} = \frac{16u^2}{2a}$ $\therefore$ $\frac{s_1}{s_2} = \frac{1}{16}$

41. (a) : For first part of penetration, by equation of motion,

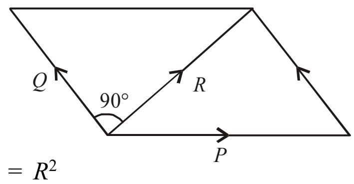
$\left(\frac{u}{2}\right)^2 = u^2 - 2a(3)$

or $3u^2 = 24a \Rightarrow u^2 = 8a$ ...(i)
For latter part of penetration,

$0 = \left(\frac{u}{2}\right)^2 - 2ax$

or $u^2 = 8ax$ ...(ii)
From (i) and (ii)
$8ax = 8a \Rightarrow x = 1$ cm

42. (b) : Resultant R is perpendicular to smaller force Q and $(P + Q)$ = 18 N
$\therefore$ $P^2 = Q^2 + R^2$ by right angled triangle



or $(P^2 - Q^2) = R^2$
or $(P + Q)(P - Q) = R^2$
or $(18)(P - Q) = (12)^2$ $[\because P + Q = 18]$
or $(P - Q) = 8$
Hence P = 13 N and Q = 5 N