CHAPTER 1

Physics and Measurement

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(a) 2.5% (b) 3.5% (c) 4.5% (d) 6%

(2018)

2. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is

(a) 0.3150 cm (b) 0.2150 cm
(c) 0.4300 cm (d) 0.0430 cm

(Online 2018)

3. The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is

(a) $\frac{3}{2}\alpha$ (b) α (c) $\frac{5}{2}\alpha$ (d) $\frac{2}{3}\alpha$

(Online 2018)

4. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, $\hbar$ and c. Which of the following correctly gives the Planck length?

(a) $G\hbar^2c^3$ (b) $G^2\hbar c$ (c) $\left(\frac{G\hbar}{c^3}\right)^{1/2}$ (d) $G^{1/2}\hbar^2c$

(Online 2018)

5. The percentage errors in quantities P, Q, R and S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity $A = \frac{P^3Q^2}{\sqrt{R}\,S}$. The maximum percentage error in the value of A will be

(a) 6.5% (b) 8.5 % (c) 6.0 % (d) 7.5 %

(Online 2018)

6. The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible, the relative uncertainty in the mass of the earth is

(a) 6×10^{-2} (b) 10^{-2}
(c) 2×10^{-2} (d) 3×10^{-2} *(Online 2018)*

7. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be

(a) $[M] = [TC^{-2}\,h]$ (b) $[M] = [T^{-1}\,C^{-2}\,h^{-1}]$
(c) $[M] = [T^{-1}\,C^{-2}\,h]$ (d) $[M] = [T^{-1}\,C^2\,h]$

(Online 2017)

8. A physical quantity P is described by the relation $P = a^{1/2}\,b^2\,c^3\,d^{-4}$

If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be

(a) 25% (b) 12% (c) 8% (d) 32%

(Online 2017)

9. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be :

(a) 92 ± 2 s (b) 92 ± 5.0 s
(c) 92 ± 1.8 s (d) 92 ± 3 s *(2016)*

10. In the following 'I' refers to current and other symbols have their usual meaning. Choose the option that corresponds to the dimensions of electrical conductivity.

(a) $M^{-1}\,L^{-3}\,T^3\,I$ (b) $M^{-1}\,L^{-3}\,T^3\,I^2$
(c) $M^{-1}\,L^3\,T^3\,I$ (d) $ML^{-3}\,T^{-3}\,I^2$

(Online 2016)

11. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation $AD = C\ln(BD)$ holds true. Then which of the combination is not a meaningful quantity?

(a) $\frac{C}{BD} - \frac{AD^2}{C}$ (b) $A^2 - B^2C^2$
(c) $\frac{A}{B} - C$ (d) $\frac{(A-C)}{D}$

(Online 2016)

12. If the capacitance of a nanocapacitor is measured in terms of a unit u made by combining the electronic charge e, Bohr radius a_0, Planck's constant h and speed of light c then

(a) $= \frac{7}{5}$ (b) $= \frac{7}{5}$
(c) $= \frac{7 \quad 5}{\quad}$ (d) $= \frac{\quad}{7 \quad 5}$

(Online 2015)

13. If electronic charge e, electron mass m, speed of light in vaccum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of

(a) $\left(\frac{}{7}\right)$ (b) $\left(\frac{}{7}\right)$

(c) $\left(\frac{}{7}\right)$ (d) $\left(\frac{7}{7}\right)$ *(Online 2015)*

14. A beaker contains a fluid of density ρ kg/m^3, specific heat S J/kg°C and viscosity η. The beaker is filled up to height h. To estimate the rate of heat transfer per unit area $-\dot{n}\,4W.$ by convection when beaker is put on a hot plate, a student proposes that it should depend on η, $\left(\frac{p\Delta\theta}{}\right)$ mzp $\left(\frac{6}{\rho}\right)$ when $\Delta\theta$ (in °C) is the difference in the temperature between the bottom and top of the fluid. In that situation the correct option for $-\dot{n}\,4W.$ is

(a) $\eta\frac{p\Delta\theta}{}$ (b) $\eta\left(\frac{p\Delta\theta}{}\right)\left(\frac{6}{\rho}\right)$

(c) $\frac{p\Delta\theta}{\eta}$ (d) $\left(\frac{p\Delta\theta}{\eta}\right)\left(\frac{6}{\rho}\right)$

(Online 2015)

15. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(a) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

(b) A meter scale.

(c) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

(d) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

(2014)

16. Let $[\in_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then

(a) $[\in_0] = [M^{-1} L^2 T^{-1} A]$ (b) $[\in_0] = [M^{-1} L^{-3} T^2 A]$

(c) $[\in_0] = [M^{-1} L^{-3} T^4 A^2]$ (d) $[\in_0] = [M^{-1} L^2 T^{-1} A^{-2}]$

(2013)

17. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is

(a) zero (b) 1%

(c) 3% (d) 6% *(2012)*

18. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 4, 4, 2 (b) 5, 1, 2

(c) 5, 1, 5 (d) 5, 5, 2. *(2010)*

19. The dimension of magnetic field in M, L, T and C (coulomb) is given as

(a) $MT^{-2}C^{-1}$ (b) $MLT^{-1}C^{-1}$

(c) MT^2C^{-2} (d) $MT^{-1}C^{-1}$. *(2008)*

20. Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?

(a) weber (Wb) (b) Wb/m^2

(c) henry (H) (d) H/m^2. *(2006)*

21. Out of the following pairs, which one does not have identical dimensions?

(a) moment of inertia and moment of a force

(b) work and torque

(c) angular momentum and Planck's constant

(d) impulse and momentum *(2005)*

22. Which one of the following represents the correct dimensions of the coefficient of viscosity?

(a) $ML^{-1}T^{-2}$ (b) MLT^{-1}

(c) $ML^{-1}T^{-1}$ (d) $ML^{-2}T^{-2}$. *(2004)*

23. The physical quantities not having same dimensions are

(a) torque and work

(b) momentum and Planck's constant

(c) stress and Young's modulus

(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$. *(2003)*

24. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are

(a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$ (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$.

(2003)

25. Identify the pair whose dimensions are equal.

(a) torque and work (b) stress and energy

(c) force and stress (d) force and work.

(2002)

ANSWER KEY

1. (c)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (b)	**11.** (a,d)	**12.** (c)
13. (c)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (d)	**20.** (c)	**21.** (a)	**22.** (c)	**23.** (b)	**24.** (c)
25. (a)											

Explanations

1. **(c) :** Density of a material is given by, $\rho = \frac{m}{V} = \frac{m}{l^3}$

For maximum error in ρ, $\frac{d\rho}{\rho} = \frac{dm}{m} + 3\frac{dl}{l}$

$\frac{d\rho}{\rho} \times 100 = \frac{dm}{m} \times 100 + 3\frac{dl}{l} \times 100$ = 1.5 + (3 × 1) = 4.5%

2. **(b) :** Least count = $\frac{0.25}{5 \times 100}$ cm = 5×10^{-4} cm

Thickness of wire = $4 \times \frac{0.25}{5}\text{ cm} + 30 \times \text{L.C.}$

= 4 × 0.05 cm + 30 × 5 × 10^{-4} cm

= 0.20 cm + 0.0150 cm = 0.2150 cm

3. **(a) :** As we know $\frac{\Delta S}{S} = 2 \times \frac{\Delta r}{r}$ and $\frac{\Delta V}{V} = 3 \times \frac{\Delta r}{r} = \frac{3}{2} \times \frac{\Delta S}{S}$

$\frac{\Delta V}{V} = \frac{3}{2}\alpha$

4. **(c) :** For planck length, $l = kG^p \hbar^q c^r$

$[M^0LT^0] = [M^{-1} L^3 T^{-2}]^p [ML^2T^{-1}]^q [LT^{-1}]^r$

$[M^0\ LT^0] = [M^{-p+q} L^{(3p+2q+r)} T^{-(2p+q+r)}]$

On comparing powers of M, L and T from both sides,

$-p + q = 0,\ 3p + 2q + r = 1,\ -(2p + q + r) = 0$

On solving these equations, $p = q = \frac{1}{2},\ r = \frac{-3}{2}$

$\therefore\ l = \left(\frac{G\hbar}{c^3}\right)^{1/2}$ (Take, $k = 1$)

5. **(a) :** Relative error in A is given by

$$\frac{\Delta A}{A} = \frac{3\Delta P}{P} + \frac{2\Delta Q}{Q} + \frac{1}{2}\frac{\Delta R}{R} + \frac{\Delta S}{S}$$

The maximum percentage error in the value of A will be

$\frac{\Delta A}{A} \times 100 = 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1.5 = 6.5\%$

6. **(c) :** From Kepler's law,

$T^2 = \frac{4\pi^2}{GM}r^3$ or $M = \left(\frac{4\pi^2}{G}\right)\frac{r^3}{T^2}$; $\frac{\Delta M}{M} = 2\frac{\Delta T}{T} + 3\frac{\Delta r}{r}$

Since $\frac{\Delta r}{r} \approx 0\ \therefore\ \left|\frac{\Delta M}{M}\right| = 2\frac{\Delta T}{T} = 2 \times 10^{-2}$

7. **(c) :** Let $m = kT^xC^yh^z$ where k is a dimensionless constant.

$\therefore\ [ML^0T^0] = [T]^x [LT^{-1}]^y [ML^2T^{-1}]^z$

$[ML^0T^0] = [M^zL^{y+2z}T^{x-y-z}] \Rightarrow z = 1, y + 2z = 0$ and $x - y - z = 0$

Solving, we get, $x = -1$, $y = -2$, $z = 1$; on putting values we get

$\therefore\ [M] = [T^{-1}C^{-2}h]$

8. **(d) :** Here, $P = a^{1/2}\, b^2\, c^3\, d^{-4}$

$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$

or $\left(\frac{\Delta P}{P} \times 100\right)\% = \left(\frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}\right) \times 100\%$

$\therefore$ Relative error in $P = \left(\frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5\right)\% = 32\%$

9. **(a) :** Here, t_1 = 90 s, t_2 = 91 s, t_3 = 95 s, t_4 = 92 s

L.C. = 1 s

Mean of the measurements, $\bar{\ } = \frac{\sum}{k}$

$\bar{\ } = \frac{>5 + >6 + >: + >7}{9} = >7\ -$

Mean deviation $= \frac{\sum|\bar{\ } - \ |}{k} = \frac{7 + 6 + 8 + 5}{9} = 63\ -$

Since the least count of the instrument is 1 s, so reported mean time = (92 ± 2) s.

10. **(b) :** Electrical conductivity = $[M^{-1}L^{-3}T^3I^2]$

11. **(a, d) :** Given, A, B, C and D have different dimensions. Also, $AD = C \ln(BD)$

log is the dimensionless, so $[B] = \frac{1}{[D]}$

Also, $[AD] = [C]$

(a) $\left[\frac{C}{BD}\right] = \frac{[C]}{1} = [C]$ $\left[\frac{AD^2}{C}\right] = \frac{[AD][D]}{[C]} = [D]$

So, $\frac{C}{BD} - \frac{AD^2}{C} = C - D$ which is not meaningful.

(b) $[B^2C^2] = [B^2][A^2D^2] = A^2[BD]^2 = [A^2]$

$\therefore$ $(A^2 - B^2C^2)$ is meaningful.

(c) $\left[\frac{A}{B}\right] = [AD] = [C]\ \therefore\ \left(\frac{A}{B} - C\right)$ is meaningful.

(d) $\left(\frac{A - C}{D}\right)$ is not meaningful as A and C both have different dimensions.

12. **(c) :** Here, capacitance $C = ke^xa_0^yh^zc^a$

$[C] = [M^{-1}L^{-2}A^2T^4]$

$[e] = [AT]$, $[a_0] = [L]$

$[c] = [L^1T^{-1}]$, $[h] = [M^1L^2T^{-1}]$

$\therefore [M^{-1}L^{-2}A^2T^4] = [AT]^x[L]^y[M^1L^2T^{-1}]^z\ [L^1T^{-1}]^a$

Comparing both sides

$x = 2$; $z = -1$, $y + 2z + a = -2$, $x - z - a = 4$

On solving these eqns, we get $x = 2$, $y = 1$, $z = -1$, $a = -1$

Also, $[C] = u$ so $u = \frac{e^2a_0}{hc}$

13. **(c) :** $[e] = [IT]$, $[m] = [M]$, $[c] = [LT^{-1}]$

$[h] = [ML^2T^{-1}]$, $[\mu_0] = [MLI^{-2}T^{-2}]$

If $\mu_0 = ke^am^bc^ch^d$

$[MLI^{-2}T^{-2}] = [IT]^a[M]^b\ [LT^{-1}]^c\ [ML^2T^{-1}]^d$

By equating powers, we get $a = -2$, $b + d = 1$
$c + 2d = 1$, $a - c - d = -2$
Solving these eqns. we get, $a = -2$; $b = 0$; $c = -1$; $d = 1$

$$\therefore\ g\mu_5 i = \left[\frac{\quad}{7}\right]$$

14. (a) : Let $\left(\frac{\dot{n}}{W}\right)$ is derived quantity which is derived from three fundamental quantities $\eta 1\left(\frac{p\Delta\theta}{}\right) m z\, p\left(\frac{6}{\rho}\right)$

By using principle of homogeneity of dimensions

$$\left[\frac{\dot{n}}{W}\right] = g\eta i\ \left[\frac{p\Delta\theta}{}\right]\left[\frac{6}{\rho}\right]$$

$$\left[\frac{\dot{n}}{W}\right] = gT^{6}\wedge^{-8} i\ ;\ [\eta] = [M^1L^{-1}T^{-1}]$$

$$\left[\frac{p\Delta\theta}{}\right] = gS^{6}\wedge^{-7} i\ ;\ \left[\frac{6}{\rho}\right] = gT^{-6}S^{7}\wedge^{7} i$$

$\therefore\quad [M^{-1}L^0T^{-3}] = [M^1L^{-1}T^{-1}]^x[M^0L^1T^{-2}]^y[M^{-1}L^2T^2]^z$
On comparing both sides
$x + 0 - z = 1$, $-x + y + 2z = 0$ and $-x - 2y + 2z = -3$
On solving these eqns, we get $x = 1$, $y = 1$, $z = 0$

$$\dashv\{1\frac{\dot{n}}{W} = \eta\ \frac{p\Delta\theta}{}$$

15. (c) : Measured value of the length of rod = 3.50 cm
So, least count of the measuring instrument must be 0.01 cm = 0.1 mm
For, vernier scale, 10 MSD = 1 cm = 10 mm
$\Rightarrow$ 1 MSD = 1 mm
Also, 9 MSD = 10 VSD
L.C. = 1 MSD – 1 VSD = (1 – 0.9) mm = 0.1 mm

16. (c) : According to Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\quad\therefore\quad \epsilon_0 = \frac{1}{4\pi}\frac{q_1q_2}{Fr^2}$$

$$[\epsilon_0] = \frac{[AT][AT]}{[MLT^{-2}][L]^2} = [M^{-1}L^{-3}T^4A^2]$$

17. (d) : $R = \dfrac{V}{I}\qquad \therefore\quad \dfrac{\Delta R}{R} = \dfrac{\Delta V}{V} + \dfrac{\Delta I}{I}$

The percentage error in R is

$$\frac{\Delta R}{R}\times 100 = \frac{\Delta V}{V}\times 100 + \frac{\Delta I}{I}\times 100 = 3\% + 3\% = 6\%$$

18. (b) : (i) All the non-zero digits are significant.

(ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

(iii) If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.

(iv) The power of 10 is irrelevant to the determination of significant figures.

According to the above rules, 23.023 has 5 significant figures.
0.0003 has 1 significant figures.
2.1×10^{-3} has 2 significant figures.

19. (d) : Lorentz force $= |\vec{F}| = |q\vec{v}\times\vec{B}|$

$$\therefore\quad [B] = \frac{[F]}{[q][v]} = \frac{MLT^{-2}}{C\times LT^{-1}} = \frac{MLT^{-2}}{CLT^{-1}} = [MT^{-1}C^{-1}]$$

20. (c) : $[ML^2Q^{-2}] = [ML^2A^{-2}T^{-2}]$
$[Wb] = [ML^2T^{-2}A^{-1}]$

$$\left[\frac{Wb}{m^2}\right] = [MT^{-2}A^{-1}]$$

$[henry] = [ML^2T^{-2}A^{-2}]$

$$\left[\frac{H}{m^2}\right] = [MT^{-2}A^{-2}]$$

Obviously henry (H) has dimensions $\dfrac{ML^2}{Q^2}$.

21. (a) : Moment of inertia $(I) = mr^2$ $\therefore$ $[I] = [ML^2]$
Moment of force $(C) = r\,F$
$\therefore\quad [C] = [r][F] = [L][MLT^{-2}]$ or $[C] = [ML^2T^{-2}]$
Moment of inertia and moment of a force do not have identical dimensions.

22. (c) : Viscous force $F = 6\pi\eta rv$

$$\therefore\quad \eta = \frac{F}{6\pi rv}\quad\text{or}\quad [\eta] = \frac{[F]}{[r][v]}$$

$$\text{or}\quad [\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]}\quad\text{or}\quad [\eta] = [ML^{-1}T^{-1}].$$

23. (b) : [Momentum] = $[MLT^{-1}]$
[Planck's constant] = $[ML^2T^{-1}]$
Momentum and Planck's constant do not have same dimensions.

24. (c) : Velocity of light in vacuum $= \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\text{or}\quad [LT^{-1}] = \left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]\quad\text{or}\quad [L^2T^{-2}] = \left[\frac{1}{\mu_0\varepsilon_0}\right]$$

$\therefore$ Dimensions of $\dfrac{1}{\mu_0\varepsilon_0} = [L^2T^{-2}]$.

25. (a) : Torque and work have the same dimensions.