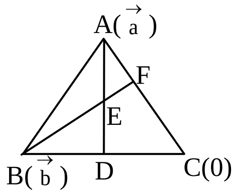
EXERCISE # 3

Part-A Subjective Type Questions

Q.1 The median AD of a triangle ABC is bisected at E and BE is produced to meet side AC in F; show that AF = $\frac{1}{3}$ AC & EF = $\frac{1}{4}$ BF.

Sol.



Let C is origin and position vector of A and B are $\vec{a}$ and $\vec{b}$ respectively.

$\Rightarrow$ position vector of D is $\frac{\vec{b}}{2}$

and position vector of E is $\frac{2\vec{a}+\vec{b}}{4}$

Now F is the point of intersection of BE and AC which equations are

$$\vec{r} = t\,\vec{a} \quad \text{...........(i)}$$

and $$\vec{r} = (1-s)\,\vec{b} + s\left(\frac{2\vec{a}+\vec{b}}{4}\right) \quad \text{........(ii)}$$

comparing the coefficient of $\vec{a}$ and $\vec{b}$

we have $t = \frac{s}{2}$ and $1 - s + \frac{s}{4} = 0$

$\Rightarrow s = \frac{4}{3}$ and $t = \frac{2}{3}$

$\therefore$ position vector of F = $\frac{2}{3}\vec{a}$

Now AF = $\frac{2}{3}\vec{a} - \vec{a} = -\frac{1}{3}\vec{a} = \frac{1}{3}\vec{AC}$

and BF = $\frac{2}{3}\vec{a} - \vec{b} = \frac{2\vec{a}-3\vec{b}}{3}$

and EF = $\frac{2}{3}\vec{a} - \frac{2\vec{a}+\vec{b}}{4} = \frac{2\vec{a}-3\vec{b}}{12}$

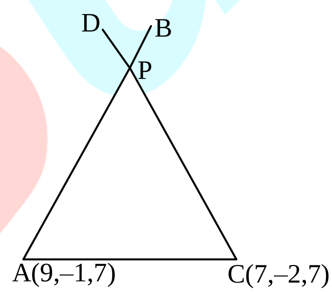
EF = $\frac{1}{4}$ BF

Q.2 ABCD is a plane quadrilateral .The diagonals AC and BD cut at P such that $\frac{AP}{PC} = \frac{4}{3}$ and $\frac{BP}{PD} = \frac{2}{3}$. Find the ratio in which AB and CD cut each other.

Sol. 10 : 7 & 7 : 5 externally

Q.3 The position vectors of two points A and C are $9\hat{i} - \hat{j} + 7\hat{k}$ and $7\hat{i} - 2\hat{j} + 7\hat{k}$ respectively. The point of intersection of vectors $\vec{AB} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{CD} = 2\hat{i} - \hat{j} + 2\hat{k}$ is P. If vectors $\vec{PQ}$ is perpendicular to $\vec{AB}$ & $\vec{CD}$ and PQ = 15, then find the position vector of Q.

Sol.



Given $\vec{AB} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{CD} = 2\hat{i} - \hat{j} + 2\hat{k}$

Equation of line passing through $\vec{A}$ and parallel to $\vec{AB}$

AB = $\vec{r} = \vec{A} + t(\vec{AB})$

$= (9\hat{i} - \hat{j} + 7\hat{k}) + t(4\hat{i} - \hat{j} + 3\hat{k})$

Similarly

CD = $\vec{r} = \vec{C} + S(\vec{CD})$

$= (7\hat{i} - 2\hat{j} + 7\hat{k}) + S(2\hat{i} - \hat{j} + 2\hat{k})$

For intersection point we compare coefficient of $\hat{i}, \hat{j}, \hat{k}$

$\Rightarrow 9 + 4t = 7 + 2s,\ -1 - t = -2 - s$

and $7 + 3t = 7 + 2s$

solving we get $t = -2$, $s = -3$

$\Rightarrow P(\hat{i} + \hat{j} + \hat{k})$

Now let $Q(x, y, z) = Q(x\hat{i} + y\hat{j} + z\hat{k})$

$= \overrightarrow{PQ} = (x-1)\,\hat{i} + (y-1)\,\hat{j} + (z-1)\,\hat{k}$

Θ $|PQ| = 15$

$\Rightarrow (x-1)^2 + (y-1)^2 + (z-1)^2 = 225$ .....(i)

Given that $\overrightarrow{PQ} \perp \overrightarrow{AB}$ and $\overrightarrow{PQ} \perp \overrightarrow{CD}$

$$\Rightarrow PQ \parallel (\overrightarrow{AB} \times \overrightarrow{CD})$$

$$\therefore\ \overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 2 & -1 & 2 \end{vmatrix}$$

$\Rightarrow \hat{i} - 2\hat{j} - 2\hat{k}$

Θ $PQ = (x-1)\hat{i} + (y-1)\,\hat{j} + (z-1)\hat{k}$

$PQ \parallel (\overrightarrow{AB} \times \overrightarrow{CD})$

$$\Rightarrow \frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-1}{-2} = \lambda \text{ say}$$

$\Rightarrow x = \lambda + 1,\ y = -2\lambda + 1,\ z = -2\lambda + 1$ .....(ii)

form (i) we get $\lambda^2 + 4\lambda^2 + 4\lambda^2 = 225$

$\Rightarrow \lambda^2 = 25 \Rightarrow \lambda = 5, -5$

from (ii) $(x, y, z) = (6, -9, -9)$ or $(-4, 11, 11)$

Q.4 The resultant of two vectors $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{a}$. If $|\vec{b}| = \sqrt{2}\,|\vec{a}|$, show that the resultant of $2\vec{a}$ & $\vec{b}$ is perpendicular to $\vec{b}$.

Q.5 In the plane of a triangle ABC, squares ACXY, BCWZ are described, in the order given, externally to the triangle on AC and BC respectively. Given that $\overrightarrow{CX} = \vec{b}$, $\overrightarrow{CA} = \vec{a}$, $\overrightarrow{CW} = \vec{x}$, $\overrightarrow{CB} = \vec{y}$ .Prove that $\vec{a}.\vec{y} + \vec{x}.\vec{b} = 0$

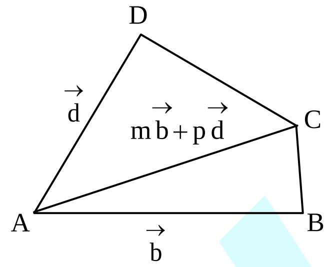
Deduce that $\overrightarrow{AW} \cdot \overrightarrow{BX} = 0$.

Q.6 Using vector, prove that for any four numbers a, b, c, d we have the inequality

$(a^2 + b^2)(c^2 + d^2) \ge (ac + bd)^2$

Q.7 ABCD is quadrilateral such that $\overrightarrow{AB} = \vec{b}$; $\overrightarrow{AD} = \vec{d}$; $\overrightarrow{AC} = m\vec{b} + p\vec{d}$, show that the area of the quadrilateral ABCD is $\frac{1}{2}|m + p|\,|\vec{b} \times \vec{d}|$.

Sol.



Area of ABCD = Area of ΔABC + Area of ΔACD

$= \frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) + \frac{1}{2}(\overrightarrow{AC} \times \overrightarrow{AD})$

$= \frac{1}{2}(\overrightarrow{AC} \times \overrightarrow{AD}) - \frac{1}{2}(\overrightarrow{AC} \times \overrightarrow{AB})$

$= \frac{1}{2}\overrightarrow{AC} \times (\overrightarrow{AD} - \overrightarrow{AB})$

$= \frac{1}{2}(m\vec{b} + p\vec{d}) \times (\vec{d} - \vec{b})$

$= \frac{1}{2}[m(\vec{b} \times \vec{d}) - p(\vec{d} \times \vec{b})]$

$= \frac{1}{2}[m(\vec{b} \times \vec{d}) + p(\vec{b} \times \vec{d})]$

$= \frac{1}{2}|m + p|\,|\vec{b} \times \vec{d}|$

Q.8 Given that $\vec{x} + \frac{1}{p^2}(\vec{p}\,.\,\vec{x})\,\vec{p} = \vec{q}$, show that $\vec{x}\,.\,\vec{p} = \frac{1}{2}\,\vec{p}\,.\,\vec{q}$ & find $\vec{x}$ in terms $\vec{p}$ and $\vec{q}$.

Sol. Given that

$$\vec{x} + \frac{1}{p^2}(\vec{p}\,.\,\vec{x})\,\vec{p} = \vec{q} \text{ ....(i)}$$

Taking dot product with $\vec{p}$ we get

$(\vec{p}\,.\,\vec{x}) + (\vec{p}\,.\,\vec{x}) = \vec{p}\,.\,\vec{q}$

$\Rightarrow \vec{x}\,.\,\vec{p} = \frac{1}{2}\,\vec{p}\,.\,\vec{q}$ ....(ii)

from (i) & (ii) we get

$\vec{x} + \frac{1}{2p^2}(\vec{p}\,.\,\vec{q})\,\vec{p} = \vec{q}$

$\Rightarrow \vec{x} = \vec{q} - \frac{1}{2p^2}(\vec{p}\,.\,\vec{q})\,\vec{p}$

Q.9 The vector $\vec{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way, find the vector in its new position.

Sol. $(4/\sqrt{2})\,\hat{i} - (1/\sqrt{2})\,\hat{j} - (1/\sqrt{2})\,\hat{k}$

Q.10 Find the distance of the point $P(\hat{i} + \hat{j} + \hat{k})$ from the plane L which passes through the three points $A(2\hat{i} + \hat{j} + \hat{k})$, $B(\hat{i} + 2\hat{j} + \hat{k})$, $C(\hat{i} + \hat{j} + 2\hat{k})$. Also find the P.V. of the foot of the perpendicular from P on the plane L.

Sol. A (2, 1, 1), B (1, 2, 1), C (1, 1, 2)

$\vec{AB} = (-1, 1, 0)$, $\vec{AC} = (-1, 0, 1)$

$\hat{n}$ is the normal to the plane $\vec{AB} \times \vec{AC}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

If Q(x, y, z) is any point on the plane

Then $\vec{QA} \cdot \vec{n} = 0$

$\Rightarrow (x-2)\,\hat{i} + (y-1)\,\hat{j} + (z-1)\,\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow x + y + z - 4 = 0$

If distance on the plane form P(1,1,1) is P then

PM = projection of $\vec{PA}$ on $\vec{n}$

$$p = \frac{\vec{PA} \cdot \vec{n}}{|\vec{n}|} = \frac{\hat{i}(i + j + k)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\because$ Foot of $\perp$ from P is M(x, y, z) PM is parallel to normal

$$\Rightarrow \frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$$

$$= \frac{[(x-1)^2 + (y-1)^2 + (z-1)^2]^{1/2}}{[1+1+1]^{1/2}} = \frac{p}{\sqrt{3}} = \frac{1}{3}$$

$$x = \frac{1}{3} + 1 = \frac{4}{3} = y = z$$

Foot of $\perp$ M $= \frac{4}{3}(\hat{i} + \hat{j} + \hat{k})$

Q.11 Given that vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, find vector $\vec{v}$ in the terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{v} \cdot \vec{a} = 0$, $\vec{v} \cdot \vec{b} = 1$ and $[\vec{v}\ \vec{a}\ \vec{b}] = 1$

Sol. $\because \vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

$\Rightarrow \vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\therefore \vec{v} = p\,\vec{a} + q\,\vec{b} + r\,(\vec{a} \times \vec{b})$

$\because \vec{v} \cdot \vec{a} = 0$

$p\,(\vec{a} \cdot \vec{a}) + 0 + 0 = 0$

$\Rightarrow p = 0$

and $\vec{v} \cdot \vec{b} = 1 \Rightarrow q\,|\vec{b}|^2 = 1$

$$\Rightarrow q = \frac{1}{|\vec{b}|^2}$$

and $[\vec{v}\ \vec{a}\ \vec{b}] = 1$

$\Rightarrow \vec{v} \cdot (\vec{a} \times \vec{b}) = 1$

$\Rightarrow r|\vec{a} \times \vec{b}|^2 = 1$

$$\Rightarrow r = \frac{1}{|\vec{a} \times \vec{b}|^2}$$

$$\Rightarrow \vec{v} = \frac{1}{|\vec{b}|^2}\,\vec{b} + \frac{1}{|\vec{a} \times \vec{b}|^2}(\vec{a} \times \vec{b})$$

Q.12 If $p\,\vec{x} + (\vec{x} \times \vec{a}) = \vec{b}$; $(p \neq 0)$ prove that

$$\vec{x} = \frac{p^2\vec{b} + (\vec{b} \cdot \vec{a})\vec{a} - p(\vec{b} \times \vec{a})}{p(p^2 + |\vec{a}|^2)}$$

Q.13 If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\vec{d}$ is a unit vector, then find the value of $|(\vec{a} \cdot \vec{d})(\vec{b} \times \vec{c}) + (\vec{b} \cdot \vec{d})(\vec{c} \times \vec{a}) + (\vec{c} \cdot \vec{d})(\vec{a} \times \vec{b})|$ independent of $\vec{d}$.

Sol. $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vector

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] \neq 0$

and $|\vec{d}|^2 = 1$

Let $\vec{A} = (\vec{a} \cdot \vec{d})(\vec{b} \times \vec{c}) + (\vec{b} \cdot \vec{d})(\vec{c} \times \vec{a}) + (\vec{c} \cdot \vec{d})(\vec{a} \times \vec{b})$

Let $\vec{a} \cdot \vec{d} = x$, $\vec{b} \cdot \vec{d} = y$, $\vec{c} \cdot \vec{d} = z$

$\Rightarrow \vec{A} = x(\vec{b} \times \vec{c}) + y(\vec{c} \times \vec{a}) + z(\vec{a} \times \vec{b})$

Taking dot product by $\vec{a}\ \vec{b}\ \vec{c}$ respectively

$\vec{A} \cdot \vec{a} = x[\vec{a}\ \vec{b}\ \vec{c}] = (\vec{a} \cdot \vec{d})[\vec{a}\ \vec{b}\ \vec{c}]$

similarly $\vec{A}\,.\,\vec{b} = (\vec{b}\,.\,\vec{d})\,[\vec{a}\;\vec{b}\;\vec{c}]$

and $\vec{A}\,.\,\vec{c} = (\vec{c}\,.\,\vec{d})[\vec{a}\;\vec{b}\;\vec{c}]$

Adding these three relation we get

$\vec{A}\,.\,(\vec{a}+\vec{b}+\vec{c})$

$= \{(\vec{a}+\vec{b}+\vec{c}).\,\vec{d}\}\,[\vec{a}\;\vec{b}\;\vec{c}]$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c})\,\{\vec{d}\,.[\vec{a}\;\vec{b}\;\vec{c}] - \vec{A}\} = 0$

$\because \vec{a}, \vec{b}, \vec{c}$ non coplanar $\Rightarrow \vec{a}+\vec{b}+\vec{c} \neq 0$

$\Rightarrow \vec{d}\,.\,[\vec{a}\;\vec{b}\;\vec{c}] = \vec{A}$

Taking mod $|\vec{A}| = |[\vec{a}\;\vec{b}\;\vec{c}]|$

$\because |\vec{d}| = 1$

Q.14 It is given that $\vec{x} = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$; $\vec{y} = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$; $\vec{z} = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ Where $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors, show that $\vec{x}$, $\vec{y}$, $\vec{z}$ also forms a non-coplanar system. Find the value of $\vec{x}.(\vec{a}+\vec{b}) + \vec{y}.(\vec{b}+\vec{c}) + \vec{z}.(\vec{c}+\vec{a})$.

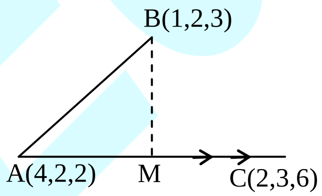
Sol. 3

Q.15 Prove or disprove the formula

$\vec{A}\times[\vec{A}\times(\vec{A}\times\vec{B})]\,.\,\vec{C} = -|\vec{A}|^2\,\vec{A}.\vec{B}\times\vec{C}$,

Q.16 Find the distance of the point B $(\hat{i}+2\hat{j}+3\hat{k})$ from the line which is passing through A $(4\hat{i}+2\hat{j}+2\hat{k})$ and which is parallel to the vector $\vec{C} = 2\hat{i}+3\hat{j}+6\hat{k}$.

Sol.



$\therefore BM^2 = AB^2 - AM^2$

$\vec{AB} = -3\hat{i}+\hat{k}$

$AB^2 = 10$

AM = projection of $\vec{AB}$ in direction of $\vec{C}$

$AM = \dfrac{\vec{AB}.\vec{C}}{|\vec{C}|}$

$= \dfrac{(-3\hat{i}+\hat{k}).(2\hat{i}+3\hat{j}+6\hat{k})}{7} = 0$

$\Rightarrow \vec{AB} \perp \vec{C}$

$BM^2 = 10 - 0 = 10$

$BM = \sqrt{10}$

Q.17 A line passes through the point A (6, 2, 2) and is parallel to the vector $\vec{p} = \hat{i} - 2\hat{j} + 2\hat{k}$. Another line passes through B (– 4, 0, –1) and is parallel to the vector $\vec{q} = 3\hat{i} - 2\hat{j} - 2\hat{k}$. Find the shortest distance between these two lines.

Sol. $\sqrt{10}$ unit

Q.18 Given four non- zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ and $(\hat{\vec{a}\,\vec{b}}) = (\hat{\vec{b}\,\vec{c}}) = \dfrac{\pi}{3}$, $(\hat{\vec{d}\,\vec{a}}) = \alpha$ and $(\hat{\vec{d}\,\vec{b}}) = \beta$, prove that $(\hat{\vec{d}\,\vec{c}}) = \cos^{-1}(\cos\beta - \cos\alpha)$.

Q.19 Consider the non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ such that no three of which are coplanar then prove that $\vec{a}[\vec{b}\;\vec{c}\;\vec{d}] + \vec{c}[\vec{a}\;\vec{b}\;\vec{d}] = \vec{b}[\vec{a}\;\vec{c}\;\vec{d}] + \vec{d}[\vec{a}\;\vec{b}\;\vec{c}]$. Hence prove that $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ represent the position vectors of the vertices of a plane quadrilateral if and only if $\dfrac{[\vec{b}\vec{c}\vec{d}]+[\vec{a}\vec{b}\vec{d}]}{[\vec{a}\vec{c}\vec{d}]+[\vec{a}\vec{b}\vec{c}]} = 1$.

Q.20 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a}\times(\vec{a}\times\vec{c}) + \vec{b} = \vec{0}$, then find the acute angle between $\vec{a}$ and $\vec{c}$.

Sol. $\vec{a}\times(\vec{a}\times\vec{c}) + \vec{b} = 0$

$\Rightarrow (\vec{a}.\vec{c})\vec{a} - (\vec{a}.\vec{a})\vec{c} + \vec{b} = 0$

$\Rightarrow 2\cos\theta\,\vec{a} - \vec{c} + \vec{b} = 0$

$\Rightarrow (2\cos\theta\ \vec{a} - \vec{c})^2 = (-\vec{b})^2$

$\Rightarrow 4|\vec{a}|^2 \cos^2\theta + |\vec{c}|^2 - 4\cos\theta\ \vec{a}\,.\,\vec{c} = |\vec{b}|^2$

$\Rightarrow 4\cos^2\theta + 4 - 8\cos\theta\,.\,\cos\theta = 1$

$\Rightarrow 4\cos^2\theta = 3$

$\Rightarrow \cos\theta = \pm\dfrac{\sqrt{3}}{2}$

But θ acute $\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = \pi/6$

Q.21 Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = 10\vec{a} + 2\vec{b}$ and $\overrightarrow{OC} = \vec{b}$ where $\vec{O}$, $\vec{A}$ and $\vec{C}$ are non-collinear points. Let p denote the area of the quadrilateral OABC, and let q denote the area of the parallelogram with OA and OC as adjacent sides. If p = kq, then find the value of k .

Sol. p = area of the quadrilateral OABC

$= \dfrac{1}{2}\,|\overrightarrow{OB} \times \overrightarrow{AC}|$

$= \dfrac{1}{2}|\overrightarrow{OB} \times (\overrightarrow{OC} - \overrightarrow{OA})|$

$= \dfrac{1}{2}|(10\vec{a} + 2\vec{b}) \times (\vec{b} - \vec{a})|$

$= \dfrac{1}{2}|10(\vec{a} \times \vec{b}) - 2(\vec{b} \times \vec{a})|$

$= \dfrac{1}{2}|12(\vec{a} \times \vec{b})| = \dfrac{12}{2}|\vec{a} \times \vec{b}| = 6|\vec{a} \times \vec{b}|$

and q = area of parallelogram with OA and OC as adjacent sides

$= |\overrightarrow{OA} \times \overrightarrow{OC}|$

$q = |\vec{a} \times \vec{b}|$

$\Rightarrow p = 6q$

$\Rightarrow k = 6$

Q.22 If vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\,.\,\vec{a} & \vec{a}\,.\,\vec{b} & \vec{a}\,.\,\vec{c} \\ \vec{b}\,.\,\vec{a} & \vec{b}\,.\,\vec{b} & \vec{b}\,.\,\vec{c} \end{vmatrix} = \vec{0}$$

Sol. Given that $\vec{a}, \vec{b}, \vec{c}$ are three coplanor vectors

$\therefore$ there exist scalars x, y, z not all zero such that $x\vec{a} + y\vec{b} + z\vec{c} = 0$ ....(i)

taking dot product (i) with $\vec{a}$ and $\vec{b}$ respectively we get

$x\vec{a}\,.\,\vec{a} + y\vec{a}\,.\,\vec{b} + z\,\vec{a}\,.\,\vec{c} = 0$ ....(ii)

and $x\vec{b}\,.\,\vec{a} + y\vec{b}\,.\,\vec{b} + z\,\vec{b}\,.\,\vec{c} = 0$.....(iii)

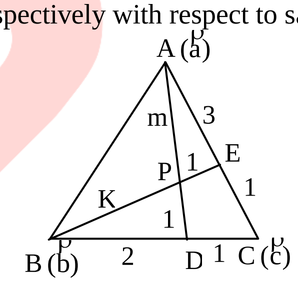
Now equation (i), (ii) & (iii) form a homogeneous system of equations where x, y, z are not all zero

$\therefore$ system must have non trivial solution and for this determinant of coefficinet matrix should be zero

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\,.\,\vec{a} & \vec{a}\,.\,\vec{b} & \vec{a}\,.\,\vec{c} \\ \vec{b}\,.\,\vec{a} & \vec{b}\,.\,\vec{b} & \vec{b}\,.\,\vec{c} \end{vmatrix} = \vec{0}$$

Q.23 In a triangle ABC, D and E are points on BC and AC respectively, such that BD = 2DC and AE = 3EC. Let P be the point of intersection of AD and BE. Find BP/PE using vector methods.

Sol. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vector of A, B, C respectively with respect to same origin then



D divides BC in the ratio 2 : 1 and E divides AC in the ratio 3 : 1

$\Rightarrow$ Position vector of D is $\dfrac{\vec{b} + 2\vec{c}}{3}$ and position vector of E is $\dfrac{\vec{a} + 3\vec{c}}{4}$

Let point of intersection P of AD and BE divides BE in the ratio k : 1 and AD in the ratio m : 1 Then position vector of P in two cases are

$$\frac{\vec{b} + k\left(\dfrac{\vec{a} + 3\vec{c}}{4}\right)}{k+1} \text{ and } \frac{\vec{a} + m\left(\dfrac{\vec{b} + 2\vec{c}}{3}\right)}{m+1}$$

Equating coefficient of P in both cases we get

$\dfrac{k}{4(k+1)} = \dfrac{1}{m+1}$ and $\dfrac{1}{k+1} = \dfrac{m}{3(m+1)}$

and $\dfrac{3k}{4(k+1)} = \dfrac{2m}{3(m+1)}$

Solving we get k = $\frac{8}{3}$

required ration BP : PE = 8: 3

Q.24 If the vectors $\vec{b}, \vec{c}, \vec{d}$ are not coplanar, then prove that the vector $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$ is parallel to $\vec{a}$.

Sol. $\vec{b}, \vec{c}, \vec{d}$ are not coplanar $\Rightarrow [\vec{b}, \vec{c}, \vec{d}] \neq 0$

we have

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$

Here

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = -(\vec{c} \times \vec{d}) \times (\vec{a} \times \vec{b})$

$= -(\vec{c} \times \vec{d} . \vec{b})\vec{a} + (\vec{c} \times \vec{d} . \vec{a})\vec{b}$

$= [\vec{a}\ \vec{c}\ \vec{d}]\vec{b} - [\vec{b}\ \vec{c}\ \vec{d}]\ \vec{a}$ ....(i)

and $(\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b})$

$= [\vec{a}\ \vec{d}\ \vec{b}]\vec{c} - [\vec{c}\ \vec{d}\ \vec{b}]\ \vec{a}$ ....(ii)

and $(\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$

$= -[\vec{a}\ \vec{c}\ \vec{d}]\vec{b} - [\vec{a}\ \vec{d}\ \vec{b}]\ \vec{c}$ ....(iii)

adding (i) (ii) & (iii) we get

given vector $= -2[\vec{b}\ \vec{c}\ \vec{d}]\vec{a} = k\vec{a}$

$\Rightarrow$ given vector is parallel to $\vec{a}$

Q.25 If $\vec{A}, \vec{B}, \vec{C}$ are vectors such that $|\vec{B}| = |\vec{C}|$, prove that

$[(\vec{A}+\vec{B}) \times (\vec{A}+\vec{C})] \times (\vec{B}\times\vec{C}) . (\vec{B}+\vec{C}) = 0$

Sol. we have

$(\vec{A} + \vec{B}) \times (\vec{A}+\vec{C}) = \vec{A}\times\vec{C} + \vec{B}\times\vec{A} + \vec{B}\times\vec{C}$

Thus

$[(\vec{A} + \vec{B}) \times (\vec{A}+\vec{C})] \times (\vec{B} \times \vec{C})$

$= [(\vec{A}\times\vec{C}) + (\vec{B} \times \vec{A}) + (\vec{B}\times\vec{C})] \times (\vec{B} \times \vec{C})$

$= (\vec{A}\times\vec{C}) \times (\vec{B}\times\vec{C}) + (\vec{B}\times\vec{A}) \times (\vec{B} \times \vec{C}) + (\vec{B} \times \vec{C}) \times (\vec{B} \times \vec{C})$

$= \{(\vec{A}\times\vec{C}).\vec{C}\}\ \vec{B} - \{(\vec{A}\times\vec{C}).\vec{B}\}\ \vec{C}$

$+ \{(\vec{B}\times\vec{A}).\vec{C}\}\ \vec{B} - \{(\vec{B}\times\vec{A}).\vec{B}\}\vec{C}$

$= [\vec{B}\ \vec{A}\ \vec{C}]\vec{B} - [\vec{A}\ \vec{C}\ \vec{B}]\vec{C}$

$\Rightarrow [\vec{A}\ \vec{C}\ \vec{B}]\{\vec{B} - \vec{C}\}$

Thus L.H.S

$= [\vec{A}\ \vec{C}\ \vec{B}](\vec{B} - \vec{C}) . (\vec{B} + \vec{C})\}$

$= [\vec{A}\ \vec{C}\ \vec{B}]\{|\vec{B}|^2 - |\vec{C}|^2\} = 0$

$\therefore$ |B| = |C|

Q.26 Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$, then prove that $|(\vec{u} \times \vec{v}) . \vec{w}| \leq 1/2$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$

Sol. $\vec{u}$ and $\vec{v}$ are unit vector $\Rightarrow |\vec{u}|^2 = |\vec{v}|^2 = 1$

Let θ be the angle between $\vec{u}$ and $\vec{w}$ then

we want to prove $|(\vec{u}\times\vec{v}).\vec{w}| \leq \frac{1}{2}$

Given $\vec{w} + (\vec{w}\times\vec{u}) = \vec{v}$ ......(i)

$\vec{w} = \vec{v} - (\vec{w}\times\vec{u})$

on squaring we get

$w^2 = 1 + (\vec{w}.1.\sin\theta)^2 - 2\vec{v}.(\vec{w}\times\vec{u})$

$\Rightarrow w^2(1-\sin^2\theta) - 1 = 2[\vec{v}\ \vec{w}\ \vec{u}] = 2[\vec{u}\ \vec{v}\ \vec{w}]$

$\Rightarrow \frac{1}{2}(w^2\cos^2\theta - 1) = (\vec{u}\times\vec{v}).\vec{w}$ ....(ii)

again squaring we get

$w^2 + w^2\sin^2\theta + 2[\vec{w}\ \vec{w}\ \vec{u}] = 1$

$w^2 = \frac{1}{1+\sin^2\theta}$

from (ii) $[\vec{u}\ \vec{v}\ \vec{w}] = \frac{1}{2}\left(\frac{\cos^2\theta}{1+\sin^2\theta} - 1\right)$

$\Rightarrow [\vec{u}\ \vec{v}\ \vec{w}] = -\frac{\sin^2\theta}{1+\sin^2\theta}$

$|\vec{u}\ \vec{v}\ \vec{w}| = \frac{\sin^2\theta}{1+\sin^2\theta} = \frac{1}{\operatorname{cosec}^2\theta + 1} \leq \frac{1}{2}$

equality hold only when sin θ = 1

$\Rightarrow \theta = \pi/2\ \ \vec{u} \perp \vec{w}$

$\Rightarrow \vec{u}.\vec{w} = 0$

we want to prove

$\Rightarrow \vec{u} \perp \vec{v} \Rightarrow \vec{u}.\vec{v} = 0$

taking dot product with $\vec{u}$ from (i) we get

$\vec{u}.\vec{w} + [\vec{u}\ \vec{w}\ \vec{u}] = \vec{u}.\vec{v}$

$\Rightarrow \vec{u}.\vec{v} = 0$

$\Rightarrow \vec{u} \perp \vec{v}$

Part-B Passage based objective questions

Passage I (Question 27 to 29)

Let $\vec{u}, \vec{v}, \vec{w}$ be three unit vectors such that

$\vec{u}+\vec{v}+\vec{w}=\vec{a}$, $\quad \vec{u}\times(\vec{v}\times\vec{w})=\vec{b}$

$(\vec{u}\times\vec{v})\times\vec{w}=\vec{c}$, $\quad \vec{a}.\vec{u}=\frac{3}{2}$, $\vec{a}.\vec{v}=\frac{7}{4}$

and $|\vec{a}|$ = 2, then

Q.27 The value of $\vec{u}.\vec{v}+\vec{u}.\vec{w}+\vec{v}.\vec{w}$ is -

(A) $\frac{1}{2}$ (B) 0 (C) 2 (D) none

Sol. **[A]**

$\therefore\ \vec{u}+\vec{v}+\vec{w}=\vec{a}$

squaring we get

$1+1+1+2(\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u})=4$

Q.28 The vector $\vec{v}$ is given by -

(A) $-4\vec{c}$ (B) $\frac{4}{3}(\vec{c}-\vec{b})$

(C) $\vec{a}+\frac{4}{3}\vec{b}+\frac{8}{3}\vec{c}$ (D) none of these

Sol. **[A]**

Multiplying (i) by $\vec{u}$ and $\vec{v}$ respectively

$1+\vec{u}.\vec{v}+\vec{u}.\vec{w}=\vec{u}.\vec{a}$

$\Rightarrow\ \vec{u}.\vec{v}+\vec{u}.\vec{w}=\frac{1}{2}$ .....(vi)

and $\vec{u}.\vec{v}+1+\vec{v}.\vec{w}=\vec{a}.\vec{v}$

$\Rightarrow\ \vec{u}.\vec{v}+\vec{v}.\vec{w}=\frac{3}{4}$ ......(vii)

Solving (ii), (iii), (vi), (vii) we get

$\vec{v}.\vec{w}=0, +\ \vec{u}.\vec{w}=-\frac{1}{4}, \vec{u}.\vec{v}=\frac{3}{4}$ .....(viii)

form (iii) & (viii) we get $-\frac{1}{4}\vec{v}=\vec{c}$

$\Rightarrow\ \vec{v}=-4\vec{c}$

Q.29 The value of $\vec{w}$ is given by -

(A) $-4\vec{c}$ (B) $\frac{4}{3}(\vec{c}-\vec{b})$

(C) $\vec{a}+\frac{4}{3}\vec{b}+\frac{8}{3}\vec{c}$ (D) none of these

Sol. **[B]**

From equation (ii) & (viii) we get

$-\frac{1}{4}\vec{v}-\frac{3}{4}\vec{w}=\vec{b}$

$\therefore$ from Q No. 21 we have

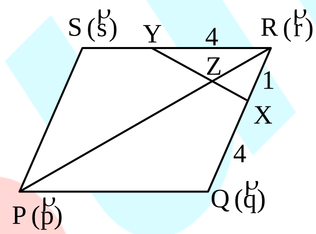
$\vec{v}=-4\vec{c}$

$\Rightarrow\ \vec{c}-\frac{3}{4}\vec{w}=\vec{b}$

$\Rightarrow\ \vec{w}=\frac{4}{3}(\vec{c}-\vec{b})$

Passage II (Question 30 to 32)

Points X and Y are taken on the sides QR and RS respectively of a parallelogram PQRS, so that QX = 4XR and RY = 4YS as shown in figure. The line XY cuts the line PR at Z.



Q.30 The ratio of XZ : ZY is -

(A) 1 : 4 (B) 21 : 4

(C) 1 : 3 (D) none of these

Sol. **[A]**

Q.31 The ratio of PZ : ZR is -

(A) 1: 4 (B) 21 : 4

(C) 1 : 3 (D) none of these

Sol. **[B]**

Q.32 The p.v. of Z is given by -

(A) $\frac{\vec{s}+\vec{r}+\vec{q}}{25}$ (B) $\frac{20\vec{s}+8\vec{r}+\vec{q}}{25}$

(C) $\frac{10\vec{s}+4\vec{r}+2\vec{q}}{25}$ (D) none of these

Sol. **[D]**

Passage III (Question 33 to 35)

Let the unit vectors $\vec{A}$ and $\vec{B}$ be perpendicular and the unit vector $\vec{C}$ be inclined at an angle θ to both $\vec{A}$ & $\vec{B}$. If $\vec{C}=\alpha\vec{A}+\beta\vec{B}+\gamma(\vec{A}\times\vec{B})$.

Q.33 Which of the following is true -

(A) $\alpha = 2\beta$ (B) $\alpha = \beta$ (C) $\alpha = 3\beta$ (D) none

Sol. **[B]**

$\vec{A}\cdot\vec{C} = \alpha + \beta(\vec{A}\cdot\vec{B}) + \gamma\vec{A}\cdot(\vec{A}\times\vec{B}) = \alpha$

and $\vec{B}\cdot\vec{C} = \beta$

from above $\vec{A}\cdot\vec{C} = \vec{B}\cdot\vec{C}$

$\Rightarrow \alpha = \beta$

Q.34 The value of γ is given by -

(A) $\gamma^2 = 1 - 2\alpha^2$ (B) $\gamma^2 = 1 + 2\alpha^2$

(C) $\gamma^2 = 1 + 2\beta$ (D) $\gamma = 1 - 2\beta^2$

Sol. **[A]**

$\vec{C}\cdot\vec{C} = 1 = 2\alpha^2 + \gamma^2|\vec{A}\times\vec{B}|^2$

$1 = 2\alpha^2 + \gamma^2(|\vec{A}|^2|\vec{B}|^2 - (\vec{A}\cdot\vec{B})^2)$

$\Rightarrow 1 = 2\alpha^2 + \gamma^2$

$\Rightarrow \gamma^2 = 1 - 2\alpha^2$

Q.35 The value of γ in terms of θ is given by -

(A) $\gamma^2 = -\cos 2\theta$ (B) $\gamma^2 = \cos 2\theta$

(C) $\gamma^2 = \dfrac{1+\cos 2\theta}{2}$ (D) none of these

Sol. **[A]**

$\therefore \gamma^2 = 1 - 2\alpha^2$

$= 1 - 2\cos^2\theta = -\cos 2\theta$

EXERCISE # 4

Old IIT-JEE questions

Q.1 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit coplanar vectors then the scalar triple product $[2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}, 2\vec{c}-\vec{a}]$= **[IIT scr. 2001]**

(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

Sol. **[A]**

$\vec{a}, \vec{b}, \vec{c}$ are unit coplanar vectors,

$2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}, 2\vec{c}-\vec{a}$ are also coplanar vectors

Thus $[2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}, 2\vec{c}-\vec{a}] = 0$

Q.2 Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on - **[IIT scr. 2001]**

(A) only x (B) only y

(C) neither x nor y (D) both x and y

Sol. **[C]**

$\vec{a} = \hat{i} - \hat{k}$, $b = x\hat{i} + \hat{j} + (1-x)\hat{k}$

and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$ then

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$= 1(1 + x - y - x(1-x) - 1(x^2 - y))$

$= 1 + x - y - x + x^2 - x^2 + y$

$= 1$

$[\vec{a}\ \vec{b}\ \vec{c}]$ depends neither x nor y

Q.3 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$ does not exceed- **[IIT-2001S]**

(A) 4 (B) 9 (C) 8 (D) 6

Sol. **[B]**

$\vec{a}, \vec{b}, \vec{c}$ are unit vectors

$\Rightarrow \vec{a}.\vec{a} = \vec{b}.\vec{b} = \vec{c}.\vec{c} = 1$

Let

$x = |\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$

$= 2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 - (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}))$

$= 6 - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

Now

$|\vec{a}+\vec{b}+\vec{c}|^2 \ge 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \ge 0$

$\Rightarrow -2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \le 3$

$\Rightarrow 6 - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \le 9$

$= x \le 9$

$\Rightarrow$ x does not exceed 9

Q.4 Find 3 dimensional vector $\vec{V}_1$, $\vec{V}_2$, $\vec{V}_3$ satisfying

$\vec{V}_1 . \vec{V}_1 = 4$, $\vec{V}_1 . \vec{V}_2 = -2$, $\vec{V}_1 . \vec{V}_3 = 6$,

$\vec{V}_2 . \vec{V}_2 = 2$, $\vec{V}_2 . \vec{V}_3 = -5$ & $\vec{V}_3 . \vec{V}_3 = 29$. **[IIT 2001]**

Sol. Many answers are possible one of the possible answers $V_1 = 2\hat{i}$, $V_2 = -\hat{i} + \hat{j}$, $V_3 = 3\hat{i} - 2\hat{j} + 4\hat{k}$

Q.5 Let $\vec{A}(t) = f_1(t)\ \hat{i} + f_2(t)\ \hat{j}$ and $\vec{B}(t) = g_1(t)\ \hat{i} + g_2(t)\hat{j}$, $t \in [0, 1]$, where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are non zero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ & $\vec{B}(1) = 2\hat{i} + 2\hat{j}$, then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some $t \in [0, 1]$. **[IIT- 2001]**

Q.6 If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is – **[IIT scr. 2002]**

(A) 45º (B) 60º

(C) $\cos^{-1}(1/3)$ (D) $\cos^{-1}(2/7)$

Sol. **[B]**

Given that $\vec{a}$ and $\vec{b}$ are unit vector

$\Rightarrow |\vec{a}| = |\vec{b}| = 1$

Given that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are $\perp^r$

$\Rightarrow (\vec{a} + 2\vec{b}).(5\vec{a} - 4\vec{b}) = 0$

$\Rightarrow 5|\vec{a}|^2 - 8|\vec{b}|^2 + 6\vec{a}.\vec{b} = 0$

$\Rightarrow 5 - 8 + 6\vec{a}.\vec{b} = 0$

$\Rightarrow 6|\vec{a}||\vec{b}|\cos\theta = 3$

$\Rightarrow \cos\theta = 1/2$

$\Rightarrow \theta = 60^o$

Q.7 Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector; then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is–

[IIT scr. 2002]

(A) – 1 (B) $\sqrt{10} + \sqrt{6}$

(C) $\sqrt{59}$ (D) $\sqrt{60}$

Sol. **[C]**

$\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{W} = \hat{i} + 3\hat{k}$

and $\vec{U}$ is a unit vector

$\Rightarrow |\vec{U}| = 1$

$[\vec{U}\ \vec{V}\ \vec{W}] = \vec{U}.(\vec{V} \times \vec{W})$

$\Rightarrow \vec{U}.(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} + 3\hat{k})$

$\Rightarrow \vec{U}.(3\hat{i} - 7\hat{j} - \hat{k})$

$= 1.\sqrt{9+49+1}\cos\theta$

$= \sqrt{59}\cos\theta$

It is maximum when $\cos\theta = 1$

$[\vec{U}\ \vec{V}\ \vec{W}]_{max} = \sqrt{59}$

Q.8 Let V be the volume of the parallelopiped formed by the vectors
$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$
$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$
if a_r, b_r, c_r where r = 1, 2, 3 are non negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$, show that $V \le L^3$. **[IIT- 2002]**

Sol. Given that

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

where a_r, b_r, c_r, r = 1, 2, 3 are all non negative real numbers

Also $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$

$\therefore$ V is volume of parallelopiped

$$V = [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\Rightarrow V = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$
$\quad - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)$........(i)

Now we know that AM $\ge$ GM

$$\therefore \frac{(a_1+b_1+c_1)+(a_2+b_2+c_2)+(a_3+b_3+c_3)}{3}$$
$\ge [(a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)]^{1/3}$

$\Rightarrow \frac{3L}{3} \ge [(a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)]^{1/3}$

$\Rightarrow L^3 \ge a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 + ...$

$\Rightarrow L^3 \ge (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) -$
$\quad (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)$

$\Rightarrow L^3 \ge V$

Q.9 If $\vec{u} = \hat{i} + a\hat{j} + \hat{k}$; $\vec{v} = \hat{j} + a\hat{k}$; $\vec{w} = a\hat{i} + \hat{k}$, then find the value of 'a' for which volume of parallelopiped formed by these three vectors as coterminous edges, is minimum. **[IIT scr. 2003]**

(A) $\sqrt{3}$ (B) 3 (C) $1/\sqrt{3}$ (D) 1/3

Sol. **[C]**

Q.10 Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ & $\vec{v}$ and $\vec{v}$ & $\vec{w}$ and $\vec{w}$ & $\vec{u}$ respectively. If $\vec{x}, \vec{y}, \vec{z}$ are the unit vectors along the bisectors of the angles α, β, γ respectively, then prove that $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]$

$= \dfrac{1}{16}\,[\vec{u}\ \vec{v}\ \vec{w}]^2.\ \sec^2\alpha/2\ .\sec^2\beta/2\ .\ \sec^2\gamma/2.$

[IIT- 2003]

Q.11 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{a}.\vec{b} = 1$ & $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$ then $\vec{b}$ is equal to - **[IIT Scr.2004]**

(A) $2\hat{i}$ (B) $\hat{i} - \hat{j} + \hat{k}$

(C) $\hat{i}$ (D) $2\hat{j} - \hat{k}$

Sol. **[C]**

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

Given $\vec{a}\,.\,\vec{b} = 1$

$x + y + z = 1$ ....(i)

and $\vec{a}\times\vec{b} = j - k$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = j - k$$

$\Rightarrow i(z - y) + j\,(x - z) + k(y - x) = j - k$

from comparing we get

$z - y = 0 \quad \Rightarrow z = y$ ....(ii)

$x - z = 1$ ....(iii)

and $x - y = 1$ ....(iv)

solving we get

$x = 1,\ y = 0,\ z = 0$

$\Rightarrow \vec{b} = \hat{i}$

Q.12 A unit vector is orthogonal to $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar to $2\hat{i} + \hat{j} + \hat{k}$ & $\hat{i} - \hat{j} + \hat{k}$ then the vector is - **[IIT Scr.2004]**

(A) $\dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (B) $\dfrac{2\hat{i}+5\hat{j}}{\sqrt{29}}$

(C) $\dfrac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (D) $\dfrac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

Sol. **[A]**

Any vector $\vec{r}$ coplanar with $\vec{a}$ and $\vec{b}$ we can written as

$\vec{r} = \vec{a} + \lambda\,\vec{b}$

$\Rightarrow \vec{r} = (2\hat{i} + \hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\Rightarrow \vec{r} = (2 + \lambda)\hat{i} + (1 - \lambda)\hat{j} + (1 + \lambda)\hat{k}$

$\therefore \vec{r}$ is orthogonal with $3\hat{i} + 2\hat{j} + 6\hat{k}$

$\Rightarrow 3(2 + \lambda) + 2(1 - \lambda) + 6(1 + \lambda) = 0$

$\Rightarrow 7\lambda + 14 = 0$

$\Rightarrow \lambda = -2$

$\Rightarrow \vec{r} = 3\hat{j} - \hat{k}$

since $\vec{r}$ is a unit vector

$\Rightarrow \vec{r} = \dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$

Q.13 $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are four distinct vectors satisfying the conditions $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ & $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$, then prove that $\vec{a}\,.\,\vec{b} + \vec{c}\,.\,\vec{d} \neq \vec{a}\,.\,\vec{c} + \vec{b}\,.\,\vec{d}$

[IIT- 2004]

Sol. Given that $\vec{a}\neq\vec{b}\neq\vec{c}\neq\vec{d}$

Such that $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ ....(i)

and $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ ....(ii)

To prove $(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) \neq 0$

Subtracting (ii) from (i) we get

$\vec{a}\times(\vec{c} - \vec{b}) = (\vec{b} - \vec{c})\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{c} - \vec{b}) = \vec{d}\times(\vec{c} - \vec{b})$

$\vec{a}\times(\vec{c} - \vec{b}) - \vec{d}\times(\vec{c} - \vec{b}) = 0$

$\Rightarrow (\vec{a} - \vec{d})\times(\vec{c} - \vec{b}) = 0$

$\Rightarrow \vec{a} - \vec{d} \parallel \vec{c} - \vec{b}$

$\angle$ between a – d and c – b is either 0º or 180º

$\Rightarrow (\vec{a} - \vec{d}).\ (\vec{c} - \vec{b}) = |\vec{a} - \vec{d}|\ |\vec{c} - \vec{b}|\ \cos 0^\circ$ (or $\cos 180^\circ$) $\neq 0$

as $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ all are distinct.

Q.14 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero, non-coplanar vectors and

$\vec{b}_1 = \vec{b} - \dfrac{\vec{b}\,.\,\vec{a}}{|\vec{a}|^2}\,\vec{a}$, $\vec{b}_2 = \vec{b} + \dfrac{\vec{b}\,.\,\vec{a}}{|\vec{a}|^2}\,\vec{a}$,

$$\vec{c}_1 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1,$$

$$\vec{c}_2 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}_1\cdot\vec{c}}{|\vec{b}_1|^2}\vec{b}_1,$$

$$\vec{c}_3 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1,$$

$$\vec{c}_4 = \vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a} - \frac{\vec{b}\cdot\vec{c}}{|\vec{b}|^2}\vec{b}_1$$, then the set of orthogonal vectors is - **[IIT Scr.2005]**

(A) $\{\vec{a}, \vec{b}_1, \vec{c}_3\}$ (B) $\{\vec{a}, \vec{b}_1, \vec{c}_2\}$

(C) $\{\vec{a}, \vec{b}_1, \vec{c}_1\}$ (D) $\{\vec{a}, \vec{b}_2, \vec{c}_2\}$

Sol. **[B]**

we observe that

$$\vec{a}\cdot\vec{b}_1 = \vec{a}\cdot\vec{b} - \left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\right)\vec{a}\cdot\vec{a}$$

$$= \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{c}_2 = \vec{a}\cdot\vec{c} - \frac{\vec{a}\cdot\vec{c}}{|\vec{a}|^2}|\vec{a}|^2 - \frac{\vec{c}\cdot\vec{b}_1}{|\vec{b}_1|}(\vec{a}\cdot\vec{b}_1)$$

$$= \vec{a}\cdot\vec{c} - \vec{a}\cdot\vec{c} - 0 = 0$$

$$\because [\vec{a}\cdot\vec{b}_1] = 0$$

and $\vec{b}_1\cdot\vec{c}_2 = \vec{b}_1\left(\vec{c} - \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{c}\cdot\vec{b}_1}{|\vec{b}_1|^2}\vec{b}_1\right)$

$$= \vec{b}_1\cdot\vec{c} - \frac{(\vec{c}\cdot\vec{a})(\vec{b}_1\cdot\vec{a})}{|\vec{a}|^2} - \frac{\vec{c}\cdot\vec{b}_1}{|\vec{b}_1|^2}|\vec{b}_1|^2$$

$$= \vec{b}_1\cdot\vec{c} - 0 - \vec{b}_1\cdot\vec{c} = 0$$

$\Rightarrow$ The set of orthogonal vector is $[\vec{a}\ \vec{b}_1\ \vec{c}_2]$

Q.15 A light ray incident along the unit vector $\hat{v}$ and the unit vector along the normal to reflecting surface at the point P is $\hat{n}$ outwards. If reflecting ray is along the unit vector $\hat{w}$, find $\hat{w}$ in terms of $\hat{v}$ and $\hat{n}$. **[IIT- 2005]**

Sol. $\hat{w} = \hat{v} - 2(\hat{n}\cdot\hat{v})\hat{n}$

Q.16 Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ & $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is of length $\frac{1}{\sqrt{3}}$ unit is- **[IIT-2006]**

(A) $4\hat{i} + \hat{j} - 4\hat{k}$ (B) $4\hat{i} - \hat{j} + 4\hat{k}$

(C) $2\hat{i} + \hat{j} - \hat{k}$ (D) $3\hat{i} + \hat{j} - 3\hat{k}$

Sol. **[B]**

Any vector $\vec{r}$ in the plane of $\vec{a}$ & $\vec{b}$ can be written as

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$= (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (1+\lambda)\hat{k}$$

Projection of $\vec{r}$ on $\vec{c}$ = $\frac{\vec{r}\cdot\vec{c}}{|\vec{c}|}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{|(1+\lambda)+(2-\lambda)-(1+\lambda)|}{\sqrt{3}}$$

$\Rightarrow |2-\lambda| = 1$

$\Rightarrow \lambda = 1$ or 3

$\Rightarrow \vec{r} = 2\hat{i} + \hat{j} + 2\hat{k}$ or $4\hat{i} - \hat{j} + 4\hat{k}$

Q.17 Let $\vec{A}$ is a vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$ then angle between vectors $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is– **[IIT-2006]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

Sol. **[B, D]**

Normal to plane P is

$\vec{n}_1 = (2\hat{j} + 3\hat{k}) \times (4\hat{j} - 3\hat{k}) = -18\hat{i}$

and Normal to plane P_2 is

$\vec{n}_2 = (\hat{j} - \hat{k}) \times (3\hat{j} + 3\hat{k}) = 3\hat{i} - 3\hat{j} - 3\hat{k}$

$\because$ $\vec{A}$ is parallel to $\pm(\vec{n}_1 \times \vec{n}_2)$

$\Rightarrow \pm(\vec{n}_1 \times \vec{n}_2) = \pm(-54\hat{j} + 54\hat{k})$

Now angle between $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is

$$\cos\theta = \pm \frac{(-54\hat{j}+54\hat{k}).(2\hat{i}+\hat{j}-2\hat{k})}{54.\sqrt{2}.3}$$

$$= \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

Q.18 The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is - **[IIT-2007]**

(A) zero (B) one

(C) two (D) three

Sol. **[C]**

Let $\vec{a} = -\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-\lambda^2\hat{j}+\hat{k}$

and $\vec{c} = \hat{i}+\hat{j}-\lambda^2\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$= \begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$= \lambda^6 - 3\lambda^2 - 2 = 0$

Let $\lambda^2 = t$

$\Rightarrow t^3 - 3t - 2 = 0$

$(t-2)(t+1)^2 = 0$

$\Rightarrow t = 2, t = -1$

$t \neq -1$

$\Rightarrow t = 2 \Rightarrow \lambda^2 = 2$

$\lambda = \pm\sqrt{2}$

two values

Q.19 Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the side of a regular hexagon.

Statement-1 : $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$

Because

Statement -2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$ **[IIT-2007]**

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1

(B) Statement-1, is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1is False, Statement-2 is True

Sol. **[C]**

Q.20 Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct ? **[IIT-2007]**

(A) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$

(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$

(C) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} \neq \vec{0}$

(D) $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ are mutually $\perp$.

Sol. **[B]**

$\vec{a} + \vec{b} + \vec{c} = 0$

Taking cross product by $\vec{a}, \vec{b}, \vec{c}$

$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times 0 = 0$

$\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$ ....(i)

and

$\vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times 0 = 0$

$\Rightarrow \vec{b} \times \vec{a} + \vec{b} \times \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ....(ii)

form (i) & (ii)

$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

$\because \vec{a} \times \vec{b} \neq 0,\ \vec{b} \times \vec{c} \neq 0\ \ \vec{c} \times \vec{a} \neq 0$

$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$

Q.21 The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = 1/2$. Then, the volume of the parallelopiped is **[IIT-2008]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Sol. **[A]**

$\therefore \vec{a}.\vec{b} = |a|\,|b| \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^o$

volume $|[\hat{a}\ \hat{b}\ \hat{c}]|$

$$= |(\hat{a} \times \hat{b}).\hat{c}| = \sqrt{\begin{vmatrix} \hat{a}.\hat{a} & \hat{a}.\hat{b} & \hat{a}.\hat{c} \\ \hat{b}.\hat{a} & \hat{b}.\hat{b} & \hat{b}.\hat{c} \\ \hat{c}.\hat{a} & \hat{c}.\hat{b} & \hat{c}.\hat{c} \end{vmatrix}}$$

$$= \frac{1}{\sqrt{2}}$$

Q.22 Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle . A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then- **[IIT-2008]**

(A) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+\hat{a}.\hat{b})^{1/2}$

(B) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+\hat{a}.\hat{b})^{1/2}$

(C) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+2\hat{a}.\hat{b})^{1/2}$

(D) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+2\hat{a}.\hat{b})^{1/2}$

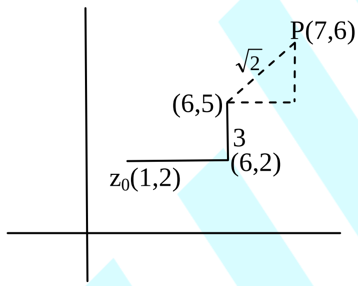
Sol. [A]

Q.23 A particle P starts from the point $z_0 = 1+2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particles moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by- **[IIT-2008]**

(A) $6 + 7\hat{i}$ (B) $-7 + 6\hat{i}$

(C) $7 + 6\hat{i}$ (D) $-6 + 7\hat{i}$

Sol. **[D]**



We see that the point P is (7, 6)

$\Rightarrow P = 7 + 6\hat{i}$

Now it is rotated by $\frac{\pi}{2}$ angle in anti clock wise since

$\Rightarrow \hat{i}(7 + 6\hat{i}) = -6 + 7\hat{i}$

Q. 24 If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$, then **[IIT 2009]**

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

(B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar

(C) $\vec{b}, \vec{d}$ are non- parallel

(D) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

Sol. **[C]**

$$(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}.\vec{c} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix} = 1$$

$$\Rightarrow \begin{vmatrix} \frac{1}{2} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix} = 1$$

$$\Rightarrow \frac{1}{2}(\vec{b}.\vec{d}) - (\vec{a}.\vec{d})(\vec{b}.\vec{c}) = 1$$

$$\left(|\vec{a}||\vec{b}|\sin\theta_1 \hat{n}_1\right).\left(|\vec{c}||\vec{d}|\sin\theta_2 \hat{n}_2\right) = 1$$

$$\Rightarrow \sin\theta_1 \sin\theta_2\ \hat{n}_1.\hat{n}_2 = 1$$

$$\theta_1 = \theta_2 = \frac{\pi}{2};\ \alpha = 0$$

$\vec{a} \perp \vec{b}$ & $\vec{c}$ & $\vec{d}$ & $\vec{n}_1 \parallel \vec{n}_2$

Q.25 Match the statements/expressions given in Column I with the values given in Column II. **[IIT 2009]**

Column I	**Column II**
(A) Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(P) $\frac{\pi}{6}$
(B) Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(Q) $\frac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i} + \hat{j}$, $\hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + \pi\hat{k}$	(R) $\frac{\pi}{3}$
(D) Angle between vectors $\vec{a}$ & $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \sqrt{3}\,\vec{c} = \vec{0}$	(S) $\frac{\pi}{2}$
	(T) π

Sol. A $\to$ Q, S; B $\to$ P, R, S, T ; C $\to$ T; D $\to$ R

Q.26 Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a - **[IIT 2010]**

(A) parallelogram, which is neither a rhombus nor a rectangle
(B) square
(C) rectangle, but not a square
(D) rhombus, but not a square

Sol.[A] $\overrightarrow{PQ} = 6i + j$

$\overrightarrow{RS} = 6i + j$

$\overrightarrow{RQ} = i - 3j$

$\overrightarrow{SP} = i - 3j$

$\left|\overrightarrow{PQ}\right| \neq \left|\overrightarrow{RQ}\right|$ ($\therefore$ not a rhombus or a rectangle)

PQ || RS
RQ || SP

Also $\overrightarrow{PQ} . \overrightarrow{RQ} \neq 0$

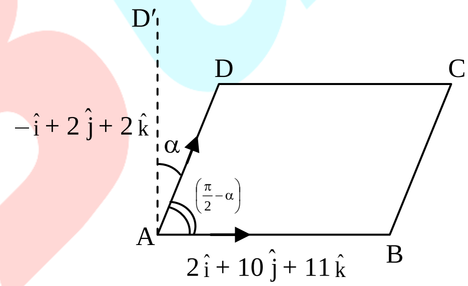
$\therefore$ PQRS is not a square

$\Rightarrow$ PQRS is a parallelogram

Q.27 Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by **[IIT 2010]**

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Sol. **[B]**



$$\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{\overrightarrow{AB}.\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|} = \frac{40}{3(15)} = \frac{8}{9}$$

$$\sin\alpha = \frac{8}{9} \Rightarrow \cos\alpha = \frac{\sqrt{17}}{9}$$

Q.28 If $\vec{a}$ and $\vec{b}$ are vector in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a} + \vec{b}).[(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})]$ is **[IIT- 2010]**

Sol. **[5]**

$|\bar{a}| = |\bar{b}| = 1 \,\&\, \bar{a} \cdot \bar{b} = 0$

$(2\bar{a} + \bar{b}) \cdot [(\bar{a} \times \bar{b}) \times (\bar{a} - 2\bar{b})]$

$= (2\bar{a} + \bar{b}) \cdot [\bar{b} + 2\bar{a}] = |\bar{b}|^2 + 4|\bar{a}|^2 = 5$

Q.29 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$

in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by **[IIT- 2011]**

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$

(C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Sol. [C] Let $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$

$\because$ $[\vec{a}\ \vec{b}\ \vec{v}] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ x & y & z \end{vmatrix} = 0$$

On solving $x = z$ ….(1)

$\because$ projection of $\vec{v}$ on $\vec{c}$ is $\frac{1}{\sqrt{3}}$

So, $\frac{1}{\sqrt{3}} = \frac{\vec{v}\cdot\vec{c}}{|\vec{c}|} \Rightarrow \frac{x-y-z}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow x - y - z = 1$ ….(2)

So solving (1) & (2)

$y = -1$ & $x = z$

Q.30 The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is /are **[IIT- 2011]**

(A) $\hat{j} - \hat{k}$ (B) $-\hat{i} + \hat{j}$

(C) $\hat{i} - \hat{j}$ (D) $-\hat{j} + \hat{k}$

Sol. [A, D]

$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is coplanar with the given vector so

$$\therefore \begin{vmatrix} x & y & z \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

So, $3x = y + z$ .....(1)

$\therefore \vec{r} \perp \hat{i} + \hat{j} + \hat{k}$

So, $\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 0$

So, $x + y + z = 0$ .....(2)

On solving (1) & (2)

So, $x = 0$ $\therefore$ $y + z = 0$

$\therefore$ (A) & (D) Satisfy

Q.31 Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}\cdot\vec{a} = 0$, then the value of $\vec{r}\cdot\vec{b}$ is. **[IIT- 2011]**

Sol. [9] $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$, $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$(\vec{r} - \vec{c}) \times \vec{b} = 0 \Rightarrow \vec{r} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

$\because$ $\vec{r}\cdot\vec{a} = 0$

$\Rightarrow$ $\vec{a}\cdot\vec{c} + \lambda\vec{b}\cdot\vec{a} = 0$

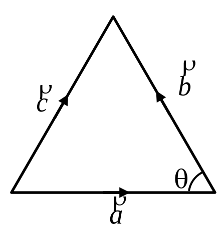
$\Rightarrow$ $\lambda = -\frac{\vec{a}\cdot\vec{c}}{\vec{b}\cdot\vec{a}} = 4$

$\Rightarrow$ $\vec{r}\cdot\vec{b} = \vec{c}\cdot\vec{b} + \lambda|\vec{b}|^2 = 9$

Q.32 Match the statements given in Column-I with the values given in Column-II. **[IIT- 2011]**

Column-I	**Column-II**
(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(P) $\pi/6$
(B) If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(Q) $2\pi/3$
(C) The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(R) $\pi/3$
(D) The maximum value of $\left|Arg\left(\frac{1}{1-z}\right)\right|$ for $\lvert z\rvert = 1, z \neq 1$ is given by	(S) π
	(T) $\pi/2$

Sol. (A) **A → q ; B → p, q, r, s, t or p ; C → s ; D → t**



$$\cos\theta = \frac{-\vec{a}\cdot\vec{b}}{|-\vec{a}||\vec{b}|} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

(B) $\int_a^b (f(x) - 3(x))dx = a^2 - b^2$

differentiating w.r.t (b).

$f(b) - 3b = -2b$

$\boxed{f(b) = b}$

So $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$; if a = b then any value of f(x) is possible

(C) $I = \frac{\pi^2}{\lambda n 3}\int_{7/6}^{5/6} \sec \pi x \, dx$

$$I = \frac{\pi^2}{\pi \lambda n 3} |\lambda n| \sec \pi x + \tan \pi x||_{7/6}^{5/6}$$

$$I = \frac{\pi}{\lambda n 3} \cdot \lambda n 3 = \pi$$

(D) $\therefore$ $|z| = 1$

$z = \cos\theta + i\sin\theta$. $\forall\ \theta \in (-\pi. \pi]$ and $\theta \neq 0$.

$$\left|Arg\ \frac{1}{(1-z)}\right|$$

$$= \left|Arg\left(\frac{1}{1-\cos\theta - i\sin\theta}\right)\right| = \left|Arg\left(\frac{1}{2} + \frac{i\cot\frac{\theta}{2}}{2}\right)\right|$$

$= \left|\frac{\pi-\theta}{2}\right|$ so maximum value is π.

Q.33 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is **[IIT- 2012]**

Sol. **[3]**

$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$

a.b + b.c + c.a = –3/2 ....(1)

$\because$ $|\vec{a}+\vec{b}+\vec{c}|^2 \geq 0$ ....(2)

a.b + b.c + c.a $\geq \frac{-3}{2}$ ....(3)

$\because$ from (1) & (3)

so $|\vec{a}+\vec{b}+\vec{c}| = 0$

$\vec{a}+\vec{b}+\vec{c} = 0$

$\vec{a} = -\vec{b}-\vec{c}$

on squaring

$1 = 2 + 2\cos B$

$\cos B = -\frac{1}{2}$ $\forall$ B = $\vec{b} \wedge \vec{c}$.

Let T = $|2\vec{a} + 5\vec{b} + 5\vec{c}|$

$= |3\vec{b} + 3\vec{c}|$

$= 3|\vec{b}+\vec{c}|$

$= 3\sqrt{2+2\cos B}$

$= 3$

Q.34 If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is **[IIT- 2012]**

(A) 0 (B) 3 (C) 4 (D) 8

Sol. **[C]**

$|\vec{a}+\vec{b}| = \sqrt{29}$

$\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$

$(\vec{a}+\vec{b}) \times (2\hat{i}+3\hat{j}+4\hat{k}) = \bar{0}$

$\vec{a}+\vec{b} = \lambda\ (2\hat{i}+3\hat{j}+4\hat{k})$

$|\vec{a}+\vec{b}| = \sqrt{4\lambda^2+9\lambda^2+16\lambda^2} = |\lambda|\ \sqrt{29}$

$\Rightarrow \lambda = 1, -1$

$\vec{a}+\vec{b} = \pm(2\hat{i}+3\hat{j}+4\hat{k})$

$(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$

$= \pm(2\hat{i}+3\hat{j}+4\hat{k}).(-7\hat{i}+2\hat{j}+3\hat{k}) = \pm 4$

EXERCISE # 5

Q.1 The number of vectors of unit length perpendicular to vectors $\vec{a}$ = (1, 1, 0) and $\vec{b}$ = (0, 1, 1) is- **[IIT-1987]**

(A) one (B) two
(C) three (D) infinite

Sol. **[B]**

Let (x, y, z) be the unit vector ⊥ to
$\vec{a}$ = (1,1,0) and $\vec{b}$ = (0, 1, 1)
⇒ x + y = 0 and y + z = 0 .....(i)
and $x^2 + y^2 + z^2 = 1$ ....(ii)
from (i) we have
x = – y and z = – y
from (ii) we get
$y^2 + y^2 + y^2 = 1$
$\Rightarrow y = \pm \frac{1}{\sqrt{3}}$
Clearly two vector possible

Q.2 Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations
$\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ then the value of the expression
$(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r}$ is equal to **[IIT-1988]**

(A) 0 (B) 1 (C) 2 (D) 3

Sol. **[D]**

Given that $\vec{a}\ \vec{b}\ \vec{c}$ are non coplanar
$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] \neq 0$
Now, $(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r}$
$= (\vec{a}+\vec{b}) . \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]} + (\vec{b}+\vec{c}) . \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$
$+(\vec{c}+\vec{a}). \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$
$= \frac{\vec{a}.\vec{b}\times\vec{c} + \vec{b}.\vec{c}\times\vec{a} + \vec{c}.\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$
using $\vec{b}.\vec{b}\times\vec{c} = \vec{c}.\vec{c}\times\vec{a} = \vec{a}.\vec{a}\times\vec{b} = 0$
$= \frac{[\vec{a}\vec{b}\vec{c}]+[\vec{a}\vec{b}\vec{c}]+[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]} = 3$

Q.3 Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is- **[IIT-1993]**

(A) the arithmetic mean of a and b
(B) the geometric mean of a and b
(C) the harmonic mean of a and b
(D) equal to zero

Sol. **[B]**

∵ Vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i}+\hat{k}$, $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar so
$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$
⇒ a(– c) – a(b – c) + c(c) = 0
$\Rightarrow c^2 - ab = 0 \Rightarrow c^2 = ab$
⇒ a,b,c are in G.P. and c is the G.M of a and b

Q.4 Let $\vec{p}$ and $\vec{q}$ be the position vectors of P and Q respectively, with respect to O and $|\vec{p}|$ = p, $|\vec{q}|$ = q. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If OR and OS are perpendicular then **[IIT-1994]**

(A) $9p^2 = 4q^2$ (B) $4p^2 = 9q^2$
(C) 9p = 4q (D) 4p = 9q

Sol. **[A]**

$\vec{OP} = \vec{p}$, $\vec{OQ} = \vec{q}$
$\vec{R} = \frac{3\vec{p}+2\vec{q}}{5}$ [∴ R divide PQ internally in the ratio 2 : 3]
and $\vec{S} = \frac{3\vec{p}-2\vec{q}}{1}$ [∵ S divide PQ externally in the ratio 2 :3]
Given $\vec{OR} \perp \vec{OS}$
$\Rightarrow \vec{OR} . \vec{OS} = 0$
$\Rightarrow \frac{3\vec{p}+2\vec{q}}{5} . \frac{3\vec{p}-2\vec{q}}{1} = 0$
$\Rightarrow 9|\vec{p}|^2 - 4|\vec{q}|^2 = 0$

$\Rightarrow 9p^2 = 4q^2$

Q.5 Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$ **[IIT-1994]**

(A) are collinear
(B) form an equilateral triangle
(C) form a scalene triangle
(D) form a right angled triangle

Sol. **[B]**

Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\vec{b} = \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$

$\vec{c} = \gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

Then $|\vec{b} - \vec{a}| = |(\beta-\alpha)\hat{i} + (\gamma-\beta)\hat{j} + (\alpha-\gamma)\hat{k}|$

$= \sqrt{2(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)}$

$|\vec{c} - \vec{b}| = |(\gamma-\beta)\hat{i} + (\alpha-\gamma)\hat{j} + (\beta-\alpha)\hat{k}|$

$= \sqrt{2(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)}$

and $|\vec{a} - \vec{c}| = |(\alpha-\gamma)\hat{i} + (\beta-\alpha)\hat{j} + (\gamma-\beta)\hat{k}|$

$= \sqrt{2(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)}$

Clearly it is a equilateral triangle

Q.6 Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30º, then $|(\vec{a} \times \vec{b}) \times \vec{c}| =$ **[IIT-1999]**

(A) 2/3 (B) 3/2 (C) 2 (D) 3

Sol. **[B]**

$|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}|\,|\vec{c}| \sin 30^\circ$

$= \frac{1}{2}|\vec{a} \times \vec{b}|\,|\vec{c}|$ ....(i)

Given $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$

we have $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{9} = 3$ ....(ii)

Given $|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\Rightarrow |\vec{c} - \vec{a}|^2 = 8$

$(\vec{c} - \vec{a}).(\vec{c} - \vec{a}) = 8$

$\Rightarrow |\vec{c}|^2 - \vec{a}.\vec{c} - \vec{c}.\vec{a} + |\vec{a}|^2 = 8$

$\Theta\ |\vec{a}| = 3$ and $\vec{a}.\vec{c} = |c|$

$\Rightarrow |c|^2 - 2|c| + 1 = 0$

$\Rightarrow (|\vec{c}| - 1)^2 = 0$

$\Rightarrow |\vec{c}| = 1$ ....(iii)

so from (i), (ii) and (iii) we get

$|(\vec{a} \times \vec{b}) \times \vec{c}| = \frac{1}{2} \times 3 \times 1 = \frac{3}{2}$

Q.7 Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a}.\vec{b})\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $|\vec{v}|$ is- **[IIT-1999]**

(A) $|\vec{u}|$ (B) $|\vec{u}| + |\vec{u}.\vec{a}|$
(C) $|\vec{u}| + |\vec{u}.\vec{b}|$ (D) $|\vec{u}| + \vec{u}.(\vec{a} + \vec{b})$

Sol. **[A,C]**

$\therefore \vec{v} = \vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta.\ \hat{n}$

$\therefore \vec{a}$ and $\vec{b}$ are unit vector

$\Rightarrow \vec{v} = \sin\theta.\ \hat{n}$

$\Rightarrow |\vec{v}| = \sin\theta$

now $\vec{u} = \vec{a} - (\vec{a}.\vec{b})\vec{b}$

$= \vec{a} - \vec{b}\cos\theta$

$\Rightarrow |\vec{u}|^2 = |\vec{a} - \vec{b}\cos\theta|^2$

$= 1 + \cos^2\theta - 2\cos\theta.\cos\theta$

$= 1 - \cos^2\theta = \sin^2\theta = |\vec{v}|^2$

$\Rightarrow |\vec{u}| = |\vec{v}|$

also $\vec{u}.\vec{b} = \vec{a}.\vec{b} - (\vec{a}.\vec{b})(\vec{b}.\vec{b})$

$= \vec{a}.\vec{b} - \vec{a}.\vec{b}$

$= 0$

$\Rightarrow |\vec{u}.\vec{b}| = 0$

$\Rightarrow \vec{v} = |\vec{u}| + |\vec{u}.\vec{b}|$

option A,C are correct.

Q.8 If the vector $\vec{a}$, $\vec{b}$ and $\vec{c}$ form the sides BC, CA and AB respectively of a triangle ABC, then - **[IIT 2000]**

(A) $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$
(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
(C) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$
(D) $\vec{a} \times \vec{b} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a} = 0$

Sol. **[B]**

Q.9 Let the vector $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ be such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively. Then the angle between P_1 and P_2 is- **[IIT 2000]**

(A) 0 (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/2$

Sol. **[A]**

Given that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$ ......(i)

P_1 is the plane determined by $\vec{a}$ and $\vec{b}$

$\therefore$ Normal vector $\vec{n}_1$ to P_1 is given by

$$\vec{n}_1 = \vec{a} \times \vec{b} \quad .....(ii)$$

Similarly P_2 is the plane determined by $\vec{c}$ and $\vec{d}$

$\therefore$ Normal vector $\vec{n}_2$ to P_2 is given by

$$\vec{n}_2 = \vec{c} \times \vec{d} \quad ......(iii)$$

from (i), (ii) & (iii) we get

$\vec{n}_1 \times \vec{n}_2 = 0$

$\Rightarrow n_1 \parallel n_2$

Hence planes will be parallel to each other

Hence angle between P_1 and P_2 is 0 (zero)

Q.10 If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and the vectors $\vec{A} = (1, a, a^2)$, $\vec{B} = (1, b, b^2)$, $\vec{C} = (1, c, c^2)$, are non-coplanar, then the product abc = ...

Sol. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$\Rightarrow (1 + abc) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

But $\vec{A} = \hat{i} + a\hat{j} + a^2\hat{k}$

$\vec{B} = \hat{i} + b\hat{j} + b^2\hat{k}$

and $\vec{C} = \hat{i} + c\hat{j} + c^2\hat{k}$

are non coplanar

$\Rightarrow [\vec{A}\,\vec{B}\,\vec{C}] \neq 0$

$\Rightarrow 1 + abc = 0$

$\Rightarrow abc = -1$

Q.11 If $\vec{A} = (1, 1, 1)$, $\vec{C} = (0, 1, -1)$ are given vectors, then a vector $\vec{B}$ satisfying the equations $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{A} \cdot \vec{B} = 3$ is.......

Sol. Given $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{C} = \hat{j} - \hat{k}$

Let $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$ ....(i)

Given $\vec{A} \times \vec{B} = \vec{C}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$

$\Rightarrow (z - y)\hat{i} + (x - z)\hat{j} + (y - x)\hat{k} = \hat{j} - \hat{k}$

$\Rightarrow z - y = 0$

$x - z = 1$

$x - y = 1$

$\Rightarrow y = z, x = 1 + z$ ....(ii)

And $\vec{A} \cdot \vec{B} = 3$

$\Rightarrow x + y + z = 3$ ....(iii)

from (ii) & (iii)

$1 + z + z + z = 3 \Rightarrow z = \frac{2}{3}$

$\Rightarrow y = \frac{2}{3}, x = \frac{5}{3}$

$\Rightarrow \vec{B} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

Q.12 The vector $-\hat{i} + \hat{j} - \hat{k}$ bisects the angle between the vectors $\vec{c}$ and $3\hat{i} + 4\hat{j}$. Determine the unit vector along $\vec{c}$.

Sol. Let unit vector $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow x^2 + y^2 + z^2 = 1$

$$\therefore -\hat{i} + \hat{j} - \hat{k} = t\left[x\hat{i} + y\hat{j} + z\hat{k} + \frac{3\hat{i} + 4\hat{j}}{5}\right]$$

comparing coefficient

$$-1 = t\left(x + \frac{3}{5}\right),\ 1 = t\left(y + \frac{4}{5}\right),\ -1 = tz$$

$\therefore x^2 + y^2 + z^2 = 1$

$$\Rightarrow \left(-\frac{1}{t} - \frac{3}{5}\right)^2 + \left(\frac{1}{t} - \frac{4}{5}\right)^2 + \left(-\frac{1}{t}\right)^2 = 1$$

$$\Rightarrow t = \frac{15}{2}$$

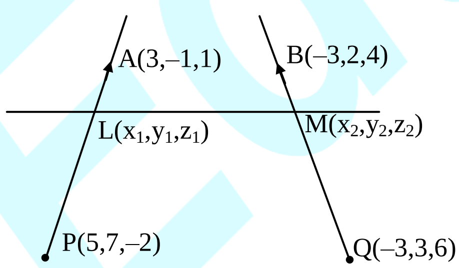
we get

$$x = -\frac{11}{15},\ y = -\frac{10}{15},\ z = -\frac{2}{15}$$

$$\vec{c} = -\frac{1}{15}(11\hat{i} + 10\hat{j} + 2\hat{k})$$

Q.13 The position vectors of the point P and Q are $5\hat{i} + 7\hat{j} - 2\hat{k}$ and $-3\hat{i} + 3\hat{j} + 6\hat{k}$ respectively. The vector $A = 3\hat{i} - \hat{j} + \hat{k}$ passes through the point P and the vector $B = -3\hat{i} + 2\hat{j} + 4\hat{k}$ passes through the point Q. A third vector $2\hat{i} + 7\hat{j} - 5\hat{k}$ intersects vectors A and B. Find the position vector of the points of intersection.

Sol.



Let vector $2\hat{i} + 7\hat{j} - 5\hat{k}$ cuts the vector $\vec{A}$ and $\vec{B}$ at L and M respectively

Let $L = (x_1, y_1, z_1)$ and $M = (x_2, y_2, z_2)$

Vector $\vec{PL}$ and $\vec{A}$ are collinear

$\therefore \vec{PL} = \lambda\vec{A}$

compare the coefficient

$$\frac{x_1 - 5}{3} = \frac{y_1 - 7}{-1} = \frac{z_1 + 2}{1} = \lambda \text{ let}$$

L is $(3\lambda + 5, -\lambda + 7, \lambda - 2)$

Similarly $\vec{QM} = \mu\vec{B}$ we get

M is $(-3\mu - 3, 2\mu + 3, 4\mu + 6)$

Again LM and vector $(2, 7, -5)$ are collinear

$$\Rightarrow \frac{x_2 - x_1}{2} = \frac{y_2 - y_1}{7} = \frac{z_2 - z_1}{-5} = \nu \text{ let}$$

$$\Rightarrow \frac{-3\mu - 3\lambda - 8}{2} = \frac{2\mu + \lambda - 4}{7} = \frac{4\mu - \lambda + 8}{-5} = \nu$$

or $3\mu + 3\lambda + 2\nu = -8$

$2\mu + \lambda - 7\nu = 4$

$4\mu - \lambda + 5\nu = -8$

solving these we get

$\lambda = \mu = \nu = -1$

L is $(2, 8, -3)$ or $2\hat{i} + 8\hat{j} - 3\hat{k}$

and M is $(0, 1, 2)$ or $\hat{j} + 2\hat{k}$

Q.14 Two triangles ABC and PQR are such the perpendiculars from A to QR, B to RP and C to PQ are concurrent. Show that the perpendicular from P to BC, Q to CA and R to AB are also concurrent. **[IIT 2000]**

Q.15 Find out whether the following pairs of lines are parallel, non-parallel and intersecting, or non-parallel and non-intersecting

(i) $\vec{r}_1 = \hat{i} + \hat{j} + 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$

$\vec{r}_2 = 2\hat{i} + \hat{j} + 3\hat{k} + \mu(-6\hat{i} + 4\hat{j} - 8\hat{k})$

(ii) $\vec{r}_1 = \hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{r}_2 = 2\hat{i} + 4\hat{j} + 6\hat{k} + \mu(2\hat{i} + \hat{j} + 3\hat{k})$

(iii) $\vec{r}_1 = \hat{i} + \hat{k} + \lambda(\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r}_2 = 2\hat{i} + 3\hat{j} + \mu(4\hat{i} - \hat{j} + \hat{k})$

Sol. (i) parallel (ii) the lines intersect at the point with p.v. $-2\hat{i} + 2\hat{j}$ (iii) lines are skew

Q.16 'O' is the origin of vectors and A is a fixed point on the circle of radius 'a' with centre O. The vector $\overrightarrow{OA}$ is denoted by $\vec{a}$. A variable point 'P' lies on the tangent at A & $\overrightarrow{OP} = \vec{r}$. Show that $\vec{a}.\vec{r} = |a|^2$. Hence if $P \equiv (x, y)$ &

$A \equiv (x_1, y_1)$ deduce the equation of tangent at A to this circle.

Sol. $xx_1 + yy_1 = a^2$

Q.17 Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60°. Suppose that $|\vec{u} - \hat{i}|$ is geometric mean of $|\vec{u}|$ and $|\vec{u} - 2\hat{i}|$ where $\hat{i}$ is the unit vector along x-axis then $|\vec{u}|$ has the value equal to $\sqrt{a} - \sqrt{b}$ where $a, b \in N$, find the value $(a + b)^3 + (a - b)^3$.

Sol. 28

Q.18 $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are the position vectors of the points $A \equiv (x, y, z)$; $B \equiv (y, -2z, 3x)$; $C \equiv (2z, 3x, -y)$ and $D \equiv (1, -1, 2)$ respectively. If $|\vec{a}| = 2\sqrt{3}$; $(\hat{\vec{a}\,\vec{b}}) = (\hat{\vec{a}\,\vec{c}})$; $(\hat{\vec{a}\,\vec{d}}) = \frac{\pi}{2}$ & $(\hat{\vec{a}\,\hat{j}})$ is obtuse, then find x, y, z.

Sol. $x = 2, y = -2, z = -2$

Q.19 If $\vec{r}$ and $\vec{s}$ are non zero constant vectors and the scalar b is chosen such that $|\vec{r} + b\vec{s}|$ is minimum, then show that the value of $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2$ is equal to $|\vec{r}|^2$.

Q.20 Given three points on the xy plane as O(0, 0), A(1, 0) and B (–1, 0). Point P is moving on the plane satisfying the condition $(\overrightarrow{PA} \cdot \overrightarrow{PB}) + 3(\overrightarrow{OA} \cdot \overrightarrow{OB}) = 0$. If the maximum and minimum value of $|\overrightarrow{PA}||\overrightarrow{PB}|$ are M and m respectively then find the value of $M^2 + m^2$.

Sol. 34

Q.21 If O is origin of reference, point $A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$; $D(\vec{a} + \vec{b})$; $E(\vec{b} + \vec{c})$; $F(\vec{c} + \vec{a})$; $G(\vec{a} + \vec{b} + \vec{c})$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ & $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then prove that these points are vertices of a cube having length of its edge equal to unity provided the matrix $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ is orthogonal. Also find the length XY such that X is the point of intersection of CM and GP, Y is the point of intersection of OQ and DN where P, Q, M, N are respectively the midpoint of sides CF, BD, GF and OB.

Sol. $\frac{\sqrt{11}}{3}$

Q.22 Given that $\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}$; $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ & $(\vec{u} \cdot \vec{R} - 10)\hat{i} + (\vec{v} \cdot \vec{R} - 20)\hat{j} + (\vec{w} \cdot \vec{R} - 20)\hat{k} = 0$. Find the unknown vector $\vec{R}$.

Sol. $-\hat{i} + 2\hat{j} + 5\hat{k}$

Q.23 (a) If $\vec{a} + \vec{b} + \vec{c} = 0$, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Deduce the sine rule for a ΔABC.

(b) Find the minimum area of the triangle whose vertices are A(–1, 1, 2); B(1, 2, 3) and C(t, 1, 1) where t is a real number.

Sol. (b) $\frac{\sqrt{3}}{2}$

Q.24 The length of the edge of the regular tetrahedron D–ABC is 'a'. Point E and F are taken on the edges AD and BD respectively such that E divides $\overrightarrow{DA}$ and F divides $\overrightarrow{BD}$ in the ratio 2 : 1 each. Then find the area of triangle CEF.

Sol. $\frac{5a^2}{12\sqrt{3}}$ sq. units

Q.25 Let $\vec{a} = \sqrt{3}\,\hat{i} - \hat{j}$ and $\vec{b} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ and $\vec{x} = \vec{a} + (q^2 - 3)\vec{b}$, $\vec{y} = -p\vec{a} + q\vec{b}$. If $\vec{x} \perp \vec{y}$, then express p as a function of q, say p = f (q), ($p \neq 0$ & $q \neq 0$) and find the intervals of monotonicity of f(q).

Sol. $p = \frac{q(q^2 - 3)}{4}$; decreasing in $q \in (-1, 1), q \neq 0$

Q.26 Let $\begin{vmatrix} (a_1 - a)^2 & (a_1 - b)^2 & (a_1 - c)^2 \\ (b_1 - a)^2 & (b_1 - b)^2 & (b_1 - c)^2 \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c)^2 \end{vmatrix} = 0$ & if the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$; $\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$; $\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$ are non coplanar, show that the vectors $\vec{\alpha}_1 = \hat{i} + a_1\hat{j} + a_1^2\hat{k}$; $\vec{\beta}_1 = \hat{i} + b_1\hat{j} + b_1^2\hat{k}$ & $\vec{\gamma}_1 = \hat{i} + c_1\hat{j} + c_1^2\hat{k}$ are coplanar.

Q.27 Given non-zero numbers x_1, x_2, x_3; y_1, y_2, y_3 and z_1, z_2 & z_3

(i) Can the given numbers satisfy

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0 \ \& \ \begin{cases} x_1x_2 + y_1y_2 + z_1z_2 = 0 \\ x_2x_3 + y_2y_3 + z_2z_3 = 0 \\ x_3x_1 + y_3y_1 + z_3z_1 = 0 \end{cases}$$

(ii) If $x_i > 0$ and $y_i < 0$ for all i = 1, 2, 3 and $P(x_1, x_2, x_3)$; $Q(y_1, y_2, y_3)$ and O(0, 0, 0), Can the triangle POQ be a right angled triangle?

Sol. NO, NO

Q.28 The p.v.'s of the four angular points of a tetrahedron are $A(\hat{j} + 2\hat{k})$; $B(3\hat{i} + \hat{k})$; $C(4\hat{i} + 3\hat{j} + 6\hat{k})$ and $D(2\hat{i} + 3\hat{j} + 2\hat{k})$. Find:

(i) the perpendicular distance from A to the line BC

(ii) the volume of the tetrahedron ABCD

(iii) the perpendicular distance from D to the plane ABC.

(iv) the shortest distance between the lines AB & CD.

Sol. (i) $\frac{6}{7}\sqrt{14}$ (ii) 6 (iii) $\frac{3}{5}\sqrt{10}$ (iv) $\sqrt{6}$

Q.29 The length of an edge of a cube $ABCDA_1B_1C_1D_1$ is equal to unity. A point E taken on the edge $\overrightarrow{AA_1}$ is such that $\left|\overrightarrow{AE}\right| = \frac{1}{3}$. A point F is taken on the edge $\overrightarrow{BC}$ such that $\left|\overrightarrow{BF}\right| = \frac{1}{4}$. If O_1 is the centre of the cube, find the shortest distance of the vertex B_1 from the plane of the ΔO_1EF.

Sol. $\frac{11}{\sqrt{170}}$

Q.30 $A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$ are the vertices of the triangle ABC such that $\vec{a} = \frac{1}{2}(2\hat{i} - \vec{r} - 7\hat{k})$; $\vec{b} = 3\vec{r} + \hat{j} - 4\hat{k}$; $\vec{c} = 22\hat{i} - 11\hat{j} - 9\vec{r}$. A vector $\vec{p} = 2\hat{j} - \hat{k}$ is such that $(\vec{r} + \vec{p})$ is parallel to $\hat{i}$ & $(\vec{r} - 2\hat{i})$ is parallel to $\vec{p}$. Show that there exists a point $D(\vec{d})$ on the line AB with $\vec{d} = 2t\hat{i} + (1 - 2t)\hat{j} + (t - 4)\hat{k}$. Also find the shortest distance C from AB.

Sol. $2\sqrt{17}$

Q.31 If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then show that the value of the scalar triple product $[n\vec{a} + \vec{b} \;\; n\vec{b} + \vec{c} \;\; n\vec{c} + \vec{a}]$ is $(n^3+1)\begin{vmatrix} \vec{a}.\hat{i} & \vec{a}.\hat{j} & \vec{a}.\hat{k} \\ \vec{b}.\hat{i} & \vec{b}.\hat{j} & \vec{b}.\hat{k} \\ \vec{c}.\hat{i} & \vec{c}.\hat{j} & \vec{c}.\hat{k} \end{vmatrix}$.

Q.32 (a) Prove that $|\vec{a} \times \vec{b}| = \sqrt{-\vec{b}.[\vec{a} \times (\vec{a} \times \vec{b})]}$

(b) Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}, \vec{b}.\vec{q} = 0$ & $(\vec{b})^2 = 1$, where μ is a scalar then prove that $|(\vec{a}.\vec{q})\vec{p} - (\vec{p}.\vec{q})\vec{a}| = |\vec{p}.\vec{q}|$.

Q.33 ABCD is a tetrahedron with pv's of its angular points as A(–5, 22, 5); B(1, 2, 3); C(4, 3, 2) and D (–1, 2, –3). If the area of the triangle AEF where the quadrilaterals ABDE and ABCF are parallelograms is $\sqrt{S}$ then find the value of S.

Sol. 110

Q.34 Given the points P(1, 1, –1), Q (1, 2, 0) and R (–2, 2, 2). Find

(a) $\overrightarrow{PQ} \times \overrightarrow{PR}$

(b) Equation of the plane containing the points P, Q and R

(i) in scalar dot product form

(ii) in parametric form

(iii) in Cartesian form

and if the plane through PQR cuts the coordinate axes at A, B, C then the area of the ΔABC.

Sol. (a) $2\hat{i} - 3\hat{j} + 3\hat{k}$

(b) (i) $\vec{r}.\vec{n} = -4$,

(ii) $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(\hat{j} + \hat{k}) + \mu(-3\hat{i} + \hat{j} + 3\hat{k})$,

(iii) $2x - 3y + 3z + 4 = 0$, area = $\frac{4\sqrt{22}}{9}$

Q.35 Find the scalars α and β if $\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a}\cdot\vec{b})\vec{b} = (4 - 2\beta - \sin\alpha)\vec{b} + (\beta^2 - 1)\vec{c}$ & $(\vec{c}\cdot\vec{c})\vec{a} = \vec{c}$ while $\vec{b}$ and $\vec{c}$ are non-zero non collinear vectors.

Sol. $\alpha = n\pi + \dfrac{(-1)^n \pi}{2}$, $n \in I$ & $\beta = 1$

Q.36 Given four points P_1, P_2, P_3 and P_4 on the coordinates plane with origin O which satisfy the condition $\overrightarrow{OP}_{n-1} + \overrightarrow{OP}_{n+1} = \dfrac{3}{2}\overrightarrow{OP}_n$, n = 2, 3

(i) If P_1, P_2 lie on the curve xy = 1, then prove that P_3 does not lie on the curve.

(ii) If P_1, P_2, P_3 lie on the circle $x^2 + y^2 = 1$, then prove that P_4 lies on this circle.

Q.37 Prove the result (Lagrange's Identity) $(\vec{p}\times\vec{q}).\,(\vec{r}\times\vec{s}) = \begin{vmatrix} \vec{p}.\vec{r} & \vec{p}.\vec{s} \\ \vec{q}.\vec{r} & \vec{q}.\vec{s} \end{vmatrix}$ & use it to prove the following. Let (ab) denote the plane formed by the lines a, b. If (ab) is perpendicular to (cd) and (ac) is perpendicular to (bd) prove that (ad) is perpendicular to (bc).

Q.38 Let $\vec{a} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$; $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$; $\vec{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

Find the numbers α, β, γ such that

$$\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \begin{bmatrix} -2 \\ -5 \\ 6 \end{bmatrix}.$$

Sol. $\alpha = -1$, $\beta = -2$, $\gamma = 3$

Q.39 Solve the following equation for the vector $\vec{p}$; $\vec{p} \times \vec{a} + (\vec{p}\cdot\vec{b})\vec{c} = \vec{b}\times\vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are non zero non coplanar vectors and $\vec{a}$ is neither perpendicular to $\vec{b}$ nor to $\vec{c}$, hence show that $\left(\vec{p}\times\vec{a} + \dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{\vec{a}\cdot\vec{c}}\vec{c}\right)$ is perpendicular to $\vec{b} - \vec{c}$.

Sol. $\left\{\vec{p} = \dfrac{[\vec{a}\;\vec{b}\;\vec{c}]}{(\vec{a}\cdot\vec{c})\,(\vec{a}\cdot\vec{b})}(\vec{a} + \vec{c}\times\vec{b}) + \dfrac{(\vec{b}\cdot\vec{c})\vec{b}}{(\vec{a}\cdot\vec{b})} - \dfrac{(\vec{b}\cdot\vec{b})\vec{c}}{(\vec{a}\cdot\vec{b})}\right\}$

Q.40 The base vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$ are given in terms of base vectors $\vec{b}_1$, $\vec{b}_2$, $\vec{b}_3$ as

$\vec{a}_1 = 2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3$;

$\vec{a}_2 = \vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3$ &

$\vec{a}_3 = -2\vec{b}_1 + \vec{b}_2 - 2\vec{b}_3$.

If $\vec{F} = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$, then express $\vec{F}$ in terms of $\vec{a}_1$, $\vec{a}_2$ & $\vec{a}_3$.

Sol. $F = 2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$

ANSWER KEY

EXERCISE # 1

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	D	B	C	A	C	B	A	B	B	A	D	B	B
Qus.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	A,C	A	D	A	C	C	A	C	D	D	B	B	C	B,C,D
Qus.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B,D	A	A	A	C	C	A	A	A,C,D	C	B	B,C	A,B,C	B	C

46. False **47.** (i) True (ii) True **48.** False **49.** False

50. $\cos^{-1}\sqrt{\frac{3}{10}}$ **51.** $5\sqrt{2}$ **52.** 0 **53.** Orthocentre **54.** $2\hat{i} - \hat{j}$

55. $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ **56.** $\vec{a}$

EXERCISE # 2

PART- A

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	C	C	C	D	B	B	D	A
Qus.	11	12	13	14	15	16	17	18	19	
Ans.	C	C	C	C	C	A	B	D	A	

PART- B

Qus.	20	21	22	23	24	25	26	27	28	29
Ans.	A,C	A	A,B	C,D	A,B	A	A,C,D	B,C,D	A,C	A,C,D

PART- C

Qus.	30	31	32	33
Ans.	B	C	D	A

PART- D

34. A $\rightarrow$ R; B $\rightarrow$ S; C $\rightarrow$ P; D $\rightarrow$ Q

35. A $\rightarrow$ P; B $\rightarrow$ P,R; C $\rightarrow$ R; D $\rightarrow$ T

EXERCISE # 3

2. 10 : 7 & 7 : 5 externally

3. (6, –9, –9) or (–4, 11, 11)

8. $\vec{x} = \vec{q} - \frac{1}{2p^2}(\vec{p}\cdot\vec{q})\vec{p}$

9. $(4/\sqrt{2})\,\hat{i} - (1/\sqrt{2})\,\hat{j} - (1/\sqrt{2})\,\hat{k}$

10. $(1/\sqrt{3})$, (4/3, 4/3, 4/3) **11.** $\vec{v} = \frac{1}{b^2}\vec{b} + \frac{1}{|\vec{a}\times\vec{b}|^2}\vec{a}\times\vec{b}$

13. $|[\vec{a}\ \vec{b}\ \vec{c}]|$ **14.** 3 **16.** $\sqrt{10}$ unit **17.** 9 units **20.** $\frac{\pi}{6}$ **21.** 6 **23.** 8 : 3

27. A **28.** A **29.** B **30.** A **31.** B **32.** D **33.** B

34. A **35.** A

EXERCISE # 4

1. A **2.** C **3.** B

4. Many answers are possible one of the possible answers $V_1 = 2\hat{i}$, $V_2 = -\hat{i} + \hat{j}$, $V_3 = 3\hat{i} - 2\hat{j} + 4\hat{k}$

6. B **7.** C **9.** C **11.** C **12.** A **14.** B

15. $\hat{w} = \hat{v} - 2(\hat{n}\ .\ \hat{v})\hat{n}$ **16.** B **17.** B, D **18.** C **19.** C

20. B **21.** A **22.** A **23.** D **24.** C

25. A $\rightarrow$ Q, S; B $\rightarrow$ P, R, S, T ; C $\rightarrow$ T; D $\rightarrow$ R **26.** A **27.** B **28.** 5

29. C **30.** A, D **31.** 9 **32.** A $\rightarrow$ Q ; B $\rightarrow$ P, Q, R, S, T or P ; C $\rightarrow$ S ; D $\rightarrow$ T

33. 3 **34.** C

EXERCISE # 5

1. B **2.** D **3.** B **4.** A **5.** B **6.** B **7.** A, C **8.** B

9. A **10.** –1 **11.** $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ **12.** $(-11/15)\hat{i} - (2/3)\hat{j} - (2/15)\hat{k}$

13. L : (2, 8, – 3), M :(0, 1, 2)

15. (i) parallel (ii) the lines intersect at the point with p.v. $-2\hat{i} + 2\hat{j}$ (iii) lines are skew

16. $xx_1 + yy_1 = a^2$ **17.** 28 **18.** x = 2, y = –2, z = –2 **20.** 34 **21.** $\frac{\sqrt{11}}{3}$ **22.** $-\hat{i} + 2\hat{j} + 5\hat{k}$

23. (b) $\frac{\sqrt{3}}{2}$ **24.** $\frac{5a^2}{12\sqrt{3}}$ sq. units **25.** $p = \frac{q(q^2-3)}{4}$; decreasing in $q \in (-1, 1)$, $q \neq 0$

27. NO, NO **28.** (i) $\frac{6}{7}\sqrt{14}$ (ii) 6 (iii) $\frac{3}{5}\sqrt{10}$ (iv) $\sqrt{6}$ **29.** $\frac{11}{\sqrt{170}}$ **30.** $2\sqrt{17}$

33. 110 **34.** (a) $2\hat{i} - 3\hat{j} + 3\hat{k}$ (b) (i) $\vec{r}\ .\ \vec{n} = -4$, (ii) $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(\hat{j} + \hat{k}) + \mu(-3\hat{i} + \hat{j} + 3\hat{k})$,

(iii) $2x - 3y + 3z + 4 = 0$, area = $\frac{4\sqrt{22}}{9}$

35. $\alpha = n\pi + \frac{(-1)^n\pi}{2}$, $n \in I$ & $\beta = 1$ **38.** $\alpha = -1$, $\beta = -2$, $\gamma = 3$

39. $\left\{\vec{p} = \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{(\vec{a}.\vec{c})\ (\vec{a}.\vec{b})}(\vec{a} + \vec{c} \times \vec{b}) + \frac{(\vec{b}.\vec{c})\vec{b}}{(\vec{a}.\vec{b})} - \frac{(\vec{b}.\vec{b})\vec{c}}{(\vec{a}.\vec{b})}\right\}$

40. $F = 2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$