VECTOR
EXERCISE # 1

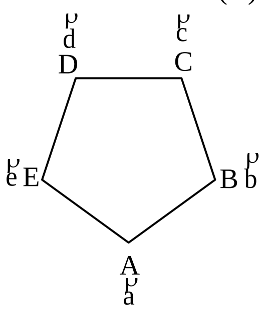
Questions based on Definition & Test of collinearity

Q.1 If ABCDE is a pentagon then the resultant of forces $\overrightarrow{AB}$, $\overrightarrow{AE}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{ED}$ and $\overrightarrow{AC}$ in terms of $\overrightarrow{AC}$ is-

(A) $2\overrightarrow{AC}$ (B) $3\overrightarrow{AC}$

(C) $5\overrightarrow{AC}$ (D) None of these

Sol. **[B]**



$\overrightarrow{AB}+\overrightarrow{AE}+\overrightarrow{BC}+\overrightarrow{DC}+\overrightarrow{ED}+\overrightarrow{AC}$

$= (\vec b-\vec a)+(\vec e-\vec a)+(\vec c-\vec b)+(\vec c-\vec d)+(\vec d-\vec e) + (\vec c-\vec a)$

$= 3(\vec c-\vec a) = 3\overrightarrow{AC}$

Q.2 Points $\vec a+\vec b+\vec c$, $4\vec a+3\vec b$, $10\vec a+7\vec b-2\vec c$ are-

(A) collinear (B) coplanar

(C) non-collinear (D) None of these

Sol. **[A]**

$\vec a+\vec b+\vec c$ = P.V. of A $\overrightarrow{AB} = 3\vec a+2\vec b-\vec c$

$4\vec a+3\vec b$ = P.V. of B $\overrightarrow{AC} = 9\vec a+6\vec b-3\vec c = 3(3\vec a+2\vec b-\vec c)$

$10\vec a+7\vec b-2\vec c$ = P.V. of C

$\overrightarrow{AB} = \lambda\,\overrightarrow{AC}$

Collinear

Q.3 If Five forces $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AE}, \overrightarrow{AF}$ act at the vertex A of a regular hexagon ABCDEF. then their resultant is (where O is the centroid of the hexagon)-

(A) $2\overrightarrow{AO}$ (B) $3\overrightarrow{AO}$ (C) $5\overrightarrow{AO}$ (D) $6\overrightarrow{AO}$

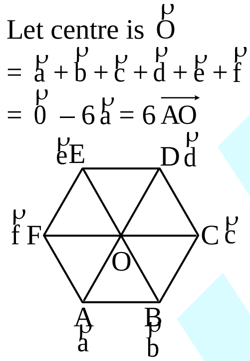
Sol.[D] $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$

$= (\vec b-\vec a)+(\vec c-\vec a)+(\vec d-\vec a)+(\vec e-\vec a)+(\vec f-\vec a)$

Let centre is $\vec O$

$= \vec a+\vec b+\vec c+\vec d+\vec e+\vec f-6\vec a$

$= \vec 0-6\vec a = 6\overrightarrow{AO}$

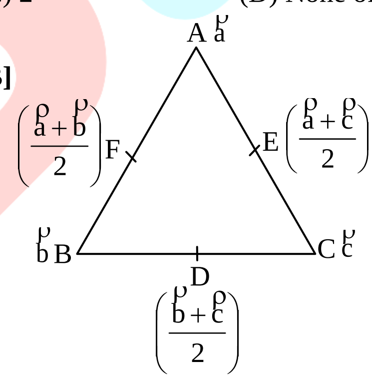


Q.4 If D, E, F are the mid points of the sides BC, CA and AB respectively of a triangle ABC and 'O' is any point then $\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}$ is-

(A) 1 (B) 0

(C) 2 (D) None of these

Sol. **[B]**



$\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF} = \left(\frac{\vec b+\vec c}{2}-\vec a\right)+\left(\frac{\vec a+\vec c}{2}-\vec b\right)+\left(\frac{\vec a+\vec b}{2}-\vec c\right)$

$= (\vec a+\vec b+\vec c)-(\vec a+\vec b+\vec c) = 0$

Q.5 If the vector $\vec b$ is collinear with the vector $\vec a = (2\sqrt{2}, -1, 4)$ and $|\vec b| = 10$, then

(A) $\vec a \pm \vec b = 0$ (B) $\vec a \pm 2\vec b = 0$

(C) $2\vec a \pm \vec b = 0$ (D) None of these

Sol. **[C]**

$\vec b = \lambda\,(2\sqrt{2}\,\hat i-\hat j+4\hat k)$

$|\vec b| = |\lambda|\,\sqrt{8+1+16}$

$10 = 5|\lambda| \Rightarrow \lambda = \pm 2$

$\vec{b} = \pm 2\vec{a} \quad \Rightarrow 2\vec{a} \pm \vec{b} = 0$

Questions based on Section formulae

Q.6 If points A(1, 2, 3), B(3, 4, 7), C(– 3, – 2, – 5) are collinear then the ratio in which B divides AC is-

(A) – 1 : 3 (B) 1 : 3
(C) 3 : 1 (D) None of these

Sol. **[A]**

A(1, 2, 3) C(– 3, – 2, –5)

let B divides AC in ratio of $\lambda : 1$

$$\left(\frac{-3\lambda+1}{\lambda+1}, \frac{-2\lambda+2}{\lambda+1}, \frac{-5\lambda+3}{\lambda+1}\right)$$

compare as in 'B' (3, 4, 7)

$\frac{-3\lambda+1}{\lambda+1} = 3$	$\frac{-2\lambda+2}{\lambda+1} = 4$	$\frac{-5\lambda+3}{\lambda+1} = 7$
$-3\lambda + 1 = 3\lambda + 3$	$-2\lambda + 2 = 4\lambda + 4$	$\lambda = \frac{-1}{3}$
$6\lambda = -2$	$6\lambda = -2$	
$\lambda = \frac{-1}{3}$	$\lambda = \frac{-1}{3}$	

Hence –1 : 3

Q.7 The position vectors of points A, B, C are respectively $\vec{a}$, $\vec{b}$, $\vec{c}$. If L divides AB in 3 : 4 & M divides BC in 2 : 1 both externally, then $\overrightarrow{LM}$ is-

(A) $4\vec{a} - 2\vec{b} + 2\vec{c}$ (B) $4\vec{a} + 2\vec{b} + 2\vec{c}$
(C) $-4\vec{a} + 2\vec{b} + 2\vec{c}$ (D) $4\vec{a} - 2\vec{b} - 2\vec{c}$

Sol. **[C]**

$\vec{L} = \frac{4\vec{a} - 3\vec{b}}{1}$, $\vec{M} = \frac{2\vec{c} - \vec{b}}{1}$

then $\overrightarrow{LM}$ = P.V of $\vec{M}$ – P.V of C

$\overrightarrow{LM} = (2\vec{c} - \vec{b}) - (4\vec{a} - 3\vec{b})$

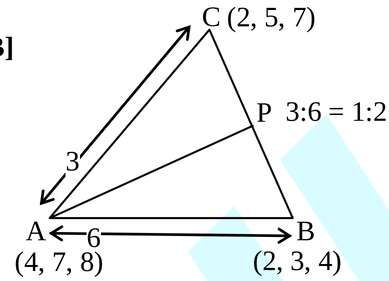
$\overrightarrow{LM} = 2\vec{c} - \vec{b} - 4\vec{a} + 3\vec{b}$

$\overrightarrow{LM} = -4\vec{a} + 2\vec{b} + 2\vec{c}$

Q.8 If A(4, 7, 8), B(2, 3, 4), C(2, 5, 7) are the position vectors of the vertices of ΔABC. Then length of angle bisector of angle A is -

(A) $\frac{3}{2}\sqrt{34}$ (B) $\frac{2}{3}\sqrt{34}$
(C) $\frac{1}{2}\sqrt{34}$ (D) $\frac{1}{3}\sqrt{34}$

Sol. **[B]**



Length $\overrightarrow{AB} = \sqrt{4+16+16} = \sqrt{36} = 6$

Length $\overrightarrow{AC} = \sqrt{4+4+1} = \sqrt{9} = 3$

So point P divides $\overrightarrow{BC}$ in Ratio of 2 : 1

B(2, 3, 4) C(2, 5, 7)

So point P

$$\left(\frac{4+2}{3}, \frac{10+3}{3}, \frac{18}{3}\right)$$

$$\left(2, \frac{13}{3}, 6\right)$$

So AP = $\sqrt{(4-2)^2 + \left(7 - \frac{13}{3}\right)^2 + (8-6)^2}$

= $\sqrt{4 + \frac{64}{9} + 4}$

= $\frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$

Questions based on Dot product and Cross product

Q.9 If $\vec{e}_1$ & $\vec{e}_2$ are non collinear unit vectors, such that $|\vec{e}_1 + \vec{e}_2| = \sqrt{3}$ then $(2\vec{e}_1 - 5\vec{e}_2).(3\vec{e}_1 + \vec{e}_2)$ is equal to

(A) $-\frac{11}{2}$ (B) $\frac{13}{2}$ (C) $\frac{2}{11}$ (D) $\frac{11}{2}$

Sol. **[A]**

$|\vec{e}_1 + \vec{e}_2| = \sqrt{3}$

$|\vec{e}_1|^2 + |\vec{e}_2|^2 + 2(\vec{e}_1.\vec{e}_2) = 3$

$1 + 1 + 2(\vec{e}_1.\vec{e}_2) = 3$

Now

$(2\vec{e}_1 - 5\vec{e}_2).(3\vec{e}_1 + \vec{e}_2)$

$= 6|\vec{e}_1|^2 + 2\vec{e}_1.\vec{e}_2 - 15\vec{e}_1.\vec{e}_2 - 5|\vec{e}_2|^2$

$\vec{e}_1.\vec{e}_2 = \frac{1}{2}$

$= 6 + 2 \times \frac{1}{2} - 15 \times \frac{1}{2} - 5$

$= 2 - \frac{15}{2} = \frac{-11}{2}$

Q.10 The vector $\vec{p}$ perpendicular to the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and satisfying the condition $\vec{p}.(2\hat{i} - \hat{j} + \hat{k}) = -6$ is

(A) $-\hat{i} + \hat{j} + \hat{k}$ (B) $3(-\hat{i} + \hat{j} + \hat{k})$

(C) $2(-\hat{i} + \hat{j} + \hat{k})$ (D) $\hat{i} - \hat{j} + \hat{k}$

Sol. **[B]**

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -2 & 3 \end{vmatrix}$$

$\vec{a} \times \vec{b} = \hat{i}(9-2) - \hat{j}(6+1) + \hat{k}(-4-3)$

$= 7(\hat{i} - \hat{j} - \hat{k})$

$\vec{p} = \lambda(\hat{i} - \hat{j} - \hat{k})$

$\vec{p}.(2\hat{i} - \hat{j} + \hat{k}) = \lambda(\hat{i} - \hat{j} - \hat{k}).(2\hat{i} - \hat{j} + \hat{k}) = -6$

$\lambda(2 + 1 - 1) = -6$

$\Rightarrow \lambda = -3$

So $\vec{p} = -3(\hat{i} - \hat{j} - \hat{k})$

$\vec{p} = 3(-\hat{i} + \hat{j} + \hat{k})$

Q.11 If $|\vec{a}| = 5$, $|\vec{a} - \vec{b}| = 8$ and $|\vec{a} + \vec{b}| = 10$, then $|\vec{b}|$ is equal to

(A) 1 (B) $\sqrt{57}$

(C) 3 (D) None of these

Sol. **[B]**

Q.12 Angle between diagonals of a parallelogram whose side are represented by $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$

(C) $\cos^{-1}\left(\frac{4}{9}\right)$ (D) $\cos^{-1}\left(\frac{5}{9}\right)$

Sol. **[A]**

Q.13 Vectors $\vec{a}$ and $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$, then $\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$ is equal to

(A) 225 (B) 250 (C) 275 (D) 300

Sol. **[D]**

$\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2 = \{-\vec{a} \times \vec{b} + 9\vec{b} \times \vec{a}\}^2$

$= \{10(\vec{b} \times \vec{a})\}^2 = (10)^2 |\vec{b}|^2 |\vec{a}|^2 \sin^2\theta$

$= 100 \times 4 \times 1 \times \sin^2\left(\frac{2\pi}{3}\right)$

$= 100 \times 4 \times \frac{3}{4} = 300$

Q.14 Unit vector perpendicular to the plane of the triangle ABC with position vectors $\vec{a}, \vec{b}, \vec{c}$ of the vertices A, B, C is

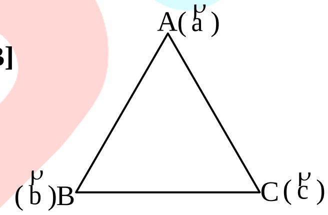
(A) $\frac{(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})}{\Delta}$

(B) $\frac{(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})}{2\Delta}$

(C) $\frac{(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})}{4\Delta}$

(D) None of these

Sol. **[B]**



$= \frac{(\vec{a} - \vec{b}) \times (\vec{a} - \vec{c})}{|(\vec{a} \times \vec{b}) \times (\vec{a} - \vec{c})|}$

$= \frac{(\vec{a} - \vec{b}) \times (\vec{a} - \vec{c})}{2\Delta}$

$= \frac{-\vec{a} \times \vec{c} - \vec{b} \times \vec{a} + \vec{b} \times \vec{c}}{2\Delta}$

$= \frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{2\Delta}$

Q.15 Given the three vectors $\vec{a} = -2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 5\hat{j}$ and $\vec{c} = 4\hat{i} + 4\hat{j} - 2\hat{k}$. The projection of the vector $3\vec{a} - 2\vec{b}$ on the vector $\vec{c}$ is

(A) 11 (B) –11

(C) 13 (D) None of these

Sol. [B] Projection of $(3\vec{a} - 2\vec{b})$ at $\vec{c}$

$= \frac{(3\vec{a} - 2\vec{b}).\vec{c}}{|\vec{c}|} = -11$

Questions based on Triple Product

Q.16 For three vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ which of the following expressions is not equal to any of the remaining three ?

(A) $\vec{u}\cdot(\vec{v}\times\vec{w})$ (B) $(\vec{v}\times\vec{w})\cdot\vec{u}$

(C) $\vec{v}\cdot(\vec{u}\times\vec{w})$ (D) $(\vec{u}\times\vec{v})\cdot\vec{w}$

Sol. **[C]**

(A) $\vec{u}\cdot(\vec{v}\times\vec{w}) = [\vec{u}\ \vec{v}\ \vec{w}]$

(B) $(\vec{v}\times\vec{w})\cdot\vec{u} = -\vec{v}\cdot(\vec{u}\times\vec{w})$

$= -(\vec{v}\times\vec{u})\cdot\vec{w}$

$= \vec{u}\cdot(\vec{v}\times\vec{w}) = [\vec{u}\ \vec{v}\ \vec{w}]$

(C) $\vec{v}\cdot(\vec{u}\times\vec{w}) = -(\vec{u}\times\vec{v})\cdot\vec{w}$

$= -\vec{u}\cdot(\vec{v}\times\vec{w}) = -[\vec{u}\ \vec{v}\ \vec{w}]$

(D) $(\vec{u}\times\vec{v})\cdot\vec{w} = \vec{u}\cdot(\vec{v}\times\vec{w}) = [\vec{u}\ \vec{v}\ \vec{w}]$

Clearly C is not equal

Q.17 Which of the following expression is meaningful ?

(A) $\vec{u}\cdot(\vec{v}\times\vec{w})$ (B) $(\vec{u}\cdot\vec{v})\cdot\vec{w}$

(C) $(\vec{u}\cdot\vec{v})\ \vec{w}$ (D) $\vec{u}\times(\vec{v}\cdot\vec{w})$

Sol. **[A,C]**

Clearly $\vec{u}\cdot(\vec{v}\times\vec{w})$ and

$(\vec{u}\cdot\vec{v})\vec{w}$ is meaningful

Option A and C are correct.

Q.18 For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$,

$(\vec{a}-\vec{b})\cdot(\vec{b}-\vec{c})\times(\vec{c}-\vec{a}) =$

(A) 0 (B) $\vec{a}\cdot\vec{b}\times\vec{c}$

(C) $2\ \vec{a}\cdot\vec{b}\times\vec{c}$ (D) None of these

Sol. **[A]**

$(\vec{a}-\vec{b})\cdot(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})$

$= (\vec{a}-\vec{b})\cdot(\vec{b}\times\vec{c} - \vec{b}\times\vec{a} + \vec{c}\times\vec{a})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) - \vec{b}\cdot(\vec{c}\times\vec{a})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{a}\times\vec{c})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) - (\vec{a}\times\vec{b})\cdot\vec{c}$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) - \vec{a}\cdot(\vec{b}\times\vec{c})$

$= 0$

Q.19 $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three non coplanar vectors, then $(\vec{A}+\vec{B}+\vec{C})\cdot((\vec{A}+\vec{B})\times(\vec{A}+\vec{C})) =$

(A) 0 (B) $[\vec{A},\vec{B},\vec{C}]$

(C) $2[\vec{A},\vec{B},\vec{C}]$ (D) $-[\vec{A},\vec{B},\vec{C}]$

Sol. **[D]**

$(\vec{A}+\vec{B}+\vec{C})\cdot((\vec{A}+\vec{B})\times(\vec{A}+\vec{C}))$

$= (\vec{A}+\vec{B}+\vec{C})\cdot(\vec{A}\times\vec{C}+\vec{B}\times\vec{A}+\vec{B}\times\vec{C})$

$= \vec{A}\cdot(\vec{B}\times\vec{C}) + \vec{B}\cdot(\vec{A}\times\vec{C}) + \vec{C}\cdot(\vec{B}\times\vec{A})$

$= \vec{A}\cdot(\vec{B}\times\vec{C}) - \vec{B}\cdot(\vec{C}\times\vec{A}) - \vec{C}\cdot(\vec{A}\times\vec{B})$

$= [\vec{A}\ \vec{B}\ \vec{C}] - [\vec{A}\ \vec{B}\ \vec{C}] - [\vec{A}\ \vec{B}\ \vec{C}]$

$= -[\vec{A}\ \vec{B}\ \vec{C}]$

Q.20 If $\vec{A},\vec{B},\vec{C}$ are three non-coplanar vectors, then $\dfrac{\vec{A}\cdot\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}\cdot\vec{B}} + \dfrac{\vec{B}\cdot\vec{A}\times\vec{C}}{\vec{C}\cdot\vec{A}\times\vec{B}} =$

(A) 0 (B) 1

(C) 2 (D) None of these

Sol. [A] ∵ $\vec{A}$, $\vec{B}$, $\vec{C}$ are non coplanar vector

$\Rightarrow [\vec{A}\ \vec{B}\ \vec{C}] \neq 0$

we know that

$\vec{A}\cdot\vec{B}\times\vec{C} = [\vec{A}\ \vec{B}\ \vec{C}]$

$\vec{B}\cdot\vec{A}\times\vec{C} = -[\vec{A}\ \vec{B}\ \vec{C}]$

$\vec{C}\cdot\vec{A}\times\vec{B} = [\vec{A}\ \vec{B}\ \vec{C}]$

$\Rightarrow \dfrac{\vec{A}\cdot\vec{B}\times\vec{C}}{\vec{C}\cdot\vec{A}\times\vec{B}} + \dfrac{\vec{B}\cdot\vec{A}\times\vec{C}}{\vec{C}\cdot\vec{A}\times\vec{B}}$

$= \dfrac{[\vec{A}\,\vec{B}\,\vec{C}]}{[\vec{A}\,\vec{B}\,\vec{C}]} + \dfrac{-[\vec{A}\,\vec{B}\,\vec{C}]}{[\vec{A}\,\vec{B}\,\vec{C}]} = 0$

Q.21 The value of $[(\vec{a}+2\vec{b}-\vec{c}),\ (\vec{a}-\vec{b}),\ (\vec{a}-\vec{b}-\vec{c})]$ is equal to the box product:

(A) $[\vec{a}\ \vec{b}\ \vec{c}]$ (B) $2[\vec{a}\ \vec{b}\ \vec{c}]$

(C) $3[\vec{a}\ \vec{b}\ \vec{c}]$ (D) $4[\vec{a}\ \vec{b}\ \vec{c}]$

Sol. **[C]**

$[(\vec{a}+2\vec{b}-\vec{c})\ (\vec{a}-\vec{b})\ (\vec{a}-\vec{b}-\vec{c})]$

$= (\vec{a}+2\vec{b}-\vec{c})\cdot\{(\vec{a}-\vec{b})\times(\vec{a}-\vec{b}-\vec{c})\}$

$= (\vec{a} + 2\vec{b} - \vec{c}).\{(\vec{a} \times \vec{a}) - (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c})$

$- (\vec{b} \times \vec{a}) + (\vec{b} \times \vec{b}) + (\vec{b} \times \vec{c})$

$= (\vec{a} + 2\vec{b} - \vec{c})\{0 - (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c})$

$+ (\vec{a} \times \vec{b}) + 0 + (\vec{b} \times \vec{c})\}$

$= (\vec{a} + 2\vec{b} - \vec{c})\{(\vec{b} \times \vec{c}) - (\vec{a} \times \vec{c})\}$

$= a.(\vec{b} \times \vec{c}) + 2\vec{b}.(\vec{b} \times \vec{c}) - \vec{c}.(\vec{b} \times \vec{c})$

$- \vec{a}(\vec{a} \times \vec{c}) - 2\vec{b}.(\vec{a} \times \vec{c}) + \vec{c}.(\vec{a} \times \vec{c})$

$= [\vec{a}\ \vec{b}\ \vec{c}] - 2[\vec{b}\ \vec{a}\ \vec{c}]$

$= [\vec{a}\ \vec{b}\ \vec{c}] + 2[\vec{a}\ \vec{b}\ \vec{c}]$

$= 3[\vec{a}\ \vec{b}\ \vec{c}]$

Q.22 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then the value of $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$ is equal to

(A) 2 (B) 4 (C) 16 (D) 64

Sol. **[C]**

Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$,

$\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

then $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 3 & -2 \\ 2 & -2 & 6 \end{vmatrix}$

$= 3(18 - 4) - 1(6 + 4) + 2(-2 - 6) = 16$

Q.23 If $\vec{b}$ and $\vec{c}$ are two non- collinear vectors such that $\vec{a} \parallel (\vec{b} \times \vec{c})$, then $(\vec{a} \times \vec{b}).(\vec{a} \times \vec{c})$ is equal to

(A) $\vec{a}^2(\vec{b}.\vec{c})$ (B) $\vec{b}^2(\vec{a}.\vec{c})$

(C) $\vec{c}^2(\vec{a}.\vec{b})$ (D) None of these

Sol. **[A]**

Q.24 Let $\vec{a} = x\hat{i} + 12\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + 2x\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$. If the ordered set $[\vec{b}\ \vec{c}\ \vec{a}]$ is left handed, then:

(A) $x \in (2, \infty)$ (B) $x \in (-\infty, -3)$

(C) $x \in (-3, 2)$ (D) $x \in \{-3, 2\}$

Sol. **[C]**

$[\vec{b}\ \vec{c}\ \vec{a}] < 0$

$\begin{vmatrix} 2 & 2x & 1 \\ 1 & 0 & 1 \\ x & 12 & -1 \end{vmatrix} < 0$

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + R_3$

$\begin{vmatrix} 2+x & 2x+12 & 0 \\ 1+x & 12 & 0 \\ x & 12 & -1 \end{vmatrix} < 0$

$-1[(24 + 12x) - (1 + x)(2x + 12)] < 0$

$[24 + 12x - 2x - 12 - 2x^2 - 12x] > 0$

$-2x^2 - 2x + 12 > 0$

$x^2 + x - 6 < 0$

$(x + 3)(x - 2) < 0$

$x \in (-3, 2)$

Q.25 If $\vec{a}, \vec{b}, \vec{c}$ be the unit vectors such that $\vec{b}$ is not parallel to $\vec{c}$ and $\vec{a} \times (2\vec{b} \times \vec{c}) = \vec{b}$, then the angle that $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$ are respectively

(A) $\frac{\pi}{3}$ & $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ & $\frac{2\pi}{3}$

(C) $\frac{\pi}{2}$ & $\frac{2\pi}{3}$ (D) $\frac{\pi}{2}$ & $\frac{\pi}{3}$

Sol. **[D]**

$\vec{a} \times (2\vec{b} \times \vec{c}) = \vec{b}$

$(\vec{a}.\vec{c})2\vec{b} - (\vec{a}.2\vec{b})\vec{c} = \vec{b}$

$\vec{b}(2\vec{a}.\vec{c} - 1) - (\vec{a}.2\vec{b})\vec{c} = 0$

$2\vec{a}.\vec{c} - 1 = 0$, $\vec{a}.2\vec{b} = 0$

so $\vec{a}.\vec{c} = \frac{1}{2}$ & $\vec{a}.2\vec{b} = 0$, $\vec{a}.\vec{b} = 0$

angle between

$\vec{a}$ & $\vec{c}$ = $\frac{\pi}{3}$ angle between $\vec{a}$ & $\vec{b}$ = $\frac{\pi}{2}$

Q.26 Vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j}$, is-

(A) $\frac{3}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$ (B) $\frac{3}{\sqrt{6}}(2\hat{i} - \hat{j} - \hat{k})$

(C) $\frac{3}{\sqrt{114}}(8\hat{i} - 7\hat{j} - \hat{k})$ (D) $\frac{3}{\sqrt{114}}(-7\hat{i} + 8\hat{j} - \hat{k})$

Sol. **[D]**

Q.27 Given unit vectors $\vec{m}, \vec{n}$ & $\vec{p}$ such that angle between $\vec{m}$ & $\vec{n}$ = angle between $\vec{p}$ and $(\vec{m}\times\vec{n}) = \frac{\pi}{6}$ then $[\vec{n}\ \vec{p}\ \vec{m}] =$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{4}$

(C) $\frac{\sqrt{3}}{5}$ (D) None of these

Sol. **[B]**

$[\vec{n}\ \vec{p}\ \vec{m}] = [\vec{p}\ \vec{m}\ \vec{n}] = \vec{p}\cdot(\vec{m}\times\vec{n})$

$= |\vec{p}|\,|\vec{m}\times\vec{n}|\cos\frac{\pi}{6}$

$= |\vec{p}|\,|\vec{m}|\,|\vec{n}|\,\sin\frac{\pi}{6}\cos\frac{\pi}{6}$

$= \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

Q.28 Let $\vec{u}, \vec{v}, \vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w} = 0$. If $|\vec{u}| = 3, |\vec{v}| = 5, |\vec{w}| = 4$. Then the value of the $\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}$ is-

(A) 47 (B) – 25 (C) 0 (D) 25

Sol. **[B]**

$\therefore\ \vec{u}+\vec{v}+\vec{w} = 0$

$\Rightarrow |\vec{u}+\vec{v}+\vec{w}|^2 = 0$

$\Rightarrow |\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2+2(\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}) = 0$

$\Rightarrow 9+25+16+2(\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}) = 0$

$\Rightarrow \vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u} = -25$

Questions based on

Straight line

Q.29 Let $\vec{a} = \hat{i}+\hat{j}$ and $\vec{b} = 2\hat{i}-\hat{k}$. The point of intersection of the lines $\vec{r}\times\vec{a} = \vec{b}\times\vec{a}$ and $\vec{r}\times\vec{b} = \vec{a}\times\vec{b}$ is

(A) $-\hat{i}+\hat{j}+2\hat{k}$ (B) $3\hat{i}-\hat{j}+\hat{k}$

(C) $3\hat{i}+\hat{j}-\hat{k}$ (D) $\hat{i}-\hat{j}-\hat{k}$

Sol. **[C]**

$\vec{r}\times\vec{a} = \vec{b}\times\vec{a}$ $\qquad$ $\vec{r}\times\vec{b} = \vec{a}\times\vec{b}$

$(\vec{r}-\vec{b})\times\vec{a} = 0$ $\qquad$ $(\vec{r}-\vec{a})\times\vec{b} = 0$

$\vec{r}-\vec{b} = \lambda\vec{a}$ $\qquad$ $\vec{r}-\vec{a} = \mu\vec{b}$

$\vec{r} = \vec{b}+\lambda\vec{a}$ ….(i) $\qquad$ $\vec{r} = \vec{a}+\mu\vec{b}$ ….(ii)

First line

$\vec{r}_1 = (2\hat{i}-\hat{k})+\lambda(\hat{i}+\hat{j})$ $\qquad$ $\vec{r}_2 = \hat{i}+\hat{j}+\mu(2\hat{i}-\hat{k})$

$\vec{r}_1 = \hat{i}(2+\lambda)+\lambda\hat{j}-\hat{k}$ $\qquad$ $\vec{r}_2 = \hat{i}(2+2\mu)+\hat{j}-\mu\hat{k}$

compare $2+\lambda = 1+2\mu$

$\lambda = 1$

$\mu = 1$

So point of intersection $3\hat{i}+\hat{j}-\hat{k}$

Q.30 If a line has a vector equation $\vec{r} = 2\hat{i}+6\hat{j}+\lambda(\hat{i}-3\hat{j})$, then which of the following statements hold good?

(A) the line is parallel to $2\hat{i}+6\hat{j}$

(B) the line passes through the point $3\hat{i}+3\hat{j}$

(C) the line passes through the point $\hat{i}+9\hat{j}$

(D) the line is parallel to XY- plane

Sol. **[B, C, D]**

$\vec{r} = 2\hat{i}+6\hat{j}+\lambda(\hat{i}-3\hat{j})$

general point on line $\{2+\lambda, 6-3\lambda, 0\}$

(B) Let point $3\hat{i}+3\hat{j}$

$2+\lambda = 3 \Rightarrow \lambda = 1$

$6-3\lambda = 3 \Rightarrow 3\lambda = 3 \Rightarrow \lambda = 1$

(C) Let point $\hat{i}+9\hat{j}$

$2+\lambda = 1 \Rightarrow \lambda = -1$

$6-3\lambda = 9 \Rightarrow -3\lambda = 3 \Rightarrow \lambda = -1$

(D) Line is parallel to $(\hat{i}-3\hat{j})$ and dot product of $(\hat{i}-3\hat{j})$ with, $\hat{k}$ is zero. Then line parallel to xy palne.

Q.31 A line passes through a point A with position vector $3\hat{i}+\hat{j}-\hat{k}$ and is parallel to the vector $2\hat{i}-\hat{j}+2\hat{k}$. If P is a point on this line such that AP = 15 units, then the position vector of the point P is/are

(A) $13\hat{i}+4\hat{j}-9\hat{k}$ (B) $13\hat{i}-4\hat{j}+9\hat{k}$

(C) $7\hat{i}-6\hat{j}+11\hat{k}$ (D) $-7\hat{i}+6\hat{j}-11\hat{k}$

Sol. **[B, D]**

$\vec{r} = (3\hat{i}+\hat{j}-\hat{k}) + \lambda(2\hat{i}-\hat{j}+2\hat{k})$

general point P $(3+2\lambda, 1-\lambda, -1+2\lambda)$

point A $(3, 1, -1)$

$|AP| = \sqrt{(2\lambda)^2+(-\lambda)^2+(2\lambda)^2} = \sqrt{9\lambda^2} = 15$

give

$\lambda = \pm 5$

So point $\vec{r} = (3\hat{i}+\hat{j}-\hat{k}) + 5(2\hat{i}-\hat{j}+2\hat{k})$

$\vec{r} = 13\hat{i}-4\hat{j}+9\hat{k}$

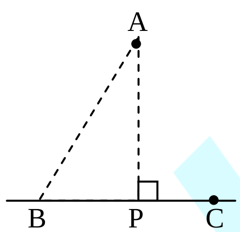
Q.32 The perpendicular distance of $\vec{A}$ (1, 4, – 2) from the segment BC where $\vec{B}$ = (2, 1, – 2) and $\vec{C}$ = (0, – 5, 1) is-

(A) $\frac{3}{7}\sqrt{26}$ (B) $\frac{6}{7}\sqrt{26}$

(C) $\frac{4}{7}\sqrt{26}$ (D) $\frac{2}{7}\sqrt{26}$

Sol. **[A]**

equation of line $\vec{r} = \vec{b} + \lambda(\vec{c}-\vec{b})$



So $|\overrightarrow{AP}| = \left|\frac{\overrightarrow{AB}\times(\vec{C}-\vec{B})}{(\vec{C}-\vec{B})}\right|$

Solve it $\frac{3}{7}\sqrt{26}$

Questions based on **Skew lines and planes**

Q.33 If line $\vec{r} = (\hat{i}-2\hat{j}-\hat{k}) + \lambda(2\hat{i}+\hat{j}+2\hat{k})$ is parallel to the plane $\vec{r}.(3\hat{i}-2\hat{j}-m\hat{k}) = 14$, then the value of m is

(A) 2 (B) –2

(C) 0

(D) can not be predicted with these information

Sol. **[A]**

Dr's of line (2, 1, 2)

Dr's of normal to plane (3, –2, – m)

So. Dot present will be zero

$(2\hat{i}+\hat{j}+2\hat{k}).(3\hat{i}-2\hat{j}-m\hat{k}) = 0$

$6-2-2m = 0 \Rightarrow 2m = 4 \Rightarrow m = 2$

Q.34 Shortest distance between the lines:

$\vec{r} = (4\hat{i}-\hat{j}) + \lambda(\hat{i}+2\hat{j}-3\hat{k})$ and

$\vec{r} = (\hat{i}-\hat{j}+2\hat{k}) + \mu(2\hat{i}+4\hat{j}-5\hat{k})$ is

(A) $6/\sqrt{5}$ (B) $12/\sqrt{5}$

(C) $18/\sqrt{5}$ (D) None of these

Sol. **[A]**

$\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}_1$

$\vec{r}_2 = \vec{a}_2 + \lambda\vec{b}_2$

line are not parallel and doesn't have any intersection point (lines are skew line)

shortest distance = $\left|\frac{(\vec{a}_1-\vec{a}_2)(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

Put values

Q.35 The distance between the line $\vec{r} = 2\hat{i}-2\hat{j}+3\hat{k} + \lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k}) = 5$ is

(A) 10/3 (B) 3/10 (C) $\frac{10}{3\sqrt{3}}$ (D) 10/9

Sol. **[C]**

Q.36 Equation of a line which passes through a point with position vector $\vec{c}$, parallel to the plane $\vec{r}.\vec{n} = 1$ & perpendicular to the line $\vec{r} = \vec{a} + t\vec{b}$ is-

(A) $\vec{r} = \vec{c} + \lambda(\vec{c}-\vec{a}) \times \vec{n}$

(B) $\vec{r} = \vec{c} + \lambda(\vec{a}\times\vec{n})$

(C) $\vec{r} = \vec{c} + \lambda(\vec{b}\times\vec{n})$

(D) $\vec{r} = \vec{c} + \lambda(\vec{b}.\vec{n})\vec{a}$

Sol. **[C]**

line $\perp$ to $\vec{n}$

line $\perp$ to $\vec{b}$

and line passes through $\vec{c}$

Then equation of line $\vec{r} = \vec{c} + \lambda(\vec{b}\times\vec{n})$

Q.37 The vectors $\vec{a} = -4\hat{i} + 3\hat{k}$, $\vec{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are co-initial. The vector $\vec{d}$ which is bisecting the angle between the vectors $\vec{a}$ and $\vec{b}$, is having the magnitude $\sqrt{6}$, is

(A) $\hat{i}+\hat{j}+2\hat{k}$ (B) $\hat{i}-\hat{j}+2\hat{k}$

(C) $\hat{i}+\hat{j}-2\hat{k}$ (D) None of these

Sol. angle Bisector = $\sqrt{6}\ \dfrac{(\hat{a}+\hat{b})}{|\hat{a}+\hat{b}|}$

Q.38 The set of values of 'm' for which the vectors $\vec{a} = m\hat{i} + (m+1)\hat{j} + (m+8)\hat{k}$, $\vec{b} = (m+3)\hat{i} + (m+4)\hat{j} + (m+5)\hat{k}$ and $\vec{c} = (m+6)\hat{i} + (m+7)\hat{j} + (m+8)\hat{k}$ are non-coplanar is

(A) R (B) R – {1}

(C) R – {1, 2} (D) ϕ

Sol. **[A]**

Vector are coplanar $\begin{vmatrix} m & m+1 & m+8 \\ m+3 & m+4 & m+5 \\ m+6 & m+7 & m+8 \end{vmatrix}$ is zero for all in $m \in R$

Q.39 If $\vec{r} = \hat{i}+\hat{j}+\lambda(2\hat{i}+\hat{j}+4\hat{k})$ & $\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k}) = 3$ are the equations of a line and a plane respectively, then which of the following is false?

(A) line is perpendicular to the plane

(B) line lies in the plane

(C) line is parallel to the plane but does not lie in the plane

(D) line cuts the plane is one point only

Sol. **[A,C,D]**

$\vec{r} = (\hat{i}+\hat{j}) + \lambda\ (2\hat{i}+\hat{j}+4\hat{k})$

$\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k}) = 3$

line parallel to $(2\hat{i}+\hat{j}+4\hat{k})$

plane $\perp$ to $(\hat{i}+2\hat{j}-\hat{k})$

(A) line is parallel to plane

(B) general point on line $(1 + 2\lambda, 1 + \lambda, 4\lambda)$ put in plane $(1 + 2\lambda)\hat{i} + (1 + \lambda)\hat{j} + 4\lambda\hat{k}$).

$(\hat{i}+2\hat{j}-\hat{k}) = 3\ \ 1 + 2\lambda + 2 + 2\lambda - 4\lambda = 3$

$\Rightarrow 3 = 3$

(C) line parallel to plane and also lies in plane

Questions based on Linearly dependency & Independency

Q.40 Which of the following system is linearly dependent-

(A) $\vec{a} = \hat{i}+\hat{j}, \vec{b} = \hat{i}+\hat{k}, \vec{c} = 3\hat{i}+3\hat{j}+2\hat{k}$

(B) $\vec{a} = -2\hat{i}-4\hat{k}, \vec{b} = \hat{i}-2\hat{j}-\hat{k}, \vec{c} = \hat{i}-4\hat{j}+3\hat{k}$

(C) $\vec{a} = \hat{i}-2\hat{j}+3\hat{k}, \vec{b} = 3\hat{i}-6\hat{j}+9\hat{k}$

(D) $\vec{a} = -2\hat{i}-4\hat{k}, \vec{b} = \hat{i}-2\hat{j}-\hat{k}, \vec{c} = \hat{i}-4\hat{j}+3\hat{k}$

Sol. **[C]**

$\vec{a} = \lambda\vec{b}$

Q.41 If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors, then which one of the following set of vectors is linearly dependent?

(A) $\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}$

(B) $\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}$

(C) $\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$

(D) None of these

Sol. **[B]**

$[\vec{a}-\vec{b}.\vec{b}-\vec{c}.\vec{c}-\vec{a}] = 0$ in all condition. So $\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}$ are coplanar. (linearly dependent)

Q.42 Points $4\hat{i}+8\hat{j}+12\hat{k}$, $2\hat{i}+4\hat{j}+6\hat{k}$, $3\hat{i}+5\hat{j}+4\hat{k}$, $5\hat{i}+8\hat{j}+5\hat{k}$ are-

(A) Linearly independent (B) coplanar

(C) Linearly dependent (D) None of these

Sol. **[B, C]**

P.V of A $(4\hat{i}+3\hat{j}+12\hat{k})$

P.V of B $(2\hat{i}+4\hat{j}+6\hat{k})$

P.V of C $(3\hat{i}+5\hat{j}+4\hat{k})$

P.V of D $(5\hat{i}+8\hat{j}+5\hat{k})$

$\overrightarrow{AB} = -2\hat{i}-4\hat{j}-6\hat{k}$

$\overrightarrow{AC} = -\hat{i}-3\hat{j}-5\hat{x}$

$\overrightarrow{AD} = \hat{i}-7\hat{k}$

$\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix} = 0$ vector $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar. (linearly dependent)

Q.43 If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are linearly independent set of vectors and $K_1\vec{a} + K_2\vec{b} + K_3\vec{c} + K_4\vec{d} = 0$, then K_1, K_2, K_3, K_4 satisfies

(A) $K_1 + K_2 + K_3 + K_4 = 0$

(B) $K_1 + K_3 = K_2 + K_4 = 0$

(C) $K_1 + K_4 = K_2 + K_3 = 0$

(D) None of these

Sol. **[A,B,C]**

$K_1 = K_2 = K_3 = K_4 = 0$

Questions based on Vector Equation

Q.44 Vector $\vec{x}$ satisfying the relation $\vec{A}\cdot\vec{x} = c$ and $\vec{A}\times\vec{x} = \vec{B}$ is

(A) $\dfrac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|}$ (B) $\dfrac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

(C) $\dfrac{c\vec{A}+(\vec{A}\times\vec{B})}{|\vec{A}|^2}$ (D) $\dfrac{c\vec{A}-2(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

Sol. **[B]**

$\vec{A}\times(\vec{A}\times\vec{x}) = \vec{A}\times\vec{B}$

$(\vec{A}.\vec{x})\vec{A}-(\vec{A}.\vec{A})\vec{x} = \vec{A}\times\vec{B}$

$c\,\vec{A}-|\vec{A}|^2\,\vec{x} = \vec{A}\times\vec{B}$

$\vec{x} = \dfrac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

Q.45 For a non- zero vector $\vec{A}$ if the equation $\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$ hold simultaneously, then:

(A) $\vec{A}$ is perpendicular to $\vec{B}-\vec{C}$

(B) $\vec{A}=\vec{B}$

(C) $\vec{B}=\vec{C}$

(D) $\vec{C}=\vec{A}$

Sol. **[C]**

$\vec{A}.(\vec{B}-\vec{C}) = 0$	$\vec{A}\times(\vec{B}-\vec{C}) = 0$
then $\vec{A} = 0$	$\vec{A} = 0$
$\vec{B} = \vec{C}$	$\vec{B} = \vec{C}$
$\vec{A}\perp(\vec{B}-\vec{C})$	$\vec{A}\parallel(\vec{B}-\vec{C})$

common $\vec{B} = \vec{C}$

➤ True or false type questions

Q.46 Points $\vec{a}-2\vec{b}+2\vec{c}$, $2\vec{a}+3\vec{b}-4\vec{c}$, $-7\vec{b}+10\vec{c}$ are collinear.

Sol. **[False]**

P.V of A $\vec{a}-2\vec{b}+2\vec{c}$

P.V of B $2\vec{a}+3\vec{b}-4\vec{c}$

P.V of C $-7\vec{b}+10\vec{c}$

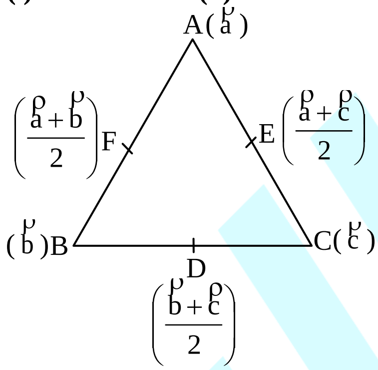
$\overrightarrow{AB}\neq\lambda\,\overrightarrow{AC}$ so points are not collinear

Q.47 If D, E, F are the mid points of the sides BC, CA and AB respectively of a triangle ABC and 'O' is any point,

(i) $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC} = \overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}$

(iii) $\overrightarrow{AD}+\frac{2}{3}\overrightarrow{BE}+\frac{1}{3}\overrightarrow{CF} = \frac{1}{2}\overrightarrow{AC}$

Sol. **(i) True** **(ii) True**



All position vector with respect to O

(i) $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC} = \vec{a}+\vec{b}+\vec{c}$

$\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF} = \dfrac{\vec{b}+\vec{c}}{2}+\dfrac{\vec{a}+\vec{c}}{2}+\dfrac{\vec{a}+\vec{b}}{2}$

$= \vec{a}+\vec{b}+\vec{c}$

(ii) $\overrightarrow{AD}+\frac{2}{3}\overrightarrow{BE}+\frac{1}{3}\overrightarrow{CF}$

$= \left(\dfrac{\vec{b}+\vec{c}}{2}-\vec{a}\right)+\dfrac{2}{3}\left(\dfrac{\vec{a}+\vec{c}}{2}-\vec{b}\right)+\dfrac{1}{3}\left(\dfrac{\vec{a}+\vec{b}}{2}-\vec{c}\right)$

$= \dfrac{\vec{b}+\vec{c}-2\vec{a}}{2}+\dfrac{\vec{a}+\vec{c}-2\vec{b}}{3}+\dfrac{\vec{a}+\vec{b}-2\vec{c}}{6}$

$= \dfrac{3\vec{b}+3\vec{c}-6\vec{a}+2\vec{a}+2\vec{c}-4\vec{b}+\vec{a}+\vec{b}-2\vec{c}}{6}$

$= \dfrac{-3\vec{a}+3\vec{c}}{6} = \dfrac{1}{2}(\vec{c}-\vec{a}) = \dfrac{1}{2}\overrightarrow{AC}$

Q.48 Let $\vec{r} = \vec{a}+\lambda\vec{\lambda}$ and $\vec{r} = \vec{b}+\mu\vec{m}$ be two lines in space where $\vec{a} = 5\hat{i}+\hat{j}+2\hat{k}$ & $\vec{b} = -\hat{i}+7\hat{j}+8\hat{k}$, $\vec{\lambda} = -4\hat{i}-\hat{j}+\hat{k}$ and $\vec{m} = 2\hat{i}-5\hat{j}-7\hat{k}$ then the position vector of a point which lies on both of these lines is $2\hat{i}+\hat{j}+\hat{k}$.

Sol. **[False]**

$\vec{r}_1 = (5\hat{i}+\hat{j}+2\hat{k})+\lambda(-4\hat{i}-\hat{j}+\hat{k})$

$\vec{r}_2 = (-\hat{i}+7\hat{j}+8\hat{k})+\mu(2\hat{i}-5\hat{j}-7\hat{k})$

$5-4\lambda = -1+2\mu$ ......(1)

$1-\lambda = 7-5\mu$ ......(2)

$2-\lambda = 8-7\mu$ ......(3)

solution of (1) & (2) doesn't satisfy eq. (3) so these two are skew lines.

Q.49 Given system of points is coplanar $3\vec{a}+2\vec{b}-5\vec{c}$, $3\vec{a}+8\vec{b}+5\vec{c}$,

$-3\vec{a} + 2\vec{b} + \vec{c}, \vec{a} + 4\vec{b} - 3\vec{c}$

Sol. **[False]**

P.V of A $(3\vec{a} + 2\vec{b} - 5\vec{c})$

P.V of B $(3\vec{a} + 8\vec{b} + 5\vec{c})$

P.V of C $(-3\vec{a} + 2\vec{b} + \vec{c})$

P.V of D $(\vec{a} + 4\vec{b} - 3\vec{c})$

$\overrightarrow{AB} = 6\vec{b} + 10\vec{c}$

$\overrightarrow{AC} = -6\vec{a} + 6\vec{c}$

$\overrightarrow{AD} = -2\vec{a} + 2\vec{b} + 2\vec{c}$

Now $\begin{vmatrix} 0 & 6 & 10 \\ -6 & 0 & 6 \\ -2 & 2 & 2 \end{vmatrix} \neq 0$. So points is not coplanar

Fill in the blanks type questions

Q.50 The vectors $\overrightarrow{AB} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{BC} = -\hat{i} + 2\hat{k}$ are the adjacent sides of a parallelogram ABCD, then the angle between the diagonals is ....

Sol. $\overrightarrow{AB} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$\overrightarrow{BC} = -\hat{i} + 2\hat{k}$

Then diagonals are

$\overrightarrow{AB} + \overrightarrow{BC} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

and $\overrightarrow{AB} - \overrightarrow{BC} = 4\hat{i} - 2\hat{j}$

$\cos\theta = \dfrac{(2\hat{i} - 2\hat{j} + 4\hat{k}).(4\hat{i} - 2\hat{j})}{\sqrt{4+4+16}\sqrt{16+4}}$

$= \dfrac{12}{\sqrt{24}\sqrt{20}}$

$\cos\theta = \sqrt{\dfrac{3}{10}}$

$\Rightarrow \theta = \cos^{-1}\sqrt{\dfrac{3}{10}}$

Q.51 Let $\vec{A}, \vec{B}, \vec{C}$ be vectors of length 3, 4, 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B} + \vec{C}$, $\vec{B}$ to $\vec{C} + \vec{A}$ and $\vec{C}$ to $\vec{A} + \vec{B}$. Then the length of vector $\vec{A} + \vec{B} + \vec{C}$ is...........

Sol. $5\sqrt{2}$

Q.52 Let $\vec{r}$, $\vec{a}$, $\vec{b}$ and $\vec{c}$ be four non zero vectors, such that

$\vec{r} \cdot \vec{a} = 0 = \vec{r} \cdot \vec{b}$, $|\vec{r} \times \vec{c}| = |\vec{r}||\vec{c}|$ then

$[\vec{a}\ \vec{b}\ \vec{c}] =$ ....

Sol. $\vec{r} \cdot \vec{a} = 0 = \vec{r} \cdot \vec{b}$ and $|\vec{r} \times \vec{c}| = |\vec{r}||\vec{c}|$

$\Rightarrow \vec{r} \perp \vec{a}$, $\vec{r} \perp \vec{b}$ and $\vec{r} \perp \vec{c}$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ coplanar

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

Q.53 A, B, C and D are four points in a plane with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively such that $(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = (\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$. The point D, then, is the .... of the triangle ABC.

Sol. Given that

$(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = 0$

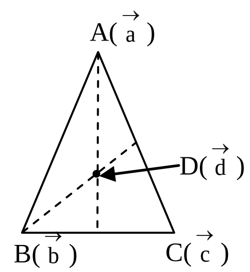
$\Rightarrow \overrightarrow{AD}.\overrightarrow{CB} = 0$

$\Rightarrow \overrightarrow{AD} \perp \overrightarrow{CB}$

and $(\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$

$\Rightarrow \overrightarrow{DB}.\overrightarrow{AC} = 0$

$\Rightarrow \overrightarrow{DB} \perp \overrightarrow{AC}$



Clearly D is the orthocenter of the ΔABC

Q.54 Let $\vec{b} = 4\hat{i} + 3\hat{j}$ & $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, are given by ....

Sol. $\vec{b} = 4\hat{i} + 3\hat{j}$ and let $\vec{c} = \alpha\hat{i} + \beta\hat{j}$

$\therefore \vec{b}$ and $\vec{c}$ are perpendicular

$\Rightarrow \vec{b}.\vec{c} = 0$

$\Rightarrow 4\alpha + 3\beta = 0$

$\frac{\alpha}{3} = \frac{\beta}{-4} = \lambda$ (let) ....(i)

Let the vector in same plane is $\vec{a} = x\hat{i} + y\hat{j}$

$\therefore$ Projection of $\vec{a}$ along $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{4x+3y}{5} = 1$

$4x + 3y = 5$ ....(ii)

$\because$ Projection of $\vec{a}$ along $\vec{c} = \frac{\vec{a}.\vec{c}}{|\vec{c}|} = 2$

$\Rightarrow \frac{x\alpha + y\beta}{\sqrt{\alpha^2+\beta^2}} = 2$

But from (i) $\alpha = 3\lambda$, $\beta = -4\lambda$

$\Rightarrow 3\lambda x - 4\lambda y = 10\lambda$

$\Rightarrow 3x - 4y = 10$ ....(iii)

Solving (ii) & (iii) we get

$x = 2, y = -1$

$\vec{a} = 2\hat{i} - \hat{j}$

Q.55 A non zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i} + \hat{j}$ and the plane determined by the $\hat{i} - \hat{j}$, $\hat{i} + \hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ is.......

Sol. Equation of plane containing vectors $\hat{i}$ and $\hat{i} + \hat{j}$ is $[\vec{r} - \hat{i}\ \ \hat{i}\ \ \hat{i} + \hat{j}] = 0$

$$\Rightarrow \begin{vmatrix} x-1 & y & z \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

$\Rightarrow z = 0$ ....(i)

and Equation of plane containing the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{k}$ is

$[\vec{r} - (\hat{i} - \hat{j})\ \ \hat{i} - \hat{j}\ \ \hat{i} - \hat{k}] = 0$

$$\Rightarrow \begin{vmatrix} x-1 & y+1 & z \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$x + y - z = 0$ ....(ii)

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since $\vec{a}$ is parallel to (i) & (ii)

$\Rightarrow a_3 = 0\ a_1 = -a_2$

$\Rightarrow \vec{a} = \hat{i} - \hat{j}$

Angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ is

$\cos\theta = \pm \frac{1.1 + 1.2}{\sqrt{2}\sqrt{9}} = \pm \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

Q.56 If $\vec{b}$ and $\vec{c}$ are any two non-collinear unit vectors and $\vec{a}$ is any vector, then

$(\vec{a}.\vec{b})\vec{b} + (\vec{a}.\vec{c})\vec{c} + \frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}(\vec{b}\times\vec{c}) =$ ....

Sol. $\vec{a}$

EXERCISE # 2

Part-A **Only single correct answer type questions**

Q.1 Let $\vec{A}$ and $\vec{B}$ be two non-parallel unit vectors in a plane. If $(\alpha\vec{A}+\vec{B})$ bisects the internal angle between $\vec{A}$ and $\vec{B}$, then α is equal to-

(A) 1/2 (B) 1 (C) 2 (D) 4

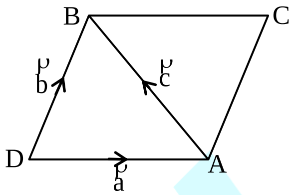
Sol. [B] $\vec{A}$ and $\vec{B}$ are unit vector so vector bisects the internal angle is $\alpha(\vec{A}+\vec{B})$ so $\alpha = 1$

Q.2 Given a parallelogram OACB. The length of the vectors $\vec{OA}$, $\vec{OB}$ and $\vec{AB}$ are a, b and c respectively. The scalar product of the vectors $\vec{OC}$ and $\vec{OB}$ is -

(A) $(a^2 - 3b^2 + c^2)/2$ (B) $(3a^2 + b^2 - c^2)/2$

(C) $(3a^2 - b^2 + c^2)/2$ (D) $(a^2 + 3b^2 - c^2)/2$

Sol. [D]



$\vec{c} = \vec{b} - \vec{a}$ ....(1)

square on both side

$c^2 = b^2 + a^2 - 2\vec{a}.\vec{b}$

$\vec{a}.\vec{b} = \dfrac{b^2+a^2-c^2}{2}$ ....(2)

$\overline{OC}.\overline{OB} = (\vec{a}+\vec{b}).\,b = a.b + b^2$

$= \dfrac{b^2+a^2-c^2}{2} + b^2$

$= \dfrac{a^2+3b^2-c^2}{2}$

Q.3 The vertices of triangle have the position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $P(\vec{r})$ is a point in the plane of Δ such that : $\vec{a}.\vec{b} + \vec{c}.\vec{r} = \vec{a}.\vec{c} + \vec{b}.\vec{r} = \vec{b}.\vec{c} + \vec{a}.\vec{r}$ then for the Δ, P is the

(A) circumcentre (B) centroid

(C) orthocentre (D) incentre

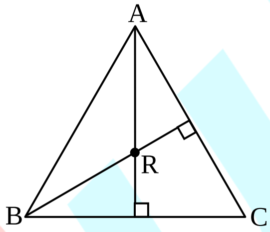
Sol. [C] $\vec{a}.\vec{b} - \vec{a}.\vec{c} = \vec{b}.\vec{r} - \vec{c}.\vec{r}$ $\quad$ $\vec{a}.\vec{b} - \vec{b}.\vec{c} = \vec{a}.\vec{r} - \vec{c}.\vec{r}$

$\vec{a}.(\vec{b}-\vec{c}) - (\vec{b}-\vec{c})\vec{r} = 0$ $\quad$ $\vec{b}(\vec{a}-\vec{c}) - \vec{r}.(\vec{a}-\vec{c}) = 0$

$(\vec{b}-\vec{c}).(\vec{a}-\vec{r}) = 0$ $\quad$ $(\vec{a}-\vec{c}).(\vec{b}-\vec{r}) = 0$

$\overline{CB}.\overline{RA} = 0$ $\quad$ $\overline{CA}.\overline{RB} = 0$

$\overline{CB} \perp \overline{RA}$ $\quad$ $\overline{AC} \perp \overline{RB}$



So R is orthocenter

Q.4 If A, B, C, D are four points in space satisfying $\overline{AB}.\overline{CD} = K[\,|\overline{AD}|^2 + |\overline{BC}|^2 - |\overline{AC}|^2 - |\overline{BD}|^2]$ then the value of K is –

(A) 2 (B) 1/3 (C) 1/2 (D) 1

Sol. **[C]**

Let $\vec{A}$ be the origin and position vector of $\vec{B}, \vec{C}, \vec{D}$ are $\vec{b}, \vec{c}, \vec{d}$ respectively

Then L.H.S

$\vec{AB}.\vec{CD} = \vec{b}.(\vec{d}-\vec{c})$

Taking R.H.S we have

$K[\,|\overline{AD}|^2 + |\overline{BC}|^2 - |\overline{AC}|^2 - |\overline{BD}|^2]$

$= K[|\vec{d}|^2 + |\vec{c}-\vec{b}|^2 - |\vec{c}|^2 - |\vec{d}-\vec{b}|^2]$

$= K[\vec{d}.\vec{d} + \vec{c}.\vec{c} + \vec{b}.\vec{b} - 2\vec{c}.\vec{b} - \vec{c}.\vec{c} - \vec{d}.\vec{d}$

$- \vec{b}.\vec{b} + 2\vec{d}.\vec{b}]$

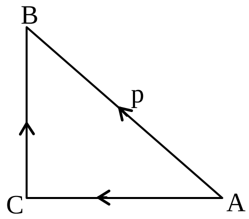
$= K[2(\vec{d}.\vec{b} - \vec{c}.\vec{b})] = 2K\,\vec{b}.(\vec{d}-\vec{c})$

$\Rightarrow K = \dfrac{1}{2}$

Q.5 If in a right angled triangle ABC, the hypotenuse AB = p, then $\vec{AB}.\vec{AC} + \vec{BC}.\vec{BA} + \vec{CA}.\vec{CB}$ is -

(A) $2p^2$ (B) $\dfrac{p^2}{2}$ (C) p^2 (D) None

Sol. **[C]**



Given $\vec{AB} = p$

$\therefore\ \vec{CA} \perp \vec{CB} \Rightarrow \vec{CA}\,.\,\vec{CB} = 0$ ....(i)

$\Rightarrow \vec{AB}\,.\,\vec{AC} + \vec{BC}\,.\,\vec{BA} + \vec{CA}\,.\,\vec{CB}$

$= \vec{AB}\,.\,\vec{AC} + \vec{BC}\,.\,\vec{BA} + 0$ using (i)

$= \vec{AB}\,.\,(\vec{AC} - \vec{BC})$

$= \vec{AB}\,.(\vec{AC} + \vec{CB})$

$= \vec{AB}\,.\,\vec{AB} = p^2$

Q.6 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non–zero and non coplanar vectors and $\vec{p}, \vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p} = \vec{a} + \vec{b} - 2\vec{c}$,
$\vec{q} = 3\vec{a} - 2\vec{b} + \vec{c}$, $\vec{r} = \vec{a} - 4\vec{b} + 2\vec{c}$.
If the volume of the parallelopiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$ is v_1 and that of the parallelopiped determined by $\vec{p}$, $\vec{q}$ and $\vec{r}$ is v_2, then $v_2 : v_1 =$

(A) 3 : 1 (B) 7 : 1
(C) 11 : 1 (D) 15 :1

Sol. **[D]**

$v_1 = [\vec{a}\ \vec{b}\ \vec{c}]$ and $v_2 = [\vec{p}\ \vec{q}\ \vec{r}]$

Θ $[\vec{p}\ \vec{q}\ \vec{r}] = \vec{p}\,.(\vec{q} \times \vec{r})$

$= (\vec{a} + \vec{b} - 2\vec{c}).\ \{(3\vec{a} - 2\vec{b} + \vec{c})$
$\times (\vec{a} - 4\vec{b} + 2\vec{c})\}$

$= (\vec{a} + \vec{b} - 2\vec{c}).\ \{-12\,(\vec{a} \times \vec{b}) + 6\,(\vec{a} \times \vec{c})$
$-2(\vec{b} \times \vec{a}) - 4(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{b}) - 4(\vec{c} \times \vec{b})\}$

$= (\vec{a} + \vec{b} - 2\vec{c}).\ \{-10(\vec{a} \times \vec{b}) + 5\,(\vec{a} \times \vec{c})\}$

$= 5\,\vec{b}\,.\,(\vec{a} \times \vec{c}) + 20\,\vec{c}\,.\,(\vec{a} \times \vec{b})$

$= -5\,\vec{a}\,.\,(\vec{b} \times \vec{c}) + 20\,\vec{a}\,.\,(\vec{b} \times \vec{c})$

$= 15\,[\vec{a}\ \vec{b}\ \vec{c}]$

$\Rightarrow \dfrac{v_2}{v_1} = \dfrac{[\vec{p}\ \vec{q}\ \vec{r}]}{[\vec{a}\ \vec{b}\ \vec{c}]} = \dfrac{15}{1}$

Q.7 A vector $\vec{x}$ is coplanar with vectors $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$ and is orthogonal to the vector $\vec{b}$. If $\vec{x}\,.\,\vec{a} = 7$ then the vector $\vec{x}$ is equal to-

(A) $(-3\hat{i} + 5\hat{j} + 6\hat{k})$ (B) $\frac{1}{2}(-3\hat{i} + 5\hat{j} + 6\hat{k})$
(C) $(3\hat{i} - 5\hat{j} - 6\hat{k})$ (D) none of these

Sol. **[B]**

$\vec{a} = -1, 1, 1,\quad \vec{b} = 2, 0, 1$

Since $\vec{x}$ is coplanar with $\vec{a}$ and $\vec{b}$

$\vec{x} = \lambda\,\vec{a} + \mu\,\vec{b}$

$\Rightarrow \vec{x} = \lambda(-\hat{i} + \hat{j} + \hat{k}) + \mu(2\hat{i} + \hat{k})$

$= \vec{x} = (-\lambda + 2\mu, \lambda, \lambda + \mu)$

$\vec{x}$ is orthogonal to $\vec{b}$

$\Rightarrow \vec{x}\,.\,\vec{b} = 0$

$\Rightarrow 2\,(-\lambda + 2\mu) + 0 + (\lambda + \mu) = 0$

$\Rightarrow -\lambda + 5\mu = 0$ ....(i)

Given $\vec{x}\,.\,\vec{a} = 7$

$-1\,(-\lambda + 2\mu) + \lambda + \lambda + \mu = 7$

$3\lambda - \mu = 7$ ....(ii)

Solving (i) & (ii) we get

$\lambda = \dfrac{5}{2}, \mu = \dfrac{1}{2}$

$\Rightarrow \vec{x} = \left(-\dfrac{5}{2} + \dfrac{2}{2}\right)\hat{i} + \dfrac{5}{2}\hat{j} + \left(\dfrac{5}{2} + \dfrac{1}{2}\right)\hat{k}$

$\vec{x} = \dfrac{1}{2}(-3\hat{i} + 5\hat{j} + 6\hat{k})$

Q.8 If $\vec{b}$ is a vector whose initial point divides the join of $5\hat{i}$ and $5\hat{j}$ in the ratio k : 1 and terminal point is origin and $|\vec{b}| \le \sqrt{37}$, then k lies in the interval -

(A) $\left[-6,-\frac{1}{6}\right]$ (B) $(-\infty,-6] \cup \left[-\frac{1}{6},\infty\right)$

(C) [0, 6] (D) None of these

Sol. **[B]**

$\vec{b}$ is a vector whose initial point divides the join to $5\hat{i}$ and $5\hat{j}$ in the ratio k : 1

$\Rightarrow$ Point $= \frac{5\hat{i}+5k\hat{j}}{k+1}$

terminal point is origin

$\Rightarrow \vec{b} = \frac{5\hat{i}+5k\hat{j}}{k+1}$

Given that $|\vec{b}| \le \sqrt{37}$

$\Rightarrow \frac{\sqrt{25+25k^2}}{k+1} \le \sqrt{37}$

$\Rightarrow 25 + 25k^2 \le 37\,(k+1)^2$

$\Rightarrow 6k^2 + 37k + 6 \ge 0$

$\Rightarrow (k+6)(6k+1) \ge 0$

$\Rightarrow k \in (-\infty,-6] \cup \left[-\frac{1}{6},\infty\right)$

Q.9 If $\vec{a}$ and $\vec{b}$ are mutually perpendicular vectors, then the projection of the vector $\left(\lambda\frac{\vec{a}}{|\vec{a}|}+m\frac{\vec{b}}{|\vec{b}|}+n\frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}\right)$ along the angle bisector of the vectors $\vec{a}$ & $\vec{b}$ may be given as-

(A) $\frac{\lambda^2+m^2}{\sqrt{\lambda^2+m^2+n^2}}$ (B) $\sqrt{\lambda^2+m^2+n^2}$

(C) $\frac{\sqrt{\lambda^2+m^2}}{\sqrt{\lambda^2+m^2+n^2}}$ (D) $\frac{\lambda+m}{\sqrt{2}}$

Sol. **[D]**

Q.10 Let co-ordinates of a point 'p' with respect to the system non-coplanar vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is (3, 2, 1). Then, co-ordinates of 'p' with respect to the system of vectors $\vec{a}+\vec{b}+\vec{c}$, $\vec{a}-\vec{b}+\vec{c}$ and $\vec{a}+\vec{b}-\vec{c}$ is -

(A) $\left(\frac{3}{2},\frac{1}{2},1\right)$ (B) $\left(\frac{3}{2},1,\frac{1}{2}\right)$

(C) $\left(\frac{1}{2},\frac{3}{2},1\right)$ (D) none of these

Sol. [A] Coordinate of P in first column

$\vec{P} = 3\vec{a}+2\vec{b}+\vec{c}$ …..(1)

coordinate of P in second column

$\vec{P} = \lambda(\vec{a}+\vec{b}+\vec{c}) + \mu(\vec{a}-\vec{b}+\vec{c}) + \gamma(\vec{a}+\vec{b}-\vec{c})$

$\vec{P} = \vec{a}(\lambda+\mu+\gamma) + \vec{b}(\lambda-\mu-\gamma) + \vec{c}(\lambda+\mu-\gamma)$ …….(2)

compare (1) & (2)

$\lambda+\mu+\gamma = 3$ so $\lambda = 3/2$

$\lambda-\mu-\gamma = 2$ $\mu = 1/2$

$\lambda+\mu-\gamma = 1$ $\gamma = 1$

Q.11 The position vectors of the points P and Q are $\vec{p}$ and $\vec{q}$ respectively. If O is the origin and R is a point in the interior of $\angle POQ$ such that OR bisects the $\angle POQ$ then unit vector along OR is

(A) $\frac{\vec{p}+\vec{q}}{|\vec{p}||\vec{q}|}$ (B) $\frac{\vec{p}}{|\vec{p}|}-\frac{\vec{q}}{|\vec{q}|}$

(C) $\frac{\left(\frac{\vec{p}}{|\vec{p}|}+\frac{\vec{q}}{|\vec{q}|}\right)}{\left|\frac{\vec{p}}{|\vec{p}|}+\frac{\vec{q}}{|\vec{q}|}\right|}$ (D) none of these

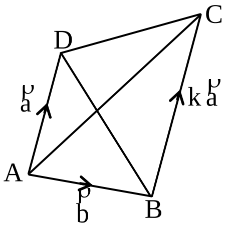
Sol. [C] Unit vector along OR is

$$\frac{\hat{p}+\hat{q}}{|\hat{p}+\hat{q}|} = \frac{\frac{\vec{p}}{|\vec{p}|}+\frac{\vec{q}}{|\vec{q}|}}{\left|\frac{\vec{p}}{|\vec{p}|}+\frac{\vec{q}}{|\vec{q}|}\right|}$$

Q.12 If $\overrightarrow{DA} = \vec{a}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{CB} = k\vec{a}$ where k > 0 and X, Y are the mid-points of DB & AC respectively, such that $|\vec{a}| = 17$ & $|\overrightarrow{XY}| = 4$, then k equal to -

(A) $\frac{8}{17}$ (B) $\frac{13}{17}$ (C) $\frac{25}{17}$ (D) $\frac{4}{17}$

Sol. [C]



Let A is origin

P.V of B is $\vec{b}$

P.V of C is $k\vec{a}+\vec{b}$

P.V of D is $\vec{a}$

P.V of X = $\frac{\vec{a}+\vec{b}}{2}$

P.V of Y = $\frac{\vec{b}+k\vec{a}}{2}$

$(\overrightarrow{XY}) = 4$ $\left|\frac{(k-1)}{2}\right| |\vec{a}| = 4$

$\frac{k-1}{2} = \frac{4}{17} \Rightarrow k-1 = \pm\frac{8}{17}$

AC = k = $1 + \frac{8}{17} = \frac{25}{17}$

$k = 1 - \frac{8}{17} = \frac{9}{17}$

Q.13 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors, no two of which are collinear. If the vector $3\vec{a} + 7\vec{b}$ is collinear with $\vec{c}$ and $3\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$, then $9\vec{a} + 21\vec{b} + 14\vec{c}$ is equal to -

(A) $\lambda\vec{a}$ (B) $\lambda\vec{c}$

(C) 0 (D) none of these

Sol. [C] $3\vec{a}+7\vec{b} = \lambda\vec{c}$ .....(1)

$3\vec{b}+2\vec{c} = \mu\vec{a}$ .....(2)

eliminate 'c' from (1) & (2)

$6\vec{a}+14\vec{b} = \lambda(\mu\vec{a}-3\vec{b})$

$\vec{a}(6-\lambda\mu)+\vec{b}(14+3\lambda) = 0$

$\vec{a}$ and $\vec{b}$ are not collinear

$6-\lambda\mu = 0$ $14 + 3\lambda = 0$

$\lambda = -14/3$ put in eq. (1)

$3\vec{a}+7\vec{b} = \frac{-14}{3}\vec{c}$

$9\vec{a}+21\vec{b} = -14\vec{c}$

$9\vec{a}+21\vec{b}+14\vec{c} = 0$

Q.14 Let $\vec{a}$ be a unit vector and $\vec{b}$ a non-zero vector not parallel to $\vec{a}$. The angles of the triangle, two of whose sides are represented by $\sqrt{3}\,(\vec{a}\times\vec{b})$ and $\vec{b}-(\vec{a}\,.\,\vec{b})\vec{a}$ are -

(A) $\pi/4, \pi/4, \pi/2$ (B) $\pi/4, \pi/3, 5\pi/12$

(C) $\pi/6, \pi/3, \pi/2$ (D) None of these

Sol. **[C]**

Given $\overrightarrow{AB} = \sqrt{3}\,(\vec{a}\times\vec{b})$, $\overrightarrow{BC} = \vec{b} - (\vec{a}\,.\,\vec{b})\,\vec{a}$

$\vec{a}$ is a unit vector

we take

$\overrightarrow{AB}\,.\,\overrightarrow{BC} = \sqrt{3}\,(\vec{a}\times\vec{b})[\vec{b}-(\vec{a}\,.\,\vec{b})\,\vec{a}\,]$

$= \sqrt{3}\,(\vec{a}\times\vec{b}).\,\vec{b} - (\vec{a}\,.\,\vec{b})\{(\vec{a}\times\vec{b}).\,\vec{a}\} = 0$

$\Rightarrow AB \perp BC \Rightarrow \angle ABC = \pi/2$

$\therefore AB^2 = 3(\vec{a}\times\vec{b})^2 = 3.b^2\sin^2\theta$

$= 3b^2\sin^2\theta$ ....(i)

and $BC^2 = (\vec{b})^2 + (\vec{a}\,.\,\vec{b})^2\,|\vec{a}|^2 - 2\,(\vec{b}\,.\,\vec{a})\,(\vec{a}\,.\,\vec{b})$

$= (\vec{b})^2 + (\vec{a}\,.\,\vec{b})^2 - 2\,(\vec{a}\,.\,\vec{b})^2$

$= (\vec{b})^2 - (\vec{a}\,.\,\vec{b})^2$

$= (\vec{b})^2\,(1-\cos^2\theta)$

$BC^2 = b^2\sin^2\theta$ ....(ii)

From (i) & (ii) $AB^2 = 3BC^2$

$\Rightarrow AB = \sqrt{3}\,BC$

$\therefore \tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}} \Rightarrow A = \frac{\pi}{6}$

and $C = \frac{\pi}{3}$

Angles are $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$

Q.15 Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$ is equal to-

(A) 0

(B) 1

(C) $\frac{1}{4}$ $(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

(D) $\frac{3}{4}$ $(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) \times (c_1^2 + c_2^2 + c_3^2)$

Sol. **[C]**

$\therefore (\vec{a} \times \vec{b}) = |\vec{a}||\vec{b}| \sin \frac{\pi}{6} . \hat{n}$

$\Rightarrow (\vec{a} \times \vec{b}) . \vec{c} = \frac{1}{2} |\vec{a}||\vec{b}| . \hat{n}\ \vec{c}$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = \frac{1}{2} |\vec{a}||\vec{b}| . 1 . 1 \cos 0^\circ$

$\because$ $\hat{n}$ is $\perp$ to both $\vec{a}$ and $\vec{b}$ and $\vec{c}$ is also a unit vector $\perp$ to both $\vec{a}$ and $\vec{b}$

$$\therefore \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = [\vec{a}\ \vec{b}\ \vec{c}\,] = \frac{1}{4} |\vec{a}|^2 |\vec{b}|^2$$

$= \frac{1}{4} (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

Q.16 Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{j} - \hat{k}$, $\vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a} . \vec{d} = 0 = [\vec{b}, \vec{c}, \vec{d}]$, then $\vec{d}$ equals

(A) $\pm \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ (B) $\pm \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

(C) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (D) $\pm \hat{k}$

Sol. **[A]**

$\because \vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{j} - \hat{k}$, $\vec{c} = \hat{k} - \hat{i}$

Let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow x^2 + y^2 + z^2 = 1$ ....(i)

$\therefore \vec{a} . \vec{d} = 0 \Rightarrow x - y = 0$

$\Rightarrow x = y$ ....(ii)

and $[\vec{b}\ \vec{c}\ \vec{d}\,] = 0$

$$\Rightarrow \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ x & y & z \end{vmatrix} = 0$$

$\Rightarrow x + y + z = 0$

$\Rightarrow 2x + z = 0$ from (ii)

$\Rightarrow z = -2x$

from (i), (ii) and (iii) we get

$x^2 + x^2 + 4x^2 = 1$

$\Rightarrow x = \pm \frac{1}{\sqrt{6}}$

$\vec{d} = \pm \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$

Q.17 If $\vec{p}$, $\vec{q}$, $\vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{p} \times ((\vec{x} - \vec{q}) \times \vec{p}) + \vec{q} \times ((\vec{x} - \vec{r}) \times \vec{q}) + \vec{r} \times ((\vec{x} - \vec{p}) \times \vec{r}) = \vec{0}$ then $\vec{x}$ is given by-

(A) $\frac{1}{2}(\vec{p} + \vec{q} - 2\vec{r})$ (B) $\frac{1}{2}(\vec{p} + \vec{q} + \vec{r})$

(C) $\frac{1}{3}(\vec{p} + \vec{q} + \vec{r})$ (D) $\frac{1}{3}(2\vec{p} + \vec{q} - \vec{r})$

Sol. **[B]**

$\vec{p}, \vec{q}, \vec{r}$ are three mutually perpendicular vector of same magnitude so

let $\vec{p} = a\hat{i}$, $\vec{q} = a\hat{j}$, $\vec{r} = a\hat{k}$

and Let $\vec{x} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\therefore \vec{p} \times (\vec{x} - \vec{q}) \times \vec{p}) = \vec{p} \times (\vec{x} \times \vec{p} - \vec{q} \times \vec{p})$

$= \vec{p} \times (\vec{x} \times \vec{p}) - \vec{p} \times (\vec{q} \times \vec{p})$

$= (\vec{p} . \vec{p})\vec{x} + (\vec{p} . \vec{x})\vec{p} - (\vec{p} . \vec{p})\vec{q} + (\vec{p} . \vec{q})\vec{p}$

$= a^2\vec{x} - a^2x_1\hat{i} - a^3\hat{j} + 0$

Similarly

$\vec{q} \times ((\vec{x} - \vec{r}) \times \vec{q}) = a^2\vec{x} - a^2y_1\hat{j} - a^3 . \hat{k}$

and $\vec{r} \times ((\vec{x} - \vec{p}) \times \vec{r}) = a^2\vec{x} - a^2z_1\hat{k} - a^3 . \hat{i}$

Put these values in equation we get

$3a^2\vec{x} - a^2 (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) - a^2 (a\hat{i} + a\hat{j} + a\hat{k}) = 0$

$\Rightarrow 3\vec{x} - \vec{x} - (\vec{p} + \vec{q} + \vec{r}) = 0$

$\Rightarrow \vec{x} = \frac{1}{2}(\vec{p} + \vec{q} + \vec{r})$

Q.18 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then-

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$

(C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

Sol. **[D]**

Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent

so $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ For some λ, μ

$\Rightarrow \hat{i} + \alpha\hat{j} + \beta\hat{k} = \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(4\hat{i} + 3\hat{j} + 4\hat{k})$

$= (\lambda + 4\mu)\hat{i} + (\lambda + 3\mu)\hat{j} + (\lambda + 4\mu)\hat{k}$

Comparing coefficient we get

$\lambda + 4\mu = 1$ ....(i)

$\lambda + 3\mu = \alpha$ ....(ii)

$\lambda + 4\mu = \beta$ ....(iii)

from (i) & (iii) we get

$\beta = 1$

Given $|\vec{c}| = \sqrt{3}$

$1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow \alpha^2 = 1 \quad \because \beta = 1$

$\Rightarrow \alpha = \pm 1$

Q.19 Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ =

(A) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$ (B) $\frac{1}{\sqrt{3}}(-\hat{i} - \hat{j} - \hat{k})$

(C) $\frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j})$ (D) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

Sol. **[A]**

$\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$

$\Rightarrow \vec{c} = x\vec{a} + y\vec{b}$ ....(i)

$\because \vec{c}$ is perpendicular to $\vec{a}$

$\Rightarrow \vec{c}\cdot\vec{a} = 0$

$\Rightarrow x\,\vec{a}\cdot\vec{a} + y\,\vec{b}\cdot\vec{a} = 0$

$\Rightarrow 6x + 3y = 0$

$\Rightarrow y = -2x$ ....(ii)

$\Rightarrow \vec{c} = x(\vec{a} - 2\vec{b}) = 3x(-\hat{j} + \hat{k})$

$\Rightarrow |\vec{c}|^2 = 9x^2(1 + 1) = 18x^2$

$\because \vec{c}$ is a unit vector $\Rightarrow |\vec{c}| = 1$

$\Rightarrow x = \pm\frac{1}{3\sqrt{2}}$

$\Rightarrow \vec{c} = \pm\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

Part-B One or more than one correct answer type questions

Q.20 Let ABC be a triangle, the position vector of whose vertices are respectively $7\hat{j} + 10\hat{k}$, $-\hat{i} + 6\hat{j} + 6\hat{k}$ & $-4\hat{i} + 9\hat{j} + 6\hat{k}$. Then ΔABC is -

(A) Isosceles (B) Equilateral

(C) Right angled (D) none of these

Sol. **[A,C]**

Let O be the origin then

$\vec{OA} = 7\hat{j} + 10\hat{k}$, $\vec{OB} = -\hat{i} + 6\hat{j} + 6\hat{k}$

$\vec{OC} = -4\hat{i} + 9\hat{j} + 6\hat{k}$

$\therefore \vec{AB} = \vec{AO} + \vec{OB} = \vec{OB} - \vec{OA} = -\hat{i} - \hat{j} - 4\hat{k}$

$\vec{AC} = \vec{AO} + \vec{OC} = \vec{OC} - \vec{OA} = -4\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{BC} = \vec{BO} + \vec{OC} = \vec{OC} - \vec{OB} = -3\hat{i} + 3\hat{j}$

$\Rightarrow AB = |\vec{AB}| = \sqrt{1+1+16} = \sqrt{18}$

$AC = |\vec{AC}| = \sqrt{16+4+16} = \sqrt{36}$

$BC = |\vec{BC}| = \sqrt{9+9} = \sqrt{18}$

Clearly triangle is isosceles

and $AB^2 + BC^2 = AC^2$

$\Rightarrow$ triangle is right angled at B

Option A,C are correct.

Q.21 Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$; $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is magnitude $\sqrt{\frac{2}{3}}$ is -

(A) $3\hat{i} + 6\hat{j} - 2\hat{k}$ (B) $2\hat{i} + 3\hat{j} + 3\hat{k}$

(C) $-2\hat{i} - \hat{j} + 2\hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

Sol. **[A]**

Q.22 Unit vectors $\hat{a}$ and $\hat{b}$ are inclined at an angle 2θ and $|\hat{a}-\hat{b}| \le 1$, if $0 \le \theta < \pi$. Then θ may belong to -

(A) $[0, \pi/6]$ (B) $(5\pi/6, \pi)$

(C) $[\pi/6, \pi/2]$ (D) $[\pi/6, 5\pi/6]$

Sol. **[A,B]**

$\therefore |\hat{a}-\hat{b}|^2 = 1 + 1 - 2\cos 2\theta$

$= 2.\ 2\sin^2\theta$

But $|\hat{a}-\hat{b}| \le 1$

$\Rightarrow 2|\sin\theta| \le 1$

$\Rightarrow |\sin\theta| \le \frac{1}{2}$

But $0 \le \theta < \pi$

$\Rightarrow \sin\theta \le \frac{1}{2}$

$\theta \in \left[0, \frac{\pi}{6}\right]$ or $\left[\frac{5\pi}{6}, \pi\right)$

option A,B are correct.

Q.23 If $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is -

(A) 0º (B) 180º (C) 135º (D) 45º

Sol. **[C,D]**

$|\vec{a}\times\vec{b}| = |\vec{a}\,.\,\vec{b}|$

$\Rightarrow |a\, b \sin\theta\ .\ \hat{n}| = |a\, b\cos\theta|$

$\Rightarrow ab\sin\theta = \pm\, a\, b\cos\theta$

$\Rightarrow \tan\theta = \pm 1$

$\Rightarrow \theta = 45^\circ, 135^\circ$

Option C, D are correct.

Q.24 If $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ then -

(A) $(\vec{a}-\vec{d}) = \lambda(\vec{b}-\vec{c})$

(B) $(\vec{a}+\vec{d}) = \lambda(\vec{b}+\vec{c})$

(C) $(\vec{a}-\vec{b}) = \lambda(\vec{c}+\vec{d})$

(D) none of these

Sol. **[A,B]**

$\therefore \vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

from options

(A) $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$= (\vec{a}\times\vec{b} - \vec{a}\times\vec{c}) - (\vec{d}\times\vec{b} - \vec{d}\times\vec{c})$

$= (\vec{a}\times\vec{b} + \vec{d}\times\vec{c}) - (\vec{a}\times\vec{c} + \vec{d}\times\vec{b})$

$= (\vec{a}\times\vec{b} - \vec{c}\times\vec{d}) - (\vec{a}\times\vec{c} - \vec{b}\times\vec{d})$

$= 0$

$\Rightarrow \vec{a}-\vec{d}$ is $\parallel$ to $\vec{b}-\vec{c}$

$\Rightarrow \vec{a}-\vec{d} = \lambda(\vec{b}-\vec{c})$

(B) $(\vec{a}+\vec{d})\times(\vec{b}+\vec{c})$

$= (\vec{a}\times\vec{b} + \vec{a}\times\vec{c}) + (\vec{d}\times\vec{b} + \vec{d}\times\vec{c})$

$= (\vec{a}\times\vec{b} + \vec{d}\times\vec{c}) + (\vec{a}\times\vec{c} + \vec{d}\times\vec{b})$

$= (\vec{a}\times\vec{b} - \vec{c}\times\vec{d}) + (\vec{a}\times\vec{c} - \vec{b}\times\vec{d})$

$= 0$

$\Rightarrow \vec{a}+\vec{d} = \lambda(\vec{b}+\vec{c})$

$\Rightarrow$ Option A, B are correct.

Q.25 The scalar $\vec{A}\,.(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$ equals

(A) 0 (B) $[\vec{A}\ \vec{B}\ \vec{C}] + [\vec{B}\ \vec{C}\ \vec{A}]$

(C) $[\vec{A}\ \vec{B}\ \vec{C}]$ (D) None of these

Sol. **[A]**

$\vec{A}\,.(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$

$(\vec{A}\,.\,\vec{B} + \vec{A}\,.\,\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$

$= \vec{A}\,.(\vec{B}\times\vec{A}) + \vec{A}\,.\,(\vec{B}\times\vec{B}) + \vec{A}\,.(\vec{B}\times\vec{C}) +$

$\vec{A}\,.(\vec{C}\times\vec{A}) + A.(\vec{C}\times\vec{B}) + \vec{A}\,.(\vec{C}\times\vec{C})$

$= 0 + 0 + [A\ B\ C] + 0 - [A\ B\ C] + 0 = 0$

Q.26 The adjacent sides of a parallelogram are represented by the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ respectively. The unit vectors parallel to the diagonals of the parallelogram are

(A) $\frac{(-\hat{i}-2\hat{j}+8\hat{k})}{\sqrt{69}}$ (B) $\frac{(-3\hat{i}+6\hat{j}-2\hat{k})}{7}$

(C) $\frac{(3\hat{i}+6\hat{j}-2\hat{k})}{7}$ (D) $\frac{(\hat{i}+2\hat{j}-8\hat{k})}{\sqrt{69}}$

Sol. **[A, C, D]**

Let the adjacent sides at parallelogram is

$AB = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $BC = \hat{i} + 2\hat{j} + 3\hat{k}$

Then the diagonal are given by

$AB - BC = \hat{i} + 2\hat{j} - 8\hat{k}$ ....(i)

and $AB + BC = 3\hat{i} + 6\hat{j} - 2\hat{k}$ ....(ii)

Unit vector parallel to the diagonal (i) & (ii) are

$\pm\frac{\hat{i}+2\hat{j}-8\hat{k}}{\sqrt{69}}$ and $\pm\frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$

Option A, C and D are correct.

Q.27 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar vectors, then the following vectors are coplanar -

(A) $\vec{a}+2\vec{b}+3\vec{c}, -2\vec{a}+3\vec{b}-4\vec{c}$, $\vec{a}-3\vec{b}+5\vec{c}$

(B) $3\vec{a}-7\vec{b}-4\vec{c}, 3\vec{a}-2\vec{b}+\vec{c}, \vec{a}+\vec{b}+2\vec{c}$

(C) $\vec{a}-2\vec{b}+3\vec{c}, -2\vec{a}+3\vec{b}-4\vec{c}, -\vec{b}+2\vec{c}$

(D) $7\vec{a}-8\vec{b}+9\vec{c}, 3\vec{a}+20\vec{b}+5\vec{c}$, $5\vec{a}+6\vec{b}+7\vec{c}$

Sol. **[B, C, D]**

Q.28 If a vector $\vec{r}$ of magnitude $3\sqrt{6}$ is directed along the bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ then $\vec{r}$ =

(A) $\hat{i}-7\hat{j}+2\hat{k}$ (B) $\hat{i}+7\hat{j}-2\hat{k}$

(C) $-\hat{i}+7\hat{j}-2\hat{k}$ (D) $\hat{i}-7\hat{j}-2\hat{k}$

Sol. **[A,C]**

$\hat{a}=\frac{1}{9}(7\hat{i}-4\hat{j}-4\hat{k})$

$\hat{b}=\frac{1}{3}(-2\hat{i}-\hat{j}+2\hat{k})$

$\vec{r}=t[\hat{a}+\hat{b}]$

$=t\left(\frac{7\hat{i}-4\hat{j}-4\hat{k}}{9}+\frac{-2\hat{i}-\hat{j}+2\hat{k}}{3}\right)$

$=t\left(\frac{\hat{i}-7\hat{j}+2\hat{k}}{9}\right)$ ....(i)

$\therefore |\vec{r}|=3\sqrt{6}$

$\Rightarrow |\vec{r}|=\frac{t}{9}|\hat{i}-7\hat{j}+2\hat{k}|$

$\Rightarrow \frac{t^2}{81}(1+49+4)=54$

$\Rightarrow t=\pm 9$

From (i) $\vec{r}=\pm(\hat{i}-7\hat{j}+2\hat{k})$

Option A, C are correct.

Q.29 The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$

(A) is a unit vector

(B) makes an angle $\frac{\pi}{3}$ with vector $(2\hat{i}-4\hat{j}+3\hat{k})$

(C) is parallel to the vector $\left(-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}\right)$

(D) is perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

Sol. **[A,C,D]**

(A) Let $\vec{a}=\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$

$|\vec{a}|^2=\frac{9}{9}=1$

$\Rightarrow \vec{a}$ is a unit vector

(B) $\cos\theta=\frac{\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k}).(2\hat{i}-4\hat{j}+3\hat{k})}{\frac{1}{3}\sqrt{4+4+1}\sqrt{4+16+9}}$

$\cos\theta=\frac{15}{3\sqrt{29}} \Rightarrow \theta=\cos^{-1}\left(\frac{5}{\sqrt{29}}\right)$

(C) $\because -\hat{i}+\hat{j}-\frac{1}{2}\hat{k}=-\frac{2}{3}(2\hat{i}-2\hat{j}+\hat{k})$

Clearly it is || to $\vec{a}$

(D) $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k}).(3\hat{i}+2\hat{j}-2\hat{k})$

$=\frac{1}{3}(6-4-2)=0$

$\vec{a}$ is $\perp$ to $(3\hat{i}+2\hat{j}-2\hat{k})$

$\Rightarrow$ option A,C,D are correct.

Part-C Assertion-Reason type questions

The following questions 30 to 33 consists of two statements each, printed as (Assertion) Statement-1 and Reason (Statement-2). While answering these questions you are to choose any one of the following four responses.

(A) If both Statement-1 and Statement-2 are true and the Statement-2 is correct explanation of the Statement-1.

(B) If both Statement-1 and Statement-2 are true but Statement-2 is not correct explanation of the Statement-1.

(C) If Statement-1 is true but the Statement-2 is false.

(D) If Statement-1 is false but Statement-2 is true

Q.30 **Statement-1 :** A vector $\vec{c}$, directed along the internal bisector of the angle between the vector $\vec{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ & $\vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, with $|\vec{c}| = 5\sqrt{6}$ is $\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$.

Statement-2 : The vector bisecting the angle of $\vec{a}$ & $\vec{b}$ is given by $\vec{c} = \lambda\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)$ $\forall\ \lambda < 0$.

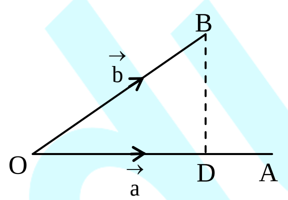
Sol. **[B]**

Q.31 **Statement-1 :** The components of a vector $\vec{b}$ along and perpendicular to a non-zero vector $\vec{a}$ are $\left(\frac{(\vec{b}.\vec{a})\vec{a}}{|\vec{a}|^2}\right)$ & $\left(\frac{\vec{a}\times(\vec{b}\times\vec{a})}{|\vec{a}|^2}\right)$ respectively.

Statement-2 : If $\vec{A}.\vec{B}$ and $\vec{C}$ are three non-coplanar vectors then

$$\frac{\vec{A}.(\vec{B}\times\vec{C})}{\vec{C}\times\vec{A}.\vec{B}} + \frac{\vec{B}\cdot\vec{A}\times\vec{C}}{\vec{C}\cdot\vec{A}\times\vec{B}} = 2$$

Sol. **[C]**



component of $\vec{b}$ along $\vec{a}$

$= \vec{OD} = (OB.\cos\theta).\ \hat{a}$

$= \left(|\vec{b}|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right).\frac{\vec{a}}{|\vec{a}|}$

$= \left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{a}$

and component $\vec{b}$ perpendicular to $\vec{a}$ is

$= \vec{DB} = \vec{b} - OD$

$= \vec{b} - \frac{(\vec{a}.\vec{b})\vec{a}}{|\vec{a}|^2}$

$\Rightarrow \frac{(\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}}{|\vec{a}|^2}$

$\Rightarrow \frac{\vec{a}\times(\vec{b}\times\vec{a})}{|\vec{a}|^2}$

Clearly assertion is true but reason is false

$\Rightarrow$ C is correct.

Q.32 **Statement-1:** Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} + 5\vec{b} + 3\vec{c} = 0$, then $\vec{a}.(\vec{b}\times\vec{c}) = \vec{b}.(\vec{a}+\vec{c})$.

Statement-2 : Box product of three coplanar vectors is 0.

Sol. **[D]**

Q.33 **Statement-1:** For non-coplanar vectors $\vec{A}, \vec{B}$ and $\vec{C}$, $|[\vec{A}\ \vec{B}\ \vec{C}]| = |\vec{A}||\vec{B}||\vec{C}|$ holds iff $\vec{A}.\vec{B} = \vec{B}.\vec{C} = \vec{C}.\vec{A} = 0$.

Statement-2 : $|\vec{A}\times\vec{B}.\vec{C}| = |A||B||C|\sin\theta$. $\cos\phi$ where θ be angle between $\vec{A}$ and $\vec{B}$ and ϕ the angle between $\vec{C}$ and $\vec{A}\times\vec{B}$.

Sol. Statement (2) is correct explanation of statement (1)

Part-D Column Matching type questions

Q.34 Match the following :

Column-I	**Column-II**
(A) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, then t, such that $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$, will be	(P) –1
(B) If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a}\times\vec{b}| = 8$, then $\vec{a}.\vec{b} =$	(Q) 4
(C) If four point A (1, 0, 3), B(–1, 3, 4), C(1, 2, 1) and	(R) 5

D(k, 2, 5) are coplanar, then k =

(D) If A, B, C, and D are four points and (S) 6

$|\overrightarrow{AB} \times \overrightarrow{CD} + \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}|$

= λ (area of the ΔBAC), then λ =

Sol. (A) **[R]**

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

$\vec{a} + t\vec{b} = (1-t)\hat{i} + (2+2t)\hat{j} + (3+t)\hat{k}$

Θ $\vec{a} + t\vec{b} \perp \vec{c}$

$\Rightarrow (\vec{a} + t\vec{b}) . \vec{c} = 0$

$\Rightarrow 3(1-t) + (2+2t) = 0$

$\Rightarrow t = 5$

(B) **[S]**

$\therefore |\vec{a} \times \vec{b}| = 8 \Rightarrow ||\vec{a}||\vec{b}|\sin\theta.\ \hat{n}| = 8$

$\Rightarrow 10|\sin\theta| = 8 \Rightarrow \sin\theta = \frac{4}{5}$ ....(i)

and $\vec{a} . \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 10.\ \frac{3}{5}$ from (i)

$\Rightarrow \vec{a} . \vec{b} = 6$

(C) **[P]**

$\vec{A} = \hat{i} + 3\hat{k}$

$\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{D} = k\hat{i} + 2\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -2\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 2\hat{i} - \hat{j} - 3\hat{k}$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = (k-1)\hat{i} + 4\hat{k}$

If $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are coplanar then $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{CD}$ also coplanar

$\Rightarrow [\overrightarrow{AB}\ \overrightarrow{BC}\ \overrightarrow{CD}] = 0$

$$\Rightarrow \begin{vmatrix} -2 & 3 & 1 \\ 2 & -1 & -3 \\ k-1 & 0 & 4 \end{vmatrix} = 0$$

$\Rightarrow -2(-4) - 3(8 + 3(k-1)) + (k-1) = 0$

$\Rightarrow 8 - 24 - 9k + 9 + k - 1 = 0$

$\Rightarrow k = -1$

(D) **[Q]**

Let $\vec{D}$ is origin and position vector of $\vec{A}, \vec{B}, \vec{C}$ are $\vec{a}, \vec{b}, \vec{c}$ Respectively we have

$|\overrightarrow{AB} \times \overrightarrow{CD} + \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}|$

$= |(\vec{b} - \vec{a}) \times (-\vec{c}) + (\vec{c} - \vec{b}) \times (-\vec{a}) + (\vec{a} - \vec{c}) \times (-\vec{b})|$

$= 2|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

= 2 [2 (area of ΔABC)]

= 4 (area of ΔABC)

$\Rightarrow \lambda = 4$

Q.35 For any three given vectors $\vec{a}, \vec{b}$ and $\vec{c}$, match the following column :

Column-I	**Column-II**
(A) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$, $\vec{c} = 10\hat{j} - \hat{k}$ and $\vec{a} \times (\vec{b} \times \vec{c}) = u\vec{a} + v\vec{b} + w\vec{c}$, then u =	(P) 0
(B) Volume of the tetrahedron whose vertices are the points with position vectors $\hat{i} - 6\hat{j} + 10\hat{k}, -\hat{i} - 3\hat{j} + 7\hat{k}$,	(Q) 1

$5\hat{i} - \hat{j} + \lambda\hat{k}$ and $7\hat{i} - 4\hat{j} + 7\hat{k}$

is 11 (units)^3 then λ =

(C) Given two vectors (R) 7

$\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$,

$\vec{b} = -2\hat{i} + 2\hat{j} - \hat{k}$ and

$\lambda = \dfrac{\text{the projection of } \vec{a} \text{ on } \vec{b}}{\text{the projection of } \vec{b} \text{ on } \vec{a}}$,

then the value of 3λ is

(D) Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that (S) $|\vec{a}||\vec{b}||\vec{c}|$

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then (T) 2

$\lambda\, \vec{b} \times \vec{a} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$,

where λ is equal to

Sol. **A → P; B → P, R; C → R; D → T**

(A)

$\vec{a} \times (\vec{b} \times \vec{c}) = u\vec{a} + v\vec{b} + w\vec{c}$

$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = u\vec{a} + v\vec{b} + w\vec{c}$

Put value compare on R.H.S and L.H.S.

(B)

P.V of A = $\hat{i} - 6\hat{j} + 10\hat{k}$ $\quad \overrightarrow{AB} = -2\hat{i} + 3\hat{j} - 3\hat{k}$

P.V of B = $-\hat{i} - 3\hat{j} + 7\hat{k}$ $\quad \overrightarrow{AC} = 4\hat{i} + 5\hat{j} + (\lambda - 10)\hat{k}$

P.V of C = $5\hat{i} - \hat{j} + \lambda\hat{k}$ $\quad \overrightarrow{AD} = 6\hat{i} + 2\hat{j} - 3\hat{k}$

P.V of D = $7\hat{i} - 4\hat{j} + 7\hat{k}$

$$\left|\frac{1}{6}[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]\right| = 11$$

$$\frac{1}{6}\begin{vmatrix} -2 & 3 & -3 \\ 4 & 5 & \lambda - 10 \\ 6 & 2 & -3 \end{vmatrix} = \pm 11$$

Then $\lambda = 0$

$\lambda = 7$

(C)

$$\lambda = \frac{\frac{\vec{a}.\vec{b}}{|\vec{b}|}}{\frac{\vec{a}.\vec{b}}{|\vec{a}|}} \Rightarrow \lambda = \frac{|\vec{a}|}{|\vec{b}|} = \frac{7}{3}$$

So $3\lambda = 7$

(D)

$\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{c} = -\vec{b}$ $\qquad \vec{b} + \vec{c} = -\vec{a}$

$\vec{b} \times \vec{a} + \vec{b} \times \vec{c} = 0$ …(1) $\qquad \vec{b} \times \vec{a} + \vec{c} \times \vec{a} = 0$ ….(2)

add (1) & (2)

$2(\vec{b} \times \vec{a}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = 0$

$\Rightarrow \lambda = 2$