Trigonometric Ratio
EXERCISE # 1

Questions based on — Relation between systems of measurement of angles

Q.1 The angle subtended at the centre of the circle of diameter 50 cm by an arc of 11 cm, is (in degree)

(A) 22º 10' (B) 23º 10'
(C) 20º 12' (D) 25º 12'

Sol.[D] Angle = $\frac{\text{arc}}{\text{radius}} = \frac{11}{25}$ radian

$= \frac{11}{25} \times \frac{180}{\pi}$ degree

$= \frac{11 \times 180 \times 7}{25 \times 22} = \frac{126}{5}$ = 25° 12′

Q.2 The angles of a triangle are in A.P. and the number of degrees in the least is to the number of radians in the greatest as 60 to π; then the angles in degree, are

(A) 24º, 60º, 96º (B) 30º, 60º, 90º
(C) 45º, 60º, 75º (D) None of these

Sol.[B] ∵ 2B = A + C ……….(i)
and A + B + C = 180° ⇒ B = 60°
Given that

$\frac{\angle A \text{ in degree}}{\angle C \text{ in radiun}} = \frac{60}{\pi}$

$\frac{\angle A.180}{\angle C.\pi} = \frac{60}{\pi}$

⇒ 3 ∠A = ∠C ………(ii)
from (i) & (ii)
4A = 120° ⇒ A = 30°
and C = 90°

Angles are 30°, 60°, 90°

Questions based on — Trigonometric ratio or functions

Q.3 If a cos θ – b sin θ = c, then a sin θ + b cos θ =

(A) $\pm\sqrt{a^2 - b^2 + c^2}$ (B) $\pm\sqrt{-a^2 + b^2 + c^2}$

(C) $\pm\sqrt{a^2 + b^2 - c^2}$ (D) None of these

Sol.[C] ∵ a cos θ – b sin θ = c

⇒ $a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$
⇒ $a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\cos\theta\sin\theta = c^2$
⇒ $a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = a^2 + b^2 - c^2$
⇒ $(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$
⇒ $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$

Q.4 If $\sin x + \sin^2 x = 1$, then
$\cos^8 x + 2\cos^6 x + \cos^4 x = ...$

(A) 0 (B) – 1
(C) 2 (D) 1

Sol. [D] $\sin x = \cos^2 x$

∴ $\cos^8 x + 2\cos^6 x + \cos^4 x = \sin^4 x + 2\sin^3 x + \sin^2 x$
$= (\sin x + \sin^2 x)^2 = 1$

Questions based on — Sign of Trigonometric ratio and allied angle

Q.5 If $\sin\theta = \frac{1}{\sqrt{2}}$ and $\frac{\pi}{2} < \theta < \pi$. Then the value of $\frac{\sin\theta + \cos\theta}{\tan\theta}$ is

(A) 0 (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Sol.[A] $\sin\theta = \frac{1}{\sqrt{2}}$ and $\frac{\pi}{2} < \theta < \pi$

then $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

So $\frac{\sin\theta + \cos\theta}{\tan\theta} = \frac{\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4}}{\tan\frac{3\pi}{4}}$

$= \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{-1} = 0$

Q.6 The expression

$3\left[\sin^4\left\{\frac{3}{2}\pi - \alpha\right\} + \sin^4(3\pi + \alpha)\right]$

$- 2\left[\sin^6\left(\frac{1}{2}\pi + \alpha\right) + \sin^6(5\pi - \alpha)\right]$ is equal to-

(A) 0 (B) 1

(C) 3 (D) sin 4α + cos 6α

Sol.[B] $3\left[\sin^4\left\{\frac{3\pi}{2}-\alpha\right\}+\sin^4(3\pi+\alpha)\right]$

$-2\left[\sin^6\left\{\frac{\pi}{2}+\alpha\right\}+\sin^6(5\pi-\alpha)\right]$

$= 3\ [\cos^4\alpha+\sin^4\alpha]-2[\cos^6\alpha+\sin^6\alpha]$

$= 3\ [1-2\sin^2\alpha\cos^2\alpha]-2[1-3\sin^2\alpha\cos^2\alpha]$

$= 3 - 2 = 1$

Q.7 If $\frac{\pi}{2} < \alpha < \pi$, $\pi < \beta < \frac{3\pi}{2}$; $\sin\alpha = \frac{15}{17}$ and $\tan\beta = \frac{12}{5}$, the value of sin (β – α) is -

(A) $\frac{-21}{221}$ (B) $\frac{21}{221}$ (C) $\frac{-171}{221}$ (D) $\frac{171}{221}$

Sol.[D] $\sin\alpha = \frac{15}{17}$, $\cos\alpha = -\frac{8}{17}$, $\tan\beta = \frac{12}{5}$,

$\sin\beta = \frac{-12}{13}$, $\cos\beta = -\frac{5}{13}$

So $\sin(\beta-\alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha$

$= \left(-\frac{12}{13}\right)\left(-\frac{8}{17}\right) - \left(-\frac{5}{13}\right)\left(\frac{15}{17}\right)$

$= \frac{96}{221}+\frac{75}{221}=\frac{171}{221}$

Questions based on Sum or Difference of angles

Q.8 $\frac{\tan^2 2\theta - \tan^2\theta}{1-\tan^2 2\theta\tan^2\theta} =$

(A) tan 3θ / tan θ (B) cot 3θ / cot θ

(C) tan 3θ tan θ (D) cot 3θ cot θ

Sol.[C] Expression we can written as

$\frac{\tan 2\theta - \tan\theta}{1+\tan 2\theta\tan\theta}\cdot\frac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}$

= tan(2θ – θ). tan (2θ + θ)

= tan θ. tan 3θ

Q.9 If tan A = 1/3 and tan B = 1/7 then the value of 2A + B is -

(A) 30º (B) 60º

(C) 45º (D) 145º

Sol.[C] ∵ $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2/3}{1-1/9} = \frac{3}{4}$

$\tan(2A+B) = \frac{\tan 2A+\tan B}{1-\tan 2A\tan B}$

$= \frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}} = \frac{21+4}{28-3}$

tan (2A + B) = 1 ⇒ 2A + B = 45°

Questions based on Transformation of product into sum & difference

Q.10 cos 20º cos 100º + cos 100º cos 140º – cos 140º cos 200º =

(A) 0 (B) 3/4

(C) –3/4 (D) – 4/3

Sol.[C] $\cos 20°\cos 100° + \cos 100°\cos 140° - \cos 140°\cos 200°$

$= \cos 20°\cos 100° - \cos 100°\cos 40° - \cos 40°\cos 20°$

$= \frac{1}{2}\left[\cos 120° + \cos 80° - \cos 140° - \cos 60° - \cos 60° - \cos 20°\right]$

$= \frac{1}{2}\left[-\frac{3}{2} + 2\cos 60°\cos 20° - \cos 20°\right] = -\frac{3}{4}$

Q.11 $2\sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Sol.[B] $2\sin\left(\frac{\pi}{2}-\frac{\pi}{12}\right)\sin\frac{\pi}{12}$

$= 2\cos\frac{\pi}{12}\sin\frac{\pi}{12} = \sin\frac{2\pi}{12} = \sin\frac{\pi}{6} = \frac{1}{2}$

Questions based on Transformation of sum or difference into product

Q.12 $\frac{\cos 12° - \sin 12°}{\cos 12° + \sin 12°} + \frac{\sin 147°}{\cos 147°} =$

(A) 1 (B) –1

(C) 0 (D) None

Sol.[C] $\frac{\cos 12° - \sin 12°}{\cos 12° + \sin 12°} + \frac{\sin 147°}{\cos 147°}$

$= \frac{\cos 12° - \cos 78°}{\cos 12° + \cos 78°} + \frac{\sin(\pi - 33°)}{\cos(\pi - 33°)}$

$= \frac{2\sin 45° \sin 33°}{2\cos 45° \cos 33°} - \frac{\sin 33°}{\cos 33°} = 0$

Q.13 If m sin θ = n sin (θ + 2α),

then tan (θ + α) cot α =

(A) $\frac{1-n}{1+n}$ (B) $\frac{m+n}{m-n}$

(C) $\frac{m-n}{m+n}$ (D) None

Sol.[B] m sin θ = n sin (θ + 2α)

$\Rightarrow \frac{\sin(\theta + 2\alpha)}{\sin\theta} = \frac{m}{n}$

use componendo & devidendo then

$\Rightarrow \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{m+n}{m-n}$

from C – D formula

$\Rightarrow \frac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha} = \frac{m+n}{m-n}$

$\Rightarrow \tan(\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$

Q.14 $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

when n is odd, is-

(A) $2\cot^n\left(\frac{A-B}{2}\right)$ (B) zero

(C) $2\tan^n\left(\frac{A-B}{2}\right)$ (D) None

Sol.[B] $\left(\frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right)^n +$

$\left(\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right)^n$

$= \left[\cot\left(\frac{A-B}{2}\right)\right]^n + \left[-\cot\left(\frac{A-B}{2}\right)\right]^n$

= 0 [∵ n is odd]

Questions based on Trigonometric Ratio of multiple angles

Q.15 $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta)$ is equal to

(A) sin 2α (B) cos 2β

(C) cos 2α (D) sin 2β

Sol.[C] $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + 2\cos^2(\alpha+\beta) - 1$

$= 2\sin^2\beta + 2\cos(\alpha+\beta)$

$[2\sin\alpha\sin\beta + \cos(\alpha+\beta)] - 1$

$= 2\sin^2\beta + 2\cos(\alpha+\beta)\cos(\alpha-\beta) - 1$

$= 2\sin^2\beta + 2\cos^2\alpha - 2\sin^2\beta - 1$

$= 2\cos^2\alpha - 1 = \cos 2\alpha$

Q.16 If tanβ = cosθ.tanα, then $\tan^2\left(\frac{\theta}{2}\right) =$

(A) $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ (B) $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$

(C) $\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$ (D) $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

Sol.[C] We know that $\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \cos\theta$

Given that $\cos\theta = \frac{\tan\beta}{\tan\alpha}$

$\Rightarrow \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \frac{\tan\beta}{\tan\alpha}$

using componendo & dividendo then solve we get

$\tan^2\theta/2 = \dfrac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$

Q.17 $\dfrac{1-\cos 4\alpha}{\sec^2 2\alpha - 1} + \dfrac{1+\cos 4\alpha}{\operatorname{cosec}^2 2\alpha - 1}$ is equal to -

(A) 0 (B) 2

(C) 1 (D) 4

Sol.[B] $\dfrac{1-\cos 4\alpha}{\sec^2 2\alpha - 1} + \dfrac{1+\cos 4\alpha}{\operatorname{cosec}^2 2\alpha - 1}$

$= \dfrac{2\sin^2 2\alpha .\cos^2 2\alpha}{1-\cos^2 2\alpha} + \dfrac{2\cos^2 2\alpha . \sin^2 2\alpha}{1-\sin^2 2\alpha}$

$= 2\cos^2 2\alpha + 2\sin^2 2\alpha = 2$

Questions based on

Conditional Trig. identities & Greatest and least value of the expression

Q.18 In a Δ ABC, $\dfrac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} =$

(A) $8 \sin\dfrac{A}{2} \sin\dfrac{B}{2} \sin\dfrac{C}{2}$

(B) $8 \cos\dfrac{A}{2} \cos\dfrac{B}{2} \cos\dfrac{C}{2}$

(C) $8 \tan\dfrac{A}{2} \tan\dfrac{B}{2} \tan\dfrac{C}{2}$

(D) $8 \cot\dfrac{A}{2} \cot\dfrac{B}{2} \cot\dfrac{C}{2}$

Sol.[A] $\dfrac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C}$

$= \dfrac{2\sin(A+B)\cos(A-B) + 2\sin C\cos C}{2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right) + 2\sin\dfrac{C}{2}\cos\dfrac{C}{2}}$

$= \dfrac{\sin C\,[\cos(A-B) - \cos(A+B)]}{\cos\dfrac{C}{2}\left[\cos\left(\dfrac{A-B}{2}\right) + \cos\left(\dfrac{A+B}{2}\right)\right]}$

$= \dfrac{2\sin A\ \sin B\ \sin C}{2\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}}$

$= 8\sin\dfrac{A}{2}\ \sin\dfrac{B}{2}\ \sin\dfrac{C}{2}$

Q.19 Maximum value of

(5 sin x – 12 cos x) (5 cos x + 12 sin x) is

(A) $\dfrac{169}{2}$ (B) $\dfrac{169}{4}$ (C) $\dfrac{196}{2}$ (D) 1

Sol.[A] (5 sin x – 12 cos x) (5 cos x + 12 sin x)

$\Rightarrow 13\sin(x-\alpha).\ 13\cos(x-\alpha)$

$\Rightarrow \dfrac{169}{2}\sin 2(x-\alpha)$

maximum value = $\dfrac{169}{2}$

Q.20 The least value of

$[\cos^2\theta - 6\sin\theta.\cos\theta + 3\sin^2\theta + 2]$ is-

(A) $4 + \sqrt{10}$ (B) $4 - \sqrt{10}$

(C) 0 (D) 4

Sol.[B] $\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$

$= 2\sin^2\theta - 6\sin\theta\cos\theta + 3 = 2\sin^2\theta - 3\sin^2\theta + 3$

$= 2\left(\dfrac{1-\cos 2\theta}{2}\right) - 3\sin 2\theta + 3$

$= 1 - \cos 2\theta - 3\sin 2\theta + 3 = 4 - (3\sin 2\theta + \cos 2\theta)$

Θ $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

$\therefore -\sqrt{10} \le 3\sin 2\theta + \cos 2\theta \le \sqrt{10}$

∴ for least value, we take

$3\sin 2\theta + \cos 2\theta \le \sqrt{10}$

∴ least value is given by $\Rightarrow 4 - \sqrt{10}$

Questions based on

Miscellaneous Points

Q.21 $96\sqrt{3}\ \sin\dfrac{\pi}{48}.\cos\dfrac{\pi}{48}.\cos\dfrac{\pi}{24}.\cos\dfrac{\pi}{12}.\cos\dfrac{\pi}{6}$ has the value

(A) 8 (B) 9

(C) 10 (D) None

Sol.[B] Θ 2sin x cos x = sin 2x

So $96\sqrt{3}\ \sin\dfrac{\pi}{48}\cos\dfrac{\pi}{48}\cos\dfrac{\pi}{24}\cos\dfrac{\pi}{12}\cos\dfrac{\pi}{6}$

$\Rightarrow 48\sqrt{3}\ \sin\dfrac{\pi}{24}\cos\dfrac{\pi}{24}\cos\dfrac{\pi}{12}\cos\dfrac{\pi}{6}$

$\Rightarrow 24\sqrt{3}\ \sin\dfrac{\pi}{12}\cos\dfrac{\pi}{12}\cos\dfrac{\pi}{6}$

$\Rightarrow 12\sqrt{3}\sin\dfrac{\pi}{6}\cos\dfrac{\pi}{6}$

$\Rightarrow 6\sqrt{3}\sin\frac{\pi}{3} = 6\sqrt{3}.\frac{\sqrt{3}}{2} = 9$

Q.22 The value of tan 6° tan 42° tan 66° tan 78° is-

(A) 1 (B) 1/2

(C) 1/4 (D) 1/8

Sol.[A] We know that

$\tan\theta.\tan(60-\theta)\tan(60+\theta) = \tan 3\theta$

So tan 6° tan 42° tan 66° tan 78°

$\Rightarrow \frac{(\tan 6°\tan 54°\tan 66°)(\tan 18°\tan 42°\tan 78°)}{\tan 18°\tan 54°}$

$\Rightarrow \frac{\tan 18°\tan 54°}{\tan 18°\tan 54°} = 1$

Q.23 The value of $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n}$ +.....to n terms is equal to

(A) 1 (B) 0

(C) $\frac{n}{2}$ (D) none

Sol.[B] $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n}$ +........ to n terms

$= \sin\frac{\pi}{n} + \sin\left(\frac{\pi}{n}+\frac{2\pi}{n}\right) + \sin\left(\frac{\pi}{n}+\frac{4\pi}{n}\right) + \ldots$ ...n terms

use $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots + \sin(\alpha+\overline{(n-1)}\beta)$

where $\alpha = \frac{\pi}{n}$ and $\beta = \frac{2\pi}{n}$

$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\frac{2\alpha+(n-1)\beta}{2}$

$= \frac{\sin\frac{n.\frac{2\pi}{n}}{2}}{\sin\frac{2\pi/n}{2}}\sin\frac{2.\frac{\pi}{n}+(n-1)\frac{2\pi}{n}}{2}$

$= \frac{\sin\pi}{\sin\frac{\pi}{n}}\sin\frac{\frac{2\pi}{n}+(n-1)\frac{2\pi}{n}}{2}$

$= 0 \times (\ldots''\ldots) = 0$

True or false type questions

Q.24 If sec (α – β), sec α and sec (α + β) are in A.P., then cos α sec (β/2) = $\sqrt{2}$.

Sol. sec (α – β), sec α, sec (α + β) are in A.P.

so 2 sec α = sec (α –β) + sec (α + β)

$2\sec\alpha = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)\cos(\alpha+\beta)}$

$2\sec\alpha = \frac{2\cos\alpha.\cos\beta}{(\cos^2\alpha-\sin^2\beta)}$

$\Rightarrow \cos^2\alpha - \sin^2\beta = \cos^2\alpha\cos\beta$

$\sin^2\beta = \cos^2\alpha(1-\cos\beta)$

$\left(2\sin\frac{\beta}{2}\cos\frac{\beta}{2}\right)^2 = \cos^2\alpha.\ 2\sin^2\beta/2$

$\cos^2\alpha = 2\cos^2\beta/2$

$\cos\alpha\sec\beta/2 = \sqrt{2}$ (True)

Q.25 If sinθ = sinϕ, then we always have sin 2θ = sin 2ϕ

Sol. sin θ = sin ϕ

then sin 2θ = sin 2ϕ

⇒ 2 sin θ cos θ = 2 sin ϕ cos ϕ

⇒ cos θ = cos ϕ is not necessary.

False.

Fill in the blanks type questions

Q.26 If α and β are the solutions of a cos θ + b sin θ = c, then

(i) sin α + sin β = ...

(ii) sin α sin β = ...

(iii) cos α + cos β = ...

(iv) cos α cos β = ...

(v) In a triangle ABC, ∠C = 90° then the equation whose roots are tan A and tan B is ......

Sol. Given a cos θ + b sin θ = c

a cos θ = c– b sin θ

$a^2(1-\sin^2\theta) = c^2 - 2bc\sin\theta + b^2\sin^2\theta$

$\Rightarrow (a^2+b^2)\sin^2\theta - 2bc\sin\theta + c^2 - a^2 = 0$

sin α, sin β are the roots so

(i) $\sin\alpha + \sin\beta = \frac{2bc}{a^2+b^2}$ and

(ii) $\sin\alpha\sin\beta = \frac{c^2-a^2}{a^2+b^2}$

Similarly change in cos θ, we get

$(a^2+b^2)\cos^2\theta - 2ac\cos\theta + c^2 - b^2 = 0$

cos α, cos β are the roots of this equation so

(iii) $\cos\alpha + \cos\beta = \frac{2ac}{a^2+b^2}$ and

(iv) $\cos\alpha\cos\beta = \frac{c^2-b^2}{a^2+b^2}$

(v) $\sin 2A\, x^2 - 2x + \sin 2A = 0$

EXERCISE # 2

Part-A Only single correct answer type questions

Q.1 Maximum value of

$\cos^3 x + \cos^3 (120^o - x) + \cos^3 (120^o + x)$ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 3/8

Sol.[C] $\because \cos^3 A = \frac{3\cos A + \cos 3A}{4}$

Given expression

$= \frac{1}{4}$[3cosx + cos3x + 3cos (120 – x) + cos (360 – 3x) + 3cos (120 + x) + cos (360 + 3x)]

$= \frac{1}{4}$[3 cosx + 3 cos3x + 6 cos120 cosx]

$= \frac{3}{4}$[cos3x]

maximum value = $\frac{3}{4}$

Q.2 $\cos^2\left(\frac{\pi}{16}\right) + \cos^2\left(\frac{3\pi}{16}\right) + \cos^2\left(\frac{5\pi}{16}\right) + \cos^2\left(\frac{7\pi}{16}\right)$

is equal to -

(A) 0 (B) 1 (C) 2 (D) 3

Sol.[C]

$\cos^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \cos^2\left(\frac{\pi}{2} - \frac{3\pi}{16}\right) + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{16}\right)$

$\Rightarrow \cos^2\frac{\pi}{16} + \sin^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \sin^2\frac{3\pi}{16}$

= 1 + 1 = 2

Q.3 If $\sin A = \frac{336}{625}$, where 450º < A < 540º, then $\sin\frac{A}{4}$ is -

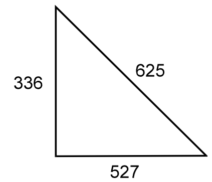
(A) 3/5 (B) – 3/5

(C) 4/5 (D) – 4/5

Sol.[C]

$\sin A = \frac{336}{625}$

450º < A < 540º



A lies in IInd quadrant

So $\cos A = -\frac{527}{625}$

$\sin A/2 = -\sqrt{\frac{1-\cos A}{2}}$

[A/2 lies in IIIrd quadrant]

$= -\sqrt{\frac{625+527}{2\times 625}} = -\frac{24}{25}$

So

$\cos A/2 = -\sqrt{1-\left(\frac{24}{25}\right)^2}$

$= -\sqrt{\frac{625-576}{625}}$ $\cos A/2 = -\frac{7}{25}$

$\because$ A/4 lies in $112.5 < \frac{A}{4} < 132^o$

[IInd quadrant]

So $\sin A/4 = \sqrt{\frac{1-\cos A/2}{2}}$

$= \sqrt{\frac{1+7/25}{2}} = \sqrt{\frac{25+7}{2\times 25}}$

$\sin\frac{A}{4} = \frac{4}{5}$

Q.4 $\sum_{r=1}^{n-1} \cos^2\left(\frac{r\pi}{n}\right)$ is equal to -

(A) $\frac{n}{2}$ (B) $\frac{n-1}{2}$

(C) $\frac{n}{2} - 1$ (D) None of these

Sol.[C]

$\sum_{r=1}^{n-1} \cos^2\frac{r\pi}{n}$

$= \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + ...+ \cos^2(n-1)\frac{\pi}{n}$

$= \frac{1}{2}\left[1+\cos\frac{2\pi}{n}+1+\cos\frac{4\pi}{n}+....+1+\cos 2(n-1)\frac{\pi}{n}\right]$

$$= \frac{1}{2}\left[(n-1) + \sum_{r=1}^{n-1} \cos\frac{2r\pi}{n}\right]$$

$$= \frac{1}{2}\,[(n-1)-1] \quad \Theta \sum_{r=1}^{n-1} \cos\frac{2r\pi}{n} = -1$$

$$= \frac{1}{2}\,(n-2)$$

Q.5 If an angle α is divided into two parts A and B such that A – B = x and tan A : tan B = k : 1, then the value of sin x is-

(A) $\left(\frac{k+1}{k-1}\right)\sin\alpha$ (B) $\left(\frac{k}{k+1}\right)\sin\alpha$

(C) $\left(\frac{k-1}{k+1}\right)\sin\alpha$ (D) None of these

Sol.[C]

α divided in two part A and B

so A + B = α and given A – B = x

Given that $\frac{\tan A}{\tan B} = \frac{k}{1}$

using componendo and dividendo, we obtain

$$\Rightarrow \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\sin(A+B)}{\sin(A-B)} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\sin\alpha}{\sin x} = \frac{k-1}{k+1}$$

$$\Rightarrow \sin x = \frac{k-1}{k+1}\sin\alpha$$

Q.6 If α, β, γ, δ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k, then the value of

$$4\sin\left(\frac{\alpha}{2}\right) + 3\sin\left(\frac{\beta}{2}\right) + 2\sin\left(\frac{\gamma}{2}\right) + \sin\left(\frac{\delta}{2}\right)$$

is equal to-

(A) $2\sqrt{1-k}$ (B) $2\sqrt{1+k}$

(C) $\frac{\sqrt{1+k}}{2}$ (D) None of these

Sol.[B]

$\alpha,\beta,\gamma,\delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k so

$\alpha = \alpha$, $\beta = \pi - \alpha$, $\gamma = 2\pi + \alpha$, $\delta = 3\pi - \alpha$

So $4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$

$$= 4\sin\frac{\alpha}{2} + 3\sin\left(\frac{\pi-\alpha}{2}\right) + 2\sin\left(\frac{2\pi+\alpha}{2}\right) + \sin\left(\frac{3\pi-\alpha}{2}\right)$$

$$= 4\sin\frac{\alpha}{2} + 3\cos\frac{\alpha}{2} - 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}$$

$$= 2\sin\frac{\alpha}{2} + 2\cos\frac{\alpha}{2}$$

$$= 2\sqrt{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$

$$= 2\sqrt{1+\sin\alpha}$$

given that $\sin\alpha = k$ so $= 2\sqrt{1+k}$

Q.7 If $\tan\left(\frac{\alpha}{2}\right)$ and $\tan\left(\frac{\beta}{2}\right)$ are the roots of the equation $8x^2 - 26x + 15 = 0$ then $\cos(\alpha + \beta)$ is equal to-

(A) $-\frac{627}{725}$ (B) $\frac{627}{725}$

(C) $-\frac{725}{627}$ (D) – 1

Sol.[A]

$\tan\frac{\alpha}{2}$, $\tan\frac{\beta}{2}$ are the roots of the equation

$8x^2 - 26x + 15 = 0$

so $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{26}{8} = \frac{13}{4}$

$\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{15}{8}$

$$\Theta \tan\frac{\alpha+\beta}{2} = \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}}{1 - \tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$$

$$= \frac{\frac{13}{4}}{1-\frac{15}{8}} = \frac{13\times 8}{4(8-15)} = -\frac{26}{7}$$

$$\cos(\alpha+\beta) = \frac{1-\tan^2\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}}$$

$$= \frac{1-\frac{676}{49}}{1+\frac{676}{49}} = \frac{49-676}{49+676} = -\frac{627}{725}$$

Q.8 cot 5º – tan 5º – 2 tan 10º – 4 tan 20º – 8 cot 40º is equal to -

(A) 0 (B) 8 tan 40º

(C) 8 tan 80º (D) None of these

Sol.[A]

We know that

cot θ – tan θ = 2 cot 2θ

so

= 2 cot 10º – 2 tan 10º – 4 tan 20º –8 cot 40º

= 4 cot 20º – 4 tan 20º – 8 cot 40º

= 8 cot 40º – 8 cot 40º = 0

Q.9 If 0º < θ < 180º then

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+...........+\sqrt{2(1+\cos\theta)}}}}}$$

there being n number of 2's , is equal to-

(A) $2\cos\frac{\theta}{2^n}$ (B) $2\cos\frac{\theta}{2^{n-1}}$

(C) $2\cos\frac{\theta}{2^{n+1}}$ (D) None of these

Sol.[A]

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+...........+\sqrt{2(1+\cos\theta)}}}}}$$

$$= \sqrt{2+\sqrt{2+...........+\sqrt{4\cos^2\theta/2}}}$$

∵ $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

$$= \sqrt{2+\sqrt{2+...........+\sqrt{2+2\cos\theta/2}}}$$

$$= \sqrt{2+\sqrt{2+...........+\sqrt{2(1+\cos\theta/2)}}}$$

$$= \sqrt{2+\sqrt{2+...........+\sqrt{4\cos^2\theta/2^2}}}$$

⋮

and so on we have

$$= \sqrt{2+2\cos\frac{\theta}{2^{n-1}}} = \sqrt{2\left(1+\cos\frac{\theta}{2^{n-1}}\right)}$$

$$= \sqrt{4\cos^2\frac{\theta}{2^n}} = 2\cos\frac{\theta}{2^n}$$

Q.10 The sign of the product sin 2 sin 3 sin 5 is -

(A) Negative (B) Positive

(C) 0 (D) None of these

Sol.[A]

We know that 1^c = 57º approx.

So sin 2 = sin (57º × 2)

= sin 114º +ve IInd quadrant

and sin 3 = sin (57º × 3)

= sin 171º +ve IInd quadrant

and sin 5 = sin (57º × 5)

= sin 285º –ve IVth quadrant

so product of sin 2. sin 3. sin 5 is

= product of +. +. – = –ve

so product of

sin 2 sin 3 sin 5 = –ve

Q.11 $\frac{\cos 20^\circ + 8\sin 70^\circ \sin 50^\circ \sin 10^\circ}{\sin^2 80^\circ}$ is equal to -

(A) 1 (B) 2

(C) $\frac{3}{4}$ (D) None of these

Sol.[B]

We know that

$$\sin\theta \sin(60+\theta)\sin(60-\theta) = \frac{1}{4}\sin 3\theta$$

so $$\frac{\cos 20^\circ + 8\sin 10^\circ \sin 70^\circ.\sin 50^\circ}{\sin^2 80^\circ}$$

$$= \frac{\cos 20^\circ + 8.\frac{1}{4}.\sin 30^\circ}{\sin^2 80^\circ}$$

$$= \frac{\cos 20^\circ + 2\sin 30^\circ}{\sin^2 80^\circ}$$

$$= \frac{\cos 20^\circ + 1}{\sin^2(90-10)} = \frac{2\cos^2 10^\circ}{\cos^2 10^\circ} = 2$$

Q.12 It is known that $\sin\beta = \frac{4}{5}$ and $0 < \beta < \pi$ then the value of $\frac{\sqrt{3}\sin(\alpha+\beta) - \frac{2}{\cos\pi/6}\cos(\alpha+\beta)}{\sin\alpha}$ is -

(A) Independent of α for all β in $(0, \pi)$

(B) $\frac{5}{13}$ for $\tan\beta < 0$

(C) $\frac{3(7+24\cot\alpha)}{15}$ for $\tan\beta > 0$

(D) None of these

Sol.[D]

Given that $\sin\beta = \frac{4}{5}$ so $\cos\beta = \frac{3}{5}$

then $\frac{\sqrt{3}\sin(\alpha+\beta) - \frac{2}{\sqrt{3}/2}\cos(\alpha+\beta)}{\sin\alpha}$

$= \frac{3(\sin\alpha\cos\beta + \cos\alpha\sin\beta) - 4(\cos\alpha\cos\beta - \sin\alpha\sin\beta)}{\sqrt{3}\sin\alpha}$

$= \frac{3\left(\frac{3}{5} + \cot\alpha.\frac{4}{5}\right) - 4\left(\cot\alpha.\frac{3}{5} - \frac{4}{5}\right)}{\sqrt{3}}$

$= \frac{9 + 12\cot\alpha - 12\cot\alpha + 16}{5\sqrt{3}}$

$= \frac{25}{5\sqrt{3}} = \frac{5}{\sqrt{3}}$

Q.13 In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $0 < k < 1$, then the measure of angle C is-

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Sol.[C]

Given equation is $\sin 3x = k$, $0 < k < 1$

since k lies between 0 and 1, the two values of 3x will be $(0, \pi)$ and will be supplementary angle so

$3A + 3B = \pi \Rightarrow A + B = \frac{\pi}{3}$

But $A + B + C = \pi \Rightarrow C = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q.14 If $A = \cos^2\theta + \sin^4\theta$, then for all values of θ

(A) $1 \le A \le 2$ (B) $13/16 \le A \le 1$

(C) $3/4 \le A \le 13/16$ (D) $3/4 \le A \le 1$

Sol.[D]

$A = \cos^2\theta + \sin^4\theta$

$\Rightarrow A = \cos^2\theta + \sin^2\theta.\sin^2\theta$

$A \le \cos^2\theta + \sin^2\theta$

$\because \sin^2\theta \le 1$

$A \le 1$ ...(1)

Again $A = \cos^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta + \sin^4\theta$

$= 1 + \sin^4\theta - \sin^2\theta + \frac{1}{4} - \frac{1}{4}$

$= \frac{3}{4} + (\sin^2\theta - 1/2)^2$

then $A \ge 3/4$ ... (2)

from (1) and (2)

$\frac{3}{4} \le A \le 1$

Q.15 The value of the expression $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{10\pi}{7} - \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$ is

(A) 0 (B) $-\frac{1}{4}$

(C) $\frac{1}{4}$ (D) $-\frac{1}{8}$

Sol.[B]

$\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{10\pi}{7} - \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$

$\Rightarrow \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\left(2\pi - \frac{4\pi}{7}\right)$

$-\sin\left(\frac{\pi}{2} - \frac{3\pi}{7}\right)\sin\left(\frac{\pi}{2} - \frac{2\pi}{7}\right)\sin\left(\frac{\pi}{2} - \frac{\pi}{7}\right)$

$\Rightarrow \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} - \cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}$

$\Rightarrow \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\left(\cos\frac{4\pi}{7} - \cos\frac{3\pi}{7}\right)$

$\Rightarrow -2\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\sin\frac{\pi}{2}\sin\frac{\pi}{14}$

$\Rightarrow \frac{-2\sin\frac{\pi}{14}\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{14}}{\cos\frac{\pi}{14}}$

$$\Rightarrow \frac{-\sin\frac{4\pi}{7}}{4\cos\frac{\pi}{14}} = \frac{-\sin\left(\pi - \frac{3\pi}{7}\right)}{4\cos\left(\frac{\pi}{2} - \frac{3\pi}{7}\right)}$$

$$= -\frac{\sin\frac{3\pi}{7}}{4\sin\frac{3\pi}{7}} = -\frac{1}{4}$$

Q.16 The minimum and maximum value of $ab\sin x + b\sqrt{(1-a^2)}\cos x + c$ ($|a| < 1$, $b > 0$) respectively are

(A) $\{b - c, b + c\}$ (B) $\{b + c, b - c\}$

(C) $\{c - b, b + c\}$ (D) None of these

Sol.[C]

$ab\sin x + b\sqrt{1-a^2}\cos x + c$

We know that the maximum and minimum value of $a\cos\theta + b\sin\theta$ is $\pm\sqrt{a^2+b^2}$

so $ab\sin x + b\sqrt{1-a^2}\cos x + c$

$\Rightarrow \pm\sqrt{a^2b^2 + b^2(1-a^2)} + c$

$= \pm\sqrt{a^2b^2 + b^2 - a^2b^2} + c = \pm b + c \Rightarrow c \pm b$

maximum value = c + b; minimum value = c – b

Q.17 $\dfrac{\sin 7x + 6\sin 5x + 17\sin 3x + 12\sin x}{\sin 6x + 5\sin 4x + 12\sin 2x} =$

(A) cos x (B) 2 cos x (C) sin x (D) 2 sin x

Sol.[B]

$$\frac{(\sin 7x + \sin 5x) + 5(\sin 5x + \sin 3x) + 12(\sin 3x + \sin x)}{\sin 6x + 5\sin 4x + 12\sin 2x}$$

Use C & D formula

$$\frac{2\sin 6x\cos x + 10\sin 4x\cos x + 24\sin 2x\cos x}{\sin 6x + 5\sin 4x + 12\sin 2x}$$

$$= 2\cos x\,\frac{(\sin 6x + 5\sin 4x + 12\sin 2x)}{(\sin 6x + 5\sin 4x + 12\sin 2x)}$$

= 2 cos x

Q.18 If product of sin 1° sin 3° sin 5°…. sin 89° $= \frac{1}{2^n}$ then n equals

(A) 44 (B) $\frac{89}{2}$

(C) 45 (D) None

Sol. [B] sin 1° . sin 3° sin 5° ...... sin 89°

= sin 1° sin 3° sin 5° .... sin 59° sin 61° sin 63° .... sin 87° sin 89°

= (sin1° sin 59° sin 61°) (sin 3° sin 57° sin 63°) ...... (sin 29° sin 31° sin 89°)

$= \frac{\sin 3^\circ}{4}\cdot\frac{\sin 9^\circ}{4}$ .... $\frac{\sin 87^\circ}{4}$ [use $\sin\theta\sin(60-\theta)\sin(60+\theta) = \frac{\sin 3\theta}{4}$]

$= \frac{1}{4^{15}}$ (sin 3° sin 57° sin 63°) (sin 9° sin 51° sin 69°) ...... (sin 27° sin 33° sin 87°)

$= \frac{1}{2^{30}}\cdot\frac{\sin 9^\circ}{4}\cdot\frac{\sin 27^\circ}{4}\cdot\ ......\ \frac{\sin 81^\circ}{4}$

$= \frac{1}{2^{40}}$ [sin 9° sin 27° sin 45° sin 63° sin 81°]

$= \frac{1}{2^{40}}$ [(sin 9° cos 9°)(sin 27° cos 27°)] $\frac{1}{\sqrt{2}}$

$= \frac{1}{2^{42}}$ [sin 18° sin 54°] $\times \frac{1}{\sqrt{2}}$

$= \frac{1}{2^{42}} \times \left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right) \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2^{89/2}}$

Q.19 If $x \in \left(\pi, \frac{3\pi}{2}\right)$ then

$4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$ equals -

(A) 2 (B) –2

(C) 3 (D) – 3

Sol. [A] $4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$

$\Rightarrow 4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^4 x + 4\sin^2 x\cos^2 x}$

$\Rightarrow 2.2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^2 x(\sin^2 x + \cos^2 x)}$

$\Rightarrow 2\left[1 + \cos\left(\frac{\pi}{4}\times 2 - \frac{x}{2}\times 2\right)\right] + \sqrt{4\sin^2 x}$

$\Rightarrow 2\left[1 + \cos\left(\frac{\pi}{2} - x\right)\right] + |2\sin x|$

$\Rightarrow 2[1 + \sin x] - 2\sin x$ $\left\{x \in \left(\pi, \frac{3\pi}{2}\right)\right\}$

$\Rightarrow 2 + 2\sin x - 2\sin x = 2$

Q.20 $1 + \text{cosec}\,\frac{\pi}{4} + \text{cosec}\,\frac{\pi}{8} + \text{cosec}\,\frac{\pi}{16}$ equals -

(A) $\cot\frac{\pi}{8}$ (B) $\cot\frac{\pi}{16}$

(C) $\cot\frac{\pi}{32}$ (D) None

Sol.[C] $1 + \text{cosec}\,\frac{\pi}{4} + \text{cosec}\,\frac{\pi}{8} + \text{cosec}\,\frac{\pi}{16}$

$= 1 + \left(\cot\frac{\pi}{8} - \cot\frac{\pi}{4}\right) + \left(\cot\frac{\pi}{16} - \cot\frac{\pi}{8}\right) + \left(\cot\frac{\pi}{32} - \cot\frac{\pi}{16}\right)$

$= 1 - \cot\frac{\pi}{4} + \cot\frac{\pi}{32}$

$= 1 - 1 + \cot\frac{\pi}{32} = \cot\frac{\pi}{32}$

Q.21 $3\tan^6\frac{\pi}{18} - 27\tan^4\frac{\pi}{18} + 33\tan^2\frac{\pi}{18}$ equals -

(A) 0 (B) 1

(C) 2 (D) 3

Sol.[B] $3\tan^6\frac{\pi}{18} - 27\tan^4\frac{\pi}{18} + 33\tan^2\frac{\pi}{18}$

Let $\theta = \frac{\pi}{18} \Rightarrow 3\theta = \frac{\pi}{6}$

$\tan 3\theta = \tan\frac{\pi}{6}$

$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \frac{1}{\sqrt{3}}$

$3\sqrt{3}\tan\theta - \sqrt{3}\tan^3\theta = 1 - 3\tan^2\theta$

square

$27\tan^2\theta + 3\tan^6\theta - 18\tan^4\theta = 1 + 9\tan^4\theta - 6\tan^2\theta$

$3\tan^6\theta - 27\tan^4\theta + 33\tan^2\theta = 1$

Part-B One or more than one correct answer type questions

Q.22 If $7\cos x - 24\sin x = \lambda\cos(x + \alpha)$, $0 < \alpha < \frac{\pi}{2}$, be true for all $x \in R$ then

(A) $\lambda = 25$ (B) $\alpha = \sin^{-1}\frac{24}{25}$

(C) $\lambda = -25$ (D) $\alpha = \cos^{-1}\frac{7}{25}$

Sol.[A, B, D]

$7\cos x - 24\sin x = \lambda\cos(x + \alpha)$

$0 < \alpha < \pi/2$

$25\left(\frac{7}{25}\cos x - \frac{24}{25}\sin x\right) = \lambda\cos(x + \alpha)$

Let $\cos\alpha = \frac{7}{25}$ then $\sin\alpha = \frac{24}{25}$

$\Rightarrow 25\cos(x + \alpha)$

so $\lambda = 25$

$\alpha = \cos^{-1}\frac{7}{25}$ and $\alpha = \sin^{-1}\frac{24}{25}$

Q.23 The set of values of $k \in R$ such that the equation $\cos 2\theta + \cos\theta + k = 0$ admits of a solution for θ is

(A) $\left[0, \frac{9}{8}\right]$ (B) $[0, \infty)$

(C) $[-2, 0]$ (D) none of these

Sol.[A]

$\cos 2\theta + \cos\theta + k = 0 \qquad k \in R$

$\Rightarrow 2\cos^2\theta + \cos\theta + k - 1 = \theta$

$\cos\theta = \frac{-1 \pm \sqrt{1 - 8(k-1)}}{4}$

$= \frac{-1 \pm \sqrt{9 - 8k}}{4}$

For real $\cos\theta$, $9 - 8k \ge 0$

$\Rightarrow k \le \frac{9}{8}$ ... (i)

and $-1 \le \frac{-1 \pm \sqrt{9 - 8k}}{4} \le 1$

$\Rightarrow -3 \le \pm\sqrt{9 - 8k} \le 5$

$\therefore -3 \le -\sqrt{9 - 8k}$ and $\sqrt{9 - 8k} \le 5$

$\Rightarrow 9 - 8k \le 9$ and $9 - 8k \le 25$

$k \ge 0$ and $k \ge -2 \Rightarrow k \ge 0$ ... (ii)

from (1) and (2) $k \in \left[0, \frac{9}{8}\right]$

Q.24 If $\sin A + \sin B + \sin C = \cos A + \cos B + \cos C = 0$, then

(A) $\cos(A - B) = -\frac{1}{2}$

(B) $\sin 2A + \sin 2B + \sin 2C = 0$

(C) $\sin^2 A + \sin^2 B + \sin^2 C = 3/2$

(D) $\cos^2 A + \cos^2 B + \cos^2 C = 3/2$

Sol.[A,C,D]

$\sin A + \sin B = -\sin C$

$\cos A + \cos B = -\cos C$

square and solve

Q.25 If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$ $(p \neq 0)$, then

(A) $\sin^2(\alpha+\beta) + p\sin(\alpha+\beta)\cos(\alpha+\beta) + q\cos^2(\alpha+\beta) = q$

(B) $\tan(\alpha+\beta) = p/q - 1$

(C) $\cos(\alpha+\beta) = 1 - q$

(D) $\sin(\alpha+\beta) = -p$

Sol.[A, B]

$\tan\alpha$ and $\tan\beta$ are the roots of

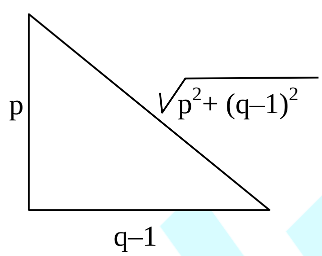
$x^2 + px + q = 0$ $\qquad p \neq 0$

so $\tan\alpha + \tan\beta = -p$

$\tan\alpha\tan\beta = q$

$\Rightarrow \tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{-p}{1-q}$

$\Rightarrow \tan(\alpha+\beta) = \dfrac{p}{q-1}$ $\qquad$ ... (i)



$\Rightarrow \sin(\alpha+\beta) = \dfrac{p}{\sqrt{p^2+(q-1)^2}}$

and $\cos(\alpha+\beta) = \dfrac{(q-1)}{\sqrt{p^2+(q-1)^2}}$

put the values in option (A), we get

$\dfrac{p^2}{p^2+(q-1)^2} + \dfrac{p^2(q-1)}{p^2+(q-1)^2} + \dfrac{q(1-q)^2}{p^2+(q-1)^2}$ and

$\cos(\alpha+\beta) = \dfrac{(q-1)}{\sqrt{p^2+(q-1)^2}}$

$= \dfrac{p^2+p^2q-p^2+q(1-q)^2}{p^2+(q-1)^2}$

$= \dfrac{q(p^2+(1-q)^2)}{p^2+(1-q)^2} = q$ $\qquad$ ... (ii)

So from (i) and (ii) A, B are correct

Q.26 If $3\sin\beta = \sin(2\alpha+\beta)$ then

(A) $[\cot\alpha + \cot(\alpha+\beta)] \times [\cot\beta - 3\cot(2\alpha+\beta)] = 6$

(B) $\sin\beta = \cos(\alpha+\beta)\sin\alpha$

(C) $2\sin\beta = \sin(\alpha+\beta)\cos\alpha$

(D) $\tan(\alpha+\beta) = 2\tan\alpha$

Sol.[A, B, C, D]

$3\sin\beta = \sin(2\alpha+\beta)$ $\qquad$ ... (i)

$3\sin\beta = \sin\alpha\cos(\alpha+\beta) + \cos\alpha\sin(\alpha+\beta)$

$3\sin\beta = [\sin\alpha\sin(\alpha+\beta)][\cot\alpha + \cot(\alpha+\beta)]$

$6\sin\beta = [\cos\beta - \cos(2\alpha+\beta)][\cot\alpha + \cot(\alpha+\beta)]$

$6 = \left[\cot\beta - \dfrac{\cos(2\alpha+\beta)}{\sin\beta}\right][\cot\alpha + \cot(\alpha+\beta)]$

$\therefore \sin\beta = \dfrac{\sin(2\alpha+\beta)}{3}$

so

$6 = [\cot\beta - 3\cot(2\alpha+\beta)][\cot\alpha + \cot(\alpha+\beta)]$

Again, $3\sin\beta = \sin(2\alpha+\beta)$

$\Rightarrow 3\sin\beta = \sin 2\alpha\cos\beta + \cos 2\alpha\sin\beta$... (ii)

$\Rightarrow 3\sin\beta = 2\sin\alpha\cos\alpha\cos\beta + (2\cos^2\alpha - 1)\sin\beta$

$4\sin\beta = 2\sin\alpha\cos\alpha\cos\beta + 2\cos^2\alpha\sin\beta$

$2\sin\beta = \cos\alpha(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$

$2\sin\beta = \cos\alpha\sin(\alpha+\beta)$

Again from (ii)

$3\sin\beta = \sin 2\alpha\cos\beta + \cos 2\alpha\sin\beta$

$\Rightarrow 3\sin\beta = 2\sin\alpha\cos\alpha\cos\beta + (1 - 2\sin^2\alpha)\sin\beta$

$\Rightarrow 2\sin\beta = 2\sin\alpha\cos\alpha\cos\beta - 2\sin^2\alpha\sin\beta$

$\sin\beta = \sin\alpha\cos(\alpha+\beta)$

Again from (i)

$3 = \dfrac{\sin(2\alpha+\beta)}{\sin\beta}$

using componendo and dividendo, we have

$\dfrac{\sin(2\alpha+\beta)+\sin\beta}{\sin(2\alpha+\beta)-\sin\beta} = \dfrac{3+1}{3-1}$

$\dfrac{2\sin(\alpha+\beta)\cos\alpha}{2\cos(\alpha+\beta)\sin\alpha} = 2$

$\tan(\alpha+\beta) = 2\tan\alpha$

so option A, B, C, D are all correct.

Q.27 If A lies between 270º & 360º and $\sin A = -\dfrac{7}{25}$, then

(A) $\sin 2A = -\dfrac{336}{625}$ $\qquad$ (B) $\cos\dfrac{A}{2} = \dfrac{\sqrt{2}}{5}$

(C) $\tan\frac{A}{2} = -\frac{1}{7}$ (D) $\sin\frac{A}{2} = -\frac{\sqrt{2}}{10}$

Sol.[A, C]

Given 270° < A < 360°

$\sin A = -\frac{7}{25}$

$\because$ A lies in IVth quadrant so

$\cos A = +\sqrt{1-\sin^2 A}$

$= \sqrt{1-\frac{49}{625}} = \sqrt{\frac{625-49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$

$\because \sin 2A = 2\sin A\cos A$

$= 2 \times \frac{-7}{25} \times \frac{24}{25} = -\frac{336}{625}$

Option (A) is correct.

$\because \cos\frac{A}{2} = -\sqrt{\frac{\cos A+1}{2}}$

$\because \frac{A}{2}$ lies in IInd quadrant.

$= -\sqrt{\frac{\frac{24}{25}+1}{2}} = -\frac{7}{5\sqrt{2}} = -\frac{7\sqrt{2}}{10}$

$\because \sin\frac{A}{2} = +\sqrt{\frac{1-\cos A}{2}}$

$\because \frac{A}{2}$ lies in IInd quadrant

$= \sqrt{\frac{1-\frac{24}{25}}{2}} = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$ & $\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \frac{\sqrt{2}/10}{-\frac{7\sqrt{2}}{10}}$

$= -\frac{\sqrt{2}\times 10}{7\sqrt{2}\times 10} = -\frac{1}{7}$

So options A and C are correct.

Q.28 If $\sin^3 x \sin 3x = \sum_{m=0}^{6} C_m \cos^m x$, where $C_0, C_1, C_2, ...C_6$ are constant, then

(A) $C_0 + C_2 + C_4 + C_6 = 0$

(B) $C_1 + C_3 + C_5 = 0$

(C) $2C_2 - 3C_6 = 0$

(D) $C_4 + 2C_6 = 0$

Sol.[A, B, C]

$\sin^3 x \sin 3x = \sum_{m=0}^{6} C_m \cos^m x$

Taking L.H.S. we have

$\sin^3 x \sin 3x$

$= \sin^3 x\,(3\sin x - 4\sin^3 x)$

$= (1-\cos^2 x)^2 (3 - 4\sin^2 x)$

$= (1 - 2\cos^2 x - \cos^4 x)(4\cos^2 x - 1)$

$= 4\cos^2 x - 1 - 8\cos^4 x + 2\cos^2 x + 4\cos^6 x - \cos^4 x$

$= -1 + 6\cos^2 x - 9\cos^4 x + 4\cos^6 x$

according to the question

we get

$C_0 = -1, C_1, = 0, C_2 = 6, C_3 = 0$

$C_4 = -9\ C_5 = 0$ and $C_6 = 4$

Then

(A) $C_0 + C_2 + C_4 + C_6$

$= -1 + 6 - 9 + 4 = 0$ correct

(B) $C_1 + C_3 + C_5 = 0 + 0 + 0 = 0$ correct

(C) $2C_2 - 3C_6 = 12 - 12 = 0$ correct

(D) $C_4 + 2C_6 = -9 + 8 = -1$ wrong

So option A, B, C are correct.

Q.29 If $\sin(x + 20°) = 2\sin x\cos 40°$ where $x \in \left(0, \frac{\pi}{2}\right)$, then which of the following is true

(A) $\tan 4x = \sqrt{3}$ (B) cosec 4x = 2

(C) $\sec\frac{x}{2} = \sqrt{6} - \sqrt{2}$ (D) $\cot\frac{x}{2} = 2 + \sqrt{3}$

Sol.[C,D]

$\sin(x + 20^o) = 2\sin x\cos 40^o$

$\sin x\cos 20^o + \cos x\sin 20^o = \sin x\cos 40^o + \sin x\cos 40^o$

$\cos x\sin 20^o = \sin x\cos 40^o + \sin x\cos 40^o - \sin x\cos 20^o$

$= \sin x\cos 40^o + \sin x[\cos 40^o - \cos 20^o]$

$= \sin x\cos 40^o + \sin x\,[2\sin 30^o \sin(-10^o)]$

$= \sin x\cos 40^o + \sin x\left(2\times\frac{1}{2}\sin(-10^o)\right)$

$= \sin x\cos 40^o - \sin x\sin 10^o$

$= \sin x\,[\sin 50^o - \sin 10^o]$

$\cos x\sin 20^o = \sin x\,[2\cos 30^o \sin 20^o]$

$\cos x = \sin x\sqrt{3}$

$\tan x = \frac{1}{\sqrt{3}}$

$\Rightarrow x = 30^o$

Putting the value of x in options we get,

$\sec\frac{x}{2} = \sqrt{6} - \sqrt{2}$ and $\cot\frac{x}{2} = 2 + \sqrt{3}$

Part-C Assertion Reason type Questions

The following questions 30 to 31 consists of two statements each, printed as Assertion and Reason. While answering these

questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Q.30 **Assertion (A) :** $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if x = y and x ≠ 0.

Reason (R) : Because sec θ decreases in III[rd] and IV[th] quadrant.

Sol. **[B]**

$\because \sec^2\theta \ge 1$

$\Rightarrow \frac{4xy}{(x+y)^2} \ge 1$

$\Rightarrow (x + y)^2 \le 4xy$

$\Rightarrow (x - y)^2 \le 0$

$\Rightarrow x = y$ where $x \ne 0$

⇒ A is true

and sec θ decreases in III[rd] and IV[th] quadrant

⇒ R is true

But R is not correct explanation of A

Q.31 **Assertion (A):** If A, B, C are the angles of a triangle such that angle A is obtuse then tan B tan C > 1.

Reason (R) : In any triangle,

$$\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}.$$

Sol. **[D]**

$A + B + C = \pi$

$\therefore A = \pi - (B + C)$

$$\tan A = -\tan(B + C) = -\left(\frac{\tan B + \tan C}{\tan B \tan C - 1}\right)$$

$$= \frac{\tan B + \tan C}{1 - \tan B \tan C}$$

$\tan A < 0 \Rightarrow 1 - \tan B \tan C < 0$

Part-D Column Matching type Questions

Q.32 In a Δ ABC,

Column 1	Column 2
(A) $\Sigma \tan A$	(P) $1 - 2\Pi \sin \frac{A}{2}$
(B) $\Sigma \tan\frac{B}{2}\tan\frac{C}{2}$	(Q) $\Pi \tan A$
(C) $\Sigma \cot\frac{A}{2}$	(R) $\Pi \cot\frac{A}{2}$
(D) $\Sigma \sin^2\frac{A}{2}$	(S) 1

Sol. **A → Q; B → S; C → R ; D → P**

Do yourself.

Q.33 The value of

Column 1	Column 2
(A) cot (π/4 + θ). cot (π/4 – θ)	(P) tan 56°
(B) sin (45° + θ) – cos (45° – θ)	(Q) 1
(C) $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$	(R) √3/2
(D) $\sin^2 75^\circ - \sin^2 15^\circ$	(S) 0

Sol. **A → Q; B → S ; C → P ; D → R**

(i) $\cot\left(\frac{\pi}{4}+\theta\right).\cot\left(\frac{\pi}{4}-\theta\right)$

$$= \frac{\cot\theta - 1}{\cot\theta + 1}.\frac{\cot\theta + 1}{\cot\theta - 1} = 1$$

(ii) sin (45° + θ) – cos (45° – θ)

= sin (45° + θ) – cos (90° – (45°+ θ))

= sin (45° + θ) – sin (45° + θ) = 0

(iii) $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$

$$= \frac{\sin(45^\circ + 11^\circ)}{\cos(45^\circ + 11^\circ)} = \tan 56^\circ$$

(iv) $\sin^2 75^\circ - \sin^2 15^\circ$

$\Rightarrow \because \sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$

$\Rightarrow \sin(75^\circ + 15^\circ)\sin(75^\circ - 15^\circ)$

$\Rightarrow \sin 90^\circ \sin 60^\circ = \frac{\sqrt{3}}{2}$

Part-E Fill in The Blanks type Questions

Q.34 If α and β are the solution of the equation a tan θ + b sec θ = c, then tan (α + β) = ...

Sol. $a\tan\theta + b\sec\theta = c$

$(a\tan\theta - c)^2 = b^2(1 + \tan^2\theta)$

$= a^2\tan^2\theta - 2ac\tan\theta + c^2 - b^2 - b^2\tan^2\theta = 0$

$= \tan^2\theta(a^2 - b^2) - 2ac\tan\theta + c^2 - b^2 = 0$ …(1)

since α, β are roots of a tanθ + 6 secθ = c

a tanα + b secα = c & a tanβ + b secβ = c

∴ tanα, tanβ are roots of …….(1)

$\therefore \tan\alpha + \tan\beta = \dfrac{2ac}{a^2 - b^2}$

$\tan\alpha + \tan\beta = \dfrac{c^2 - b^2}{a^2 - b^2}$

$\therefore \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$$= \frac{\dfrac{2ac}{a^2 - b^2}}{1 - \dfrac{c^2 - b^2}{a^2 - b^2}} = \frac{2ac}{a^2 - b^2 - c^2 + b^2} = \frac{2ac}{a^2 - c^2}$$

Q.35 $\dfrac{1}{\cos 290^\circ} + \dfrac{1}{\sqrt{3}\sin 250^\circ} =$.............

Sol. $\dfrac{1}{\sin 20} - \dfrac{1}{\sqrt{3}\cos 20} = \dfrac{\sqrt{3}\cos 20 - \sin 20}{\sqrt{3}\sin 20 \cos 20}$

$$= \frac{4}{\sqrt{3}}\left(\frac{\dfrac{\sqrt{3}}{2}\cos 20 - \dfrac{1}{2}\sin 20}{2\sin 20 \cos 20}\right) = \frac{4}{\sqrt{3}}\frac{\sin 40}{\sin 40} = \frac{4}{\sqrt{3}}$$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 If $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = (a^2 - b^2)$, and

$\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$, show that

$(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

Sol. $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$

$ax\sin\theta + by\cos\theta = (a^2 - b^2)\cos\theta\sin\theta$ ...(1)

and $\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$

$ax\sin^3\theta - by\cos^3\theta = 0$ ... (2)

Multiplying (1) by $\cos^2\theta$ and adding in (2), we get

$ax(\sin\theta\cos^2\theta + \sin^3\theta) = (a^2 - b^2)\cos^3\theta\sin\theta$

$ax = (a^2 - b^2)\cos^3\theta$ ...(3)

Similarly,

$by = (a^2 - b^2)\sin^3\theta$ ... (4)

from (3) and (4)

$\cos\theta = \left(\frac{ax}{a^2 - b^2}\right)^{1/3}$

$\sin\theta = \left(\frac{by}{a^2 - b^2}\right)^{1/3}$

Squaring and adding, we get

$(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

Q.2 Prove that $\frac{1 - 2\cos^2\alpha}{2\tan\left(\alpha - \frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4} + \alpha\right)} = 1.$

Sol. Taking L.H.S.

$$\frac{1 - 2\cos^2\alpha}{2\tan(\alpha - \pi/4)\sin^2(\pi/4 + \alpha)}$$

$$= \frac{\sin^2\alpha - \cos^2\alpha}{2\tan(\alpha - \pi/4)\left(\frac{\sin\alpha + \cos\alpha}{\sqrt{2}}\right)^2}$$

$$= \frac{(\sin\alpha + \cos\alpha)(\sin\alpha + \cos\alpha)}{2\tan(\alpha - \pi/4)\frac{(\sin\alpha + \cos\alpha)^2}{2}}$$

$$= \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} \cdot \frac{1}{\tan(\alpha - \pi/4)}$$

$$= \frac{\tan\alpha - 1}{\tan\alpha + 1} \cdot \frac{1}{\tan(\alpha - \pi/4)}$$

$$= \frac{\tan(\alpha - \pi/4)}{\tan(\alpha - \pi/4)} = 1$$

Hence proved.

Q.3 Prove that

$$\frac{\sin(A-B)}{\cos A\cos B} + \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A} = 0$$

Sol. Taking L.H.S.

$$\frac{\sin(A-B)}{\cos A\cos B} + \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A}$$

$$= \sum \frac{\sin(A-B)}{\cos A\cos B} = \sum \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}$$

$$= \sum(\tan A - \tan B)$$

$= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A$

$= 0$ Hence proved.

Q.4 Prove that

$$\frac{\cos 3\theta + 2\cos 5\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 5\theta} = \cos 2\theta - \sin 2\theta\tan 3\theta$$

Sol. Taking L.H.S.

$$\frac{\cos 3\theta + 2\cos 5\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 5\theta}$$

$$= \frac{\cos 3\theta + \cos 7\theta + 2\cos 5\theta}{\cos\theta + \cos 5\theta + 2\cos 3\theta}$$

$$= \frac{2\cos 5\theta.\cos 2\theta + 2\cos 5\theta}{2\cos 3\theta\cos 2\theta + 2\cos 3\theta}$$

$= \dfrac{\cos 5\theta}{\cos 3\theta} = \dfrac{\cos(3\theta + 2\theta)}{\cos 3\theta}$

$= \dfrac{\cos 3\theta \cos 2\theta - \sin 3\theta \sin 2\theta}{\cos 3\theta}$

$= \cos 2\theta - \tan 3\theta \sin 2\theta$, Hence proved

Q.5 Prove that

$$\left(\frac{\tan^2 \frac{\alpha - \pi}{4} - 1}{\tan^2 \frac{\alpha - \pi}{4} + 1} + \cos \frac{\alpha}{2} . \cot 4\alpha \right) \sec \frac{9}{2} \alpha$$

$= \text{cosec } 4\alpha$.

Sol. Taking L.H.S.

$$\left[\frac{\tan^2 \left(\frac{\alpha - \pi}{4} \right) - 1}{\tan^2 \left(\frac{\alpha - \pi}{4} \right) + 1} + \cos \frac{\alpha}{2} \cot 4\alpha \right] \sec \frac{9}{2} \alpha$$

$$\left[\frac{\cos \frac{\alpha}{2} \cos 4\alpha - \sin \frac{\alpha}{2} \sin 4\alpha}{\sin 4\alpha} \right] \sec \frac{9}{2} \alpha$$

$$\Rightarrow \left[-\cos \left(\frac{\alpha - \pi}{2} \right) + \cos \frac{\alpha}{2} \frac{\cos 4\alpha}{\sin 4\alpha} \right] \sec \frac{9}{2} \alpha$$

$$\Rightarrow \frac{\cos \frac{9}{2} \alpha . \sec \frac{9}{2} \alpha}{\sin 4\alpha} = \text{cosec } 4\alpha$$,Hence proved.

Q.6 If $\cos\theta = \dfrac{a}{b+c}$, $\cos\phi = \dfrac{b}{a+c}$ and $\cos\psi = \dfrac{c}{a+b}$, where θ, ϕ, ψ lies between 0 and π then prove that $\tan^2\left(\dfrac{\theta}{2}\right) + \tan^2\left(\dfrac{\phi}{2}\right) + \tan^2\left(\dfrac{\Psi}{2}\right) = 1$.

Sol. $\cos\theta = \dfrac{a}{b+c}$, $\cos\phi = \dfrac{b}{a+c}$ $\cos\psi = \dfrac{c}{a+b}$

then we know that

$\cos\theta = \dfrac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2}$

$\dfrac{a}{b+c} = \dfrac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2}$

$\dfrac{a}{b+c} + \dfrac{a}{b+c} \tan^2\theta/2 = 1 - \tan^2\theta/2$

$\Rightarrow \tan^2 \theta/2 \left(\dfrac{a}{b+c} + 1 \right) = 1 - \dfrac{a}{b+c}$

$\Rightarrow \tan^2 \theta/2 = \dfrac{b+c-b}{a+b+c}$ ... (i)

Similarly $\tan^2 \dfrac{\phi}{2} = \dfrac{a+c-b}{a+b+c}$ ... (ii)

and $\tan^2 \dfrac{\Psi}{2} = \dfrac{a+b-c}{a+b+c}$ ... (iii)

then $\tan^2 \dfrac{\theta}{2} + \tan^2 \dfrac{\phi}{2} + \tan^2 \dfrac{\Psi}{2}$

$= \dfrac{b+c-a}{a+b+c} + \dfrac{a+c-b}{a+b+c} + \dfrac{a+b-c}{a+b+c}$

$= \dfrac{a+b+c}{a+b+c} = 1$

Hence proved.

Q.7 Prove that the identity,

$\cos\left(\dfrac{3\pi}{2} + 4\alpha\right) + \sin(3\pi - 8\alpha) - \sin(4\pi - 12\alpha)$

$= 4 \cos 2\alpha . \cos 4\alpha . \sin 6\alpha$.

Sol. Taking L.H.S.

$\cos\left(\dfrac{3\pi}{2} + 4\alpha\right) + \sin(3\pi - 8\alpha) - \sin(4\pi - 12\alpha)$

$= \sin 4\alpha + \sin 8\alpha + \sin 12\alpha$

$= 2 \cos 6\alpha \cos 2\alpha + 2 \sin 6\alpha \cos 6\alpha$

$= 2 \sin 6\alpha (\cos 2\alpha + \cos 6\alpha)$

$= 2 \sin 6\alpha . 2 \cos 4\alpha \cos 2\alpha$

$= 4 \cos 2\alpha . \cos 4\alpha \sin 6\alpha$

Hence proved

Q.8 Prove that $\dfrac{\cos 4\alpha \tan 2\alpha - \sin 4\alpha}{\cos 4\alpha \cot 2\alpha + \sin 4\alpha} = -\tan^2 2\alpha$

Sol. Taking LHS

$= \dfrac{\cos 4\alpha \tan 2\alpha - \sin 4\alpha}{\cos 4\alpha \cot 2\alpha + \sin 4\alpha} = \dfrac{\tan 2\alpha - \tan 4\alpha}{\cot 2\alpha + \tan 4\alpha}$

$= \tan 2\alpha \left(\dfrac{\tan 2\alpha - \tan 4\alpha}{1 + \tan 2\alpha \tan 4\alpha} \right)$

$= \tan 2\alpha \tan(2\alpha - 4\alpha) = -\tan^2 2\alpha$

Hence proved.

Q.9 Prove that $\dfrac{\cot\alpha - \tan\alpha}{2\sin\alpha + \cos(90^o + 3\alpha) + \sin 5\alpha}$

$= \text{cosec}\,\alpha\ \text{cosec}\,4\alpha.$

Sol. Taking L.H.S.

$$= \frac{\dfrac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cos\alpha}}{2\sin\alpha - \sin 3\alpha + \sin 5\alpha}$$

$$= \frac{\dfrac{2\cos 2\alpha}{\sin 2\alpha}}{2\sin\alpha + 2\cos 4\alpha \sin\alpha}$$

$$= \frac{2\cos 2\alpha}{2\sin 2\alpha \sin\alpha(1 + \cos 4\alpha)}$$

$$= \frac{\cos 2\alpha}{2\sin 2\alpha \sin\alpha . \cos^2 2\alpha} = \frac{1}{\sin\alpha \sin 4\alpha}$$

$= \text{cosec}\,\alpha\ \text{cosec}\,4\alpha$

Q.10 Prove that

$$(\cos^6\alpha - \sin^6\alpha) = \frac{(3 + \cos^2 2\alpha)\cos 2\alpha}{4}$$

Sol. Taking L.H.S.

$\cos^6\alpha - \sin^6\alpha = (\cos^2\alpha - \sin^2\alpha)^3$

$\quad - 3\cos^2\alpha\sin^2\alpha(\cos^2\alpha - \sin^2\alpha)$

Θ $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

$\Rightarrow \cos^3 2\alpha + \dfrac{3}{4}\sin^2 2\alpha \cos 2\alpha$

$= \dfrac{\cos 2\alpha}{4}(4\cos^2 2\alpha + 3\sin^2 2\alpha)$

$= \dfrac{\cos 2\alpha}{4}(3 + \cos 2\alpha)$

Q.11 Prove the identity, $\sin 2\alpha(1 + \tan 2\alpha . \tan\alpha) + \dfrac{1 + \sin\alpha}{1 - \sin\alpha} = \tan 2\alpha + \tan^2\left(\dfrac{\pi}{4} + \dfrac{\alpha}{2}\right)$

Sol. Taking L.H.S.

$$\sin 2\alpha\left(1 + \frac{\sin 2\alpha . \sin\alpha}{\cos 2\alpha \cos\alpha}\right) + \frac{\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)^2}{\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)^2}$$

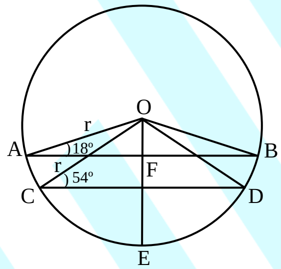
$$= \frac{\tan 2\alpha}{\cos\alpha}(\cos 2\alpha \cos 3\alpha + \sin 2\alpha \sin\alpha) + \left(\frac{1 + \tan\frac{\alpha}{2}}{1 - \tan\frac{\alpha}{2}}\right)^2$$

$$= \frac{\tan 2\alpha}{\cos\alpha}\cos(2\alpha - \alpha) + \left[\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right]^2$$

$= \tan 2\alpha + \tan^2\left(\dfrac{\pi}{4} + \dfrac{\alpha}{2}\right)$ Hence proved

Q.12 Two parallel chords of a circle, which are on the same side of the centre, subtend angles of 72º and 144º respectively at the centre. Prove that the perpendicular distance between the chords is half the radius of the circle.

Sol.



Given $\angle AOB = 144^o$

$\angle COD = 72^o$

Let radius = r

we want to find EF

from fig. $r \sin 54^o = OE$

and $r \sin 18^o = OF$

$EF = OE - OF = r(\sin 54^o - \sin 18^o)$

$= 2r\,\dfrac{\cos 36^o \sin 18^o \cos 18^o}{\cos 18^o}$

$= 2r\,\dfrac{\cos 36^o \sin 36^o}{2\cos 18^o}$

$= \dfrac{r \sin 72^o}{2\cos 18^o} = \dfrac{r}{2}\dfrac{\cos 18^o}{\cos 18^o} = \dfrac{r}{2}$ Hence proved.

Q.13 If $\cos\theta = \dfrac{\cos u - e}{1 - e\cos u}$.

Prove that $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 + e}{1 - e}}\ \tan\dfrac{u}{2}$

Sol. Given $\cos\theta = \dfrac{\cos u - e}{1 - e\cos u}$

We know that

$\tan^2\dfrac{\theta}{2} = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

$$\tan^2\frac{\theta}{2} = \frac{1 - \dfrac{\cos u - e}{1 - e\cos u}}{1 + \dfrac{\cos u - e}{1 - e\cos u}}$$

$$= \frac{1 - e\cos u - \cos u + e}{1 - e\cos u + \cos u - e}$$

$$= \frac{(1+e)-\cos u(1+e)}{(1-e)+\cos u(1-e)}$$

$$= \frac{(1+e)(1-\cos u)}{(1-e)(1+\cos u)}$$

$$\tan^2\frac{\theta}{2} = \frac{1+e}{1-e}\tan^2\frac{u}{2}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1+e}{1-e}}\tan\frac{u}{2}$$

Hence proved.

Q.14 If $\tan\theta = \frac{\tan\alpha+\tan\beta}{1+\tan\alpha.\tan\beta}$, show that

$$\sin 2\theta = \frac{\sin 2\alpha+\sin 2\beta}{1+\sin 2\alpha.\sin 2\beta}$$

Sol. $\tan\theta = \frac{\tan\alpha+\tan\beta}{1+\tan\alpha\tan\beta}$

$$= \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta} = \frac{\sin(\alpha+\beta)}{\cos(\alpha-\beta)}$$

$$\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{\frac{2\sin(\alpha+\beta)}{\cos(\alpha-\beta)}}{1+\frac{\sin^2\theta(\alpha+\beta)}{\cos^2(\alpha-\beta)}} = \frac{2\sin(\alpha+\beta)\cos(\alpha-\beta)}{\cos^2(\alpha-\beta)+\sin^2(\alpha+\beta)}$$

$$\sin 2\theta = \frac{\sin 2\alpha+\sin 2\beta}{1-\sin^2(\alpha-\beta)+\sin^2(\alpha+\beta)}$$

$\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

$$\sin 2\theta = \frac{\sin 2\alpha+\sin 2\beta}{1+\sin 2\alpha\sin 2\beta}$$

Q.15 If $\frac{\sin\beta}{\sin(2\alpha+\beta)} = \frac{n}{m}$ ($|m| > |n|$) then prove that

$$\frac{1+\frac{\tan\beta}{\tan\alpha}}{m+n} = \frac{1-\tan\alpha\tan\beta}{m-n}$$

Sol. Taking L.H.S.

$$\frac{\sin\alpha}{\sin(2\alpha+\beta)} = \frac{n}{m} \qquad |n| > |m|$$

Using componendo and dividendo, we have

$$\frac{\sin(2\alpha+\beta)-\sin\beta}{\sin(2\alpha+\beta)+\sin\beta} = \frac{m-n}{m+n}$$

$$= \frac{2\cos(\alpha+\beta)\sin\alpha}{2\sin(\alpha+\beta)\cos\alpha} = \frac{m-n}{m+n}$$

$$= \frac{(\cos\alpha\cos\beta-\sin\alpha\sin\beta)\sin\alpha}{(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\cos\alpha} = \frac{m-n}{m+n}$$

$$= \frac{(1-\tan\alpha\tan\beta)\tan\alpha}{(\tan\alpha+\tan\beta)} = \frac{m-n}{m+n}$$

$$= \frac{1-\tan\alpha\tan\beta}{m-n} = \frac{1+\frac{\tan\beta}{\tan\alpha}}{m+n}$$ Hence proved.

Q.16 Prove that cot 16º. cot 44º + cot 44º . cot 76º – cot 76º . cot 16º = 3

Sol. Taking L.H.S.

cot 16° cot 44° + cot 44° cot 76° – cot 76° cot 16°

$$= \left(\frac{\cos 16°\cos 44°}{\sin 16°\sin 44°}-1\right)+\left(\frac{\cos 44°\cos 76°}{\sin 44°\sin 76°}-1\right)-\left(\frac{\cos 76°\cos 16°}{\sin 76°\sin 16°}+1\right)+3$$

$$= \frac{\cos(44°+16°)}{\sin 16°\sin 44°}+\frac{\cos(76°+44°)}{\sin 44°\sin 76°}-\frac{\cos 60°}{\sin 76°\sin 16°}+3$$

$$= \frac{1}{2}\left[\left(\frac{\sin 76°-\sin 16°}{\sin 16°\sin 44°\sin 76°}\right)-\frac{\sin 44°}{\sin 76°\sin 16°\sin 44°}\right]+3$$

$$= \frac{1}{2}\left[\frac{2\cos 46°\sin 30°-\sin 44°}{\sin 16°\sin 44°\sin 76°}\right]+3$$

$$= \frac{1}{2}\left[\frac{\cos 46°-\sin 44°}{\sin 16°\sin 44°\sin 76°}\right]+3$$

$\because \cos 46° = \sin 44° = 0 + 3 = 3$

Q.17 Calculate without using trigonometric tables

(a) [2 cos 40º – cos 20º]/sin20º

(b) $2\sqrt{2}\sin 10°\left[\frac{\sec 5°}{2}+\frac{\cos 40°}{\sin 5°}-2\sin 35°\right]$

Sol. (a) $\frac{2\cos 40°-\cos 20°}{\sin 20°}$

$$= \frac{\cos 40°+\cos 40°-\sin 20°}{\sin 20°}$$

$$= \frac{\cos 40°+2\sin 30°\sin(-10°)}{\sin 20°}$$

$$= \frac{\cos 40°-\sin 10°}{\sin 20°} \qquad \because \sin 30° = \frac{1}{2}$$

$$= \frac{\sin 50°-\sin 10°}{\sin 20°} \qquad \because \cos(\pi/2-1) = \sin\theta$$

$$= \frac{2\cos 30°\sin 20°}{\sin 20°} = \sqrt{3}$$

(b) $2\sqrt{2}\sin 10°\left[\frac{\sin 5°}{2}+\frac{\cos 40°}{\sin 5°}-2\sin 35°\right]$

$= 2\sqrt{2}$ [sin 5° + 2 cos 5° cos 40° – 2 sin 10° sin 35°]

$= 2\sqrt{2}$ [sin 5° + cos 45° + cos 35° – (cos 25° – cos 45°)]

$= 2\sqrt{2}$ [sin 5° + 2 cos 45° + cos 35° – cos 25°]

$= 2\sqrt{2}$ [sin 5° + $\sqrt{2}$ + 2 sin 30° sin (–5°)]

$= 2\sqrt{2}$ [sin 5° + $\sqrt{2}$ – sin 5°] ∵ sin 30° $= \frac{1}{2}$ = 4

Part-B Passage based objective questions

Passage # 1 (Q. 18 to 20)

In a ΔABC, if cos A cos B cos C = $\frac{\sqrt{3}-1}{8}$ and sin A sin B sin C = $\frac{3+\sqrt{3}}{8}$, then

On the basis of above passage, answer the following questions :

Q.18 The value of tan A + tan B + tan C is-

(A) $\frac{3+\sqrt{3}}{\sqrt{3}-1}$ (B) $\frac{\sqrt{3}+4}{\sqrt{3}-1}$

(C) $\frac{6-\sqrt{3}}{\sqrt{3}-1}$ (D) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-1}$

Sol. **[A]**

We know that

tan (A + B+ C) = $\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$

But A + B + C = π

so tan (A + B + C) = 0

so tan A + tan B + tan C = tan A tan B tan C

$= \frac{\sin A\sin B\sin C}{\cos A\cos B\cos C} = \frac{3+\sqrt{3}}{\sqrt{3}-1}$

Q.19 The value of tan A tan B + tan B tan C + tan C tan A is-

(A) $5-4\sqrt{3}$ (B) $5+4\sqrt{3}$

(C) $6+\sqrt{3}$ (D) $6-\sqrt{3}$

Sol. **[B]**

tan A tan B + tan B tan C + tan C tan A

$= \frac{\sin A\sin B}{\cos A\cos B}+\frac{\sin B\sin C}{\cos B\cos C}+\frac{\sin C\sin A}{\cos C\cos A}$

$\Rightarrow \frac{\sin A\sin B\cos C+\cos A\sin B\sin C+\sin A\cos B\sin C}{\cos A\cos B\cos C}$

∵ cos (A + B + C)

= cos A cos B cos C – cos A sin B sin C – sin A cos B sin C – sin A sin B cos C

$\Rightarrow \frac{\cos A\cos B\cos C-\cos(A+B+C)}{\cos A\cos B\cos C}$

$= 1-\frac{\cos(A+B+C)}{\cos A\cos B\cos C}$

$\Rightarrow 1+\frac{8}{\sqrt{3}-1} = 1+4\sqrt{3}+4 = 5+4\sqrt{3}$

Q.20 The value of tan A, tan B and tan C are-

(A) 1, $\sqrt{3}$, $\sqrt{2}$ (B) 1, $\sqrt{3}$, 2

(C) 1, 2, $\sqrt{3}$ (D) 1, $\sqrt{3}$, $2+\sqrt{3}$

Sol. **[D]**

From above equation, clearly x = 1 is the root of the equation so

$(x-1)(x^2-(2+2\sqrt{3})x+(3+2\sqrt{3})) = 0$

$(x-1)(x-\sqrt{3})(x-(2+\sqrt{3})) = 0$

$x = 1, \sqrt{3}, 2+\sqrt{3}$

$\Rightarrow$ tan A = 1, tan B = $\sqrt{3}$, tan C = $2+\sqrt{3}$

Passage # 2 (Q. 21 to 23)

Consider the cubic equation $x^3-(1+\cos\theta+\sin\theta)x^2+(\cos\theta\sin\theta+\cos\theta+\sin\theta)x-\sin\theta\cos\theta = 0$ where x_1, x_2, x_3 are roots.

On the basis of above passage, answer the following questions :

Q.21 The value of $x_1^2+x_2^2+x_3^2$ is

(A) 1 (B) 2cosθ

(C) 2sinθ (D) 2

Sol.[D] $x^3-(1+\cos\theta+\sin\theta)x^2+(\cos\theta\sin\theta+\cos\theta+\sin\theta)x-\sin\theta\cos\theta = 0$

$(x-1)(x^2-(\cos\theta+\sin\theta)x+\sin\theta\cos\theta) = 0$

$(x-1)(x^2-x\cos\theta-x\sin\theta+\sin\theta\cos\theta) = 0$

$(x-1)(x-\cos\theta)(x-\sin\theta) = 0$

$x_1 = 1, x_2 = \cos\theta, x_3 = \sin\theta$

$x_1^2+x_2^2+x_3^2$

$= 1^2+\cos^2\theta+\sin^2\theta$

$= 1 + 1$ [∵ $\cos^2\theta+\sin^2\theta = 1$]

= 2

Q.22 Number of values of θ in [0, 2π] for which at least two roots are equal

(A) 3 (B) 4 (C) 5 (D) 6

Sol.[C] $x^3-(1+\cos\theta+\sin\theta)x^2+(\cos\theta\sin\theta+\cos\theta+\sin\theta)x-\sin\theta\cos\theta = 0$

$(x-1)(x^2-(\cos\theta+\sin\theta)x+\sin\theta\cos\theta) = 0$

$(x-1)(x^2-x\cos\theta-x\sin\theta+\sin\theta\cos\theta) = 0$

(x – 1) (x – cos θ) (x – sin θ) = 0
$x_1 = 1, x_2 = \cos\theta, x_3 = \sin\theta$
Given cos θ = 1 or sin θ = 1 or sin θ = cos θ or cos θ = sin θ = 1
If cos θ = 1, θ = 0º, 360º
sin θ = 1, θ = 90º
sin θ = cos θ or tan θ = 1
⇒ θ = 45º, 225º
cos θ = sin θ = 1 is Not possible at the same time.
∴ 5 values of θ at which value of two roots are equal.

Q.23 Greatest possible difference between two of roots if θ ∈ [0, 2π] is

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Sol.[A] $x^3 - (1 + \cos\theta + \sin\theta)x^2 + (\cos\theta\sin\theta + \cos\theta + \sin\theta)x - \sin\theta\cos\theta = 0$
$(x - 1)(x^2 - (\cos\theta + \sin\theta)x + \sin\theta\cos\theta) = 0$
$(x - 1)(x^2 - x\cos\theta - x\sin\theta + \sin\theta\cos\theta) = 0$
(x – 1) (x – cos θ) (x – sin θ) = 0
$x_1 = 1, x_2 = \cos\theta, x_3 = \sin\theta$
Difference of roots can be
= |1 – cos θ|, |1 – sin θ|, |sin θ – cos θ|
greater difference
If |1 – cos θ | = |1 – (–1)| = 2
|1 – sin θ | = 2
$|\sin\theta - \cos\theta| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$
greatest difference between two roots is 2

Passage # 3 (Q. 24 to 26)

The graph of all 6 trigonometric functions are drawn from $\left(0, \frac{\pi}{2}\right)$. Let the graph y = sinx be A_1; y = cos x be A_2; y = tan x be A_3; y = cot x be A_4; y = sec x be A_5 and y = cosec x be A_6

On the basis of above passage, answer the following questions:

Q.24 A vertical line is drawn through intersection of A_2 and A_3 intersecting A_1 and A_6 at P and Q respectively. The length of PQ is-

(A) 1 (B) $\frac{\sqrt{5}-1}{2}$

(C) $\sqrt{2}$ (D) $\frac{\sqrt{5}+1}{2}$

Sol. **[A]**
Intersection of A_2 and A_3
∵ cos x = tan x
$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{\sqrt{5}-1}{2} \Rightarrow x = \sin^{-1}\frac{\sqrt{5}-1}{2}$

∵ $A_1 \Rightarrow y = \sin x \Rightarrow y = \sin\left(\sin^{-1}\frac{\sqrt{5}-1}{2}\right) = \frac{\sqrt{5}-1}{2}$

P is $\left(\sin^{-1}\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}-1}{2}\right)$

similarly Q is $\left(\sin^{-1}\frac{\sqrt{5}-1}{2}, \frac{2}{\sqrt{5}-1}\right)$

$$PQ = \sqrt{\left(\frac{\sqrt{5}-1}{2}, \frac{2}{\sqrt{5}-1}\right)^2} = 1$$

Q.25 A vertical line is drawn through intersection of A_3 and A_6 intersecting A_2 and A_5 at R and S respectively. Then length of RS is-

(A) 1 (B) $\frac{\sqrt{5}-1}{2}$

(C) $\sqrt{2}$ (D) $\frac{\sqrt{5}+1}{2}$

Sol. **[A]**
Intersection of A_3 and A_6
∵ tan x = cosec x
$\Rightarrow \cos^2 x + \cos x - 1 = 0$

$\Rightarrow \cos x = \frac{\sqrt{5}-1}{2} \Rightarrow x = \cos^{-1}\frac{\sqrt{5}-1}{2}$

$A_2 \Rightarrow y = \cos x \Rightarrow y = \cos\cos^{-1}\frac{\sqrt{5}-1}{2} = \frac{\sqrt{5}-1}{2}$

R is $\left(\cos^{-1}\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}-1}{2}\right)$

Similarly S is $\left(\cos^{-1}\frac{\sqrt{5}-1}{2}, \frac{2}{\sqrt{5}-1}\right)$

$$RS = \sqrt{\left(\frac{\sqrt{5}-1}{2} - \frac{2}{\sqrt{5}-1}\right)^2} = 1$$

Q.26 A horizontal line is drawn through intersection A_5 and A_6 to intersect A_3 and A_4 at C and D respectively. Then length of CD is-

(A) $\tan^{-1}2$ (B) $\tan^{-1} 2\sqrt{2}$

(C) $\cot^{-1} 2\sqrt{2}$ (D) $\cot^{-1}2$

Sol. **[C]**
∵ A_5 and A_6 intersect at

$x = \frac{\pi}{4}$

Horizontal line is $y = \sqrt{2}$

$A_3 \Rightarrow y = \tan x \Rightarrow x = \tan^{-1}\sqrt{2}$

C is $(\tan^{-1}\sqrt{2}, \sqrt{2})$

Similarly D is $(\cot^{-1}\sqrt{2}, \sqrt{2})$

$CD = \sqrt{(\tan^{-1}\sqrt{2} - \cot^{-1}\sqrt{2})^2}$

$= |\tan^{-1}\sqrt{2} - \cot^{-1}\sqrt{2}|$

$= \cot^{-1} 2\sqrt{2}$

EXERCISE # 4

Old IIT-JEE questions

Q.1 In any ΔABC, prove that

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right)$$

$$= \cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$$ **[IIT 2000]**

Sol. $A + B + C = \pi \Rightarrow \frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

Taking tan of both side

$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \cot\frac{C}{2}$$

$$\frac{1}{\cot\frac{A}{2}} + \frac{1}{\cot\frac{B}{2}} = \left[1 - \frac{1}{\cot\frac{A}{2}\cot\frac{B}{2}}\right]\cot\frac{C}{2}$$

$$\cot\frac{B}{2} + \cot\frac{A}{2} = \left[\cot\frac{A}{2}\cot\frac{B}{2} - 1\right]\cot\frac{C}{2}$$

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$

Q.2 Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ **[IIT Scr.2000]**

(A) ≥ 0 only when $\theta \geq 0$

(B) ≤ 0 for all real θ

(C) ≥ 0 for all real θ

(D) ≤ 0 only when $\theta \leq 0$

Sol. **[C]**

$f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= 2\sin\theta\sin 2\theta\cos\theta \quad = \sin^2 2\theta$

which is positive for all real θ.

Q.3 If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals- **[IIT Scr. 2001]**

(A) $2(\tan\beta + \tan\gamma)$ (B) $\tan\beta + \tan\gamma$

(C) $\tan\beta + 2\tan\gamma$ (D) $2\tan\beta + \tan\gamma$

Sol. **[C]**

Given that $\alpha + \beta = \pi/2$ and $\beta + \gamma = \alpha$

so $\tan(\alpha + \beta) = \tan\pi/2$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{0}$$

$\tan\alpha\tan\beta = 1$ ... (i)

and $\tan\alpha = \tan(\beta + \gamma)$

$$\tan\alpha = \frac{\tan\beta + \tan\gamma}{1 - \tan\beta\tan\gamma}$$

$\Rightarrow \tan\alpha - \tan\alpha\tan\beta\tan\gamma = \tan\beta + \tan\gamma$

but $\tan\alpha\tan\beta = 1$

so $\tan\alpha = \tan\beta + 2\tan\gamma$

Q.4 The maximum value of $(\cos\alpha_1).\cos(\alpha_2)$ .... $(\cos\alpha_n)$, under the restrictions $0 \leq \alpha_1.\alpha_2$ ....$\alpha_n \leq \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2).(\cot\alpha_3)$...... ...$(\cot\alpha_n) = 1$ is **[IIT Scr. 2001]**

(A) $\frac{1}{2^{n/2}}$ (B) $\frac{1}{2^n}$ (C) $\frac{1}{2n}$ (D) 1

Sol. **[A]**

$\cot\alpha_1.\cot\alpha_2$...............$\cot\alpha_n = 1$

$\cos\alpha_1\cos\alpha_2$.......$\cos\alpha_n = \sin\alpha_1\sin\alpha_2$........$\sin\alpha_n$

$\cos^2\alpha_1.\cos^2\alpha_2$ .....$\cos^2\alpha_n = \sin^2\alpha,$

$\sin^2\alpha_2$.......$\sin^2\alpha_n$

$2^n(\cos^2\alpha_1$ ....$\cos^2\alpha_n) = 2\sin^2\alpha$, ...,$2\sin^2\alpha_n$

$= (1 - \cos 2\alpha_1)(1 - \cos 2\alpha_2)$......$(1 - \cos 2\alpha_n)$

This gives maximum value = 1

So, $\cos^2\alpha_1\cos^2\alpha_2$ ....$\cos^2\alpha_n = \frac{1}{2^n}$

$\Rightarrow \cos\alpha_1\cos\alpha_2$ ......$\cos\alpha_n = \frac{1}{2^{n/2}}$

Q.5 If θ and ϕ are acute angles such that $\sin\theta = \frac{1}{2}$ and $\cos\phi = \frac{1}{3}$ then $\theta + \phi$ lies in- **[IIT Scr. 2004]**

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

(C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left(\frac{\pi}{6}, \pi\right]$

Sol. **[B]**

Given $\sin\theta = \frac{1}{2}$ and $\cos\phi = \frac{1}{3}$

$\theta = \frac{\pi}{6}$ and $0 < \cos\phi < \frac{1}{2}$ $\because 0 < \frac{1}{3} < \frac{1}{2}$

$\theta = \frac{\pi}{6}$ and $\cos^{-1}(0) > \phi > \cos^{-1}\frac{1}{2}$

$\because \cos x$ is decreasing in $(0, \pi/2)$

$\theta = \frac{\pi}{6}$ and $\frac{\pi}{3} < \phi < \frac{\pi}{2} = \frac{\pi}{2} < (\theta + \phi) < \frac{2\pi}{3}$

$(\theta + \phi) \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

Q.6 $\cos(\alpha + \beta) = \frac{1}{e}$, $\cos(\alpha - \beta) = 1$ find no. of ordered pair of (α, β), $-\pi \le \alpha, \beta \le \pi$

[IIT Scr. 2005]

(A) 0 (B) 1 (C) 2 (D) 4

Sol. **[D]**

Since $\cos(\alpha - \beta) = 1$

$\Rightarrow \alpha - \beta = 2n\pi \quad \Rightarrow -2\pi < \alpha - \beta < 2\pi$

$\because \alpha \ge -\pi, \beta \le \pi \quad \therefore \alpha - \beta = 0$

Thus $\cos(\alpha + \beta) = \frac{1}{e}$

$\cos 2\alpha = \frac{1}{e} < 1$ which is true for four values of α.

Q.7 If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$ then **[IIT -2009]**

(A) $\tan^2 x = \frac{2}{3}$

(B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$

(D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

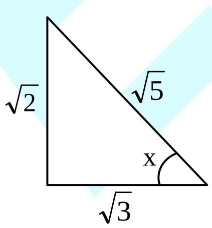
Sol. **[A, B]**

$\frac{(\sin x)^4}{2} + \frac{(\cos x)^4}{3} = \frac{1}{5}$

$3 - 6\cos^2 x + 5(\cos x)^4 = \frac{6}{5}$ Let $\cos x = t$

$25t^4 - 30t^2 + 9 = 0$

$t^2 = \frac{3}{5}$,



$\tan^2 x = \frac{2}{3} \Rightarrow (\sin x)^8 = \left(\frac{2}{5}\right)^4 = \frac{16}{625}$

$(\cos x)^8 = \left(\frac{\sqrt{3}}{\sqrt{5}}\right)^4 = \frac{81}{625}$

$\frac{(\sin x)^8}{8} + \frac{(\cos x)^8}{27} = \frac{1}{125}$

Q.8 For $0 < \theta < \frac{\pi}{2}$, the solution (s) of

$$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

is (are) : **[IIT -2009]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

Sol. **[C, D]**

$$\frac{1}{\sin(\pi/4)} \sum_{m=1}^{6} \frac{\sin\left[\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)}$$

$$\frac{1}{\sin(\pi/4)} \sum_{m=1}^{6} \left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right]$$

$$\Rightarrow \frac{1}{\sin\pi/4}\left[\cot\theta - \cot\left(\theta + \frac{\pi}{4}\right)\right] + \left[\cot\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{2\pi}{4}\right)\right] + \ldots\ldots\ldots + \left[\cot\left(\theta + \frac{5\pi}{4}\right) - \cot\left(\theta + \frac{6\pi}{4}\right)\right]$$

$$= \frac{1}{\sin\pi/4}\left[\cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right)\right] = 4\sqrt{2}$$

$\cot\theta + \tan\theta = 4$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = 4$

$\sin 2\theta = \frac{1}{2} \Rightarrow \quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}$

Q.9 The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is **[IIT -2010]**

Sol. $\frac{1}{\sin^2\theta + \cos^2\theta + 4\cos^2\theta + 3\sin\theta\cos\theta}$

$= \frac{1}{1 + 3\sin\theta\cos\theta + 4\cos^2\theta}$

$= \frac{1}{1 + 2(1 + \cos 2\theta) + \sin\theta\cos\theta + \sin 2\theta}$

$= \dfrac{1}{3+2\cos 2\theta+\sin 2\theta+\sin\theta\cos\theta}$

$= \dfrac{2}{6+4\cos 2\theta+2\sin 2\theta+2\sin\theta\cos\theta}$

$= \dfrac{2}{6+4\cos 2\theta+3\sin 2\theta}$

maximum value of 4 cos 2θ + 3 sin 2θ

$= \sqrt{(4)^2+(3)^2} = 5$

minimum value of 4 cos 2θ + 3 sin 2θ = –5

for question max. value = $\dfrac{2}{6-5} = \dfrac{2}{1} = 2$

Q.10 Let θ, φ ∈ [0, 2π] be such that

$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\dfrac{\theta}{2}+\cot\dfrac{\theta}{2}\right)\cos\varphi - 1,$

$\tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\dfrac{\sqrt{3}}{2}$. Then φ cannot satisfy **[IIT -2012]**

(A) $0 < \varphi < \dfrac{\pi}{2}$ (B) $\dfrac{\pi}{2} < \varphi < \dfrac{4\pi}{3}$

(C) $\dfrac{4\pi}{3} < \varphi < \dfrac{3\pi}{2}$ (D) $\dfrac{3\pi}{2} < \varphi < 2\pi$

Sol.[A, C, D]

tan (2π – θ) > 0

$\Rightarrow 0 < 2\pi-\theta < \dfrac{\pi}{2}$ or $\pi < 2\pi-\theta < \dfrac{3\pi}{2}$

$\Rightarrow \dfrac{3\pi}{2} < \theta < 2\pi$ or $\dfrac{\pi}{2} < \theta < \pi$ ....(1)

Also $-1 < \sin\theta < -\dfrac{\sqrt{3}}{2}$

$\dfrac{3\pi}{2} < \theta < \dfrac{5\pi}{3}$ .....(2)

from (1) & (2)

$\theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right)$ .....(3)

Now, $2\cos\theta\,(1-\sin\phi) = \sin^2\theta\left(\tan\dfrac{\theta}{2}+\cot\dfrac{\theta}{2}\right)\cos\phi - 1$

$\Rightarrow \cos\theta + \dfrac{1}{2} = \sin(\theta+\varphi)$ ....(4)

Now, $\theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right)$ from (3)

so $\cos\theta \in \left(0, \dfrac{1}{2}\right)$

$\sin(\theta+\varphi) \in \left(\dfrac{1}{2}, 1\right)$

Now, check option

(A) if $0 < \varphi < \dfrac{\pi}{2}$

then $\theta+\varphi \in \left(\dfrac{3\pi}{2}, \dfrac{11\pi}{6}\right)$ & $\sin(\theta+\varphi) \notin \left(\dfrac{1}{2}, 1\right)$

Similarly check option B, C, D.

EXERCISE # 5

Q.1 Prove that

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

[IIT 1988]

Sol. Taking L.H.S.

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$

We know that

$\tan\alpha + 2\cot 2\alpha = \cot\alpha$

$= \tan\alpha + 2\tan 2\alpha + 4(\tan 4\alpha + 2\cot 8\alpha)$

$= \tan\alpha + 2(\tan 2\alpha + 2\cot 4\alpha)$

$= \tan\alpha + 2\cot 2\alpha = \cot\alpha$

R.H.S. Hence proved.

Q.2 Find the value of

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}.$$

[IIT 1991]

Sol. **[A]**

$\Theta\ \sin\frac{7\pi}{14} = \sin\frac{\pi}{2} = 1$

and $\sin\frac{9\pi}{14} = \sin\left(\pi - \frac{5\pi}{14}\right) = \sin\frac{5\pi}{14}$

and $\sin\frac{11\pi}{14} = \sin\left(\pi - \frac{3\pi}{14}\right) = \sin\frac{3\pi}{14}$

and $\sin\frac{13\pi}{14} = \sin\left(\pi - \frac{\pi}{14}\right) = \sin\frac{\pi}{14}$

so given expression, we can write

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$$

$$= \left(\sin\frac{\pi}{14}\sin\left(\frac{\pi}{2} - \frac{4\pi}{14}\right)\sin\left(\frac{\pi}{2} - \frac{2\pi}{14}\right)\right)^2$$

$$= \left(\sin\frac{\pi}{14}\cos\frac{4\pi}{14}\cos\frac{2\pi}{14}\right)^2$$

$$= \left(\frac{2\cos\frac{\pi}{14}\sin\frac{\pi}{14}\cos\frac{2\pi}{14}\cos\frac{4\pi}{14}}{2\cos\frac{\pi}{14}}\right)^2$$

$$= \left(\frac{2\sin\frac{2\pi}{14}\cos\frac{2\pi}{14}\cos\frac{4\pi}{14}}{4\cos\frac{\pi}{14}}\right)^2$$

$$= \left(\frac{2\sin\frac{4\pi}{14}\cos\frac{4\pi}{14}}{8\cos\frac{\pi}{14}}\right)^2$$

$$= \left(\frac{\sin\frac{8\pi}{14}}{8\cos\frac{\pi}{14}}\right)^2 = \left(\frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{14}\right)}{8\cos\frac{\pi}{14}}\right)^2$$

$$= \left(\frac{\cos\frac{\pi}{14}}{8\cos\frac{\pi}{14}}\right)^2 = \frac{1}{64}$$

Q.3 Let A, B, C be three angles such that A = π/4 and tan B tan C = p. Find all possible values of p such that A, B, C are the angles of a triangle. **[IIT 1997]**

Sol. Given $\angle A = \frac{\pi}{4}$ and tan B tan C = P

We know A + B + C = π

$B + C = \frac{3\pi}{4}$ = tan B tan C = P

$\Rightarrow \tan B \tan\left(\frac{3\pi}{4} - B\right) = P$

$\Rightarrow \tan B\left(\frac{-1 - \tan B}{1 - \tan B}\right)$ $\quad \Theta\ \tan\frac{3\pi}{4} = -1$

$\Rightarrow \tan^2 B + (1 - P)\tan B + P = 0 \quad D \ge 0$

$\Rightarrow (1 - P)^2 - 4P \ge 0 \Rightarrow P^2 - 6P + 1 \ge 0$

roots are $\alpha = 3 - \sqrt{2}$ $\quad \beta = 3 + 2\sqrt{2}$

$(P - \alpha)(P - \beta) \ge 0$ $P \le \alpha$ or $P \ge \beta$

So $P \in (-\infty, 3 - \sqrt{2}\,] \cup [3 + 2\sqrt{2}\,, \infty)$

Q.4 Prove that a triangle ABC is equilateral if and only if $\tan A + \tan B + \tan C = 3\sqrt{3}$.**[IIT 1998]**

Sol. If the triangle is equilateral, then A = B = C = 60°

$\Rightarrow \tan A + \tan B + \tan C = 3\tan 60° = 3\sqrt{3}$

conversely assume that

$\tan A + \tan B + \tan C = 3\sqrt{3}$

But in ΔABC, A + B + C = π

A + B = π – C

Taking tan of both side $\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\Rightarrow \tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow \tan A \tan B \tan C = 3\sqrt{3}$

None of the tan A, tan B, tan C can be negative so ΔABC cannot be obtuse angle triangle.

Also A.M. ≥ G.M.

$\frac{\tan A + \tan B + \tan C}{3} \ge [\tan A \tan B \tan C]^{1/3}$

$\sqrt{3} \ge \sqrt{3}$ not possible

So equality hold only when A = B = C or when triangle is equilateral.

Q.5 If $\cos^2\theta = \frac{m^2 - 1}{3}$ & $\tan^3\frac{\theta}{2} = \tan\alpha$.

Prove that $\cos^{2/3}\alpha + \sin^{2/3}\alpha = \left(\frac{2}{m}\right)^{2/3}$.

Sol. $\cos^2\theta = \frac{m^2 - 1}{3}$ and $\tan^3\frac{\theta}{2} = \tan\alpha$

$\because \tan^3\frac{\theta}{2} = \tan\alpha$

$\Rightarrow \tan\frac{\theta}{2} = \tan^{1/3}\alpha$

$\Rightarrow \tan^2\frac{\theta}{2} = \tan^{2/3}\alpha$

$\Rightarrow \tan^{2/3}\alpha = \tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{1 - \frac{\sqrt{m^2-1}}{\sqrt{3}}}{1 + \frac{\sqrt{m^2-1}}{\sqrt{3}}}$

$\Rightarrow \frac{\sin^{2/3}\alpha}{\cos^{2/3}\alpha} = \frac{\sqrt{3} - \sqrt{m^2-1}}{\sqrt{3} + \sqrt{m^2-1}}$

$\Rightarrow \frac{\sin^{2/3}\alpha}{\sqrt{3} - \sqrt{m^2-1}} = \frac{\cos^{2/3}\alpha}{\sqrt{3} + \sqrt{m^2-1}}$

$\Rightarrow \frac{a}{p} = \frac{b}{q}$ (Let)

Now apply roles of ratio proportion

$\frac{a}{p} = \frac{b}{q} = \frac{a+b}{p+q} = \left(\frac{a^3+b^3}{p^3+q^3}\right)^{1/3}$

$p + q = 2\sqrt{3}$, $p^3 + q^3 = 6\sqrt{3}m^2$

$a^3 + b^3 = \sin^2\alpha + \cos^2\alpha = 1$

$a + b = \sin^{2/3}\alpha + \cos^{2/3}\alpha$

$= 2\sqrt{3}\left(\frac{1}{6\sqrt{3}m^2}\right)^{1/3} = \left[\frac{24\sqrt{3}}{6\sqrt{3}m^2}\right]^{1/3} = \left(\frac{2}{3}\right)^{2/3}$

$\therefore \sin^{2/3}\alpha + \cos^{2/3}\alpha = \left(\frac{2}{m}\right)^{2/3}$

Q.6 If $\tan\left(\frac{\pi}{4} + \frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4} + \frac{x}{2}\right)$. Prove that

$\sin y = \sin x\left[\frac{3 + \sin^2 x}{1 + 3\sin^2 x}\right]$.

Sol. We know that

$\tan\left(\frac{\pi}{4} + \alpha\right) = \frac{1 + \tan\alpha}{1 - \tan\alpha} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}$

$\Rightarrow \tan^2\left(\frac{\pi}{4} + \alpha\right) = \frac{1 + \sin 2\alpha}{1 - \sin 2\alpha}$

Given $\tan\left(\frac{\pi}{4} + \frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4} + \frac{x}{2}\right)$

$\Rightarrow \frac{1 + \sin y}{1 - \sin y} = \frac{(1 + \sin x)^3}{(1 - \sin x)^3}$

using componendo and dividendo, we get

$\frac{2\sin y}{2} = \frac{2(3\sin x + \sin^3 x)}{2(1 + 3\sin^2 x}$

$\sin y = \frac{\sin x(3 + \sin^2 x)}{(1 + 3\sin^2 x)}$ Hence proved.

Q.7 If A + B + C = 2S, then prove that,

sin (S – A) + sin (S – B) + sin (S – C)

$-\sin S = 4\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$.

Sol. Given A + B + C = 2S

Taking L.H.S.

sin (S –A) + sin (S – B) + sin (S – C) – sin S

using C & D formula

$= 2\sin\left(\frac{2s-(A+B)}{2}\right)\cos\frac{B-A}{2}$

$+ 2\cos\left(\frac{2S-C}{2}\right)\sin\left(\frac{-C}{2}\right)$

$\Rightarrow 2\sin\frac{C}{2}\cos\frac{B-A}{2} - 2\cos\frac{A+B}{2}\sin\frac{C}{2}$

$= 2\sin\frac{C}{2}\left(\cos\frac{B-A}{2} - \cos\frac{A-B}{2}\right)$

$= 2\sin\frac{C}{2}\left(2\sin\frac{B}{2}\sin\frac{A}{2}\right)$

$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ Hence proved.

Q.8 If $\alpha + \beta + \gamma + \delta = 2\pi$.

Prove that $\cos\alpha + \cos\beta + \cos\gamma + \cos\delta$

$+ 4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right) = 0.$

Sol. Given $\alpha + \beta + \gamma + \delta = 2\pi$

Taking L.H.S., we have

$\cos\alpha + \cos\beta + \cos\gamma + \cos\delta + 4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$

Using Componendo & Dividendo formula

$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\cos\frac{\gamma+\delta}{2}\cos\frac{\gamma-\delta}{2} +$

$4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$

$\gamma + \delta = 2\pi - (\alpha + \beta)$

so $\cos\frac{\gamma+\delta}{2} = \cos\left(\pi - \frac{\alpha+\beta}{2}\right) = -\cos\left(\frac{\alpha+\beta}{2}\right)$

$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\gamma-\delta}{2}\right)\right] + 4$

$\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right) = 2\cos\left(\frac{\alpha+\beta}{2}\right)$

$\left[2\cos\left(\frac{\alpha+\gamma-(\beta+\delta)}{4}\right)\sin\frac{(\gamma+\beta)-(\alpha+\delta)}{4}\right] + 4$

$\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$

$\because \beta + \delta = 2\pi - (\alpha + \gamma)$ and $\gamma + \beta = 2\pi - (\alpha + \delta)$

so, $\sin\frac{\alpha+\gamma-(\beta+\delta)}{4}$

$= \sin\frac{2(\alpha+\gamma)-2\pi}{4} = -\sin\left(\frac{\pi}{2} - \frac{\alpha+\gamma}{2}\right)$

$= -\cos\frac{\alpha+\gamma}{2}$ and $\sin\frac{\gamma+\beta-(\alpha+\delta)}{4}$

$= \sin\left(\frac{2\pi-2(\alpha+\delta)}{4}\right) = \cos\left(\frac{\alpha+\delta}{2}\right)$

Then $\Rightarrow 2\cos\left(\frac{\alpha+\beta}{2}\right)$

Q.9 Show that $\tan\frac{\pi}{16} = \sqrt{4+2\sqrt{2}} - (\sqrt{2} + 1)$.

Sol. $\frac{\pi}{16} = 11\frac{1}{4}^{o}$

$\tan 22½^{o} = \frac{1-\cos 45^{o}}{\sin 45^{o}} = \sqrt{2} - 1 = \frac{1}{\sqrt{2}+1}$ ...(i)

Let $A = 11\frac{1}{4}^{o}$, $2A = 22½^{o}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

from (i),

$\frac{1}{\sqrt{2}+1} = \frac{2t}{1-t^2}$ Let tan A = t

$1 - t^2 = 2\sqrt{2}\,t + 2t$

$t^2 + 2(\sqrt{2} + 1)t - 1 = 0$

$\because t = \tan 11\frac{1}{4}^{o}$ +ve = 0

so $t = \frac{-2(\sqrt{2}+1)+2\sqrt{3+2\sqrt{2}+1}}{2}$

$\tan\frac{\pi}{16} = \sqrt{4+2\sqrt{2}} - (\sqrt{2} + 1)$ Hence proved.

Q.10 If $A + B + C = 2\pi$, prove that

$\sin^3 A + \sin^3 B + \sin^3 C = 3\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)$

$\sin\left(\frac{C}{2}\right) - \sin\left(\frac{3A}{2}\right).\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right).$

Sol. Given $A + B + C = 2\pi$

Taking L.H.S. we have $\sin^3 A + \sin^3 B + \sin^3 C$

$\because \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

So $\Rightarrow \frac{1}{4}$ [3(sin A + sin B + sin C) –

(sin3A + sin 3B + sin 3C)]

$$\Rightarrow \frac{1}{4}\left[3\left\{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+\sin(2\pi-(A+B)\right\}\right.$$

$$\left.-\left(2\sin\frac{3A+3B}{2}\cos\frac{3A-3B}{2}+\sin(6\pi-(3A+3B))\right)\right]$$

$$\Rightarrow \frac{1}{4}\left[3\left\{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}-2\sin\frac{A+B}{2}\cos\frac{A+B}{2}\right\}\right.$$

$$\left.-\left(2\sin\frac{3A+3B}{2}\cos\frac{3A-3B}{2}-2\sin\frac{3A+3B}{2}\cos\frac{3A+3B}{2}\right)\right]$$

$$= \frac{1}{4}\left[6\sin\frac{A+B}{2}\left\{\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right\}\right.$$

$$\left.-2\sin\frac{3A+3B}{2}\left\{\cos\frac{3A-3B}{2}-\cos\frac{3A+3B}{2}\right\}\right]$$

$$= \frac{1}{2}\left[3\sin\frac{C}{2} : 2\sin\frac{A}{2}\sin\frac{B}{2}-\sin\frac{3C}{2}\right.$$

$$\left.\left(2\sin\frac{3A}{2}\sin\frac{3B}{2}\right)\right]$$

$$= 3\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} - \sin\frac{3A}{2}\sin\frac{3B}{2}\sin\frac{3C}{2}$$

Hence proved.

Q.11 Prove that $(4\cos^2 9° - 3)(4\cos^2 27° - 3) = \tan 9°$.

LHS = $(4\cos^2 9° - 3)(4\cos^2 27° - 3)$

$= \frac{\cos 27°}{\cos 9°} \times \frac{\cos 81°}{\cos 27°}$ [use $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$]

$= \frac{\cos 81°}{\cos 9°} = \frac{\sin 9°}{\cos 9°} = \tan 9°$

Q.12 In ΔABC, prove that $\cos A + \cos B + \cos C \le \frac{3}{2}$.

Sol. In a ΔABC

A + B + C = π

$\therefore \cos A + \cos B + \cos C - \frac{3}{2} = 2\cos\left(\frac{A+B}{2}\right)$

$\cos\left(\frac{A-B}{2}\right) - \frac{3}{2} + 1 - 2\sin^2\frac{C}{2}$

$= -2\sin^2\frac{C}{2} + 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - \frac{1}{2}$

$= -2\left[\sin^2\frac{C}{2} - \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)\right] - \frac{1}{2}$

$$= -2\left[\sin\frac{C}{2} - \frac{\cos\left(\frac{A-B}{2}\right)}{2}\right]^2 + \frac{\cos^2\left(\frac{A-B}{2}\right)}{2} - \frac{1}{2}$$

$$= -2\left[\sin\frac{C}{2} - \frac{\cos\left(\frac{A-B}{2}\right)}{2}\right]^2 - \frac{\sin^2\left(\frac{A-B}{2}\right)}{2} \le 0$$

$\therefore \cos A + \cos B + \cos C \le \frac{3}{2}$

Q.13 If A , B & C are angles of a triangle then prove that $\tan^2\left(\frac{A}{2}\right) + \tan^2\left(\frac{B}{2}\right) + \tan^2\left(\frac{C}{2}\right) \ge 1$.

Sol. In a ΔABC

A + B + C = π

$\Rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$

$\Rightarrow \tan\left(\frac{A}{2}+\frac{B}{2}+\frac{C}{2}\right) = \tan\frac{\pi}{2}$

on solving, we get

$\Sigma\tan\frac{A}{2}\tan\frac{B}{2} = 1$ .....(1)

Now we know

$(x-y)^2 + (y-z)^2 + (z-x)^2 \ge 0$

$\Rightarrow x^2 + y^2 + z^2 \ge xy + yz + zx$

so

$\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \ge \Sigma\tan\frac{A}{2}\tan\frac{B}{2}$ .....(2)

from (1) & (2)

$\Sigma\tan^2\frac{A}{2} \ge 1$

Q.14 Prove that $\frac{\tan 2^n\theta}{\tan\theta} = (1 + \sec 2\theta).(1 + \sec 2^2\theta)$ $(1 + \sec 2^3\theta)$ …….. $(1 + \sec 2^n\theta)$.

Sol. RHS

(1 + sec 2θ) (1 + sec2^2θ) (1 + sec2^3θ) ..... (1 + sec2^nθ)

$$= \left(\frac{1+\cos 2\theta}{\cos 2\theta}\right)\left(\frac{1+\cos 4\theta}{\cos 4\theta}\right)\left(\frac{1+\cos 8\theta}{\cos 8\theta}\right)\cdots\left(\frac{1+\cos 2^n\theta}{\cos 2^n\theta}\right)$$

$$= \frac{2\cos^2\theta . 2\cos^2 2\theta . 2\cos^2 4\theta ......2\cos^2 2^{n-1}\theta}{\cos 2\theta . \cos 4\theta \;.\; \cos 8\theta .....\cos 2^n\theta}$$

$$= \frac{\cos\theta(2\cos\theta)(2\cos 2\theta)(2\cos 4\theta).....(2\cos 2^{n-1}\theta)}{\cos 2^n\theta}$$

$$= \frac{\cos\theta(2\sin\theta\cos\theta)\;(2\cos 2\theta)(2\cos 4\theta)......(2\cos 2^{n-1}\theta)}{\sin\theta \;.\; \cos 2^n\theta}$$

$$= \frac{\cos\theta.\sin 2\theta(2\cos 2\theta)(2\cos 4\theta)......(2\cos 2^{n-1}\theta)}{\sin\theta \;.\; \cos 2^n\theta}$$

$$= \frac{\cos\theta.\sin 4\theta(2\cos 4\theta)......(2\cos 2^{n-1}\theta)}{\sin\theta \;.\; \cos 2^n\theta}$$

M M M M

M M M M

$$= \frac{\cos\theta(2\sin 2^{n-1}\theta\cos 2^{n-1}\theta)}{\sin\theta \;.\; \cos 2^n\theta}$$

$$= \cot\theta \;\frac{\sin 2^n\theta}{\cos 2^n\theta} = \cot\theta \;.\; \tan 2^n\theta$$

Q.15 Show that

$$\frac{1}{2}[\tan 27x - \tan x] = \frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x}.$$

Sol. $\frac{\sin x}{\cos 3x} = \frac{2\sin x\cos x}{2\cos x\cos 3x} = \frac{\sin 2x}{2\cos x.\cos 3x} = \frac{\sin(3x-x)}{2\cos x\cos 3x} = \frac{1}{2}[\tan 3x - \tan x]$

similarly

$$\frac{\sin 3x}{\cos 9x} = \frac{\tan 9x - \tan 3x}{2}$$

$$\frac{\sin 9x}{\cos 27x} = \frac{\tan 27x - \tan 9x}{2}$$

Add. All

ANSWER KEY

EXERCISE # 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	B	C	D	A	B	D	C	C	C	B	C	B	B	C
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**							
Ans.	C	B	A	A	B	B	A	B							

24. True **25.** False

26. (i) $\frac{2bc}{a^2+b^2}$, (ii) $\frac{c^2-a^2}{a^2+b^2}$, (iii) $\frac{2ac}{a^2+b^2}$ (iv) $\frac{c^2-b^2}{a^2+b^2}$, (v) $\sin 2A\ x^2 - 2x + \sin 2A = 0$

EXERCISE # 2

PART-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	C	C	C	B	A	A	A	A	B	D	C	D	B
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**									
Ans.	C	B	B	A	C	B									

PART-B

Q.No.	22	23	24	25	26	27	28	29
Ans.	A,B,D	A	A,C,D	A,B	A,B,C,D	A,C	A,B,C	C,D

PART-C

Q.No.	30	31
Ans.	B	D

PART-D

32. A $\rightarrow$ Q, B $\rightarrow$ S, C $\rightarrow$ R, D $\rightarrow$P **33.** A$\rightarrow$Q, B$\rightarrow$S, C $\rightarrow$ P, D $\rightarrow$ R

PART-E

34. $\frac{2ac}{a^2-c^2}$ **35.** $\frac{4}{\sqrt{3}}$

EXERCISE # 3

17. (a) $\sqrt{3}$ (b) 4 **18.** (A) **19.** (B) **20.** (D) **21.** (D)
22. (C) **23.** (A) **24.** (A) **25.** (A) **26.** (C)

EXERCISE # 4

2. C **3.** C **4.** A **5.** B **6.** D **7.** A, B
8. C, D **9.** 2 **10.** A, C, D

EXERCISE # 5

2. $\frac{1}{64}$ **3.** $(-\infty, 0) \cup [(\sqrt{2} + 1)^2, \infty)$