BINOMIAL THEOREM

Expansion of binomial theorem

1. The value of $(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$ is

(A) 252 (B) 352
(C) 452 (D) 532

2. In the expansion of the following expression

$1+(1+x)+(1+x)^2+.....+(1+x)^n$

the coefficient of $x^k (0\le k\le n)$ is

(A) ${}^{n+1}C_{k+1}$ (B) ${}^nC_k$
(C) ${}^nC_{n-k-1}$ (D) None of these

3. The larger of $99^{50}+100^{50}$ and 101^{50} is

(A) $99^{50}+100^{50}$ (B) Both are equal
(C) 101^{50} (D) None of these

4. $(1+x)^n-nx-1$ divisible (where $n\in N$)

(A) by $2x$
(B) by x^2
(C) by $2x^3$
(D) All of these

5. If $T_0, T_1, T_2, ...T_n$ represent the terms in the expansion of $(x+a)^n$, then $(T_0-T_2+T_4-....)^2+(T_1-T_3+T_5-....)^2=$

(A) (x^2+a^2) (B) $(x^2+a^2)^n$
(C) $(x^2+a^2)^{1/n}$ (D) $(x^2+a^2)^{-1/n}$

General term, Coefficient of any power of *x*, Independent term, Middle term and Greatest term and Greatest coefficient

6. In $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ if the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is $\frac{1}{6}$, then $n=$

(A) 7 (B) 8
(C) 9 (D) None of these

7. If coefficient of $(2r+3)^{th}$ and $(r-1)^{th}$ terms in the expansion of $(1+x)^{15}$ are equal, then value of *r* is

(A) 5 (B) 6
(C) 4 (D) 3

8. If x^4 occurs in the r^{th} term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then $r=$

(A) 7 (B) 8
(C) 9 (D) 10

9. If the $(r+1)^{th}$ term in the expansion of $\left(\sqrt[3]{\frac{a}{\sqrt{b}}}+\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{21}$ has the same power of *a* and *b*, then the value of *r* is

(A) 9 (B) 10
(C) 8 (D) 6

10. If the third term in the binomial expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then the rational value of *m* is

(A) 2 (B) $1/2$
(C) 3 (D) 4

11. The first 3 terms in the expansion of $(1+ax)^n$ $(n\ne 0)$ are 1, $6x$ and $16x^2$. Then the value of *a* and *n* are respectively

(A) 2 and 9 (B) 3 and 2
(C) 2/3 and 9 (D) 3/2 and 6

12. If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in A.P., then $r=$

(A) 6 (B) 7
(C) 8 (D) 9

13. Coefficient of *x* in the expansion of $\left(x^2+\frac{a}{x}\right)^5$ is

(A) $9a^2$ (B) $10a^3$
(C) $10a^2$ (D) $10a$

14. If the coefficients of p^{th}, $(p+1)^{th}$ and $(p+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P., then

(A) $n^2 - 2np + 4p^2 = 0$

(B) $n^2 - n(4p+1) + 4p^2 - 2 = 0$

(C) $n^2 - n(4p+1) + 4p^2 = 0$

(D) None of these

15. In the expansion of $\left(\frac{a}{x} + bx\right)^{12}$, the coefficient of x^{-10} will be

(A) $12a^{11}$ (B) $12b^{11}a$

(C) $12a^{11}b$ (D) $12a^{11}b^{11}$

16. The coefficient of x^{53} in the following expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} . 2^m$ is

(A) ${}^{100}C_{47}$ (B) ${}^{100}C_{53}$

(C) $-{}^{100}C_{53}$ (D) $-{}^{100}C_{100}$

17. The coefficient of x^{32} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is

(A) ${}^{15}C_5$ (B) ${}^{15}C_6$

(C) ${}^{15}C_4$ (D) ${}^{15}C_7$

18. If the coefficients of x^7 and x^8 in $\left(2 + \frac{x}{3}\right)^n$ are equal, then *n* is

(A) 56 (B) 55

(C) 45 (D) 15

19. The coefficient of x^3 in the expansion of $\left(x - \frac{1}{x}\right)^7$ is

(A) 14 (B) 21

(C) 28 (D) 35

20. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and – 6 respectively, then *m* is

(A) 6 (B) 9

(C) 12 (D) 24

21. If x^m occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{2n}$, then the coefficient of x^m is

(A) $\frac{(2n)!}{(m)!(2n-m)!}$

(B) $\frac{(2n)!3!3!}{(2n-m)!}$

(C) $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$

(D) None of these

22. If coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the binomial expansion of $(1+x)^n$ are in A.P., then $n^2 - 9n$ is equal to

(A) – 7 (B) 7

(C) 14 (D) – 14

23. In the expansion of $(1 + x + x^3 + x^4)^{10}$, the coefficient of x^4 is

(A) ${}^{40}C_4$ (B) ${}^{10}C_4$

(C) 210 (D) 310

24. If coefficients of $(2r+1)^{th}$ term and $(r+2)^{th}$ term are equal in the expansion of $(1+x)^{43}$, then the value of r will be

(A) 14 (B) 15

(C) 13 (D) 16

25. If the coefficient of 4^{th} term in the expansion of $(a+b)^n$ is 56, then *n* is

(A) 12 (B) 10

(C) 8 (D) 6

26. If in the expansion of $(1+x)^{21}$, the coefficients of x^r and x^{r+1} be equal, then r is equal to

(A) 9 (B) 10
(C) 11 (D) 12

27. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ will be

(A) 3/2 (B) 5/4
(C) 5/2 (D) None of these

28. The term independent of x in the expansion of $\left(\frac{1}{2}x^{1/3}+x^{-1/5}\right)^8$ will be

(A) 5 (B) 6
(C) 7 (D) 8

29. In the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$,the term independent of x is

(A) ${}^9C_3.\frac{1}{6^3}$ (B) ${}^9C_3\left(\frac{3}{2}\right)^3$
(C) ${}^9C_3$ (D) None of these

30. The term independent of x in $\left(2x-\frac{1}{2x^2}\right)^{12}$ is

(A) – 7930 (B) – 495
(C) 495 (D) 7920

31. In the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term independent of x is

(A) ${}^{15}C_6 2^6$ (B) ${}^{15}C_5 2^5$
(C) ${}^{15}C_4 2^4$ (D) ${}^{15}C_8 2^8$

32. The term independent of x in the expansion of $\left(x^2-\frac{1}{x}\right)^9$ is

(A) 1 (B) –1
(C) – 48 (D) None of these

33. The term independent of x in the expansion of $\left(2x+\frac{1}{3x}\right)^6$ is

(A) $\frac{160}{9}$ (B) $\frac{80}{9}$
(C) $\frac{160}{27}$ (D) $\frac{80}{3}$

34. The term independent of x in the expansion $\left(x^2-\frac{1}{3x}\right)^9$ is

(A) $\frac{28}{81}$ (B) $\frac{28}{243}$
(C) $-\frac{28}{243}$ (D) $-\frac{28}{81}$

35. The term independent of x in the expansion of $\left(2x-\frac{3}{x}\right)^6$ is

(A) 4320 (B) 216
(C) – 216 (D) – 4320

36. The coefficient of middle term in the expansion of $(1+x)^{10}$ is

(A) $\frac{10!}{5!6!}$ (B) $\frac{10!}{(5!)^2}$
(C) $\frac{10!}{5!7!}$ (D) None of these

37. The middle term in the expansion of $(1+x)^{2n}$ is

(A) $\frac{(2n)!}{n!}x^2$

(B) $\frac{(2n)!}{n!(n-1)!}x^{n+1}$

(C) $\frac{(2n)!}{(n!)^2}x^n$

(D) $\frac{(2n)!}{(n+1)!(n-1)!}x^n$

38. The greatest coefficient in the expansion of $(1+x)^{2n+2}$ is

(A) $\frac{(2n)!}{(n!)^2}$ (B) $\frac{(2n+2)!}{\{(n+1)!\}^2}$

(C) $\frac{(2n+2)!}{n!(n+1)!}$ (D) $\frac{(2n)!}{n!(n+1)!}$

39. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is

(A) $\frac{25840}{9}$ (B) $\frac{24840}{9}$

(C) $\frac{26840}{9}$ (D) None of these

40. If *n* is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also, is

(A) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (B) $\frac{n+1}{n} < x < \frac{n}{n+1}$

(C) $\frac{n}{n+4} < x < \frac{n+4}{4}$ (D) None of these

Properties of binomial coefficients

41. $C_1 + 2C_2 + 3C_3 + 4C_4 + ... + nC_n =$

(A) 2^n (B) $n.\ 2^n$

(C) $n.\ 2^{n-1}$ (D) $n.\ 2^{n+1}$

42. $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + ... =$

(A) $\frac{2^{n+1}}{n+1}$ (B) $\frac{2^{n+1}-1}{n+1}$

(C) $\frac{2^n}{n+1}$ (D) None of these

43. $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} =$

(A) $\frac{2^n}{n+1}$ (B) $\frac{2^n - 1}{n+1}$

(C) $\frac{2^{n+1}-1}{n+1}$ (D) None of these

44. $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ... =$

(A) $\frac{2^n}{n!}$; for all even values of *n*

(B) $\frac{2^{n-1}}{n!}$; for all values of *n i.e.*, all even odd values

(C) 0

(D) None of these

45. The sum to $(n+1)$ terms of the following series $\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + ...$ is

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n+2}$

(C) $\frac{1}{n(n+1)}$ (D) None of these

46. If *a* and *d* are two complex numbers, then the sum to $(n+1)$ terms of the following series

$aC_0 - (a+d)C_1 + (a+2d)C_2 - ....$ is

(A) $\frac{a}{2^n}$ (B) na

(C) 0 (D) None of these

47. If $(1+x)^{15} = C_0 + C_1x + C_2x^2 + ... + C_{15}x^{15}$, then $C_2 + 2C_3 + 3C_4 + ... + 14C_{15} =$

(A) 14.2^{14} (B) $13.2^{14}+1$

(C) $13.2^{14}-1$ (D) None of these

48. The value of $\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+.....$ is equal to

(A) $\frac{2^n-1}{n+1}$ (B) $n.2^n$

(C) $\frac{2^n}{n}$ (D) $\frac{2^n+1}{n+1}$

49. In the expansion of $(1+x)^n$ the sum of coefficients of odd powers of *x* is

(A) 2^n+1 (B) 2^n-1

(C) 2^n (D) 2^{n-1}

50. $C_0-C_1+C_2-C_3+.....+(-1)^n C_n$ is equal to

(A) 2^n (B) 2^n-1

(C) 0 (D) 2^{n-1}

51. Coefficients of $x^r[0\le r\le (n-1)]$ in the expansion of

$(x+3)^{n-1}+(x+3)^{n-2}(x+2)$
$+(x+3)^{n-3}(x+2)^2+...+(x+2)^{n-1}$

(A) ${}^nC_r(3^r-2^n)$

(B) ${}^nC_r(3^{n-r}-2^{n-r})$

(C) ${}^nC_r(3^r+2^{n-r})$

(D) None of these

52. If the sum of the coefficients in the expansion of $(\alpha^2x^2-2\alpha x+1)^{51}$ vanishes, then the value of α is

(A) 2 (B) –1

(C) 1 (D) – 2

53. If $x+y=1$, then $\sum_{r=0}^{n} r^2\ {}^nC_r x^r y^{n-r}$ equals

(A) *nxy* (B) $nx(x+yn)$

(C) $nx(nx+y)$ (D) None of these

54. The value of

${}^{4n}C_0+{}^{4n}C_4+{}^{4n}C_8+....+{}^{4n}C_{4n}$ is

(A) $2^{4n-2}+(-1)^n2^{2n-1}$ (B) $2^{4n-2}+2^{2n-1}$

(C) $2^{2n-1}+(-1)^n2^{4n-2}$ (D) None of these

55. The sum of the last eight coefficients in the expansion of $(1+x)^{15}$ is

(A) 2^{16} (B) 2^{15}

(C) 2^{14} (D) None of these

56. If $(1+x)^n=C_0+C_1x+C_2x^2+....+C_nx^n$, then the value of

$C_0+2C_1+3C_2+....+(n+1)C_n$ will be

(A) $(n+2)2^{n-1}$ (B) $(n+1)2^n$

(C) $(n+1)2^{n-1}$ (D) $(n+2)2^n$

57. The value of ${}^{15}C_0^2-{}^{15}C_1^2+{}^{15}C_2^2-....-{}^{15}C_{15}^2$ is

(A) 15 (B) – 15

(C) 0 (D) 51

58. $2C_0+\frac{2^2}{2}C_1+\frac{2^3}{3}C_2+....+\frac{2^{11}}{11}C_{10}$

(A) $\frac{3^{11}-1}{11}$ (B) $\frac{2^{11}-1}{11}$

(C) $\frac{11^3-1}{11}$ (D) $\frac{11^2-1}{11}$

59. If $(1+x)^n=C_0+C_1x+C_2x^2+....+C_nx^n$, then $C_0C_2+C_1C_3+C_2C_4+C_{n-2}C_n$ equals

(A) $\frac{(2n)!}{(n+1)!(n+2)!}$

(B) $\frac{(2n)!}{(n-2)!(n+2)!}$

(C) $\frac{(2n)!}{(n)!(n+2)!}$

(D) $\frac{(2n)!}{(n-1)!(n+2)!}$

60. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then the value of $C_0 + C_2 + C_4 + C_6 + ...$ is

(A) 2^{n-1} (B) 2^{n-1}

(C) 2^n (D) $2^{n-1} - 1$

61. The sum of coefficients in $(1+x-3x^2)^{2134}$ is

(A) – 1 (B) 1

(C) 0 (D) 2^{2134}

62. The sum of coefficients in the expansion of $(1+x+x^2)^n$ is

(A) 2 (B) 3^n

(C) 4^n (D) 2^n

63. The sum of the coefficients in the expansion of $(1+x-3x^2)^{3148}$ is

(A) 7 (B) 8

(C) – 1 (D) 1

64. If $a_k = \frac{1}{k(k+1)}$, for $k = 1, 2, 3, 4, ..., n$, then $\left(\sum_{k=1}^{n} a_k\right)^2 =$

(A) $\left(\frac{n}{n+1}\right)$

(B) $\left(\frac{n}{n+1}\right)^2$

(C) $\left(\frac{n}{n+1}\right)^4$

(D) $\left(\frac{n}{n+1}\right)^6$

65. In the expansion of $(1+x)^5$, the sum of the coefficient of the terms is

(A) 80 (B) 16

(C) 32 (D) 64

Binomial theorem for any index

66. The coefficient of x^3 in the expansion of $\frac{(1+3x)^2}{1-2x}$ will be

(A) 8 (B) 32

(C) 50 (D) None of these

67. If $|x| < 1$ then the coefficient of x^n in the expansion of $(1 + x + x + x^2 + …)^2$ will be

(A) 1 (B) n

(C) $n+1$ (D) None of these

68. If $|x| > 1$, then $(1+x)^{-2} =$

(A) $1 - 2x + 3x^2 - ….$

(B) $1 + 2x + 3x^2 - ….$

(C) $1 - \frac{2}{x} + \frac{3}{x^2} - ….$

(D) $\frac{1}{x^2} - \frac{2}{x^3} + \frac{3}{x^4} - ….$

69. If $|x| < 1$, then in the expansion of $(1+2x+3x^2+4x^3+....)^{1/2}$, the coefficient of x^n is

(A) n (B) $n+1$

(C) 1 (D) – 1

70. The approximate value of $(7.995)^{1/3}$ correct to four decimal places is [

(A) 1.9995 (B) 1.9996

(C) 1.9990 (D) 1.9991

71. $1 - \frac{1}{8} + \frac{1}{8}.\frac{3}{16} - \frac{1.3.5}{8.16.24} + ... =$

(A) $\frac{2}{5}$ (B) $\frac{\sqrt{2}}{5}$

(C) $\frac{2}{\sqrt{5}}$ (D) None of these

72. If $(r+1)^{th}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of *r* is

(A) 5 (B) 6

(C) 4 (D) 7

73. The coefficient of x^n in the expansion of $(1-2x+3x^2-4x^3+.....)^{-n}$ is

(A) $\frac{(2n)!}{n!}$ (B) $\frac{(2n)!}{(n!)^2}$

(C) $\frac{1}{2}\frac{(2n)!}{(n!)^2}$ (D) None of these

74. The coefficient of x^n in the expansion of $(1-9x+20x^2)^{-1}$ is

(A) 5^n-4^n (B) $5^{n+1}-4^{n+1}$

(C) $5^{n-1}-4^{n-1}$ (D) None of these

75. The coefficient of x^n in the expansion of $\frac{1}{(1-x)(3-x)}$ is

(A) $\frac{3^{n+1}-1}{2.3^{n+1}}$ (B) $\frac{3^{n+1}-1}{3^{n+1}}$

(C) $\left(\frac{3^{n+1}-1}{3^{n+1}}\right)$ (D) None of these

76. The coefficient of x^n in the expansion of $(1+x+x^2+....)^{-n}$ is

(A) 1 (B) $(-1)^n$

(C) *n* (D) $n+1$

77. If $y=3x+6x^2+10x^3+....$, then the value of *x* in terms of *y* is

(A) $1-(1-y)^{-1/3}$

(B) $1-(1+y)^{1/3}$

(C) $1+(1+y)^{-1/3}$

(D) $1-(1+y)^{-1/3}$

78. The coefficient of x in the expansion of $[\sqrt{1+x^2}-x]^{-1}$ in ascending powers of *x*, when $|x|<1$, is

(A) 0 (B) $\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) 1

79. $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+..........=$

(A) $\sqrt{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$

80. If *x* is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is

(A) 7^{th} term (B) 5^{th} term

(C) 8^{th} term (D) 6^{th} term

Multinomial theorem, Terms free from radical sign in the expansion of ($a^{1/p}+b^{1/q}$), Problems regarding to three/four consecutive terms or coefficients

81. In the expansion of $(5^{1/2}+7^{1/8})^{1024}$, the number of integral terms is

(A) 128 (B) 129

(C) 130 (D) 131

82. The number of terms which are free from radical signs in the expansion of $(y^{1/5}+x^{1/10})^{55}$ is

(A) 5 (B) 6

(C) 7 (D) None of these

83. Let $R=(5\sqrt{5}+11)^{2n+1}$ and $f=R-[R]$, where [.] denotes the greatest integer function. The value of $R.f$ is

(A) 4^{2n+1} (B) 4^{2n}

(C) 4^{2n-1} (D) 4^{-2n}

84. The greatest integer less than or equal to $(\sqrt{2}+1)^6$ is

(A) 196 (B) 197

(C) 198 (D) 199

85. If number of terms in the expansion of $(x-2y+3z)^n$ are 45, then n=

(A) 7 (B) 8

(C) 9 (D) None of these

86. Find the value of

$$\frac{(18^3+7^3+3.18.7.25)}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$$

(A) 1 (B) 5

(C) 25 (D) 100

87. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=$

(A) $\frac{a_2}{a_2+a_3}$ (B) $\frac{1}{2}\frac{a_2}{(a_2+a_3)}$

(C) $\frac{2a_2}{a_2+a_3}$ (D) $\frac{2a_3}{a_2+a_3}$

88. The number of integral terms in the expansion of $(5^{1/2}+7^{1/6})^{642}$ is

(A) 106 (B) 108

(C) 103 (D) 109

89. The expression $(2+\sqrt{2})^4$ has value, lying between

(A) 134 and 135 (B) 135 and 136

(C) 136 and 137 (D) None of these

90. The digit in the unit place of the number $(183!)+3^{183}$ is

(A) 7 (B) 6

(C) 3 (D) 0