

HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $(1-2x+5x^2)^n$
For sum of coefficients put $x = 1$
 $\therefore a = (1-2+5)^n = 4^n$
 $(1+x)^{2n}$
For sum of coefficients put $x = 1$;
 $b = (1+x)^{2n} = 2^{2n} = (2^2)^n = 4^n$
 $\therefore a = b$
9. Given $(1+x)^n = a+b$ (i)
then, $(1-x)^n = a-b$ (ii)
Multiplying equation (i) & (ii)
we get, $(1-x^2)^n = a^2 - b^2$
11. Let $b = \sum_{r=0}^n \frac{r}{{}^nC_r} = \sum_{r=0}^n \frac{n-(n-r)}{{}^nC_r}$
 $= n \sum_{r=0}^n \frac{1}{{}^nC_r} - \sum_{r=0}^n \frac{n-r}{{}^nC_r}$
 $= na_n - \sum_{r=0}^n \frac{n-r}{{}^nC_{n-r}}$ $\rightarrow {}^nC_r = {}^nC_{n-r}$
 $= na_n - b$
 $\Rightarrow 2b = na_n \Rightarrow b = \frac{na_n}{2}$
13. $P = {}^{2n}C_n$ and $Q = {}^{2n-1}C_n$
14. $\left(\frac{47}{4}\right) + \sum_{j=1}^5 \binom{52-j}{3} = \binom{x}{y}$
L.H.S.
 ${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$
 $= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$
Using property ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ we get
 $= {}^{52}C_4 = \binom{x}{y} \Rightarrow x = 52, y = 4.$
15. Let $x = \frac{1}{2}$, then the sum of the given series
 $= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \text{ to } \infty = \log_e(1+x)$
 $= \log_e\left(1+\frac{1}{2}\right) = \log_e\left(\frac{3}{2}\right)$

17. $(1+x)^{10} = a_0 + a_1 + a_2x^2 + \dots + a_{10}x^{10}$
Put $x = i$,
 $(1+i)^{10} = a_0 - a_2 + a_4 + \dots + a_{10} + i(a_1 - a_3 + \dots + a_9)$
 $a_0 - a_2 + a_4 + \dots + a_{10} = \text{real part of } (1+i)^{10} = 2^5 \cos 10\pi/4$ (i)
 $a_1 - a_3 + \dots = \text{imaginary part of } (1+i)^{10} = 2^5 \sin 10\pi/4$ (ii)
 $(i)^2 + (ii)^2 = 2^{10}$

18. Given expression can be rewritten as

$$\frac{2}{2^7 \sqrt{4x+1}} \left[{}^7C_1(\sqrt{4x+1}) + {}^7C_3(\sqrt{4x+1})^3 + \dots + {}^7C_7(\sqrt{4x+1})^7 \right]$$

$$\frac{1}{2^6} \left[{}^7C_1 + {}^7C_3(\sqrt{4x+1})^2 + \dots + {}^7C_7(\sqrt{4x+1})^6 \right]$$

$\therefore$ Last term becomes $(4x+1)^3$

Hence degree is 3

21. $\left(\sum_{r=0}^{10} {}^{10}C_r \right) \left(\sum_{k=0}^{10} (-1)^k \frac{{}^{10}C_k}{2^k} \right)$
 $= ({}^{10}C_0 + \dots + {}^{10}C_{10}) \left({}^{10}C_0 - \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{2^2} - \dots + \frac{{}^{10}C_{10}}{2^{10}} \right)$
 $= 2^{10} \times \left(1 - \frac{1}{2} \right)^{10} = 1$

23. Let $R' = (5\sqrt{5} - 11)^{31}$

$$\text{Now } R - R' = (5\sqrt{5} + 11)^{31} - (5\sqrt{5} - 11)^{31}$$

$$\Rightarrow R - R' = \text{Integer}$$

$$\Rightarrow I + f - R' = \text{Integer}$$

$$\Rightarrow f - R' \text{ is an Integer but } -1 < f - R' < 1$$

$$\text{so } f - R' = 0 \Rightarrow f = R'$$

$$\text{so } R.f = R.R' = (5\sqrt{5} + 11)^{31} (5\sqrt{5} - 11)^{31}$$

$$= 4^{31} = 2^{62}$$

25. $(4+x+7x^2) \left(x - \frac{3}{x} \right)^{11}$

$$\Rightarrow 4 \left(x - \frac{3}{x} \right)^{11} + x \left(x - \frac{3}{x} \right)^{11} + 7x^2 \left(x - \frac{3}{x} \right)^{11}$$

term independent of x in above will be

EXERCISE - 2

Part # I : Multiple Choice

$$4 \times \text{coefficient of } x^0 \text{ in expansion of } \left(x - \frac{3}{x}\right)^{11}$$

$$+ 1 \times \text{coefficient of } x^{-1} \text{ in expansion of } \left(x - \frac{3}{x}\right)^{11}$$

$$+ 7 \times \text{coefficient of } x^{-2} \text{ in expansion of } \left(x - \frac{3}{x}\right)^{11}$$

$$\rightarrow x^r \text{ in expansion of } \left(x - \frac{3}{x}\right)^{11}$$

$$= {}^{11}C_r x^{11-r} \left(\frac{-3}{x}\right)^r \Rightarrow {}^{11}C_r \cdot x^{11-2r} \cdot (-3)^r$$

$$x^0 \text{ will not exist in expansion of } \left(x - \frac{3}{x}\right)^{11} \text{ for integral } r.$$

$$x^{-1} \text{ will occur at } r = 6$$

$$\therefore \text{coefficient of } x^{-1} = {}^{11}C_6 (-3)^6 = 3^6 \cdot {}^{11}C_6$$

$$\text{Also } x^2 \text{ will not exist in expansion of } \left(x - \frac{3}{x}\right)^{11} \text{ for integral } r.$$

$$\therefore \text{term independent of } x \text{ in expansion will be } = 3^6 \cdot {}^{11}C_6$$

$$26. T_2 = {}^nC_1 (a^{1/3})^{n-1} (a^{3/2}) = 14a^{5/2} \Rightarrow n = 14$$

$$\therefore \frac{{}^nC_3}{{}^nC_2} = 4$$

$$29. \text{ Calculate } m = \frac{n+1}{1 + \left| \frac{a}{b} \right|} \text{ as in } (a+b)^n$$

$$m = \frac{13+1}{1 + \left| \frac{2x}{5y} \right|} = \frac{14}{1+2} = \frac{14}{3}$$

$$m \text{ is not integer so greatest term is } T_{[m]+1}$$

$$T_5 = {}^{13}C_4 (2x)^9 (5y)^4$$

$$= {}^{13}C_4 \cdot 20^9 \cdot 10^4$$

$$[\rightarrow x = 10, y = 2]$$

$$1. (1 + 2x^2 + x^4)(1+x)^n = A_0 + A_1x + A_2x^2 + \dots$$

$$\text{Here } A_0 = 1, A_1 = n, A_2 = 2 + {}^nC_2$$

$$\text{Given } A_0, A_1, A_2 \text{ are in A.P.}$$

$$\therefore n - 1 = 2 + \frac{n(n-1)}{2} - n$$

$$\Rightarrow n^2 - 5n + 6 = 0 \Rightarrow n = 2, 3$$

$$10. \left(4^{1/3} + \frac{1}{6^{1/4}}\right)^{20}$$

$$\Rightarrow T_{r+1} = {}^{20}C_r (4^{1/3})^{20-r} (6^{-1/4})^r$$

$$\text{For rational terms}$$

$$20 - r = 3k \text{ \& } r = 4p, \text{ where } k, p \in \mathbb{I}$$

$$\Rightarrow r = 20 \text{ \& } r = 8$$

$$\therefore \text{no. of rational terms} = 2$$

$$\therefore \text{no. of irrational terms} = 19$$

$$13. 7^9 + 9^7 = (8-1)^9 + (8+1)^7 = {}^9C_0(8)^9 - {}^9C_1(8)^8 + {}^9C_2(8)^7$$

$$- {}^9C_3(8)^6 + {}^9C_4(8)^5 - {}^9C_5(8)^4 + {}^9C_6(8)^3 - {}^9C_7(8)^2 + {}^9C_8(8) - {}^9C_9$$

$$+ {}^7C_0(8)^7 + {}^7C_1(8)^6 + {}^7C_2(8)^5 + {}^7C_3(8)^4 + {}^7C_4(8)^3 + {}^7C_5(8)^2 + {}^7C_6(8) + {}^7C_7$$

$$\text{This is divisible by } 64 \text{ \& } 16$$

$$14. \left(x^3 + 3 \cdot 2^{-\log_2 x^3}\right)^{11} = \left(x^3 + \frac{3}{x^3}\right)^{11}$$

$$T_{r+1} = {}^{11}C_r (x^3)^{11-r} \left(\frac{3}{x^3}\right)^r$$

$$= {}^{11}C_r (x)^{33-6r} (3)^r$$

$$\text{Now } 33 - 6r = 2 \Rightarrow 6r = 31 \text{ (not possible)}$$

$$33 - 6r = -3 \Rightarrow r = 6$$

$$33 - 6r = 3 \Rightarrow r = 5$$

$$\therefore \frac{\text{coeff. of } x^3}{\text{coeff. of } x^{-3}} = \frac{{}^{11}C_5 3^5}{{}^{11}C_6 3^6} = \frac{1}{3}$$

$$17. \text{ Constant term in } P_1(x) \text{ is } 4$$

$$\text{If the constant term in } P_k(x) \text{ is also } 4, \text{ then}$$

$$P_k(x) = 4 + a_1x + a_2x^2 + \dots$$

$$\text{and } P_{k+1}(x) = (P_k(x) - 2)^2 = (a_1x + a_2x^2 + \dots + 2)^2$$

$$18. (1 + x + x^2 + x^3)^{100}$$

$$= a_0 + a_1x + a_2x^2 + \dots + a_{300}x^{300} \dots (i)$$

$$\text{put } x = 1$$

$$\Rightarrow 4^{100} = a_0 + a_1 + \dots + a_{300}$$

$$\text{So divisible by } 2^{10}$$

$$\text{Put } x = -1$$

$$\Rightarrow 0 = a_0 - a_1 + a_2 - \dots + a_{300}$$

$$\Rightarrow a_0 + a_2 + a_4 + \dots = a_1 + a_3 + \dots$$

replace x by $\frac{1}{x}$ we get $(x^3 + x^2 + x + 1)^{100}$

$$= a_0 x^{300} + a_1 x^{299} + \dots + a_{300} \quad \dots (ii)$$

by (i) & (ii)

$$\Rightarrow a_0 = a_{300}, a_1 = a_{299}, \dots \text{ so 'B' is true}$$

Diff. (i) we get $100(1 + x + x^2 + x^3)^{99} (1 + 2x + 3x^2)$

$$= a_1 + 2a_2x + \dots + 300x^{299}a_{300}$$

put $x = 0$

$$\Rightarrow 100 = a_1$$

21. The number of term in the expansion of $(1+x)^{2n}$ is $2n+1$ (odd), its middle term is $(n+1)^{\text{th}}$ term

$$\text{coefficient} = {}^{2n}C_n = \frac{2n!}{n!n!} = \frac{1.2.3 \dots (2n-1).2n}{n!n!}$$

$$= \frac{(1.3.5 \dots 2n-1)(2.4.6 \dots 2n)}{n!n!}$$

$$= \frac{(1.3.5 \dots 2n-1)2^n (1.2.3 \dots n)}{n!n!}$$

$$= \frac{(1.3.5 \dots 2n-1)2^n}{n!}$$

Part # II : Assertion & Reason

3. Using expansion we get

$$\frac{(14!)}{r_1! \times r_2! \times r_3! \times r_4!} (a^{r_1} \cdot b^{r_2} \cdot c^{r_3} \cdot d^{r_4})$$

$$\text{where } r_1 + r_2 + r_3 + r_4 = 14$$

$$\Rightarrow r_1 = 1, r_2 = 8, r_3 = 3, r_4 = 2$$

$$\therefore \text{Coefficient of } ab^8c^3d^4 \text{ is } \frac{14!}{1!8!3!2!}$$

$\therefore$ Statement I is true & statement II explain I

4. Statement 1 : $(\sqrt{2}-1)^2 = 2 + 1 - 2\sqrt{2} = 3 - 2\sqrt{2}$
- $$= \sqrt{9} - \sqrt{8}$$

$$\text{and } (\sqrt{2}-1)^3 = 2\sqrt{2} - 1 - 3\sqrt{2}(\sqrt{2}-1)$$

$$= -7 + 5\sqrt{2} = \sqrt{50} - \sqrt{49}$$

and so on

Statement 1 is correct (can be proved by induction)

Statement 2 is also correct but not correct explanation of statement 1 since any integral power of $(\sqrt{2}-1)$ will have rational and irrational part. The irrational part will have only one surd $\sqrt{2}$. Thus $(\sqrt{2}-1)^n = A + B\sqrt{2}$, where A and B are integers.

8. Statement-1 : Let $I + f = (\sqrt{5} + 2)^n = {}^nC_0 (\sqrt{5})^n (2)^0 + {}^nC_1 (\sqrt{5})^{n-1} (2)^1 + \dots$

$$f = (\sqrt{5} - 2)^n = {}^nC_0 (\sqrt{5})^n 2^0 - {}^nC_1 (\sqrt{5})^{n-1} (2)^1$$

$$+ \dots I + f - f' = 2$$

$$\left[{}^nC_1 (\sqrt{5})^{n-1} 2 + {}^nC_3 (\sqrt{5})^{n-3} 2^3 + \dots + {}^nC_n 2^n \right]$$

$$\Rightarrow I + f - f'$$

$$\left[{}^nC_1 (\sqrt{5})^{n-1} 2 + {}^nC_3 (\sqrt{5})^{n-3} \cdot 2^3 + \dots + {}^nC_{n-2} 5 \cdot 2^{n-2} \right] + 2^{n+1}$$

Here $f - f' = 0 \Rightarrow I - 2^{n+1}$ is clearly divisible by $20n$ hence true.

Statement-2 is obviously true

EXERCISE - 3

Part # I : Matrix Match Type

1. (A) $(2n+1)(2n+3)(2n+5) \dots (4n-1)$

$$\begin{aligned} &= \frac{(2n!)(2n+1)(2n+2)(2n+3)(2n+4) \dots (4n-1)(4n)}{(2n!)(2n+2)(2n+4)(2n+6) \dots (4n)} \\ &= \frac{(4n!)(n!)}{(n!)(2n)!2^n(n+1)(n+2) \dots (2n)} \\ &= \frac{(4n!)(n!)}{2^n \cdot (2n)!(2n)!} \end{aligned}$$

$$\begin{aligned} \text{(B)} \quad \sum_{r=1}^n r \frac{{}^nC_r}{{}^nC_{r-1}} &= \frac{r \cdot n!}{r!(n-r)!} \cdot \frac{(r-1)!(n-r+1)!}{n!} \\ &= \sum_{r=1}^n (n-r+1) \\ &= n(n+1) - \frac{n(n+1)}{2} = \frac{n(n+1)}{2} \end{aligned}$$

$$\begin{aligned} \text{(C)} \quad (C_0+C_1)(C_1+C_2)(C_2+C_3) \dots (C_{n-1}+C_n) \\ = m C_1 C_2 \dots C_{n-1} \\ = \left(\frac{C_0}{C_1}+1\right) \left(\frac{C_1}{C_2}+1\right) \left(\frac{C_2}{C_3}+1\right) \dots \left(\frac{C_{n-1}}{C_n}+1\right) = m \\ = \frac{n+1}{n} \cdot \frac{n+1}{n-1} \cdot \frac{n+1}{n-2} \dots \frac{n+1}{1} = m \\ \frac{(n+1)^n}{n!} = m \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad \sum_{i=1}^n \left\{ \sum_{j=1}^n i C_i C_j + \sum_{j=1}^n j C_i C_j \right\} \\ = \sum_{i=1}^n i C_i (2^n - 1) + \sum_{i=1}^n n C_i 2^{n-1} \\ = n \cdot 2^{n-1} (2^n - 1) + n \cdot 2^{n-1} (2^n - 1) \\ = n \cdot 2^n (2^n - 1) \end{aligned}$$

3. (A) $I + f = (7 + 4\sqrt{3})^{2n}$

$$\text{Here } (7 - 4\sqrt{3})^{2n} = f' = 1 - f \Rightarrow (I + f)(1 - f) = 1$$

$$\begin{aligned} \text{(B)} \quad T_2 &= {}^nC_1 (x)^{n-1} \cdot a = 240 \quad \dots\dots\text{(i)} \\ T_3 &= {}^nC_2 (x)^{n-2} a^2 = 720 \quad \dots\dots\text{(ii)} \\ T_4 &= {}^nC_3 (x)^{n-3} a^3 = 1080 \quad \dots\dots\text{(iii)} \end{aligned}$$

From (i) and (ii)

$$\text{Here } \frac{{}^nC_1 (x)^{n-1} a}{{}^nC_2 x^{n-2} a^2} = \frac{2x}{(n-1)a} = \frac{240}{720} = \frac{1}{3}$$

$$\Rightarrow 6x = (n-1)a$$

From (ii) and (iii)

$$\Rightarrow 9x = 2(n-2)a$$

$$\text{On dividing } \frac{3}{2} = \frac{2(n-2)}{(n-1)}$$

$$\Rightarrow 3n - 3 = 4n - 8$$

$$\Rightarrow n = 5$$

$$\text{(C)} \quad C_0 C_4 - C_1 C_3 + C_2 C_2 - C_3 C_1 + C_4 C_0 = 2 - 2.4.4 + 6.6 = 6$$

$$\begin{aligned} \text{(D)} \quad \sqrt{\frac{x}{x+y}} \sqrt{\frac{x}{x-y}} &= \left(\frac{1}{1+\frac{y}{x}} \right)^{1/2} \left(\frac{1}{1-\frac{y}{x}} \right)^{1/2} \\ &= \left(1 - \frac{y^2}{x^2} \right)^{-1/2} = 1 + \frac{1}{2} \cdot \frac{y^2}{x^2} \\ &\Rightarrow k = 2 \end{aligned}$$

Part # II : Comprehension

Comprehension-1

$$1. \quad (1 + 4x + 4x^2)^n = a_0 + a_1 x + a_2 x^2 + \dots + a_{2n} x^{2n}$$

$$9^n = a_0 + a_1 + a_2 + \dots + a_{2n} \quad \dots\dots\text{(i)}$$

$$1 = a_0 - a_1 + a_2 - \dots + a_{2n} \quad \dots\dots\text{(ii)}$$

adding (i) & (ii) we get

$$9^n + 1 = 2 \sum_{r=0}^n a_{2r}$$

2. Subtracting (ii) from (i) we get

$$a^n - 1 = 2 \sum_{r=1}^n a_{2r-1}$$

3. a_{2n-1} = coefficient of x^{2n-1} in

$$(1 + 4x + 4x^2)^n = (1 + 2x)^{2n}$$

$$T_{r+1} = {}^{2n}C_r (2x)^r$$

$$a_{2n-1} = {}^{2n}C_{2n-1} \cdot 2^{2n-1} = 2n \cdot 2^{2n-1} = 2n \cdot 2^{2n}$$

$$4. \quad a_2 = {}^{2n}C_2 \cdot 2^2 = 2n(2n-1) \cdot 2 = 8n^2 - 4n$$

Comprehension-2

1. The expression $(2+x)^2(3+x)^3(4+x)^4 = (x+2)(x+2)$

$$(x+3)(x+3)(x+3)(x+4)(x+4)(x+4)(x+4)$$

$$= x^9 + (2+2+3+3+3+3+4+4+4+4)x^8 + \dots\dots\dots$$

$$\Rightarrow \text{Co-efficient of } x^8 = 29$$

2. Expression = $x \cdot x^2 \cdot x^3 \dots \dots \dots x^{20}$

$$\left(1 - \frac{1}{x}\right) \left(1 - \frac{2}{x^2}\right) \left(1 - \frac{3}{x^3}\right) \dots \dots \left(1 - \frac{20}{x^{20}}\right)$$

$$\text{Let } E = \left(1 - \frac{1}{x}\right) \left(1 - \frac{2}{x^2}\right) \left(1 - \frac{3}{x^3}\right) \dots \dots \left(1 - \frac{20}{x^{20}}\right)$$

Now Co-efficient of x^{203} in original expression

⇒ Co-efficient of x^{-7} in E.

$$\begin{aligned} \text{But } E &= 1 - \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} + \dots\right) + \\ &\quad \left(\frac{1}{x} \cdot \frac{6}{x^6} + \frac{2}{x^2} \cdot \frac{5}{x^5} + \frac{3}{x^3} \cdot \frac{4}{x^4} + \dots\right) \\ &\quad - \left(\frac{1}{x} \cdot \frac{2}{x^2} \cdot \frac{4}{x^4} + \dots\right) \end{aligned}$$

$$= \text{Co-efficient of } x^{-7} = -7 + 6 + 10 + 12 - 8 = 13$$

3. The Co-efficient of $x^{98} = (1.2 + 2.3 + \dots \dots \dots 99.100)$
= Sum of product of first 100 natural numbers taken two at a time

$$= \frac{1}{2} [(1+2+3+\dots+100)^2 - (1^2+2^2+3^2+\dots+100^2)]$$

Comprehension-3

1. $\sum_{0 \leq i < j \leq n} {}^nC_i \cdot {}^nC_j$

$$= \frac{\left(\sum_{i=0}^n \sum_{j=0}^n {}^nC_i \cdot {}^nC_j\right) - \sum_{i=0}^n ({}^nC_i)^2}{2}$$

$$= \frac{\left(\sum_{i=0}^n {}^nC_i \cdot 2^n\right) - \sum_{i=0}^n ({}^nC_i)^2}{2} = \frac{2^n 2^n - \sum_{i=0}^n ({}^nC_i)^2}{2}$$

$$= \frac{2^{2n} - 2^n C_n}{2}$$

2. $\sum_{m=0}^n \sum_{p=0}^m {}^nC_m \cdot {}^mC_p = \sum_{m=0}^n {}^nC_m \left(\sum_{p=0}^m {}^mC_p\right)$

$$= \sum_{m=0}^n {}^nC_m (2^m) = 3^n$$

3. $\sum_{0 \leq i \leq j \leq n} ({}^nC_i + {}^nC_j)$

$$= \frac{\left(\sum_{i=0}^n \sum_{j=0}^n ({}^nC_i + {}^nC_j)\right) - \sum_{i=0}^n 2 \cdot {}^nC_i}{2}$$

$$= \frac{\left(\sum_{i=0}^n \left(\sum_{j=0}^n {}^nC_i + \sum_{j=0}^n {}^nC_j\right)\right) - 2 \times 2^n}{2}$$

$$= \frac{\left(\sum_{i=0}^n \left({}^nC_i \sum_{j=0}^n 1 + 2^n\right)\right) - 2^{n+1}}{2}$$

$$= \frac{\left(\sum_{i=0}^n ({}^nC_i (n+1) + 2^n)\right) - 2^{n+1}}{2}$$

$$= \frac{(n+1) \sum_{i=0}^n {}^nC_i + 2^n \sum_{i=0}^n 1 - 2^{n+1}}{2}$$

$$= \frac{(n+1)2^n + 2^n(n+1) - 2^{n+1}}{2} = (n+1)2^n - 2^n = n2^n$$

EXERCISE - 4

Subjective Type

1. $r = 5$ or 9

2. 7th term from beginning $T_7 = {}^nC_6 (2)^{\frac{n-6}{3}} \left(\frac{1}{3}\right)^2$

7th term from the end $T_{n-5} = {}^nC_{n-6} (2)^2 \left(\frac{1}{3}\right)^{\frac{n-6}{3}}$

$$\frac{T_7}{T_{n-5}} = \frac{1}{6} = \frac{2^{\left(\frac{n-6}{3}-2\right)}}{\left(\frac{1}{3}\right)^{\left(\frac{n-6}{3}-2\right)}} \Rightarrow \frac{1}{6} = (6)^{\frac{n-12}{3}}$$

$$\Rightarrow \frac{n-12}{3} = -1 \Rightarrow n = 9$$

3. $\rightarrow \sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$

Let $y = x - 3 \Rightarrow y + 1 = x - 2$
so the given expression reduces to :

$$\sum_{r=0}^{2n} a_r (1+y)^r = \sum_{r=0}^{2n} b_r \cdot y^r$$

$$\Rightarrow a_0 + a_1(1+y) + a_2(1+y)^2 + \dots + a_{2n}(1+y)^{2n} \\ = b_0 + b_1y + \dots + b_{2n} \cdot y^{2n}$$

using $a_{k=1}$ for all $k \geq n$, then we get

$$\Rightarrow a_0 + a_1(1+y) + a_2(1+y)^2 + \dots + a_{n-1}(1+y)^{n-1} \\ + (1+y)^n + (1+y)^{n+1} + \dots + (1+y)^{2n} \\ = b_0 + b_1y + \dots + b_n y^n + \dots + b_{2n} \cdot y^{2n}$$

Compare the co-efficients of y^n on both sides we get

$${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + \dots + {}^{2n}C_n = b_n$$

$$\Rightarrow {}^{n+1}C_{n+1} + {}^{n+1}C_n + {}^{n+2}C_n + \dots + {}^{2n}C_n = b_n$$

$$(\text{use } {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$$

$${}^{n+2}C_{n+1} + {}^{n+2}C_n + \dots + {}^{2n}C_n = b_n$$

(adding the first two terms).

$\Rightarrow$ If we combine terms on LHS, finally we get

$${}^{2n+1}C_{n+1} = b_n$$

4. 101^{50}

5. LHS $= {}^nC_r + {}^{n+1}C_r + {}^{n+2}C_r + \dots + {}^{n-2}C_r + {}^{n-1}C_r$
 $= {}^{r+1}C_{r+1} + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^{n-2}C_r + {}^{n-1}C_r$
 $\quad (\rightarrow {}^nC_r = {}^{n-1}C_{r+1})$
 $S = {}^{r+2}C_{r+1} + {}^{r+3}C_{r+1} + \dots = {}^nC_{r+1} \quad (\rightarrow {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

8. $40x(1-x)^{39} + 2 \times \frac{40}{2} x^2 (1-x)^{35} + 3 \times \frac{40}{3} x^3 (1-x)^{37} \\ + \dots + 40 \cdot x^{40}$
 $= 40x[(1-x)^{39} + x(1-x)^{38} + x^2(1-x)^{37} + \dots + x^{39}]$
 $= 40x[(1-x) + x]^{39} = 40x = ax + b$
 $\Rightarrow a = 40 \text{ \& } b = 0$

9. $n = 12$

11. $S = \sum_{r=0}^n \frac{{}^nC_r \cdot 2^{r+2}}{(r+1)(r+2)} = \frac{1}{n+1} \sum_{r=0}^n \frac{{}^{n+1}C_{r+1} \cdot 2^{r+2}}{r+2}$
 $= \frac{1}{(n+1)(n+2)} \sum_{r=0}^n {}^{n+2}C_{r+2} \cdot 2^{r+2}$

$$(\rightarrow (1+2)^{n+2} = {}^{n+2}C_0 + {}^{n+2}C_1 2^1 + \dots + \sum_{r=0}^n {}^{n+2}C_{r+2})$$

$$S = \frac{1}{(n+1)(n+2)} \{3^{n+2} - 1 - 2n - 4\} = \frac{3^{2n+2} - 2n - 5}{(n+1)(n+2)}$$

13. $S = {}^nC_0 \sin(0x) \cos nx + {}^nC_1 \sin x \cos(n-1)x + \dots$
 $+ {}^nC_n \sin nx \cos(0x) \dots \dots \dots$ (i)
 $S = {}^nC_0 \sin nx \cos(0x) + {}^nC_1 \sin(n-1)x \cos x + \dots$
 $+ {}^nC_n \sin(0x) \cos nx \dots \dots \dots$ (ii)

Add (i) & (ii)

$$2S = ({}^nC_0 + {}^nC_1 + \dots + {}^nC_n) \sin nx \Rightarrow S = 2^{n-1} \sin nx$$

14. $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$
 $\Rightarrow x(1+x)^n = C_0 x + C_1 x^2 + \dots + C_n x^{n+1}$

Differentiating w.r.t. x

$$(1+x)^n + n x (1+x)^{n-1} = C_0 + 2C_1 x + \dots + (n+1) C_n x^n$$

Putting $x = -1$

$$C_0 - 2C_1 + \dots + (-1)^n (n+1) C_n = 0$$

15. (i) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$

$\Rightarrow (x+1)^n = C_0 x^n + C_1 x^{n-1} + \dots + C_n$

$C_0 C_3 + C_1 C_4 + \dots + C_{n-3} C_n$

= Co-efficient of x^{n-3} in $(1+x)^{2n} = {}^{2n}C_{n-3}$

(ii) $C_0 C_r + C_1 C_{r+1} + \dots + C_{n-r} C_n$

= co-efficient of x^{n-r} in $(1+x)^{2n} = {}^{2n}C_{n-r}$

(iii) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$

$\Rightarrow (x-1)^n = C_0 x^n - C_1 x^{n-1} + \dots + (-1)^n C_n$

$C_0^2 - C_1^2 + \dots + (-1)^n C_n^2$

= co-efficient of x^n in $(x^2-1)^n = 0$ (if n is odd)

= ${}^nC_{n/2}(-1)^{n/2}$ (if n is even)

16. $\frac{1}{{}^{2n+1}C_r} + \frac{1}{{}^{2n+1}C_{r+1}} = \frac{r!(2n+1-r)!}{(2n+1)!} + \frac{(r+1)!(2n-r)!}{(2n+1)!}$

= $\frac{r!(2n-r)!}{(2n+1)!} \{2n+1-r+r+1\}$

= $\frac{2n+2}{2n+1} \cdot \frac{r!(2n-r)!}{2n!} = \frac{2n+2}{2n+1} \cdot \frac{1}{{}^{2n}C_r}$

17. $y = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \text{to } \infty \right) = \log_e (1+x)$

$\Rightarrow e^y = (1+x)$

$\Rightarrow x = (e^y - 1) = \left[\left(1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots \right) - 1 \right]$

$\Rightarrow x = \left(y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots \text{to } \infty \right)$

18. $\sum_{r=0}^{n-2} \binom{n-1}{r} \binom{n}{r+2} = \binom{2n-1}{n-2}$

L.H.S. $\sum_{r=0}^{n-2} {}^{n-1}C_r \cdot {}^nC_{r+2}$

$\sum_{r=0}^{n-2} {}^{n-1}C_{n-r-1} \cdot {}^nC_{r+2}$

Coefficient of x^{n+1} in the expansion of

$(1+x)^{n-1}(1+x)^n$ i.e. $(1+x)^{2n-1}$

= ${}^{2n-1}C_{n+1} = {}^{2n-1}C_{n-2} = \binom{2n-1}{n-2}$

19. $\frac{1}{2} \left(\frac{1}{5} \right)^2 + \frac{2}{3} \left(\frac{1}{5} \right)^3 + \frac{3}{4} \left(\frac{1}{5} \right)^4 + \dots$

= $\left(1 - \frac{1}{2} \right) \left(\frac{1}{5} \right)^2 + \left(1 - \frac{1}{3} \right) \left(\frac{1}{5} \right)^3 + \left(1 - \frac{1}{4} \right) \left(\frac{1}{5} \right)^4 + \dots$

= $\left(\frac{1}{5} \right)^2 + \left(\frac{1}{5} \right)^3 + \left(\frac{1}{5} \right)^4 + \dots$

- $\left[\frac{1}{2} \left(\frac{1}{5} \right)^2 + \frac{1}{3} \left(\frac{1}{5} \right)^3 + \frac{1}{4} \left(\frac{1}{5} \right)^4 + \dots \right]$

= $\left[\frac{1}{5} + \left(\frac{1}{5} \right)^2 + \left(\frac{1}{5} \right)^3 + \left(\frac{1}{5} \right)^4 + \dots \right]$

- $\left[\frac{1}{5} + \frac{1}{2} \left(\frac{1}{5} \right)^2 + \frac{1}{3} \left(\frac{1}{5} \right)^3 + \frac{1}{4} \left(\frac{1}{5} \right)^4 + \dots \right]$

adding and subtracting $\frac{1}{5}$

= $\frac{\frac{1}{5}}{1 - \frac{1}{5}} - \left(-\log_e \left(1 - \frac{1}{5} \right) \right) = \frac{1}{4} + \log_e \frac{4}{5}$

EXERCISE - 5

Part # 1 : AIEEE/JEE-MAIN

1. $(1 + 2x + 3x^2 + \dots)^{-3/2} = [(1 - x)^{-2}]^{-3/2} = (1 - x)^3$

So, coefficient of x^5 in $(1 + 2x + 3x^2 + \dots)^{-3/2}$

= coefficient of x^5 in $(1 - x)^3 = 0$.

2. $(r + 1)^{\text{th}}$ term of $(\sqrt{3} + \sqrt[8]{5})^{256}$

i.e., $T_{r+1} = {}^{256}C_r (3)^{(256-r)/2} (5)^{r/8}$

The terms are integral, if $\frac{256-r}{2}$ and $\frac{r}{8}$ are both positive integer.

$\Rightarrow r = 0, 8, 16, 24, 32, \dots, 256$

Hence total terms are 33.

3. $\rightarrow (r + 1)^{\text{th}}$ term in the expansion of $(1 + x)^{27/5}$

$$= \frac{\frac{27}{5} \left(\frac{27}{5} - 1 \right) \dots \left(\frac{27}{5} - r + 1 \right)}{r!} x^r$$

Now this term will be negative, if the last factor in numerator is the only negative factor.

$\Rightarrow \frac{27}{5} - r + 1 < 0 \Rightarrow \frac{32}{5} < r \Rightarrow 6.4 < r$

$\Rightarrow$ least value of r is 7.

Thus first negative term will be 8th.

4. Coefficient of middle term in $(1 + \alpha x)^4 = {}^4C_2 \alpha^2$
coefficient of middle term in $(1 - \alpha x)^6 = {}^6C_3 (-\alpha)^3$

${}^4C_2 \alpha^2 = -{}^6C_3 \alpha^3 \Rightarrow -\frac{6}{20} = \alpha \Rightarrow \alpha = -\frac{3}{10}$

5. $(1 - x)(1 - x)^n = (1 - x)^n + x(1 - x)^n$

Coefficient of $x^n = (-1)^n + (-1)^{n-1} \cdot n = (-1)^n (1 - n)$

6. $s_n = \sum_{r=0}^n \frac{1}{{}^nC_r} \Rightarrow t_n = \sum_{r=0}^n \frac{r}{{}^nC_r}$

$\Rightarrow t_n = \sum_{r=0}^n \frac{n-r}{{}^nC_r}$

$\therefore 2t_n = n \sum_{r=0}^n \frac{1}{{}^nC_r} = ns_n$

$\Rightarrow \frac{t_n}{s_n} = \frac{n}{2}$

7. $(1 + y)^m$

$\Rightarrow T_r = {}^mC_{r-1} \cdot y^{r-1}$

$\Rightarrow T_{r+1} = {}^mC_r \cdot y^r$

$\Rightarrow T_{r+2} = {}^mC_{r+1} \cdot y^{r+1}$
 ${}^mC_{r-1} + {}^mC_{r+1} = 2 {}^mC_r$

$\Rightarrow \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = 2$

$\Rightarrow m^2 - m(4r + 1) + 4r^2 - 2 = 0$

8. ${}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + \dots + {}^{50}C_3 = {}^{56}C_4$

9. $\frac{(1+x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{1/2}}$
 $\Rightarrow \frac{\left(1 + \frac{3}{2}x + \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{x^2}{2!}\right) - \left(1 + \frac{3x}{2} + \frac{3.2}{2} \cdot \frac{x^2}{4}\right)}{(1-x)^{1/2}}$

$= \left(-\frac{3}{8}x^2\right) (1-x)^{-1/2} = -\frac{3}{8}x^2$

10. $(1 - ax)^{-1} (1 - bx)^{-1}$

$= (1 + ax + (ax)^2 + \dots) (1 + bx + (bx)^2 + \dots)$

so, $a_n = a^n + a^{n-1}b + a^{n-2}b^2 + \dots + b^n$

$= a^n \frac{\left(1 - \left(\frac{b}{a}\right)^{n+1}\right)}{1 - \frac{b}{a}} = \frac{a^{n+1} - b^{n+1}}{b - a}$

11. $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \dots$

$(1 - my + {}^mC_2 y^2 - \dots) (1 + ny + {}^nC_2 y^2 - \dots)$

$= 1 + a_1 y + a_2 y^2 + \dots$

$a_1 = n - m = 10 \dots (i)$

$\Rightarrow a_2 = {}^mC_2 + {}^nC_2 - mn = 10 \dots (ii)$

solving (i) & (ii), we get $(m, n) \equiv (35, 45)$

12. $S = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \dots + {}^{20}C_{10}$

We know, ${}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{20} = 0$

$\Rightarrow 2({}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9) + {}^{20}C_{10} = 0$

$\therefore {}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9 = -\frac{1}{2} {}^{20}C_{10}$

So, $S = \frac{1}{2} {}^{20}C_{10}$



13. **Statement -1** : $\sum_{r=0}^n (r+1) {}^n C_r = \sum_{r=0}^n r \cdot {}^n C_r + \sum_{r=0}^n {}^n C_r$
 $= n \cdot 2^{n-1} + 2^n = (n+2) 2^{n-1}$

Statement-2 : $\sum_{r=0}^n (r+1) {}^n C_r x^r = \sum_{r=0}^n r \cdot {}^n C_r x^r + \sum_{r=0}^n {}^n C_r x^r$
 $= xn(1+x)^{n-1} + (1+x)^n$

14. $S_1 = \sum_{j=1}^{10} j(j-1) \cdot \frac{10(10-1)}{j(j-1)} {}^8 C_{j-2}$

$\Rightarrow S_1 = 9 \times 10 \sum_{j=2}^{10} {}^8 C_{j-2} \Rightarrow S_1 = 90 \cdot 2^8$

$S_2 = \sum_{j=1}^{10} j \cdot \frac{10}{j} {}^9 C_{j-1} = 10 \cdot 2^9$

$S_3 = \sum_{j=1}^{10} (j(j-1) + j) {}^{10} C_j$

$= \sum_{j=1}^{10} j(j-1) {}^{10} C_j + \sum_{j=1}^{10} j {}^{10} C_j$

$= 90 \sum_{j=2}^{10} {}^8 C_{j-2} + 10 \sum_{j=1}^{10} {}^9 C_{j-1}$

$= 90 \times 2^8 + 10 \times 2^9 = (45 + 10) \cdot 2^9 = (45 + 10) \cdot 2^9 = 55 \cdot 2^9$

so statement-1 is true and statement 2 is false.

Hence correct option is (2)

15. $(1-x-x^2+x^3)^6$
 $(1-x)^6 (1-x^2)^6$
 $({}^6 C_0 - {}^6 C_1 x^1 + {}^6 C_2 x^2 - {}^6 C_3 x^3 + {}^6 C_4 x^4 - {}^6 C_5 x^5 + {}^6 C_6 x^6)$
 $({}^6 C_0 - {}^6 C_1 x^2 + {}^6 C_2 x^4 - {}^6 C_3 x^6 + {}^6 C_4 x^8 + \dots + {}^6 C_6 x^{12})$
 Now coefficient of $x^7 = {}^6 C_1 {}^6 C_3 - {}^6 C_3 {}^6 C_2 + {}^6 C_5 {}^6 C_1$
 $= 6 \times 20 - 20 \times 15 + 36 = 120 - 300 + 36 = 156 - 300$
 $= -144$

16. $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n} = 2[{}^{2n} C_1 (\sqrt{3})^{2n-1}$
 $+ {}^{2n} C_3 (\sqrt{3})^{2n-3} + {}^{2n} C_5 (\sqrt{3})^{2n-5} + \dots]$
 = which is an irrational number

17. $\left((x^{1/3} + 1) - \left(\frac{\sqrt{x} + 1}{\sqrt{x}} \right) \right)^{10} \Rightarrow (x^{1/3} - x^{-1/2})^{10}$

$\Rightarrow T_{r+1} = {}^{10} C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0$

$\Rightarrow 20 - 2r - 3r = 0 \Rightarrow r = 4$

$T_5 = {}^{10} C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$

20. Number to terms is $2n + 1$ which is odd but it is given 28.
 If we take $(x + y + z)^n$ then number of terms is $n + {}^2 C_2 = 28$
 Hence $n = 6$

$\left(1 - \frac{2}{x} + \frac{4}{x^2} \right)^6 = a_0 + a_1 x + a_2 x^2 + \dots + a_6 x^6$

Sum of coefficients can be obtained by $x = 1$

$(1 - 2 + 4)^6 = 3^6 = 729$

So according to what the examiner is trying to ask option 3 can be correct.

Part # II : IIT-JEE ADVANCED

1. $(1+x)^m (1-x)^n$

$3 = {}^m C_0 {}^n C_1 (-1) + {}^m C_1 {}^n C_0$

or $3 = m - n \dots (i)$

$-6 = {}^m C_0 {}^n C_2 + {}^m C_2 {}^n C_0 - {}^m C_1 {}^n C_1$

or -6

$= \frac{n(n-1)}{2} + \frac{m(m-1)}{2} - mn \dots (ii)$

from (i) & (ii) $n = 9$ & $m = 12$

2. ${}^n C_r + 2 \cdot {}^n C_{r-1} + {}^n C_{r-2} = {}^n C_r + {}^n C_{r-1} + {}^n C_{r-1} + {}^n C_{r-2}$
 $= {}^{n+1} C_r + {}^{n+1} C_{r-1} = {}^{n+2} C_r$

3. ${}^n C_m + {}^{n-1} C_m + {}^{n-2} C_m + \dots + {}^m C_m$
 = Co-efficient of x^m in $(1+x)^n + (1+x)^{n-1} + \dots + (1+x)^m$

$= \text{Co-efficient of } x^m \text{ in } (1+x)^m \left[\frac{(1+x)^{n-m+1} - 1}{x} \right] = {}^{n+1} C_{m+1}$

$S = {}^n C_m + 2 \cdot {}^{n-1} C_m + 3 \cdot {}^{n-2} C_m + \dots$

$\Rightarrow S = \text{Co-efficient of } x^m \text{ in } (1+x)^n + 2 \cdot (1+x)^{n-1} + 3 \cdot (1+x)^{n-2} + \dots$

Let $S' = (1+x)^n + 2 \cdot (1+x)^{n-1} + 3 \cdot (1+x)^{n-2} + \dots + (n-m+1)(1+x)^m \dots (i)$

$\Rightarrow \frac{S'}{(1+x)} = (1+x)^{n-1} + 2 \cdot (1+x)^{n-2} + \dots + (n-m+1)(1+x)^{m-1}$

.....(ii)



from (i) – (ii)

$$\Rightarrow \frac{xS'}{1+x} = (1+x)^n + (1+x)^{n-1} + \dots + (1+x)^m - (n-m+1)(1+x)^{m-1}$$

$$\Rightarrow \frac{xS'}{1+x} = (1+x)^m \left[\frac{(1+x)^{n-m+1} - 1}{x} \right] - (n-m+1)(1+x)^{m-1}$$

$$\Rightarrow S' = \frac{(1+x)^{n+2} - (1+x)^{m+1}}{x^2} - \frac{(n-m+1)(1+x)^m}{x}$$

$$\Rightarrow S = \text{Co-efficient of } x^m \text{ in } S' = {}^{n+2}C_{m+2}$$

4. $(a-b)^n$
 $T_5 + T_6 = {}^nC_4(a)^{n-4}(-b)^4 + {}^nC_5(a)^{n-5}(-b)^5 = 0$

$$\Rightarrow a + \frac{n-4}{5}(-b) = 0 \Rightarrow a/b = \frac{n-4}{5}$$

5. $S = \sum_{i=0}^m \binom{10}{i} \binom{20}{m-i} = {}^{10}C_0 \cdot {}^{20}C_m + {}^{10}C_1 \cdot {}^{20}C_{m-1} + \dots$

$$\Rightarrow S = \text{coefficient of } x^m \text{ in } (1+x)^{10} (1+x)^{20} = {}^{30}C_m$$

S is maximum when $m = 15$

6. $(1+t^2)^{12} (1+t^{12}+t^{24}+t^{36}) = (1+t^{12}+t^{24}) (1+t^2)^{12}$
 coefficient of $t^{24} = {}^{12}C_{12} + {}^{12}C_6 + {}^{12}C_0 = {}^{12}C_6 + 2$

7. $S = 2^k {}^nC_0 - {}^nC_k - 2^{k-1} {}^nC_1 - {}^{n-1}C_{k-1} + 2^{k-2} {}^nC_2 - {}^{n-2}C_{k-2} + \dots$

$$\Rightarrow S = \sum_{r=0}^k (-1)^r {}^nC_r - {}^{n-r}C_{k-r} \cdot 2^{k-r}$$

$$\Rightarrow S = \sum_{r=0}^k (-1)^r \frac{n!}{r!(n-r)!} \times \frac{(n-r)! \cdot 2^{k-r}}{(n-k)!(k-r)!}$$

$$= \sum_{r=0}^k (-1)^r \cdot 2^{k-r} \frac{n!}{k!(n-k)!} \times \frac{k!}{r!(k-r)!}$$

$$= 2^k {}^nC_k \left(1 - \frac{1}{2}\right)^k = {}^nC_k$$

8. ${}^{(n-1)}C_r = (k^2-3) {}^nC_{r+1}$ or ${}^{(n-1)}C_{n-(r+1)} = (k^2-3) {}^nC_{n-(r+1)}$

$$1 \geq k^2 - 3 > 0 \Rightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$$

9. $S = {}^{30}C_0 - {}^{30}C_{20} + {}^{30}C_1 - {}^{30}C_{19} + {}^{30}C_2 - {}^{30}C_{18} + \dots$

$S = \text{Co-efficient of } x^{20} \text{ in } (1-x)^{30} (1+x)^{30}$

$$S = \text{Co-efficient of } x^{20} \text{ in } (1-x^2)^{30} = {}^{30}C_{10}$$

10. $B_{10} \sum_{r=1}^{10} A_r B_r - C_{10} \sum_{r=1}^{10} (A_r)^2 = {}^{20}B_{10} ({}^{30}C_{20} - 1) - {}^{30}C_{10}$

$$({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$$

13. Coeff. x^2

$${}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^{49}C_2 + {}^{50}C_2 \cdot m^2 = (3n+1) {}^{51}C_3$$

$${}^3C_3 + {}^3C_2 + {}^4C_2 + \dots + {}^{49}C_2 + {}^{50}C_2 \cdot m^2 = (3n+1) {}^{51}C_3$$

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$$\Rightarrow {}^{50}C_3 + {}^{50}C_2 \cdot m^2 = (3n+1) {}^{51}C_3$$

$$\Rightarrow {}^{50}C_3 + {}^{50}C_2 + (m^2-1) {}^{50}C_2 = 3n \cdot \frac{51}{3} \cdot {}^{50}C_2 + {}^{51}C_3$$

$$\Rightarrow {}^{51}C_3 + (m^2-1) {}^{50}C_2 = 51n \cdot {}^{50}C_2 + {}^{51}C_3$$

$$m^2 - 1 = 51n \Rightarrow m^2 = 51n + 1$$

min value of m^2 for $51n + 1$ is integer for $n = 5$

MOCK TEST

1. Using rationalizing

$$= \left(\sqrt{2x^2+1} + \sqrt{2x^2-1} \right)^6 + \left(\sqrt{2x^2+1} - \sqrt{2x^2-1} \right)^6$$

$$= 2 \left({}^6C_0 (2x^2+1)^3 + {}^6C_2 (2x^2+1)^2 (2x^2-1) + \dots + {}^6C_4 (2x^2+1)(2x^2-1)^2 + {}^6C_6 (2x^2-1)^3 \right)$$

clearly '6'

2. (B)

$$t_{r+1} = {}^{21}C_r \left(\frac{a}{b} \right)^{\frac{21-r}{3}} \frac{b^{r/3}}{a^{r/6}} = {}^{21}C_r a^{\frac{42-3r}{6}} b^{\frac{2r-21}{3}}$$

$\therefore 42 - 3r = 4r - 42$ i.e. $r = 12$

$\therefore 13^{\text{th}}$ term contains same powers of a and b

3. Co-efficient of x^{15} in $(1+x+x^3+x^4)^n$

$$= \text{Co-efficient of } x^{15} \text{ in } (1+x^3)^n (1+x)^n$$

$$= {}^nC_0 {}^nC_{15} + {}^nC_1 {}^nC_{12} + {}^nC_2 {}^nC_9 + {}^nC_3 {}^nC_6 + {}^nC_4 {}^nC_3 + {}^nC_5 {}^nC_0$$

4. (B)

$$E = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \dots (x - \alpha_n)$$

where $\alpha_1 = 1, \alpha_2 = 2$ etc

$$= x^n - \left(\sum \alpha_i \right) x^{n-1} + \left(\sum \alpha_i \alpha_j \right) x^{n-2} + \dots$$

Hence co-efficient of x^{n-2} = sum of all the products of the first 'n' natural numbers taken two at a time

$$= \frac{(1+2+3+\dots+n)^2 - (1^2+2^2+\dots+n^2)}{2}$$

$$= \frac{n(n^2-1)(3n+2)}{24}$$

5. Co-efficient of $x^n = {}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_n = 2^{2n}$

6. (C)

Let $x-1=t$, then

$$\sum_{r=0}^{2n} a_r t^r = \sum_{r=0}^{2n} b_r (t-1)^r$$

$$\therefore a_n = \text{coefficient of } t^n \text{ in } \sum_{r=0}^{2n} b_r (t-1)^r$$

$$= \text{coefficient of } t^n \text{ in } (b_0 + b_1(t-1) + \dots + b_n(t-1)^n + b_{n+1}(t-1)^{n+1} + \dots + b_{2n}(t-1)^{2n})$$

$$= b_n {}^nC_0 + b_{n+1} {}^{n+1}C_1 (-1)^1 + b_{n+2} {}^{n+2}C_2 (-1)^2 + \dots + b_{2n} {}^{2n}C_n (-1)^n$$

$$= (-1)^{n-n} \cdot {}^nC_0 + (-1)^{n+1-n+1} \cdot {}^{n+1}C_1 + \dots + (-1)^{2n-n+n} {}^{2n}C_n$$

$$= {}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \dots + {}^{2n}C_n = {}^{2n+1}C_{n+1}$$

$$= {}^{2n+1}C_n$$

7. $\sum_{r=1}^n \left(\sum_{p=0}^{r-1} {}^nC_r {}^rC_p 2^p \right) = \sum_{r=1}^n {}^nC_r \sum_{p=0}^{r-1} {}^rC_p 2^p$

$$= \sum_{r=1}^n {}^nC_r [{}^rC_0 + {}^rC_1 \cdot 2 + \dots + {}^rC_{r-1} 2^{r-1}]$$

$$= \sum_{r=1}^n {}^nC_r (3^r - 2^r) = 4^n - 3^n$$

8. (B)

$$2(1+x^3)^{100} = \sum_{k=0}^{100} \left(a_k x^k - \cos \frac{\pi}{2} (x+k) \right)$$

$$= \sum_{k=0}^{100} a_k \cdot x^k - \sum_{k=0}^{100} \cos \frac{\pi}{2} (x+k)$$

$$= \sum_{k=0}^{100} a_k \cdot x^k -$$

$$\left[\cos \frac{\pi}{2} x + \cos \left(\frac{\pi}{2} x + \frac{\pi}{2} \right) + \cos \left(\frac{\pi}{2} x + \pi \right) + \cos \left(\frac{\pi}{2} x + \frac{3\pi}{2} \right) + \dots \right]$$

$$2(1+x^3)^{100} = \sum_{k=0}^{100} a_k \cdot x^k - (0)$$

put $x = -1$ $0 = a_0 - a_1 + a_2 - a_3 + \dots$

put $x = 1$ $2^{101} = a_0 + a_1 + a_2 + a_3$

add

$$2^{101} = 2[a_0 + a_2 + a_4 + \dots]$$

$$a_0 + a_2 + a_4 + \dots + a_{100} = 2^{100}$$

9. $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

Multiply it by x

$$x(1+x)^n = {}^nC_0 x + {}^nC_1 x^2 + {}^nC_2 x^3 + \dots + {}^nC_n x^{n+1}$$

Differentiate w.r. to x and put $x = -3$

$$n x (1+x)^{n-1} + (1+x)^n$$

$$= {}^nC_0 + 2 {}^nC_1 x + 3 {}^nC_2 x^2 + 4 {}^nC_3 x^3 + \dots + (n+1) {}^nC_n x^n$$

So answer, $-3n(-2)^{n-1} + (-2)^n = (-2)^n \left(1 + \frac{3n}{2} \right)$

$$= (-1)^n 2^n \left(\frac{3n}{2} + 1 \right)$$

10. (B)

$$S_1 : (1+x)^n = {}^nC_0 + {}^nC_1 x + \dots + {}^nC_n x^n \quad \dots\dots(i)$$

$$(x+1)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + \dots + {}^nC_n \quad \dots\dots(ii)$$

coefficient of x^{n-1} in the product of two series

$${}^nC_0 \cdot {}^nC_1 + {}^nC_1 \cdot {}^nC_2 + \dots = {}^{2n}C_{n-1}$$

$$S_2 : (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n \quad \dots\dots(i)$$

$$(x+1)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + \dots + {}^nC_n \quad \dots\dots(ii)$$

coefficient of x^n in the product of two series

$$C_0^2 + C_1^2 + \dots = {}^{2n}C_n$$

$$S_3 : (1+x)^{11} = {}^{11}C_0 + {}^{11}C_1 x + \dots + {}^{11}C_{11} x^{11}$$

$$(1-x)^{11} = {}^{11}C_0 - {}^{11}C_1 x + \dots + {}^{11}C_{11} (-1)^{11} x^{11}$$

Coefficient of x^{11} in product of two series

$${}^{11}C_0 \cdot {}^{11}C_{11} - {}^{11}C_1 \cdot {}^{11}C_{10} + \dots + (-1)^{11} \cdot {}^{11}C_{11}$$

$${}^{11}C_0 = \text{coefficient of } x^{11} \text{ in the expansion of } (1-x^2)^{11} = 0$$

$$S_4 : \sum_{r=0}^n \frac{{}^{2r+1}C_r}{r+1} = \frac{1}{(n+1)} \sum_{r=0}^n {}^{n+1}C_{r+1} 2^{r+1}$$

$$= \frac{1}{n+1} ({}^{n+1}C_0 2^0 + {}^{n+1}C_1 2^1 + \dots + {}^{n+1}C_{n+1} 2^{n+1} - {}^{n+1}C_0)$$

$$= \frac{3^{n+1} - 1}{n+1}$$

$$11. n^n \left(\frac{n+1}{2} \right)^{2n} = \left(\frac{\left(\frac{n(n+1)}{2} \right)^2}{n} \right)^n$$

$$= \left(\frac{1^3 + 2^3 + \dots + n^3}{n} \right)^n$$

$$\Rightarrow \frac{1^3 + 2^3 + \dots + n^3}{n} \geq \sqrt[n]{(n!)^3}$$

12. (A, B, C)

We have $a_1 = 2$ and for $n \geq 2$,

$$a_n = \left(1 + \frac{1}{n} \right)^n = {}^nC_0 + {}^nC_1 \left(\frac{1}{n} \right) + {}^nC_2 \left(\frac{1}{n} \right)^2 + \dots$$

$$+ {}^nC_r \left(\frac{1}{n} \right)^r + \dots + {}^nC_n \left(\frac{1}{n} \right)^n$$

$$= 1 + 1 + \frac{n(n-1)}{2!} \frac{1}{n^2} + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}$$

$$\frac{1}{n^r} + \dots + \frac{n(n-1)\dots 2.1}{n!} \frac{1}{n^n} \quad \dots\dots(i)$$

$$= 2 + \frac{1}{2!} \left(1 - \frac{1}{n} \right) + \frac{1}{3!} \left(1 - \frac{1}{n} \right) \left(1 - \frac{2}{n} \right) + \dots$$

$$+ \frac{1}{r!} \left(1 - \frac{1}{n} \right) \left(1 - \frac{2}{n} \right) \dots \left(1 - \frac{r-1}{n} \right)$$

$$+ \dots + \frac{1}{n!} \left(1 - \frac{1}{n} \right) \left(1 - \frac{2}{n} \right) \dots \left(1 - \frac{n-1}{n} \right)$$

Hence from (i), $a_n \geq 2$ for all $n \in \mathbb{N}$. Also

$$a_n \leq 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{r!} + \dots + \frac{1}{n!}$$

For $2 \leq r \leq n$, we have $r! = 1 \cdot 2 \cdot 3 \dots r \geq 2^{r-1}$.

$$\text{Thus, } a_n \leq 1 + 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{r-1}} + \dots + \frac{1}{2^{n-1}}$$

$$= 1 + \frac{1 - (1/2^n)}{1 - (1/2)} = 1 + 2 \left(1 - \frac{1}{2^n} \right) = 3 - \frac{1}{2^{n-1}}$$

$$a_n \leq 3 - \frac{1}{2^{n-1}} < 3 \quad \forall \quad n \geq 1 \quad \Rightarrow \quad a_n < 4 \quad \forall \quad n \geq 1$$

$$13. a_n = \frac{(1000)(1000)\dots\dots(1000)}{1.2\dots\dots n}$$

$$a_{999} = a_{1000}$$

a_n is maximum for $n = 999$ and $n = 1000$

$$14. y = (1-x)^{-1} (1+x)^n$$

$$\Rightarrow y = (1+x+x^2+\dots\infty)(1+x)^n$$

$$y = (1+x)^n + x(1+x)^n + x^2(1+x)^n + \dots$$

$$\text{Co-efficient of } x^r = {}^nC_r + {}^nC_{r-1} + \dots + {}^nC_0 = 2^n$$

$$r \geq n \quad (\text{As } {}^nC_{n+1}) = 0$$

15. (A,C,D)

$$I + f = (4 + \sqrt{15})^n$$

$$\text{Let } g = (4 - \sqrt{15})^n, \text{ then } 0 < g < 1$$

$$I + f = {}^nC_0 4^n + {}^nC_1 4^{n-1} \sqrt{15} + {}^nC_2 4^{n-2} 15 + {}^nC_3 4^{n-3} (\sqrt{15})^3 + \dots$$

$$g = {}^nC_0 4^n - {}^nC_1 4^{n-1} \sqrt{15} + {}^nC_2 \cdot 4^{n-2} \cdot 15 - {}^nC_3 4^{n-3} (\sqrt{15})^3 + \dots$$

$$\therefore I + f + g = 2({}^nC_0 4^n + {}^nC_2 4^{n-2} \cdot 15 + \dots) = \text{even integer}$$

$$\rightarrow 0 < f + g < 2 \Rightarrow f + g = 1 \Rightarrow 1 - f = g$$

thus I is an odd integer

$$1 - f = g = (4 - \sqrt{15})^n$$

$$(1 + f)(1 - f) = (1 + f) \cdot g = 1$$

$$16. (D) \rightarrow (a + b + c)^n = \sum \frac{n!}{p! \cdot q! \cdot r!} \cdot a^p b^q c^r,$$

$$p + q + r = n$$

in statement-1 $p + q + r$ exceeds n

$$17. (D) \sum_{r=0}^n r {}^n C_r p^r q^{n-r} = np \sum_{r=0}^n {}^{n-1} C_{r-1} p^{r-1} q^{n-r} \\ = np (q + p)^{n-1} = np$$

18. (A)

If $n \geq 4$, then term containing $x^{\frac{n(n+1)}{2}-4}$ is $(-4)x^{1+2+3+5+6+7+\dots+n} + (-1)(-3)x^{2+4+5+6+\dots+n}$

$\therefore$ coefficient of $x^{\frac{n(n+1)}{2}-4}$ is $-4 + (-1)(-3) = -1$
Statement : 2 is true and it explains statement-1

19. (A)

The number of ways of selecting committee of r persons among 40 women and 60 men $= {}^{100}C_r$
This will assume greatest value at $r = 50$

20. (A)

$1 + x = {}^nC_n + {}^nC_{n-1} + {}^{n+1}C_{n-1} + \dots + {}^{2n}C_{n-1} = {}^{2n+1}C_n$
Since $2n + 1$ and n are co-prime for every natural number n .
 $\therefore {}^{2n+1}C_n$ is divisible by $2n + 1$
 $\therefore \frac{x+1}{2n+1}$ is an integer

21. (A) $\rightarrow$ (p), (B) $\rightarrow$ (q), (C) $\rightarrow$ (t), (D) $\rightarrow$ (s)

(A) $({}^m C_1 {}^n C_m - {}^m C_2 {}^{2n} C_m + {}^m C_3 {}^{3n} C_m - \dots (-1)^{m-1} {}^m C_m {}^{mn} C_m)$
= Coefficient of x^m in the expansion of $({}^m C_1 (1+x)^n - {}^m C_2 (1+x)^{2n} + {}^m C_3 (1+x)^{3n} \dots + (-1)^{m-1} {}^m C_m \cdot (1+x)^{mn})$
= Coefficient of x^m in the expansion of $({}^m C_0 - [{}^m C_0 - {}^m C_1 (1+x)^n + {}^m C_2 (1+x)^{2n} - {}^m C_3 (1+x)^{3n} + \dots + (-1)^m {}^m C_m (1+x)^{mn}])$
= Coefficient of x^m in the expansion of $(1 - (1 - (1+x)^n)^m)$
= Coefficient of x^m in the expansion of $-[1 - (1+x)^n]^m$

(B) ${}^n C_m + {}^{n-1} C_m + {}^{n-2} C_m + \dots + {}^m C_m$ is the coefficient of x^m in the expansion of

$$(1+x)^n + (1+x)^{n-1} + (1+x)^{n-2} + \dots + (1+x)^m \\ = (1+x)^m [1 + (1+x) + (1+x)^2 + \dots + (1+x)^{n-m}] \\ = (1+x)^m \left(\frac{1 - (1+x)^{n-m+1}}{1 - (1+x)} \right) = \frac{(1+x)^{n+1} - (1+x)^m}{x}$$

Thus the given expression is equal to the coefficient of x^m in the expansion of $\frac{(1+x)^{n+1}}{x}$

$$(C) (1+x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + {}^n C_n x^n \quad \dots (A)$$

$$(1+x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + {}^n C_n x^n \quad \dots (B)$$

multiplying eq. (A) and (B) and equating coefficient of x^n on both the sides.

$$\text{coefficient of } x^n \text{ in the expansion of } (1+x)^n (1+x)^n \\ = {}^n C_0 {}^n C_n + {}^n C_1 {}^n C_{n-1} + {}^n C_2 {}^n C_{n-2} + \dots + {}^n C_n {}^n C_0$$

$$\therefore \text{Coefficient of } x^n \text{ in the expansion of } (1+x)^{2n} \\ = {}^{2n} C_n$$

$$(D) {}^{2m} C_m = \text{Coefficient of } x^m \text{ in the expansion of } (1+2x)^n \\ {}^{2m-1} C_{m-1} = \text{Coefficient of } x^{m-1} \text{ in the expansion of } (1+2x)^{n-1}$$

$$= \text{Coefficient of } x^m \text{ in the expansion of } x(1+2x)^{n-1}$$

$\therefore$ given expression = coefficient of x^m in the expansion of

$${}^n C_0 (1+2x)^n - {}^n C_1 x(1+2x)^{n-1} + {}^n C_2 x^2 (1+2x)^{n-2} - \dots$$

$$= \text{coefficient of } x^m \text{ in the expansion of } (1+2x-x)^n = {}^n C_m$$

22. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (s), (D) $\rightarrow$ (q)

$$(A) (1+x)^5)^{20} = (1+x)^{100}$$

$$\text{no. of terms} = 101 = \lambda$$

$$3^{101} = 3 \cdot 3^{100} = 3 \cdot (9^{50}) = 3 \cdot (10-1)^{50}$$

$$= 3 \cdot (1-10)^{50} = 3(1 - {}^{50} C_1 \cdot (10) + {}^{50} C_2 \cdot (10)^2 - \dots)$$

$$= 3(1 - 500 + 122500 + \dots) = 3[1 + \text{some number with zero at unit and tenth place}]$$

$$= 3[\dots 01] = \dots 03$$

$$O + T = 3$$



(B) $8 \cdot \left\{ \frac{(8+1)^n}{8} \right\} = 8 \cdot \left\{ \frac{1}{8} + \frac{C_1 \cdot 8 + C_2 \cdot 8^2 + \dots}{8} \right\} = 8 \cdot \frac{1}{8} = 1$

(C) $(x + \sqrt{3x^2 + 1})^7 = {}^7C_0 x^7 + {}^7C_1 x^6 + \dots$

$(\sqrt{3x^2 + 1})^1 + {}^7C_2 x^5 \cdot (\sqrt{3x^2 + 1})^2 + \dots$

$(x - \sqrt{3x^2 + 1})^7 = {}^7C_0 x^7 - {}^7C_1 x^6 \cdot (\sqrt{3x^2 + 1}) + \dots$

$\Rightarrow 2 \left({}^7C_1 x^6 \cdot \sqrt{3x^2 + 1} + {}^7C_3 x^4 \cdot (\sqrt{3x^2 + 1})^3 + \dots \right)$

${}^7C_5 x^2 (\sqrt{3x^2 + 1})^5 + {}^7C_7 x^0 (\sqrt{3x^2 + 1})^7$

multiply by $\sqrt{3x^2 + 1} =$

$({}^7C_1 x^6 \cdot (3x^2 + 1) + {}^7C_5 x^4 (3x^2 + 1)^2 + {}^7C_7 x^2 (3x^2 + 1)^3 + \dots)$

Degree = 8

(D) $\frac{(2002+1)}{1.2002} \cdot \frac{(2001+2)}{2.2001} \cdot \frac{(2000+3)}{3.2000} + \dots$

$\frac{(1002+1001)}{1001.1002}$

$\frac{(2003)^{1001}}{(2002)!}$

23.

1 (C)

$\sum_{r=0}^{6m} {}^{6m}C_r 2^{r/2}$ put $x = \sqrt{2} = (1 + \sqrt{2})^{6m}$

$= (3 + 2\sqrt{2})^{3m}$

2 (B)

$\sum_{r=0}^{3m} (-1)^r {}^{6m}C_{2r}$

$= \frac{1}{2} \left[\left(\sqrt{2} \right)^{6m} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)^{6m} + \left(\sqrt{2} \right)^{6m} \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right)^{6m} \right]$

$= 2^{3m} \cdot \cos \frac{3m\pi}{2} = \begin{cases} 0 & \text{if } m \text{ is odd} \\ (-1)^{\frac{m}{2}} 2^{3m} & \text{if } m \text{ is even} \end{cases}$

3 (A)

$\sum_{r=1}^{3m} (-3)^{r-1} {}^{6m}C_{2r-1}$

$= \frac{1}{\sqrt{3}i}$

$\left\{ \sqrt{3}i {}^{6m}C_1 + (\sqrt{3}i)^3 {}^{6m}C_3 + (\sqrt{3}i)^5 {}^{6m}C_5 + \dots + (\sqrt{3}i)^{6m-1} {}^{6m}C_{6m-1} \right\}$

$(1 + \sqrt{3}i)^{6m} = {}^{6m}C_0 + \sqrt{3}i {}^{6m}C_1 + (\sqrt{3}i)^2 {}^{6m}C_2 + (\sqrt{3}i)^3 {}^{6m}C_3 + \dots$

$(1 - \sqrt{3}i)^{6m} = {}^{6m}C_0 - \sqrt{3}i {}^{6m}C_1 + (\sqrt{3}i)^2 {}^{6m}C_2 - (\sqrt{3}i)^3 {}^{6m}C_3 + \dots$

$\therefore (1 + \sqrt{3}i)^{6m} - (1 - \sqrt{3}i)^{6m} = 2 \left[\sqrt{3}i {}^{6m}C_1 + (\sqrt{3}i)^3 {}^{6m}C_3 + \dots \right]$

$\therefore$ given expression

$\frac{1}{2\sqrt{3}i} \left[(1 + \sqrt{3}i)^{6m} - (1 - \sqrt{3}i)^{6m} \right]$

$= \frac{2^{6m}}{2\sqrt{3}i} (\cos 2m\pi + i \sin 2m\pi - \cos 2m\pi + i \sin 2m\pi)$
 $= 0$

24.

1 $S_2(n) + S_1(n) = \sum n^2 + \sum n = \sum n(n+1) = 1.2 + 2.3 + 3.4 + \dots + n(n+1) = B(2, n)$

2 $S_3(n) + 3S_2(n) + 2S_1(n) - 2S_1(n)$
 $= \sum n^3 + 3\sum n^2 + 2\sum n - 2\sum n = \sum n(n+1)(n+2) - 2\sum n$
 $= B(3, n) - 2B(1, n)$

3 $(1+x)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k x + {}^{k+1}C_{k-1} x^2 + {}^{k+1}C_{k-2} x^3 + \dots + {}^{k+1}C_0 x^{k+1}$

Put $x = 1, 2, \dots, n$

$2^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot 1 + {}^{k+1}C_{k-1} \cdot 1^2 + {}^{k+1}C_{k-2} \cdot 1^3 + \dots + {}^{k+1}C_0 \cdot 1^{k+1}$

$3^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot 2 + {}^{k+1}C_{k-1} \cdot 2^2 + \dots + {}^{k+1}C_0 \cdot 2^{k+1}$

$(1+n)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot n + {}^{k+1}C_{k-1} \cdot n^2 + \dots + {}^{k+1}C_0 n^{k+1}$

$2^{k+1} + 3^{k+1} + \dots + (1+n)^{k+1} = {}^{k+1}C_{k+1} S_0(n) + {}^{k+1}C_k S_1(n) + {}^{k+1}C_{k-1} S_2(n) + \dots + {}^{k+1}C_0 S_{(k+1)}(n)$

$2^{k+1} + 3^{k+1} + \dots + (n+1)^{k+1} = {}^{k+1}C_{k+1} S_0(n) + {}^{k+1}C_k S_1(n) + \dots + {}^{k+1}C_1 S_k(n) + 1^{k+1} + 2^{k+1} + 3^{k+1} + \dots + n^{k+1}$

So $(n+1)^{k+1} - 1$

25.

$$\begin{aligned} x^3 - 1 &= (x-1)(x^2+x+1) \\ x = 1, \omega, \omega^2 \quad \text{or} \quad x = \omega, \omega^2, \omega^3 \\ x = 1 &: C_0 + C_1 + C_2 + C_3 + C_4 + \dots = 2^n \\ x = \omega &: C_0 + C_1\omega + C_2\omega^2 + C_3\omega^3 + C_4\omega^4 + \dots = (1+\omega)^n \\ x = \omega^2 &: C_0 + C_1\omega^2 + C_2\omega^4 + C_3\omega^6 + \dots = (1+\omega^2)^n \end{aligned}$$

$$3(C_0 + 0 + 0 + C_3 + 0 + 0 + C_6 + \dots) = 2^n + (-\omega)^n + (-\omega^2)^n = 2^n + (-1)^n + (-1)^n$$

$$\therefore C_0 + C_3 + C_6 + \dots = \frac{2^n + 2(-1)^n}{3}$$

$$x^4 - 1 = 0 \Rightarrow x = \pm 1, \pm i$$

$$\therefore \text{Sum of values of } x = 1 + (-1) + i + (-i) = 0$$

26. (7)

$$\text{If } (1-x^3)^n = \sum_{r=0}^n a_r x^r (1-x)^{3n-2r}$$

$$\Rightarrow [(1-x)(1+x+x^2)]^n = (1-x)^n \sum_{r=0}^n a_r x^r [(1-x)^2]^{n-r}$$

$$\Rightarrow [3x + (1-x)^2]^n = \sum_{r=0}^n a_r x^r [(1-x)^2]^{n-r}$$

$$\Rightarrow \sum_{r=0}^n {}^nC_r (3x)^r [(1-x)^2]^{n-r} = \sum_{r=0}^n a_r x^r [(1-x)^2]^{n-r}$$

comparing the coefficients of like power of x on both sides, we get $a_r = {}^nC_r \cdot 3^r$.

$$\therefore a_1 = 3 \cdot {}^nC_1, a_2 = 9 \cdot {}^nC_2 \text{ and } a_3 = 27 \cdot {}^nC_3$$

$$\therefore a_1, a_2, a_3 \text{ are in A.G.P. iff } {}^nC_1, {}^nC_2, {}^nC_3 \text{ are in A.P.}$$

$$\therefore n = 7$$

27. $(1-x)^{-n} = a_0 + a_1x + a_2x^2 + \dots + a_nx^n = \sum_{r=0}^n {}^{n+r-1}C_r x^r$

$$\begin{aligned} a_0 + a_1 + \dots + a_n &= {}^{n-1}C_0 + {}^nC_1 + {}^{n+1}C_2 + \dots + {}^{2n-1}C_n \\ &= {}^{n-1}C_{n-1} + {}^nC_{n-1} + {}^{n+1}C_{n-1} + \dots + {}^{2n-1}C_{n-1} \\ &= {}^nC_n + {}^{n+1}C_{n-1} + {}^{n+2}C_{n-1} + \dots + {}^{2n-1}C_{n-1} = {}^{2n}C_n \\ &= \{ {}^nC_r + {}^{n+1}C_{r-1} = {}^{n+1}C_r \} \end{aligned}$$

28. (1)

Any term of the expansion is of the form

$$\frac{6!}{a!b!c!} (5^{1/3}x)^a (3^{1/2}y)^b z^c$$

a, b, c non-negative integers and $a+b+c=6$. For rational coefficients 'a' must be multiple of 3 and b must be multiple of 2.

The following are the possibilities

a	b	c
0	0	6
0	2	4
0	4	2
0	6	0
3	0	3
3	2	1
6	0	0

Sum of coefficients

$$= \frac{6!}{6!} + \frac{6!}{2!4!} \cdot 3 + \frac{6!}{4!2!} \cdot 3^2 + \frac{6!}{6!} \cdot 3^3 + \frac{6!}{3!3!} \cdot 5 + \frac{6!}{3!2!} \cdot 5 \cdot 3 + \frac{6!}{6!} \cdot 5^2 = 1233$$

29. $32^{32} = (2^5)^{32} = 2^{160}$

$$(3-1)^{160} = 3\lambda + 1$$

$$32^{32^{32}} = (2^5)^{(3\lambda+1)} = 2^{(15\lambda+3)+2} = 4 \cdot (2^3)^{(5\lambda+1)} = 4(7+1)^\beta = 4(7\mu+1)$$

$\therefore$ remainder is 4

30. (2)

$$x^3 - 3xy^2 = 2005$$

$$\Rightarrow \left[\left(\frac{x}{y} \right)^3 - 3 \left(\frac{x}{y} \right) \right] = \frac{2005}{y^3} \times 2004 \quad \dots (i)$$

$$y^3 - 3x^2y = 2004$$

$$\Rightarrow \left[1 - 3 \left(\frac{x}{y} \right)^2 \right] = \frac{2004}{y^3} \times 2005 \quad \dots (ii)$$

subtract (i) & (ii) & put $\frac{x}{y} = t$

$$2004t^3 + 6015t^2 - 6012t - 2005 = 0 \quad \begin{matrix} \nearrow t_1 \\ \rightarrow t_2 \\ \searrow t_3 \end{matrix}$$

$$\frac{y_1 \cdot y_2 \cdot y_3}{(y_1 - x_1)(y_2 - x_2)(y_3 - y_3)} = \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$$

$$= \frac{1}{1 + (t_1t_2 + t_2t_3 + t_3t_1) - (t_1 + t_2 + t_3) - t_1t_2t_3}$$

put values = 1002

