THREE D

EXERCISE # 1

Questions based on Distance between two points

Q.1 The points A(5, –1, 1); B(7, –4, 7); C(1, –6, 10) and D(–1, –3, 4) are vertices of a -
(A) square (B) rhombus
(C) rectangle (D) none of these

Sol. **[B]**

$AB = \sqrt{4+9+36} = 7$

BC = 7

CD = 7

DA = 7

$AC = \sqrt{16+25+81}$

$= \sqrt{122}$

$BD = \sqrt{64+1+9}$

$= \sqrt{74}$

rhombus.

Q.2 Points (1, 2, 3); (3, 5, 7) and (–1, –1, –1) are-
(A) Vertices of a equilateral triangle
(B) Vertices of a right angle triangle
(C) Vertices of a isosceles triangle
(D) Collinear

Sol. **[D]**

$\overrightarrow{AB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{BC} = -4\hat{i} - 6\hat{j} - 8\hat{k}$

$\overrightarrow{BC} = -2\overrightarrow{AB}$

Collinear point.

Questions based on Section formula

Q.3 The ratio in which the segment joining the points (2, 4, 5), (3, 5, –4) is divided by the yz-plane is-
(A) – 2 : 3 (B) 2 : 3 (C) 3 : 2 (D) – 3 : 2

Sol. **[A]**

$\dfrac{2+\lambda(3)}{\lambda+1} = 0$

$\lambda = -\dfrac{2}{3}$

Q.4 The ratio in which the segment joining (1, 2, –1) and (4, –5, 2) is divided by the plane 2x – 3y + z = 4 is-
(A) 7 : 3 (B) 3 : 7 (C) 3 : 5 (D) None

Sol. **[B]**

λ : 1

$2\left(\dfrac{1+4\lambda}{1+\lambda}\right) - 3\left(\dfrac{2-5\lambda}{\lambda+1}\right) + \left(\dfrac{-1+2\lambda}{\lambda+1}\right) = 4$

$\Rightarrow 2 + 8\lambda - 6 + 15\lambda - 1 + 2\lambda = 4\lambda + 4$

$21\lambda = 9$

$\lambda = \dfrac{3}{7}$

Q.5 If distance of any point from z-axis is thrice its distance from xy-plane, then its locus is-
(A) $x^2 + y^2 - 9z^2 = 0$ (B) $y^2 + z^2 - 9x^2 = 0$
(C) $x^2 - 9y^2 + z^2 = 0$ (D) $x^2 + y^2 + z^2 = 0$

Sol. **[A]**

(x, y, z)

$\sqrt{x^2 + y^2} = 9z$

Q.6 The co-ordinates of the point where the line joining the points (2, –3, 1), (3, –4, –5) cuts the plane 2x + y + z = 7 are-
(A) (2, 1, 0) (B) (3, 2, 5)
(C) (1, –2, 7) (D) None of these

Sol. **[C]**

Direction ratio of the line

$= (x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

= (1,– 1,– 6)

This line passes through (2,– 3, 1)

Then its equation

$\dfrac{x-2}{1} = \dfrac{y+3}{-1} = \dfrac{z-1}{-6} = r$ say

Coordinate of any point on this line is

(r + 2, – r – 3, – 6 r + 1)

This point lie on the plane

2x + y + z = 7

Then 2(r + 2) + (– r – 3) + 6r + 1) = 7

$\Rightarrow -5r = 5 \Rightarrow r = -1$

coordinate of required point is (1, – 2, 7)

Q.7 A point moves in such a way that sum of square of its distances from the co-ordinate axis are 36, then distance of these given point from origin are-
(A) 6 (B) $2\sqrt{3}$ (C) $3\sqrt{2}$ (D) None

Sol. **[C]**

Distance of the point form

x axis = $\sqrt{y^2+z^2}$

y axis = $\sqrt{x^2+z^2}$

z axis = $\sqrt{x^2+y^2}$

Sum of square = 36

$\Rightarrow 2(x^2 + y^2 + z^2) = 18$

$\Rightarrow x^2 + y^2 + z^2 = 18$

distance of this point form the origin

P = $\sqrt{x^2+y^2+z^2}$

P = $\sqrt{18} = 3\sqrt{2}$

Questions based on Direction cosines & direction ratio's

Q.8 The d.c's of a line whose direction ratios are 2, 3, –6, are-

(A) $\frac{2}{7},\frac{3}{7},\frac{-6}{7}$ (B) $\frac{-2}{7},\frac{3}{7},\frac{-6}{7}$

(C) $\frac{2}{7},\frac{-3}{7},\frac{-6}{7}$ (D) None of these

Sol. **[A]**

$\vec{r} = 2\hat{i}+3\hat{j}-6\hat{k}$

DC's = $\frac{2}{7},\frac{3}{7},\frac{-6}{7}$

Q.9 The projections of a line segment on x, y and z axes are respectively 3, 4 and 5, then the length and direction cosines of the line segment is

(A) $5\sqrt{2}$; $\frac{3}{5\sqrt{2}},\frac{4}{5\sqrt{2}},\frac{1}{\sqrt{2}}$

(B) $3\sqrt{2}$; $\frac{3}{3\sqrt{2}},\frac{4}{5\sqrt{2}},\frac{1}{\sqrt{2}}$

(C) $5\sqrt{2}$; $\frac{3}{5\sqrt{2}},\frac{4}{3\sqrt{2}},\frac{1}{\sqrt{2}}$

(D) $3\sqrt{2}$; $\frac{3}{5\sqrt{2}},\frac{4}{5\sqrt{2}},-\frac{1}{\sqrt{2}}$

Sol. **[A]**

$\frac{\vec{r}_1\cdot\hat{i}}{|\hat{i}|} = 3$

$r_1 = 3, r_2 = 4, r_3 = 5$

$\vec{r} = 3\hat{i}+4\hat{j}+5\hat{k}$

$|\vec{r}| = 5\sqrt{2}$

DC's = $\left(\frac{3}{5\sqrt{2}},\frac{4}{5\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

Q.10 The direction cosines of a line equally inclined with the coordinate axes are -

(A) (1, 1, 1) or (–1, –1, –1)

(B) $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ or $\left(-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(D) none of these

Sol. **[B]**

$3\lambda^2 = 1$

$\lambda = \pm\frac{1}{\sqrt{3}}$

$\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ or $\left(-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right)$

Questions based on Angle between two lines

Q.11 Direction ratios of two lines are a, b, c and $\frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$. The lines are -

(A) Mutually perpendicular

(B) Parallel

(C) Coincident

(D) None of these

Sol. **[B]**

we have

$a_1 = a, b_1 = b, c_1 = c$

and $a_2 = \frac{1}{bc}$, $b_2 = \frac{1}{ca}$, $c_2 = \frac{1}{ab}$

θ be the angle between them. Then

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}}{\sqrt{a^2+b^2+c^2}\sqrt{\frac{1}{b^2c^2}+\frac{1}{c^2a^2}+\frac{1}{a^2b^2}}} = 1$$

$\Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 0$

both lines are parallel.

Q.12 If λ, m, n and λ′, m′, n′ be the direction cosines of two lines which include an angle θ, then -

(A) $\cos\theta = \lambda\lambda' + mm' + nn'$
(B) $\sin\theta = \lambda\lambda' + mm' + nn'$
(C) $\cos\theta = mm' + m'n + n\lambda' + n'\lambda + \lambda m' + \lambda' m$
(D) $\sin\theta = mn' + m'n + n\lambda' + n'\lambda + \lambda m' + \lambda' m$

Sol. **[A]**

λ, m, n and λ', m', n'

be the direction cosine of two lines then

$\cos\theta = \lambda\lambda' + mm' + nn'$

Questions based on Projection problems

Q.13 P ≡ (x_1, y_1, z_1) and Q ≡ (x_2, y_2, z_2) are two points if direction cosines of a line AB are λ, m, n then projection of PQ on AB aare -

(A) $\frac{1}{\lambda}(x_2 - x_1) + \frac{1}{m}(y_2 - y_1) + \frac{1}{n}(z_2 - z_1)$

(B) $\lambda(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

(C) $\frac{1}{\lambda mn}[\lambda(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)]$

(D) None of these

Sol. **[B]**

$PQ = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$AB = \lambda(\lambda\hat{i} + m\hat{j} + n\hat{k})$

Projection

$= \frac{\overrightarrow{PQ}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$= \lambda(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

Q.14 If the line OP of length r makes an angle α with x-axis and lies in the xz plane, then the coordinate of P are -

(A) (r cos α, 0, r sin α) (B) (0, 0, r sin α)
(C) (0, 0, r cos α) (D) (r cos α, 0, 0)

Sol. **[A]**

Line OP of length r make an angle α with x-axis and in the xz plane then coordinate of P is

(r cosα, 0, r cos($\pi/2-\alpha$))

$\Rightarrow$ (r cosα, 0, r sinα)

Q.15 If coordinates of point P, Q, R, S are respectively (6, 3, 2) ; (5, 1, 4) ; (3, 4, –7) and (0, 2, 5) then the projection of PQ on RS are-

(A) $\frac{31}{\sqrt{157}}$ (B) $\frac{131}{\sqrt{157}}$ (C) $\frac{\sqrt{13}}{7}$ (D) $\frac{13}{\sqrt{7}}$

Sol. **[A]**

For PQ (x_2-x_1), (y_2-y_1), (z_2-z_1)

= – 1, – 2, 2

For RS (x_2-x_1), (y_2-y_1), (z_2-z_1)

= – 3, – 2, 12

Then for RS sirection cosine

$\lambda = \frac{-3}{\sqrt{157}}$, $m = \frac{-2}{\sqrt{157}}$, $n = \frac{12}{\sqrt{157}}$

Projection of PQ on RS

$= \lambda(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

$= \frac{3}{\sqrt{157}} + \frac{4}{\sqrt{157}} + \frac{24}{\sqrt{157}} = \frac{31}{\sqrt{157}}$

Questions based on Equation of straight line in space

Q.16 The angle between two lines $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z-4}{-1}$ and $\frac{x-4}{1} = \frac{y+4}{2} = \frac{z+1}{2}$ is-

(A) $\cos^{-1}(2/9)$ (B) $\cos^{-1}(4/9)$
(C) $\cos^{-1}(5/9)$ (D) $\cos^{-1}(7/9)$

Sol. **[B]**

$\cos\theta = \frac{2+4-2}{3\,.\,3} = \frac{4}{9}$

Q.17 The equation of a line passing through the origin and parallel to the line whose direction ratios are 1, –1, 2 is -

(A) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$

(B) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$

(C) $\frac{x}{1/\sqrt{6}} = \frac{y}{-1/\sqrt{6}} = \frac{z}{2/\sqrt{6}}$

(D) $\frac{x-7}{1/2} = \frac{y+7}{-1/2} = \frac{z-14}{1}$

Sol. **[A]**

$\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$

Q.18 $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{0}$ is -

(A) parallel to yz plane
(B) parallel to zx plane
(C) perpendicular to z axis
(D) parallel to z axis

Sol. **[C]**

$\perp$ to z-axis

Q.19 The point in which the join of (–9, 4, 5) and (11, 0, –1) is met by the perpendicular from the origin is-

(A) (2, 1, 2) (B) (2, 2, 1)
(C) (1, 2, 2) (D) None of these

Sol. **[C]**

Direction ratio of the line joining the point (–9, 4, 5) and (11, 0, –1) is

= (20, – 4, – 6)

it is passes through (–9, 4, 5)

It's equation is

$$\frac{1+9}{20} = \frac{y-4}{-4} = \frac{z-5}{-6} = r \text{ let}$$

any point on this line given as

(20r – 9, –4r + 4, –6r + 5) ….(i)

Direction ratio of the line passes trough origin and point (i) we have

(20r – 9, –4r + 4, 6r + 5)

It is perpendicular with (20,– 4, – 6)

$\Rightarrow$ 20(20r – 9) – 4 (4r + 4) – 6 (6r + 5) = 0

$\Rightarrow r = \frac{1}{2}$

point is (1, 2, 2)

Q.20 If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are at right angles, then the value of k will be –

(A) $-\frac{10}{7}$ (B) $-\frac{7}{10}$ (C) –10 (D) –7

Sol. **[A]**

Given that

$a_1 = -3, b_1 = 2k, c_1 = 2$

$a_2 = 3k\ b_2 = 1\ ,c_2 = -5$

both line are at right angle then

$\cos\theta = 0$

$\Rightarrow a_1a_2+b_1b_2+c_1c_2 = 0$

$\Rightarrow$ – 9k + 2k – 10 = 0

$\Rightarrow$ –7k = 10

$\Rightarrow k = -\frac{10}{7}$

Q.21 The equation of straight line passing through the points (a, b, c) and (a – b, b – c, c – a), is -

(A) $\frac{x-a}{a-b} = \frac{y-b}{b-c} = \frac{z-c}{c-a}$

(B) $\frac{x-a}{b} = \frac{y-b}{c} = \frac{z-c}{a}$

(C) $\frac{x-a}{a-b} = \frac{y-b}{b} = \frac{z-c}{c}$

(D) $\frac{x-a}{2a-b} = \frac{y-b}{2b-c} = \frac{z-c}{2c-a}$

Sol. **[B]**

Point (a, b, c) and (a – b,b – c, c – a)

direction cosine is (b, c, a)

$\Rightarrow$ equation of line passing through (a, b, c) and dr's (b, c, a) is

$\frac{x-a}{b} = \frac{y-b}{c} = \frac{z-c}{a}$

Questions based on **Equation of plane**

Q.22 The normal form of the plane 2x + 6y + 3z = 1, is-

(A) $\frac{2}{7}x + \frac{6}{7}y + \frac{3}{7}z = \frac{1}{7}$

(B) $\frac{2}{7}x + \frac{6}{7}y - \frac{3}{7}z = \frac{1}{7}$

(C) $\frac{2}{7}x - \frac{6}{7}y - \frac{3}{7}z = \frac{1}{7}$

(D) $\frac{2}{7}x - \frac{6}{7}y + \frac{3}{7}z = \frac{1}{7}$

Sol. **[A]**

$\frac{2}{7}x + \frac{6}{7}y + \frac{3}{7}z = \frac{1}{7}$

Q.23 A point which lie in yz plane, the sum of co-ordinate is 3, if distance of point from xz plane is twice the distance of point from xy plane, then co-ordinates are -

(A) (1, 2, 0) (B) (0, 1, 2)
(C) (0, 2, 1) (D) (2, 0, 1)

Sol. **[C]**

0 + y + z = 3

y = 2z

$z = \frac{1}{3}, y = \frac{2}{3}$

Q.24 A point located in space is moves in such a way that sum of algebraic distance from xy and yz plane is equal to distance from zx plane the locus of the point are -

(A) x – y + z = 2 (B) x + y – z = 0
(C) x + y – z = 2 (D) x – y + z = 0

Sol. **[D]**

x + z = y

Q.25 A plane meets the co-ordinate axes in A, B, C such that the centroid of the triangle is the point $(1, r, r^2)$, the equation of the plane is -
(A) $x + ry + r^2z = 3r^2$ (B) $r^2x + ry + z = 3r^2$
(C) $x + ry + r^2z = 3$ (D) $r^2x + ry + z = 3$

Sol. **[B]**
Let the point on axis is
A(a, 0, 0), B(0, b, 0) and C(0, 0, c)
then centroid is
$$\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$
But given centroid is $(1, r, r^2)$
$\Rightarrow a = 3, b = 3r, c = 3r^2$
equation of plane in intercept form is
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$
$$\Rightarrow \frac{x}{3} + \frac{y}{3r} + \frac{z}{3r^2} = 1$$
$\Rightarrow r^2x + ry + z = 3r^2$

Q.26 The plane $x - 2y + 7z + 21 = 0$
(A) contains the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$
(B) contains the point (0, 7, –1)
(C) is perpendicular to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{7}$
(D) is parallel to the plane $x - 2y + 7z = 0$

Sol. **[C]**
(A) $1(-3) - 2(2) + 7(1) = 0$
(–1, 3, –2) has on the plane
(C) $\cos\theta = \frac{1+4+49}{\sqrt{53}\,\sqrt{53}} = 1$

plane is perpendicular to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{7}$

Q.27 If the line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ lies in the plane $4x + 4y - kz - d = 0$, then the value of k and d, are-
(A) 3, 5 (B) 5, 3 (C) 2, 5 (D) 5, 2

Sol. **[B]**
$4(2) + 4(3) - k(4) = 0$
$k = 5$
(3, 4, 5) lies on plane
$4(3) + 4(4) - 5(5) - d = 0$
$d = 3$

➤ True or false type questions

Q.28 The foot of the perpendicular from (a, b, c) on the line $x = y = z$ is the point (r, r, r) where $3r = a + b + c$.

Sol. **[True]**
Dc's of line (1,1,1)
Foot of perpendicular from (a, b, c)
on the line is (r,r,r)
then DC's of perpendicular is
$(r - a, r - b, r - c)$
$\Rightarrow (r - a)1 + (r - b) 1 + (r - c).1 = 0$
$\Rightarrow 3r = a + b + c$

Q.29 The line $x - 2y + 4z + 4 = 0$, $x + y + z - 8 = 0$ intersects the plane $x - y + 2z + 1 = 0$ at the point (2, 5, 1).

Sol. **[True]**
Given line is (symetnic form)
$$\frac{x-4}{-2} = \frac{y-4}{1} = \frac{z-0}{1} = r$$
any point on this line is
$(2r + 4, r + 4, r)$
this point on the plane so
$-2r + 4 - r - 4 + 2r + 1 = 0$
$\Rightarrow r = 1$
Point are (2, 5, 1)

➤ Fill in the blanks type questions

Q.30 The planes $bx - ay = n$, $cy - bz = \lambda$, $az - cx = m$ intersect in a line if ....

Sol. $bx - ay = n$
$cy - bz = \lambda$
$z = 0, \quad y = \lambda/c$
$$x = \frac{a\lambda}{cb} + \frac{n}{b}$$
$az - cx = m$
$$-c\left(\frac{a\lambda}{cb} + \frac{n}{b}\right) = m$$
$a\lambda + cn + bm = 0$

Q.31 If a plane cuts off intercepts OA = a, OB = b, OC = c from the coordinate axes, then the area of the triangle ABC is .....

Sol. OA = a, OB = b, OC = c
Length of sides are
$\sqrt{a^2+b^2}$, $\sqrt{b^2+c^2}$, $\sqrt{c^2+a^2}$
respectively
$$s = \frac{\sqrt{a^2+b^2} + \sqrt{b^2+c^2} + \sqrt{c^2+a^2}}{2}$$
use $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Put the values and solving , we get

$$D = \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}$$

EXERCISE # 2

Part-A **Only single correct answer type questions**

Q.1 OABC is a tetrahedron whose vertices are O (0, 0, 0); A (a, 2, 3); B (1, b, 2) and C (2, 1, c) if its centroid is (1, 2, –1) then distance of point (a, b, c) from origin are -

(A) $\sqrt{14}$ (B) $\sqrt{107}$

(C) $\sqrt{107/14}$ (D) None of these

Sol. **[B]**

Centroid of tetrahedron is

Given point O(0,0,0), A(a,2,3), B(1,b,2) and C(2,1,c)

Centroid is (1,2,– 4)

Put value and solving we get

(a, b, c) ≡ (1, 5, – 9)

distance of (a, b, c) from origin

$= \sqrt{1+25+81} = \sqrt{107}$

Q.2 A point P (x, y, z) moves parallel to z-axis. Which of the three variable x, y, z remain fixed ?

(A) x and y (B) y and z

(C) x and z (D) none of these

Sol. **[A]**

Point P(x, y, z) moves parallel to z axis so x and y are fixed.

Q.3 A line segment (vector) has length 21 and direction ratios as 2, –3, 6. If the line makes obtuse angle with x - axis, the components of the line (vector) are -

(A) –6, 9, –18 (B) 2, –3, 6

(C) 6, – 9, 18 (D) –18, 27, – 54

Sol. **[A]**

Let the components of the line are (a,b,c)

then $a^2 + b^2 + c^2 = (21)^2$ …….(i)

Also $\frac{a}{2} = \frac{b}{-3} = \frac{c}{6} = \lambda$ say

$\Rightarrow a = 2\lambda, b = -2\lambda, c = 6\lambda$

form (i) we get

$\Rightarrow 4\lambda^2 + 9\lambda^2 + 36\lambda^2 = (21)^2$

$\Rightarrow \lambda^2 = \frac{21\times 21}{49} = 9$

$\Rightarrow \lambda = \pm 3$

since line makes obtuse angle with x-axis

$\Rightarrow a = 2\lambda < 0$

$\Rightarrow \lambda = -3$

$\Rightarrow$ required components are

(–6, 9, –18)

Q.4 A line passes through the points (6, – 7, –1) and (2, –3, 1). The direction cosines of the line so directed that the angle made by it with positive direction of x-axis is acute, are -

(A) $\frac{2}{3}, \frac{-2}{3}, \frac{-1}{3}$ (B) $\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}$

(C) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ (D) $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

Sol. **[A]**

Direction ratio = $x_2 - x_1, y_2 - y_1, z_2 - z_1$

= (– 4, 4, 2) or (+ 4, – 4, – 2)

$\Rightarrow$ (–2, 2, 2) or (2, – 2, – 1)

But $(2)^2 + (-2)^2 + (-1)^2 = 9$

Direction cosine

$= \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ or $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

But line make acute angle with positive direction of x-axis so

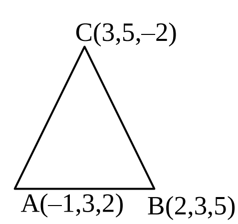
$\lambda = \cos\alpha$ is positive

$\Rightarrow$ direction cosine = $\left(\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right)$

Q.5 If the coordinates of the vertices of a triangle ABC be A(–1, 3, 2), B(2, 3, 5) and C(3, 5, –2), then ∠A is equal to-

(A) 45º (B) 60º (C) 90º (D) 30º

Sol. **[C]**



Direction ratio of the line AC is (+ 4, 2, – 4) and AB is (3, 0, 3)

$\Rightarrow \cos\theta = \frac{12+0-12}{\sqrt{16+4+16}\sqrt{9+9}}$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \pi/2$

Q.6 The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is-

(A) $\sqrt{30}$ (B) $2\sqrt{30}$ (C) $5\sqrt{30}$ (D) $3\sqrt{30}$

Sol. **[D]**

$$\text{S.D.} = \frac{\begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{6^2+15^2+3^2}}$$

$$= \frac{|6(-4-2)-15(12+3)-3(6-3)|}{\sqrt{36+225+9}}$$

$$= \frac{|-36-225-9|}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$$

Q.7 The plane passing through the point (–2, –2, 2) and containing the line joining the points (1, 1, 1) and (1, –1, 2) makes intercepts on the co-ordinates axes, the sum of whose length is-

(A) 3 (B) 4 (C) 6 (D) 12

Sol. **[D]**

Any pane passing through the point (– 2,– 2, 2) is

$a(x + 2) + b(y + 2) + c (z – 2) = 0$ ....(i)

equation of line joining the given points

$\frac{x-1}{0} = \frac{y-1}{2} = \frac{z-1}{-1}$ ....(ii)

plane containing the line

so (1, 1, 1) lie on the plane and normal is perpendicular to the line which direction-ratio is (0, 2, –1)

$\therefore$ 3a + 3b – c = 0

and a. 0 + 2b – c = 0

intercept on the coordinate axis is

$8, \frac{8}{3}, \frac{8}{6}$ respectively

sum = $8 + \frac{8}{3} + \frac{4}{3} = 12$

Q.8 If the plane x – 3y + 5z = d passes through the point (1, 2, 4), then the intercepts cut by it on the axes of x, y, z are respectively-

(A) 15, –5, 3 (B) 1, –5, 3

(C) –15, 5, –3 (D) 1, –6, 20

Sol. **[A]**

Plane x – 3y + 5z = d passes through

(1, 2, 4) $\Rightarrow$ d = 15

$\Rightarrow$ x – 3y + 52 = 15 then the length of intercept cut by it on the axis x,y,z are respectively 15, –5, 3

Q.9 In three dimensional space, the equation 3y + 4z = 0 represents-

(A) A plane containing x-axis

(B) A plane containing y-axis

(C) A plane containing z-axis

(D) A line with direction ratios 0, 3, 4

Sol. **[A]**

3y + 4z = 0

Clearly it is equation of a plane containg x-axis.

Part-B One or more than one correct answer type questions

Q.10 The coordinates of a point, square of whose distance from the origin is 90 is -

(A) (5, 4, 7) (B) (–1, 8, 5)

(C) (4, – 5, –7) (D) (0, 9, 3)

Sol. **[A, B, C, D]**

For any point (x, y, z) distance from origin

$\Rightarrow \sqrt{x^2+y^2+z^2} = \sqrt{90}$ given

$\Rightarrow x^2 + y^2 + z^2 = 90$

from option A,B,C,D all are correct

Q.11 If λ_1, m_1, n_1 and λ_2, m_2, n_2 are D.C.'s of the two lines inclined to each other at an angle θ, then the D.C.'s of the internal and external bisectors of the angle between these lines are-

(A) $\frac{\lambda_1+\lambda_2}{2\sin(\theta/2)}, \frac{m_1+m_2}{2\sin(\theta/2)}, \frac{n_1+n_2}{2\sin(\theta/2)}$

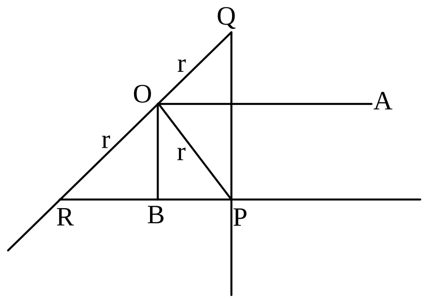
(B) $\frac{\lambda_1+\lambda_2}{2\cos(\theta/2)}, \frac{m_1+m_2}{2\cos(\theta/2)}, \frac{n_1+n_2}{2\cos(\theta/2)}$

(C) $\frac{\lambda_1-\lambda_2}{2\sin(\theta/2)}, \frac{m_1-m_2}{2\sin(\theta/2)}, \frac{n_1-n_2}{2\sin(\theta/2)}$

(D) $\frac{\lambda_1-\lambda_2}{2\cos(\theta/2)}, \frac{m_1-m_2}{2\cos(\theta/2)}, \frac{n_1-n_2}{2\cos(\theta/2)}$

Sol. **[B, C]**

$\therefore \lambda_1 \lambda_2 + m_1m_2 + n_1n_2 = \cos\theta$



Through origin O draw two lines parallel to given lines and take two points on each at distance r from O and a point R on OQ produce so that OR = r

Then coordinates of P, Q, R are

$(\lambda_1 r, m_1 r, n_1 r)$, $(\lambda_2 r, m_2 r, n_2 r)$, $(-\lambda_2 r, -m_2 r, -n_2 r)$ respectively

If A,B be the mid point of PQ and PR then OA and OB are along the bisectors of the lines

Dr's of OA are $\lambda_1 + \lambda_2$, $m_1 + m_2$, $n_1 + n_2$

Dr's of OB are $\lambda_1 - \lambda_2$, $m_1 - m_2$, $n_1 - n_2$

Now $\sum(\lambda_1+\lambda_2)^2 = 1 + 1 + 2\cos\theta$

$= 4\cos^2\theta/2$

and $\sum(\lambda_1-\lambda_2)^2 = 1 + 1 - 2\cos\theta$

$= 4\sin^2\theta/2$

So, D. C. of internal and external bisectors are as given in B,C respectively.

Q.12 The equation of a line passing through the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and in the direction of the vector $3\hat{i} + 4 - 5\hat{k}$ is -

(A) $\vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$, where λ is a parameter

(B) $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$

(C) $4x - 3y = 17$, $5y + 4z = 1$

(D) $\vec{r} = 3\hat{i} + 4\hat{j} - 5\hat{k} + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k})$, where λ is a parameter

Sol. **[A, B, C]**

Vector form

equation of line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} - 4\hat{j} + 5\hat{k})$

where λ is a parameter

Cartesian from

Here (x1,y1,z1) = (2,–3, 4) and

Parallel to the straight line whose dr's are (3, 4, 5) is

$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$

$\Rightarrow 4x - 3y = 17$

or $5y + 4z = 1$

or $5x + 3z = 22$

$\Rightarrow$ option A,B,C are correct.

Q.13 The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if -

(A) k = 0 (B) k = – 1 (C) k = – 3 (D) k = 3

Sol. **[A, C]**

The given lines are coplanar if

$$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow k^2 + 3k = 0$

$\Rightarrow k = 0, -3$

Q.14 The lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y+0}{0} = \frac{z+1}{3}$

(A) do not intersect

(B) intersect

(C) intersect at (4, 0, –1)

(D) intersect at (1, 1, –1)

Sol. **[B, C]**

Q.15 The equation of a line passing through the origin and parallel to the line whose direction ratio are 1, – 1, 2 is -

(A) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$

(B) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$

(C) $\frac{x}{1/\sqrt{6}} = \frac{y}{-1/\sqrt{6}} = \frac{z}{2/\sqrt{6}}$

(D) $\frac{x-7}{1/2} = \frac{y+7}{-1/2} = \frac{z-14}{1}$

Sol. **[A, B, C, D]**

Q.16 The equation of a plane L is given by $x + 2y - 2z = 9$, then -

(A) Intercept made by L on x-axis is 9 units in length

(B) Intercept made by L on y-axis = 9/2 units in length

(C) Intercept made by L on z-axis is 9/2 units in length

(D) direction cosines of the normal to the plane are 1/3, 2/3, –2/3

Sol. **[A, B, C, D]**

$L : x + 2y - 2z = 9$

Intercept made by L on x-axis is 9 unit

Intercept made by L on y-axis is $\frac{9}{2}$ unit

Intercept made by L on z-axis is $\frac{9}{2}$ unit

Direction ratio of the normal are (1, 2, –2)

Direction cosines of the normal are

$$\left(\frac{1}{\sqrt{1+4+4}}, \frac{2}{\sqrt{1+4+4}}, \frac{-2}{\sqrt{1+4+4}}\right)$$

$$\Rightarrow \left(\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$$

$\Rightarrow$ Option A,B,C,D all are correct.

Q.17 The equation of the line passing through $3\hat{i} - 5\hat{j} + 7\hat{k}$ and perpendicular to the plane $3x - 4y + 5z = 8$ is -

(A) $\frac{x-3}{3} = \frac{y+5}{-4} = \frac{z-7}{5}$

(B) $\frac{x-3}{3} = \frac{y+4}{-5} = \frac{z-5}{7}$

(C) $\vec{r} = 3\hat{i} - 5\hat{j} + 7\hat{k} + \lambda(3\hat{i} - 4\hat{j} + 5\hat{k})$

(D) $\vec{r} = 3\hat{i} - 4\hat{j} + 5\hat{k} + \mu(3\hat{i} - 5\hat{j} + 7\hat{k})$

(λ, μ are parameter)

Sol. **[A, C]**

Part-C Column Matching type questions

Q.18

Column-I	Column-II
(A) $\frac{x-2}{4} = \frac{y-3}{7} = \frac{z-4}{6}$	(P) lies in $3x + 2y + 6z - 12=0$
(B) $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-4}{-2}$	(Q) is parallel to $2x + 6y - 2z = 3$
(C) $\frac{x}{-2} = \frac{y}{-3} = \frac{z}{4}$	(R) is perpendicular to $4x+7y+6z = 0$
(D) $\frac{x-1}{4} = \frac{y}{7} = \frac{z+1}{6}$	(S) passes through (–2, –3, 4).

Sol. A $\rightarrow$ R; B $\rightarrow$ P, S; C $\rightarrow$ S; D $\rightarrow$ R

Q.19 Equation of a plane

Column-I	Column-II
(A) through the origin and (1, 1, 1)	(P) $2x + 3y - 4z = 5$
(B) perpendicular to the plane $2x + 3y + 4z = 5$	(Q) $3x - 2y + 4z = 7$
(C) parallel to the plane $3x - 2y + 4z = 5$	(R) $4x + 4y - 5z = 3$
(D) containing the line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$	(S) $x - 2y + z = 0$

Sol. A $\rightarrow$ S; B $\rightarrow$ R, S; C $\rightarrow$ Q; D $\rightarrow$ R, S

Q.20 The equation of a plane passing through the line of intersection of the planes $2x + 3y - 4z = 1$, $3x - y + z + 2 = 0$ is

$2x + 3y - 4z - 1 + \lambda(3x - y + z + 2) = 0$

Column-I	Column-II
(A) $\lambda = 1/2$	(P) it is parallel to $14x - y = 0$
(B) $\lambda = 29$	(Q) it makes an intercept of 4 on the positive x-axis
(C) $\lambda = 4$	(R) is passes through the origin
(D) $\lambda = -1/2$	(S) it is perpendicular to $2x + 3y - 4z = 0$

Sol. **[A $\rightarrow$** R, **B $\rightarrow$** S, **C $\rightarrow$** P, **D $\rightarrow$** Q**]**

(A) $\lambda = 1/2$

The equation is

$7x + 5y + 7z = 0$

Clearly this is passes through origin

(B) $\lambda = 29$ then

$89x - 26y + 25z + 57 = 0$

$\therefore$ $89 \times 2 - 26 \times 3 - 25 \times 4 = 0$

Clearly it is perpendicular to $2x + 2y - 4z = 0$

(C) $\lambda = 4$ the

$14x - y + 7 = 0$

Clearly it is parallel to $14x - y = 0$

(D) $\lambda = -\frac{1}{2}$ then

$x + 7y - 9z - 4 = 0$

$\Rightarrow \frac{x}{4} + \frac{y}{4/7} - \frac{z}{4/9} = 1$

it makes an intercept of 4 on the positive x-axis

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Show that the lines whose direction-cosines are given by $\lambda + 2m + 3n = 0$ and $3\lambda m - 4\lambda n + mn = 0$ are perpendicular.

Sol. By solving equations we get

$(\lambda_1, m_1, n_1) = (2\sqrt{2} - 3, -\sqrt{2}, 1)$ and

$(\lambda_2, m_2, n_2) = (-2\sqrt{2} - 3, \sqrt{2}, 1)$

$\therefore\ \lambda_1\lambda_2 + m_1m_2 + n_1n_2 = 0$

The angle between them is $\frac{\pi}{2}$

Q.2 Find the condition so that the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other.

Sol. Equations of the given lines can be written in the symmetric form is

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$$

and $\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{x-d'}{c'}$

which are perpendicular if $aa' + 1 + cc' = 0$

$\Rightarrow aa' + cc' = -1$

Q.3 Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$, also find equation of line of shortest distance.

Sol.

$$\text{S.D.} = \frac{\begin{vmatrix} -1 & -2 & -2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}}{\sqrt{(15-16)^2 + (10-12)^2 + (8-9)^2}}$$

$$= \frac{-1(15-16) + 2(10-12) - 2(8-9)}{\sqrt{1+4+1}} = \frac{1}{\sqrt{6}}$$

Let $P(2r_1 + 1, 3r_1 + 2, 4r_1 + 3)$ and

$Q(3r_2 + 2, 4r_2 + 4, 5r_2 + 5)$

be the points of S.D.so

Direction ratio of PQ are

$(2r_1 - 3r_2 - 1, 3r_1 - 4r_2 + 4, 5r_2 + 5)$

Since it is perpendicular to the given lines

$\Rightarrow 2(2r_1 - 3r_2 - 1) + 3(3r_1 - 4r_2 - 2) + 4(4r_1 + 5r_2 - 2) = 0$

and $3(2r_1 - 3r_2 - 1) + 4(3r_1 - 4r_2 - 2) + 5(4r_1 - 5r_2 - 2) = 0$

solving we get $P\left(\frac{5}{3}, 3, \frac{13}{3}\right)$ & $Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Direction ratio of PQ is $\left(\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

Equation of line is

$$\frac{x - 3/2}{1/6} = \frac{y - \frac{10}{3}}{-\frac{1}{3}} = \frac{z - \frac{25}{6}}{1/6}$$

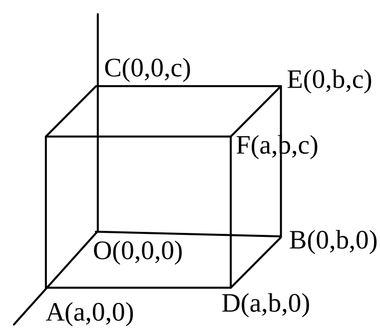
$\Rightarrow 6x - 9 = 10 - 3y = 6z - 25$

Q.4 Prove that the shortest distances between a diagonal of a rectangular parallelopiped and its edges not meeting it, are

$$\frac{bc}{\sqrt{b^2+c^2}},\ \frac{ca}{\sqrt{c^2+a^2}},\ \frac{ab}{\sqrt{a^2+b^2}}$$

where, a, b, c are lengths of the edges.

Sol.



Equation of diagonal of is

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

and equation of edge BD is

$$\frac{x}{a} = \frac{y-b}{0} = \frac{z}{0}$$

shortest distance between them

$$\text{S.D.} = \frac{\begin{vmatrix} 0 & b & 0 \\ a & b & c \\ a & 0 & 0 \end{vmatrix}}{\sqrt{(0)^2 + (ab)^2 + (ac)^2}}$$

$$= \frac{abc}{\sqrt{a(b^2+c^2)}} = \frac{bc}{\sqrt{b^2+c^2}}$$

Similarly we can find shortest distance between of and AD is $\frac{ca}{\sqrt{a^2+c^2}}$ and between OF and AH is $\frac{ab}{\sqrt{a^2+b^2}}$

Q.5 If a variable plane is at a constant distance p from the origin and meets the axes in A, B and C then find the locus of the centroid of tetrahedron OABC.

Sol. Let equation of plane is

$\lambda x + my + nz = p$

where $\lambda^2 + m^2 + n^2 = 1$

plane be written as

$$\frac{x}{p/\lambda} + \frac{y}{p/m} + \frac{z}{p/n} = 1$$

which meets the axes at

$$A\left(\frac{p}{4\lambda}, \frac{p}{4m}, \frac{p}{4n}\right) \equiv (x, y, z)$$

form (i)

$$\frac{p^2}{16x^2} + \frac{p^2}{16y^2} + \frac{p^2}{16y^2} = 1$$

$\Rightarrow x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$

Q.6 A variable plane makes intercepts on the co-ordinate axes the sum of whose squares is constant and equal to k^2. Show that the locus of the foot of the perpendicular from origin to the plane is $(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2)^2 = k^2$.

Sol. Let equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ .....(i)

form given condition $a^2 + b^2 + c^2 = k^2$ ......(ii)

equation of line which direction ratio are

$\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$ and passing through the origin is

$$\frac{x}{1/a} = \frac{y}{1/b} = \frac{z}{1/c} = r \text{ ....(iii)}$$

any point on this line is

$$\left(\frac{r}{a}, \frac{r}{b}, \frac{r}{c}\right) \quad \text{................(iv)}$$

This point lie on the circle if

$$r\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) = 1$$

$$\Rightarrow r = \frac{1}{a^{-2} + b^{-2} + c^{-2}}$$

Put the value of r in (iv) we get

$$x = \frac{a^{-1}}{\sum a^{-2}},\ y = \frac{b^{-1}}{\sum a^{-2}},\ z = \frac{c^{-1}}{\sum a^{-2}}$$

$$\Rightarrow x^2 + y^2 + z^2 = \frac{1}{a^{-2} + b^{-2} + c^{-2}} \text{ ....(v)}$$

and $x^{-2} + y^{-2} + z^{-2} = (a^{-2} + b^{-2} + c^{-2})(a^2 + b^2 + c^2)$

$$\Rightarrow x^{-2} + y^{-2} + z^{-2} = \frac{1}{(x^2 + y^2 + z^2)^2} k^2$$

$\Rightarrow (x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2) = k^2$

Q.7 Find the equation of the planes passing through the line of intersection of the planes $3x - y - 4z = 0$ & $x + 3y + 6 = 0$ whose distance from the origin is 1.

Sol. Equation of plane passing through the line of intersection of given plane is

$x + 3y + 6 + \lambda(3x - y - 4z) = 0$

$\Rightarrow (1 + 3\lambda)x + (3 - \lambda)y - 4\lambda z + 6 = 0$ ....(1)

Perpendicular distance of (i) from (0, 0, 0) is 1

$$\Rightarrow \frac{6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (4\lambda)^2}} = 1$$

$\Rightarrow 26\lambda^2 + 10 = 36$

from (i) we get

$4x + 2y + 4z + 6 = 0$

and $-2x + 4y + 4z + 6 = 0$

or $2x + y - 2z + 3 = 0$ and $x - 2y - 2z - 3 = 0$

Q.8 Prove that, the planes $x = cy + bz$, $y = az + cx$, $z = bx + ay$ pass through one line, if $a^2 + b^2 + c^2 + 2abc = 1$.

Sol. Let l, m, n be the direction ratios of the line lying in three planes, then the line is perpendicular to the normals to these planes.

So $\lambda - cm - bn = 0$

and $-c\lambda + m - an = 0$

and $-b\lambda - am + n = 0$

eliminating l, m, n form these equations we get

$$\begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$$

$(1 - a^2) + c(- c - ab) - b(ac + b) = 0$

$1 - a^2 - c^2 - b^2 - 2abc = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

Q.9 Find the volume of the tetrahedron included between the plane $3x + 4y - 5z - 60 = 0$ and the coordinate planes.

Sol. Equation of the given plane is

$$\frac{x}{20} + \frac{y}{15} + \frac{z}{-12} = 1$$

which meets the coordinate axes in points

A(20, 0, 0), B(0, 15, 0), c(0, 0, –12)

and coordinate of the origine are (0, 0, 0)

volume of the tetrahedron OABC is

$$= \frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 20 & 0 & 0 & 1 \\ 0 & 15 & 0 & 1 \\ 0 & 0 & -12 & 1 \end{vmatrix}$$

$$= \left|\frac{1}{6} \times 20 \times 15 \times (-12)\right| = 600$$

Q.10 If L is the line $\frac{x-1}{2} = \frac{y}{-1} = \frac{z+2}{1}$, find the direction cosines of the projection of L on the plane $2x + y - 3z = 4$ and the equation of the plane through L parallel to the line $2x + 5y + 3z = 4$, $x - y - 5z = 6$.

Sol. $< \frac{2}{\sqrt{6}}\ \frac{-1}{\sqrt{6}}\ \frac{1}{\sqrt{6}} >$, $3x + 4y - 2z = 7$

Part-B Passage based objective questions

Passage I (Question 11 to 13)

The vector equation of a plane is a relation satisfied by position vectors of all the points on the plane. If P is a plane and $\hat{n}$ is a unit vector through origin which is perpendicular to the plane P then vector equation of the plane must be $\vec{r} \cdot \hat{n} = d$ where d represents perpendicular distance of plane P from origin.

On the basis of above information, answer the following questions.

Q.11 If A is a point with position vector $\vec{a}$ then perpendicular distance of A from the plane $\vec{r} \cdot \hat{n} = d$ must be -

(A) $|d + \vec{a} \cdot \hat{n}|$ (B) $|d - \vec{a} \cdot \hat{n}|$

(C) $||\vec{a}| - d|$ (D) None of these

Sol. **[B]**

The equation of the line through A and Perpendicular to the plane is

$\vec{r} = \vec{a} + t\,\hat{n}$

This will be meet the plane at a point for which

$(\vec{a} + t\,\hat{n}) \cdot \hat{n} = d$

$\Rightarrow t = d - \vec{a} \cdot \hat{n}$ ....(i)

Now the foot of the parpendicular L form the point A to the plane $\vec{r} \cdot \hat{n} = d$

Foot $= \vec{b} = \vec{a} + (d - \vec{a} \cdot \hat{n})\,\hat{n}$ ....(ii)

Length of perpendicular AL = $|\vec{AL}|$

= |Position vector of L – position vector of A|

$= |\vec{a} + (d - \vec{a} \cdot \hat{n})\,\hat{n} - \vec{a}|$

$= |d - \vec{a} \cdot \hat{n}|$ $\because |\hat{n}| = 1$

Q.12 If $\vec{b}$ be the foot of the perpendicular from A to the plane $\vec{r} \cdot \hat{n} = d$ then $\vec{b}$ must be -

(A) $\vec{a} + (d - \vec{a} \cdot \hat{n})\,\hat{n}$

(B) $\vec{a} - (d - \vec{a} \cdot \hat{n})\,\hat{n}$

(C) $\vec{a} + \vec{a} \cdot \hat{n}$

(D) None of these

Sol. **[A]**

From Q 14 we get

$\vec{b} = \vec{a} + (d - \vec{a} \cdot \hat{n})\,\hat{n}$

Q.13 The position vector of the image of the point $\vec{a}$ in the plane $\vec{r} \cdot \hat{n} = d$ must be $(d \neq 0)$

(A) $-\vec{a} \cdot \hat{n}$ (B) $\vec{a} - 2(d - \vec{a} \cdot \hat{n})\,\hat{n}$

(C) $\vec{a} + 2(d - \vec{a} \cdot \hat{n})\,\hat{n}$ (D) None of these

Sol. **[C]**

Let position vector of the image of the point $\vec{a}$ in the plane $\vec{r} \cdot \hat{n} = d$ is $\vec{p}$

then we have

$$\frac{\vec{p}+\vec{a}}{2} = \vec{a} + (d - \vec{a}\,.\,\hat{n})\,\hat{n}$$

$$\Rightarrow \vec{p} = \vec{a} + 2(d - \vec{a}\,.\,\hat{n})\,\hat{n}$$

Passage II (Question 14 to 16)

Equation of lines are $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ both are coplanar.

On the basis of above information, answer the following questions.

Q.14 The point of intersection of lines is -

(A) (2, 4, +3) (B) (–2, 4, –3)

(C) (2, 4, –3) (D) None of these

Sol. **[C]**

Any point on the lines (i) & (v) we have

$(3r_1 - 4, 5r_1 - 6, -2r_1 + 1)$

and $(11r_2 + 2, 10r_2 + 4, -13r_2 - 3)$

these represent same point os solving we get

$r_1 = 2, r_2 = 0$

point is (2, 4, –3)

Q.15 The equation of plane containing both lines is-

(A) $45x - 17y - 25z + 53 = 0$

(B) $45x - 17y + 25z + 53 = 0$

(C) $45x - 17y + 25z - 53 = 0$

(D) None of these

Sol. **[B]**

Equation of plane containing line (i) & (ii) is

$$\begin{vmatrix} x+4 & y+6 & z-1 \\ 3 & 5 & -2 \\ 11 & 10 & -13 \end{vmatrix} = 0$$

$-45(x + 4) + 17(y + 6) - 25(z - 1) = 0$

$\Rightarrow -45x + 17y - 25z - 53 = 0$

$\Rightarrow 45x - 17y + 25z + 53 = 0$

Q.16 Direction ratio of line of intersection of $3x - 2y + z + 5 = 0$ & $2x + 3y + 4z - 4 = 0$ are -

(A) (10, 11, –13) (B) (11, –10, 13)

(C) (11, 10, –13) (D) None of these

Sol. **[C]**

from equation (iii) we get direction

ration is (11, 10, – 13)

Passage III (Question 17 to 19)

Let two planes $P_1 \equiv 2x + y - z = 2$ and $P_2 = x - 2y - z = 3$ are given.

On the basis of above information, answer the following questions.

Q.17 The equation of obtuse angle bisector of above two planes-

(A) $x + 3y - 1 = 0$ (B) $3x - y - 2z = 5$

(C) $x + 3y + 1 = 0$ (D) None of these

Sol. **[C]** obtuse angle bisector of above planes is

$$\frac{2x+y-z-2}{\sqrt{4+1+1}} = \frac{x-2y-z-3}{\sqrt{1+4+1}}$$

$\Rightarrow x + 3y + 1 = 0$

Q.18 The equation of bisector of angle not containing origin-

(A) $x + 3y - 1 = 0$ (B) $3x - y - 2z = 5$

(C) $x + 3y + 1 = 0$ (D) None of these

Sol. **[B]**

equation of planes

$-2x - y + z + 2 = 0, -x + 2y + z + 3 = 0$

then $2 - 2 + 1 > 0$ origin lies in obtuse angle eequation of biector of angle not containing the the origin is

$$= \frac{-2x-y+z+2}{\sqrt{4+1+1}} = -\frac{-x-2y+z+3}{\sqrt{1+4+1}}$$

$\Rightarrow 3x - y - 2z = 5$

Q.19 The equation of plane passing through (1, 0, –2) and perpendicular to both planes $P_1 = 0$ and $P_2 = 0$

(A) $3x + y + 5z + 7 = 0$

(B) $3x - y + 5z + 7 = 0$

(C) $3x - y - 5z - 13 = 0$

(D) None of these

Sol. **[B]**

Let direction ratio of required plane be (λ, m, n)

so $2\lambda + m - n = 0$ and $\lambda - 2m - n = 0$

solving we get $\frac{\lambda}{3} = \frac{m}{-1} = \frac{n}{5}$

equation of plane passing through (1, 0, –2) and has direction ratio (3, –1, 5) is

$3(x - 1) - 1(y - 0) + 5(z + 2) = 0$

$\Rightarrow 3x - y + 5z + 7 = 0$

EXERCISE # 4

Old IIT-JEE questions

Q.1 If line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, then the value of k = ? **[IIT Scr.2003]**

(A) $k = -1$ (B) $k = 7$
(C) $k = -7$ (D) No value of k

Sol. **[B]**

If the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$

then $2 - 4 + 2 = 0$

and $-8 + 8 + k - 7 = 0$

$\Rightarrow k = 7$

Q.2 Find the equation of the plane passing through the points A (1, 2,0), B (5, 0, 1) and C(4, 1, 1). Also, determine the point Q such that the line segment joining the points P (2, 1, 6) and Q is perpendicular to the plane and is bisected by it. **[IIT Mains 2003]**

Sol. Equation of the plane passing through A(1, 2, 0), B (5, 0, 1) and C(4, 1, 1)

$$\begin{vmatrix} x-1 & y-2 & z \\ 4 & -2 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z = 3$

$\therefore$ Q is perpendicular to the $x + y - 2z = 3$ and mid point of PQ lies on the plane

Let Q be (α, β, γ)

Equation of PM passing through (2, 1, 6) and perpendicular to the plane is

$$\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = l$$

$\therefore$ Q lies on it so

$$\frac{\alpha-2}{1} = \frac{\beta-1}{1} = \frac{\gamma-6}{-2} = \lambda$$

$\Rightarrow \alpha = \lambda + 2, \beta = \lambda + 1, \gamma = -2\lambda + 6$

mid point of PQ is M

$$\left(\frac{\lambda+2+2}{2}, \frac{\lambda+1+1}{2}, \frac{-2\lambda+6+6}{2}\right)$$

$$\Rightarrow M\left(\frac{\lambda+4}{2}, \frac{\lambda+2}{2}, \frac{-2\lambda+12}{2}\right)$$

M lies on the plane so

$$\frac{\lambda+4}{4} + \frac{\lambda+2}{2} - \frac{-2\lambda+12}{2} = 3$$

$\Rightarrow \lambda = 4$

$\therefore$ Point Q is (6, 5, –2)

Q.3 Two lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} \ \& \ \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$$

intersect at a point then k is – **[IIT Scr. 2004]**

(A) 3/2 (B) 9/2 (C) 2/9 (D) 2

Sol. **[B]**

Let any point on the lines be

$(2r_1 + 1, 3r_1 - 1, 4r + 1)$

and $(r_2 + 3, 2r_2 + k, r_2)$

If lines intersect at a point then both points must be same

$2r_1 + 1 = r_2 + 3, 3r_1 - 1 = 2r_2 + k, 4r_1 + 1 = r_2$

$\Rightarrow r_1 = -\frac{3}{2}, r_2 = -5$

then $k = \frac{9}{2}$

Q.4 A plane is parallel to two lines whose direction ratios are (1, 0, – 1) and (– 1, 1, 0) and it passes through the point (1, 1,1), cuts the axis at A, B, C, then find the volume of the tetrahedron OABC. **[IIT Mains 2004]**

Sol. Plane parallel to two lines whese direction ratio are (1, 0, –1) and (–1, 1, 0) so its normal is perpendicular to both the lines.

Plane passes through the point (1,1,1) os its equation is

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} = 0$$

$\Rightarrow x + y + z = 3$

it is cuts the axes at

A(3, 0, 0), B(0, 3, 0), C (0, 0, 3)

Volume of the tetradron OABC is

$$= \frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{6}|27| = \frac{9}{2}\ (\text{unit})^3$$

Q.5 T is a parallelopiped in which A, B, C and D are vertices of base and A', B', C' and D' are corresponding vertices of top. T is now compressed to S with face ABCD remaining same and A', B', C', D' shifted to A", B", C", D" in S. The volume of parallelopiped S is reduced to 90% of T. Prove that locus of A" is a plane.

[IIT Mains 2004]

Q.6 Two planes P_1 and P_2 pass through origin. Two lines L_1 and L_2 also passing through origin are such that L_1 lies on P_1 but not on P_2, L_2 lies on P_2 but not on P_1. A, B, C are three points other than origin, then prove that the permutation [A' B' C'] of [ABC] exists, such that

(i) A lies on L_1, B lies on P_1 not on L_1, C does not lie on P_1

(ii) A' lies on L_2, B lies on P_2 not on L_2, C' does not lies on P_2. **[IIT-2004]**

Q.7 A plane at a unit distance from origins cuts at three axes at P, Q, R points. Δ PQR has centroid at (α, β, γ) point & satisfies to $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=k$, then k = **[IIT. Scr. 2005]**

(A) 9 (B) 1 (C) 3 (D) 4

Sol. **[A]**

Let equation of plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad \text{..................(i)}$$

which meets the axes at P,Q, R then

P(a, 0, 0), Q(0, b, 0), R(0, 0, c)

$\Rightarrow$ centroid of ΔPQR is $\left(\frac{\alpha}{3},\frac{b}{3},\frac{c}{3}\right)$

It satisfy the relation

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$$

$$\Rightarrow \frac{9}{a^2}+\frac{9}{b^2}+\frac{9}{c^2}=k$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{k}{9} \qquad \text{.............(ii)}$$

Also given that the distance of plane (i) form origin is 1 unit

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \qquad \text{................(iii)}$$

from (2) & (3)

$$\frac{k}{9}=1 \Rightarrow k=9$$

Q.8 Find the equation of the plane at distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, – 1) and containing the line $2x - y + z - 3 = 0 = 3x + y + z - 5$.

[IIT Mains 2005]

Sol. Given line is

$2x - y + z - 3 = 0 = 3x + y + z - 5$

Which is intersection line of two planes

$2x - y + z - 3 = 0$

and $3x + y + z - 5 = 0$

any plane containing this line is

$(2x - y + z - 3) + \lambda(3x + y + z - 5) = 0$

$\Rightarrow (3\lambda + 2)x + (\lambda - 1)y + (\lambda + 1)z + (-5\lambda - 3) = 0$

Given that its distance from (2, 1, –1) is $\frac{1}{\sqrt{6}}$

$$\therefore \left|\frac{(3\lambda+2)2+(\lambda-1)1+(\lambda+1)(-1)+(-5\lambda-3)}{\sqrt{(3\lambda+2)^2+(\lambda-1)^2+(\lambda+1)^2}}\right|=\frac{1}{\sqrt{6}}$$

Squaring and solving we get

$\lambda = 0$ or $-\frac{24}{5}$

required equations of planes are

$2x - y + z - 3 = 0$

and $62x + 29y + 19z - 105 = 0$

Q.9 A plane passes through (1, –2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The distance of the plane from point (1, 2, 2) is – **[IIT-2006]**

(A) $2\sqrt{2}$ (B) 0 (C) 1 (D) $\sqrt{2}$

Sol. **[A]**

Equation of the plane through the point (1,–2, 1) and perpendicular to the planes 2x – 2y + z = 0 and x – y + 2z = 4 is

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow x + y + 1 = 0$

its distance from the point (1, 2, 2) is

$$= \left|\frac{1+2+1}{\sqrt{1+1}}\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Q.10 Match the following : **[IIT-2006]**

Column -I	**Column -II**
(A) $\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then tan t =	(P) 0
(B) a line perpendicular to x + 2y + 2z = 0 and passes through (0, 1, 0) then the perpendicular distance of the line from origin is	(Q) $\frac{\sqrt{5}}{3}$
(C) not available	(R) 1
(D) not available	(S) 2/3

Sol. $A \to R,\ B \to S$

Q.11 Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5 **[IIT-2007]**

Statement-1 : The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.

Statement-2 : The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes.

(A) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1

(C) Statement–1 is true, Statement–2 is false

(D) Statement–1 is false, Statement–2 is true

Sol. **[D]**

Given planes are

3x – 6y – 2z = 15

and 2x + y – 2z = 5

Let direction cosine of line of intersection is (λ, m, n) so

$3\lambda - 6m - 2n = 0$

and $2\lambda + m - 2n = 0$

solving we get $\frac{\lambda}{14} = \frac{m}{2} = \frac{n}{15}$

Also 3x – 6y – 15 = 0

and 2x + y – 5 = 0 Put z = 0

solving we get $\frac{x}{3} = \frac{y}{-1} = 1$

line is $\frac{x-3}{14} = \frac{y+1}{2} = \frac{z}{15}$

Parametric equations are

x = 3 + 14t, y = – 1 + 2t, z = 15t

parametric equations are

x = 3 + 14t, y = – 1 + 2t, z = 15 t

Stetment-1 is false

vector parallel to the line of intersection of given plane is $14\hat{i} + 2\hat{j} + 15\hat{k}$

Stetment-2 is true

Q.12 Consider the following linear equations

ax + by + cz = 0, bx + cy + az = 0,

cx + ay + bz = 0 **[IIT-2007]**

Match the conditions/expressions in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column-I

(A) $a + b + c \neq 0$ & $a^2 + b^2 + c^2 = ab + bc + ca$

(B) $a + b + c = 0$ & $a^2 + b^2 + c^2 \neq ab + bc + ca$

(C) $a + b + c \neq 0$ & $a^2 + b^2 + c^2 \neq ab + bc + ca$

(D) $a + b + c = 0$ & $a^2 + b^2 + c^2 = ab + bc + ca$

Column-II

(P) The equations represent planes meeting only at a single point

(Q) The equations represent the line x = y = z.

(R) The equations represent identical planes

(S) The equations represent the whole of the three dimensional space.

Sol. $A \to R$; $B \to Q$; $C \to P$; $D \to S$

Passage (Question 13 to 15)

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

Q.13 The unit vector perpendicular to both L_1 and L_2 is- **[IIT-2008]**

(A) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(C) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

Sol. **[B]**

Vector perpendicular to both L_1 and L_2 is

$$\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= -\hat{i}-7\hat{j}+5\hat{k}$$

Unit vector is = $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

Q.14 The shortest distance between L_1 and L_2 is **[IIT-2008]**

(A) 0 (B) $\frac{17}{\sqrt{3}}$ (C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

Sol. **[D]**

$$\text{S.D.} = \frac{\begin{vmatrix} 3 & 0 & 4 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}}{\sqrt{(3-4)^2+(9-2)^2+(6-1)^2}}$$

$$= \frac{3(3-4)+4(6-1)}{\sqrt{1+49+25}}$$

$$= \frac{-3+20}{\sqrt{75}} = \frac{17}{5\sqrt{3}}$$

Q.15 The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is- **[IIT-2008]**

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$ (C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

Sol. **[C]**

Equation of the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines is

$$\begin{vmatrix} x+1 & y+2 & z+1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow x - 7y - 5z + 10 = 0$

It's distance form the point (1,1,1) is

$$\left|\frac{1+7-5+10}{\sqrt{1+49+25}}\right| = \frac{13}{\sqrt{75}}$$

Q.16 Consider three planes **[IIT-2008]**

$P_1 : x - y + z = 1$; $P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$.

Let L_1, L_2, L_3 be the lines intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

Statement-1 : At least two of the lines L_1, L_2 and L_3 are non- parallel and

Statement-2 : The three planes do not have a common point.

(A) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1

(C) Statement–1 is true, Statement–2 is false

(D) Statement–1 is false, Statement–2 is true

Sol. **[D]**

The given three planes intersect on parallel lines.

Q.17 Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is- **[IIT-2009]**

(A) 1/4 (B) –1/4 (C) 1/8 (D) –1/8

Sol. **[A]**

P (3, 2, 6)

$Q \equiv (1 - 3\mu, \mu - 1, 2 + 5\mu)$

$\overline{PQ} = (-3\mu - 2, \mu - 3, 5\mu - 4)$

D.R. of plane is (1, –4, 3)

$(-3\mu - 2) + (-4\mu + 12) + (15\mu - 12) = 0$

$8\mu = 2 \Rightarrow \mu = \dfrac{1}{4}$

Alternative

$\overrightarrow{PQ} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$

$\qquad\qquad - (3\hat{i} + 2\hat{j} + 6\hat{k})$

$\vec{n} = \hat{i} - 4\hat{j} + 3\hat{k}$

Dot product = 0 $\Rightarrow$ 1 + 4 + 6 + μ (– 3 – 4 + 15)

– (3 – 8 – 18)

$\Rightarrow \quad 11 + 8\mu - 13 = 0$

$\Rightarrow \quad 8\mu = 2$

$\mu = \dfrac{1}{4}$

Q.18 A line with positive direction cosines passes through the point P(2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals : **[IIT- 2009]**

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

Sol. **[C]**

$\dfrac{x-2}{1} = \dfrac{y+1}{1} = \dfrac{z-2}{1} = \lambda \qquad P \equiv (2, -1, 2)$

$Q \equiv (2 + \lambda, -1 + \lambda, 2 + \lambda)$

$2(2 + \lambda) + (\lambda - 1) + (\lambda + 2) = 9$

$\Rightarrow 4 + 2\lambda + \lambda - 1 + \lambda + 2 = 9$

$\Rightarrow 4\lambda = 4 \Rightarrow \lambda = 1$

$Q \equiv (3, 0, 3)$

$PQ \equiv \sqrt{1+1+1} = \sqrt{3}$

Q.19 Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations 3x – y – z = 0, –3x + z = 0, –3x + 2y + z = 0. Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is..... **[IIT-2009]**

Sol. **[7]**

$\because$ 3x – y – z = 0, 3x – z = 0 , 3x –2y –z = 0

On solving these three y = 0

z = 3x

so $x^2 + y^2 + z^2 \le 100$

$x^2 + 0 + 9x^2 \le 100$

$x^2 \le 10 \Rightarrow |x| = 0, 1, 2, 3$

so total no. of different points possible are 7

(0, 0, 0), (–1, 0, 1), (–1, 0, –1), (2, 0, 2),

(–2, 0, –2), (3, 0, 3), (–3, 0, –3)

Q. 20 Equation of the plane containing the straight line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{4}$ and perpendicular to the plane containing the straight lines $\dfrac{x}{3} = \dfrac{y}{4} = \dfrac{z}{2}$ and $\dfrac{x}{4} = \dfrac{y}{2} = \dfrac{z}{3}$ is **[IIT-2010]**

(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0

(C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

Sol. **[C]**

Plane passing through origin (0, 0, 0) and normal vector to plane is perpendicular to $3\hat{i} + 4\hat{j} + 2\hat{k}$, $4\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} + 3\hat{j} + 4\hat{k}$ i.e. normal vector to plane is $\hat{i} - 2\hat{j} + \hat{k}$ so equation to plane is x – 2y + z = 0.

Q.21 The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solution, is- **[IIT-2010]**

(A) 0 (B) $2^9 - 1$ (C) 168 (D) 2

Sol. **[A]**

Q.22 If the distance of the point P (1, –2, 1) from the plane x + 2y –2z = α, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is **[IIT-2010]**

(A) $\left(\dfrac{8}{3}, \dfrac{4}{3}, -\dfrac{7}{3}\right)$ (B) $\left(\dfrac{4}{3}, -\dfrac{4}{3}, \dfrac{1}{3}\right)$

(C) $\left(\dfrac{1}{3}, \dfrac{2}{3}, \dfrac{10}{3}\right)$ (D) $\left(\dfrac{2}{3}, -\dfrac{1}{3}, \dfrac{5}{2}\right)$

Sol. **[A]**

$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = \lambda$

foot $(1+\lambda, -2+2\lambda, 1-2\lambda)$

$(1+\lambda) + 2(-2+2\lambda) = 10$

$-2(1-2\lambda)$

$1 + \lambda - 4 + 4\lambda - 2 + 4\lambda = 10$

$9\lambda = 15$, $\Rightarrow \lambda = 5/3$

foot $= \left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

$\frac{|1-4-2-\alpha|}{3} = 5$

$|\alpha+5| = 15$

$\alpha = 10$ (correct),

-20 (wrong)

Q.23 Match the statements in **Column-I** with the values in **Column-II**. **[IIT-2010]**

Column-I	**Column-II**
(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(P) –4
(B) The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are,	(Q) 0
(C) Non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b} = 0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c}) = 0$ and $2\lvert\vec{b}+\vec{c}\rvert = \lvert\vec{b}-\vec{a}\rvert$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are	(R) 4
(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(S) 5
	(T) 6

Sol. **A $\to$ T ; B $\to$ P, R ; C $\to$ Q, S ; D $\to$ R**

(A) Let $P \equiv (\lambda+2, 1-2\lambda, \lambda-1)$

$Q \equiv (2\mu + \frac{8}{3}; -\mu-3, \mu+1)$

equation of line PQ

$\vec{r} = (\lambda+2)\hat{i} + (1-2\lambda)\hat{j} + (\lambda-1)\hat{k} + \alpha$

$((2\mu - \lambda + \frac{2}{3})\hat{i} + (2\lambda-\mu-4)\hat{j} + (\mu+2-\lambda)\hat{k})$

$\therefore$ This line passing through origin so.

$\lambda + 2 + \alpha(2\mu - \lambda + \frac{2}{3}) = 0$

$1 - 2\lambda + \alpha(2\lambda - \mu - 4) = 0$

$\lambda - 1 + \alpha(\mu - \lambda + 2) = 0$

on solving above three $\mu = \frac{1}{3}$ & $\lambda = 3$

So $P \equiv (5, -5, 2)$ & $Q \equiv (\frac{10}{3}, \frac{-10}{3}, \frac{4}{3})$

So $PQ = \sqrt{6} \Rightarrow (PQ)^2 = 6$

(B) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\frac{3}{5}$

$\tan^{-1}\frac{6}{x^2-8} = \tan^{-1}\frac{3}{4}$

$\Rightarrow x^2 - 8 = 8$

$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

(C) $|\vec{b}|^2 + \vec{b}\cdot\vec{c} = \vec{a}\cdot\vec{c}$ …. (1)

put $\vec{a} = \mu\vec{b} + 4\vec{c}$ $\forall\ \vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{b}\cdot\vec{c} = -\frac{\mu}{4}|\vec{b}|^2$

…(2)

from (1) and (2)

$\frac{b^2}{c^2} = \frac{16}{4-\mu+\mu^2}$ … (3)

$\therefore 2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$ and $\vec{a} = \mu\vec{b} + 4\vec{c}$

$\frac{b^2}{c^2} = \frac{12}{3-2\mu+\mu^2}$ … (4)

from (3) and (4)

m = 0,5

(D) $f(x) = \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}} = \frac{\sin 5x}{\sin x} + \frac{\sin 4x}{\sin x}$

$I = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{4}{\pi}\int_0^{\pi} f(x)\,dx$

$= \frac{4}{\pi}\int_0^{\pi} \frac{\sin 5x}{\sin x}$

$= \frac{8}{\pi}\int_0^{\pi/2} \frac{\sin 5x}{\sin x}\,dx$

$= \frac{8}{\pi}\int_0^{\pi/2} \frac{\sin(3x+2x)}{\sin x}\,dx = \frac{8}{\pi}\int_0^{\pi/2}(1+2\cos 4x)dx$

= 4

Q.24 If the distance between the plane Ax – 2y + z = d and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |d| is ..... **[IIT-2010]**

Sol. **[6]**

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$= \hat{i}(-1) - \hat{j}(-2) + \hat{k}(-1)$

Plane is normal to vector $\hat{i} - 2\hat{j} + \hat{k}$

1(X – 1) – 2 (Y –2) + 1(Z – 3) = 0

X – 2Y + Z = 0

$\sqrt{6} = \frac{|d|}{\sqrt{6}} \Rightarrow |d| = 6$

Q.25 The point P is the intersection of the straight line joining the points Q(2, 3,5) and R(1, –1, 4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is **[IIT-2012]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Sol. **[A]**

Equation of line

$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1} = \lambda$

General points {λ + 2, 4λ + 3, λ + 5}

Intersection point with plane

5(λ + 2) – 4(4λ + 3) – (λ + 5) = 1

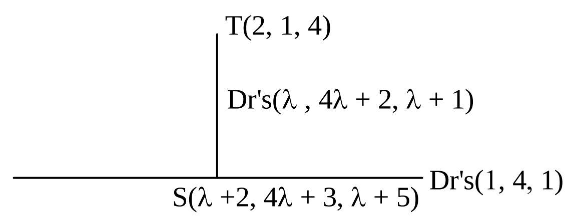
5λ + 10 – 16λ – 12 – λ – 5 = 1

–12λ – 8 = 0

$\lambda = -\frac{8}{12} = -\frac{2}{3}$

Point $\left[\frac{-2}{3}+2, -\frac{8}{3}+3, \frac{-2}{3}+5\right]$

P $\left[\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right]$



Now

λ + 4 (4λ + 2) + (λ + 1) = 0

λ + 16λ + 8 + λ + 1 = 0

18λ = –9

$\lambda = -\frac{1}{2}$

Points $\left(\frac{-1}{2}+2, -2+3, -\frac{1}{2}+5\right)$

$\left(\frac{3}{2}, 1, \frac{9}{2}\right)$

Distance at PS = $\sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2 + \left(\frac{1}{3}-1\right)^2 + \left(\frac{13}{3}-\frac{9}{2}\right)^2}$

PS = $\sqrt{\frac{1}{36}+\frac{4}{9}+\frac{1}{36}} = \sqrt{\frac{1+16+1}{36}} = \sqrt{\frac{18}{36}} = \frac{1}{\sqrt{2}}$

Q.26 The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and

$x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, – 1) is **[IIT-2012]**

(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}\,y = 1 - \sqrt{2}$

Sol. [A] Equation of plane passing through intersecting of plane P_1 & P_2

is $P_1 + \lambda P_2 = 0$

$(1 + \lambda) x + (2 - \lambda) y + (3 + \lambda) z - 2 - 3\lambda = 0$

distance of plane from pt (3, 1, –1) is $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{|3+3\lambda+2-\lambda-3-\lambda-2-3\lambda|}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3-\lambda)^2}}$$

on solving $\lambda = -\frac{7}{2}$

so equation of plane is

$$\left(1-\frac{7}{2}\right)x + \left(2+\frac{7}{2}\right)y + \left(3-\frac{7}{2}\right)z - 2 + \frac{21}{2} = 0$$

$5x - 11y + z = 17$

Q.27 If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is (are)

[IIT-2012]

(A) $y + 2z = -1$ (B) $y + z = -1$

(C) $y - z = -1$ (D) $y - 2z = -1$

Sol. [B, C]

If these two lines are coplanar then shortest distance between them = 0

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0$$

$k = 2$ or -2

so lines are
$$\left.\begin{array}{l} \frac{x-1}{2} = \frac{y+1}{2} = \frac{z}{2} \\ \text{and } \frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{2} \end{array}\right\} \text{set (i)}$$

OR

$$\left.\begin{array}{l} \frac{x-1}{2} = \frac{y+1}{-2} = \frac{z}{2} \\ \text{and } \frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{-2} \end{array}\right\} \text{set (ii)}$$

the plane which contain these set of line should contain the points (1, –1, 0) and (–1, –1, 0) which is satisfied by all the four options and

$(2\hat{i} + 2\hat{j} + 2\hat{k})$ & $(5\hat{i} + 2\hat{j} + 2\hat{k})$ OR

$(2\hat{i} - 2\hat{j} + 2\hat{k})$ & $(5\hat{i} + 2\hat{j} - 2\hat{k})$ are perpendicular to normal of plane

For first set option (C) is correct.

For second set option (B) is correct.

EXERCISE # 5

Q.1 The shortest distance between the z-axis and the line, $x + y + 2z - 3 = 0$, $2x + 3y + 4z - 4 = 0$ is :

(A) 1 (B) 2
(C) 3 (D) none of these

Sol. **[B]**

Q.2 The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$ if c is equal to

(A) ± 1 (B) $\pm \frac{1}{3}$
(C) $\pm\sqrt{5}$ (D) none of these

Sol. **[C]**

Q.3 Minimum value of $x^2 + y^2 + z^2$ when $ax + by + cz = p$ is

(A) $\frac{p}{\Sigma a}$ (B) $\frac{p^2}{\Sigma a^2}$ (C) $\frac{\Sigma a^2}{p}$ (D) 0

Sol. **[B]**

Q.4 The lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$, $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar and these determine a single plane if $\alpha\gamma \neq \beta\delta$. Equation of the plane in which they lie is

(A) $x + y + z = 0$ (B) $x - y + z = 0$
(C) $x - 2y + z = 0$ (D) $x + y - 2z = 0$

Sol. **[C]**

Q.5 The square of the perpendicular distance of a point P(p, q, r) from a line through A(a, b, c) and whose direction cosine are λ, m, n is

(A) $\Sigma\{(q-b)n - (r-c)m\}^2$
(B) $\Sigma\{(q+b)n - (r+c)m\}^2$
(C) $\Sigma\{(q-b)n + (r-c)m\}^2$
(D) none of these

Sol. **[A]**

Q.6 The coplanar points A, B, C, D are $(2 - x, 2, 2)$, $(2, 2 - y, 2)$, $(2, 2, 2 - z)$ and $(1, 1, 1)$ respectively. Then :

(A) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ (B) $x + y + z = 1$
(C) $\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z} = 1$
(D) none of these

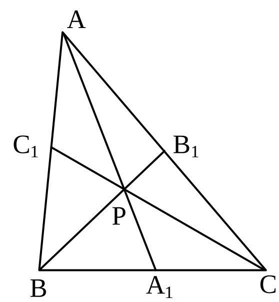
Sol. **[A]**

Q.7 The direction ratios of a normal to the plane through (1, 0, 0), (0, 1, 0), which makes an angle of $\pi/4$ with the plane $x + y = 3$ are

(A) $(1, \sqrt{2}, 1)$ (B) $(1, 1, \sqrt{2})$
(C) (1, 1, 2) (D) $(\sqrt{2}, 1, 1)$

Sol. **[D]**

Q.8 In the adjacent figure, 'P' is any arbitrary interior point of the triangle ABC such that the lines AA_1, BB_1 and CC_1 are concurrent at P. Value of $\frac{PA_1}{AA_1} + \frac{PB_1}{BB_1} + \frac{PC_1}{CC_1}$ is always equal to



(A) 1 (B) 2
(C) 3 (D) none of these

Sol. **[A]**

Q.9 Equation of the sphere with centre on the positive z-axis which passes through the circle $x^2 + y^2 = 4$, $z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3 is :

(A) $x^2 + y^2 + z^2 - 6x - 4 = 0$
(B) $x^2 + y^2 + z^2 - 6y - 4 = 0$
(C) $x^2 + y^2 + z^2 - 6z - 4 = 0$
(D) $x^2 + y^2 - 6x - 6y - 4 = 0$

Sol. **[C]**

Q.10 P is any point on the plane $\lambda x + my + nz = p$. A point Q taken on the line OP (where O is the origin) such that OP.OQ = p^2. Show that the locus of Q is $p(\lambda x + my + nz) = x^2 + y^2 + z^2$.

Q.11 Find the equations of the straight line passing through the point (1, 2, 3) to intersect the straight line x + 1 = 2(y – 2) = z + 4 and parallel to the plane x + 5y + 4z = 0.

Sol. $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-3}$

Q.12 Let P = (1, 0, –1); Q = (1, 1, 1) and R = (2, 1, 3) are three points.

(a) Find the area of the triangle having P, Q and R as its vertices.

(b) Give the equation of the plane through P, Q and R in the form ax + by + cz = 1.

(c) Where does the plane in part (b) intersect the y-axis.

(d) Give parametric equations for the line through R that is perpendicular to the plane in part (b).

Sol. (a) $\frac{3}{2}$; (b) $\frac{2x}{3} + \frac{2y}{3} - \frac{z}{3} = 1$; (c) $\left(0, \frac{3}{2}, 0\right)$;

(d) x = 2t + 2; y = 2t + 1 and z = – t + 3

Q.13 Find the equations to the line of greatest slope through the point (7, 2, –1) in the plane x – 2y + 3z = 0 assuming that the axes are so placed that the plane 2x + 3y – 4z = 0 is horizontal.

Sol. $\frac{x-7}{22} = \frac{y-2}{5} = \frac{z+1}{-4}$

Q.14 Let L be the line given by $\vec{r} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and let P be the point (2, –1, 1). Also suppose that E be the plane containing three non collinear points A(0, 1, 1); B(1, 2, 2) and C(1, 0, 1). Find

(a) Distance between the point P and the line L.

(b) Equation of the plane E.

(c) Equation of the plane F containing the line L and the point P.

(d) Acute angle between the plane E and F.

(e) Volume of the parallelopiped by A, B, C and the point D(–3, 0, 1)

Sol. (a) $\sqrt{3}$; (b) x + y – 2z + 1 = 0; (c) x – 2y + z = 5; (d) $\pi/3$; (e) 4

Q.15 The position vectors of the four angular points of a tetrahedron OABC are (0, 0, 0); (0, 0, 2); (0, 4, 0) and (6, 0, 0) respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the value of 'r'.

Sol. 2/3

Q.16 The line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

Sol. $\frac{x-7}{3} = \frac{y-2}{6} = \frac{z-4}{2}$; $\frac{x-7}{2} = \frac{y-2}{-3} = \frac{z-4}{6}$

Q.17 Find the foot and hence the length of the perpendicular from the point (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-z}{5}$. Also find the equation of the plane in which the perpendicular and the given straight line lie.

Sol. (9, 13, 15); 9x – 4y – z = 14

Q.18 Find the equation of the line which is reflection of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane 3x – 3y + 10z = 26.

Sol. $\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$

Q.19 Consider the plane

$$E : \vec{r} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Let F be the plane containing the point A(–4, 2, 2) and parallel to E. Suppose the point B is on the plane E such that B has a minimum distance from the point A. If C(–3, 0, 4) lies in the plane F. Find the area of the angle ABC.

Sol. 9/2

Q.20 Through a point $P(x', y', z')$, a plane is drawn at right angles to OP ($= r$) to meet the coordinates axes in A, B and C. Show that the area of the triangle ABC is $r^5/2x'y'z'$.

Q.21 Find the locus of the centroid of the tetrahedron of constant volume $64k^3$, formed by the three co-ordinates planes and a variable plane.

Sol. $xyz = 6k^3$

Q.22 A line with direction cosines proportional to $(2, 7, -5)$ is drawn to intersect the lines $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ & $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$. Find the coordinate of the points of intersection and the length intercepted on it. Also find the equation of intersecting straight line.

Sol. $\frac{x-2}{2} = \frac{y-8}{7} = \frac{z+3}{-5}$

Q.23 Find the equation of the sphere which has centre at the origin and touches the line $2(x + 1) = 2 - y = z + 3$.

Sol. $9(x^2 + y^2 + z^2) = 5$

Q.24 A variable plane $\lambda x + my + nz = p$ (where λ, m, n are direction cosines) intersects with co-ordinate axes at points A, B and C respectively. Show that the foot of normal on the plane from origin is the orthocentre of triangle ABC and hence find the coordinates of circumcentre of triangle ABC.

Sol. $\left(\frac{p-\lambda^2 p}{2\lambda}, \frac{p-m^2 p}{2m}, \frac{p-n^2 p}{2n}\right)$

Q.25 Prove that the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{1}$ lies in the plane $3x + 4y + 6z + 7 = 0$. If the plane is rotated about the line till the plane passes through the origin then find the equation of the plane in the new position.

Sol. $x + y + z = 0$

Q.26 A sphere has an equation $|\vec{r} - \vec{a}|^2 + |\vec{r} - \vec{b}|^2 = 72$ where $\vec{a} = \hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + 2\hat{k}$. Find :

(i) the centre of the sphere

(ii) the radius of the sphere

(iii) perpendicular distance from the centre of the sphere to the plane $\vec{r}.(2\hat{i} + 2\hat{j} - \hat{k}) = -3$.

Sol. (i) $\left(\frac{3}{2}, \frac{7}{2}, -2\right)$ (ii) $\sqrt{\frac{39}{2}}$ (iii) 5 unit

Q.27 Find the equation of the sphere which is tangential to the plane $x - 2y - 2z = 7$ at $(3, -1, -1)$ and passes through the point $(1, 1, -3)$.

Sol. $x^2 + (y - 5)^2 + (z - 5)^2 = 81$

Passage (Question 28 to 30)

Let $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ be two planes, where $d_1, d_2 > 0$. Then origin lies in acute angle if $a_1a_2 + b_1b_2 + c_1c_2 < 0$ and origin lies in obtuse angle if $a_1a_2 + b_1b_2 + c_1c_2 > 0$. Further point (x_1, y_1, z_1) and origin both lie either in acute angle or in obtuse angle, if $(a_1x_1 + b_1y_1 + c_1z_1 + d_1)(a_2x_1 + b_2y_1 + c_2z_1 + d_2) > 0$, one of (x_1, y_1, z_1) and origin lie in acute angle and the other in obtuse angle, if $(a_1x_1 + b_1y_1 + c_1z_1 + d_1)(a_2x_1 + b_2y_1 + c_2z_1 + d_2) < 0$.

Q.28 Given the planes $2x + 3y - 4z + 7 = 0$ and $x - 2y + 3z - 5 = 0$, if a point P is $(1, -2, 3)$, then

(A) O and P both lie in acute angle between the planes

(B) O and P both lie in obtuse angle

(C) O lies in acute angle, P lies in obtuse angle

(D) O lies in obtuse angle, P lies in acute angle

Sol. **[B]**

Q.29 Given the planes $x + 2y - 3z + 5 = 0$ and $2x + y + 3z + 1 = 0$. If a point P is $(2, -1, 2)$, then

(A) O and P both lie in acute angle between the planes

(B) O and P both lie in obtuse angle

(C) O lies in acute angle, P lies in obtuse angle

(D) O lies in obtuse angle, P lies in acute angle

Sol. **[C]**

Q.30 Given the planes $x + 2y - 3z + 2 = 0$ and $x - 2y + 3z + 7 = 0$, if the point P is $(1, 2, 2)$, then

(A) O and P both lie in acute angle between the planes

(B) O and P both lie in obtuse angle

(C) O lies in acute angle, P lies in obtuse angle

(D) O lies in obtuse angle, P lies in acute angle

Sol. **[A]**

ANSWER KEY

EXERCISE # 1

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	A	B	A	C	C	A	A	B	B	A	B	A	A
Qus.	16	17	18	19	20	21	22	23	24	25	26	27			
Ans.	B	A,B,C,D	C	C	A	B	A	C	D	B	A,B,C,D	B			

28. True **29.** True **30.** $a\lambda + bm + cn = 0$ **31.** $\frac{1}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

EXERCISE # 2

(PART- A)

Qus.	1	2	3	4	5	6	7	8	9
Ans.	B	A	A	A	C	D	D	A	A

(PART- B)

Qus.	10	11	12	13	14	15	16	17
Ans.	A,B,C,D	B,C	A,B,C	A,C	B,C	A,B,C,D	A,B,C,D	A,C

(PART- C)

18. $A \to R$; $B \to P, S$; $C \to S$; $D \to R$ **19.** $A \to S$; $B \to R, S$; $C \to Q$; $D \to R, S$

20. $A \to R$,; $B \to S$; $C \to P$; $D \to Q$

EXERCISE # 3

2. $aa' + cc' = -1$ **3.** $\frac{1}{\sqrt{6}}$; $6x - 9 = 10 - 3y = 6z - 25$

5. $x^{-2} + y^{-2} + z^{-2} = 16\,p^{-2}$ **9.** 600 **10.** $<\frac{2}{\sqrt{6}}\ \frac{-1}{\sqrt{6}}\ \frac{1}{\sqrt{6}}>$, $3x + 4y - 2z = 7$

11. B **12.** A **13.** C **14.** C **15.** B **16.** C **17.** C **18.** B **19.** B

EXERCISE # 4

1. B **2.** (6, 5, –2) **3.** B **4.** $\frac{9}{2}$ $(\text{unit})^3$ **7.** A **8.** $2x - y + z - 3 = 0$ and $62x + 29y + 19z - 105 = 0$

9. A **10.** $A \to R$, $B \to S$ **11.** D **12.** $A \to R$; $B \to Q$; $C \to P$; $D \to S$ **13.** B **14.** D

15. C **16.** D **17.** A **18.** C **19.** 7 units **20.** C **21.** A **22.** A

23. $A \to T$; $B \to P, R$; $C \to Q, S$; $D \to R$ **24.** 6 **25.** A **26.** A **27.** B, C

EXERCISE # 5

1. B **2.** C **3.** B **4.** C **5.** A **6.** A **7.** B

8. A **9.** C **11.** $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-3}$

12. (a) $\frac{3}{2}$; (b) $\frac{2x}{3}+\frac{2y}{3}-\frac{z}{3} = 1$; (c) $\left(0,\frac{3}{2},0\right)$; (d) x = 2t + 2; y = 2t + 1 and z = – t + 3

13. $\frac{x-7}{22}=\frac{y-2}{5}=\frac{z+1}{-4}$

14. (a) $\sqrt{3}$; (b) x + y – 2z + 1 = 0; (c) x – 2y + z = 5; (d) $\pi/3$; (e) 4

15. 2/3

16. $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$; $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

17. (9, 13, 15); 9x – 4y – z = 14

18. $\frac{x-4}{9}=\frac{y+1}{-1}=\frac{z-7}{-3}$

19. 9/2

21. $xyz = 6k^3$

22. $\frac{x-2}{2}=\frac{y-8}{7}=\frac{z+3}{-5}$

23. $9(x^2 + y^2 + z^2) = 5$

24. $\left(\frac{p-\lambda^2 p}{2\lambda},\frac{p-m^2 p}{2m},\frac{p-n^2 p}{2n}\right)$

25. x + y + z = 0

26. (i) $\left(\frac{3}{2},\frac{7}{2},-2\right)$ (ii) $\sqrt{\frac{39}{2}}$ (iii) 5 unit

27. $x^2 + (y-5)^2 + (z-5)^2 = 81$

28. B **29.** C **30.** A