QUADRATIC EQUATION
EXERCISE # 1

Question based on Quadratic Equation & Nature of roots

Q.1 The roots of the equation

$x^2 - 2\sqrt{2}\,x + 1 = 0$ are-

(A) real and different

(B) imaginary and different

(C) real and equal

(D) rational and different

Sol.[A] Roots = $\frac{2\sqrt{2} \pm \sqrt{8-4}}{2}$

$\sqrt{2} \pm 1$

Roots are real and different

Q.2 The roots of the equation

$(b + c)x^2 - (a + b + c)\,x + a = 0$

$(a, b, c \in Q,\ b + c \neq a)$ are-

(A) irrational and different

(B) rational and different

(C) imaginary and different

(D) real and equal

Sol.[B] Θ $a, b, c \in Q,\ \ b + c \neq a$

$\Rightarrow a + b + c \in Q,\ b + c \in Q$

Θ $D = (a + b + c)^2 - 4(b + c)$

$= (a - b - c)^2$

Clearly roots are rational & different

Q.3 If the roots of the equation $ax^2 + x + b = 0$ be real and different, then the roots of the equation $x^2 - 4\sqrt{ab}\ \ x + 1 = 0$ will be-

(A) rational (B) irrational

(C) real (D) imaginary

Sol.[D] $ax^2 + x + b = 0$ has roots real & different

$\therefore$ $D > 0 \Rightarrow 1 - 4\,ab > 0$

$x^2 - 4\ \sqrt{ab}x\ \ + 1 = 0$

$D = 16\,ab - 4$

$= 4\,(4ab - 1) = -4\,(1 - 4\,ab)$

$\Rightarrow D < 0$

$\therefore$ roots are Imaginary

Q.4 The number of real solution of the equation $\left(\frac{9}{10}\right)^x = -3 + x - x^2$ is-

(A) 1 (B) 2 (C) 0 (D) 3

Sol. **[C]**

$\left(\frac{9}{10}\right)^x = -\,(x^2 - x + 3)$

$\left(\frac{9}{10}\right)^x = -\,\{(x - \frac{1}{2})^2 + \frac{11}{4}\}$

LHS is always positive while RHS is negative

Hence LHS $\neq$ RHS

$\therefore$ No solution

Q.5 If $a < c < b$ then the roots of the equation $(a - b)^2\,x^2 + 2\,(a + b - 2c)\,x + 1\ = 0$ are-

(A) imaginary

(B) real

(C) one real and one imaginary

(D) equal and imaginary

Sol. **[A]**

$a < c < b,\quad (a - b)^2\,x^2 + 2\,(a + b - 2c)\,x + 1 = 0$

$D = 4(a + b - 2c)^2 - 4\,(a - b)^2$

$D = 4[(a + b - 2c + a - b)\,(a + b - 2c - a + b)]$

$D = 4[(2a - 2c)\,(2b - 2c)]$

$D = 16[(a - c)\,(b - c)]$ $\quad\left\{\begin{array}{l}\Theta\ a - c < 0\\ \&\ b - c > 0\end{array}\right.$

$D = 16\,[(-ve)\,(+ve)]$

$D = -ve$

$D\ < 0$

roots are Imaginary.

Q.6 If λ, m, n are real and $\lambda \neq m$, the roots of the equation $(\lambda - m)\,x^2 - 5\,(\lambda + m)\,x - 2\,(\lambda - m) = 0$ are-

(A) real & equal (B) complex

(C) real and unequal (D) None of these

Sol. **[C]**

$(\lambda - m)\,x^2 - 5\,(\lambda + m)\,x - 2\,(\lambda - m) = 0$

$D = 25\,(\lambda + m)^2 - 4\,(\lambda - m)\,(-\,2\,(\lambda - m))$

$= 25\,(\lambda + m)^2 + 8(\lambda - m)^2$

$= (5)^2\,(\lambda + m)^2 + 8\,(\lambda - m)^2$ always positive

$D > 0$ roots are real & unequal.

Q.7 If a, b, c are three distinct positive real numbers then the number of real roots of $ax^2 + 2b|x| - c = 0$ is

(A) 4 (B) 2

(C) 0 (D) None of these

Sol. **[B]**

$ax^2 + 2b|x| - c = 0$

where a > 0, b > 0, c > 0

$D = 4b^2 + 4ac$

D > 0 Roots are Real & different (two roots)

Q.8 The number of real solutions of

$x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$ is-

(A) 0 (B) 1

(C) 2 (D) infinite

Sol. **[A]**

$x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$

for exp. defined $x^2 - 4 \neq 0 \Rightarrow x \neq \pm 2$

on solving we get x = 2 (Not possible)

$\therefore$ No root

Q.9 If $x = 2 + 2^{1/3} + 2^{2/3}$, then the values of $x^3 - 6x^2 + 6x$ is-

(A) –2 (B) 3 (C) 4 (D) 2

Sol. **[D]**

$x = 2 + 2^{1/3} + 2^{2/3}$

$(x - 2) = 2^{1/3} + 2^{2/3}$ ...(1)

$(x - 2)^3 = (2^{1/3} + 2^{2/3})^3$

$\Rightarrow x^3 - 8 - 6x(x - 2) = 2 + 4 + 3.2(x - 2)$ from (1)

$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$

$\Rightarrow x^3 - 6x^2 + 6x = 8 - 6 = 2$ Ans

Q.10 If b and c are odd integers, then the equation $x^2 + bx + c = 0$ has-

(A) two odd roots

(B) two integer roots, one odd and one even

(C) no integer roots

(D) None of these

Sol. **[C]**

$x^2 + bx + c = 0 \Rightarrow D = b^2 - 4c$

$x = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$

$x = \frac{-b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$ fractional roots

Q.11 The roots of the quadratic equation $(a + b - 2c)x^2 - (2a - b - c)x + (a - 2b + c) = 0$ are-

(A) a + b + c & a – b + c

(B) 1/2 and a –2b + c

(C) a – 2b + c & 1/(a + b –2c)

(D) None of these

Sol.[D] Here

a + b – 2c – 2a + b + c + a – 2b + c = 0

$\Rightarrow$ one roots is 1 and other is $\frac{a - 2b + c}{a + b - 2c}$

Q.12 Sum of roots of the equation $(x + 3)^2 - 4|x + 3| + 3 = 0$ is-

(A) 4 (B) 12 (C) – 12 (D) – 4

Sol.[C] Let | x + 3 | = t

$\Rightarrow t^2 - 4t + 3 = 0$

$\Rightarrow$ t = 1, 3

$\Rightarrow$ | x + 3 | = 1, | x + 3 | = 3

$\Rightarrow$ (x + 3) = ± 1 and x + 3 = ± 3

$\Rightarrow$ x = – 2, – 4 and x = 0, – 6

Sum = – 2 – 4 – 6 = – 12

Question based on Sum and Product of the roots

Q.13 If α, β are roots of the equation $x^2 + px - q = 0$ & γ, δ are roots of $x^2 + px + r = 0$, then the value of (α – γ)(α – δ) is-

(A) p + r (B) p – r (C) q – r (D) q + r

Sol. [D] α, β are the roots of $x^2 + px - q = 0$

$\Rightarrow \alpha + \beta = -p, \ \alpha\beta = -q$

γ, δ are the roots of $x^2 + px + r = 0$

$\Rightarrow \gamma + \delta = -p, \gamma\delta = r$

therefore

$(\alpha - \gamma)(\alpha - \delta)$

$= \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta$

$= \alpha^2 + p\alpha + r$

$\because$ α is root of $x^2 + px - q = 0$

$\Rightarrow \alpha^2 + p\alpha = q$

$\Rightarrow (\alpha - \gamma)(\alpha - \delta) = q + r$

Q.14 If α, β are roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 . (2\beta - 35)^3$ is equal to-

(A) 1 (B) 8

(C) 64 (D) None of these

Sol. [C] $\because$ α, β are roots of $2x^2 - 35x + 2 = 0$

$\Rightarrow 2\alpha - 35 = -\dfrac{2}{\alpha}$ and $2\beta - 35 = -\dfrac{2}{\beta}$

$\Rightarrow (2\alpha - 35)^3 . (2\beta - 35)^3$

$= \left(-\dfrac{8}{\alpha^3}\right)\left(-\dfrac{8}{\beta^3}\right) = \dfrac{64}{(\alpha\beta)^3}$

$= 64$ [$\because \alpha\beta = 1$]

Q.15 For the roots of the equation
$a - bx - x^2 = 0$; ($a > 0$, $b > 0$) which statement is true-

(A) positive and same sign

(B) negative and same sign

(C) greater root in magnitude negative and opposite in signs

(D) greater root in magnitude positive and opposite in signs

Sol. [C] Let roots are α and β then from equation

$-x^2 - bx + a = 0$

we have $\alpha\beta = -a < 0$

both the roots are in opposite sign and

$\alpha + \beta = -b$

$\Rightarrow$ both roots are in opposite sign and greater root in magnitude is negative

Q.16 The value of 'a' for which the sum of the squares of the roots of $2x^2 - 2(a-2)x - a - 1 = 0$ is least is

(A) 1 (B) 3/2 (C) 2 (D) –1

Sol. [B]

Let $y = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$y = \left(\dfrac{2(a-2)}{2}\right)^2 + \dfrac{2(a+1)}{2}$

$y = a^2 - 3a + 5$

$\dfrac{dy}{da} = 2a - 3$

for max., mini. we have

$\dfrac{dy}{da} = 0 \Rightarrow a = \dfrac{3}{2}$

$\dfrac{d^2y}{da^2} = 2$ minimum $\Rightarrow a = 3/2$

Q.17 If α, β are roots of $Ax^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$ then p is equal to-

(A) $\dfrac{B^2 - 4AC}{A^2}$ (B) $\dfrac{2AC - B^2}{A^2}$

(C) $\dfrac{B^2 - 2AC}{A^2}$ (D) $\dfrac{4AC - B^2}{A^2}$

Sol. **[B]**

$\alpha + \beta = \dfrac{-B}{A}$ $\qquad \alpha^2 + \beta^2 = -p$

$\alpha\beta = \dfrac{C}{A}$ $\qquad \alpha^2\beta^2 = q$

$\therefore \alpha^2 + \beta^2 = -p$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = -p \Rightarrow \dfrac{B^2}{A^2} - \dfrac{2C}{A} = -p$

$\Rightarrow \dfrac{B^2 - 2AC}{A^2} = -p \Rightarrow p = \dfrac{2AC - B^2}{A^2}$ Ans.

Question based on **Theory of Equations**

Q.18 If α, β are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is-

(A) $x^2 - 11x + 30 = 0$

(B) $(x - 3)^2 - 5(x - 3) + 6 = 0$

(C) Both (A) and (B)

(D) None of these

Sol.[C] Clearly

Replace x by $x - 3$ in given equation we get

$(x - 3)^2 - 5(x - 3) + 6 = 0$

$\Rightarrow x^2 - 11x + 30 = 0$

$\Rightarrow$ both A and B are correct

Q.19 If α, β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is-

(A) $x^2 + 4x + 1 = 0$ (B) $x^2 - 4x + 4 = 0$

(C) $x^2 - 4x - 1 = 0$ (D) $x^2 + 2x + 3 = 0$

Sol.[D] $\because \alpha, \beta$ are the roots of

$x^2 - 3x + 5 = 0$

$\Rightarrow \alpha^2 - 3\alpha = -5,\ \beta^2 - 3\beta = -5$

given roots are

$(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7) \Rightarrow 2$ & 2

equation $x^2 - 4x + 4 = 0$

Q.20 Let $\alpha, \beta, \gamma, \delta$ be the roots of $x^4 + x^2 + 1 = 0$. Then the equation whose roots are $\alpha^2, \beta^2, \gamma^2, \delta^2$ are-

(A) $(x^4 - x + 1)^2 = 0$ (B) $(x^2 + x + 1)^2 = 0$

(C) $(x^4 - x^2 + 1) = 0$ (D) $(x^2 + x + 1) = 0$

Sol.[B] $\because \alpha, \beta, \gamma, \delta$ are the roots of

$x^4 + x^2 + 1 = 0$ ……(1)

then the equation whose roots are

$\alpha^2, \beta^2, \gamma^2, \delta^2$

$\Rightarrow$ Replace $x \to \sqrt{x}$ in (1) we get

$(\sqrt{x})^4 + (\sqrt{x})^2 + 1 = 0$

$\Rightarrow x^2 + x + 1 = 0$

squaring we get equation

$(x^2 + x + 1)^2 = 0$

Q.21 If α, β, γ are the roots of $x^3 + 8 = 0$, then the equation whose roots are α^2, β^2 and γ^2 is-

(A) $x^3 - 8 = 0$ (B) $x^3 - 16 = 0$

(C) $x^3 + 64 = 0$ (D) $x^3 - 64 = 0$

Sol.[D] Replace $x \to \sqrt{x}$ in given equation we get

$(x)^{3/2} = -8$

Squaring we get

$x^3 = 64$

$\Rightarrow x^3 - 64 = 0$

Q.22 If the roots of the equation, $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equation $x^3 - Ax^2 + Bx - C = 0$, where A, B, C, P & Q are constants then the value of A + B + C =

(A) 18 (B) 19

(C) 20 (D) None of these

Sol.[A] ∵ roots of $x^3 + Px^2 + Qx - 19 = 0$ ……(i)

are each one more than the roots of

$x^3 - Ax^2 + Bx - C = 0$ ……(ii)

So in (i) if we replace x → x + 1 then we get (ii)

there fore

$(x + 1)^3 + P(x + 1)^2 + Q(x + 1) - 19 = 0$

$\Rightarrow x^3 + 3x^2 + 3x + 1 + Px^2 + 2Px + P + Qx + Q - 19 = 0$

$\Rightarrow x^3 + (3 + P)x^2 + (3 + 2P + Q)\,x + 1 + P + Q - 19 = 0$

it is represent (ii) So comparing we get

$A = -(3 + P)$, $B = 3 + 2P + Q$

$C = -(1 + P + Q - 19)$

$\Rightarrow A + B + C = 18$

Q. 23 If α, β, γ, δ are roots of

$x^4 - 100x^3 + 2x^2 + 4x + 10 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

is equal to

(A) $\frac{2}{5}$ (B) $\frac{1}{10}$ (C) 4 (D) $-\frac{2}{5}$

Sol.[D] α, β, γ, δ are roots of

$x^4 - 100x^3 + 2x^2 + 4x + 10 = 0$

$= \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{\Sigma\alpha\beta\gamma}{\alpha\beta\gamma\delta}$

$= -\frac{4}{10} = -\frac{2}{5}$

Question based on Common Roots

Q.24 If one root of the equations $x^2 + 2x + 3k = 0$ and $2x^2 + 3x + 5k = 0$ is common then the value of k is-

(A) 1, 2 (B) 0, –1

(C) 1, 3 (D) None of these

Sol.[B] Let α be the common root then

$\alpha^2 + 2\alpha + 3k = 0$ and $2\alpha^2 + 3\alpha + 5k = 0$

$\Rightarrow \frac{\alpha^2}{10k - 9k} = \frac{\alpha}{6k - 5k} = \frac{1}{3 - 4}$

$\Rightarrow k^2 = -k$

$\Rightarrow k(k + 1) = 0$

$\Rightarrow k = 0, -1$

Q.25 If the equations $2x^2 + x + k = 0$ & $x^2 + \frac{x}{2} - 1 = 0$ have 2 common roots then the value of k is-

(A) 1 (B) 3 (C) –1 (D) –2

Sol.[D] Equation have both the roots are common then

$\frac{2}{1} = \frac{1}{1/2} = \frac{k}{-1} \Rightarrow k = -2$

Q.26 If equation $x^2 + ax + bc = 0$ and $x^2 + bx + ca = 0$ have one root common then their remaining roots are-

(A) a, b (B) b, a (C) b, c (D) c, a

Sol. **[B]**

Let common root is α and let 1st eqn has α, β roots & 2nd eqn has α,γ roots

$\Rightarrow \alpha^2 + a\alpha + bc = 0$ …(1)

and $\alpha^2 + b\alpha + ca = 0$ …(2)

(1) – (2) gives

$\alpha(a - b) + c(b - a) = 0$

$\Rightarrow \alpha(a - b) = c(a - b)$

$\Rightarrow \alpha = c$

∴ Product of 1st eqn $= \alpha\beta = bc$

$\Rightarrow c.\beta = bc$

$\beta = b$ Ans.

and product of 2nd eqn $\alpha.\gamma = ca$

$\Rightarrow c.\gamma = c.a$

$\Rightarrow \gamma = a$ Ans.

∴ Remaining roots are b & a Ans.

Q.27 If $f(x) = 4x^2 + 3x - 7$ and α is a common root of the equation $x^2 - 3x + 2 = 0$ and $x^2 + 2x - 3 = 0$ then the value of f(α) is -

(A) 3 (B) 2 (C) 1 (D) 0

Sol. **[D]**

$f(x) = 4x^2 + 3x - 7$ (given)

$\alpha^2 - 3\alpha + 2 = 0$ ...(1)

$\alpha^2 + 2\alpha - 3 = 0$ ...(2)

(1) & (2)

$\Rightarrow -5\alpha + 5 = 0 \Rightarrow \alpha = 1$

$\therefore f(\alpha) = 4\alpha^2 + 3\alpha - 7$

$f(1) = 4 + 3 - 7 = 0$ Ans.

Q.28 If $x^2 + x - 1 = 0$ and $2x^2 - x + \lambda = 0$ have a common root then-

(A) $\lambda^2 - 7\lambda + 1 = 0$ (B) $\lambda^2 + 7\lambda - 1 = 0$

(C) $\lambda^2 + 7\lambda + 1 = 0$ (D) $\lambda^2 - 7\lambda - 1 = 0$

Sol.[C] Let common root be α then

$\alpha^2 + \alpha - 1 = 0$ and $2\alpha^2 - \alpha + \lambda = 0$

$\Rightarrow \frac{\alpha^2}{\lambda - 1} = \frac{\alpha}{-2-\lambda} = \frac{1}{-3}$

$\Rightarrow (-2-\lambda)^2 = (\lambda - 1)(-3)$

$\Rightarrow \lambda^2 + 4\lambda + 4 = -3\lambda + 3$

$\Rightarrow \lambda^2 + 7\lambda + 1 = 0$

Question based on

Maximum and minimum value of Quadratic Expression

Q.29 The minimum value of the expression $4x^2 + 2x + 1$ is-

(A) 1/4 (B) 1/2 (C) 3/4 (D) 1

Sol. [C] Minimum value $= -\frac{D}{4a}$ $[\because a > 0]$

$= -\frac{4-16}{16} = \frac{12}{16} = \frac{3}{4}$

Q.30 The range of the values of $\frac{x}{x^2+4}$ for all real value of x is-

(A) $\frac{-1}{4} \le y \le \frac{1}{4}$ (B) $\frac{-1}{2} \le y \le \frac{1}{2}$

(C) $\frac{-1}{6} \le y \le \frac{1}{6}$ (D) None of these

Sol. [A] Let $y = \frac{x}{x^2+4} \Rightarrow x^2y - x + 4y = 0$

x is real $\Rightarrow D \ge 0$

$1 - 16y^2 \ge 0 \Rightarrow y^2 \le \frac{1}{16} \Rightarrow |y| \le \frac{1}{4}$

Q.31 The expression $\frac{x^2+2x+1}{x^2+2x+7}$ lies in the interval; $(x \in R)$

(A) [0, –1] (B) $(-\infty, 0] \cup [1, \infty)$

(C) [0, 1) (D) None of these

Sol. [C] Let $y = \frac{x^2+2x+1}{x^2+2x+7}$

$\Rightarrow x^2(y-1) + 2x(y-1) + 7y - 1 = 0$

x is real $\Rightarrow D \ge 0, y \ne 1$

$4(y-1)^2 - 4(y-1)(7y-1) \ge 0$

$\Rightarrow (y-1)(y-1-7y+1) \ge 0$

$\Rightarrow y(y-1) \le 0 \Rightarrow [0, 1)$

Q.32 For real values of x, $2x^2 + 5x - 3 > 0$, if-

(A) $x < -2$ (B) $x > 0$

(C) $x > 1$ (D) None of these

Sol. [C] $2x^2 + 5x - 3 > 0$

$\Rightarrow 2x^2 + 6x - x - 3 > 0 \Rightarrow (2x-1)(x+3) > 0$

$\Rightarrow x < -3$ and $x > \frac{1}{2}$

from option $x > 1$ is correct

Question based on

Location of roots

Q.33 If $c > 0$ and $b > c$ then $x^2 + bx - c = 0$ will have-

(A) exactly one root between 0 and 1

(B) both roots between 0 and 1

(C) no root between 0 and 1

(D) None of these

Sol. [A] $x^2 + bx - c = 0,\ c > 0,\ b > c$

$0 + 0 - c = -$ ve

and $1 + b - c > 0 \Rightarrow b - c > 0$ (+ve)

clearly one root between 0 and 1

Now $\alpha\beta = -c = -$ve

$\alpha + \beta = -b = -$ve

$\Rightarrow$ one root is –ve and one is +ve

from option A is correct

Q.34 If both the roots of the equation $x^2 - 9x + a = 0$ are positive and one is greater than 3 and other is less than 3, then all possible values of a is-

(A) $0 < a < 18$ (B) $-1 < a < 2$

(C) $-18 < a < 0$ (D) None of these

Sol. [A] Both root are positive

$\Rightarrow \alpha\beta = a = +$ve $\Rightarrow a > 0$

clearly 3 lies between roots

$\Rightarrow 9 - 27 + a < 0 \Rightarrow a < 18$

$\Rightarrow 0 < a < 18$

Q.35 The number of integral values of m, for which the roots of $x^2 - 2mx + m^2 - 1 = 0$ will lie between –2 and 4 is

(A) 2 (B) 0 (C) 3 (D) 1

Sol. [C] Roots lie between –2 and 4 so

$\Rightarrow f(-2) > 0$ and $f(4) > 0$

$\Rightarrow 4 + 4m + m^2 - 1 > 0$ and $16 - 8m + m^2 - 1 > 0$

$\Rightarrow m^2 + 4m + 3 > 0$ and $m^2 - 8m + 15 > 0$

$\Rightarrow (m + 1)(m + 3) > 0$ and $(m - 5)(m - 3) > 0$

$\Rightarrow (-\infty, -3) \cup (-1, \infty) \cap (-\infty, 3) \cup (5, \infty)$

$\Rightarrow m \in (-\infty, -3) \cup (-1, 3) \cup (5, \infty)$

But $-2 < m < 4 \Rightarrow m \in (-2, 4)$

$\Rightarrow m \in (-1, 3)$

integral values = 0, 1, 2 = three values

Q.36 If α, β are the roots of the quadratic equation $(p^2 + p + 1)x^2 + (p - 1)x + p^2 = 0$ such that unity lies between the roots then the set of values of p is

(A) ϕ

(B) $p \in (-\infty, -1) \cup (0, \infty)$

(C) $p \in (-1, 0)$

(D) $(-1, 1)$

Sol. [C] Clearly $f(1) < 0$

$\Rightarrow p^2 + p + 1 + p - 1 + p^2 < 0$

$\Rightarrow 2p^2 + 2p < 0$

$\Rightarrow p(p + 1) < 0$

$\Rightarrow p \in (-1, 0)$

Q.37 The set of values of 'p' for which the expression $x^2 - 2px + 3p + 4$ is negative for at least one real x is-

(A) ϕ (B) $(-1, 4)$

(C) $(-\infty, -1) \cup (4, \infty)$ (D) $\{-1, 4\}$

Sol. **[C]**

$x^2 - 2p\,x + 3p + 4 < 0$

$\therefore D > 0$

$4p^2 - 4(3p + 4) > 0$

$\Rightarrow p^2 - 3p - 4 > 0$

$\Rightarrow p^2 - 4p + p - 4 > 0$

$\Rightarrow p(p - 4) + 1(p - 4) > 0$

$\Rightarrow (p - 4)(p + 1) > 0$

$\therefore p \in (-\infty, -1) \cup (4, \infty)$ Ans.

Q.38 The least integral value of a for which the equation $x^2 - 2(a - 1)x + (2a + 1) = 0$ has both the roots positive is-

(A) 3 (B) 4 (C) 1 (D) 5

Sol. **[B]**

Let $f(x) = x^2 - 2(a - 1)x + (2a + 1)$

then f (x) = 0 have both roots positive, if

(i) Discriminant ≥ 0

(ii) Sum of the roots > 0

(iii) f (0) > 0

$D \geq 0 \Rightarrow 4(a - 1)^2 - 4(2a + 1) \geq 0$

$\Rightarrow a^2 - 2a + 1 - 2a - 1 \geq 0$

$\Rightarrow a^2 - 4a \geq 0 \Rightarrow a(a - 4) \geq 0$

$\Rightarrow a \leq 0$ or $a \geq 4$ …(1)

sum of roots > 0

$\Rightarrow 2(a - 1) > 0$

$\Rightarrow a > 1$ …(2)

$f(0) > 0 \Rightarrow (2a + 1) > 0$

$\Rightarrow a > -1/2$ …(3)

from (1), (2) & (3) we get

$a \geq 4$

∴ least integral value of a is 4 Ans.

Q.39 If α, β are the roots of the equation $x^2 - 3x + a = 0$, $a \in R$ and $\alpha < 1 < \beta$ then-

(A) $a \in (-\infty, 2)$ (B) $a \in (-\infty, 9/4)$

(C) $a \in (2, 9/4)$ (D) None of these

Sol. [A] Θ $\alpha < 1 < \beta$ then

$f(1) < 0$

$\Rightarrow 1 - 3 + a < 0 \Rightarrow a < 2$

$\Rightarrow a \in (-\infty, 2)$

EXERCISE # 2

Part-A **Only single correct answer type questions**

Q.1 The number of values of a for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x is -

(A) 0 (B) 2

(C) 1 (D) 3

Sol. **[C]**

$(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$

is an identity in x,

$\Rightarrow a^2 - 3a + 2 = 0$

$\Rightarrow (a - 1)(a - 2) = 0$

$\Rightarrow a = 1, 2$ … (1)

Also $a^2 - 5a + 6 = 0$

$\Rightarrow (a - 2)(a - 3) = 0$

$\Rightarrow a = 2, 3$ …(2)

and $a^2 - 4 = 0$

$a = \pm 2$ … (3)

from (1), (2) & (3) we get a = 2

Hence no. of values of a is one. Ans.

Q.2 If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + ...}}}$, then -

(A) $-2 < x < 3$ (B) $2 < x < 3$

(C) $x = 3$ (D) $x > 3$

Sol. **[C]** $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + ...}}}$

$\Rightarrow x = \sqrt{6 + x} \Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = -2, 3$

$\Rightarrow x = -2$ (Not possible)

$\therefore x = 3$ Ans.

Q.3 Let α, β be the roots of the equation $ax^2 + 2bx + c = 0$ and γ, δ be the roots of the equation $px^2 + 2qx + r = 0$. If α, β, γ, δ are in G.P., then-

(A) $q^2 ac = b^2 pr$ (B) $qac = bpr$

(C) $c^2 pq = r^2 ab$ (D) $p^2 ab = a^2 qr$

Sol. **[A]**

$\alpha + \beta = \frac{-2b}{a}$ & $\gamma + \delta = \frac{-2q}{p}$

$\alpha\beta = \frac{c}{a}$ & $\gamma\delta = \frac{r}{p}$

Θ α, β, γ, δ are in G.P.

$\Rightarrow \frac{\alpha}{\beta} = \frac{\beta}{\gamma} = \frac{\gamma}{\delta}$

$\Rightarrow \frac{\alpha}{\beta} = \frac{\gamma}{\delta}$

$\Rightarrow \frac{\alpha}{\gamma} = \frac{\beta}{\delta}$

$\Rightarrow \frac{\alpha + \beta}{\gamma + \delta} = \sqrt{\frac{\alpha\beta}{\gamma\delta}}$

$\Rightarrow \frac{-2b/a}{\frac{-2q}{p}} = \sqrt{\frac{c/a}{r/p}}$

$\Rightarrow \frac{bp}{aq} = \sqrt{\frac{cp}{ra}}$

Squaring both sides

$\frac{b^2p^2}{a^2q^2} = \frac{cp}{ra}$

$b^2pr = q^2ac$ Ans.

Q.4 The set of values of p for which $(p - 2)x^2 + 7x + p^2 - 4p = 0$ has roots of opposite signs are-

(A) $0 < p < 2$ (B) $2 < p < 4$

(C) $p < 0$ (D) $0 < p > 4$

Sol. **[B, C]**

Product of roots = negative

$\Rightarrow \frac{p^2 - 4p}{p - 2} < 0 \Rightarrow \frac{p(p-4)}{(p-2)} < 0$

$p \in (2, 4)$, $2 < p < 4$ Ans.

Q.5 If α, β, γ are the roots of the equation $x^3 - x - 1 = 0$, then the value of $\sum\left(\frac{1+\alpha}{1-\alpha}\right)$ is -

(A) –3 (B) –5

(C) –7 (D) None of these

Sol. **[C]**

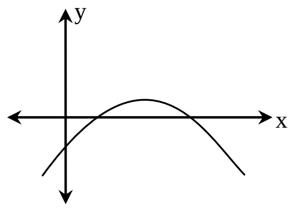
Put $\alpha + \beta + \gamma = 0$, $\alpha\beta + \beta\gamma + \gamma\alpha = -1$, $\alpha\beta\gamma = 1$

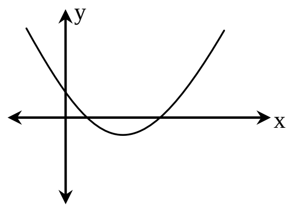
Q.6 Graph of the function $f(x) = Ax^2 - BX + C$, where

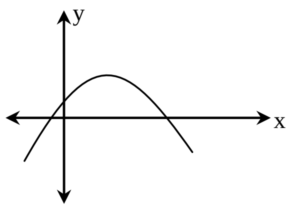
$A = (\sec\theta - \cos\theta)(\mathrm{cosec}\,\theta - \sin\theta)(\tan\theta + \cot\theta)$,

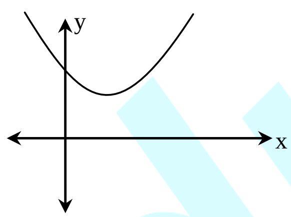
$B = (\sin\theta + \mathrm{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 - (\tan^2\theta + \cot^2\theta)$

& C = 12, is represented by

(A) 

(B) 

(C) 

(D) 

Sol. **[B]**

$$\Theta\ A = \frac{\sin^2\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin\theta}\cdot\frac{1}{\sin\theta\cos\theta} = 1$$

$B = \sin^2\theta + \mathrm{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2 - \tan^2\theta - \cot^2\theta$

$= 5 + 1 + \cot^2\theta + 1 + \tan^2\theta - \tan^2\theta - \cot^2\theta = 7$

and C = 12

function is $f(x) = x^2 - 7x + 12$

$f(x) = (x - 3)(x - 4)$

clearly graph is option B

Q.7 If the roots of the quadratic equation $x^2 + 6x + b = 0$ are real and distinct and they differ by atmost 4 then the least value of b is-

(A) 5 (B) 6 (C) 7 (D) 8

Sol. **[A]**

roots are real

$\Rightarrow 36 - 4b > 0 \Rightarrow b < 9$

they differ by at most 4

$\Rightarrow |\alpha - \beta| < 4 \Rightarrow |\alpha - \beta|^2 < 16$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta < 16$

$\Rightarrow 36 - 4b < 16 \Rightarrow b > 5$

Least value of b is 5

Q.8 If p and q are distinct reals, then $2\{(x - p)(x - q) + (p - x)(p - q) + (q - x)(q - p)\} = (p - q)^2 + (x - p)^2 + (x - q)^2$ is satisfied by-

(A) no value of x

(B) exactly one value of x

(C) exactly two value of x

(D) infinite values of x

Sol. **[D]**

$2\{(x - p)(x - q) + (p - q)(p - q)\} = (p - q)^2 + (x - p)^2 + (x - q)^2$

$\Rightarrow (p - q)^2 + 2(x - p)(x - q) = (x - p)^2 + (x - q)^2$

$\Rightarrow (x - p - x + q)^2 = (p - q)^2$

$\Rightarrow (p - q)^2 = (p - q)^2$

$\Rightarrow$ Infinite solution

Q.9 The set of values of 'a' for which $f(x) = ax^2 + 2x(1 - a) - 4$ is negative for exactly three integral values of x, is-

(A) (0, 2) (B) (0, 1]

(C) [1, 2) (D) $[2, \infty)$

Sol. **[C]**

$f(x) = ax^2 + 2x(1 - a) - 4$

roots are 2, $-\dfrac{2}{a}$

$\Rightarrow$ two integer values of x is 0 and 1 for third value we have

$-2 \le -\frac{2}{a} < -1$

$\Rightarrow -2 \le -\frac{2}{a} \Rightarrow a \ge 1$

and $-\frac{2}{a} < -1 \Rightarrow a < 2$

$\Rightarrow a \in [1, 2)$

Q.10 Set of the values of parameter 'm' for which every solution of the inequality $x^2 - 3x + 2 \le 0$ is also a solution of the inequality is $2x^2 - (m + 1)x - (2m + 3) < 0$, is

(A) $\left(\frac{3}{4}, \infty\right)$ (B) $\left(-\frac{2}{3}, \infty\right)$

(C) $\left(-\frac{2}{3}, \frac{3}{4}\right)$ (D) $(-\infty, \infty)$

Sol. **[A]**

$\because x^2 - 3x + 2 \le 0$

$\Rightarrow (x - 2)(x - 1) \le 0$

$\Rightarrow x \in [1, 2]$

therefore at x = 1

$2 - m - 1 - 2m - 3 < 0$

$-3m + 4 < 0$

$\Rightarrow m > \frac{3}{4}$

Solution is $m \in \left(\frac{3}{4}, \infty\right)$

Q.11 The number of possible value of 'α' for which the expression $y = \frac{\alpha x^2 + 7x - 2}{\alpha + 7x - 2x^2}$ has atleast one common linear factor in numerator and denominator, is-

(A) 0 (B) 1 (C) 2 (D) 3

Sol. **[D]**

If one factor is common in numerator and denominator therefore factor

Let p is common factor then

$\alpha p^2 + 7p - 2$ and $-2p^2 + 7p + \alpha$

$\Rightarrow \frac{p^2}{7\alpha + 14} = \frac{p}{4 - \alpha^2} = \frac{1}{7\alpha + 14}$

Solving we get $\alpha = 2, \frac{96}{50}$

and If both the factor are common then

$\frac{\alpha}{-2} = \frac{7}{7} = \frac{-2}{\alpha} \Rightarrow \alpha = -2$

$\Rightarrow 2, \frac{96}{50}$ and -2 are three values of α

Q.12 The set of values of 'a' for which the inequality $x^2 - (a + 2)x - (a + 3) < 0$ is satisfied by atleast one positive real x, is-

(A) $[-3, \infty)$ (B) $(-3, \infty)$

(C) $(-\infty, -3)$ (D) $(-\infty, 3]$

Sol. **[B]**

$\because f(x) < 0$ but $A > 0$

$\Rightarrow D > 0$

$\Rightarrow (a + 2)^2 + 4(a + 3) > 0$

$\Rightarrow (a + 4)^2 > 0 \Rightarrow a \in (-\infty, \infty)$

roots are

$x = \frac{(a+2) \pm \sqrt{(a+2)^2 + 4(a+3)}}{2}$

$= \frac{(a+2) \pm (a+4)}{2}$

$\Rightarrow -2, (a + 3)$

from question we have

$a + 3 > 0$

$a > -3$

$\Rightarrow a \in (-3, \infty)$

Part-B One or more than one correct answer type questions

Q.13 The roots of the equation,

$(x^2 + 1)^2 = x(3x^2 + 4x + 3)$, are given by-

(A) $2 - \sqrt{3}$ (B) $(-1 + i\sqrt{3})/2$

(C) $2 + \sqrt{3}$ (D) $(-1 - i\sqrt{3})/2$

Sol. **[A,B,C,D]**

$(x^2 + 1)^2 = x(3x^2 + 4x + 3)$

$\Rightarrow x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$

Let roots are $\alpha, \beta, \gamma, \delta$

$\Rightarrow \alpha + \beta + \gamma + \delta = 3$

and $\alpha\beta\gamma\delta = 1$

from option A, B, C, D,

Satisfy these conditions

$\Rightarrow$ option A, B, C, D are correct

Q.14 The number of real solutions of the equation $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$ are, where $t = x^2 - 2|x|$

(A) 0 (B) 2 (C) 4 (D) 6

Sol. **[C]**

Let $(15+4\sqrt{14})^t = p \Rightarrow p^2 - 30p + 1 = 0$

$\Rightarrow p = 15 \pm 4\sqrt{14} \Rightarrow t = \pm 1$

$\Theta\ t = x^2 - 2|x|$

at $t = 1 \Rightarrow x^2 - 2|x| - 1 = 0 \Rightarrow x = \pm (1+\sqrt{2})$

at $t = -1 \Rightarrow x^2 - 2|x| + 1 = 0 \Rightarrow x = \pm 1$

Number of real solution are 4

Q.15 Let $x^2 - px + q = 0$, where $p \in R$, $q \in R$ have the roots α, β such that $\alpha + 2\beta = 0$ then-

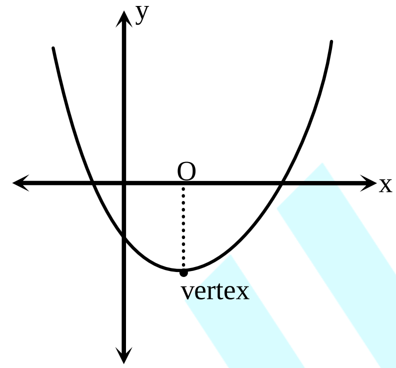
(A) $2p^2 + q = 0$ (B) $2q^2 + p = 0$

(C) $q < 0$ (D) None of these

Sol. **[A, C]**

$x^2 - px + q = 0$ and	$\alpha^2 - p\alpha + q = 0$
$\alpha + \beta = p$	$\alpha + 2\beta = 0$ (given)
$\alpha\beta = q$	$p + \dfrac{q}{\alpha} = 0$
$\alpha + 2.\dfrac{q}{\alpha} = 0$	$p = \dfrac{-q}{\alpha}$
$\alpha^2 + 2q = 0$	$p^2 = \dfrac{q^2}{\alpha^2}$
$q = \dfrac{-\alpha^2}{2}$	$p^2 = \dfrac{q^2}{-2q} = \dfrac{-q}{2}$
$q < 0$	$2p^2 + q = 0$

Q.16 Graph of $y = ax^2 + bx + c = 0$ is given adjacently. What conclusions can be drawn from this graph-



(A) $a > 0$ (B) $b < 0$

(C) $c < 0$ (D) $b^2 - 4ac > 0$

Sol. **[A,B]**

From graph

$a > 0$

$\Theta\ \alpha + \beta = -\dfrac{b}{a}$]

But $\alpha + \beta$ +ve $\Rightarrow -\dfrac{b}{a} > 0 \Rightarrow b < 0$

$\Theta\ \alpha\beta = \dfrac{c}{a}$

But $\alpha\beta < 0 \Rightarrow \dfrac{c}{a} < 0 \Rightarrow c < 0$

roots are real $\Rightarrow D > 0 \Rightarrow b^2 - 4ac > 0$

$\Rightarrow$ option A, B, C, D all are correct

Q.17 Equation $2x^2 - 2(2a + 1)x + a(a + 1) = 0$ has one root less than 'a' and other root greater than 'a', if

(A) $0 < a < 1$ (B) $-1 < a < 0$

(C) $a > 0$ (D) $a < -1$

Sol. **[C,D]**

If one root is less than a and other is greater than a then

$f(a) < 0 \quad \Theta\ 2 > 0$

and $D > 0$

$\Rightarrow 2a^2 - 4a^2 - 2a + a^2 + a < 0$

$\Rightarrow -a^2 - a < 0 \Rightarrow a(a + 1) > 0$

$\Rightarrow a < -1, a > 0$

For D solving we get $D > 0\ \forall a$

$\Rightarrow$ option C, D are correct

Q.18 If $\left(\sqrt{(49+20\sqrt{6})}\right)^{\sqrt{a\sqrt{a\sqrt{a\ldots\ldots\infty}}}}$ +

$\left(5-2\sqrt{6}\right)^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x\ldots\ldots\infty}}}} = 10$

where $a = x^2 - 3$, then x is-

(A) $-\sqrt{2}$ (B) $\sqrt{2}$

(C) –2 (D) 2

Sol. **[D]**

We can write the equation as

$(5+2\sqrt{6})^{a} + (5-2\sqrt{6})^{x^2-3} = 10$

$\Rightarrow (5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$

Let $(5+2\sqrt{6})^{x^2-3} = t$

$\Rightarrow t^2 - 10t + 1 = 0$

$\Rightarrow t = 5 \pm 2\sqrt{6}$

$\Rightarrow x^2 - 3 = \pm 1$

$\Rightarrow x = \pm\sqrt{2}\,,\ x = \pm 2$

But $x \neq -2,\ -\sqrt{2}$ and $\sqrt{2}$

$\Rightarrow x = 2$ only

Q.19 The correct statement is/ are-

(A) If x_1 and x_2 are roots of the equation $2x^2 - 6x - b = 0$; $(b > 0)$ then $\frac{x_1}{x_2} + \frac{x_2}{x_1} < -2$

(B) Equation $ax^2 + bx + c = 0$ has real roots if $a < 0$ and $c > 0$

(C) If $P(x) = ax^2 + bx + c$ & $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$ then P(x). Q(x) has at least two real roots.

(D) If both the roots of the equation $(3a + 1)x^2 - (2a + 3b)x + 3 = 0$ are infinite then $a = 0$ and $b \in R$.

Sol. **[A,B,C]**

(A) $\frac{x_1}{x_2} + \frac{x_2}{x_1}$

$= \frac{x_1^2 + x_2^2}{x_1 x_2} = \frac{(x_1+x_2)^2 - 2x_1x_2}{x_1x_2}$ …..(1)

Now, $2x^2 - 6x - b = 0$

x_1 and x_2 are roots of the above equation.

So, $x_1 + x_2 = -\frac{(-6)}{2} = 3$ ….(2)

$x_1 x_2 = \frac{-b}{2}$ ….(3)

In equation (1)

After putting the value from equation (2) & (3), we get

$$\frac{(3)^2 - 2\times\frac{-b}{2}}{\frac{-b}{2}} = \frac{-2(9+b)}{b}$$

$$= -2\left(\frac{9}{b}+1\right) < -2 \quad \forall\, b \in R^+$$

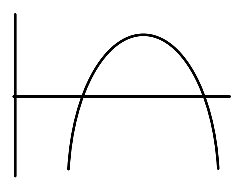
(B) $ax^2 + bx + c = 0$

$a < 0$ and $c > 0$

$f(0) = c > 0$

So, Graph must cut the x-axis,

Hence the equation has real roots



(C) $P(x) = ax^2 + bx + c$

$Q(x) = -ax^2 + bx + c$ {$ac \neq 0$ Given}

Discriminant Δ of $P(x) = b^2 - 4ac$

Δ of $Q(x) = b^2 + 4ac$

Now, For $\rightarrow$ P(x) . Q(x)

Case - I

Let $ac < 0$ then,

$b^2 - 4ac > 0$

Hence two real roots

Case II :

Let $ac > 0$ then

$b^2 + 4ac > 0$

Hence two real roots

Since, ac can not. be +ve and –ve simultaneously.

So, minimum real roots of the

P(x) . Q(x) = 2

(D) $(3a + 1)x^2 – (2a + 3b)x + 3 = 0$

for the roots to be infinite

coefficient of $x^2 = 0$

$\Rightarrow 3a + 1 = 0$

$$a = -\frac{1}{3}$$

& the statement is wrong.

Q.20 If $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \alpha_6$, then the equation $(x – \alpha_1)(x – \alpha_3)(x – \alpha_5) + 3(x – \alpha_2)(x – \alpha_4)(x – \alpha_6) = 0$ has

(A) three real roots

(B) no real root in $(–\infty, \alpha_1)$

(C) one real root in (α_1, α_2)

(D) no real root in (α_5, α_6)

Sol. **[A,B,C]**

$P(x) = (x – \alpha_1)(x – \alpha_3)(x – \alpha_5) + 3(x – \alpha_2)(x – \alpha_4)(x – \alpha_6) = 0$

(I) If $x = \alpha_1$

then,

$P(x) = 0 + 3(\alpha_1 – \alpha_2)(\alpha_1 – \alpha_4)(\alpha_1 – \alpha_6) < 0$

–ve –ve –ve

(II) If $x = \alpha_2$

$P(x) = (\alpha_2 – \alpha_1)(\alpha_2 – \alpha_3)(\alpha_2 – \alpha_5) > 0$

+ve –ve –ve

(III) If $x = \alpha_3$

$P(x) = 3(\alpha_3 – \alpha_2)(\alpha_3 – \alpha_4)(\alpha_3 – \alpha_6) > 0$

+ve –ve –ve

(IV) If $x = \alpha_4$

$P(x) = (\alpha_4 – \alpha_1)(\alpha_4 – \alpha_3)(\alpha_4 – \alpha_5) < 0$

+ve +ve –ve

(V) If $x = \alpha_5$

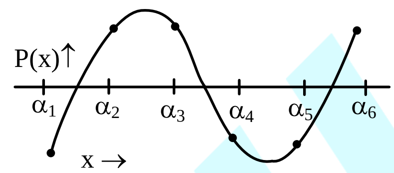
$P(x) = 3(\alpha_5 – \alpha_2)(\alpha_5 – \alpha_4)(\alpha_5 – \alpha_6) < 0$

+ve +ve –ve

(VI) If $x = \alpha_6$

$P(x) = (\alpha_6 – \alpha_1)(\alpha_6 – \alpha_3)(\alpha_6 – \alpha_5) > 0$

+ve +ve +ve



$\because$ Graph cuts x-axis three times, So, the given equation has three real roots.

(B) No real roots in $(–\infty, \alpha_1)$

(C) One real root in (α_1, α_2)

Q.21 If equations $(a + 2)x^2 + bx + c = 0$ & $2x^2 + 3x + 4 = 0$ have a common root where a, b, c $\in$ N, then-

(A) $b^2 – 4ac < 0$

(B) minimum value of a + b + c is 16

(C) $b^2 < 4ac + 8c$

(D) minimum value of a + b + c = 7

Sol. **[B,C]**

$\because$ Equation $2x^2 + 3x + 4 = 0$ has imaginary roots so both the roots are common

$\Rightarrow a + 2 = 2,\ b = 3,\ c = 4$

$\Rightarrow a = 0,\ b = 3,\ c = 4$

minimum value of a + b + c = 7

and $b^2 < 4c(a + 2)$

$\Rightarrow b^2 < 4ac + 8c$

$\Rightarrow$ option C, D are correct

Q.22 If one of the roots of $x^2 – bx + c = 0$; b, c $\in$ Q is $\sqrt{7-4\sqrt{3}}$ then-

(A) $\log_b c = 0$ (B) b + c = 5

(C) $\log_c b = 0$ (D) bc = –4

Sol. **[A,B]**

One root is $\sqrt{7-4\sqrt{3}} = \sqrt{4-4\sqrt{3}+3} = (2-\sqrt{3})$

other root is $2 + \sqrt{3}$

$\because b = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$c = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$

$\Rightarrow \log_b c = \log_4 1 = 0$, $b + c = 5$

$\Rightarrow$ option A, B are correct

Q.23 If α, β and γ are the roots of the equation $x^3 - 3x + 1 = 0$ then

(A) $\Pi(\alpha + 1) = -3$

(B) $\Sigma(\alpha + 1) = 0$

(C) $\Sigma(\alpha + 1)(\beta + 1) = -3$

(D) $\Sigma\alpha^2 = 6$

Sol. **[A,D]**

α, β, γ are the roots of $x^3 - 3x + 1 = 0$

then equation whose roots are

$\alpha + 1, \beta + 1, \gamma + 1$ is

$(x - 1)^3 - 3(x - 1) + 1 = 0$

$x^3 - 3x^2 + 3x - 1 - 3x + 3 + 1 = 0$

$\Rightarrow x^3 - 3x^2 + 3 = 0$

Clearly $\Pi(\alpha + 1) = -3$

& $\sum(\alpha + 1) = 3$

& $\sum(\alpha + 1)(\beta + 1) = 0$

and equation whose roots are $\alpha^2, \beta^2, \gamma^2$ is

$x^{3/2} - 3x^{1/2} + 1 = 0$

$\Rightarrow x^3 - 6x^2 + 9x - 1 = 0$

$\Rightarrow \sum\alpha^2 = 6$

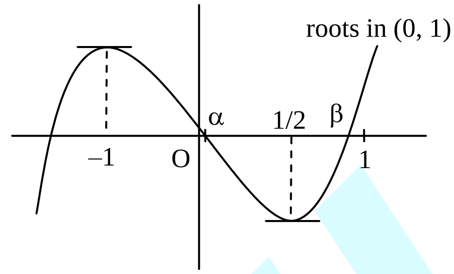
$\Rightarrow$ option A, D are correct

Q.24 The real values of λ for which the equation, $4x^3 + 3x^2 - 6x + \lambda = 0$ has two distinct real roots in [0, 1] lie in the interval-

(A) $(0, \infty)$ (B) $(3, \infty)$

(C) $(-5, 7/4)$ (D) $\left[0, \frac{7}{4}\right)$

Sol. [D]



f(x) has two distinct roots in [0, 1]

$f(x) = 4x^3 + 3x^2 - 6x + \lambda$

$f'(x) = 12x^2 + 6x - 6$

$= 6(2x^2 + x - 1)$

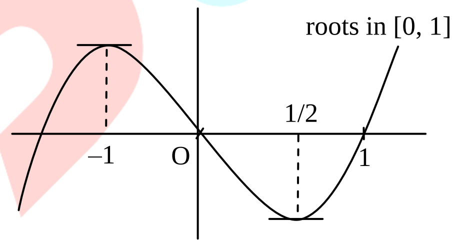
$= 6((2x - 1)(x + 1))$

$f'(x) > 0 \Rightarrow x \in (-\infty, -1) \cup \left(\frac{1}{2}, \infty\right)$

$f'(x) < 0 \Rightarrow x \in \left(-1, \frac{1}{2}\right)$

$f'(x) = 0 \Rightarrow x = \frac{1}{2}, -1$

or roots in [0, 1]



from both the graphs,

$f(0) \geq 0 \Rightarrow \lambda \geq 0$

$f(1) \geq 0 \Rightarrow \lambda \geq -1$

$f(-1) > 0 \Rightarrow -4 + 3 + 6 + \lambda > 0$

$\Rightarrow \lambda = -5$

$f(1/2) < 0 \Rightarrow \lambda < 7/4$

So, $\lambda \in [0, 7/4)$

Q.25 If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $a(2x + 1)^2 + b(2x + 1)(x - 1) + c(x - 1)^2 = 0$ are

(A) $\frac{2\alpha+1}{\alpha-1}, \frac{2\beta+1}{\beta-1}$ (B) $\frac{2\alpha-1}{\alpha+1}, \frac{2\beta-1}{\beta+1}$

(C) $\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$ (D) $\frac{2\alpha+3}{\alpha-1}, \frac{2\beta+3}{\beta-1}$

Sol. **[C]**

Given equation is

$$a\left(\frac{2x+1}{x-1}\right)^2 + b\left(\frac{2x+1}{x-1}\right) + c = 0$$

then $\frac{2x+1}{x-1} = \alpha$

$\Rightarrow x = \frac{\alpha+1}{\alpha-2}$

roots of the equation is

$\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$

Q.26 If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is-

(A) $x^2 + 10x - 11 = 0$ (B) $11x^2 + 10x + 1 = 0$

(C) $x^2 + 10x + 11 = 0$ (D) $11x^2 - 10x + 1 = 0$

Sol. **[B]**

Θ $x = \frac{1}{2\alpha - 3}$

$\Rightarrow \alpha = \frac{1}{2x} + \frac{3}{2}$

Replace x by $\left(\frac{1}{2x} + \frac{3}{2}\right)$ in $2x^2 + 4x - 5 = 0$

we have $2\left(\frac{1+3x}{2x}\right)^2 + 4\left(\frac{1+3x}{2x}\right) - 5 = 0$

Solving we get

$11x^2 + 10x + 1 = 0$

Q.27 If α, β are the roots of the equation $px^2 - qx + r = 0$, then the equation whose roots are $\alpha^2 + \frac{r}{p}$ and $\beta^2 + \frac{r}{p}$ is

(A) $p^3x^2 + pq^2x + r = 0$

(B) $px^2 - qx + r = 0$

(C) $p^3x^2 - pq^2x + q^2r = 0$

(D) $px^2 + qx - r = 0$

Sol. **[C]**

Θ $x = \alpha^2 + \frac{r}{p}$

$\Rightarrow \alpha = \sqrt{x - \frac{r}{p}}$

Replace x by $\sqrt{x - \frac{r}{p}}$ in $pr^2 - qx + r = 0$

we have

$p\left(x - \frac{r}{p}\right) + r = q\sqrt{x - \frac{r}{p}}$

Squaring and solving we get

$p^3x^2 - pq^2x + q^2r = 0$

Part-C Assertion-Reason type questions

The following questions 28 to 32 consists of two statements each, printed as Statement (1) & Statement (2). While answering these questions you are to choose any one of the following four responses.

(A) **If both Statement (1) and Statement (2) are true & the Statement (2) is correct explanation of the Statement (1).**

(B) **If both Statement (1) and Statement (2) are true but Statement (2) is not correct explanation of the Statement (1).**

(C) **If Statement (1) is true but the Statement (2) is false.**

(D) **If Statement (1) is false but Statement (2) is true**

Q.28 **Statement (1) :** If a and b are integers and roots of $x^2 + ax + b = 0$ are rational then they must be integers.

Statement (2): If the coefficient of x^2 in a quadratic equation is unity then its roots must be integers

Sol.[C] Reason is obviously false since the roots of a quadratic equation whose leading coefficient is unity may not even be real (e.g. $x^2 + 3x + 3 = 0$)

Assertion is true since if the roots are rational then roots = $\frac{-a \pm \sqrt{a^2 - ab}}{2}$.

If both a & b are even then the numerator must be even and therefore roots should be integers. If a is odd then also roots should be integers. If a is odd then also roots will be even.

Θ roots = $\frac{odd \pm odd}{2}$ = integer

If a is even, n is odd, then again roots are integers. If both a & b are odd then roots can not be rational which contradicts the given fact that roots are rational. Assertion is true.

Q.29 **Statement (1) :** The equation $(x - a)(x - c) + \lambda(x - b)(x - d) = 0$ where $a < b < c < d$ has non real roots if $\lambda > 0$.

Statement (2) : The equation $(a, b, c \in R)$ $ax^2 + bx + c = 0$ has non real roots if $b^2 - 4ac < 0$.

Sol.[D] Reason is obviously true. To test the assertion.

Let $f(x) = (x - a)(x - c) + \lambda(x - b)(x - d) = 0$

then $f(a) = \lambda(a - b)(a - d)$

$f(c) = \lambda(c - b)(c - d)$

If $\lambda > 0$, then $f(a) > 0$, $f(c) < 0$

$\Rightarrow$ There is a root between a & c

Thus assertion A is false.

Q.30 **Statement (1) :** If equation $ax^2 + bx + c = 0$; $(a, b, c \in R)$ and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c = 2 : 3 : 4$

Statement (2) : if $p + iq$ is one root of the quadratic equation with real coefficients then $p - iq$ will be the other root; $p, q \in R$, $i = \sqrt{-1}$

Sol.[A] Θ Equation $2x^2 + 3x + 4 = 0$ have imagining roots so both roots are common

$\Rightarrow a : b : c = 2 : 3 : 4$

$\Rightarrow$ Statement (1) and Statement (2) both true and (2) is correct explanation of (1)

Q.31 **Statement (1) :** If $a + b + c > 0$ & $a < 0 < b < c$, then the roots of the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$ are of both negative.

Statement (2) : If both roots are negative, then sum of roots < 0 and product of roots > 0.

Sol.[D] Equation

$a[x^2 - x(b + c) + bc] + b[x^2 - x(a + c) + ac] + c[x^2 - x(a + b) + ab] = 0$

$\Rightarrow x^2(a + b + c) - 2x(ab + bc + ca) + 3abc = 0$

Sum of roots = $\frac{2(ab + bc + ca)}{a + b + c}$

product of roots = $\frac{3abc}{a + b + c} < 0$

Θ $a + b + c > 0$ so we can not say that sum of roots are +ve or –ve so

$\Rightarrow$ Statement (1) is false and clearly Statement (2) is true

Q.32 **Statement (1) :** Let $(a_1, a_2, a_3, a_4, a_5)$ denote a re-arrangement of (1, –4, 6, 7, –10). Then the equation $a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$ has at least two real roots.

Statement (2) : If $ax^2 + bx + c = 0$ & $a + b + c = 0$, (i.e. in a polynomial the sum of coefficients is zero) then $x = 1$ is root of $ax^2 + bx + c = 0$.

Sol.[A] Θ $a_1 + a_2 + a_3 + a_4 + a_5 = 1 - 4 + 6 + 7 - 10 = 0$

therefore if sum of coefficients is zero than one root is 1. Here is four degree polynomial so it one root is real then any other root also real because imaginary roots always occur in pair.

$\Rightarrow$ Statement (1) and (2) both are true and (2) is correct explanation of (1)

Part-D Column Matching type questions

Q.33 Consider the equation $x^2 + 2(a - 1)x + a + 5 = 0$, where 'a' is a parameter. Match of the real values of 'a' so that the given equation has

Column I	**Column II**
(A) imaginary roots	(P) $\left(-\infty, -\frac{8}{7}\right)$
(B) one root smaller than 3 and other root greater	(Q) (–1, 4)

than 3

(C) exactly one root in the (R) $\left(-\frac{4}{3},-\frac{8}{7}\right)$

interval (1, 3) & 1 and 3 are not the root of the equation

(D) one root smaller than 1 (S) $\left(-\infty,-\frac{4}{3}\right)$

and other root greater than 3

Sol. **$A \to Q,\ B \to P,\ C \to R,\ D \to S$**

(A) $D < 0 \Rightarrow 4(a-1)^2 - 4(a+5) < 0$

$\Rightarrow a^2 - 2a + 1 - a - 5 < 0$

$\Rightarrow a^2 - 3a - 4 < 0$

$(a-4)(a+1) < 0$

$\Rightarrow a \in (-1, 4)$

(B) $\because f(3) < 0$

$\Rightarrow 9 + 6(a-1) + a + 5 < 0$

$\Rightarrow 7a + 8 < 0$

$\Rightarrow a < -\frac{8}{7} \Rightarrow a \in \left(-\infty,-\frac{8}{7}\right)$

(C) $\because f(3) < 0$ and $f(1) > 0$

$1 + 2a - 2 + a + 5 > 0$

$\Rightarrow 3a + 4 > 0 \Rightarrow a > -\frac{4}{3}$

$\Rightarrow a \in \left(-\frac{4}{3},\infty\right) \cap \left(-\infty,-\frac{8}{7},\right)$

$\Rightarrow a \in \left(-\frac{4}{3},-\frac{8}{7}\right)$

(D) $f(3) < 0$ and $f(1) < 0$

$\Rightarrow a \in \left(-\infty,-\frac{4}{3}\right) \cap \left(-\infty,-\frac{8}{7}\right)$

$\Rightarrow a \in \left(-\infty,-\frac{4}{3}\right)$

Q.34 **Column I** **Column II**

(A) $Q_1(x) = x^2 - mx + 1$ is negative for values of x in (1, 2), if m lies in the interval (P) (–3/2, 1/2)

(B) $Q_2(x) = x^2 + 2(m-1)x + m + 5$ is positive for all x if m lies in the interval (Q) (5/2, ∞)

(C) If $\frac{2x-1}{2x^3+3x^2+x}$ is positive, then x lies in the interval (R) (1/2, 5/2)

(D) The interval of x for which $x^{12} - x^9 + x^4 - x + 1 > 0$ (S) (–∞, –3/2)

Sol **$A \to Q$, ; $B \to R$; $C \to Q, R, S$; $D \to P, Q, R, S$**

(A) $f(1) < 0,\ f(2) < 0$

$\Rightarrow m > 2,\ m > \frac{5}{2}$

and $D > 0 \Rightarrow m^2 > 4 \Rightarrow |m| > 2$

$\Rightarrow m \in \left(\frac{5}{2}, \infty\right)$

(B) $D < 0$

$\Rightarrow 4(m-1)^2 - 4(m+5) < 0$

$\Rightarrow m^2 - 2m + 1 - m - 5 < 0$

$\Rightarrow m^2 - 3m - 4 < 0$

$\Rightarrow (m-4)(m+1) < 0$

$\Rightarrow m \in (-1, 4)$

(C) $\frac{2x-1}{x(x-1)(2x+1)} > 0$

$\Rightarrow x \in (-\infty, -1) \cup \left(\frac{-1}{2},0\right) \cup \left(\frac{1}{2},\infty\right)$

(D) $x^{12} - x^9 + x^4 - x + 1 > 0$

$x^9(x^3-1) + x(x^3-1) + 1 > 0$

$\Rightarrow x(x^8+1)(x^3-1) + 1 > 0$

this is true for $x \in (-\infty, \infty)$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 If one roots of the equation $ax^2 + bx + c = 0$ be the square of the other show that
$b^3 + a^2c + ac^2 = 3abc$

Sol. $ax^2 + bx + c = 0$ (Let α, α^2 are roots)

$\alpha + \alpha^2 = \frac{-b}{a}$ ...(1)

$\alpha^3 = \frac{c}{a}$ ...(2)

$(1)^3 \Rightarrow (\alpha + \alpha^2) = \frac{-b^3}{a^3}$

$\Rightarrow \alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2) = \frac{-b^3}{a^3}$

$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} + \frac{3c}{a}\left(\frac{-b}{a}\right) = \frac{-b^3}{a^3}$

$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} - \frac{3bc}{a^2} = \frac{-b^3}{a^3}$

$\Rightarrow a(ac + c^2 - 3bc) = -b^3$

$a^2c + ac^2 - 3abc = -b^3 = 0$

$\Rightarrow b^3 + a^2c + ac^2 = 3abc$ Ans.

Q.2 If the difference of the roots of the equation $x^2 + ax + b = 0$ is equal to the difference of the roots of the equation $x^2 + bx + a = 0$, if $a \neq b$ then prove that $a + b + 4 = 0$.

Sol. $x^2 + ax + b = 0$ (let α, β), $x^2 + bx + a = 0$ (γ, δ)

$\alpha - \beta = \gamma - \delta$

$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(\gamma+\delta)^2 - 4\gamma\delta}$ ...(1)

we have $\alpha + \beta = a$, $\alpha\beta = b$ and

$\gamma + \delta = -b$, $\gamma\delta = a$

then from eqn. (1)

$\sqrt{a^2 - 4b} = \sqrt{b^2 - 4a}$

$a^2 - 4b = b^2 - 4a$

$a^2 - b^2 = 4(b - a)$

$(a + b)(a - b) = 4(b - a) = -4(a - b)$

$\Rightarrow a + b = -4$

$\Rightarrow a + b + 4 = 0$ Ans.

Q.3 A and B solve an equation $x^2 + px + q = 0$. In solving A commits a mistake in reading p and finds the roots 2 and 6 and B commits a mistake in reading q and finds the roots 2 and – 9. Find the correct roots.

Sol. $x^2 + px + q = 0$

product = q = 12

sum = – p = – 7

p = 7

$\therefore x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow (x + 4)(x + 3) = 0$

$\Rightarrow x = -4, -3$ Ans.

Q.4 Given $x, y \in R$; $x^2 + y^2 > 0$. If the maximum and minimum value of expression $E = \frac{x^2 + y^2}{x^2 + xy + 4y^2}$ are M and m and A denotes the average of M and m. Compute (2007) A.

Sol. 1.338

Q.5 If x be real and $0 < b < c$ show that $\frac{x^2 - bc}{2x - b - c}$ can not lie between b and c.

Sol. Let $y = \frac{x^2 - bc}{2x - b - c}$

$\Rightarrow x^2 - bc = 2xy - (b + c)y$

$\Rightarrow x^2 - 2yx + (b + c)y - bc = 0$

$\because$ x is real , $\therefore D \geq 0$

$(-2y)^2 - 4.1[(b + c)y - bc] \geq 0$

$\Rightarrow y^2 - (b + c)y + bc \geq 0$

$\Rightarrow (y - b)(y - c) \geq 0$

$\Rightarrow$ y has no real value between b and c. Ans.

Q.6 Find all the values of the parameters a, for which $\frac{ax^2 + 3x - 4}{a + 3x - 4x^2}$ takes all real values for real values of x.

Sol. $y = \frac{ax^2 + 3x - 4}{a + 3x - 4x^2}$

$\Rightarrow ax^2 + 3x - 4 + 4yx^2 - 3xy - ay = 0$

$\Rightarrow x^2(a + 4y) + 3x(1 - y) - (4 + ay) = 0$

$\because$ x is real, $\therefore D \geq 0$

$9(1 - y)^2 + 4(a + 4y)(4 + ay) \geq 0$

$\Rightarrow 9 + 9y^2 - 18y + 16a + 4a^2y + 64y + 16ay^2 \geq 0$

$\Rightarrow y^2(9 + 16a) + y(4a^2 + 64 - 18) + 16a + 9 \geq 0$

$\Rightarrow B^2 - 4AC < 0$ & the sign is same as of $(9 + 16a)$ which is to be +ve.

$\Rightarrow y^2(9 + 16a) + 2y(2a^2 + 23) + (16a + 9) \geq 0$

$\therefore 4(2a^2 + 23)^2 - 4(16a + 9) < 0$

and $(9 + 16a) > 0$

$\Rightarrow 16(a + 4)^2(a^2 - 8a + 7) < 0$

and $16a + 9 > 0$

$\Rightarrow (a + 4)^2 (a - 1)(a - 7) < 0$

and $16a + 9 > 0$

$\Rightarrow (a - 1)(a - 7) < 0$

$1 < a < 7$

$a \in (1, 7)$ Ans.

Q.7 Find the values of 'm' for which $(m - 2)x^2 + 8x + m + 4 > 0$ for all real x.

Sol. $(m - 2)x^2 + 8x + (m + 4) > 0$ for all $x \in R$

$m - 2 > 0,\ D \le 0$

$m > 2,\ 64 - 4(m - 2)(m + 4) \le 0$

$\Rightarrow 16 - (m^2 + 2m - 8) \le 0$

$\Rightarrow 16 - m^2 - 2m + 8 \le 0$

$\Rightarrow m^2 + 2m - 24 \ge 0$

$\Rightarrow m^2 + 6m - 4m - 24 \ge 0$

$\Rightarrow m(m + 6) - 4(m + 6) \ge 0$

$\Rightarrow (m + 6)(m - 4) \ge 0$

$\Rightarrow m \ge 4$ or $m \le -6$

$\Rightarrow m > 4 \qquad \Rightarrow (4, \infty)$ Ans.

Q.8 $x^2 - (m - 3)x + m = 0$ ($m \in R$) be a quadratic equation. Find the value of 'm' for which

(a) both roots are real & distinct

(b) both roots are equal

(c) one root is smaller than 2. The other root is greater than 2

(d) both roots are greater than 2

(e) both roots are smaller than 2

(f) exactly one root lie in the interval (1, 2)

(g) both roots lie in the interval (1, 2)

(h) at least one root lie in the interval (1, 2)

(i) one root is greater than 2, the other root is smaller than 1

(j) at least one root is greater than 2.

Sol. $x^2 - (m - 3)x + m = 0$ ($m \in R$)

(a) both roots are real & distinct

$\Rightarrow D > 0$

$\Rightarrow (m - 3)^2 - 4m > 0$

$\Rightarrow m^2 - 6m - 4m + 9 > 0$

$\Rightarrow m^2 - 9m - m + 9 > 0$

$\Rightarrow m(m - 9) - 1(m - 9) > 0$

$\Rightarrow (m - 1)(m - 9) > 0$

$$-\infty \quad \overset{+}{} \quad 1 \quad \overset{-}{} \quad 9 \quad \overset{+}{} \quad +\infty$$

$\Rightarrow m \in (-\infty, 1) \cup (9, \infty)$ Ans.

(b) both roots are equal

$\Rightarrow D = 0$

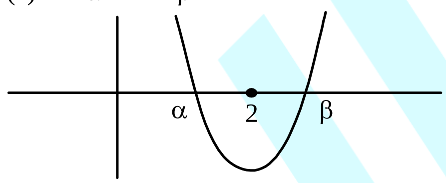
$\Rightarrow (m - 3)^2 - 4m = 0$

$\Rightarrow m^2 - 10m + 9 = 0$

$\Rightarrow m = 1$ & 9

$\Rightarrow m \in \{1, 9\}$ Ans.

(c) Let $\alpha < 2$ & $\beta > 2$



$\Rightarrow D > 0$ & $f(2) < 0$

from $D > 0$

$m \in (-\infty, -1) \cup (9, \infty)$ ...(1)

$f(2) = 4 - 2m + 6 + m < 0$

$= 10 - m < 0$

$\Rightarrow m > 10$

$\Rightarrow m \in (10, \infty)$ ...(2)

from (1) & (2) ; $m \in (10, \infty)$ Ans.

(d) both roots are greater than 2

(i) $D \ge 0$ (ii) $f(2) > 0$

(iii) sum of roots > 4

(i) $D \ge 0 \Rightarrow m \in (-\infty, -1) \cup (9, \infty)$...(1)

(ii) $f(2) > 0 \Rightarrow 4 - 2m + m + 6 > 0$

$\Rightarrow 10 - m > 0$

$\Rightarrow m < 10$

$\Rightarrow m \in (-\infty, 10)$ ...(2)

(iii) $m - 3 > 4$

$\Rightarrow m > 7$

$\Rightarrow m \in (7, \infty)$ ...(3)

$\therefore m \in [9, 10)$ Ans.

(e) Both roots are smaller than 2

(i) $D > 0$ & $f(2) > 0$ and sum of roots < 4

$\therefore m - 3 < 4$

$m < 7$

(i) one root is greater than 2, the other root is smaller than 1

(j) at least one root is greater than 2.

Q.9 Find all 'm' for which $f(x) = x^2 - (m - 3)x + m > 0$ for all values of 'x' in [1, 2]

Sol. $f(x) = x^2 - (m - 3)x + m > 0$

$D < 0$

$(m - 3)^2 - 4m < 0$

$\Rightarrow m^2 - 6m - 4m + 9 < 0 \Rightarrow m^2 - 9m - m + 9 < 0$

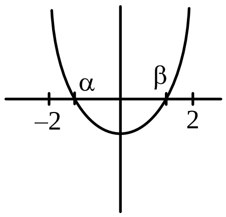
$\Rightarrow m(m - 9) - 1(m - 9) < 0$

$\Rightarrow (m - 1)(m - a) < 0 \Rightarrow m \in (1, 9)$

Q.10 If the quadratic equation $ax^2 + bx + c = 0$ has real roots, of opposite signs in the interval $(-2, 2)$ then prove that $1 + \frac{c}{4a} - \left|\frac{b}{2a}\right| > 0$.

Sol. $ax^2 + bx + c = 0$

Case: I



$a > 0,\ f(2) > 0$

$\Rightarrow 4a + 2b + c > 0$

Divide both sides by 4a

$\Rightarrow 1 + \frac{2b}{4a} + \frac{c}{4a} > 0$ ....(i)

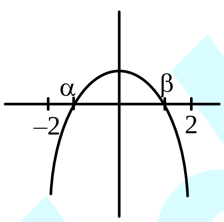
Similarly $f(-2) > 0$

$1 - \frac{b}{2a} + \frac{c}{4a} > 0$ ....(ii)

So, combining (i) & (ii)

$1 + \frac{c}{4a} - \left|\frac{b}{2a}\right| > 0$

Case : II



$a < 0$

$f(2) = 4a + 2b + c < 0$

On dividing by 4a

$\Rightarrow 1 + \frac{b}{2a} + \frac{c}{4a} > 0$ ....(i)

Similarly, $f(-2) = 1 - \frac{b}{2a} + \frac{c}{4a} > 0$ ...(ii)

So, on combining (i) & (ii), we get

$1 + \frac{c}{4a} - \left|\frac{b}{2a}\right| > 0$

Q.11 If α, β are the roots of the equation $x^2 - 2x + 3 = 0$ obtain the equation whose roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2,\ \beta^3 - \beta^2 + \beta + 5$.

Sol. α, β are the roots of $x^2 - 2x + 3 = 0$

Given roots are

$\alpha^3 - 3\alpha^2 + 5\alpha - 2,\ \beta^3 - \beta^2 + \beta + 5$

$\Rightarrow \alpha(\alpha^2 - 2\alpha + 3 - \alpha + 2) - 2,$

$\beta(\beta^2 - 2\beta + 3 + \beta - 2) + 5$

$\Rightarrow -\alpha^2 + 2\alpha - 2,\ \beta^2 - 2\beta + 5$

$\Rightarrow -(\alpha^2 - 2\alpha + 3 - 1),\ \beta^2 - 2\beta + 3 + 2$

$\Rightarrow 1, 2$

Equation is $x^2 - 3x + 2 = 0$

Q.12 Find the range of values of a, such that $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

Sol. $\because x^2 - 8x + 32 > 0\ \forall\ x$

$\Rightarrow ax^2 + 2x(a + 1) + 9a + 4 < 0$

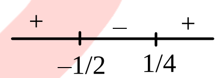
$\Rightarrow a < 0$ and $D < 0$

$\Rightarrow 4(a + 1)^2 - 4a(9a + 4) < 0$

$\Rightarrow a^2 + 2a + 1 - 9a^2 - 4a < 0$

$\Rightarrow 8a^2 + 2a - 1 > 0$

$\Rightarrow (4a - 1)(2a + 1) > 0$



$\Rightarrow a \in \left(-\infty, -\frac{1}{2}\right)$ $\because a < 0$

Q.13 The equation $x^2 - ax + b = 0$ & $x^3 - px^2 + qx = 0$, where $b \neq 0$, $q \neq 0$, have one common root and the second equation has two equal roots. Prove that $2(q + b) = ap$.

Sol. $x^2 - ax + b = 0$

$x^3 - px^2 + qx = 0$

$\Rightarrow x(x^2 - px + q) = 0$

$\therefore x = 0$

And other, two roots are equal

So, $\frac{-B}{2A} = \frac{p}{2}$

$\therefore \alpha = \frac{p}{2}$ .....(i)

$\alpha \cdot \alpha = q$

$\alpha^2 = q$

$\Rightarrow \frac{p^2}{4} = q$ ...(2)

Using (1)

Now $\because x^2 = ax + b = 0$

$\Rightarrow \alpha^2 - a\alpha + b = 0$ ($\because \alpha$ is common root)

$\Rightarrow \frac{p^2}{4} - \frac{p}{2}a + b = 0$

$\Rightarrow \frac{p^2}{4} + b = \frac{pa}{2}$

$\Rightarrow 2(q + b) = ap$ (using (2)).

Q.14 Find the values of K so that the quadratic equation $x^2 + 2(K - 1)x + K + 5 = 0$ has atleast one positive root.

Sol. $\because D \ge 0$

$\Rightarrow 4(k-1)^2 - 4(k+5) \ge 0$

$\Rightarrow k^2 - 2k + 1 - k - 5 \ge 0$

$\Rightarrow k - 3k - 4 \ge 0$

$\Rightarrow (k-4)(k+1) \ge 0$

$\Rightarrow k \in (-\infty, -1] \cup [4, \infty)$

and $-\frac{b}{2a} > 0$

$\Rightarrow \frac{-2(k-1)}{2} > 0$

$\Rightarrow k - 1 < 0 \Rightarrow k < 1$

therefore $k \le -1$

Q.15 Solve the following equations where $x \in R$.

(a) $(x - 1)|x^2 - 4x + 3| + 2x^2 + 3x - 5 = 0$

(b) $|x^2 + 4x + 3| + 2x + 5 = 0$

(c) $|x + 3|(x + 1) + |2x + 5| = 0$

Sol. (a) $(x - 1)|x^2 - 4x + 3| + 2x^2 + 3x - 5 = 0$

Case 1 : $x \ge 3$

$(x-1)(x-1)(x-3) + (2x+5)(x-1) = 0$

$x^2 - 4x + 3 + 2x + 5 = 0$

$x^2 - 2x + 8 = 0$

no solution

Case 2 : $1 \le x < 3$

$(x-1)^2(3-x) + (2x+5)(x-1) = 0$

$-3 - x^2 + 4x + 2x + 5 = 0$

$\Rightarrow x^2 - 6x - 2 = 0$

no solution $x = 1$

Case 3 : $x < 1$ no solution

(b) $|x^2 + 4x + 3| + 2x + 5 = 0$

Case 1 : $x \ge -1$ or $x \le -3$

$|x^2 + 4x + 3| + 2x + 5 = 0$

$x^2 + 4x + 3 + 2x + 5 = 0$

$x^2 + 6x + 8 = 0$

$x = -2, -4$

$x \ne -2, x = -4$

Case 2 : $-3 \le x \le -1$

$x^2 + 2x - 2 = 0$

$x = -(1 + \sqrt{3})$

So, $x = \{-4, -(1 + \sqrt{3})$

(c) $|x + 3|(x + 1) + |2x + 5| = 0$

Case 1 : $x \ge -\frac{5}{2}$

$x^2 + 6x + 8 = 0$

$\Rightarrow x = -4, -2$

$\Rightarrow x = -2$

Case 2 : $x < -3 \Rightarrow$ solving, $x = -4$

Case 3 : $-3 \le x < -\frac{5}{2}$

$x^2 + 2x - 2 = 0$

$x = -(1 + \sqrt{3})$

$x = \{-4, -(1 + \sqrt{3}), -2\}$

Q.16 If α, β, γ are the roots of the equation $9x^3 - 7x + 6 = 0$, then find the equation whose roots are $3\alpha + 2, 3\beta + 2, 3\gamma + 2$.

Sol. $\because x = 3\alpha + 2 \Rightarrow \alpha = \frac{x-2}{3}$

Replace x by $\frac{x-2}{3}$ is given equation we get

$$9\left(\frac{x-2}{3}\right)^3 - 7\left(\frac{x-2}{3}\right) + 6 = 0$$

$(x-2)^3 - 7(x-2) + 18 = 0$

Solving we get

$x^3 - 6x^2 + 5x + 24 = 0$

Q.17 If one root of the equation $x^3 + 2x^2 + px + q = 0$ is $i\alpha$ then prove that equation which has one root 2 is $x^3 - 2x^2 + px - q = 0$.

Sol. Let $x^3 + 2x^2 + px + q = 0$ have roots $i\alpha, -i\alpha, \beta$

other root will be $-i\alpha$.

$i\alpha - i\alpha + \beta = -2$

$\Rightarrow \beta = -2$ is the third root

So, $-8 + 8 - 2p + q = 0 \Rightarrow q - 2p = 0$

$x^3 - 2x^2 + px - q = 0$

put x = 2

8 – 8 + 2p –q

0 = 0

Hence β = 2 is a root

Q.18 If $2^{2x+3} + 3^{2x+1} = 10.6^x$ then find x.

Sol. $8.4^x + 3.9^x = 10.6^x$

$\Rightarrow 8.\left(\frac{2}{3}\right)^x + 3.\left(\frac{3}{2}\right)^x = 10$

Let $\left(\frac{2}{3}\right)^x = t$

$\Rightarrow 8t^2 - 10t + 3 = 0$

$\Rightarrow 8t^2 - 6t - 4t + 3 = 0$

$\Rightarrow (2t - 1)(4t - 3) = 0$

$\Rightarrow t = \frac{1}{2}, \frac{3}{4}$

$\Rightarrow \left(\frac{2}{3}\right)^x = \frac{1}{2}$ and $\left(\frac{2}{3}\right)^x = \frac{3}{4}$

$\Rightarrow x = \log_{2/3}\frac{1}{2}$ and $x = \log_{2/3}\frac{3}{4}$

Q.19 Find the set of real values of 'm' for which the equation $\left(\frac{x}{1+x^2}\right)^2 - (m-3)\left(\frac{x}{1+x^2}\right) + m = 0$ has real roots.

Sol. $\left[-\frac{7}{2}, \frac{5}{6}\right]$

Q.20 If $a^2 + c^2 > ab$ and $b^2 > 4c^2$, for real x, show that $\frac{x+a}{x^2+bx+c^2}$ cannot lie between two limits.

Sol. $a^2 + c^2 > ab$ and $b^2 > 4c^2$

Let $y = \frac{x+a}{x^2+bx+c^2}$

$\Rightarrow x^2y + by\,x + c^2y - x - a = 0$

$\Rightarrow x^2y + x(by - 1) + c^2y - a = 0$

$\because$ x is real, $\therefore D \geq 0$

$\Rightarrow (by - 1)^2 - 4y(c^2y - a) \geq 0$

$\Rightarrow b^2y^2 + 1 - 2by - 4c^2y^2 + 4ay \geq 0$

$\Rightarrow (b^2 - 4c^2)y^2 + y(4a - 2b) + 1 \geq 0$

$$\Rightarrow y \geq \frac{-(4a-2b) \pm \sqrt{(4a-2b)^2 - 4(b^2-4c^2)}}{2(b^2-4c^2)}$$

$$\Rightarrow y \geq \frac{-(4a-2b) \pm \sqrt{16a^2 + 4b^2 - 16ab - 4b^2 + 16c^2}}{2(b^2-4c^2)}$$

$$\Rightarrow y \geq \frac{(2b-4a) \pm 4\sqrt{a^2+c^2-ab}}{2(b^2-4c^2)}$$

$$\Rightarrow y \geq \frac{(b-2a) \pm 2\sqrt{a^2+c^2-ab}}{(b^2-4c^2)}$$

$\therefore$ y cannot lie between two limits.

Q.21 If the three equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, find a & b and also the roots of the equation.

Sol. Let α be the common root then

$\alpha^2 + a\alpha + 12 = 0$ ……..(1)

$\alpha^2 + b\alpha + 15 = 0$ ……..(2)

$\alpha^2 + (a + b)\alpha + 36 = 0$ ……..(3)

(1) + (2) – (3) we get

$\alpha^2 - 9 = 0$

$\Rightarrow \alpha = \pm 3 \Rightarrow \alpha = 3$

then roots of equation are respectively

(3, 4), (3, 5), (3, 12)

$\Rightarrow a = -7,\ b = -8$

Q.22 Two roots of a bi-quadratic $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ have their product equal to (–32). Find the value of k.

Sol. $\because \alpha\beta = -32$

$\alpha\beta\gamma\delta = -1984 \Rightarrow \gamma\delta = 62$

$\alpha + \beta + \gamma + \delta = 18$

$\alpha\beta + \beta\gamma + \gamma\delta + \alpha\gamma + \alpha\delta + \beta\delta = k$

$\alpha\beta\gamma + \beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\delta = -200$

Solving we get k = 86

Q.23 Find the true set of values of p for which the equation $p.2^{\cos^2 x} + p.2^{-\cos^2 x} - 2 = 0$ has real roots.

Sol. $\left[\frac{4}{5}, 1\right]$

Part-B Passage based objective questions

PASSAGE- 1 (Q. 24 to Q.26)

Let $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$ be a quadratic polynomial in x, a be any real number.

On the basis of above information, answer the following questions:

Q.24 If x- coordinate of vertex of parabola y= f(x) is less than 0 and f(x) has minimum value 3 for $x \in [0, 2]$, then value of a is

(A) $1 + \sqrt{2}$ (B) $1 - \sqrt{2}$

(C) $1 - \sqrt{3}$ (D) $1 + \sqrt{3}$

Sol. **[B]**

$\because \frac{4a}{8} < 0 \Rightarrow a < 0$

Minimum value takes at zero

So $a^2 - 2a + 2 = 3 \Rightarrow a = 1 - \sqrt{2}$ $[\because a < 0]$

Q.25 If y = f(x) takes minimum value 3 on [0, 2] and x- coordinate of vertex is greater than 2, then value of a is

(A) $5 - \sqrt{10}$ (B) $10 - \sqrt{5}$

(C) $5 + \sqrt{10}$ (D) $10 + \sqrt{5}$

Sol. **[C]**

$\because \frac{4a}{8} > 2 \Rightarrow a > 4$

Minimum value takes at 2 therefore

$16 - 8a + a^2 - 2a + 2 = 3$

$\Rightarrow a^2 - 10a + 15 = 0 \Rightarrow a = 5 + \sqrt{10}$ $[\because a > 4]$

Q.26 If at least one root of f(x) = 0 lies in [0, 2], then the value of a belongs to

(A) $[1, 5 - \sqrt{7}) \cup [5 - \sqrt{7}, 5 + \sqrt{7})$

(B) $[1, 5 + \sqrt{7}]$

(C) $(\sqrt{7} - 5, \sqrt{7} + 5) \cup (5 + \sqrt{7}, \infty)$

(D) $(\sqrt{7} - 5, \infty)$

Sol. **[B]**

$\because D \geq 0$

$\Rightarrow 16a^2 - 16(a^2 - 2a + 2) \geq 0$

$\Rightarrow 2a - 2 \geq 0 \Rightarrow a \geq 1 \because f(0) . f(2) \leq 0$

$\Rightarrow (a^2 - 2a + 2)(a^2 - 10a + 18) \leq 0$

$\because a^2 - 2a + 2$ always +ve

$\Rightarrow a^2 - 10a + 18 \leq 0 \Rightarrow a \in [5 - \sqrt{7}, 5 + \sqrt{7}]$

$\Rightarrow a \in [1, 5 + \sqrt{7}]$

PASSAGE- 2 (Q. 27 to Q.29)

Let $f_1(x) = a_1x^2 + b_1x + c_1$, $f_2(x) = a_2x^2 + b_2x + c_2$ be quadratic functions with real coefficients. Sum of roots of $f_1(x) = 0$ is equal to sum of roots of $f_2(x) = 0$. Range of $y = f_1(x)$ can be $[2, \infty)$ or $[-2, \infty)$. Range of $y = f_2(x)$ can be $(-\infty, -2]$ or $(-\infty, 2]$

On the basis of above information, answer the following questions:

Q.27 If arithmetic mean of roots of $f_1(x) f_2(x) = 0$ is equal to 1, then

(A) $b_1 + 2a_1 = 0$, $b_2 + 2a_2 \neq 0$

(B) $b_1 + 2a_1 \neq 0$, $b_2 + 2a_2 = 0$

(C) $b_1 + 2a_1 = 0$, $b_2 + 2a_2 = 0$

(D) $a_1b_2 + a_2b_1 = 4a_1a_2$

Sol. **[C]**

$f_1(x) = 0$

α & β are the roots of this equation

$f_2(x) = 0$

γ & δ are the roots of this equation

And, if A.M. of $f_1(x) . f_2(x) = 0$ is 1, then

$$\frac{\alpha + \beta + \gamma + \delta}{4} = 1$$

$$\Rightarrow \frac{\frac{-b_1}{a_1} + \frac{-b_2}{a_2}}{4} = 1 \quad \text{....(i)}$$

$$\Rightarrow \frac{-b_1a_2 - b_2a_1}{4a_1a_2} = 1$$

$\Rightarrow 4a_1a_2 = -b_1a_2 - b_2a_1$

Given :- $\frac{b_1}{a_1} = \frac{b_2}{a_2}$

$\therefore$ In equation (i)

$\frac{-2b_1}{4a_1} = 1 \Rightarrow -2b_1 = 4a_1$

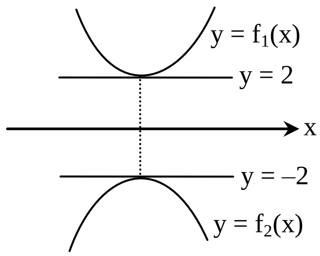
$\Rightarrow 4a_1 + 2b_1 = 0$

$\Rightarrow 2(2a_1 + b_1) = 0$

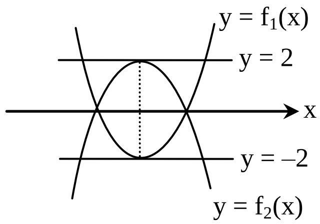
Similarly, $2(2a_2 + b_2) = 0$

Q.28 Which of the following can be possible graphs of $y = f_1(x)$ and $y = f_2(x)$

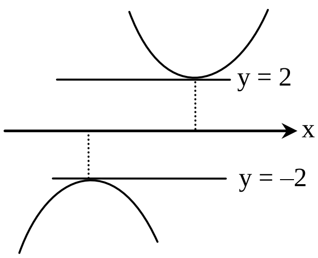
(i)



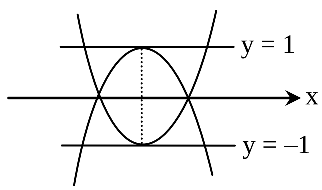
(ii)



(iii)



(iv)



(A) (i), (ii) (B) (i), (ii), (iii)

(C) (i), (ii), (iv) (D) (i), (ii), (iii), (iv)

Sol. **[A]**

$\frac{b_1}{a_1} = \frac{b_2}{a_2}$ (Given)

$\Rightarrow \frac{b_1}{2a_1} = \frac{b_2}{2a_2}$

Θ x-components of vertex are same, so only (i) and (ii) graph is possible

Q.29 If $y = f_2(x)$ passes through (1, –2) and $f_1(x) = 0$ has a negative root then

(A) $a_2c_2 < 0$ (B) $a_1c_1 < 0$

(C) $b_1c_1 < 0$ (D) $b_2c_2 > 0$

Sol. **[B]**

$f_1(x) = a_1x^2 + b_1x + c_1$

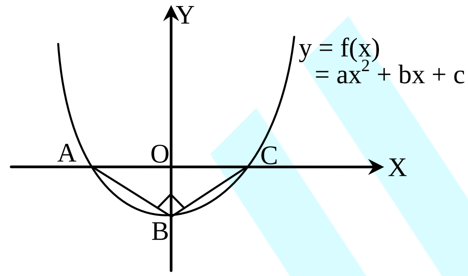
This equation has a negative root then,

$\alpha . \beta < 0$

$\Rightarrow \frac{c_1}{a_1} < 0 \Rightarrow c_1a_1 < 0$

PASSAGE- 3 (Q. 30 to Q.32)

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuous AC = $4\sqrt{2}$ units, then



Q.30 y = f(x) is given by

(A) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$ (B) $y = \frac{x^2}{2} - 2$

(C) $y = x^2 - 8$ (D) $y = x^2 - 2\sqrt{2}$

Sol. **[C]**

Here, AC = $4\sqrt{2}$

And Θ ΔABC is an isosceles right angle Δ

So, AO = OC (By congruent property)

$\therefore$ OA = OC = $2\sqrt{2}$

So roots are $\alpha = -2\sqrt{2}$

$\beta = 2\sqrt{2}$

Hence, $y = f(x) = x^2 - (-2\sqrt{2} + 2\sqrt{2})x$

$+ (-2\sqrt{2} . 2\sqrt{2})$

$= x^2 - 8$

$\Rightarrow y = x^2 - 8$

Q.31 Minimum value of y = f(x) is

(A) $2\sqrt{2}$ (B) $-2\sqrt{2}$

(C) 2 (D) –2

Sol. **[B]**

B = Minimum value of f(x)

So, in ΔABC

AB = BC

∠BAC = ∠BCA = 45°

(angles opp. equal, sides are equal)

⇒ ΔAOB

$\tan 45° = \frac{OB}{AO}$

OB = AO = $-2\sqrt{2}$

Q.32 Number of integral value of k for which $\frac{k}{2}$ lies between the roots of f(x) = 0, is

(A) 9 (B) 10 (C) 11 (D) 12

Sol. **[C]**

$\alpha = -\ 2\sqrt{2}$, $\beta = 2\sqrt{2}$ (As solved above)

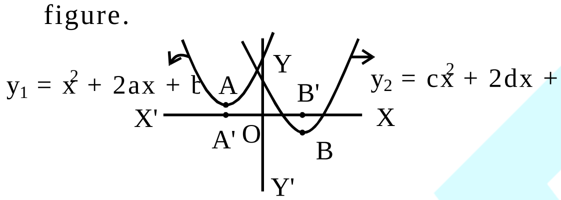
If k/2 lies b/w $-2\sqrt{2}$ & $2\sqrt{2}$

then k lies b/w $-\ 4\sqrt{2}$ and $4\sqrt{2}$

So, number of integral value of k = 11

PASSAGE- 4 (Q. 33 to Q.35)

We are define here two quadratic expression $y_1 = x^2 + 2ax + b$ & $y_2 = cx^2 + 2dx + 1$ where a, b, c, d are real numbers. The graph of y_1 and y_2 are shown in the figure.



Here also given AA′ = BB′ and OA′ = OB′

On the basis of above information, answer the following:

Q.33 Which statement is correct?

(A) $a^2 - d^2 = c - d$ (B) $a - b = c - d$

(C) $a^2 + d^2 = c + b$ (D) None of these

Sol. **[D]**

For y_1, $\Delta_1 = 4a^2 - 4b$

For y_2, $\Delta_2 = 4d^2 - 4c$

And, $\frac{-\Delta_1}{4} = \frac{-\Delta_2}{4c}$ (Given AA' = BB')

$\Rightarrow \frac{4a^2 - 4b}{4} = -\left(\frac{4d^2 - 4c}{4c}\right)$

$\Rightarrow c\ (a^2 - b) = -\ (d^2 - c) = c - d^2$

$\Rightarrow c\ (a^2 - b) = c - d^2$

So, no option is possible

Q.34 The sum of all the roots of the equation $y_1 = 0$ and $y_2 = 0$ is -

(A) 0 (B) $-\ 2a + 2d$

(C) $-\ 2a - \frac{2}{c}\ d$ (D) None of these

Sol. **[C]**

$y_1 = x^2 + 2ax + b$

Let α & β be its two roots

$y_2 = cx^2 + 2dx + 1$

Let γ & δ be its two roots

then, $(\alpha + \beta) + (\gamma + \delta) = -\ 2a + \frac{-2d}{c}$

$= -\ 2a - \frac{2}{c}\ d$

Q.35 Which statement is correct?

(A) ac = d (B) ad = c

(C) ac = – d (D) None of these

Sol. **[C]**

OA' = OB' (Given)

$\Rightarrow \frac{-2a}{2} = \frac{-2d}{2c}$

$\Rightarrow a = \frac{-d}{c}$

$\Rightarrow ac = -\ d$

EXERCISE # 4

Old IIT-JEE questions

Q.1 For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the roots is square of the other, then p is equal to- **[IIT Sc.-2000]**

(A) 1/3 (B) 1 (C) 3 (D) 2/3

Sol. **[C]**

Let roots are α, α^2

$\alpha + \alpha^2 = \frac{-p}{3}$ …(1)

$\Rightarrow \alpha^3 = 3/3 = 1$

$\Rightarrow \alpha = 1, \omega, \omega^2$

put $\alpha = \omega$ in (1), we get

$\omega + \omega^2 = \frac{-p}{3}$

$-1 = \frac{-p}{3}$

p = 3 Ans.

Q.2 If α and β ($\alpha < \beta$), are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then- **[IIT Sc.-2000]**

(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$

(C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

Sol. **[B]**

$\alpha + \beta = -b$ (negative)

$\alpha\beta = c$ (negative)

$\because$ $c < 0 \Rightarrow \alpha, \beta$ are in opposite sign, but $\alpha < \beta$

$\therefore$ $\alpha < 0, \beta < 0$ …(1)

Now since $\alpha + \beta < 0, \alpha < 0, \beta > 0 \Rightarrow |\alpha| > \beta$

$\therefore$ $\alpha < 0 < \beta < |\alpha|$ Ans.

Q.3 If b > a, then the equation $(x - a)(x - b) - 1 = 0$, has-

(A) both roots in [a, b]

(B) both roots in $(-\infty, a)$

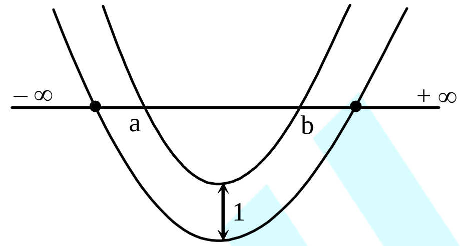
(C) both roots in $(b, +\infty)$

(D) one root in $(-\infty, a)$ and other in $(b, +\infty)$

[IIT Sc.-2000]

Sol. **[D]**

$(x - a)(x - b) - 1 = 0$, b > a



clearly one root lies in $(-\infty, a)$ & other (b, ∞).

Q.4 If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$, and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant d, then the value of $\frac{b^2 - 4ac}{a^2}$ is **[IIT-2000]**

(A) $\frac{B^2 - 4AC}{A^2}$ (B) $\frac{B^2 + 4AC}{A^2}$

(C) $\frac{A^2 - 4BC}{A^2}$ (D) None of these

Sol.[A] We know that

$\alpha - \beta = \alpha - \beta$

$\Rightarrow \alpha - \beta = \alpha - \beta + \delta - \delta$ (Add & subtract δ)

$\Rightarrow \alpha - \beta = (\alpha + \delta) - (\beta + \delta)$

Squaring both sides

$(\alpha - \beta)^2 = [(\alpha + \delta) - (\beta + \delta)]^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = \left(\overline{\alpha + \delta} + \overline{\beta + \delta}\right)^2 - 4(\alpha + \delta)(\beta + \delta)$

$\Rightarrow \left(\frac{-b}{a}\right)^2 - \frac{4a}{c} = \left(\frac{-\beta}{A}\right)^2 - \frac{4C}{A}$

$\because$ $\alpha + \beta = -b/a, \alpha\beta = c/a, \alpha + \delta + (\beta + \delta) = \frac{-\beta}{A}, (\alpha + \delta)(\beta + \delta) = \frac{C}{A}$

$\Rightarrow \frac{b^2}{a^2} - \frac{4c}{a} = \frac{B^2}{A^2} - \frac{4C}{A}$

$\Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$ (Proved)

Q.5 Let a,b,c be real numbers with $a \neq 0$ and α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **[IIT-2001]**

(A) $\alpha\beta^2$ (B) $\alpha^2\beta$

(C) both (A) and (B) (D) None of these

Sol. $ax^2 + bx + c = 0$ has roots α & β

$\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = c/a$

Now, $a^3 x^2 + abcx + c^3 = 0$

divide by c^2 we get

$\frac{a^3}{c^2} x^2 + \frac{abcx}{c^2} + \frac{c^3}{c^2} = 0$

$\Rightarrow a\left(\frac{ax}{c}\right)^2 + b\left(\frac{ax}{c}\right) + c = 0$

$\Rightarrow \frac{ax}{c} = \alpha, \beta$ are the roots

$\Rightarrow x = \frac{c}{a}\alpha, \frac{c}{a}\beta$ are the roots

$\Rightarrow x = \alpha\beta\alpha, \alpha\beta\beta$ are the roots

$\Rightarrow x = \alpha^2\beta, \alpha\beta^2$ are the roots

Q.6 Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are- **[IIT Sc.-2001]**

(A) –2, –32 (B) – 2, 3

(C) – 6, 3 (D) – 6, –32

Sol. **[A]**

$\alpha + \beta = 1$ and $\gamma + \delta = 4$

$\alpha\beta = p$ and $\gamma\delta = q$

Θ $\alpha, \beta, \gamma, \delta$ are in G.P. let c.r. = r

$\Rightarrow \alpha, \alpha r, \alpha r^2, \alpha r^3$

Θ $\alpha + \alpha r = 1$ and $\alpha r^2 + \alpha r^3 = 4$

$\alpha(1 + r) = 1$ $\alpha r^2 (1 + r) = 4$

$\Rightarrow \alpha(1 \pm 2) = 1$ $r^2.1 = 4$

$\Rightarrow \alpha(1 \pm 2) = 1$ $r = \pm 2$

$\Rightarrow \alpha = \frac{1}{1 \pm 2} = \frac{1}{3}, -1$ $\Rightarrow r = -2$

Θ p, q are integers

$\therefore$ we takes

$\alpha = -1$ & $r = -2$

$\Rightarrow \alpha = -1, \beta = 2, \gamma = -4, \delta = 8$

$\therefore$ p = – 2, q = – 32 Ans.

Q.7 Find 'a' for which the equation $x^2 + (a - b)x + (1 - a - b) = 0$ has two distinct and unequal roots for $\forall$ b$\in$ R? **[IIT-2003]**

(A) greater than 1 (B) greater than 2

(C) less than 1 (D) None of these

Sol. $x^2 + (a - b)x + (1 - a - b) = 0$ has real & unequal roots

$\Rightarrow D > 0$

$\Rightarrow (a - b)^2 - 4(1)(1 - a - b) > 0$

$\Rightarrow a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

Now to find the values of a for which equation has unequal real roots for $b \in R$

or $b^2 + b(4 - 2a) + (a^2 + 4a - 4) > 0$ is true for all b when $D < 0$

$\Rightarrow (4 - 2a)^2 - 4(a^2 + 4a - 4) < 0$

$\Rightarrow 16 - 16a + 4a^2 - 4a^2 - 16a + 16 < 0$

$\Rightarrow -32a + 32 < 0$

$\Rightarrow a > 1$ Ans.

Q.8 If one root of the equation $x^2 + px + q = 0$ is square of the other then for any p & q, it will satisfy the relation- **[IIT Sc.-2004]**

(A) $p^3 - q(3p - 1) + q^2 = 0$

(B) $p^3 - q(3p + 1) + q^2 = 0$

(C) $p^3 + q(3p - 1) + q^2 = 0$

(D) $p^3 + q(3p + 1) + q^2 = 0$

Sol. **[A]**

Let the roots be α, α^2

Θ $\alpha + \alpha^2 = -p$ and $\alpha.\alpha^2 = q$

$\Rightarrow \alpha(1 + \alpha) = -p \Rightarrow \alpha^3(1 + \alpha^3 + 3\alpha(1+\alpha)) = -p^3$

$\Rightarrow q(1 + q + 3(-p)) = -p^3$

$\Rightarrow p^3 - q(3p - 1) + q^2 = 0$

Q.9 Let $x^2 + 2ax + 10 - 3a > 0$ for every real value of x, then- **[IIT Sc.-2004]**

(A) a > 5 (B) a < –5

(C) –5 < a < 2 (D) 2 < a < 5

Sol. **[C]**

Θ $x^2 + 2ax + 10 - 3a > 0 \ \forall x \in R$

$\therefore$ Discriminant < 0

$\therefore 4a^2 - 4(10 - 3a) < 0$

$\Rightarrow a^2 + 3a - 10 < 0$

$\Rightarrow (a + 5)(a - 2) < 0$

$-5 < a < 2$ Ans.

Q.10 Let a, b, c be sides of a triangle and any two of them are not equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real then- **[IIT-2006]**

(A) $\frac{4}{3} < \lambda < \frac{5}{3}$ (B) $\frac{1}{3} < \lambda < \frac{5}{3}$

(C) $\lambda > \frac{5}{3}$ (D) $\lambda < \frac{4}{3}$

Sol. **[D]**

$\because$ a, b , c are sides of a triangle.

$\therefore$ a + b > c, b + c > a, c + a > b

or $|a - b| < |c| \Rightarrow a^2 + b^2 - 2ab < c^2$

similarly $|b - c| < |a| \Rightarrow b^2 + c^2 - 2bc < a^2$

and $|c - a| < |b| \Rightarrow c^2 + a^2 - 2ca < b^2$

on adding, we get

$2(a^2 + b^2 + c^2) - 2(ab + bc + ca) < a^2 + b^2 + c^2$

$\Rightarrow a^2 + b^2 + c^2 < 2(ab + bc + ca)$

$\Rightarrow \frac{a^2+b^2+c^2}{ab+bc+ca} < 2$ …(1)

Roots of the given equation are real

$\therefore 4(a + b + c)^2 - 4.3\lambda(ab + bc + ca) \geq 0$

$\Rightarrow (a + b + c)^2 - 3\lambda(ab + bc + ca) \geq 0$

$\Rightarrow \frac{a^2+b^2+c^2}{ab+bc+ca} > 3\lambda - 2$ …(2)

from (1) & (2), we get

$3\lambda - 2 < 2$

$3\lambda < 4, \quad \lambda < \frac{4}{3}$ Ans.

Q.11 If roots of $x^2 - 10cx - 11d = 0$ are a, b and the roots of $x^2 - 10ax - 11b = 0$ are c, d, then the value of a + b + c + d is equal to (a, b, c, d are different numbers) …… **[IIT-2006]**

Sol. Root of $x^2 - 10cx - 11d = 0$ are a & b therefore a + b = 10 c and ab = – 11 d

Similarly c & d are the roots of $x^2 - 10ax - 11b = 0$

$\Rightarrow$ c + d = 10 a & cd = – 11 b

$\Rightarrow$ a + b + c + d = 10 (a + c) and abcd = 121 bd

$\Rightarrow$ b + d = 9(a + c) and ac = 121

Also, $a^2 - 10ac - 11d = 0$ & $c^2 - 10ac - 11b = 0$

$\Rightarrow a^2 + c^2 - 20ac - 11(b + d) = 0$

$\Rightarrow (a + c)^2 - 22 \times 121 - 99(a + c) = 0$

$\Rightarrow$ (a + c) = 121 or – 22

for a + c = – 22, we get a = c

$\therefore$ rejecting this value and take a + c = 121

$\therefore$ (a + b + c + d) = 10 (a + c)

= 10 × 121

= 1210 Ans.

Q.12 Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is– **[IIT-2007]**

(A) $\frac{2}{9}$ (p – q) (2q – p) (B) $\frac{2}{9}$ (q – p) (2p – q)

(C) $\frac{2}{9}$ (q – 2p) (2q – p) (D) $\frac{2}{9}$ (2p – q) (2q – p)

Sol. **[D]**

$\because \alpha$ is a root of $x^2 - px + r = 0$

$\Rightarrow \alpha^2 - p\alpha + r = 0$

Again $\frac{\alpha}{2}$ is a root of $x^2 - qx + r = 0$

$\Rightarrow \frac{\alpha^2}{4} - \frac{q\alpha}{2} + r = 0$

on eliminating r

$\Rightarrow \alpha^2 - p\alpha = \frac{\alpha^2}{4} - \frac{q\alpha}{2}$ and $\frac{3\alpha^2}{4} = \alpha\left(p - \frac{q}{2}\right)$

$\Rightarrow \alpha = \frac{4}{3}\left(p - \frac{q}{2}\right)$

since α + β = p we have

$\beta = p - \frac{4}{3}\left(p - \frac{q}{2}\right) = p - \frac{4p}{3} + \frac{2q}{3} = \frac{2q}{3} - \frac{p}{3} = \left(\frac{2q-p}{3}\right)$

Now $r = \alpha\beta = \frac{2}{9}$ (2p – q) (2q – p) Ans.

Q.13 Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$ **[IIT-2007]**

Match the expressions/statements in Column-I with expression/statements in Column-II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column-I

(A) If –1 < x < 1, then f(x) satisfies

(B) If 1 < x < 2, then f(x) satisfies

(C) If 3 < x < 5, then f(x) satisfies

(D) If x > 5, then f(x) satisfies

Column-II

(P) 0 < f(x) < 1

(Q) f(x) < 0

(R) f(x) > 0

(S) f(x) < 1

Sol. **A → P,R,S; B → Q, S; C → Q, S; D → P,R,S**

$f(x) = \frac{x^2-6x+5}{x^2-5x+6}$

$f(x) = \frac{(x-1)(x-5)}{(x-2)(x-3)}$

$f'(x) = \frac{x^2+2x-11}{x^2-5x+6}$

$f'(x) = 0 \Rightarrow x = -1 \pm 2\sqrt{3}$

$\therefore$ f $(-1-2\sqrt{3}) = \dfrac{12-8\sqrt{3}}{12-7\sqrt{3}} < 1$

and f $(-1+2\sqrt{3}) = \dfrac{12+8\sqrt{3}}{12+7\sqrt{3}} > 1$

(i) $2 < -1 + 2\sqrt{3} < 3$

(ii) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} f(x) = 1$

(iii) y = 1, x = 2, x = 3 are the asymptotes

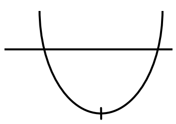
(iv) curve cuts x-axis at (1, 0), (5, 0)

(v) f (–1) = 1

Q.14 Find the smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real and have values at least 4. **[IIT-2009]**

(A) 1 (B) 2 (C) 0 (D) 4

Sol. [2]



4

$D > 0 \Rightarrow 64k^2 - 64(k^2 - k + 1) > 0$

$64k^2 - 64k^2 + 64k - 64 > 0$

$k > 1$

$-\dfrac{B}{2A} > 4 \Rightarrow 4k > 4 \Rightarrow k > 1$

$f(4) \geq 0$

$16 - 32k + 16k^2 - 16k + 16 \geq 0$

$\Rightarrow 16k^2 - 48k + 32 \geq 0$

$k^2 - 3k + 2 \geq 0 \Rightarrow (k-1)(k-2) \geq 0 \Rightarrow k \leq 1, k \geq 2$

so $k \in [2, \infty)$

so smallest integer value of k is 2.

Q.15 Let p & q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ as its roots is - **[IIT-2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Sol.[B] $\alpha + \beta = -p$ ....(1)

$\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = q$

$\Rightarrow (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta) = q$

$\Rightarrow (-p)(p^2 - 3\alpha\beta) = q$

$\alpha\beta = \dfrac{q + p^3}{3p}$ .....(2)

Now $S = \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

(Sum of root) $S = \dfrac{p^3 - 2q}{p^3 + q}$ using (1) and (2)

(Product of root) $P = \dfrac{\alpha}{\beta}.\dfrac{\beta}{\alpha} = 1$

Q.16 Let α and β be the roots of $x^2 - 6x - 2 = 0$, with α > β. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is **[IIT 2011]**

(A) 1 (B) 2

(C) 3 (D) 4

Sol.[C] $\therefore x^2 - 6x - 2 = 0$ has roots α, β

So, $\alpha^2 - 2 = 6\alpha$ & $\beta^2 - 2 = 6\beta$

$a_n = \alpha^n - \beta^n$

So, $\dfrac{a_{10} - 2a_8}{2a_9} = \dfrac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$

$= \dfrac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$

$= \dfrac{\alpha^8(6\alpha) - \beta^8(6\beta)}{2(\alpha^9 - \beta^9)} = 3.$

Q.17 A value of b for which the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$, have one root in common is **[IIT 2011]**

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$

(C) $i\sqrt{5}$ (D) $\sqrt{2}$

Sol. [B] $x^2 + bx - 1 = 0$ … (i)

$x^2 + x + b = 0$ … (ii)

(i) – (ii) we get $x = \dfrac{b+1}{b-1}$

Put this value in (i)

$\left(\dfrac{b+1}{b-1}\right)^2 + b\left(\dfrac{b+1}{b-1}\right) - 1 = 0$

$\Rightarrow b^3 + 3b = 0$

$\Rightarrow b(b^2 + 3) = 0$

$\Rightarrow b = 0$ or $b = \pm i\sqrt{3}$

EXERCISE # 5

Q.1 Solution of

$(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \dfrac{4}{2-\sqrt{3}}$ are

(A) $1 \pm \sqrt{3}$, 1 (B) $1 \pm \sqrt{2}$, 1

(C) $1 \pm \sqrt{3}$, 2 (D) $1 \pm \sqrt{2}$, 2

Sol. **[B]**

Q.2 For a ≤ 0, determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$ **[IIT-1986]**

Sol. Here a ≤ 0

and we know $|x-a| = \begin{cases} (x-a), & x > a \\ -(x-a), & x < a \end{cases}$

Thus $x^2 - 2a|x-a| - 3a^2 = 0$ arises two cases :

Case-I : x ≥ a

$\Rightarrow x^2 - 2a(x-a) - 3a^2 = 0$

$\Rightarrow x^2 - 2ax - a^2 = 0$

or $x = a \pm \sqrt{2}\,a$

{as, $a(1+\sqrt{2}) < a$ and $a(1-\sqrt{2}) > a$}

since a ≤ 0 is given

∴ neglecting $x = a(1+\sqrt{2})$ as x ≥ a

Hence, when x ≥ a and a < 0

$\Rightarrow x = a(1-\sqrt{2})$ …. (1)

Case-II : x < a

$\Rightarrow x^2 + 2a(x-a) - 3a^2 = 0$

$\Rightarrow x^2 + 2ax - 5a^2 = 0$ or $x = -a \pm \sqrt{6}\,a$

{as, $a(\sqrt{6}-1) < a$ and $a(-1-\sqrt{6}) > a$}

∴ neglecting $x = a(-1-\sqrt{6})$

$\Rightarrow x = a(\sqrt{6}-1)$ ... (2)

from (1) & (2), we get

$x = (a(1-\sqrt{2}), a(\sqrt{6}-1)]$ Ans.

Q.3 If a, b and c are distinct positive numbers, then show that the expression

(b + c – a) (c + a – b) (a + b – c) – abc is negative. **[IIT-1986]**

Sol. As we know that

A.M. > G.M.

$\Rightarrow \dfrac{(b+c-a)+(c+a-b)}{2} > ((b+c-a)(c+a-b))^{1/2}$

$c > ((b+c-a)(c+a-b))^{1/2}$ …(i)

$b > ((a+b-c)(b+c-a))^{1/2}$ …(ii)

$a > ((a+b-c)(c+a-b))^{1/2}$ …(iii)

Multiplying (i), (ii) and (iii), we get

abc > (a + b – c) (b + c – a) (c + a – b)

Hence, (a + b – c) (b + c – a) (c + a – b) – abc < 0

Therefore the given expression is negative.

Q.4 If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \le 0$ then show that a, b, c, d are in G.P. **[IIT-1987]**

Sol. **[B]**

$(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \le 0$

$\Rightarrow (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \le 0$

$\Rightarrow (ap-b)^2 + (bp-c)^2 + (cp-d)^2 \le 0$

$\Rightarrow (ap-b)^2 + (bp-c)^2 + (cp-d)^2 = 0$

$\Rightarrow$ ap – b = 0 and bp – c = 0 and cp – d = 0

$\Rightarrow \dfrac{b}{a} = p, \dfrac{c}{b} = p$ and $\dfrac{d}{c} = p$

$\Rightarrow \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = p$

$\Rightarrow$ a, b, c, d, are in G.P.

Q.5 Solve $|x^2 + 4x + 3| + 2x + 5 = 0$. **[IIT-1988]**

Sol. $|x^2 + 4x + 3| + 2x + 5 = 0$

Case-I: $x^2 + 4x + 3 > 0 \Rightarrow (x < -3$ or $x > -1)$

∴ $|x^2 + 4x + 3| + 2x + 5 = 0$

$\Rightarrow x^2 + 4x + 3 + 2x + 5 = 0$

$\Rightarrow x^2 + 6x + 8 = 0$

$\Rightarrow (x + 4)(x + 2) = 0$

$\Rightarrow x = -4, -2$ (but $x < -3$ or $x > -1$)

∴ x = – 4 is the only solution. … (1)

Case-II : $x^2 + 4x + 3 < 0 \Rightarrow -3 < x < -1$

∴ $|x^2 + 4x - 3| + 2x + 5 = 0$

$\Rightarrow -x^2 - 4x - 3 + 2x + 5 = 0$

$\Rightarrow -x^2 - 2x + 2 = 0$

$\Rightarrow x^2 + 2x - 2 = 0$

$\Rightarrow (x^2 + 2x + 1) = 3$ or $(x + 1)^2 = 3$

∴ $|x + 1| = \sqrt{3}$

∴ $x = -1 - \sqrt{3}\;\; -1 + \sqrt{3}$ but $x \in (-3, -1)$

∴ $x = -1 - \sqrt{3}$ is only solution … (2)

Thus, from (1) & (2)

$x = -4$ and $(-1-\sqrt{3})$ are the only solutions.

Q.6 Show that the equation

$x^{3/4(\log_2 x)^2+\log_2 x-5/4} = \sqrt{2}$ has exactly three solutions. **[IIT-1989]**

Sol. $x^{3/4(\log_2 x)^2+\log_2 x-5/4} = \sqrt{2}$

Let $\log_2 x = y$

$(\frac{3}{4}y^2 + y - \frac{5}{4})y = \frac{1}{2}$

$\Rightarrow 3y^3 + 4y^2 - 5y - 2 = 0$

or $(y-1)(3y^2 + 7y + 2) = 0$

$\Rightarrow (y-1)(3y+1)(y+2) = 0$

$\Rightarrow \log_2 x = 1, \frac{-1}{3}, -2$

$\Rightarrow x = 2, \sqrt[3]{1/2}, \frac{1}{4}$ Ans.

Q.7 Let a, b, c be real numbers, $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$. β is the root of $a^2x^2 - bx - c = 0$ & $0 < \alpha < \beta$, then show that the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies $\alpha < \gamma < \beta$. **[IIT-1989]**

Sol. As α is roots of $a^2x^2 + bx + c = 0$

$\Rightarrow a^2\alpha^2 + b\alpha + c = 0$

β is root of $a^2x^2 - bx - c = 0$

$\Rightarrow a^2\beta^2 - b\beta - c = 0$

$\Rightarrow f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c = a^2\alpha^2 - 2a^2\alpha^2$

$= -a^2\alpha^2$ {using (1)}

$f(\beta) = a^2\beta^2 + 2b\beta + 2c = a^2\beta^2 + 2a^2\beta^2 = 3a^2\beta^2$

$= f(\alpha) f(\beta) < 0$ {using (2)}

$\therefore$ f (x) must have a root lying in the open interval (α, β)

$\therefore \alpha < \gamma < \beta$

Q.8 If α & β are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then show that the equation $x^2 - 4qx + 4q^2 - r = 0$ has always two real roots. **[IIT-1989]**

Sol. Since α, β are roots of $x^2 + px + q = 0$ and α^4, β^4 are roots of $x^2 - rx + s = 0$

$\Rightarrow \alpha + \beta = -p, \alpha\beta = q, \alpha^4\beta^4 = r$ and $\alpha^4\beta^4 = s$

Thus if roots of $x^2 - 4qx +\_ (2q^2 - r) = 0$ has roots α' and β'

$\Rightarrow \alpha'\beta' = (2q^2 - r)$

$= 2(\alpha\beta)^2 - (\alpha^4 + \beta^4)$

$= -\{\alpha^4 + \beta^4 - 2\alpha^2\beta^2\}$

$= -\{\alpha^2 - \beta^2\}^2 < 0$

as (product of the roots) < 0

$\Rightarrow$ roots are real and of opposite sign.

Q.9 The product of n positive numbers is unity. Then show that their sum is never less than n. **[IIT-1991]**

Sol. Since product of n positive numbers is unity

$\Rightarrow x_1, x_2, x_3 \ldots\ldots x_n = 1$ …(i)

each x is > 0 {using A.M. ≥ G.M.}

$\frac{x_1 + x_2 + \ldots.. x_n}{n} \geq (x_1, x_2 \ldots\ldots x_n)^{1/n}$

$\Rightarrow x_1 + x_2 \ldots\ldots + x_n \geq n(1)^{1/n}$ {using (i)}

$\Rightarrow$ Sum of n positive number is never less than n

Q.10 If p, q are roots of the equation $x^2 + px + q = 0$, then find the value of p. **[IITSc.-95]**

Sol. $x^2 + px + q = 0$

$p + q = -p$ …(1)

$pq = q$ …(2)

from (1) $\Rightarrow 2p = q$

$p = \frac{q}{2}$

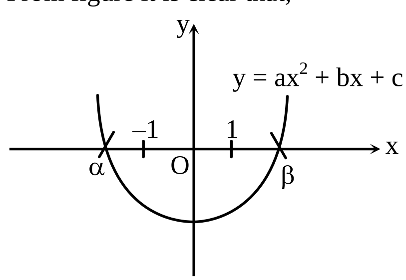
from (2) $\Rightarrow pq - q = 0$

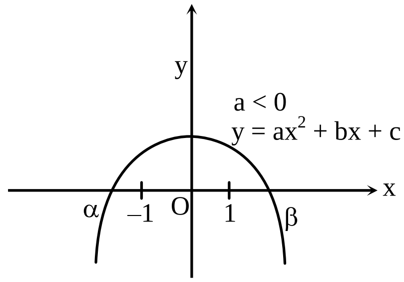
$q(p-1) = 0 \Rightarrow q = 0$ or $p = 1$

when $q = 0, p = 0$

Q.11 Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$. **[IIT-1995]**

Sol. From figure it is clear that,





If a > 0, then f(–1) < 0 and f(1) < 0 and if a < 0, f(–1) > 0 and f(1) > 0
In both cases, a f(–1) < 0 and a f(1) < 0
$\Rightarrow$ a(a – b + c) < 0 and a(a + b + c) < 0
Dividing by a^2,

$$\Rightarrow 1 - \frac{b}{a} + \frac{c}{a} < 0 \text{ and } 1 + \frac{b}{a} + \frac{c}{a} < 0$$

combining beta

$$\Rightarrow 1 \pm \frac{b}{a} + \frac{c}{a} < 0$$

$$\Rightarrow 1 + \left|\frac{b}{a}\right| + \frac{c}{a} < 0 \text{ (Proved)}$$

Q.12 Find the number of solution of the equation $\sqrt{(x+1)} - \sqrt{(x-1)} = \sqrt{(4x-1)}$.

[IIT-97 can.]

Sol. On squaring both sides

$\Rightarrow (x + 1) + (x - 1) - 2\sqrt{x^2 - 1} = 4x - 1$

$\Rightarrow 2x - 2\sqrt{x^2 - 1} = 4x - 1$

$\Rightarrow 1 - 2x = 2\sqrt{x^2 - 1}$

$\Rightarrow 1 + 4x^2 - 4x = 4x^2 - 4$

$\Rightarrow 4x = 5$

$\Rightarrow x = 5/4$

but x = 5/4 does not satisfy the given equation
Hence No solution.

Q.13 Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the lengths of the sides of the quadrilateral, prove that
$2 \le a^2 + b^2 + c^2 + d^2 \le 4$. **[IIT-1997]**

Sol.

Q.14 Prove that the values of the function $\frac{\sin x \ \cos 3x}{\sin 3x \ \cos x}$ do not lie between 1/3 and 3 for any real x. **[IIT-1997]**

Sol. Let $y = \frac{\sin x \cos 3x}{\sin 3x \cos x} = \frac{\tan x}{\tan 3x}$

$$\Rightarrow y = \frac{\tan x}{\tan 3x} = \frac{\tan x(1 - 3\tan^2 x)}{3\tan x - \tan^3 x}$$

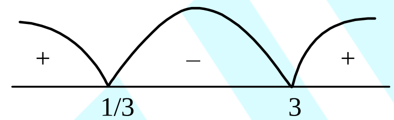
$$= \frac{1 - 3\tan^2 x}{3 - \tan^2 x} \quad \because x \ne 0$$

Put tan x = t

$$\Rightarrow y = \frac{1 - 3t^2}{3 - t^2} \Rightarrow 3y - t^2 y = 1 - 3t^2$$

$\Rightarrow 3y - 1 = t^2 y - 3t^2 \Rightarrow 3y - 1 = t^2 (y - 3)$

$$\Rightarrow \frac{3y - 1}{y - 3} = t^2 \Rightarrow \frac{3y - 1}{y - 3} > 0 \ (\because t^2 > 0)$$



$$\Rightarrow y < \frac{1}{3} \text{ or } y > 3$$

This show that y cannot lie between $\frac{1}{3}$ and 3.

Q.15 If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then show that a < 2. **[IIT-1999]**

Sol. $x^2 - 2ax + a^2 + a - 3 = 0$
$\because$ roots are real & less than 3

$$\Rightarrow D \ge 0, f(3) > 0, -\frac{b}{2a} < 3$$

$\Rightarrow b^2 - 4ac \ge 0 \ \Rightarrow 4a^2 - 4(a^2 + a - 3) \ge 0$ &

$a^2 - 5a + 6 > 0, \ \frac{2a}{2} < 3$

$\Rightarrow -a + 3 \ge 0$ & $(a - 2)(a - 3) > 0, a < 3$

$\Rightarrow a \le 3, a < 2$ or $a > 3, a < 3 \Rightarrow a < 2$ Ans.

ANSWER KEY

EXERCISE # 1

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Ans.	A	B	D	C	A	C	B	A	D	C	D	C	D	C	C	B	B	C	B	B	D
Qus.	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39			
Ans.	A	D	B	D	B	D	C	C	A	C	C	A	A	C	C	C	B	A			

EXERCISE # 2

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	A	B,C	C	B	A	D	C	A	D	B	A,B,C,D	C	A,C	A,B,C,D	C,D	D	A,B,C	A,B,C
Qus.	21	22	23	24	25	26	27	28	29	30	31	32								
Ans.	B,C	A,B	A,D	D	C	B	C	C	D	A	D	A,C								

33. $A \rightarrow Q$; $B \rightarrow P$; $C \rightarrow R$; $D \rightarrow S$ **34.** $A \rightarrow Q$; $B \rightarrow R$; $C \rightarrow Q,R,S$; $D \rightarrow P,Q,R,S$

EXERCISE # 3

3. $x = -3, -4$ **4.** 1338 **6.** $[1, 7]$ **7.** $(4,\infty)$

8. **(a)** $(-\infty, 1) \cup (9, \infty)$ **(b)** $\{1, 9\}$ **(c)** $(10, \infty)$ **(d)** $[9, 10)$ **(e)** $(-\infty, 1]$

(f) $(10, \infty)$ **(g)** No value possible **(h)** $(10, \infty)$ **(i)** No solution **(j)** $[9, \infty)$

9. $m < 10$ **11.** $x^2 - 3x + 2 = 0$ **12.** $a \in (-\infty, -1/2)$

14. $k \le -1$ **15.** (a) $x = 1$ (b) $x = -4$ or $-(1 + \sqrt{3})$ (c) $x = -2$ or -4 or $-(1 + \sqrt{3})$

16. $x^3 - 6x^2 + 5x + 24 = 0$ **18.** $\log_{2/3}3/4, \log_{2/3}1/2$ **19.** $\left[-\frac{7}{2}, \frac{5}{6}\right]$

21. $a = -7, b = -8$; (3, 4) ; (3, 5), (3, 12) **22.** $k = 86$ **23.** $\left[\frac{4}{5}, 1\right]$

24. (B) **25.** (C) **26.** (B) **27.** (C) **28.** (A) **29.** (B)

30. (A) **31.** (B) **32.** (C) **33.** (D) **34.** (A) **35.** (C)

EXERCISE # 4

1. (C) **2.** (B) **3.** (D) **5.** (C) **6.** (A) **7.** (A)

8. (A) **9.** (C) **10.** (D) **11.** 1210 **12.** (D)

13. A → P,R,S ; B → Q, S; C → Q, S ; D → P, R, S **14.** (A) **15.** (B)

16. (C) **17.** (B)

EXERCISE # 5

1. (B) **2.** $\{a \pm a\sqrt{2}, -a \pm a\sqrt{6}\}$

5. $-4, -1 - \sqrt{3}$ **10.** p = 1 or 0 **12.** No solution