

## Properties of Integers

### What are Integers?

Integers are defined as the set of all whole numbers but they also include negative numbers. Therefore, integers can be negative, i.e, -5, -4, -3, -2, -1, positive 1, 2, 3, 4, 5, and even include 0. An integer can never be a fraction, a decimal, or a percent.

The integer set is denoted by the symbol “Z”. The set of integers are defined as:

$$Z = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

Integers Examples: -57, 0, -12, 19, -82, etc.

The integers can be represented as:

$$Z = \{.....-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 .....\}$$

On the number, line integers are represented as follows

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Here 0 is at the center of the number line and is called the origin.

All integers to the left of the origin (0) are negative integers prefixed with a minus(-) sign and all numbers to the right are positive integers prefixed with positive(+) sign, they can also be written without + sign.

In this article we will study different properties of integers.

### Properties of Integers

Integers have 5 main properties they are:

- Closure Property
- Associative Property
- Commutative Property
- Distributive Property

- Identity Property

### Property 1: Closure Property

Closure property of integers under addition and subtraction states that the sum or difference of any two integers will always be an integer i.e. if  $p$  and  $q$  are any two integers,  $p + q$  and  $p - q$  will also be an integer.

Example :  $7 - 4 = 3$

$$7 + (-4) = 3;$$

Closure property of integers under multiplication states that the product of any two integers will be an integer i.e. if  $p$  and  $q$  are any two integers,  $pq$  will also be an integer.

**Example :**  $5 \times 7 = 35$  ;

$$(-4) \times (7) = -28, \text{ which are integers.}$$

Division of integers doesn't hold true for the closure property, i.e. the quotient of any two integers  $p$  and  $q$ , may or may not be an integer.

**Example :**  $(-3) \div (-12) = \frac{1}{4}$  , is not an integer.

### Property 2: Associative Property

Associative property refers to grouping. Associative property rules can be applied for addition and multiplication. If the associative property for addition and multiplication operation is carried out regardless of the order of how they are grouped, the result remains constant.

- Associative Property for Addition states that if

$$(l + n) + m = l + (n + m)$$

For example:  $(2 + 5) + 4 = 2 + (5 + 4)$  the answer for both the possibilities will be 11.

- Associative Property for Multiplication states that if

$$(l \times n) \times m = l \times (n \times m)$$

For example  $(2 \times 3) \times 5 = 2 \times (3 \times 5)$  the answer for both the possibilities will be 30.

Thus we can apply the associative rule for addition and multiplication but it does not hold true for subtraction and division.

### **Property 3: Commutative Property**

Commutative law states that when any two numbers say  $x$  and  $y$ , in addition gives the result as  $z$ , then if the position of these two numbers is interchanged we will get the same result  $z$ . Thus, we can say that commutative property states that when two numbers undergo swapping the result remains unchanged. For example,  $5 + 4 = 9$  if it is written as  $4 + 5$  then also it will give the result 9. Similarly, the commutative property holds true for multiplication. But it does not hold true for subtraction and division.

Example:

$$7 + 2 = 2 + 7 = 9$$

$$5 + 21 = 21 + 5 = 26$$

$$5 + 28 + 43 = 43 + 5 + 28 = 76$$

### **Property 4: Distributive Property**

Distribute, the name itself implies that to divide something given equally.

Distributive property means to divide the given operations on the numbers so that the equation becomes easier to solve. It states that “multiplication is distributed over addition.”

For instance, take the equation  $a(b + c)$

when we apply distributive property we have to multiply  $a$  with both  $b$  and  $c$  and then add i.e  $a \times b + a \times c = ab + ac$ .

Let us understand this concept with distributive property examples

$$\text{For example } 3(2 + 4) = 3(6) = 18$$

Or

By distributive law

$$\begin{aligned}
 3(2 + 4) &= 3 \times 2 + 3 \times 4 \\
 &= 6 + 12 \\
 &= 18
 \end{aligned}$$

Here we are distributing the process of multiplying 3 evenly between 2 and 4. We observe that whether we follow the order of the operation or distributive law the result is the same.

### **Property 5: Identity Property**

Identity property states that when any zero is added to any number it will give the same given number. Zero is called additive identity.

For any integer  $p$ ,

$$p + 0 = p$$

The multiplicative identity property for integers says that whenever a number is multiplied by the number 1 it will give the integer itself as the result. Hence 1 is called the multiplicative identity for a number.

For any integer  $p$ ,

$$p \times 1 = p = 1 \times p$$

If any integer multiplied by 0, the result will be zero:

$$x \times 0 = 0 = 0 \times x$$

If any integer multiplied by -1, the result will be opposite of the number:

$$x \times (-1) = -x = (-1) \times x$$

### **Solved Examples**

**Example 1:** Show that -37 and 25 follow commutative property under addition.

**Solution :**

Let  $a = -37$   $b = 25$

Commutative property states that

$$a + b = b + a$$

$$\text{L.H.S} = a + b$$

$$= -37 + 25$$

$$= -12$$

$$\text{R.H.S} = b + a$$

$$= 25 + (-37)$$

$$= 25 - 37$$

$$= -12$$

So, L.H.S = R.H.S, i.e  $a + b = b + a$

This means the two integers hold true commutative property under addition.

**Example 2:** Show that (-6), (-2) and (5) are associative under addition.

**Solution :**

Let  $a = -6$ ;  $b = -2$  and  $c = 5$

Associative property for addition states that

$$a + (b + c) = (a + b) + c$$

$$\text{L.H.S} = -6 + (-2 + 5)$$

$$= -6 + 3$$

$$= -3$$

$$\text{R.H.S} = (-6 + (-2)) + 5$$

$$= (-6 - 2) + 5$$

$$= -8 + 5$$

$$= -3$$

So, L.H.S = R.H.S, i.e  $a + (b + c) = (a + b) + c$

This proves that all three integers follow associative property under addition