Powers, Indices and Surds

$$(243)^{-\frac{3}{5}} = (3 \times 3 \times 3 \times 3 \times 3)^{-\frac{3}{5}}$$

$$= (3^5)^{-\frac{3}{5}} \quad [(a^m)^n = a^{mn}]$$

$$= 3^{5 \times \left(-\frac{3}{5}\right)}$$

$$= 3^{-3}$$

$$= \frac{1}{3^3} \quad [a^{-n} = \frac{1}{a^n}]$$

$$= \frac{1}{3 \times 3 \times 3}$$

$$= \frac{1}{27}$$

(ii)

$$\frac{1}{(27)^{-\frac{2}{3}}} = 27^{\frac{2}{3}} \quad [a^{-n} = \frac{1}{a^n}]$$

$$27^{\frac{2}{3}} = (3 \times 3 \times 3)^{\frac{2}{3}}$$

$$= (3^3)^{\frac{2}{3}} \quad [(a^m)^n = a^{mn}]$$

$$= 3^2$$

$$= 3 \times 3$$

$$= 9$$

(iii)

$$(-32)^{\frac{2}{5}} = ((-2) \times (-2) \times (-2) \times (-2) \times (-2))^{\frac{2}{5}}$$

$$= ((-2)^5)^{\frac{2}{5}}$$

$$= (-2)^{5 \times \frac{2}{5}} \quad [(a^m)^n = a^{mn}]$$

$$= (-2)^2$$

$$= (-2) \times (-2)$$

$$= 4$$

Solution 2

(i)

$$a^{\frac{4}{3}} \div a^{-\frac{2}{3}} = \frac{a^{\frac{4}{3}}}{a^{-\frac{2}{3}}}$$

$$= a^{\frac{4}{3}-\left(-\frac{2}{3}\right)} \quad [\frac{a^m}{a^n} = a^{m-n}]$$

$$= a^{\frac{4}{3}+\frac{2}{3}}$$

$$= a^{\frac{6}{3}}$$

$$= a^2$$

(ii)

$$a^{-5} \times a^0 \times a^5 = a^{-5+0+5} \quad [a^m \times a^n = a^{m+n}]$$

$$= a^0$$

$$= 1 \quad [a^0 = 1]$$

(iii)

$$\left(a^{-1} + b^{-1}\right) \div \left(a^{-2} - b^{-2}\right) = \left(\frac{1}{a} + \frac{1}{b}\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right) \quad [a^{-n} = \frac{1}{a^n}]$$

$$= \frac{\left(\frac{1}{a} + \frac{1}{b}\right)}{\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}$$

$$= \frac{\left(\frac{a+b}{ab}\right)}{\left(\frac{b^2 - a^2}{a^2 b^2}\right)}$$

$$= \left(\frac{a+b}{ab}\right) \times \left(\frac{a^2 b^2}{b^2 - a^2}\right)$$

$$= \left(\frac{a+b}{ab}\right) \times \left(\frac{a \times a \times b \times b}{(b+a)(b-a)}\right)$$

$$= \frac{ab}{b-a}$$

Solution 3

(i)

$$3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}} = 3^3 \times (3 \times 3 \times 3 \times 3 \times 3)^{-\frac{2}{3}} \times (3 \times 3)^{-\frac{1}{3}}$$

$$= 3^3 \times \left(3^5\right)^{-\frac{2}{3}} \times \left(3^2\right)^{-\frac{1}{3}}$$

$$= 3^3 \times 3^{\left(-\frac{10}{3}\right)} \times 3^{-\frac{2}{3}} \qquad [(a^m)^n = a^{mn}]$$

$$= 3^{3-\frac{10}{3}-\frac{2}{3}} \qquad [a^m \times a^n \times a^p = a^{m+n+p}]$$

$$= 3^{\frac{9-10-2}{3}}$$

$$= 3^{\frac{9-12}{3}}$$

$$= 3^{-\frac{3}{3}}$$

$$= 3^{-1}$$

$$= \frac{1}{3}$$

(ii)

$$5^{-4} \times (125)^{\frac{5}{3}} \div (25)^{-\frac{1}{2}} = 5^{-4} \times (5 \times 5 \times 5)^{\frac{5}{3}} \div (5 \times 5)^{-\frac{1}{2}}$$

$$= 5^{-4} \times \left(5^3\right)^{\frac{5}{3}} \div \left(5^2\right)^{-\frac{1}{2}}$$

$$= 5^{-4} \times \left(5^{3 \times \frac{5}{3}}\right) \div \left(5^{2 \times \left(-\frac{1}{2}\right)}\right)$$

$$= \frac{5^{-4} \times 5^5}{5^{-1}}$$

$$= \frac{5^{5-4}}{5^{-1}}$$

$$= \frac{5^1}{5^{-1}}$$

$$= 5^{1-(-1)}$$

$$= 5^2$$

$$= 5 \times 5$$

$$= 25$$

(iii)

$$\left(\frac{27}{125}\right)^{\frac{2}{3}} \times \left(\frac{9}{25}\right)^{-\frac{3}{2}} = \left(\frac{3\times3\times3}{5\times5\times5}\right)^{\frac{2}{3}} \times \left(\frac{3\times3}{5\times5}\right)^{-\frac{3}{2}}$$

$$= \left[\left(\frac{3}{5}\right)^{3}\right]^{\frac{2}{3}} \times \left[\left(\frac{3}{5}\right)^{2}\right]^{-\frac{3}{2}}$$

$$= \left(\frac{3}{5}\right)^{3\times\frac{2}{3}} \times \left(\frac{3}{5}\right)^{2\times\left(-\frac{3}{2}\right)}$$

$$= \left(\frac{3}{5}\right)^{2} \times \left(\frac{3}{5}\right)^{-3}$$

$$= \left(\frac{3}{5}\right)^{2-3}$$

$$= \left(\frac{3}{5}\right)^{-1}$$

$$= \frac{1}{\frac{3}{5}}$$

$$= \frac{5}{3}$$

(iv)

$$7^0 \times (25)^{-\frac{3}{2}} - 5^{-3} = 7^0 \times (5\times5)^{-\frac{3}{2}} - 5^{-3}$$

$$= 7^0 \times (5^2)^{-\frac{3}{2}} - \frac{1}{5^3}$$

$$= 7^0 \times 5^{2\times\left(-\frac{3}{2}\right)} - \frac{1}{5^3}$$

$$= 7^0 \times 5^{-3} - \frac{1}{5^3}$$

$$= 1 \times 5^{-3} - \frac{1}{5^3}$$

$$= \frac{1}{5^3} - \frac{1}{5^3}$$

$$= \frac{1-1}{5\times5\times5}$$

$$= \frac{0}{125}$$

$$= 0$$

(v)

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2\times2\times2\times2}{3\times3\times3\times3}\right)^{-\frac{3}{4}} \times \left(\frac{7\times7}{3\times3}\right)^{\frac{3}{2}} \div \left(\frac{7\times7\times7}{6\times6\times6}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2}{3}\right)^{4}\right]^{-\frac{3}{4}} \times \left[\left(\frac{7}{3}\right)^{2}\right]^{\frac{3}{2}} \div \left[\left(\frac{7}{6}\right)^{3}\right]^{\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{4\times\left(-\frac{3}{4}\right)} \times \left(\frac{7}{3}\right)^{2\times\frac{3}{2}} \div \left(\frac{7}{6}\right)^{3\times\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{7}{3}\right)^{3} \div \left(\frac{7}{6}\right)^{2}$$

$$= \frac{1}{\left(\frac{2}{3}\right)^{3}} \times \left(\frac{7}{3}\right)^{3} \times \frac{1}{\left(\frac{7}{6}\right)^{2}}$$

$$= \frac{1}{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}} \times \frac{7}{3} \times \frac{7}{3} \times \frac{7}{3} \times \frac{1}{\frac{7}{6}\times\frac{7}{6}}$$

$$= \frac{1\times3\times3\times3}{2\times2\times2} \times \frac{7}{3} \times \frac{7}{3} \times \frac{7}{3} \times \frac{1\times6\times6}{7\times7}$$

$$= \frac{7\times3\times3}{2}$$

$$= \frac{63}{2}$$

$$= 31.5$$

Solution 4

(i)

$$\left(8x^3 \div 125y^3\right)^{\frac{2}{3}} = \left(\frac{8x^3}{125y^3}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2x\times2x\times2x}{5y\times5y\times5y}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2x}{5y}\right)^{3}\right]^{\frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^{3\times\frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^{2}$$

$$= \frac{2x}{5y} \times \frac{2x}{5y}$$

$$= \frac{4x^2}{25y^2}$$

(ii)

$$(a+b)^{-1}.\left(a^{-1}+b^{-1}\right)=\frac{1}{(a+b)}\times\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$=\frac{1}{(a+b)}\times\left(\frac{b+a}{ab}\right)$$

$$=\frac{1}{(a+b)}\times\frac{(a+b)}{ab}$$

$$=\frac{1}{ab}$$

(iii)

$$\frac{5^{n+3}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}=\frac{5^{n+1}\times5^{2}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}$$

$$=\frac{5^{n+1}\times\left(5^{2}-6\right)}{5^{n}\times(9-4)}$$

$$=\frac{5^{n}\times5^{1}\times(25-6)}{5^{n}\times(9-4)}$$

$$=\frac{5^{1}\times19}{5}$$

$$=19$$

(iv)

$$\left(3x^{2}\right)^{-3}\times\left(x^{9}\right)^{\frac{2}{3}}=\frac{1}{\left(3x^{2}\right)^{3}}\times x^{9\times\frac{2}{3}}$$

$$=\frac{1}{3^{3}x^{2\times3}}\times x^{6}$$

$$=\frac{1}{27x^{6}}\times x^{6}$$

$$=\frac{1}{27}$$

Solution 5

(i)

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}=\sqrt{\frac{1}{2}\times\frac{1}{2}}+(0.1\times 0.1)^{-\frac{1}{2}}-(3\times 3\times 3)^{\frac{2}{3}}$$
$$=\frac{1}{2}+\left[(0.1)^2\right]^{-\frac{1}{2}}-\left(3^2\right)^{\frac{2}{3}}$$
$$=\frac{1}{2}+(0.1)^{2\times\left(-\frac{1}{2}\right)}-3^{3\times\frac{2}{3}}$$
$$=\frac{1}{2}+(0.1)^{-1}-3^2$$
$$=\frac{1}{2}+\frac{1}{0.1}-9$$
$$=\frac{1}{2}+\frac{10}{1}-9$$
$$=\frac{1+20-18}{2}$$
$$=\frac{3}{2}$$
$$=1\frac{1}{2}$$

(ii)

$$\left(\frac{27}{8}\right)^{\frac{2}{3}}-\left(\frac{1}{4}\right)^{-2}+5^0=\left(\frac{3\times 3\times 3}{2\times 2\times 2}\right)^{\frac{2}{3}}-\left(\frac{1\times 1}{2\times 2}\right)^{-2}+5^0$$
$$=\left[\left(\frac{3}{2}\right)^3\right]^{\frac{2}{3}}-\left[\left(\frac{1}{2}\right)^2\right]^{-2}+1$$
$$=\left(\frac{3}{2}\right)^{3\times\frac{2}{3}}-\left(\frac{1}{2}\right)^{2\times(-2)}+1$$
$$=\left(\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^{-4}+1$$
$$=\frac{3}{2}\times\frac{3}{2}-\frac{1}{\left(\frac{1}{2}\right)^4}+1$$
$$=\frac{9}{4}-\frac{1}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}+1$$
$$=\frac{9}{4}-\frac{1}{\frac{1}{16}}+1$$
$$=\frac{9}{4}-16+1$$
$$=\frac{9-64+4}{4}$$
$$=\frac{-51}{4}$$

Solution 6

(i)

$$\left(\frac{3^{-4}}{2^{-8}}\right)^{\frac{1}{4}} = \left(\frac{2^8}{3^4}\right)^{\frac{1}{4}}$$

$$= \frac{\left(2^8\right)^{\frac{1}{4}}}{\left(3^4\right)^{\frac{1}{4}}}$$

$$= \frac{2^{8\times\frac{1}{4}}}{3^{4\times\frac{1}{4}}}$$

$$= \frac{2^2}{3}$$

$$= \frac{4}{3}$$

(ii)

$$\left(\frac{27^{-3}}{9^{-3}}\right)^{\frac{1}{5}} = \left(\frac{9^3}{27^3}\right)^{\frac{1}{5}}$$

$$= \left(\frac{\left(3^2\right)^3}{\left(3^3\right)^3}\right)^{\frac{1}{5}}$$

$$= \left[\left(\frac{3^2}{3^3}\right)^3\right]^{\frac{1}{5}}$$

$$= \left[\left(\frac{1}{3}\right)^3\right]^{\frac{1}{5}}$$

$$= \left(\frac{1}{3}\right)^{3\times\frac{1}{5}}$$

$$= \frac{1}{3^{\frac{3}{5}}}$$

$$(32)^{-\frac{2}{5}} \div (125)^{-\frac{2}{3}} = \frac{(32)^{-\frac{2}{5}}}{(125)^{-\frac{2}{3}}}$$

$$= \frac{(125)^{\frac{2}{3}}}{(32)^{\frac{2}{5}}}$$

$$= \frac{(5 \times 5 \times 5)^{\frac{2}{3}}}{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}}$$

$$= \frac{\left(5^3\right)^{\frac{2}{3}}}{\left(2^5\right)^{\frac{2}{5}}}$$

$$= \frac{5^2}{2^2}$$

$$= \frac{25}{4}$$

$$= 6\frac{1}{4}$$

(iii)

$$
\begin{aligned}
\left[1-\left\{1-(1-n)^{-1}\right\}^{-1}\right]^{-1} &= \frac{1}{\left[1-\left\{1-(1-n)^{-1}\right\}^{-1}\right]^{+1}} \\
&= \frac{1}{1-\dfrac{1}{1-(1-n)^{-1}}} \\
&= \frac{1}{1-\dfrac{1}{1-\dfrac{1}{(1-n)}}} \\
&= \frac{1}{1-\dfrac{1}{\dfrac{1(1-n)-1}{(1-n)}}} \\
&= \frac{1}{1-\dfrac{1}{\dfrac{1-n-1}{(1-n)}}} \\
&= \frac{1}{1-\dfrac{1}{\dfrac{-n}{(1-n)}}} \\
&= \frac{1}{1-\dfrac{(1-n)}{-n}} \\
&= \frac{1}{1+\dfrac{(1-n)}{n}} \\
&= \frac{1}{\dfrac{n+(1-n)}{n}} \\
&= \frac{1}{\dfrac{n+1-n}{n}} \\
&= \frac{n}{1} \\
&= n
\end{aligned}
$$

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Solution 7

$2160 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^4 \times 3^3 \times 5^1 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^a \times 3^b \times 5^c = 2^4 \times 3^3 \times 5^1$

Comparing powers of 2,3 and 5 on the both sides of equation, we have

a=4;b=3 and c=1

Hence value of $3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$

$$= 3 \times 3 \times 3 \times 3 \times \frac{1}{2^3} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{2 \times 2 \times 2} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{8} \times \frac{1}{5}$$

$$= \frac{81}{40}$$

$$= 2\frac{1}{40}$$

Solution 8

$1960 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2 \times 2 \times 2 \times 5 \times 7 \times 7 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^a \times 5^b \times 7^c = 2^3 \times 5^1 \times 7^2$

Comparing powers of 2,5 and 7 on the both sides of equation, we have

a=3;b=1 and c=2

Hence value of $2^{-a} \times 7^b \times 5^{-c} = 2^{-3} \times 7^1 \times 5^{-2}$

$$= \frac{1}{2^3} \times 7 \times \frac{1}{5^2}$$

$$= \frac{1}{8} \times 7 \times \frac{1}{5 \times 5}$$

$$= \frac{7}{200}$$

Solution 9

(i)

$$\frac{8^{3a}\times 2^{5}\times 2^{2a}}{4\times 2^{11a}\times 2^{-2a}} = \frac{\left(2^{3}\right)^{3a}\times 2^{5}\times 2^{2a}}{2^{2}\times 2^{11a}\times 2^{-2a}}$$

$$= \frac{2^{3\times 3a}\times 2^{5}\times 2^{2a}}{2^{2}\times 2^{11a}\times 2^{-2a}}$$

$$= \frac{2^{9a}\times 2^{5}\times 2^{2a}}{2^{2}\times 2^{11a}\times 2^{-2a}}$$

$$= 2^{9a+5+2a-2-11a+2a}$$

$$= 2^{2a+3}$$

(ii)

$$\frac{3\times 27^{n+1}+9\times 3^{3n-1}}{8\times 3^{3n}-5\times 27^{n}} = \frac{3\times (3\times 3\times 3)^{n+1}+3\times 3\times 3^{3n-1}}{2\times 2\times 2\times 3^{3n}-5\times (3\times 3\times 3)^{n}}$$

$$= \frac{3\times \left(3^{3}\right)^{n+1}+3^{2}\times 3^{3n-1}}{2^{3}\times 3^{3n}-5\times \left(3^{3}\right)^{n}}$$

$$= \frac{3\times 3^{3n+3}+3^{3n+1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$= \frac{3^{3n+3+1}+3^{3n+1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$= \frac{3^{3n+4}+3^{3n+1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$= \frac{3^{3n}\times 3^{4}+3^{3n}\times 3^{1}}{2^{3}\times \left(3^{3}\right)^{n}-5\times \left(3^{3}\right)^{n}}$$

$$= \frac{3^{3n}\left(3^{4}+3^{1}\right)}{\left(3^{3}\right)^{n}(8-5)}$$

$$= \frac{3^{3n}\left(3^{4}+3^{1}\right)}{3^{3n}\times 3}$$

$$= \frac{3\times 3\times 3\times 3+3}{3}$$

$$= \frac{81+3}{3}$$

$$= \frac{84}{3}$$

$$= 28$$

Solution 10

$$\left(\frac{a^m}{a^{-n}}\right)^{m-n} \times \left(\frac{a^n}{a^{-\ell}}\right)^{n-\ell} \times \left(\frac{a^\ell}{a^{-m}}\right)^{\ell-m}$$

$$= \left(a^m \times a^n\right)^{m-n} \times \left(a^n \times a^\ell\right)^{n-\ell} \times \left(a^\ell \times a^m\right)^{\ell-m}$$

$$= \left(a^{m+n}\right)^{m-n} \times \left(a^{n+\ell}\right)^{n-\ell} \times \left(a^{\ell+m}\right)^{\ell-m}$$

$$= a^{m^2-n^2} \times a^{n^2-\ell^2} \times a^{\ell^2-m^2}$$

$$= a^{m^2-n^2+n^2-\ell^2+\ell^2-m^2}$$

$$= a^0$$

$$= 1$$

Solution 11

$$a = x^{m+n}.x^l$$

$$b = x^{n+l}.x^m$$

$$c = x^{l+m}.x^n$$

LHS

$$a^{m-n}.b^{n-l}.c^{l-m}$$

$$= (x^{m+n}.x^l)^{m-n}.(x^{n+l}.x^m)^{n-l}.(x^{l+m}.x^n)^{l-m} \text{ [Substituting a,b,c in LHS]}$$

$$= x^{(m+n)(m-n)}.x^{l(m-n)}.x^{(n+l)(n-l)}.x^{m(n-l)}.x^{(l+m)(l-m)}.x^{n(l-m)}$$

$$= x^{m^2-n^2+ml-nl+n^2-l^2+mn-nl+l^2-m^2+nl-mn}$$

$$= x^0$$

$$= 1 = \text{RHS}$$