PROGRESSION

EXERCISE # 1

Question based on Arithmetic Progression

Q.1 If the ratio of the sum of n terms of two AP's is 2n : (n+1), then ratio of their 8th terms is-

(A) 15 : 8 (B) 8 : 13
(C) n : (n– 1) (D) 5 : 17

Sol. **[A]**

$\because S_n = \frac{n}{2}[a + (n-1)d]$

$$\frac{S_{n_1}}{S_{n_2}} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2b_1 + (n-1)d_2]} = \frac{2n}{n+1}$$

$$\Rightarrow \frac{a_1 + \frac{(n-1)}{2}d_1}{b_1 + \frac{(n-1)}{2}d_2} = \frac{2n}{n+1} \quad \ldots(1)$$

For T_8 we know $\frac{n-1}{2} = 7 \Rightarrow n = 15$

Put n = 15 in (1) we get

$$\frac{(T_8)_1}{(T_8)_2} = \frac{30}{16} = \frac{15}{8}$$

Q.2 The sum of n terms of an AP is $3n^2 + 5n$. The number of term which equals 164 is-

(A) 13 (B) 21
(C) 27 (D) None of these

Sol. **[C]**

$S_n = 3n^2 + 5n$
$T_n = S_n - S_{n-1}$
$= 3n^2 + 5n - [3(n-1)^2 + 5(n-1)] = 164$ given
$\Rightarrow 6n - 2 = 164$
$\Rightarrow n = 27$

Q.3 If a, b, c be the Ist, 3rd and nth terms respectively of an A.P., then sum to n terms is –

(A) $\frac{c+a}{2} + \frac{c^2-a^2}{b-a}$ (B) $\frac{c+a}{2} - \frac{c^2-a^2}{b-a}$

(C) $\frac{c+a}{2} + \frac{c^2+a^2}{b-a}$ (D) $\frac{c+a}{2} + \frac{c^2+a^2}{b+a}$

Sol. **[A]**

Given A = a, a + 2d = b, a + (n – 1) d = c

Solving these we get

$d = \frac{b-a}{2}$, $n = \frac{2(c-a)}{b-a} + 1$

$$S_n = \frac{n}{2}[a + c] = \left[\frac{2(c-a)}{b-a} + 1\right]\frac{(a+c)}{2}$$

$$S_n = \frac{c^2-a^2}{b-a} + \frac{c+a}{2}$$

Q.4 If $a_1, a_2, a_3, \ldots\ldots\ldots$is an A.P. such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$ then $a_1 + a_2 + a_3 + \ldots\ldots + a_{23} + a_{24}$ is equal to-

(A) 909 (B) 75
(C) 750 (D) 900

Sol. **[D]**

We know that in an A.P. $a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15} = a_{12} + a_{13}$

So $3(a_{12} + a_{13}) = 225 \Rightarrow a_{12} + a_{13} = 75$

Therefore

$a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots + a_{23} + a_{24}$
$= 12(a_{12} + a_{13}) = 12 \times 75 = 900$

Q.5 The sum of all even positive integers less then 200 which are not divisible by 6 is –

(A) 6534 (B) 6354
(C) 6543 (D) 6454

Sol. **[A]**

all even integer less than 200 is
2,4, 6, ………. 198 ⇒ n = 99

$S_{99} = \frac{99}{2}[2 + 198] = 9900$

integer which are divisible by 6 is
6, 12, 18, ……….. 198 ⇒ n = 33

$S_{33} = \frac{33}{2}[6 + 198] = 33 \times 102 = 3366$

Sum of all integer which are not divisible by 6 is
= 9900 – 3366 = 6534

Question based on Arithmetic Mean

Q.6 If x, y, z are in AP, a is AM between x and y and b is AM between y and z; then AM between a and b will be-

(A) $\frac{1}{3}(x + y + z)$ (B) z

(C) x (D) y

Sol. **[D]**

x y, z are in A.P. ⇒ x + z = 2y

Given $\frac{x+y}{2}$ = a and $\frac{y+z}{2}$ = b

$\Rightarrow \frac{a+b}{2} = \frac{x+2y+z}{4} = \frac{4y}{4} = y$

Q.7 If n AM's are inserted between 1 and 31 and ratio of 7^{th} and $(n-1)^{th}$ A.M. is 5 : 9, then n equals-

(A) 12 (B) 13 (C) 14 (D) None

Sol. **[C]**

$d = \frac{30}{n+1}$

$\Theta \ \frac{1+7d}{1+(n-1)d} = \frac{5}{9} \Rightarrow \frac{1+7\frac{30}{n+1}}{1+(n-1)\frac{30}{n+1}} = \frac{5}{9}$

Solving we get

146 n = 2044 ⇒ n = 14

Question based on Supposition of terms in A.P.

Q.8 If the angles of a quadrilateral are in A.P. whose common difference is 10º, then the angles of the quadrilateral are-

(A) 65º, 85º, 95º, 105º (B) 75º, 85º, 95º, 105º

(C) 65º, 75º, 85º, 95º (D) 65º, 95º, 105º, 115º

Sol. **[B]**

Let angles are a, a + d, a + 2d, a + 3d given that d = 10 and Sum of angles = 360º

⇒ 4a + 60 = 360 ⇒ a = 75º

angles are 75º, 85º, 95º, 105º

Q.9 Divide 20 into four parts which are in A.P., such that the product of the first and fourth is to the product of the second and third is 2 : 3 -

(A) 2, 4, 6, 8 (B) 3, 5, 7, 9

(C) 4, 6, 8, 10 (D) None of these

Sol. **[A]**

Let parts are

a – 3d, a – d, a + d, a + 3d

then from question

a – 3d + a – d + a + d + a + 3d = 20

⇒ 4a = 20 ⇒ a = 5

and $\frac{a^2-9d^2}{a^2-d^2} = \frac{2}{3}$

$\Rightarrow 3\,(25 - 9d)^2 = 2\,(25 - d^2)$

$\Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

Parts are 2, 4, 6, 8 or 8, 6, 4, 2

Question based on Properties of A.P.

Q.10 If a^2 (b+ c) , b^2 (c+ a), c^2 (a+ b) are in A.P., then-

(A) a,b,c are in A.P (B) ab + bc + ca = 0.

(C) a,b,c are in G.P (D) ab – bc – ca = 0

Sol. **[A,B]**

$a^2(b + c), b^2(c + a), c^2(a + b)$ are in A.P.

$\Rightarrow b^2(c + a) - a^2(b + c) = c^2(a + b) - b^2(c + a)$

$\Rightarrow b^2c - a^2c + ab^2 - a^2b = ac^2 - ab^2 + bc^2 - cb^2$

$\Rightarrow c(b^2 - a^2) + ab(b - a) = a(c^2 - b^2) + bc(c - b)$

$\Rightarrow (b - a)(ab + bc + ca) = (c - b)(ab + bc + ca)$

$\Rightarrow (ab + bc + ca)(b - a - c + b) = 0$

$\Rightarrow ab + bc + ca = 0$ or $2b = a + c$

option A, B are correct.

Q.11 The sum of the series

$1.3^2 + 2.\ 5^2 + 3.7^2 + ...$ upto 20 terms is-

(A) 188090 (B) 180890

(C) 189820 (D) None of these

Sol. **[A]**

$1.3^2 + 2.5^2 + 3.7^2 + \ldots\ldots$ upto 20 terms

$= \sum_{r=1}^{20} n(2n+1)^2 = \sum_{r=1}^{20}(4n^3+4n^2+n)$

$= 4\sum_{r=1}^{20} n^3 + 4\sum_{r=1}^{20} n^2 + \sum_{r=1}^{20} n$

$= 4\left(\frac{20\times 21}{2}\right)^2 = 4\left(\frac{20\times 21\times 41}{6}\right) + \frac{20\times 21}{2}$

$= (420)^2 + 11480 + 210 = 188090$

Q.12 The sum to infinity of the series $\frac{1}{2.4} + \frac{1}{4.6} + \frac{1}{6.8} + \frac{1}{8.10} + \ldots\ldots$.is-

(A) 1/4 (B) 1/8 (C) 1/2 (D) 1/16

Sol. **[A]**

$\frac{1}{2.4} + \frac{1}{4.6} + \frac{1}{6.8} + \frac{1}{8.10} = \ldots\ldots..$

$= \sum_{r=1}^{\infty} \frac{1}{2n(2n+2)}$

$= \frac{1}{4}\sum_{r=1}^{\infty}\frac{1}{n(n+1)} = \frac{1}{4}\sum_{r=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \frac{1}{4}\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots\right] = \frac{1}{4}$

Question based on

Geometric Progression

Q.13 A GP consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying odd places, the common ratio will be equal to-

(A) 2 (B) 3 (C) 4 (D) 5

Sol. **[C]**

Let G.P. is a, ar, ar^2, ……. ar^{2n-1}

$$\text{Sum} = \frac{a(1-r^{2n})}{1-r} = S_1$$

terms occupying odd places is

a, ar^2, ar^4 …….

$$\text{Sum} = \frac{a(1-r^{2n})}{1-r^2} = S_2$$

Given $S_1 = 5S_2$

$$\frac{a(1-r^{2n})}{1-r} = \frac{5a(1-r^{2n})}{1-r^2}$$

$\Rightarrow 1 + r = 5 \Rightarrow r = 4$

Q.14 If in a geometric progression $\{a_n\}$, $a_1 = 3$, $a_n = 96$ and $S_n = 189$, then the value of n is-

(A) 5 (B) 6

(C) 7 (D) 8

Sol. $a_1 = 3$, $a_1r^{n-1} = 96$, $\dfrac{a_1(1-r^n)}{1-r} = 189$

$\Rightarrow r^{n-1} = 32$ and $\dfrac{1-r^n}{1-r} = 63$

$\Rightarrow \dfrac{1-32r}{1-r} = 63 \Rightarrow r = 2$

$\because\ 2^{n-1} = 2^5 \Rightarrow n = 6$

Q.15 In any G.P. the first term is 2 and last term is 512 and common ratio is 2, then 5^{th} term from end is-

(A) 16 (B) 32

(C) 64 (D) None of these

Sol. **[B]**

a = 2, $ar^{n-1} = 512$, r = 2

5^{th} term from end is = $\dfrac{T_n}{r^{5-1}} = \dfrac{512}{2^4} = 2^5 = 32$

Q.16 If the sum of an infinite G.P. be 3 and the sum of the squares of its term is also 3, then its first term and common ratio are-

(A) 3/2, 1/2 (B) 1/2, 3/2

(C) 1, 1/2 (D) None of these

Sol. **[A]**

G.P. is a, ar, ar^2 ………….

$S_\infty = \dfrac{a}{1-r} = 3$ given …(1)

Sum of square at its term is

$= \dfrac{a^2}{1-r^2} = 3$ given …(2)

From (1) and (2)

$9(1-r)^2 = 3(1-r^2)$

$\Rightarrow 3(1-r) = (1+r) \Rightarrow 4r = 2 \Rightarrow r = \dfrac{1}{2}$

$\because\ a = 3(1-r) = 3\left(1-\dfrac{1}{2}\right) = \dfrac{3}{2}$

$\Rightarrow a = \dfrac{3}{2}$, $r = \dfrac{1}{2}$

Q.17 Sum $\dfrac{1}{5} + \dfrac{1}{7} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \ldots$ to ∞ =

(A) 5/12 (B) 3/4 (C) 7/12 (D) 3/49

Sol. **[A]**

$$\frac{1}{5} + \frac{1}{7} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots\ \infty$$

$$= \left(\frac{1}{5} + \frac{1}{5^2} + \ldots\ldots\infty\right) + \left(\frac{1}{7} + \frac{1}{7^2} + \ldots\ldots\infty\right)$$

$$= \frac{1}{5}\left(1 + \frac{1}{5} + \frac{1}{5^2} + \ldots\ldots\infty\right) + \frac{1}{7}\left(1 + \frac{1}{7} + \frac{1}{7^2} + \ldots\ldots\infty\right)$$

$$= \frac{1}{5}\left(\frac{1}{1-\frac{1}{5}}\right) + \frac{1}{7}\left(\frac{1}{1-\frac{1}{7}}\right)$$

$$= \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

Q.18 The sum of infinite number of terms of a decreasing G.P. is 4 and the sum of the squares of its terms to infinity is $\dfrac{16}{3}$, then the G.P is –

(A) 2, 1, 1/2, 1/4,…. (B)1/2, 1/4, 1/8,……
(C) 2, 4, 8, …. (D) None of these

Sol. **[A]**

Let G.P. is a, ar, ar^2 ………..

Sum of its term = $\frac{a}{1-r} = 4$ …(1)

sum of square of its term

$= \frac{a^2}{1-r^2} = \frac{16}{3}$ …(2)

from (1) and (2)

$\frac{16(1-r)^2}{1-r^2} = \frac{16}{3}$

$\Rightarrow 3(1-r) = 1 + r \Rightarrow r = \frac{1}{2}$

From (1) $a = 4\left(1-\frac{1}{2}\right) = 2$

G.P. is 2, 1, $\frac{1}{2}, \frac{1}{4}$, ……….

Q.19 The sum of 10 terms of the series .7 + .77 + .777 + ...is-

(A) $\frac{7}{9}\left(89+\frac{1}{10^{10}}\right)$ (B) $\frac{7}{81}\left(89+\frac{1}{10^{10}}\right)$

(C) $\frac{7}{81}\left(89+\frac{1}{10^{9}}\right)$ (D) None of these

Sol. **[B]**

$= \frac{7}{10} + \frac{77}{100} + \frac{777}{1000} + \ldots\ldots$ 10 terms

$= \frac{7}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots\ldots\right]$

$= \frac{7}{9}\left[1-\frac{1}{10}+1-\frac{1}{100}+1-\frac{1}{1000}+\ldots\ldots\right]$

$= \frac{7}{9}\left[10-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\ldots\ldots 10\text{ terms}\right)\right]$

$= \frac{7}{9}\left[10-\frac{\frac{1}{10}\left(1-(1/10)^{10}\right)}{1-\frac{1}{10}}\right]$

$= \frac{7}{81}\left[90-1+\frac{1}{10^{10}}\right] = \frac{7}{81}\left[89+\frac{1}{10^{10}}\right]$

Q.20 If 0 < x, y, a, b < 1, then the sum of the infinite terms of the series $\sqrt{x}(\sqrt{a}+\sqrt{x}) + \sqrt{x}(\sqrt{ab}+\sqrt{xy}) + \sqrt{x}(b\sqrt{a}+y\sqrt{x}) + \ldots$ is-

(A) $\frac{\sqrt{ax}}{1+\sqrt{b}}+\frac{x}{1+\sqrt{y}}$ (B) $\frac{\sqrt{x}}{1+\sqrt{b}}+\frac{\sqrt{x}}{1+\sqrt{y}}$

(C) $\frac{\sqrt{x}}{1-\sqrt{b}}+\frac{\sqrt{x}}{1-\sqrt{y}}$ (D) $\frac{\sqrt{ax}}{1-\sqrt{b}}+\frac{x}{1-\sqrt{y}}$

Sol. **[D]**

We can break in to two series of given series as follows

$(\sqrt{ax} + \sqrt{axb} + b\sqrt{ax} + \ldots\ldots\infty)$

$+ (x + x\sqrt{y} + xy + \ldots\ldots\infty)$

$= \sqrt{ax}\,(1 + \sqrt{b} + b + \ldots\ldots\infty) + x\,(1 + \sqrt{y} + y + \ldots\ldots\infty)$

$= \frac{\sqrt{ax}}{1-\sqrt{b}} + \frac{x}{1-\sqrt{y}}$

Q.21 The sum to n terms of the series

$\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+\ldots$ is equal to-

(A) $1-(1/3)^n$ (B) $2-\frac{1}{2}(2/3)^n$

(C) $n-n(1/3)^n$ (D) None of these

Sol. **[D]**

$\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+\ldots$ up to terms

$\Rightarrow \left(1-\frac{1}{3}\right)+\left(1-\frac{1}{9}\right)+\left(1-\frac{1}{27}\right)+\left(1-\frac{1}{81}\right) + \ldots\ldots\text{n terms}$

$\Rightarrow (1+1+1+\ldots\text{ n terms}) - \left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27} + \ldots\ldots\text{ n terms}\right)$

$= n - \frac{\frac{1}{3}\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$

$= n - \frac{1}{2}\left(1-\frac{1}{3^n}\right)$ Ans.

Question based on **Geometric Mean**

Q.22 The product of three geometric means between 4 and 1/4 will be -

(A) 4 (B) 2

(C) – 1 (D) 1

Sol. **[D]**

Three G.M. be inserted between 2 and 32

So $r = \left(\frac{32}{2}\right)^{\frac{1}{4}} = (2^4)^{1/4} = 2$

Third geometric mean = ar^3 $= 2.2^3 = 16$

Q.23 If the A.M. is twice the G.M. of the numbers a and b, then a : b will be-

(A) $\frac{\sqrt{3}+2}{\sqrt{3}-2}$ (B) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

(C) $\frac{\sqrt{3}-2}{\sqrt{3}+2}$ (D) None of these

Sol. **[B]**

Given that

$\frac{a+b}{2} = 2\sqrt{ab} \Rightarrow a^2 + b^2 + 2ab = 16\ ab$

$\Rightarrow a^2 + b^2 = 14\ ab \Rightarrow \frac{a}{b} + \frac{b}{a} - 14 = 0$

Let $\frac{a}{b} = t \Rightarrow t^2 - 14\ t + 1 = 0$

$t = \frac{14 \pm \sqrt{196-4}}{2} = 7 + 4\sqrt{3}$

$\Rightarrow \frac{a}{b} = (2 + \sqrt{3})^2 = (2 + \sqrt{3})^2 \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$

$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

Q.24 If one A.M. A and two G.M.'s p and q be inserted between two given numbers, then $\frac{p^2}{q} + \frac{q^2}{p} =$

(A) A (B) 2A (C) A/2 (D) A^2

Sol. **[B]**

Let numbers are a and b

then $A = \frac{a+b}{2}$

and $r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$\Rightarrow p = ar = a\left(\frac{b}{a}\right)^{\frac{1}{3}}$ and $q = a\left(\frac{b}{a}\right)^{2/3}$

then $\frac{p^2}{q} + \frac{q^2}{p} = \frac{a^2r^2}{ar^2} + \frac{a^2r^4}{ar}$

$= a + ar^3$

$= a + b$

$\because r^3 = \frac{b}{a}$

$= 2A$

Question based on Supposition of terms in G.P.

Q.25 If the product of three numbers in GP is 3375 and their sum is 65, then the smallest of these numbers is-

(A) 3 (B) 5 (C) 4 (D) 6

Sol. **[B]**

Let numbers are $\frac{a}{r}$, a, ar

then $\frac{a}{r}$. a. ar = 3375 $\Rightarrow a^3 = 3375$

$\Rightarrow a = 15$

and $\frac{a}{r} + a + ar = 65$

$\Rightarrow 15r^2 + 15r - 65r + 15 = 0$

$\Rightarrow 15r^2 - 50\ r + 15 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0 \Rightarrow r = 3, \frac{1}{3}$

Numbers are 5, 15, 45

Smallest number is 5

Q.26 Three numbers whose sum is 15 are in A.P. If 1,4,19 be added to them respectively the resulting numbers are in G.P. Then the numbers are-

(A) 2, 5 , 8 (B) 26, 5, – 16

(C) 36, 5, –16 (D) None of these

Sol. **[A,B]**

Let three numbers are

$a - d, a, a + d \Rightarrow a - d + a + a + d = 15 \Rightarrow a = 5$

Numbers are $5 - d, 5, 5 + d$

from question

$\Rightarrow 6 - d, 9, 24 + d$ are in G.P. $\Rightarrow 81 = (6-d)(24+d)$

$\Rightarrow d^2 + 18d - 63 = 0 \Rightarrow d = -21, d = 3$

Numbers are 2, 5, 8 or 26, 5, – 16

Q.27 Four numbers are such that the first three are in A.P. while the last three are in G.P. If the first number is 6 and common ratio of G.P. is 1/2, then the numbers are -

(A) 6, 8, 4, 2 (B) 6, 10, 14, 7

(C) 6, 9, 12, 6 (D) 6, 4, 2, 1

Sol. **[D]**

a, b, c, d are four numbers

a, b, c, are in A.P. $\Rightarrow 2b = a + c$

b, c, d, are in G.P. $\Rightarrow c^2 = bd$

$\Rightarrow \frac{c}{b} = \frac{d}{c} = \frac{1}{2}$

Θ a = 6 and r = 1/2 $\Rightarrow b = 2c$

$4c - c = a$

$3c = a$

$\therefore$ $c = \frac{a}{3} = \frac{6}{3} = 2$

$\therefore b = 4$ $\therefore d = \frac{c}{2} = \frac{2}{2} = 1$

$\therefore$ a = 6, b = 4, c = 2, d = 1

6, 4, 2, 1 Ans.

Question based on Properties of G.P.

Q.28 If x, y, z are in G.P. then $x^2 + y^2$, $xy + yz$, $y^2 + z^2$ are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Sol. **[B]**

Let $x^2 + y^2$, $xy + yz$, $y^2 + z^2$ are in G.P.

Then $(xy + yz)^2 = (x^2 + y^2)(y^2 + z^2)$

$\Rightarrow x^2y^2 + y^2z^2 + 2xy^2z = x^2y^2 + y^2z^2 + y^4 + x^2z^2$

$\Rightarrow y^4 + y^2z^2 - 2xy^2z = 0$

$\Rightarrow y^2 = xz$ $\Rightarrow$ x, y, z are in G.P. given

Q.29 If a, b, c, d are in G.P. then a + b, b + c, c + d are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Sol. **[B]**

Θ a, b, c, d are in G.P.

$\Rightarrow ad = bc$ and $b^2 = ac$ and $c^2 = bd$

$\Rightarrow b^2 + c^2 = ac + bd$

$\Rightarrow (b + c)^2 = ac + bd + 2bc$

$\Rightarrow (b + c)^2 = ac + bd + bc + ad$ [Θ ad = bc]

$\Rightarrow (b + c)^2 = (a + b)(c + d)$

$\Rightarrow$ (a + b), (b + c), (c + d) are in G.P.

Q.30 The fractional value of $0.\dot{1}\dot{2}\dot{5}$ is-

(A) 125/999 (B) 23/990

(C) 61/550 (D) None of these

Sol. **[A]**

Fractional value of $0.\dot{1}\dot{2}\dot{5}$ is

$= \frac{125}{999}$

Q.31 If a, b, c are in G.P. then-

(A) $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right) = a^3 + b^3 + c^3$

(B) $(a^2 - b^2)(b^2 + c^2) = (b^2 - c^2)(a^2+b^2)$

(C) $a^2b^2c^2\left(\frac{1}{a^3}-\frac{1}{b^3}-\frac{1}{c^3}\right) = a^3 + b^3 + c^3$

(D) $(a^2 + b^2)(b^2 + c^2) = (b^2 + c^2)(a^2+b^2)$

Sol. **[A,B]**

Θ $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)$

$= \frac{b^2c^2}{a} + \frac{a^2c^2}{b} + \frac{a^2b^2}{c}$

$= b^2\left(\frac{c^2}{a}+\frac{a^2}{c}\right) + \frac{a^2c^2}{b}$

$= \frac{b^2(c^3+a^3)}{b^2} + \frac{b^4}{b}$ [Θ $b^2 = ac$] $= a^3 + b^3 + c^3$

Again $(a^2 - b^2)(b^2 + c^2)$

$= a^2b^2 + a^2c^2 - b^4 - b^2c^2$

$= a^2b^2 + b^4 - a^2c^2 - b^2c^2$ [Θ $b^2 = ac$]

$= b^2(a^2 + b^2) - c^2(a^2 + b^2)$

$= (b^2 - c^2)(a^2 + b^2)$

$\Rightarrow$ option A, B are correct.

Question based on Arithmetico Geometric Progression

Q.32 Sum to infinite of the series

$1 + \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + ...$ is-

(A) 5/4 (B) 6/5

(C) 25/16 (D) 16/9

Sol. **[C]**

$1 + \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + \ldots\ldots\ldots\ldots$

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$= \frac{1}{1-\frac{1}{5}} + \frac{1.\frac{1}{5}}{\left(1-\frac{1}{5}\right)^2} = \frac{5}{4} + \frac{5}{16} = \frac{25}{16}$

Q.33 The sum of the infinite series

$\frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \ldots \infty$ is-

(A) 23 (B) 32

(C) 36 (D) None of these

Sol. $S = \frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \ldots \infty$

$\frac{S}{2} = \frac{1.3}{2^2} + \frac{3.5}{2^3} + \ldots\ldots\infty$

$\frac{S}{2} = \frac{1.3}{2} + \frac{3.4}{2^2} + \frac{5.4}{2^3} + \ldots\ldots\infty$

$S = 1.3 + 4\underbrace{\left(\frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \ldots\ldots\infty\right)}_{S_1}$

$S_1 = \frac{3}{2} + \frac{5}{2^2} + \ldots\ldots\infty$

$\frac{S_1}{2} = \frac{3}{2^2} + \ldots\ldots\infty$

$\frac{S_1}{2} = \frac{3}{2} + \frac{2}{2^2} + \frac{2}{2^3} + \ldots\infty = \frac{3}{2} + \left(\frac{1}{2} + \frac{1}{2^2} + \ldots\ldots\infty\right)$

$\frac{S_1}{2} = \frac{3}{2} + \frac{1/2}{1-\frac{1}{2}} = \frac{3}{2} + 1 = \frac{5}{2}$

$S_1 = 5$

$S = 1 \times 3 + 4 \times 5 = 23$

Question based on

Harmonic Progression

Q.34 If fourth term of an HP is 3/5 and its 8^{th} term is 1/3, then its first term is-

(A) 2/3 (B) 3/2

(C) 1/4 (D) None of these

Sol. **[B]**

From A.P.

$a + 3d = \frac{5}{3}$

and $a + 7d = 3$

Solving we get $d = \frac{1}{3}$, $a = \frac{2}{3}$

Then first term of H.P. $= \frac{3}{2}$

Q.35 If first and second terms of a HP are a and b, then its n^{th} term will be-

(A) $\frac{ab}{b+(n-1)\,ab}$ (B) $\frac{ab}{b+(n-1)\,(a+b)}$

(C) $\frac{ab}{b+(n-1)\,(a-b)}$ (D) None of these

Sol. **[C]**

Ist term of A.P. $= \frac{1}{a}$

2nd term of A.P. $= \frac{1}{b}$

$T_n = \frac{1}{a} + (n-1)\left(\frac{a-b}{ab}\right)$

$= \frac{b+(n-1)(a-b)}{ab}$

nth term of H.P. $= \frac{ab}{b+(n-1)(a-b)}$

Q.36 If a, b, c be in A.P. and b, c, d be in H.P., then

(A) ad = bc (B) a + d = b + c

(C) ac = bd (D) None of these

Sol. **[A]**

a, b, c are in A.P.

$\Rightarrow 2b = a + c$ …(1)

b, c, d are in H.P.

$\Rightarrow c = \frac{2bd}{b+d}$ …(2)

from (1) and (2) we have

$c = \frac{(a+c)d}{b+d}$

$\Rightarrow bc + cd = ad + cd \Rightarrow bc = ad$

Q.37 If 2 (y – a) is the H.M. between y – x and y – z then x – a, y –a, z – a are in-

(A) A.P. (B) G.P.

(C) H.P (D) None of these

Sol. **[B]**

$2(y-a) = \frac{2(y-x)(y-z)}{y-x+y-z}$

Solving we get

$y^2 - 2ay = xz - ax - az$

$\Rightarrow (y-a)^2 = xz - ax - az + a^2$

$= x(z-a) - a(z-a)$

$\Rightarrow (y-a)^2 = (x-a)(z-a)$

$\Rightarrow (x-a), (y-a), (z-a)$ are in G.P.

Q.38 If $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is H.M. between a and b, then value of n is-

(A) 1 (B) –1

(C) 2 (D) –2

Sol. **[B]**

$$\text{H.M} = \frac{2ab}{a+b} = \frac{2}{a^{-1}+b^{-1}}$$

Clearly n = – 1.

Q.39 If a, b, c are in geometric series, then $\log_a 10, \log_b 10, \log_c 10$ are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Sol. **[C]**

a, b, c, are in G.P.

$\Rightarrow \log_{10} a, \log_{10} b, \log_{10} c$ are in A.P.

$\Rightarrow \dfrac{1}{\log_{10} a}, \dfrac{1}{\log_{10} b}, \dfrac{1}{\log_{10} c}$ are in H.P.

$\Rightarrow \log_a 10, \log_b 10, \log_c 10, \log_c 10$ are in H.P.

Question based on Relation between A.M., G.M., H.M.

Q.40 Let a_n = product of the first n natural numbers. Then for all $n \in N$ -

(A) $n^n \geq a_n$ (B) $\left(\dfrac{n+1}{2}\right)^n \geq n!$

(C) $n^n \geq a_{n+1}$ (D) None of these

Sol. **[A,B]**

Q.41 If a, b, c, d are four positive numbers then-

(A) $\left(\dfrac{a}{b}+\dfrac{b}{c}\right)\left(\dfrac{c}{d}+\dfrac{d}{e}\right) \leq 4.\sqrt{\dfrac{a}{e}}$

(B) $\left(\dfrac{a}{b}+\dfrac{c}{d}\right)\left(\dfrac{b}{c}+\dfrac{d}{e}\right) \geq 4.\sqrt{\dfrac{a}{e}}$

(C) $\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{d}+\dfrac{d}{e}+\dfrac{e}{a} \geq 5$

(D) $\dfrac{b}{a}+\dfrac{c}{b}+\dfrac{d}{c}+\dfrac{e}{d}+\dfrac{a}{e} \geq \dfrac{1}{5}$

Sol.[B,C] $\dfrac{a}{b}+\dfrac{b}{c} \geq 2\sqrt{\dfrac{a}{c}}$ [AM ≥ GM]

$\dfrac{c}{d}+\dfrac{d}{e} \geq 2\sqrt{\dfrac{c}{e}}$ [AM ≥ GM]

multiply $\left(\dfrac{a}{b}+\dfrac{b}{c}\right)\left(\dfrac{c}{d}+\dfrac{d}{e}\right) \geq 4\sqrt{\dfrac{a}{c}\times\dfrac{c}{e}} = 4\sqrt{\dfrac{a}{e}}$

EXERCISE # 2

Part-A **Only single correct answer type questions**

Q.1 If 9 A.M.'s and H.M.'s are inserted between the 2 and 3 and if the harmonic mean H is corresponding to arithmetic mean A, then $A + \frac{6}{H}$ equal to-

(A) 1 (B) 3 (C) 5 (D) 6

Sol. **[C]**

$A = \frac{2+3}{2} = \frac{5}{2}$

$H = \frac{2.2.3}{2+3} = \frac{12}{5}$

$\Rightarrow A + \frac{6}{H} = \frac{5}{2} + \frac{5}{2} = 5$

Q.2 The sum of n terms of an A.P. is an (n – 1). The sum of the squares of these terms is-

(A) $a^2 n^2 (n-1)^2$

(B) $\frac{a^2}{6} n (n-1) (2n-1)$

(C) $\frac{2a^2}{3} n (n-1) (2n-1)$

(D) $\frac{2a^2}{3} n (n+1) (2n+1)$

Sol. **[C]**

$S_n = an (n-1)$ $\therefore$ $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = a (n-1) \{n - n + 2\}$ $\therefore$ $T_n = 2a (n-1)$

$(T_n)^2 = [2a (n-1)]^2 = 4a^2 (n-1)^2$

$(T_n)^2 = 4a^2 n^2 - 8a^2 n + 4a^2$

$\therefore \Sigma T_n^2 = 4a^2 \Sigma n^2 - 8a^2 \Sigma n + 4a^2 \Sigma 1$

$= 4a^2 \frac{n(n+1)(2n+1)}{6} - 8a^2 \frac{n(n+1)}{2} + 4a^2 n$

$= 4a^2 n\left(\frac{(n+1)(2n+1)}{6} - (n+1) + 1\right)$

$= \frac{2a^2 n}{3} (2n^2 - 3n + 1) = \frac{2a^2 n}{3} (n-1)(2n-1)$

Q.3 In the following two A.P.'s how many terms are identical? 2, 5, 8, 11.... to 60 terms; 3, 5, 7, .. 50 terms

(A) 15 (B) 16 (C) 17 (D) 18

Sol. **[C]**

2, 5, 8, 11, …to 60 terms $\Rightarrow$ 2, 5, 8, 11, …. 179.

3, 5, 7, …. to 50 terms $\Rightarrow$ 3, 5, 7, … 101.

Since the L.C.M. of the common differences of two Ap's is 6 therefore, we get a common term on adding 6 to the previous common term. Here 5 is the first common term which is followed by 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, 89, 95, 101.

Hence total Identical terms = 17 Ans.

Q.4 If p^{th}, q^{th} and r^{th} terms of an A.P. are in G.P., then the common ratio of G.P. is-

(A) $\frac{q-r}{p-q}$ (B) $\frac{r-q}{p-q}$

(C) $\frac{q-r}{q-p}$ (D) $\frac{q-p}{q-r}$

Sol. **[A]**

T_p. T_q, T_r are in G.P.

$\Rightarrow \frac{T_q}{T_p} = \frac{T_r}{T_q} = R$ (common ratio)

$\Rightarrow \frac{a+(q-1)d}{a+(p-1)d} = \frac{a+(r-1)d}{a+(q-1)d} = R$

$\Rightarrow \frac{d(q-1-r+1)}{d(p-1-q+1)} = R$

$\Rightarrow \frac{q-r}{p-q} = R$

$\therefore$ Common ratio $= \frac{q-r}{p-q}$ Ans.

Q.5 If the roots of cubic equation $ax^3 + bx^2 + cx + d = 0$ are in G.P., then-

(A) $c^3a = b^3d$ (B) $ca^3 = bd^3$

(C) $a^3b = c^3d$ (D) $ab^3 = cd^3$

Sol. **[A]**

Let roots are α, β, γ which are in G.P.

$\Rightarrow \beta^2 = \alpha\gamma$

$\Rightarrow \alpha + \beta + \gamma = \frac{-b}{a}$

$\alpha\beta\gamma = \frac{-d}{a}$

$\beta^3 = \frac{-d}{a}$

β is roots of $ax^3 + bx^2 + cx + d = 0$

$\Rightarrow a\beta^3 + b\beta^2 + c\beta + d = 0$

$$\Rightarrow a\left(\frac{-d}{a}\right) + b\left(\frac{-d}{a}\right)^{2/3} + c\left(\frac{-d}{a}\right)^{1/3} + d = 0$$

$$\Rightarrow b\left(\frac{d}{a}\right)^{2/3} = c\left(\frac{d}{a}\right)^{1/3}$$

$$\Rightarrow b^3 \frac{d^2}{a^2} = c^3 \frac{d}{a}$$

$$\Rightarrow b^3 d = c^3 a$$

Q.6 Let a and b be roots of $x^2 - 3x + p = 0$ and let c and d be roots of $x^2 - 12x + q = 0$ where a, b, c, d form an increasing G.P. then the ratio of

$q + p : q - p$ is equal to -

(A) 8 : 7 (B) 11 : 10

(C) 17 : 15 (D) None of these

Sol. **[C]**

a, b are roots of $x^2 - 3x + p = 0$

$\Rightarrow a + b = 3, ab = p$

c, d are roots of $x^{-2} - 12x + q = 0$

$\Rightarrow c + d = 12$ and $cd = q$

Now, a, b, c, d are in G.P.

$$\Rightarrow \frac{b}{a} = \frac{d}{c} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\Rightarrow \frac{(a-b)^2}{(a+b)^2} = \frac{(c-d)^2}{(c+d)^2}$$

$$\Rightarrow 1 - \frac{4ab}{(a+b)^2} = 1 - \frac{4cd}{(c+d)^2}$$

$$\Rightarrow \frac{ab}{(a+b)^2} = \frac{cd}{(c+d)^2}$$

$$\Rightarrow \frac{p}{9} = \frac{q}{144} \Rightarrow \frac{p}{1} = \frac{q}{16}$$

$$\Rightarrow \frac{p}{q} = \frac{1}{16} \Rightarrow \frac{p+q}{q-p} = \frac{17}{15}$$

$$\Rightarrow \frac{p+q}{q-p} = \frac{17}{15} = 17 : 15 \text{ Ans.}$$

Q.7 If S denotes the sum of infinity and S_n the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ such that $S - S_n < \frac{1}{1000}$, then the least value of n is-

(A) 11 (B) 9 (C) 10 (D) 8

Sol. **[A]**

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

$$S = \frac{1}{1 - \frac{1}{2}} = 2$$

$$S_n = \frac{1\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}} = 2\left(1 - \frac{1}{2^n}\right) = 2 - 2^{1-n}$$

$$\because S - S_n < \frac{1}{1000}$$

$\therefore 2 - 2 + 2^{1-n} < 10^{-3}$

$\Rightarrow 2^{1-n} < 10^{-3}$

$(1 - n) \log 2 < -3 \log 10$

$(n - 1) \log 2 > 3 \log 10$

$$(n - 1) > \frac{3 \log 10}{\log 2}$$

$$n - 1 > \frac{3}{0.3010}$$

$n - 1 > 9$

$n > 10$

$\therefore n = 11$

$\therefore$ least value of n is equal to 11

Q.8 The least value of 'a' for which $5^{1+x} + 5^{1-x}$, a/2, $25^x + 25^{-x}$ are three consecutive terms of an AP is

(A) 1 (B) 5

(C) 12 (D) None of these

Sol. **[C]**

$5^{1+x} + 5^{1-x}$, $\frac{a}{2}$, $25^x + 25^{-x}$ are in A.P.

$\Rightarrow 2(a/2) = (5^{1+x} + 5^{1-x}) + (25^x + 25^{-x})$

$\Rightarrow a = (5.5^x + 5.5^{-x}) + (5^{2x} + 5^{-2x})$

$$\Rightarrow a = 5\left(5^x + \frac{1}{5^x}\right) + \left(5^{2x} + \frac{1}{5^{2x}}\right)$$

We know that the sum of a positive real number and its reciprocal is always greater than or equal to 2.

$$\therefore 5^x + \frac{1}{5^x} \geq 2 \text{ and } 5^{2x} + \frac{1}{5^{2x}} \geq 2 \text{ for all x}$$

$\Rightarrow 5\left(5^x + \frac{1}{5^x}\right) + \left(5^{2x} + \frac{1}{5^{2x}}\right) \geq 5 \times 2 + 2$ for all x

$\Rightarrow a \geq 12$

$\therefore$ a =12

Q.9 If a, b, c are in G.P. then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in -

(A) A.P. (B) G.P.

(C) H.P (D) None of these

Sol. **[A]**

a, b, c are in G.P. $\Rightarrow b^2 = ac \Rightarrow b \Rightarrow \sqrt{ac}$

$\Rightarrow ax^2 + 2b\,x + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}\,x + c = 0$

$\Rightarrow \left(\sqrt{a}x + \sqrt{c}\right)^2 = 0$

$\Rightarrow x = \frac{-\sqrt{c}}{\sqrt{a}}$

Putting $x = \frac{-\sqrt{c}}{\sqrt{a}}$ in $dx^2 + 2e\,x + f = 0$, we get

$d.\frac{c}{a} - 2e\frac{\sqrt{c}}{\sqrt{a}} + f = 0$

$\Rightarrow \frac{dc}{a} + f = 2e\frac{\sqrt{c}}{\sqrt{a}}$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{\sqrt{ca}} = \frac{2e}{b}$ (Θ $\sqrt{ac} = b$)

$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2\frac{e}{b}$

$\Rightarrow 2\frac{e}{b} = \frac{d}{a} + \frac{f}{c}$

$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Q.10 A certain number is inserted between the number 3 and the unknown number so that the three numbers form an A.P. If the middle term is diminished by 6 then the number are in G.P. The unknown number can be -

(A) 3 (B) 15 (C) 18 (D) 25

Sol. **[A]**

3, P, K are in A.P.

P ↑ (Inserted number) K ↑ (Unknown number)

$\Rightarrow 2P = 3 + K$ …(1)

$\Rightarrow K = 2P - 3$

and 3, P – 6, K are in G.P.

$\Rightarrow (P - 6)^2 = 3K$ …(2)

from (1) & (2), we get

$(P - 6)^2 = 3(2P - 3)$

$\Rightarrow P^2 - 18P + 45 = 0$

$\Rightarrow P = 3, 15$

$\therefore$ Unknown number K = 2P – 3 when P = 3

K = 2 × 3 – 3

K = 6 – 3

K = 3 Ans.

When P = 15

K = 2 × 15 – 3

= 27

$\therefore$ 3, 3, 3 A.P. 3, 15, 27 A.P.

3, –3, 3 G.P. 3, 9, 27 G.P.

$\therefore$ K = 3 and 27 Ans.

Q.11 If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \text{.........upto } \infty = \frac{\pi^2}{6}$ then, $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{....}\infty = -$

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{8}$ (C) $\frac{\pi^2}{4}$ (D) π^2

Sol. **[B]**

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\infty = \frac{\pi^2}{6}$

$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ..\infty\right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + ..\infty\right)$

$= \frac{\pi^2}{6}$

$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ..\infty\right) + \frac{1}{2^2}$

$\Rightarrow \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + ..\infty\right) = \frac{\pi^2}{6}$

$$\Rightarrow \left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..\infty\right) + \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{6}$$

$$\Rightarrow \left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..\infty\right) = \frac{\pi^2}{6} - \frac{\pi^2}{24}$$

$$\Rightarrow \left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..\infty\right) = \frac{4\pi^2-\pi^2}{24} = \frac{3\pi^2}{24}$$

$$\Rightarrow \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..\infty = \frac{\pi^2}{8} \text{ Ans.}$$

Q.12 Let the numbers a_1, a_2, a_3 ..... a_n constitute a geometric progression. If $S = a_1 + a_2 + ... + a_n$, $T = \frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}$ and $P = a_1 a_2 a_3 ..... a_n$ then P^2 is equal to -

(A) $\left(\frac{S}{T}\right)^n$ (B) $\left(\frac{T}{S}\right)^n$

(C) $\left(\frac{2S}{T}\right)^n$ (D) $\left(\frac{2T}{S}\right)^n$

Sol. **[A]**

$a_1, a_2, a_3, \ldots a_n$ are in G.P.

$S = a_1 + a_2 + a_3 + \ldots + a_n$

$T = \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_n}$

$P = a_1 a_2 a_3 \ldots a_n$

Let $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, ... $a_n = ar^{n-1}$

$\therefore$ $S = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$

$S = a\left(\frac{r^n-1}{r-1}\right)$ and $P = a^n r^{\frac{n(n-1)}{2}}$

Also $T = \frac{1}{a}\frac{\{(1/r)^n - 1\}}{\left(\frac{1}{r}-1\right)} = \frac{1}{a}\left(\frac{1-r^n}{1-r}\right)\frac{1}{r^{n-1}}$

$\Rightarrow T = \frac{1}{a}\left(\frac{r^n-1}{r-1}\right)\frac{1}{r^{n-1}}$

$\therefore \frac{S}{T} = a\left(\frac{r^n-1}{r-1}\right) a \left(\frac{r-1}{r^n-1}\right) r^{n-1}$

$\Rightarrow \frac{S}{T} = a^2 r^{n-1}$

$\Rightarrow \left(\frac{S}{T}\right)^n = a^{2n} r^{n(n-1)} \Rightarrow \left(\frac{S}{T}\right)^n = \left[a^n r^{n\frac{(n-1)}{2}}\right]^2 = P^2$

$\Rightarrow \left(\frac{S}{T}\right)^n = P^2 \therefore P^2 = \left(\frac{S}{T}\right)^n$ Ans.

Q.13 If a, b, c are in H.P. then $\frac{1}{a} + \frac{1}{b+c}, \frac{1}{b} + \frac{1}{a+c}, \frac{1}{c} + \frac{1}{a+b}$ are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Sol. **[C]**

We have to prove that

$\frac{a+b+c}{a(b+c)}, \frac{a+b+c}{b(c+a)}, \frac{a+b+c}{c(a+b)}$ are in H.P.

Taking reciprocal and cancelled (a + b +c), we get a(b + c), b(c + a), c(a + b) are in A.P.

or (ab + bc + ca) – bc, Σab – ca, Σab – ab are in H.P.

or –bc, – ca, – ab are in A.P.

or $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. [divide by – abc]

or a, b, c are in H.P.

Q.14 The value of $\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$ is-

(A) $\frac{n}{\sqrt{a}+\sqrt{a+nx}}$ (B) $\frac{\sqrt{a+nx}-\sqrt{a}}{x}$

(C) $\frac{n(\sqrt{a+nx}-a)}{x}$ (D) None of these

Sol. **[A]**

$$\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$$

Since $\frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$

$$= \frac{\sqrt{a+rx}-\sqrt{a+(r-1)x}}{a+rx-a-(r-1)x}$$

$$= \frac{1}{x}\left[\sqrt{a+rx}-\sqrt{a+(r-1)x}\right]$$

$\therefore t_1 + t_2 + t_3 + \ldots + t_n$ is given by

$$\frac{1}{x}\left[\{\sqrt{a+x}-\sqrt{a}\}+\{\sqrt{a+2x}-\sqrt{a+x}\} +\ldots\ldots+\{\sqrt{a+nx}-\sqrt{a+(n-1)x}\}\right]$$

$$= \frac{1}{x}\left[\sqrt{a+nx}-\sqrt{a}\right] = \frac{a+nx-a}{x(\sqrt{a+nx}+\sqrt{a})}$$

$= \dfrac{n}{\sqrt{a+\sqrt{a+nx}}}$

Q.15 If $2\cdot{}^nP_1$, ${}^nP_2$, ${}^nP_3$, are three consecutive terms of an A.P. then they are-

(A) in G.P. (B) in H.P.

(C) equal (D) All of these

Sol. **[D]**

$\because\ 2\cdot{}^nP_1 , {}^nP_2 , {}^nP_3$ are in A.P.

$\Rightarrow 2\cdot{}^nP_2 = 2.{}^nP_1 + {}^nP_3$

or $2.\dfrac{n!}{(n-2)!} = 2.\dfrac{n!}{(n-1)!} + \dfrac{n!}{(n-3)!}$

or 2n (n – 1) = 2n + n (n – 1) (n – 2)

or 2 (n – 1) = 2 + (n – 1)(n – 2)

or n = 2, 3

clearly $n \neq 2$, so n = 3

$\therefore$ The numbers are $2.{}^3P_1$, ${}^3P_2$, ${}^3P_3$

= 2.3, 3.2, 3.2.1

= 6, 6, 6

Part-B **One or more than one correct answer type questions**

Q.16 S_r denotes the sum of the first r terms of an AP. Then $S_{3n} : (S_{2n} - S_n)$ is -

(A) n (B) 3n

(C) 3 (D) independent of n

Sol. **[C, D]**

$S_r = \dfrac{r}{2}$ [2a + (r – 1)d] where a is first term and d is common difference.

$\therefore S_{3n} = \dfrac{3n}{2}$ [2a + (3n – 1)d] …(i)

and $S_{2n} - S_n$

$= \dfrac{2n}{2}$ [2a + (2n – 1)d] – $\dfrac{n}{2}$ [2a + (n – 1)d]

$= \dfrac{n}{2}$ [2a + (3n – 1)d] …(ii)

Divide (i) by (ii), we get

$$\frac{S_{3n}}{S_{2n}-S_n} = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(3n-1)d]} = 3 \text{ Ans.}$$

Q.17 If $\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = an^4 + bn^3 + cn^2 + dn + e$ then -

(A) $a = \dfrac{1}{12}$ (B) $b = \dfrac{1}{6}$

(C) $d = \dfrac{1}{6}$ (D) e = 0

Sol. **[A, C, D]**

$$\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = \sum_{k=1}^{n}\frac{k(k+1)(2k+1)}{6}$$

$$= \frac{1}{6}\sum_{k=1}^{n}(2k^3 + 3k^2 + k)$$

$$= \frac{1}{3}\left\{\frac{n(n+1)}{2}\right\}^2 + \frac{1}{2}\,\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\,\frac{n(n+1)}{2}$$

a = coefficient of $n^4 = \dfrac{1}{3}.\dfrac{1}{4} = \dfrac{1}{12}$

b = coefficient of $n^3 = \dfrac{1}{3}.\dfrac{1}{2}+\dfrac{1}{6} = \dfrac{1}{6}+\dfrac{1}{6}=\dfrac{1}{3}$

Again, $\dfrac{1}{3}\left\{\dfrac{n(n+1)}{2}\right\}^2 + \dfrac{1}{2}\,\dfrac{n(n+1)(2n+1)}{6} + \dfrac{1}{6}\,\dfrac{n(n+1)}{2}$

$$= \frac{1}{3}.\left\{\frac{n^2(n^2+2n+1)}{4}\right\} + \frac{1}{12}n(2n^2+n+2n+1) + \frac{n(n+1)}{12}$$

$$= \frac{1}{12}\{n^4 + 2n^3 + n^2\} + \frac{1}{12}\{2n^3 + 3n^2 + n\} + \frac{1}{12}(n + n^2)$$

$= \frac{1}{12}.n^4 + 2n^3\left(\frac{1}{12}+\frac{1}{12}\right) + \left(\frac{n^2}{12}+\frac{3n^2}{12}+\frac{n^2}{12}\right)$

$+\left(\frac{n}{12}+\frac{n}{12}\right)$

$= \frac{1}{12}.n^4 + \frac{1}{3}n^3 + \frac{5}{12}n^2 + \frac{1}{6}.n$

$\therefore$ a = $\frac{1}{12}$, b = $\frac{1}{3}$, c = $\frac{5}{12}$, d = $\frac{1}{6}$, e = 0

$\therefore$ a = $\frac{1}{12}$, d = $\frac{1}{6}$, and e = 0 Ans.

Q.18 If $a^x = b^y = c^z$ and x, y, z are in GP then $\log_c b$ is equal to-

(A) $\log_b a$ (B) $\log_a b$

(C) z/y (D) None of these

Sol. **[A, C]**

$a^x = b^y = c^z$ = k (let)

$\because$ x, y, z are in G.P.

$\Rightarrow y^2 = xz$ …(1)

$\because a^x = k$

$\therefore$ x log a = log k

$\therefore x = \frac{\log k}{\log a} = \log_a k$

Similarly

y = $\log_b k$, z = $\log_c k$

Put values of x, y, z in eq^n (1)

$\Rightarrow (\log_b k)^2 = \log_a k \,.\, \log_c k$

$\Rightarrow \frac{(\log k)^2}{(\log b)^2} = \frac{(\log k)^2}{\log a \log c}$

$\Rightarrow (\log b)^2 = \log a \log c$

$\Rightarrow \frac{\log b}{\log c} = \frac{\log a}{\log b}$

$\Rightarrow \log_c b = \log_b a$ Ans.

and $\frac{z}{y} = \frac{\log_c k}{\log_b k} = \frac{\log k}{\log c} \times \frac{\log b}{\log k} = \frac{\log b}{\log c}$

$= \log_c b$

$\therefore \log_b a$ & $\frac{z}{y}$ are correct answers.

Q.19 Let f(x) = $\frac{1-x^{n+1}}{1-x}$ and

g(x) = $1 - \frac{2}{x} + \frac{3}{x^2} - \ldots + (-1)^n \frac{n+1}{x^n}$. Then the constant term in f ′(x) × g(x) is equal to-

(A) $\frac{n(n^2-1)}{6}$ when n is even

(B) $\frac{n(n+1)}{2}$ when n is odd

(C) $-\frac{n}{2}$ (n + 1) when n is even

(D) $-\frac{n(n-1)}{2}$ when n is odd

Sol. **[B, C]**

f(x) = $\frac{1-x^{n+1}}{1-x}$

$= 1 + x + x^2 + \ldots + x^n$

$\therefore f'(x).g(x) = (1 + 2x + 3x^2 + \ldots + n\,x^{n-1})$.

$\left(1-\frac{2}{x}+\frac{3}{x^2}\ldots+(-1)^n\frac{n+1}{x^n}\right)$

$\therefore$ required constant term $= 1 - 2^2 + 3^2 \ldots + (-1)^{n-1} n^2$

$\therefore 1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ldots\ldots - n^2$

= (1 + 2) (1 – 2) + (3 + 4) (3 – 4) + …

= – (1 + 2 + 3 + 4 + 5 + … n terms)

$= -\frac{n}{2}$ [n + 1] when n is even.

when n is odd,

sum = $\frac{n(n+1)}{2}$

Q.20 If a, b & c are distinct positive real which are in H.P., then the quadratic equation $ax^2 + 2bx + c = 0$ has-

(A) two non-real roots such that their sum is real

(B) two purely imaginary roots

(C) two non-real roots such that their product is real

(D) None of these

Sol. **[A, C]**

$ax^2 + 2bx + c = 0$

$D = 4\,(b^2 - ac)$

a, b, c are in H.P.

$b = \dfrac{2ac}{a+c}$ & $\dfrac{4a^2c^2}{(a+c)^2} - ac$

$ac = \dfrac{ab+bc}{2}$

$= \dfrac{4a^2c^2 - ac\left(a^2+c^2+2ac\right)}{(a+c)^2}$

$= -\dfrac{ac}{(a+c)^2}\,(a-c)^2 = -\,ve$

$x = \dfrac{-2b \pm \sqrt{4b^2-4ac}}{2.a}$

$x = \dfrac{-b}{a} \pm \dfrac{\sqrt{b^2-ac}}{a}$

$x = \dfrac{-b}{a} \pm i\,K$

Q.21 If AM of the number 5^{1+x} and 5^{1-x} is 13 then the set of possible real values of x is -

(A) $\left\{5, \dfrac{1}{5}\right\}$ (B) $\{1, -1\}$

(C) $\{x|x^2-1 = 0,\ x \in R\}$ (D) None of these

Sol. **[B, C]**

$\dfrac{5^{1+x}+5^{1-x}}{2} = 13$

$\Rightarrow 5.5^x + 5.5^{-x} = 26$

$\Rightarrow 5\left(5^x + \dfrac{1}{5^x}\right) = 26$

Let $5^x = t$

$\Rightarrow 5t^2 - 26\,t + 5 = 0$

$\Rightarrow (t-5)\,(5t-1) = 0$

$\Rightarrow t = 5,\ t = \dfrac{1}{5}$

$\Rightarrow 5^x = 5$ or $5^x = \dfrac{1}{5} = 5^{-1}$

$\Rightarrow x = 1$ or $x = -1$

Hence $x = \{-1, 1\}$

$\{x|x^2-1=0, x\in R\}$ Ans.

Q.22 If a, b, c are in H.P. then $\dfrac{1}{b-a} + \dfrac{1}{b-c} =$

(A) $\dfrac{2}{b}$ (B) $\dfrac{2}{a+c}$

(C) $\dfrac{1}{a} + \dfrac{1}{c}$ (D) None of these

Sol **.[A, C]**

a, b, c, are in H.P.

$\therefore\ b = \dfrac{2ac}{a+c}$

$\therefore\ \dfrac{1}{b-a} + \dfrac{1}{b-c}$

$= \dfrac{1}{\dfrac{2ac}{a+c}-a} + \dfrac{1}{\dfrac{2ac}{a+c}-c}$

$= \dfrac{a+c}{2ac-a^2-ac} + \dfrac{a+c}{2ac-ac-c^2}$

$= \dfrac{a+c}{a(c-a)} + \dfrac{a+c}{c(a-c)}$

$= (a+c)\left[\dfrac{1}{a(c-a)} - \dfrac{1}{c(c-a)}\right]$

$= \dfrac{a+c}{c-a}\left[\dfrac{1}{a} - \dfrac{1}{c}\right] = \dfrac{a+c}{c-a}\,.\,\dfrac{c-a}{ac} = \dfrac{a+c}{ac}$

$= \dfrac{1}{c} + \dfrac{1}{a}$ Ans.

Since $\dfrac{a+c}{ac}$ is equal to $\dfrac{2}{b}$ also

Θ $b = \dfrac{2ac}{a+c} \Rightarrow \dfrac{a+c}{ac} = \dfrac{2}{b}$ Ans.

➤ True or false type questions

Q.23 Equal numbers are always in A.P., G.P. and H.P.

Sol. No $\left(\underset{(\text{zero s})}{'0','0','0',\ldots\ldots}\text{are not in G.P. or H.P.}\right)$

Q.24 If $\dfrac{a+be^y}{a-be^y} = \dfrac{b+ce^y}{b-ce^y} = \dfrac{c+de^y}{c-de^y}$ then a, b, c, d are in H.P.

Sol. Θ $\dfrac{a+be^y}{a-be^y} = \dfrac{b+ce^y}{b-ce^y}$

$\Rightarrow \dfrac{2a}{2be^y} = \dfrac{2b}{2ce^y} \Rightarrow b^2 = ac$

and $\dfrac{b+ce^y}{b-ce^y} = \dfrac{c+de^y}{c-de^y}$

$\Rightarrow \dfrac{2b}{2ce^y} = \dfrac{2c}{2de^y} \Rightarrow c^2 = bd$

$\Rightarrow$ a, b, c, d are in G.P.

Q.25 There does not exist a H.P. all of whose terms are irrational.

Sol. **(False)**

There does not exist a H.P. all of whose terms are irrational. This statement is false.

Fill in the blanks type questions

Q.26 If t_n denotes the n^{th} term of the series $2 + 3 + 6 + 11 + 18 + \ldots\ldots$ then t_{50} is …………

Sol. $S = 2 + 3 + 6 + 11 + 18 + \ldots\ldots\ldots\ldots + T_{50}$

$S = \quad 2 + 3 + 6 + 11 + \ldots\ldots\ldots\ldots\ldots + T_{50}$

$\Rightarrow T_{50} = 2 + (1 + 3 + 5 + \ldots\ldots\ldots T_{49})$

$T_{50} = (2 + 49^2)$

Q.27 If $S_n = n^2a + \frac{n}{4}(n-1)\,d$ is the sum of first n terms of A.P., then common difference is……………..

Sol. $S_1 = a + 0 = a$

$S_2 = 4a + \frac{1}{2}\,d$

$T_1 = a,\ T_2 = S_2 - S_1 = 3a + \frac{1}{2}\,d$

$\Rightarrow D = T_2 - T_1 = 2a + \frac{1}{2}\,d$

Q.28 If x > 0 then the expression $\frac{x^{100}}{1+x+x^2+x^3+\ldots.+x^{200}}$ is always less than or equal to ………………..

Sol. $E = \frac{x^{100}}{1+x+x^2+\ldots..x^{200}}$

$= \frac{1}{(x^{-100}+x^{100})+(x^{-99}+x^{99})+\ldots\ldots(x^{-1}+x)} + 1$

AM $\geq$ GM

$\frac{x^{-100}+x^{100}}{2} \geq (x^{-100}\times x^{100})^{1/2} = 1$

So, $x^{-100} + x^{100} \geq 2$ etc

Hence $E \leq \frac{1}{\underset{(100\ \text{terms})}{(2+2+\ldots\ldots+2)}+1} = \frac{1}{201}$

Part-C Assertion-Reason type questions

The following questions 29 to 32 consists of two statements each, printed as Statement (1) and Statement (2). While answering these questions you are to choose any one of the following four responses.

(A) If both Statement (1) and Statement (2) are true and the Statement (2) is correct explanation of the Statement (1).

(B) If both Statement (1) and Statement (2) are true but Statement (2) is not correct explanation of the Statement (1).

(C) If Statement (1) is true but the Statement (2) is false.

(D) If Statement (1) is false but Statement (2) is true

Q.29 **Statement (1) :** 1, 2, 4, 8, ……. is a G.P., 4,8, 16, 32 is a G.P. and 1 + 4, 2 + 8, 4 + 16, 8 + 32,….. is also a G.P.

Statement (2) : Let general term of a G.P. with common ratio r be T_{k+1} and general term of another G.P. with common ratio r be T'_{k+1}, then the series whose general term $T''_{k+1} = T_{k+1} + T'_{k+1}$ is also a G. P. with common ratio r.

Sol.[A] Clearly $T''_{k+1} = T_{k+1} + T'_{k+1}$

$\Rightarrow$ option A is correct.

Q.30 **Statement (1) :** 3, 6, 12 are in G.P., then 9, 12, 18 are in H.P.

Statement (2) : If middle term is added in 3 consecutive terms of a G.P., resultant will be in H.P.

Sol.[A] True

Q.31 **Statement (1):** There exists an A.P. whose three terms are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

Statement (2): There exists distinct real numbers p, q, r satisfying $\sqrt{2} = A + (p-1)d$, $\sqrt{3} = A + (q-1)d$, $\sqrt{5} = A + (r-1)d$

Sol.[B] If we could show that reason R is false then assertion A will also be false. Indeed if R is true then

$\sqrt{2} - \sqrt{3} = (p-q)d$,

$\sqrt{3} - \sqrt{5} = (q-r)d$,

on dividing $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{5}} = \frac{p-q}{q-r}$

$\Rightarrow$ rational = irrational

$\Rightarrow$ Both A and R are false.

Q.32 **Statement (1) :** If three positive numbers in G.P. represent sides of a triangle then the common ratio of G.P. must lie between $\frac{\sqrt{5}-1}{2}$ and $\frac{\sqrt{5}+1}{2}$.

Statement (2) : Three positive real numbers can form a triangle if sum of any two sides is greater than the third.

Sol.[A] The assertion A can be proved by taking the intersection of the inequalities.

$a > 0,\ ar > 0,\ ar^2 > 0$

at $ar > ar^2$, $ar + ar^2 > a$, $ar^2 + a > ar$

The inequalities follow from reason.

Part-D Column Matching type questions

Q.33 Match the column

	Column-I	**Column-II**
(A)	If $\log_5 2$, $\log_5(2^x - 5)$ and $\log_5(2^x - 7/2)$ are in A.P., then value of 2x is equal to	(P) 6
(B)	Let S_n denote sum of first n terms of an A.P. If $S_{2n} = 3S_n$, then $\frac{S_{3n}}{S_n}$ is	(Q) 9
(C)	Sum of infinite series $4 + \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3} + \ldots$ is	(R) 3
(D)	The length, breadth, height of a rectangular box are in G.P., The volume is 27, the total surface area is 78. Then the length is	(S) 1

Sol. A → P, B → P, C → Q, D → Q, R, S

(A) $2\log_5(2^x - 5) = \log_5 2 + \log_5\left(2^x - \frac{7}{2}\right)$

$\Rightarrow (2^x - 5)^2 = 2.2^x - 7$

Let $2^x = t$

$\Rightarrow t^2 - 12t + 32 = 0$

$\Rightarrow (t - 4)(t - 8) = 0$

$\Rightarrow t = 4, 8$

$\Rightarrow 2^x = 2^2, 2^3 \Rightarrow x = 2, 3$

But x = 2 impossible

So $x = 3 \Rightarrow 2x = 6$

(B) $\because \frac{2n}{2}[2a + (2n-1)d] = \frac{3n}{2}[2a + (n-1)d]$

$\Rightarrow 2a = (n + 1)d$ …(1)

$$\Rightarrow \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]}$$

from (1) we get

$\frac{S_{3n}}{S_n} = 6$

(C) $4\left[1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots\right]$

$$= 4\left[\frac{1}{1-\frac{1}{3}} + \frac{\frac{1}{3}}{\left(1-\frac{1}{3}\right)^2}\right] = 4.\frac{9}{4} = 9$$

(D) Let $\frac{a}{r}$, a, ar be the sides of rectangular box then

$\frac{a}{r}.a.ar = 27 \Rightarrow a = 3$

and $2\left(\frac{a^2}{r} + a^2 r + a^2\right) = 78$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow (3r - 1)(r - 3) = 0$

$\Rightarrow r = 3, \frac{1}{3}$

Sides are 1, 3, 9 or 9, 3, 1

Length is 1 or 3 or 9.

Q.34 Match the following

	Column-I	**Column-II**
(A)	Suppose that $F(n + 1) = \frac{2F(n)+1}{2}$ for n = 1, 2, 3,…and F(1) = 2. Then F(101) equals	(P) 42
(B)	If $a_1, a_2, a_3, \ldots a_{21}$ are in A.P. and $a_3 + a_5 + a_{11} + a_{17} + a_{19} = 10$ then the value of $\sum_{i=1}^{21} a_i$ is	(Q)1620
(C)	10^{th} term of the sequence S = 1 + 5 + 13 + 29+….., is	(R) 52
(D)	The sum of all two digit numbers which are not divisible by 2 or 3 is	(S) 2045

Sol. A → R, B → P, C → S, D → Q

(A) $F(2) = \frac{5}{2}$, $F(3) = \frac{6}{2}$, $f(4) = \frac{7}{2}$

we get $F(101) = \frac{104}{2} = 52$

(B) $\because a_3 + a_{19} = a_5 + a_{17} = 2a_{11}$

$\Rightarrow 5a_{11} = 10 \Rightarrow a_{11} = 2$

$\sum_{i=1}^{21} a_i = 10(a_1 + a_{21}) + a_{11} = 21a_{11} = 42$

(C) $S = 1 + 5 + 13 + 29 + \ldots\ldots T_{10}$

$S = \quad 1 + 5 + 13 + \ldots\ldots + T_{10}$

Subtracting we get

$T_{10} = 1 + 4 + 8 + 16 + \ldots\ldots T_9$

= 1 + 4 (1 + 2 + 4 + ……. 9 term)

$= 1 + \frac{4(2^9 - 1)}{2-1} = 2045$

(D) Sum of all two digit number = 4905

sum of all two digit number which is divisible by 2 or 3 is = 2430 + 1665 – 810 = 3285

sum of all two digit number which is not divisible by 2 or 3 is = 4905 – 3285 = 1620

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Let $s_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \ldots + \frac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots\ldots+n^3}$; n = 1, 2, 3,........ Then s_n is not greater than.

Sol. $s_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \ldots + \frac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots\ldots+n^3}$;

n = 1, 2, 3,........

we have $t_n = \frac{1+2+3+\ldots+n}{1^3+2^3+3^3+\ldots+n^3}$

$= \frac{\Sigma n}{\Sigma n^3} = \frac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{\frac{n(n+1)}{2}}{\frac{n^2(n+1)^2}{4}}$

$= \frac{2}{n(n+1)} = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$\Rightarrow \sum_{k=1}^{n} t_k = 2\left\{\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n-1}\right)\right\}$

$= 2\left(1-\frac{1}{n+1}\right) = \frac{2n}{n+1}$

$\Rightarrow s_n = \frac{2n}{n+1} = 2\left(1-\frac{1}{n+1}\right) = 2 - \frac{2}{n+1}$

$\Rightarrow s_n = 2 - \frac{2}{n+1} \Rightarrow s_n = 2 - \frac{2}{n+1} < 2$

when $n \to \infty$, $s_n = 2$

$\therefore$ s_n is not greater than 2 Ans.

Q.2 If there be m AP's beginning with unity whose common difference are 1, 2, 3,......m respectively . Show that the sum of their n^{th} terms is (m/2) [mn – m + n +1]

Sol. Sum of n^{th} term is given by

= {1 + (n – 1) 1} + {1 + (n – 1)2}

+ {1 + (n – 1)3} + … + {1 + (n – 1)m}

= (1 + 1 + 1 + … m times) + (n – 1) [1 + 2 + 3 + … + m]

$= m + (n-1)\frac{m}{2}[1+m]$

$= \frac{2m+(n-1)(m+m^2)}{2}$

$= \frac{m}{2}[2 + (n-1)(1+m)]$

$= \frac{m}{2}[2 + n + mn - 1 - m]$

$= \frac{m}{2}[mn - m + n + 1]$ Ans.

Q.3 All terms of the arithmetic progression are natural numbers. The sum of its nine consecutive terms. beginning with the first, is larger than 200 and smaller than 220. Find the progression if its second term is equal to 12.

Sol. a, a + d, a + 2d ...

$s_9 = \frac{9}{2}[2a + 8d]$

$200 < s_9 < 220$

$200 < \frac{9}{2}[2a + 8d] < 220$

$\frac{200}{9} < a + 4d < \frac{220}{9}$ …(1)

Also given that

a + d = 12 …(2)

from (1) and (2) we get

$\frac{200}{9} < 12 + 3d < \frac{220}{9}$

$\frac{200-108}{9.3} < d < \frac{220-108}{9.3}$

$\frac{92}{27} < d < \frac{112}{27}$

3.40 < d < 4.14

$\Rightarrow$ d = 4

$\therefore$ a = 12 – 4 = 8

$\therefore$ series is 8, 12, 16, … Ans.

Q.4 Show that if $(b-c)^2$, $(c-a)^2$, $(a-b)^2$ are in A.P. then 1/(b – c) , 1/(c – a) , 1/(a – b) are also in A.P.

Sol. $(b-c)^2$, $(c-a)^2$, $(a-b)^2$ are in A.P.

$\Rightarrow (c-a)^2 - (b-c)^2 = (a-b)^2 - (c-a)^2$

$\Rightarrow$ (c – a + b – c) (c – a – b + c) = (a – b + c – a) (a – b – c + a)

$\Rightarrow$ (b – a) (2c – a – b) = (c – b) (2a – b – c)

$\Rightarrow$ (b – c) (b + c – 2a) = (a – b) (a + b – 2c)…(1)

Now $\frac{1}{c-a}-\frac{1}{b-c}=\frac{1}{a-b}-\frac{1}{c-a}$

$\Rightarrow \frac{a+b-2c}{b-c} = \frac{c+b-2a}{a-b}$

$\Rightarrow$ (a – b) (a + b – 2c) = (b – c) (b + c – 2a)…(2)

from (1) & (2), it is true. (Hence proved.)

Q.5 Show that the sum of the term in the n^{th} bracket (1) (3, 5) (7, 9,11) ....is n^3.

Sol. The successive group contains number of terms 1, 2, 3, ...

Therefore nth group contains n terms which are in A.P. and whose common difference is 2. Now we have to find first term. Successive group contains first term 1, 3, 7, 13, … whose successive difference are 2, 4, 6, … which are in A.P.

$s = 1 + 3 + 7 + 13 + \ldots + T_n$ …(1)

$s = \quad 1 + 3 + 7 + \ldots + T_{n-1} + T_n$ …(2)

on subtracting (1) and (2).

$0 = 1 + (2 + 4 + 6 + \ldots (n-1) \text{ terms}) - T_n$

$\therefore T_n = 1 + [(n-1)/2]\,[2.2 + (n-2).2]$

$T_n = 1 + (n-1)n = n^2 - n + 1$

The terms of nth group form an A.P. for which

$a = n^2 - n + 1, d = 2, n = n$

$\therefore s_n = \frac{n}{2}\,[2(n^2 - n + 1) + (n-1).2]$

$= n[n^2 - n + 1 + n - 1]$

$= n.n^2$

$= n^3$ (Hence proved)

Q.6 The sum of the series

$1+\left(1+\frac{1}{2}\right)\frac{1}{3}+\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2\right)\left(\frac{1}{3}\right)^2+\ldots$to

infinite terms is -

Sol. Let

$s = 1+\left(1+\frac{1}{2}\right)\frac{1}{3}+\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2\right)\left(\frac{1}{3}\right)^2$

$+\ldots \infty$ terms …(1)

$\frac{1}{3}s = \left(\frac{1}{3}\right) \quad +\left(1+\frac{1}{2}\right)\left(\frac{1}{3}\right)^2+\ldots$ …(2)

on subtracting eq^n (1) and (2).

$\left(s-\frac{1}{3}s\right) = 1 + \left(\frac{1}{3}\right)\left(1+\frac{1}{2}-1\right) +$

$\left(\frac{1}{3}\right)^2\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2-1-\frac{1}{2}\right)+\ldots$

$\Rightarrow \frac{2}{3}s = 1+\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^3\left(\frac{1}{2}\right)^3+\ldots$

$\Rightarrow \frac{2}{3}s = 1+\frac{1}{6}+\left(\frac{1}{6}\right)^2+\left(\frac{1}{6}\right)^3+\ldots$

$\Rightarrow \frac{2}{3}s = \frac{1}{1-\frac{1}{6}}$

$\Rightarrow \frac{2}{3}s = \frac{1}{5/6}$

$\Rightarrow s = \frac{6}{5}\times\frac{3}{2}$

$\Rightarrow s = \frac{9}{5}$ Ans.

Q. 7 Find the n^{th} term and the sum to n terms of the sequence-

(A) 1 + 5 + 13 + 29 + 61 + ...

(B) 6 + 13 + 22 + 33 + ...

(C) The sum of infinite terms of the progression $1+ 3x + 5x^2 + 7x^3 + \ldots\ldots (x<1)$ is-

(D) Sum the series to n terms and to ∞. $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots\ldots.$

Sol. (A) $S = 1 + 5 + 13 + 29 \ldots\ldots.. T_n$

$S = \quad 1 + 5 + 13 + \ldots\ldots\ldots T_{n-1} + T_n$

On Subtracting

$0 = 1 + 4 + 8 + 16 + \ldots\ldots\ldots T_n - T_{n-1} - T_n$

$T_n = 1 + \underbrace{(4 + 8 + 16 + \ldots\ldots\ldots.. T_n - T_{n-1})}_{n-1}$

$T_n = 1 + \frac{4(2^{n-1}-1)}{2-1} = 1 + 4(2^{n-1} - 1) = 2^{n+1} - 3$

$S = \Sigma 2^{n+1} - 3 = \frac{2^2(2^n-1)}{2-1} - 3n = 2^{n+2} - 4 - 3n$

(B) $S = 6 + 13 + 22 + 33 + \ldots.T_n$

$S = \quad 6 + 13 + 22 + \ldots. T_{n-1} + T_n$

$T_n = 6 + (7 + 9 + 11 + \ldots\ldots. (T_n - T_{n-1}))$

$\longleftarrow n-1 \longrightarrow$

$$T_n = 6 + \frac{n-1}{2}(14 + (n-2)2)$$
$$= 6 + (n-1)(n+5)$$
$$= n^2 + 4n + 1$$
$$S = \Sigma n^2 + 4\Sigma n + \Sigma 1$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} + n$$

(C) $S = 1 + 3x + 5x^2 + 7x^3 + \ldots\ldots\ldots \infty$

$Sx = x + 3x^2 + 5x^3 + \ldots\ldots. \infty$

$S(1-x) = 1 + 2x(1 + x + x^2 + \ldots.. \infty)$

$$S(1-x) = 1 + \frac{2x}{1-x} = \frac{1+x}{1-x}$$

$$\Rightarrow S = \frac{1+x}{(1-x)^2}$$

(D) $S = 1 + \frac{4}{5} + \frac{7}{5^2} + \ldots\ldots\ldots \infty$

$$\frac{S}{5} = \frac{1}{5} + \frac{4}{5^2} + \ldots\ldots. \infty$$

$$\frac{4S}{5} = 1 + \frac{3}{5}\left(1 + \frac{1}{5} + \ldots..\infty\right)$$

$$= 1 + \frac{3}{5\left(1-\frac{1}{5}\right)}$$

$$= 1 + \frac{3}{4} = \frac{7}{4}$$

$$S = \frac{35}{16}$$

Q.8 Find the sum of the series upto n terms $1.3.5 + 3.5.7 + 5.7.9 + \ldots.$

Sol. The r^{th} term of the series is given by

$t_r = (2r-1)(2r+1)(2r+3)$

$t_r = 8r^3 + 12r^2 - 2r - 3$

$$\therefore S_n = 8\sum_{r=1}^{n} r^3 + 12\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r - 3n$$

$$= 8\left[\frac{n(n+1)}{2}\right]^2 + 12\,\frac{n(n+1)(2n+1)}{6} -$$

$$2\,\frac{n(n+1)}{2} - 3n$$

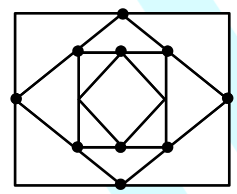
$= 2n^2(n+1)^2 + 2n(n+1)(2n+1) - n(n+1) - 3n$

$= n(2n^3 + 4n^2 + 2n + 4n^2 + 6n + 2 - n - 1 - 3)$

$= n(2n^3 + 8n^2 + 7n - 2)$ Ans.

Q.9 A square is drawn by joining the mid- points of the sides of a given square. A third square is drawn inside the second square in the same way and this process continuous indefiniting. If a side of the first square is 4 cm determine the sum of the area of all the squares.

Sol. If a side of any square is x cm, then the side of the square obtained by joining its mid-points is given by



$$\sqrt{\left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2} = \frac{x}{\sqrt{2}} \text{ cm}$$

and such its area is

$$\left(\frac{x}{\sqrt{2}}\right)^2 = \frac{x^2}{2} \text{ cm}^2$$

Now the area of the first square is $4^2 = 16$ sq cm. the area of the second square is 859. cm, the area of the third square is 4 square cm and so on. Hence the sum of the areas is given by

$16 + 8 + 4 + 2 + \ldots$ upto infinite

$$= \frac{16}{1-(1/2)}$$

$$= \frac{16}{1/2}$$

$= 16 \times 2$

$= 32\text{ cm}^2$ Ans.

Q.10 The value of xyz is 15/2 or 18/5 according as the series a, x, y, z, b is an A.P. or H.P. Find the values of a and b assuming them to be positive integer.

Sol. a = 1, b = 3 or b = 1, a = 3

Q.11 If three positive numbers a, b , c are in HP, then prove that $\frac{a+b}{2a-b} + \frac{c+b}{2c-b} > 4$.

Sol. Θ a, b, c, are positive numbers are in H.P.

$$\therefore \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \quad \ldots (1)$$

$$\text{LHS} = \frac{\frac{1}{b}+\frac{1}{a}}{\frac{2}{b}-\frac{1}{a}} + \frac{\frac{1}{b}+\frac{1}{c}}{\frac{2}{b}-\frac{1}{c}}$$

$$= \frac{\frac{1}{b}+\frac{1}{a}}{\frac{1}{c}} + \frac{\frac{1}{b}+\frac{1}{c}}{\frac{1}{a}} \text{ from (1)}$$

$$= \frac{c}{b}+\frac{c}{a}+\frac{a}{b}+\frac{a}{c} = \frac{a+c}{b}+\frac{a}{c}+\frac{c}{a}$$

Now, A.M. > G.M.

$$\Rightarrow \frac{\frac{a}{c}+\frac{c}{a}}{2} > \sqrt{\frac{a}{c}.\frac{c}{a}}$$

or $\frac{a}{c}+\frac{c}{a} > 2$

A > H

$$\Rightarrow \frac{a+c}{2} > \frac{2ac}{a+c} = b \text{ using (1)}$$

$$\Rightarrow \frac{a+c}{b} > 2$$

$$\therefore \text{LHS} = \frac{a+c}{b} + \left(\frac{a}{c}+\frac{c}{a}\right)$$

= > 2 + > 2

= > (2 + 2)

= > 4

Q.12 The value of x + y + z is 15, if a, x, y, z, b are in AP while the value of : (1/x) + (1/y) + (1/z) is 5/3 y if a , x, y , z, b are in HP. Find a and b.

Sol. We know that the sum of n arithmetic means between two numbers is equal to n times of A.M. of these two numbers. There are three A.M.'s x, y, z between a and b.

$$x + y + z = 3\left(\frac{a+b}{2}\right) = 15$$

or a + b = 10 … (1)

a, x, y, z, b are in H.P.

$\therefore \frac{1}{a},\frac{1}{x},\frac{1}{y},\frac{1}{z},\frac{1}{b}$ are in A.P.

$$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$= \frac{3}{2}\left(\frac{a+b}{ab}\right)$$

$$= 3\left(\frac{a+b}{2ab}\right)$$

or $\frac{5}{3} = \frac{3}{2ab}.10$ from (1)

$\therefore$ ab = 9 …(2)

from (1) & (2), we get

$$a + \frac{9}{a} = 10$$

$\Rightarrow a^2 - 10a + 9 = 0$

$\Rightarrow a^2 - 9a - a + 9 = 0$

$\Rightarrow$ a = 9, 1

Similarly, $b^2 - 10b + 9 = 0$

$\Rightarrow$ b = 1, 9

$\therefore$ a = 1, b = 9 & a = 9, b = 1 Ans

Q.13 Sum the following series to n terms and to infinity-

(i) $\frac{1}{1.3.5}+\frac{1}{3.5.7}+\frac{1}{5.7.9}+\ldots$

(ii) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$ (iii) $\sum_{r=1}^{n}\frac{1}{4r^2-1}$

Sol. (i)

$$\frac{1}{1.3.5}+\frac{1}{3.5.7}+\frac{1}{5.7.9}+\ldots$$

$$T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}$$

put 2n = 1, –1, –3 and divide in partial fractions.

$$T_n = \frac{1}{8(2n-1)} - \frac{2}{8(2n+1)} + \frac{1}{8(2n+3)}$$

$$= \frac{1}{8}\left[\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)-\left(\frac{1}{2n+1}-\frac{1}{2n+3}\right)\right]$$

Now put n = 1, 2, 3, … n and on adding, we get

$$S_n = \frac{1}{8}\left[\left(1-\frac{1}{2n+1}\right)-\left(\frac{1}{3}-\frac{1}{2n+3}\right)\right]$$

$$= \frac{1}{8}\left[\frac{2n}{2n+1}-\frac{2n}{3(2n+3)}\right]$$

$$= \frac{n}{4}\left[\frac{4n+8}{3(2n+1)(2n+3)}\right]$$

$$= \frac{n(n+2)}{3(2n+1)(2n+3)} \text{ Ans.}$$

Now, we have to find sum of infinite terms

$$S_n = \frac{n^2\left(1+\frac{2}{n}\right)}{3.4n^2\left(1+\frac{1}{2n}\right)\left(1+\frac{3}{2n}\right)} = \frac{\left(1+\frac{2}{n}\right)}{12\left(1+\frac{1}{2n}\right)\left(1+\frac{3}{2n}\right)}$$

when $n \to \infty$

$S_\infty = \frac{1}{12}.\frac{1}{1} = \frac{1}{12}$

(iv) $\sum_{r=1}^{n} \frac{1}{4r^2-1}$

$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} \Rightarrow \sum_{r=1}^{n} \frac{1}{2}\left[\frac{1}{2r-1} - \frac{1}{2r+1}\right]$

$\Rightarrow S_n = \frac{1}{2}\left[1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} + \ldots - \frac{1}{2n+1}\right]$

$\Rightarrow S_n = \frac{1}{2}\left[1 - \frac{1}{2n+1}\right]$

$\Rightarrow S_n = \frac{1}{2}\left[\frac{2n+1-1}{2n+1}\right]$

$\Rightarrow S_n = \frac{n}{2n+1}$ Ans.

when $n \to \infty$, $S_n = \frac{n}{n\left(2 + \frac{1}{n}\right)}$

$S_\infty = \frac{1}{2 + \frac{1}{n}}$

$S_\infty = \frac{1}{2}$ Ans.

Q.14 Find the sum of n terms of the sequence

$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots$

Sol. Let T_n be the n^{th} term of the series

$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots$

$\therefore T_n = \frac{n}{1+n^2+n^4} = \frac{n}{(1+n^2)^2 - n^2} =$

$\frac{n}{(n^2+n+1)(n^2-n+1)}$

$\therefore T_n = \frac{1}{2}\left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right]$

$= \frac{1}{2}\left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)}\right]$

Now, $\sum_{r=1}^{n} T_r = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{1+1.2}\right]$

$+ \frac{1}{2}\left[\frac{1}{1+1.2} - \frac{1}{1+2.3}\right] + \frac{1}{2}\left[\frac{1}{1+2.3} - \frac{1}{1+3.4}\right]$

$+ \frac{1}{2}\left[\frac{1}{1+2.3} - \frac{1}{1+3.4}\right] + \ldots +$

$\frac{1}{2}\left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)}\right]$

$= \frac{1}{2}\left[1 - \frac{1}{1+n(n+1)}\right]$

$= \frac{n(n+1)}{2(n^2+n+1)}$ Ans.

Q.15 Obtain the sum of

$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{4}{x^4+1} + \ldots\ldots\ldots\ldots + \frac{2^n}{x^{2^n}+1}$

Sol. $\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{4}{x^4+1} + \ldots\ldots\ldots\ldots + \frac{2^n}{x^{2^n}+1}$

$\frac{1}{x-1} - \frac{1}{x-1} + \frac{1}{x+1} + \frac{2}{x^2+1} + \frac{4}{x^4+1}$

$\ldots\ldots + \frac{2^n}{x^{2^n}+1}$

$\frac{1}{x-1} - \left(\frac{1}{x-1} - \frac{1}{x+1}\right) + \frac{2}{x^2+1} -$

$\frac{4}{x^4+1} \ldots\ldots + \frac{2^n}{x^{2^n}+1}$

$= \frac{1}{x-1} - \left(\frac{2}{x^2-1} - \frac{2}{x^2+1}\right) + \frac{4}{x^4+1} \ldots \frac{2^n}{x^{2^n}+1}$

$\Rightarrow \frac{1}{x-1} - \frac{2^{n+1}}{x^{2^{n+1}}-1}$ Ans.

Q.16 Find the sum of the n terms of the sequence, whose general term is given by

$$a_r = \frac{r^5 + 6r^4 + 11r^3 + 6r^2 + 4r + 6}{r^4 + 6r^3 + 11r^2 + 6r}$$

Sol. $a_r = \frac{r^5 + 6r^4 + 11r^3 + 6r^2 + 4r + 6}{r^4 + 6r^3 + 11r^2 + 6r}$

$= r + \frac{4r}{r(r+1)(r+2)(r+3)} + \frac{6}{r(r+1)(r+2)(r+3)}$

$$a_r = r + \frac{4}{2}\left(\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)}\right)$$

$$+ \frac{6}{3}\left(\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)}\right)$$

$$S = \sum_{r=1}^{n} a_r$$

$$S = \frac{n(n+1)}{2} +$$

$$2\left(\frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \ldots \frac{1}{(n+1)(n+2)} - \frac{1}{(n+2)(n+3)}\right) +$$

$$2\left(\frac{1}{1.2.3} - \frac{1}{2.3.4} + \frac{1}{2.3.4} - \frac{1}{3.4.5} + \ldots \frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)}\right)$$

$$S = \frac{n(n+1)}{2} + 2\left(\frac{1}{6} - \frac{1}{(n+2)(n+3)}\right)$$

$$+ 2\left(\frac{1}{6} - \frac{1}{(n+1)(n+2)(n+3)}\right)$$

Simplify Yourself

Part-B Passage based objective questions

Passage-1 (Q. 17 to Q.19)

The arithmetic mean of two positive numbers p and q exceeds their geometric mean by 1/2 and their G.M. exceeds their H.M. by 1/4, the minimum value of the quadratic expression of the form $x^2 + \alpha x + \beta$ whose zeros are p and q is 'm'. Also if $(1 - b)(1 + 2a + 4a^2 + 8a^3 + 16a^4 + 32a^5) = 1 - b^6$ $(b \neq 1)$ and b/a = n, then answer the following questions:

Sol. Given that

$$\frac{p+q}{2} - \frac{1}{2} = \sqrt{pq}$$

$\Rightarrow [(p + q) - 1]^2 = 4pq$ …(1)

and $\frac{2pq}{p+q} + \frac{1}{4} = \sqrt{pq}$

$\Rightarrow [8pq + (p + q)]^2 = 16(p + q)^2 pq$ …(2)

Solving (1) and (2) we get

$p + q = 2$ and $pq = \frac{1}{4}$

$\because$ zeroes of $x^2 + \alpha x + \beta$ is p and q

$\Rightarrow \alpha = -(p + q), \beta = pq$

$\alpha = -2, \beta = \frac{1}{4}$

Expression is $x^2 - 2x + \frac{1}{4}$

and $b = 2a \Rightarrow n = 2$

Q.17 The quadratic equation whose roots are p, q is
(A) $4x^2 - 8x + 1 = 0$ (B) $4x^2 + 8x - 1 = 0$
(C) $2x^2 - 8x + 1 = 0$ (D) $2x^2 + 8x - 1 = 0$

Sol.[A] Equation whose roots are p and q is

$x^2 - 2x + \frac{1}{4} = 0 \Rightarrow 4x^2 - 8x + 1 = 0$

Q.18 The value of 'm' is

(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$

Sol. [B] Minimum value of $x^2 - 2x + \frac{1}{4}$ is $-\frac{4-1}{4} = -\frac{3}{4}$.

Q.19 Value of 'n' in terms of 'm' is

(A) $\frac{8m}{3}$ (B) $\frac{8}{3m}$ (C) $-\frac{8m}{3}$ (D) $\frac{-8}{3m}$

Sol. [C] $n = 2 \Rightarrow n = 2\left(-\frac{3}{4}\right).\left(-\frac{4}{3}\right)$

$\Rightarrow n = -\frac{8m}{3}$ or $-\frac{3}{2m}$

Passage-2 (Q. 20 to Q.22)

In a sequence of (4n + 1) terms the first (2n +1) terms are in A.P. whose common difference is 2, and the last (2n +1) terms are in G.P. whose common ratio 0.5. If the middle terms of the AP and GP are equal, then

Sol. A.P. is a, a + 2, a + 4, ……. (a + 4n)

& G.P. is $(a+4n), \frac{(a+4n)}{2}, \left(\frac{1+4n}{2^2}\right), \ldots \left(\frac{a+4n}{2^{2n}}\right)$

middle term of A.P. = middle term of G.P.

$\Rightarrow a + 2n = \frac{a+4n}{2^n} \Rightarrow a = \frac{4n - 2n.2^n}{2^n - 1}$ …(1)

Q.20 Middle term of the sequence is

(A) $\frac{n.2^{n+1}}{2^n - 1}$ (B) $\frac{n.2^{n+1}}{2^{2n} - 1}$
(C) $n.2^n$ (D) None of these

Sol. [A] Middle term of sequence is
$T_{2n+1} = a + 4n$ from (1) we get

$T_{2n+1} = \frac{n.2^{n+1}}{2^n - 1}$

Q.21 First term of the sequence is

(A) $\frac{4n+2n.2^n}{2^n-1}$ (B) $\frac{4n-2n.2^n}{2^n-1}$

(C) $\frac{2n-n.2^n}{2^n-1}$ (D) $\frac{2n+n.2^n}{2^n-1}$

Sol. [B] First term is a

From (1) a = $\frac{4n-2n.2^n}{2^n-1}$

Q.22 Middle term of the GP is

(A) $\frac{2^n}{2^n-1}$ (B) $\frac{n.2^n}{2^n-1}$

(C) $\frac{n}{2^n-1}$ (D) $\frac{2n}{2^n-1}$

Sol. [D] Middle term of G.P. = $\frac{a+4n}{2^n}$

From (1) we get

$T_{middle} = \frac{2n}{2^n-1}$

Passage-3 (Q. 23 to Q.25)

Let $A_1, A_2, A_3, \ldots, A_m$ be arithmetic means between –2 and 1027 and $G_1, G_2, G_3, \ldots, G_n$ be geometric means between 1 and 1024. Product of geometric means is 2^{45} and sum of arithmetic means is 1025 × 171.

Sol. ∵ $A_1, A_2 \ldots\ldots A_m$ be arithmetic means between – 2 and 1027

$\Rightarrow d = \frac{1029}{m+1}$

and $G_1\ G_2 \ldots G_n$ be the geometric means between 1 and 1024

$\Rightarrow r = (1024)^{\frac{1}{n+1}}$

Sum of A.M's = $\left(\frac{-2+1027}{2}\right)$ m

$1025 \times 171 = \frac{1025}{2}$ m ...(1)

Product of G.M's = $(\sqrt{1\times1024})^n$

$2^{45} = 2^{\frac{10n}{2}} = 2^{5n}$ ...(2)

Q.23 The value of n, m is

(A) 7, 340 (B) 9,342

(C) 11, 344 (D) None of these

Sol. [B] Value of n, m is

From (1) and (2) m = 342, n = 9

Q.24 The value of $G_1 + G_2 + G_3 + \ldots + G_n$ is

(A) 1022 (B) 2044

(C) 512 (D) None of these

Sol. [A] ∵ r = 2, a = 1

$G_1 + G_2 + \ldots + G_9 = 2 + 2^2 + 2^3 + \ldots + 2^9$

$= \frac{2(2^9-1)}{2-1} = 2 \times 511 = 1022$

Q.25 The numbers $2A_{171}$, $G_{25}+1$, $2A_{172}$ are in

(A) A.P. (B) G.P.

(C) H.P. (D) A.G.P.

Sol. [A] $2A_{171} = 2(-2 + 171 \times 3) = 2(511) = 1022$

∵ d = 3

$G_5^2 + 1 = (2^5)^2 + 1 = 1025$

$2A_{172} = 2(-2 + 172 \times 3) = 1028$

Clearly $\frac{1022+1028}{2}$ = 1025 are in A.P.

EXERCISE # 4

Old IIT-JEE Questions

Q.1 Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are- **[IIT Sc. 2001]**

(A) – 2, – 32 (B) – 2, 3

(C) – 6, 3 (D) – 6, – 32

Q.2 Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are - **[IIT Sc.-2001]**

(A) Not in A.P./G.P./H.P.

(B) in A.P.

(C) in G.P.

(D) in H.P.

Sol. **[D]**

a, b, c, d are in A.P.

$\Rightarrow \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ are in A.P.

$\Rightarrow \frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$ are in A.P.

$\Rightarrow \frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ are in A.P.

$\Rightarrow$ abc, abd, acd, bcd are in H.P. Ans.

Q.3 If the sum of the first 2n terms of the A.P. 2, 5, 8,.... is equal to the sum of the first n terms of the A.P.57, 59, 61,.... then n equals- **[IIT Sc. 2001]**

(A) 10 (B) 12 (C) 11 (D) 13

Sol. **[C]**

2, 5, 8, … 57, 59, 61, …

$\frac{2n}{2}[4 + (2n - 1)3] = \frac{n}{2}[114 + (n - 1)2]$

$8 + 12n - 6 = 114 + 2n - 2$

$10\,n = 110$

n = 11 Ans.

Q.4 Let a_1, a_2 ...be positive real numbers in geometric progression for each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean and harmonic mean of $a_1, a_2, ..., a_n$. Find an expression for the geometric mean of $G_1, G_2 ...... G_n$ in terms of $A_1, A_2, ... A_n, H_1, H_2, ..., H_n$ **[IIT-2001]**

Sol. Let a be the first term and r be the common ratio of the G.P., $a_1, a_2, a_3 \ldots$ then

$a_k = ar^{k-1}$ for k = 1, 2, 3, …

It is given that a_1, a_2, a_3, .. are positive real numbers, therefore a > 0 and r > 0

Now two cases are :

Case-I : When r = 1

In this case, we have $a_1 = a_2 = \ldots = a_n = a$

$\therefore A_n = \frac{1}{n}(a_1 + a_2 + \ldots + a_n) = a$

$G_n = (a_1a_2a_3 \ldots a_n)^{1/n} = a$

and, $\frac{1}{H_n} = \frac{\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}}{n} = \frac{1}{a}$

$\therefore H_n = a$

Also $A_nH_n = a^2 = G_n^2$

Now, let G be the geometric mean of $G_1, G_2, \ldots G_n$ then

$G = (G_1, G_2, \ldots G_n)^{1/n}$

$= \left(\sqrt{A_1H_1}\sqrt{A_2H_2}\sqrt{A_3H_3}..\sqrt{A_nH_n}\right)^{1/n}$

$= (A_1A_2\,A_3 \ldots..A_n.\ H_1H_2H_3 \ldots H_n)^{1/2n}$

Case – II When $r \neq 1$

We have,

$A_n = \frac{1}{n}(a_1 + a_2 + \ldots + a_n) = \frac{1}{n}(a + ar + \ldots + ar^{n-1})$

$= \frac{1}{n}\left\{a\left(\frac{1-r^n}{1-r}\right)\right\}$

$G_n = (a_1a_2 \ldots a_n)^{1/2}$

$= (a.\ ar. \ldots ar^{n-1})^{1/n}$

$= \{a^n\ r^{\frac{n(n-1)}{2}}\}^{1/2}$

$= ar^{\frac{n-1}{2}}$

and, $\frac{1}{H_n} = \frac{1}{n}\left(\frac{1}{a_1}+\frac{1}{a_2}+\ldots\frac{1}{a_n}\right)$

$= \frac{1}{n}\left(\frac{1}{a}+\frac{1}{ar}+\ldots\frac{1}{ar^{n-1}}\right)$

$= \frac{1}{n}\left[\frac{1}{a}\left\{\frac{1-\left(\frac{1}{r}\right)^n}{1-\frac{1}{r}}\right\}\right]$

$= \frac{1}{n}\left(\frac{1-r^n}{1-r}\right) - \frac{1}{ar^{n-1}}$

$= \frac{1}{n}\left(\frac{1-r^n}{1-r}\right) . \frac{1}{ar^{n-1}}$

$H_n \Rightarrow \frac{n(1-r)ar^{n-1}}{1-r^n}$

Thus, $A_n H_n = \frac{a(1-r^n)}{n(1-r)} . \frac{n(1-r)ar^{n-1}}{1-r^n}$

$= a^2 r^{n-1} = G_n^2$

Let G be the geometric mean of $G_1, G_2, G_3, \ldots G_n$ then,

$G = (G_1 G_2 G_3 \ldots G_n)^{1/n}$

$= \left(\sqrt{A_1H_1}\sqrt{A_2H_2}\sqrt{A_3H_3}\ldots\sqrt{A_nH_n}\right)^{1/n}$

$= (A_1 A_2 A_3 \ldots A_n . H_1H_2H_3 \ldots H_n)^{1/2n}$

$= (A_1H_1A_2H_2A_3H_3\ldots)^{1/2n}$ Ans.

Q.5 If a_1 , a_2, ..., a_n are positive real numbers whose product is a fixed number c , then the minimum value of $a_1 + a_2 + \ldots\ldots + a_{n-1} + 2a_n$ is –

[IIT Sc.2002]

(A) $n (2c)^{1/n}$ (B) $(n+1)c^{1/n}$

(C) $2nc^{1/n}$ (D) $(n+1) (2c)^{1/n}$

Sol. **[A]**

Using A.M. ≥ G.M.

$\Rightarrow \frac{a_1 + a_2 + \ldots + 2a_n}{n} \geq (a_1 a_2 \ldots 2a_n)^{1/n}$

$\Rightarrow a_1 + a_2 + \ldots + 2a_n \geq n(2 a_1 a_2 \ldots a_n)^{1/n}$

$\geq n(2 c)^{1/n}$ Ans.

Q. 6 Suppose a, b & c are in A.P. and a^2, b^2, c^2 are in G.P. If a< b < c and a + b+ c = $\frac{3}{2}$, then the value of **'a'** is - **[IIT Sc.2002]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$

(C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

Sol. **[D]**

We have $b = \frac{a+c}{2}$ … (1)

and $b^4 = a^2c^2$ ... (2)

$(1) \Rightarrow 2b = \frac{3}{2} - b$

$\Rightarrow 3b = \frac{3}{2}$

$\Rightarrow b = \frac{1}{2}$ $(\because a + b + c = \frac{3}{2})$

$(2) \Rightarrow b^2 = \pm ac \Rightarrow \frac{1}{4} = \pm ac \Rightarrow ac = \pm \frac{1}{4}$ …(3)

$a + b + c = \frac{3}{2} \Rightarrow a + \frac{1}{2} + c = \frac{3}{2} \Rightarrow a + c = 1$…(4)

$a + c = 1$ and $ac = \frac{1}{4}$ Implies that

a, c are roots of $x^2 - x + \frac{1}{4} = 0$

$\therefore$ a, c = $\frac{1}{2}$, $\frac{1}{2}$ i.e. a = c

This is impossible because a < b < c.

Also, a + c = 1 and ac = $-\frac{1}{4}$ implies that a, c are roots of $x^2 - x - \frac{1}{4} = 0$

$\therefore a, c = \frac{1 \pm \sqrt{2}}{2}$

$\therefore a = \frac{1-\sqrt{2}}{2}$, $c = \frac{1+\sqrt{2}}{2}$ because a < b < c

$\therefore a = \frac{1-\sqrt{2}}{2} = \frac{1}{2} - \frac{\sqrt{2}}{2} = \frac{1}{2} - \frac{1}{\sqrt{2}}$ Ans.

Q.7 Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression,

show that $\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2} = \frac{(2a+b)(a+2b)}{9ab}$

[IIT -2002]

Q.8 If a, b, c are in A.P. & a^2, b^2, c^2 are in H.P. then prove that a = b = c or a, b, – c/2 are in G.P. **[IIT -2003]**

Sol. It is given that a, b, c are in A.P. and a^2, b^2, c^2 are in H.P.

$\therefore 2b = a + c$ …(i)

and $b^2 = \frac{2a^2c^2}{a^2+c^2}$ …(ii)

Now, $2b = a + c$ and $b^2 = \frac{2a^2c^2}{a^2+c^2}$

$\Rightarrow \left(\frac{a+c}{2}\right)^2 = \frac{2a^2c^2}{a^2+c^2}$

$\Rightarrow (a + c)^2 (a^2 + c^2) = 8a^2c^2$

$\Rightarrow (a^2 + c^2 + 2ac)(a^2 + c^2) = 8a^2c^2$

$\Rightarrow (a^2 + c^2)^2 + 2ac(a^2 + c^2) = 8a^2c^2$

$\Rightarrow (a^2 + c^2)^2 + 2ac(a^2 + c^2) + a^2c^2 = 9a^2c^2$

$\Rightarrow (a^2 + c^2 + ac)^2 = 9a^2c^2$

$\Rightarrow a^2 + c^2 + ac = \pm 3ac$

$\Rightarrow a^2 + c^2 = -4ac$

$\Rightarrow a^2 + c^2 + 2ac = -2ac$

$\Rightarrow (a + c)^2 = -2ac$

$\Rightarrow 4b^2 = -2ac$ from (i)

$\Rightarrow b^2 = -\frac{ac}{2}$

$\Rightarrow$ a, b, – c/2 are in G.P.

and, $a^2 + c^2 = 2ac$

$\Rightarrow (a - c)^2 = 0$

$\Rightarrow a = c$

$\Rightarrow$ a = b = c (Proved)

Q.9 An infinite G.P., with first term x and sum of the series is 5 then - **[IIT Scr.2004]**

(A) $x \geq 10$ (B) $0 < x < 10$

(C) $x < -10$ (D) $-10 < x < 0$

Sol. **[B]**

First term of infinite G.P. is x, and sum = 5

Let common ratio of infinite G.P. is r.

$\therefore$ Sum $= \frac{a}{1-r} = \frac{x}{1-r}$ where $|r| < 1$

Θ sum = 5 given

$\therefore \frac{x}{1-r} = 5$

$\Rightarrow r = 1 - \frac{x}{5}$

$\Theta\ |r| < 1 \Rightarrow -1 < r < 1$

$\Rightarrow -1 < 1 - \frac{x}{5} < 1 \Rightarrow -2 < -\frac{x}{5} < 0$

$\Rightarrow -10 < -x < 0 \Rightarrow 0 < x < 10$ Ans

Q.10 If a, b, c are positive real numbers, then prove that $(1+a)^7 (1+b)^7 (1+c)^7 \geq 7^7 a^4 b^4 c^4$. **[IIT - 2004]**

Sol. Given that a, b, c are positive real numbers

We have to prove that $(1 + a)^7 (1 + b)^7 (1 + c)^7 \geq 7^7a^4b^4c^4$

Consider L.H.S. $= (1 + a)^7 (1 + b)^7 (1 + c)^7$

$= [(1 + a)(1 + b)(1 + c)]^7$

$[1 + a + b + c + ab + bc + ca + abc]^7 \geq$

$[a + b + c + ab + bc + ca + abc]^7$ … (1)

Now, we know that AM $\geq$ GM using if for positive numbers a, b, c, ab, bc, ca and abc, we get

$\frac{a+b+c+ab+bc+ca+abc}{7} \geq (a^4b^4c^4)^{1/7}$

$\Rightarrow (a + b + c + ab + bc + ca + abc)^7 \geq 7^7 (a^4b^4c^4)$

from (1) & (2), we get

$[(1 + a)(1 + b)(1 + c)]^7 \geq 7^7 (a^4b^4c^4)$ (Proved)

Q.11 In the quadratic equation $ax^2 + bx + c = 0$, $\Delta = b^2 - 4ac$ and $(\alpha + \beta)$; $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then- **[IIT Scr-2005]**

(A) $\Delta \neq 0$ (B) $b.\Delta = 0$ (C) $c.\Delta = 0$ (D) $\Delta = 0$

Sol. **[C]**

α, β are roots of $ax^2 + bx + c = 0$

$\alpha + \beta = -b/a$, $\alpha\beta = c/a$

$\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P.

$\Rightarrow (\alpha^2 + \beta^2)^2 = (\alpha + \beta)(\alpha^3 + \beta^3)$

$\Rightarrow [(\alpha+\beta)^2 - 2\alpha\beta]^2 = (\alpha + \beta)[(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)]$

$\Rightarrow \left[\frac{b^2}{a^2} - \frac{2c}{a}\right]^2 = \frac{-b}{a}\left[\frac{-b^3}{a^3} + 3\frac{c}{a}.\frac{b}{a}\right]$

$\Rightarrow (b^2 - 2ac)^2 = b^4 - 3ab^2c$

$\Rightarrow ac(b^2 - 4ac) = 0$

$\Rightarrow c\Delta = 0$ ($\Theta\ a \neq 0$)

Q.12 Let $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$ then find the natural number n_0 such that $b_n > a_n$, $n > n_0$, is……… **[IIT-2006]**

Sol. $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots (-1)^{n-1}\left(\frac{3}{4}\right)^n$

$a_n = \frac{\frac{3}{4}\left[1-\left(-\frac{3}{4}\right)^n\right]}{1+\frac{3}{4}} = \frac{3}{7}\left[1-\left(-\frac{3}{4}\right)^n\right]$ $b_n > a_n$

$\Rightarrow 1 - a_n > a_n$

$\Rightarrow 2a_n < 1$

$\Rightarrow \frac{6}{7}\left[1-\left(-\frac{3}{4}\right)^n\right] < 1$

$\Rightarrow \left(-\frac{3}{4}\right)^n > -\frac{1}{6}$

$\Rightarrow$ for n = 3 & 5, inequality fails and for n = 6 the inequality holds.

Hence minimum $n_0 = 5$ Ans

Passage -1 (Q. 13 to 15) [IIT-2007]

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r – 1). Let $T_r = V_{r+1} - V_{r-2}$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ………

Q.13 The sum $V_1 + V_2 + \ldots + V_n$ is-

(A) $\frac{1}{12}$ n (n + 1) (3n² – n + 1)

(B) $\frac{1}{12}$ n (n + 1) (3n² + n + 2)

(C) $\frac{1}{2}$ n (2n² – n + 1)

(D) $\frac{1}{3}$ (2n³ – 2n + 3)

Sol. **[B]**

$V_1 = \frac{1}{2}[2.1 + (4-1).1]$

$V_2 = \frac{2}{2}[2.2 + (2-1).3]$

M M M

$V_r = \frac{r}{2}[2.r + (r-1).(2r-1)]$

$\therefore V_1 + V_2 + V_3 + \ldots + V_n$

$= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$

$= \frac{n(n+1)}{12}(3n^2 + n + 2)$ Ans.

Q.14 T_r is always -

(A) an odd number (B) in even number

(C) a prime number (D) a composite number

Sol **[D]**

$V_r = a_1 + a_2 + \ldots + a_r$

$= \frac{r}{2}[2r + (r-1)(2r-1)]$

$= \frac{r}{2}[2r^2 - r + 1]$

$= r^3 - \frac{r^2}{2} + \frac{r}{2}$

$T_r = V_{r+1} - V_r - 2$

$Q_r = T_{r+1} - T_r$

$V_1 + V_2 + V_3 + \ldots + V_n$

$\sum_{r=1}^{n} r^3 - \frac{r^2}{2} + \frac{r}{2}$

$= \left[\frac{n(n+1)}{2}\right]^2 - \frac{1}{2}\frac{n(n+1)(2n+1)}{6}$

$+ \frac{1}{2}\left(\frac{n(n+1)}{2}\right)$

$= \frac{n(n+1)}{4}\left[n(n+1) - \frac{(2n+1)}{3} + 1\right]$

$= \frac{n(n+1)}{4}\left[\frac{3n^2 + n + 2}{3}\right]$

$T_r = V_{r+1} - V_r - 2$

$= (r+1)^3 - \frac{(r+1)^2}{2} + \frac{(r+1)}{2} - \left(r^3 - \frac{r^2}{2} + \frac{r}{2}\right) - 2$

$= 3r^2 + 2r - 1$

$= (3r - 1)(r + 1)$

$\therefore T_r$ is a composite number.

Q.15 Which one of the following is a correct statement ?

(A) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5

(B) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6

(C) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = \ldots$

Sol. **[B]**

$Q_r = T_{r+1} - T_r$

$Q_r = 3(r+1)^2 + 2(r+1) - 1 - 3r^2 - 2r + 1$

$= 3(r^2 + 2r + 1) + 2r + 2 - 3r^2 - 2r$

$= 6r + 2 + 2$

$= 6r + 5$

$\therefore Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference = 6

Passage- 2 (Q. 16 to 18) [IIT-200]

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For n ≥ 2, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Q.16 Which one of the following statement is correct

(A) $G_1 > G_2 > G_3 > \ldots$ (B) $G_1 < G_2 < G_3 < \ldots$

(C) $G_1 = G_2 = G_3 = \ldots$

(D) $G_1 < G_3 < G_5 < \ldots$ & $G_2 > G_4 > G_6 > \ldots$

Sol. **[C]**

Since A.M., G.M., and H.M. of two positive numbers are in G.P.

$\Rightarrow G_n = \sqrt{A_{n-1}H_{n-1}} = G_{n-1}$

This implies that $G_1 = G_2 = G_3 = \ldots$

Q.17 Which one of the following statements is correct?

(A) $A_1 > A_2 > A_3 > \ldots$

(B) $A_1 < A_2 < A_3 < \ldots$
(C) $A_1 > A_3 > A_5 > \ldots$ & $A_2 < A_4 < A_6 < \ldots$
(D) $A_1 < A_3 < A_5 < \ldots$ & $A_2 > A_4 > A_6 > \ldots$

Sol. **[A]**

$A_n = \frac{A_{n-1} + H_{n-1}}{2}$ Since A.M. ≥ H.M.

$\Rightarrow A_n = \frac{A_{n-1} + H_{n-1}}{2} \le A_{n-1}$

$\Rightarrow A_1 > A_2 > A_3 > \ldots$

Q.18 Which one of the following statement is correct?
(A) $H_1 > H_2 > H_3 > \ldots$
(B) $H_1 < H_2 < H_3 < \ldots$
(C) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$
(D) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

Sol. **[B]**

$\frac{2}{H_n} = \frac{1}{H_{n-1}} + \frac{1}{A_{n-1}}$ $\qquad \frac{1}{H_{n-1}} > \frac{1}{A_{n-1}}$

$\Rightarrow \frac{1}{H_{n-1}} + \frac{1}{A_{n-1}} < \frac{2}{H_{n-1}}$

$\Rightarrow \frac{1}{H_n} < \frac{1}{H_{n-1}}$ $\qquad \Rightarrow H_{n-1} < H_n$

$\Rightarrow H_1 < H_2 < H_3 < \ldots$

Q.19 Suppose for distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT -1

The number b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

STATEMENT- 2

The numbers b_1, b_2, b_3, b_4 are in H.P. **[IIT 2008]**

(A) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.
(B) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1
(C) Statement–1 is True, Statement–2 is False
(D) Statement–1 is False, Statement–2 is True

Sol. **[C]**

Given that, $b_1 = a_1$
$b_2 = a_1 + a_2$
$b_3 = a_1 + a_2 + a_3$
$b_4 = a_1 + a_2 + a_3 + a_4$

$\Rightarrow b_1, b_2, b_3$ and b_4 are neither in A.P., G.P. nor in H.P.

Q.20 If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms, is : **[IIT-2009]**

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Sol.[C] $T_n = S_n - S_{n-1}$
$= cn^2 - c(n-1)^2$
$= cn^2 - cn^2 + 2cn - c$
$= 2cn - c$
$T_n^2 = c^2 (2n-1)^2 = c^2 (4n^2 - 4n + 1)$
$\sum T_n^2 = c^2 [4\sum n^2 - 4 \sum n + n]$

$= c^2 \left[\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right]$

$= \frac{nc^2}{6} [8n^2 + 12n + 4 - 12n - 12 + 6]$

$= \frac{nc^2}{6} [8n^2 - 2]$

$= \frac{nc^2(4n^2-1)}{3}$

Q. 21 Let $a_1, a_2, a_3, \ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to **[IIT- 2010]**

Sol. [0] $\because a_{k-1} = \frac{a_k + a_{k-2}}{2}$

so $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$

$\Rightarrow \Sigma(a + (r-1)d)^2 = 11 \times 90$
$\Rightarrow \Sigma(a^2 + 2ad(r-1) + (r-1)^2 d^2) = 11 \times 90$

$11a^2 + 2ad \frac{10 \times 11}{2} + \frac{10 \times 11 \times 21}{6} d^2 = 11 \times 90$

so on solving $d = -3$

so $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$

$= \frac{11}{2} \cdot \frac{1}{11} \cdot (2 \times a_1 + (11-1)(-3))$

$= \frac{1}{2}(30 - 30) = 0$

Q.22 Let S_k, k = 1, 2, ……, 100, denote the sum of the infinite geometric series whose first term is

$k\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is –

[IIT - 2010]

Sol.[4] $S_k = \frac{K}{\lfloor K}$

$\sum_{K=1}^{100} |(k^2 - 3k + 1)S_k|$

$= 1 + 1 + \sum_{K=3}^{100}\left|\frac{(k^2-3k+1)}{\lfloor k-1}\right|$

$= 2 + \sum\left|\frac{k-1}{\lfloor k-2} - \frac{k}{\lfloor k-1}\right|$

$= 2 + 2 - \frac{100}{\lfloor 99} = 4$

Q.23 Let $a_1, a_2, a_3, ..., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is.

[IIT - 2011]

Sol.[9] $a_1 = 3$

$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[2a_1+(5n-1)d]}{\frac{n}{2}[2a_1+(n-1)d]}$

$= \frac{5[(6-d)+5nd]}{(6-d)+nd}$

Θ $\frac{S_{5n}}{S_n}$ is independent of n so d = 6

So $a_2 = a_1 + d = 3 + 6 = 9$

Q.24 The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is.

[IIT - 2011]

Sol. [8] A.M. ≥ G.M.

$\frac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+1+a^{8}+a^{10}}{8} \ge (a^{-5}.$

$a^{-4}. a^{-3}. a^{-3}. a^{-3}. 1. a^8 .a^{10})^{1/8}$

$a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10} \ge 8$

so minimum value is 8

Q.25 Let $a_1, a_2, a_3,$ .... be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[IIT - 2012]**

(A) 22 (B) 23
(C) 24 (D) 25

Sol. [D] $a_1 = 5$ $a_{20} = 25$

$T_1 = \frac{1}{5}$ $T_{20} = \frac{1}{25}$

$T_{20} = \frac{1}{5} + 19D = \frac{1}{25}$

$D = \left(\frac{1}{25} - \frac{1}{5}\right)\frac{1}{19}$

$= -\frac{20}{5(25)(19)}$

$T_n = \frac{1}{5} - \frac{(n-1)(20)}{(125)(19)}$

$= \frac{(25)(19)-(n-1)(20)}{(125)(19)} < 0$

$(25)(19) < (n-1)(20)$

$n - 1 > \frac{(25)(19)}{(20)}$

$n > \frac{5(19)}{4} + 1$

$n > \frac{95}{4} + 1$

$n > 23.75 + 1$

$n > 24.75$

$n = 25$

EXERCISE # 5

Q.1 The sum of three distinct numbers in G.P. is αS and the sum of their squares is S^2, show that $\alpha^2 \in]\frac{1}{3}, 1 [\cup]1,3[$ **[IIT 1986]**

Q.2 If the first and the $(2n-1)^{th}$ terms of an A.P., a G.P. and a H.P. are equal an their n^{th} terms are a, b and c respectively then find relation between a, b and c. **[IIT-1988]**

Sol. Consider the A.P. since a is equidistant from the first term α and last term β of the A.P.

$\Rightarrow \alpha, a, \beta$ are in A.P.

$\Rightarrow$ a is the A.M. of α and β

$\therefore a = \frac{\alpha+\beta}{2}$

Similary b and c are the G.M. and H.M. of α and β, respectively then

$b = \sqrt{\alpha\beta}$ and $c = \frac{2\alpha\beta}{\alpha+\beta}$

$\therefore (G.M.)^2 = (A.M.)(H.M.)$

$\therefore b^2 = ac$

and A.M. $\geq$ G.M. $\geq$ H.M.

$\therefore a \geq b \geq c$ Ans.

Q.3 If $\log_3 2$, $\log_3(2^x - 5)$, and $\log_3\left(2^x - \frac{7}{2}\right)$ are in arithmetic progression, determine the value of x. **[IIT 1990]**

Sol. x = 3

Q.4 Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2$ p. **[IIT 1991]**

Sol. Let two numbers be a and b and $A_1, A_2, \ldots A_n$ be n arithmetic means between a and b. Then a, A_1, A_2, …….. , A_n, b is A.P. with common difference $d = \frac{b-a}{n+1}$.

$\therefore p = A_1 = a + d = a + \frac{b-a}{n+1} \Rightarrow p = \frac{ma+b}{n+1}$ …(1)

Let $H_1, H_2 \ldots. H_n$ be n harmonic means between a and b

$\therefore \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots \frac{1}{H_n}. \frac{1}{b}$ is an A.P. with common difference $D = \frac{(a-b)}{(n+1)ab}$

$\therefore \frac{1}{q} = \frac{1}{a} + D$

$\Rightarrow \frac{1}{q} = \frac{1}{a} = \frac{(a-b)}{(n+1)ab} \Rightarrow \frac{1}{q} = \frac{nb+a}{(n+1)ab}$

$\Rightarrow q = \frac{(n+b)ab}{nb+a}$ …(2)

From (1), we get

$b = (n+1)p - na$ putting in (2), we get

$q[n(n+1)p - n^2a + a] = (n+1)a[(n+1)p - na]$

$\Rightarrow n(n+1)a^2 - \{(n+1)^2 p + (n^2-1)q\}a + n(n+1)pq = 0$

$\Rightarrow na^2 - \{(n+1)p + (n-1)q\}a + npq = 0$

Since 'a' is real, therefore

$\{(n+1)p + (n-1)q\}^2 - 4n^2 pq > 0$

$\Rightarrow (n+1)^2p^2 + (n-1)^2q^2 + 2(n^2-1)pq - 4n^2pq \geq 0$

$\Rightarrow (n+1)^2p^2 + (n-1)^2q^2 + 2(n^2+1)pq \geq 0$

$\Rightarrow q^2 - \frac{2(n^2+1)}{(n-1)^2} pq + \left(\frac{n+1}{n-1}\right)^2 p^2 \geq 0$

$\Rightarrow q^2 - \left\{1+\left(\frac{n+1}{n-1}\right)^2\right\}pq + \left(\frac{n+1}{n-1}\right)^2 p^2 \geq 0$

$\Rightarrow (q-p)\left\{q - \left(\frac{n+1}{n-1}\right)^2 p\right\} > 0$

$\Rightarrow q < p$ or $q > \left(\frac{n+1}{n-1}\right)^2 p$

$\Theta \left\{\left(\frac{n+1}{n-1}\right)^2 p > p\right\}$

Hence, q cannot lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$

Q.5 If $S_1, S_2, S_3, \ldots, S_n$ are the sums of infinite geometric series whose first terms are 1, 2, 3, ..., n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\frac{1}{n+1}$, respectively, then find the values of $S_1^2 + S_2^2 + S_3^2 + ... + S_{2n-1}^2$.

[IIT 1991]

Sol. Consider an infinite G.P. with first term 1, 2, 3 .. , n and common ratios as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{n+1}$

$\therefore S_1 = \frac{1}{1-1/2} = 2$

$S_2 = \frac{1}{1-1/3} = 3$

$S_{2n-1} = \frac{2n-1}{1-1/2n} = 2n$

Thus $S_1^2 + S_2^2 + S_3^2 + ... S^2_{2n-1}$

$= 2^2 + 3^2 + 4^2 + \ldots + (2n)^2$

$= \frac{1}{6}(2n)(2n+1)(4n+1) - 1$

Q.6 For any odd integer n ≥ 1 ;
$n^3 - (n-1)^3 + ... (-1)^{n-1} 1^3 =$ .... **[IIT-1996]**

Sol. Since n is odd integer, $(-1)^{n-1} = 1$ and n – 1, n – 3, n – 5 ,… are even integers.

We have, $n^3 - (n-1)^3 + (n-2)^3 - (n-3)^3 + \ldots + (-1)^{n-1} 1^3$

$= n^3 + (n-1)^3 + (n-2)^3 + \ldots + 1^3 - 2[(n-1)^3 + (n-3)^3 + \ldots + 2^3]$

$= n^3 + (n-1)^3 + (n-2)^3 + \ldots + 1^3 - 2 \times 2^3 \left[\left(\frac{n-1}{2}\right)^3 + \left(\frac{n-3}{2}\right)^3 + \ldots + 1^3\right]$

(∵ n – 1, n – 3, … are even integers)

$= \left[\frac{n(n+1)}{2}\right]^2 - 16\left[\frac{1}{2}\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)\right]^2$

$= \frac{1}{4} n^2 (n+1)^2 - 16 \frac{(n-1)^2(n+1)^2}{16 \times 4}$

$= \frac{1}{4}(n+1)^2 [n^2 - (n-1)^2]$

$= \frac{1}{4}(n+1)^2 (2n-1)$ Ans.

Q.7 The three real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in A.P. Find the intervals in which β and γ lie. **[IIT -1996]**

Sol. Since x_1, x_2, x_3 are in A.P. therefore, let $x_1 = a - d$, $x_2 = a$ and $x_3 = a + d$ and x_1, x_2, x_3 are the roots of $x^3 - x^2 + \beta x + \gamma = 0$

we have,

$\Sigma\alpha = a - d + a + a + d = 1$ …(1)

$\Sigma\alpha\beta = (a-d)a + a(a+d) + (a-d)(a+d) = \beta$ …(2)

$\alpha\beta\gamma = (a-d)\,a\,(a+d) = -\gamma$ … (3)

From (1), we get $3a = 1 \Rightarrow a = 1/3$

From (2), we get $3a^2 - d^2 = \beta$

$\Rightarrow 3\left(\frac{1}{3}\right)^2 - d^2 = \beta$

$\Rightarrow \frac{1}{3} - \beta = d^2$

$\Rightarrow \frac{1}{3} - \beta \ge 0 \quad \because d^2 \ge 0$

$\Rightarrow \beta \le \frac{1}{3} \Rightarrow \beta \in (-\infty, \frac{1}{3}]$

from (3), $a(a^2 - d^2) = -\gamma$

$\Rightarrow \frac{1}{3}\left(\frac{1}{9} - d^2\right) = -\gamma \Rightarrow \frac{1}{27} - \frac{1}{3}d^2 = -\gamma$

$\Rightarrow \gamma + \frac{1}{27} = \frac{1}{3}d^2$

$\Rightarrow \gamma + \frac{1}{27} \ge 0$

$\Rightarrow \gamma \ge -\frac{1}{27}$

$\Rightarrow \gamma \in [-\frac{1}{27}, \infty)$

Hence $\beta \in (-\infty, \frac{1}{3})$ and $\gamma \in [-1/27, \infty)$

Q.8 Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then $\frac{y^3 + z^3}{xyz} = \ldots\ldots$ **[IIT-1997]**

Sol. 2

Q.9 Let p and q are roots of the equation $x^2 - 2x + A = 0$ and r, s are roots of $x^2 - 18x + B = 0$ if $p < q < r < s$ are in A.P. then find the value of A and B. **[IIT-1997]**

Q.10 Let $a_1, a_2, \ldots\ldots a_{10}$ be in A.P. and $h_1, h_2, \ldots\ldots h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then find the value of a_4h_7. **[IIT- 1999]**

Sol. **[D]**

Let d be the common difference of the A.P.

$\therefore a_{10} = a_0 + 9d$

$\Rightarrow 3 = 2 + 9d$

$\Rightarrow d = 1/9$

Let d be the common difference of the corresponding A.P. of the H.P.

$$\therefore \frac{1}{h_{10}} = \frac{1}{h_1} + 9D$$

$$\Rightarrow \frac{1}{3} = \frac{1}{2} - 9D \Rightarrow D = -\frac{1}{54}$$

Now $a_4 = a_1 + 3d = 2 + 3(1/9) = 7/3$

$$\text{and } \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} + 6\left(-\frac{1}{54}\right) = \frac{7}{18}$$

$$\therefore a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6 \text{ Ans.}$$

Q.11 The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. If the inverse of its common ratio is an integer, find all possible values of the common ratio, n and the first term of the series. **[REE-1999]**

Sol. $s_\infty = \dfrac{a}{1-r} = 162$

$$s_n = \frac{a(1-r^n)}{1-r} = 160$$

on dividing, we get

$$1 - r^n = \frac{160}{162} = \frac{80}{81}$$

$$\therefore 1 - \frac{80}{81} = r^n$$

$$\text{or } r^n = \frac{1}{81}$$

$$\Rightarrow \left(\frac{1}{r}\right)^n = 81$$

$\because \dfrac{1}{r}$ is an integer and n also an integer

$\therefore \dfrac{1}{r} = 3, 9, \text{ or } 81$ for which n = 4, 2 or 1

$$\therefore a = 162\left(1-\frac{1}{3}\right) \text{ or } 162\left(1-\frac{1}{9}\right) \text{ or }$$

$$162\left(1-\frac{1}{81}\right)$$

$\Rightarrow$ a = 108 or 144 or 160. Ans.

Q.12 The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. **[IIT-2000]**

Sol. Let $a - 3d$, $a - d$, $a + d$, $a + 3d$ be any four consecutive terms of an A.P. with common difference 2d.

Hence $P = (2d)^4 + (a - 3d)(a - d)(a + d)(a + 3d)$

$= 16d^4 + (a^2 - 9d^2)(a^2 - d^2)$

$= (a^2 - 5d^2)^2$

Now, $a^2 - 5d^2 = a^2 - 9d^2 + 4d^2$

$= (a - 3d)(a + 3d) + (2d)^2$

$= I \,.\, I + I^2 = 2I^2$ $\quad\because$ 2d is an integer

Where I is an integer

Thus, $P = (I)^2$ = Integer

Q.13 Find the sum of the integers from 1 to 100 which are not divisible by 3 or 5 is- **[IIT -2000]**

Sol. 1 – 100

Total sum : $1 + 2 + \ldots. 100 = \dfrac{100 \times 101}{2} = 5050$

numbers divisible by 3

3, 6, 9 99

$99 = 3 + (n - 1)3 \Rightarrow n = 33$

$$S_3 = \frac{33}{2}(6 + 32(3)) = 1683$$

numbers divisible by 5

5, 10, ...100

$S_5 = \frac{20}{2}(10 + 19 \times 5) = 1050$

numbers divisible by 15

15, 30 90

$S_{15} = \frac{6}{2}(105) = 315$

Sum = 5050 – (1683 + 1050) + 315

= 2632

Q.14 Find the sum to n terms of the series

$$1 - \frac{4}{2} + \frac{7}{2^2} - \frac{10}{2^3} + \ldots\ldots$$

Sol. $S = 1 - \frac{4}{2} + \frac{7}{2^2} - \frac{10}{2^3} + \ldots$ n terms

Put $r = -\frac{1}{2}$

$S = 1 + 4r + 7r^2 + 10r^3 + \ldots\ldots (3n-2)r^{n-1}$

$S = 1 + 4r + 7r^2 + \ldots\ldots\ldots (3n-5)r^{n-2} + (3n-2)r^{n-1}$

$Sr = r + 4r^2 + 7r^3 + \ldots\ldots\ldots (3n-5)r^{n-1} + (3n-2)r^n$

$S(1-r) = 1 + 3r + 3r^2 + 3r^3 + \ldots\ldots 3r^{n-1} - (3n-2)r^n$

$S(1-r) = 1 + 3r\underbrace{(1 + r + r^2 \ldots\ldots r^{n-2})}_{(n-1)\text{ terms}}$

$$S(1-r) = 1 + 3r(1)\left(\frac{1-r^{n-1}}{1-r}\right) - (3n-2)r^n$$

$$S = \frac{1}{1-r} + \frac{3r(1-r^{n-1})}{(1-r)^2} - \frac{(3n-2)r^n}{1-r}$$

Put $r = -\frac{1}{2}$ and solve.

Q.15 Let $x = 1 + 3a + 6a^2 + 10a^3 + \ldots\ldots\ldots\ldots\ |a| < 1$, $y = 1 + 4b + 10b^2 + 20b^3 + \ldots\ldots\ldots\ldots\ |b| < 1$. Then find $S = 1 + 3(ab) + 5(ab)^2 + \ldots\ldots\ldots$in terms of x and y.

Sol. **[A]**

$x = 1 + 3a + 6a^2 + 10a^3 + \ldots$ …(1)

$\therefore ax = a + 3a^2 + 6a^3$ …(2)

on subtracting (1) and (2).

$\therefore x(1-a) = 1 + 2a + 3a^2 + 4a^3 + \ldots$

The series is A.G.P.

$$S_\infty = \frac{A}{1-R} + \frac{dR}{(1-R)^2}$$

$$\therefore x(1-a) = \frac{1}{1-a} + \frac{a}{(1-a)^2} = \frac{1}{(1-a)^2}$$

$$\therefore x = \frac{1}{(1-a)^3}$$

$\therefore (1-a)^3 = x^{-1}$ or $a = 1 - x^{-1/3}$

similarly, $b = 1 - y^{-1/4}$

$$\therefore S = \frac{1}{1-ab} + \frac{2ab}{(1-ab)^2}$$

$$S = \frac{1+ab}{(1-ab)^2}$$

$$S = \frac{1+(1-x^{-1/3})(1-y^{-1/4})}{\{1-(1-x^{-1/3})(1-y^{-1/4})\}^2}$$ Ans.

ANSWER KEY

EXERCISE # 1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	A	D	A	D	C	B	A	A,B	A	A	C	B	B	A	A	A	B	D
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	D	D	B	B	B	A	D	B	B	A	A,B	C	A	B	C	A	B	B	C	A,B
Que.	41																			
Ans.	B,C																			

EXERCISE # 2

1. (C) **2.** (C) **3.** (C) **4.** (A) **5.** (A) **6.** (C) **7.** (A)

8. (C) **9.** (A) **10.** (A) **11.** (B) **12.** (A) **13.** (C) **14.** (A)

15. (D) **16.** (C, D) **17.** (A, C) **18.** (A, C) **19.** (B, C) **20.** (A, C) **21.** (B, C)

22. (A, C) **23.** (False) **24.** (False) **25.** (False) **26.** $(49^2 + 2)$ **27.** $(2a + \frac{d}{2})$ **28.** $\left(\frac{1}{201}\right)$

29. (A) **30.** (A) **31.** (B) **32.** (A)

33. (**A** $\rightarrow$ P, **B** $\rightarrow$ P, **C** $\rightarrow$ Q, **D** $\rightarrow$ S) **34.** (**A** $\rightarrow$ R, **B** $\rightarrow$ P, **C** $\rightarrow$ S, **D** $\rightarrow$ Q)

EXERCISE # 3

1. 2 **3.** 8, 12, 16, ... **6.** 9/5

7. **(A) (i)** $2^{n+1} - 3$; $2^{n+2} - 4 - 3n$ **(B)** $n^2 + 4n + 1$; $n(n + 1)(2n + 13) + n$;

(C) $\left(\frac{1+x}{1-x}\right)^2$; **(D)** $\frac{35}{16} - \frac{12n+7}{16.5^{n-1}}$ & $\frac{35}{16}$

8. $n(2n^3 + 8n^2 + 7n - 2)$ **9.** 32 cm^2 **10.** a = 1, b = 3 or b = 1, a = 3

12. a = 1, b = 9 or b = 1, a = 9

13. **(i)** $s_n = (1/12) - [1/\{4(2n+1)(2n+3)\}]$; $s_\infty = 1/12$

(ii) $(1/5)\, n(n + 1)(n + 2)(n + 3)(n + 4)$ **(iii)** $n/(2n + 1)$

14. $n(n + 1)/2(n^2 + n + 1)$ **15.** $\frac{1}{x-1} - \frac{2^{n+1}}{x^{2^{n+1}} - 1}$

16. $\frac{n(3n^3 + 15n^2 + 25n + 25)}{6(n+1)(n+3)}$ **17.** (A) **18.** (B) **19.** (C) **20.** (A)

21. (B) **22.** (D) **23.** (B) **24.** (A) **25.** (A)

EXERCISE # 4

1. (A) **2.** (D) **3.** (C) **4.** $(G_1G_2 \ldots G_n)^{1/n} = [A_1H_1 . A_2H_2 \ldots A_n H_n]^{1/2n}$

5. (A) **6.** (D) **9.** (B) **11.** (C) **12.** 5 **13.** (B) **14.** (D)

15. (B) **16.** (C) **17.** (A) **18.** (B) **19.** (C) **20.** (C) **21.** 0

22. 4 **23.** 9 **24.** 8 **25.** (D)

EXERCISE # 5

2. $ac - b^2 = 0$ & $a \geq b \geq c$ **3.** $x = 3$ **5.** $\frac{n(2n+1)(4n+1)-3}{3}$

6. $\frac{1}{4}(2n-1)(n+1)^2$ **7.** $\beta \in (-\infty, 1/3]$ & $\gamma \in [-1/27, \infty)$ **8.** 2 **10.** 6

11. $r = \frac{1}{3}, \frac{1}{9}, \frac{1}{81}$; $n = 4, 2, 1$; $a = 108, 144, 160$ **13.** 2632 **14.** $n(-1/2)^{n-1}$

15. $\frac{1+(1-x^{-1/3})(1-y^{-1/4})}{\{1-(1-x^{-1/3})(1-y^{-1/4})\}^2}$