PROBABILITY

EXERCISE # 1

Question based on Classical definition of probability

Q.1 A bag contains 20 tickets numbered 1 to 20. Two tickets are drawn at random. The probability that both the numbers on the ticket are prime is -

(A) $\frac{4}{95}$ (B) $\frac{14}{95}$ (C) $\frac{17}{95}$ (D) $\frac{9}{95}$

Sol. **[B]**

Total case = ${}^{20}C_2 = \frac{20.19}{2} = 190$

Prime no.s = 2, 3, 5, 7, 11, 13, 17, 19 = 8

Favourable case = ${}^{8}C_2 = 28$

Required probability = $\frac{28}{190} = \frac{14}{95}$

Q.2 A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is –

(A) $\frac{3}{11}$ (B) $\frac{2}{11}$ (C) $\frac{4}{11}$ (D) $\frac{7}{11}$

Sol. **[C]**

PROBABILITY

vowel = O, A, I, I

Total case = ${}^{11}C_1 = 11$

Favourable case = ${}^{4}C_1 = 4$

Required probability = $\frac{4}{11}$

Q.3 A bag contains 4 red and 4 white balls. Three balls are drawn at random. The odd against these balls being all white will be ?

(A) 11 : 1 (B) 13 : 1 (C) 13 : 2 (D) 11 : 5

Sol. **[B]**

Total case = ${}^{8}C_3 = 56$

Favourable case x = ${}^{4}C_3 = 4$

unfavourable case y = 56 – 4 = 52

odds against = $\frac{y}{x} = \frac{52}{4} = \frac{13}{1}$ = 13 : 1

Question based on Addition and multiplication theorem of Probability

Q.4 There are three events E_1, E_2 and E_3, one of which must, and only one can happen. The odds are 7 to 4 against E_1 and 5 to 3 against E_2. Then the odds against E_3 is –

(A) 65 : 23 (B) 65 : 32

(C) 55 : 76 (D) 23 : 65

Sol. **[A]**

odds against $E_1 = \frac{7}{4}$

probability of $E_1 = \frac{4}{11}$

odds against $E_2 = \frac{5}{3}$

probability of $E_2 = \frac{3}{8}$

Probability of $E_3 = 1 - [P(E_1) + P(E_2)]$

$= 1 - \left[\frac{4}{11} + \frac{3}{8}\right] = 1 - \frac{65}{88} = \frac{23}{88}$

odds against $E_3 = \frac{88-23}{23} = \frac{65}{23}$ = 65 : 23

Q.5 From a pack of well shuffled cards, one card is drawn randomly. A gambler bets that it is either a diamond or a king. The odds in favour of his winning the bet will be ?

(A) 9 : 4 (B) 4 : 9

(C) 5 : 7 (D) 9 : 7

Sol. **[B]**

Total case = 52

Favourable case x = 13 + 3 = 16

unfavourable case y = 52 – 16 = 36

odds in favour = $\frac{x}{y} = \frac{16}{36} = \frac{4}{9}$ = 4 : 9

Q.6 In a given race, the odds in favour of horses A, B, C, D are 1 : 3, 1 : 4, 1 : 5 and 1 : 6 respectively, then the probability that one of them wins the race is –

(A) 319/420 (B) 219/420

(C) 391/420 (D) 291/420

Sol. **[A]**

$\frac{P(A)}{1-P(A)} = \frac{1}{3} \Rightarrow P(A) = \frac{1}{4}$

$\frac{P(B)}{1-P(B)} = \frac{1}{4} \Rightarrow P(B) = \frac{1}{5}$

$$\frac{P(C)}{1-P(C)} = \frac{1}{5} \Rightarrow P(C) = \frac{1}{6}$$

$$\frac{P(D)}{1-P(D)} = \frac{1}{6} \Rightarrow P(D) = \frac{1}{7}$$

$P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D)$

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{319}{420}$$

Q.7 A dice is thrown twice. The probability of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw is ?

(A) 2/3 (B) 1/3 (C) 2/5 (D) 1/5

Sol. **[B]**

$P(A \cap B) = P(A) . P(B)$ (Independent)

$$= \frac{1}{2} . \frac{2}{3} = \frac{1}{3}$$

Q.8 The probability of student A passing an examination is $\frac{2}{9}$ & of student B passing is $\frac{5}{9}$. Assuming the two events. 'A passes', 'B passes' as independent. Then the probability of

(i) Only A passing the examination is –

(A) 8/81 (B) 7/81 (C) 5/81 (D) 4/81

(ii) Only one of them passing the examination is –

(A) 31/81 (B) 43/81 (C) 53/81 (D) 8/81

Sol. **[A, B]**

(i) $P(A) = \frac{2}{9}$, $P(\overline{B}) = 1 - \frac{5}{9} = \frac{4}{9}$

only a passing then

probability = $P(A) . P(\overline{B}) = \frac{2}{9} . \frac{4}{9} = \frac{8}{81}$

(ii) $P(A) = \frac{2}{9}$, $P(\overline{A}) = \frac{7}{9}$

$P(B) = \frac{5}{9}$, $P(\overline{B}) = \frac{4}{9}$

only one of them passing then

probability = $P(A)\, P(\overline{B}) + P(\overline{A})\, P(B)$

$$= \frac{2}{9} . \frac{4}{9} + \frac{7}{9} . \frac{5}{9} = \frac{8}{81} + \frac{35}{81} = \frac{43}{81}$$

Q.9 A bag contains 3 white, 3 black and 2 red balls one by one three balls are drawn without replacing them. The probability that the third ball is red, is –

(A) 3/4 (B) 1/4 (C) 3/7 (D) 5/7

Sol. **[B]**

Let R is the event that ball is red and A is the event that ball is white or black so probability that third ball is red

$P(E) = P(RAR) + P(ARR) + P(AAR)$

$$P(E) = \frac{2}{8} . \frac{6}{7} . \frac{1}{6} + \frac{6}{8} . \frac{2}{7} . \frac{1}{6} + \frac{6}{8} . \frac{5}{7} . \frac{2}{6}$$

$$= \frac{1}{28} + \frac{1}{28} + \frac{5}{28} = \frac{7}{28} = \frac{1}{4}$$

Q.10 The probability of a number n showing in a throw of a dice marked 1 to 6 is proportional to n. Then the probability of the number 3 showing in a throw is -

(A) 1/2 (B) 1/6 (C) 1/7 (D) 1/21

Sol. **[C]**

$\because$ Probability of a number n showing is proportional to n so let probability

= n, 2n, 3n, 4n, 5n, 6n

$\Rightarrow$ n + 2n + 3n + 4n + 5n + 6n = 1

$\Rightarrow n = \frac{1}{21}$

probability of showing 3 is

$$= 3n = 3 . \frac{1}{21} = \frac{1}{7}$$

Q.11 If A and B are two events such that

$P(A) = \frac{3}{5}$ and $P(B) = \frac{7}{10}$, then -

(A) $P(A \cap B) \geq \frac{3}{10}$

(B) $P(A \cap B) \leq \frac{7}{10}$

(C) $\frac{3}{5} < P(A \cup B) \leq \frac{7}{10}$

(D) $\frac{3}{5} < P(A \cup B) \leq \frac{7}{10}$

Sol. **[A, B]**

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{3}{5} + \frac{7}{10} - P(A \cup B) \geq \frac{13}{10} - 1 \quad (\because P(A \cup B) \leq 1)$$

$\Rightarrow P(A \cap B) \geq \frac{3}{10}$

$P(A \cap B) \leq P(B) = \frac{7}{10}$

$P(A \cup B) > P(A) = \frac{3}{5}$

$\Rightarrow P(A \cup B) \geq P(B) = \frac{7}{10}$

Q.12 If M & N are any two events, then which one of the following represents the probability of the occurrence of exactly one of them ?

(A) $P(M) + P(N) - 2P(M \cap N)$

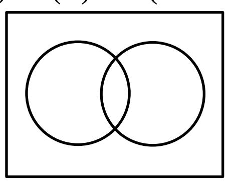
(B) $P(M) + P(N) - P(M \cap N)$

(C) $P(\overline{M}) + P(\overline{N}) - 2P(\overline{M} \cap \overline{N})$

(D) $P(M \cap \overline{N}) + P(\overline{M} \cap N)$

Sol. **[A, C, D]**

$P(M) + P(N) - 2P(M \cap N)$



$P(M \cap \overline{N}) + P(\overline{M} \cap N)$

Q.13 If M & N are independent events such that $0 < P(M) < 1$ & $0 < P(N) < 1$, then :

(A) M & N are mutually exclusive

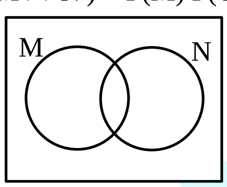
(B) M & $\overline{N}$ are independent

(C) $\overline{M}$ & $\overline{N}$ are independent

(D) $P(M/N) + P(\overline{M}/N) = 1$

Sol. **[B,C,D]**

$P(M \cap \overline{N}) = P(M)\, P(\overline{N})$



$M = (M \cap N') \cup (M \cap N)$

$P(M) = P(M \cap N') + P(M \cap N)$

$P(M) = P(M \cap N') + P(M)\, P(N)$

$P(M \cap N') = P(M)\, \{1 - P(N)\}$

$P(M \cap N') = P(M)\, P(N')$

Question based on Probability of at least one of the n independent events

Q.14 A can solve 75 % of the problems of a book and B can solve 60 % of the problem of the same book. The probability that at least one can solve a problem of the book chosen at random, is ?

(A) 9/17 (B) 9/10

(C) 1/10 (D) 9/17

Sol. **[B]**

$P(A) = \frac{75}{100} = \frac{3}{4}$, $P(B) = \frac{60}{100} = \frac{3}{5}$

Required probability $= 1 - P(\overline{A})\, P(\overline{B})$

$$= 1 - \left(\frac{1}{4}.\frac{2}{5}\right) = 1 - \frac{1}{10} = \frac{9}{10}$$

Q.15 4 coins are tossed. Then the probability of getting at least one head is –

(A) 1/16 (B) 15/16

(C) 3/16 (D) 5/16

Sol. **[B]**

$P(H) = \frac{1}{2}$, $P(\overline{H}) = \frac{1}{2}$

getting at least one head then

probability $= 1 - [P(\overline{H})]^4 = 1 - \frac{1}{16} = \frac{15}{16}$

Question based on Conditional probability

Q.16 A dice is thrown twice and the sum of the numbers appearing is observed to be 6. What is the probability that the number 4 has appeared at least once ?

(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

Sol. **[B]**

P(A) = number is 4

P(B) = sum is 6

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{5/36} = \frac{2}{5}$$

Q.17 A pair of dice is thrown. If 5 appears on at least one of the dice, then the probability that the sum is 10 or greater is –

(A) 1/11 (B) 2/11 (C) 3/11 (D) 4/11

Sol. **[C]**

Total case

= (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (1,5) (2,5) (3,5) (4,5) (6,5) = 1

Favourable case = (5,5) (5,6) (6,5) = 3

Required probability $= \frac{3}{11}$

Q.18 Let $P(A) = 0.4$ and $P(B/A) = 0.5$. The probability $P(\overline{A} \cup \overline{B})$ is equal to -

(Hint : $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B)$)

(A) 0.5 (B) 0.7 (C) 0.8 (D) 0.2

Sol. **[C]**

Given P(A) = 0.4 and $P\left(\frac{B}{A}\right) = 0.5$

$\Theta\ P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow P(A\cap B) = 0.5 \times 0.4 = 0.2$

$\Theta\ P(\overline{A} \cup \overline{B}) = 1 - P(A\cap B) = 1 - 0.2 = 0.8$

Q.19 A coin is tossed twice and the four possible outcomes are assumed to be equally likely. If A is the event, 'both head and tail have appeared' and B the event,' at most one tail is observed,' then the value of P(B/A) is –

(A) 1 (B) 2 (C) 1/2 (D) 1/4

Sol. **[A]**

$P(A) = \frac{2}{4} = \frac{1}{2}$

$P(B) = \frac{3}{4}$

$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1/2}{1/2} = 1$

Q.20 A, B and C are contesting the election for the post of secretary of a club which does not allow ladies to become members. The probabilities of A, B and C winning the election are $\frac{1}{3}$, $\frac{2}{9}$ & $\frac{4}{9}$ respectively. The probabilities of introducing the clause of admitting lady members to the club by A, B and C are 0.6, 0.7 and 0.5 respectively. The probability that ladies will be taken as members in the club after the election is -

(A) 26/45 (B) 5/9 (C) 19/45 (D) none

Sol. **[A]**

$P(A) = \frac{3}{9}$, $P(B) = \frac{2}{9}$, $P(C) = \frac{4}{9}$

P(A) + P(B) + P(C) = 1

ME & EE

$P(R) = \frac{3}{9}\left(\frac{6}{10}\right) + \frac{2}{9}\left(\frac{7}{10}\right) + \frac{4}{9}\left(\frac{5}{10}\right)$

$= \frac{52}{90} = \frac{26}{45}$

Question based on Binomial Distributions of probability

Q.21 A dice is thrown three times. Getting a 3 or a 6 is considered success. Then the probability of at least two success is ?

(A) 20/27 (B) 7/27 (C) 5/27 (D) 1/27

Sol. **[B]**

$p = \frac{2}{6} = \left(\frac{1}{3}\right),\ q = \frac{4}{6} = \left(\frac{2}{3}\right)$

2 success + 3 success

$^3C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 + {}^3C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$

$= \frac{6}{27} + \frac{1}{27} = \frac{7}{27}$

Q.22 Two dice are thrown together 4 times. The probability that both dice will show same numbers twice is –

(A) 25/216 (B) 35/216

(C) 45/216 (D) 55/216

Sol. **[A]**

$P(A) = \frac{6}{36}$ $P(B) = \frac{30}{36}$

$^4C_2 \times \left(\frac{6}{36}\right)^2 \times \left(\frac{30}{36}\right)^2$

$\Rightarrow 6 \times \frac{1}{36} \times \frac{25}{36}$

$\Rightarrow \frac{25}{216}$

Q.23 A dice is thrown 2n + 1 times, n ∈ N. The probability that faces with even numbers show odd number of times is -

(A) $\frac{2n+1}{4n+3}$ (B) less than $\frac{1}{2}$

(C) greater than $\frac{1}{2}$ (D) none of these

Sol. **[D]**

$^{2n+1}C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{2n} + {}^{2n+1}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{2n-2} + \ldots\ldots + {}^{2n+1}C_{2n+1}\left(\frac{1}{2}\right)^{2n+1}$

$\Rightarrow \left(\frac{1}{2}\right)^{2n+1}[C_1 + C_3 + C_5 + \ldots\ldots + C_{2n+1}]$

$C_0 + C_1 + C_2 + \ldots\ldots + C_{2n+1} = 2^{2n+1}$ ....(1)

$C_0 - C_1 + C_2 + \ldots\ldots + C_{2n+1} = 0$ ....(2)

(1) – (2)

$\Rightarrow C_1 + C_3 + \ldots\ldots + C_{2n+1} = 2^{2n}$

$\left(\frac{1}{2}\right)^{2n+1}(2^{2n}) = \frac{1}{2}$

Q.24 One hundred identical coins, each with probability, p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

(A) 1/2 (B) 49/101

(C) 50/101 (D) 51/101

Sol. **[D]**

Let x be the number of coins showing heads. Let x be a binomial variate with parameters n = 100 and p

$P(x = 50) = P(x = 51)$

$\Rightarrow {}^{100}C_{50}\, p^{50}(1 - p)^{50} = {}^{100}C_{51}\, p^{51}(1 - p)^{49}$

$$\Rightarrow \frac{\lfloor 100}{\lfloor 50 \lfloor 50} . \frac{\lfloor 51 \lfloor 49}{\lfloor 100} = \frac{p}{1-p}$$

$$\Rightarrow \frac{p}{1-p} = \frac{51}{50} \quad \text{or} \quad p = \frac{51}{101}$$

Question based on Expectation

Q.25 From a bag containing 2 rupee coins and 3 ten paise coins, a person is allowed to draw 2 coins indiscriminately, Then the value of his expectation is -

(A) 72 paise (B) 82 paise

(C) 92 paise (D) 62 paise

Sol. [C]

Q.26 A had in his pocket a rupee coin and four ten paise coins, taking out two coins at random, he promises to given them to B and C. What is the worth of B's expectation ?

(A) 28 paise (B) 38 paise

(C) 48 paise (D) 58 paise

Sol. **[A]**

Question based on Baye's theorem

Q.27 Suppose there are three urns containing 2 white and 3 black balls, 3 white and 2 black balls and 4 white and 1 black ball respectively. There is equal probability of each urn being chosen. One white ball has been drawn, the probability that it was drawn from the first urn is –

(A) 1/9 (B) 2/9

(C) 4/9 (D) 5/9

Sol. **[B]**

2W 3B   3W 2B   4W 1B

$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$

$P(A/E_1) = \frac{2}{5}$, $P(A/E_3) = \frac{4}{5}$

$P(A/E_2) = \frac{3}{5}$

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{3} \times \frac{2}{5} + \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{4}{5}}$$

$$= \frac{2}{9}$$

Q.28 In a bolt factory, machines A, B and C manufacture 25% 35%, 40% respectively of the total of their output 5% 4% and 2% are defective. A bolt is drawn and is found to be defective. What are the probabilities that it was manufactured by the machines A, B and C ?

(A) 25/69, 28/69, 16/69

(B) 28/69, 25/69, 16/69

(C) 16/69, 28/69, 25/69

(D) 29/69, 39/69, 49/69

Sol. **[A]**

$P(E_1) = \frac{25}{100}$, $P(A/E_1) = \frac{5}{100}$

$P(E_2) = \frac{35}{100}$, $P(A/E_2) = \frac{4}{100}$

$P(E_3) = \frac{40}{100}$, $P(A/E_3) = \frac{2}{100}$

$$P\left(\frac{E_1}{A}\right) = \frac{25 \times 5}{125 + 140 + 80} = \frac{125}{345}$$

$$P\left(\frac{E_2}{A}\right) = \frac{140}{345} = \frac{28}{69}$$

$$P\left(\frac{E_3}{A}\right) = \frac{80}{345} = \frac{16}{69}$$

Q.29 A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, what is the probability that the missing card is black ?

(A) 1/3 (B) 2/3

(C) 1/4 (D) 3/4

Sol. **[B]**

E_1 = Missing card is red

E_2 = Missing card is black

A = Drawing 13 cards

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{^{26}C_{13}}{^{51}C_{13}}}{\frac{1}{2}\times\frac{^{25}C_{13}}{^{51}C_{13}} + \frac{1}{2}\times\frac{^{26}C_{13}}{^{51}C_{13}}}$$

Q.30 The contents of three urns 1, 2 and 3 are as follows :

1W,	2R	3B balls
2W,	3R	1B balls
3W,	1R	2B balls

An urn is chosen at random and from it two balls are drawn at random and are found to be "one red and one white". Then the probability that they came from the second urn, is-

(A) 5/11 (B) 6/11 (C) 7/11 (D) 9/11

Sol. **[B]**

Question based on

Miscellaneous Topics (Multinomial theorem, geometric probability etc)

Q.31 3 letters are placed in 3 envelopes randomly. The probability that only one letter goes in right envelope is ?

(A) 1/2 (B) 1/3 (C) 1/4 (D) 3/4

Sol. **[A]**

Q.32 A person throws two dice, one the common cube and the other a regular tetrahedron, the number on the lowest face being taken in the case of the tetrahedron. What is the chance that the sum of the numbers thrown is not less than 5 ?

(A) 1/4 (B) 3/4 (C) 3/16 (D) 3/8

Sol. **[B]**

Q.33 A and B throw with 3 dice ; if A throws 8, what is the B's chance of throwing a higher number ?

(A) 7/27 (B) 20/27 (C) 5/27 (D) 19/27

Sol. **[B]**

Q.34 The probability of getting a sum of 12 in four throws of an ordinary dice is -

(A) $\frac{1}{6}\left(\frac{5}{6}\right)^3$ (B) $\left(\frac{5}{6}\right)^4$

(C) $\frac{1}{36}\left(\frac{5}{6}\right)^2$ (D) none of these

Sol. **[A]**

Q.35 A sphere is circumscribed over a cube. Then the probability that a point lies inside the sphere, lies outside the cube.

(A) $1 - \frac{2}{\pi\sqrt{3}}$ (B) $1 + \frac{2}{\pi\sqrt{3}}$

(C) $1 - \frac{3}{\pi\sqrt{2}}$ (D) $1 - \frac{5}{\pi\sqrt{3}}$

Sol. **[A]**

Q.36 A given line segment is divided at random into three parts. What is the probability that they form sides of a possible triangle ?

(A) 3/4 (B) 1/4 (C) 3/16 (D) 1/16

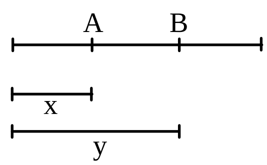
Sol. **[B]**

Q.37 On a line segment of length L two points are taken at random, the probability that the distance between them is at least K. Where K < L.

(A) $\left(\frac{L-K}{L}\right)^2$ (B) $\left(\frac{L+K}{L}\right)^2$

(C) $\left(\frac{L+K}{2L}\right)^2$ (D) $\left(\frac{L-K}{2L}\right)^2$

Sol. **[A]**



$$\left.\begin{matrix} 0 < x < L \\ 0 < y < L \end{matrix}\right\} \text{Total}$$

fav.

$|x - y| > k$

Q.38 A line segment of length a is divided in two parts at random by taking a point on it. The probability that no part is greater than b, where 2b > a is –

(A) $\frac{2b-a}{b}$ (B) $\frac{2b-a}{a}$

(C) $\frac{a-2b}{a}$ (D) $\frac{a+2b}{a}$

Sol. **[B]**

Q.39 A cloth of length 10 metres is to be randomly distributed among three brothers, the probability that no one gets more than 4 metres of cloth is –

(A) $\frac{24}{25}$ (B) $\frac{23}{25}$ (C) $\frac{1}{25}$ (D) $\frac{7}{25}$

Sol. **[C]**

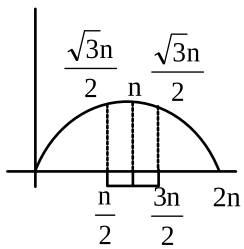
➢ True or false type questions

Q.40 If 3 events A, B, C are exhaustive, then $P(A) + P(B) + P(C) = 1$.

Sol. **False.**

Q.41 The sum of two positive quantities is equal to 2n. The probability that their product is not less than 3/4 times their greatest product is $\frac{1}{2}$.

Sol. **[True]**



$x + y = 2n$

$\max (xy) = n^2$

$xy \geq \frac{3}{4} n^2$

$$P(A) = \frac{\text{length of fav.cases}}{\text{length of total}} = \frac{n}{2n} = \frac{1}{2}$$

Q.42 A fair coin is tossed 5-times, then the probability of getting at most 2 heads is $\frac{1}{2}$.

Sol. **[True]**

Q.43 $P\left(\frac{\bar{A}}{\bar{B}}\right) = \left(\frac{1 - P(A \cup B)}{P(\bar{B})}\right)$

Sol. **[True]**

➢ Fill in the blanks type questions

Q.44 Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. If one ball is now drawn at random from urn A, the probability that it is found to be red is.........................

Sol. **Case I :** Red from A to B and Red from B to A then probability of drawing Red from A is

$$= \frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} = \frac{18}{110} = \frac{9}{55}$$

Case II : Red from A to B and Black from B to A then probability is

$$= \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} = \frac{18}{110} = \frac{9}{55}$$

Case III : Black from A to B and Red from B to A then probability is

$$= \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} = \frac{28}{275}$$

Case IV : Black from A to B and Black from B to A then probability is

$$\frac{4}{10} \times \frac{7}{11} \times \frac{6}{10} = \frac{42}{275}$$

Required probability

$$= \frac{9}{55} + \frac{9}{55} + \frac{28}{275} + \frac{42}{275}$$

$$= \frac{45 + 45 + 28 + 42}{275} = \frac{160}{275} = \frac{32}{55}$$

Q.45 A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is.........

Sol. Favourable cases of getting a sum of 5 is (1, 4), (2, 3) (3, 2), (4, 1)

$$P(A) = \frac{4}{36} = \frac{1}{9}$$

Favourable cases of getting a sum of 7 is (1, 6), (2, 5) (3, 4) (4, 3), (5, 2) (6, 1)

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

Θ events are mutually exclusive so probability of getting a sum 5 or 7 is $= \frac{1}{9} + \frac{1}{6} = \frac{5}{18}$

Probability of getting a sum neither 5 nor 7 is

$$= 1 - \frac{5}{18} = \frac{13}{18}$$

Now we get a sum of 5 before a sum of 7 then required probability

$$= \frac{1}{9} + \frac{13}{18} \times \frac{1}{9} + \left(\frac{13}{18}\right)^2 \frac{1}{9} + \left(\frac{13}{18}\right)^3 \frac{1}{9} + \ldots\infty$$

$$= \frac{1/9}{1 - \frac{13}{18}} = \frac{1}{9} \times \frac{18}{5} = \frac{2}{5}$$

EXERCISE # 2

Part-A Only single correct answer type questions

Q.1 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the same boy does not receive both the pens is -

(A) $\frac{5}{11}$ (B) $\frac{7}{11}$ (C) $\frac{2}{3}$ (D) $\frac{8}{11}$

Sol. **[D]**

$$\text{Books} \begin{cases} \to 4 \\ \to 3 \\ \to 3 \end{cases}$$

$$\left[\frac{10!}{4!3!3!2!}\right] \times 3! \times 2!$$

$$\text{Total} = \frac{12\,!}{4!\,4!\,4!\,3!} \times 3\,!$$

Q.2 A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4^{th} time on the 7^{th} draw is -

(A) 5/64 (B) 27/32

(C) 5/32 (D) ½

Sol. **[C]**

Probability of drawing a white ball is = $\frac{1}{2}$

Probability of not drawing a white ball is = $\frac{1}{2}$

we want to drawn a white ball 4^{th} time in 7^{th} draw so a white ball drawn in 7^{th} draw and 3 white balls are drawn in first 6 draws

So required probability

$= {}^6C_3\, p^3 q^3 \,.\, p$

$= {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 . \frac{1}{2} = \frac{5}{32}$

Q.3 A bag contains 12 pairs of socks, 4 socks are picked up at random. The probability that there is at least one pair is -

(A) 41/161 (B) 40/161

(C) 8/25 (D) 7/25

Sol. **[A]**

Total number of ways of choosing 4 socks out of 12 pair (24 socks) is = 24 × 23 × 22 × 21

The number of way in which no pair is selected is

= 24 × 22 × 20 × 18

Then the probability of not getting a pair is

$$= \frac{24 \times 22 \times 20 \times 18}{24 \times 23 \times 22 \times 21} = \frac{120}{161}$$

Hence the probability of getting at least one pair is

$$= 1 - \frac{120}{161} = \frac{41}{161}$$

Q.4 A card is drawn from the pack. The card is replaced and the pack is reshuffled. If this is done six times, the probability that 2 hearts, 2 diamonds and 2 black cards are drawn is -

(A) $90\left(\frac{1}{4}\right)^6$ (B) $\frac{45}{2}\left(\frac{3}{4}\right)^4$

(C) $\frac{90}{2^{10}}$ (D) none of these

Sol. **[C]**

$P(H) = \frac{1}{4}$, $P(D) = \frac{1}{4}$ & $P(B) = \frac{1}{2}$

If drawing cords is 2 heart, 2 diamonds and 2 Black cord then

$$\text{probability} = \frac{1}{4} . \frac{1}{4} \times \frac{1}{4} . \frac{1}{4} \times \frac{1}{2} . \frac{1}{2} = \frac{1}{2^{10}}$$

$$\text{Total number of arrangement} = \frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2} = 90$$

$$\text{Required probability} = \frac{90}{2^{10}}$$

Q.5 5 girls and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15, then the probability that end seats are occupied by the girls and between any two girls odd number of boys sit is -

(A) $\dfrac{20\times10!\times5!}{15!}$ (B) $\dfrac{10\times10!\times5!}{15!}$

(C) $\dfrac{20\times10!\times5!}{25!}$ (D) $\dfrac{10\times10!\times5!}{25!}$

Sol. **[A]**

There are four gaps in between the girls where boys can sit.

Let the number of boys in these gaps be

$2a + 1, 2b + 1, 2c + 1, 2d + 1$ then

$2a + 1 + 2b + 1 + 2c + 1 + 2d + 1 = 10$

$\Rightarrow a + b + c + d = 3$

number of solution of above equation

= coffi. of x^3 in $(1 - x)^{-4}$

$= {}^6C_3 = 20$

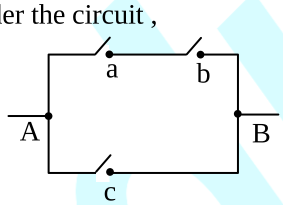
Thus boys and girls can sit in

$20 . \lfloor\underline{10} . \lfloor\underline{5}$ ways

Total ways = $\lfloor\underline{15}$

Required probability = $\dfrac{20\lfloor\underline{10}\lfloor\underline{5}}{\lfloor\underline{15}}$

Q.6 Consider the circuit ,



If the probability that each switch is closed is p, then find the probability of circuit flowing through AB

(A) $p^2 + p$ (B) $p^3 + p - 1$

(C) $p^3 + p$ (D) $p^2 + p + 1$

Sol. **[A]**

$P[(a \cap b) \cup c]$

$= P(A \cap B) + P(C) - P(A \cap B \cap C)$

$= P(A) . P(B) + P(C) - P(A) . P(B) . P(C)$

$= p^2 + p - p^3$

$2\,P(\overline{A})\,P(B)\,P(C)$

$\Rightarrow 2(1 - p)\,p^2$

$\Rightarrow 2p^2 - 2p^3$

Total

$= 3p^2 - 3p^3 + p$

Q.7 x_1, x_2, x_3 .... x_{50} are fifty real numbers such that $x_r < x_{r+1}$ for r = 1, 2, 3, ..., 49. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number is -

(A) $\dfrac{{}^{20}C_2 \times {}^{30}C_2}{{}^{50}C_5}$ (B) $\dfrac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$

(C) $\dfrac{{}^{19}C_2 \times {}^{31}C_3}{{}^{50}C_5}$ (D) none of these

Sol. **[B]**

$n(S) = {}^{50}C_5$, $n(E) = {}^{30}C_2 \times {}^{19}C_2$

$P(E) = \dfrac{{}^{30}C_2 . {}^{19}C_2}{{}^{50}C_5}$

Q.8 A number is chosen at random from the numbers 10 to 99. By seeing the number a man will laugh if product of the digits is 12. If he chooses three numbers with replacement then the probability that he will laugh at least once will be -

(A) $1 - \left(\dfrac{44}{45}\right)^3$ (B) $1 - \left(\dfrac{43}{45}\right)^3$

(C) $\left(\dfrac{44}{45}\right)^3$ (D) $\left(\dfrac{43}{45}\right)^3$

Sol. **[B]**

$n(S) = 90$

Number whose product is 12 is

{26, 34, 43, 62} = 4 number

Total number when the man will not laugh is $90 - 4 = 86$

Probability that the man not laugh = $\dfrac{86}{90} = \dfrac{43}{45}$

He tries three time then probability that he will not laugh = $\left(\dfrac{43}{45}\right)^3$

So the probability that he will laugh at least one time = $1 - \left(\dfrac{43}{45}\right)^3$

Q.9 A dice is rolled three times. The probability of getting a larger number than the previous number each time is -

(A) 15/216 (B) 5/54

(C) 13/216 (D) 1/18

Sol. **[B]**

Total number of elementary events

$= 6 \times 6 \times 6 = 216$

Clearly the second number has to be greater than unity. If the second number is i then first can be chosen in (i – 1) ways and the third (6 – i) ways so

ways for choosen three number = $(i-1)(6-i)$

But second number vary from 2 to 5, so favourable number of elementary events

$$= \sum_{i=2}^{5} (i-1)\,(6-i)$$

$= 1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1 = 20$

Required probability = $\frac{20}{216} = \frac{5}{54}$

Q.10 A man firing at a distant target has 10% chance of hitting the target in each shot. The number of times he must fire at the target to have about 50% chance of hitting the target is -

(A) 11 (B) 9 (C) 7 (D) 5

Sol. **[C]**

Probability of hitting the target in one

fire = p = $\frac{1}{10}$

$\therefore$ probability of not hitting the target in

on fire = q = $\frac{9}{10}$

Let the man take n fire then

the probability of hitting the target at least once

= 1 – (P(not hitting the target n times))

$= 1 - {}^nC_0\, p^0 q^n$

$= 1 - \left(\frac{9}{10}\right)^n$

Now $1 - \left(\frac{9}{10}\right)^n > \frac{50}{100}$ or $\left(\frac{9}{10}\right)^n < \frac{1}{2}$

But

$\frac{9}{10} > \frac{1}{2}, \left(\frac{9}{10}\right)^2 > \frac{1}{2} ..... \left(\frac{9}{10}\right)^6 > \frac{1}{2}, \left(\frac{9}{10}\right)^7 < \frac{1}{2}$

Least value of n = 7

Q.11 The minimum number of times a fair coin must be tossed so that the probability of getting at least one head is at least 0.8 is -

(A) 7 (B) 6

(C) 5 (D) 3

Sol. **[D]**

Let coin be tossed n times.

Let x be the number of heads obtained.

Then x follows a binomial distribution with parameters n and p = $\frac{1}{2}$

so we have

$p(x \geq 1) \geq 0.8 \Rightarrow 1 - p(x = 0) \geq 0.8$

$\Rightarrow p(x = 0) \leq 0.2$

$\Rightarrow {}^nC_0 \left(\frac{1}{2}\right)^n \leq 0.2 = \frac{2}{10} = \frac{1}{5}$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{5} \Rightarrow 5 \leq 2^n$

Least value of n = 3

Q.12 A bag contains four tickets numbered 00, 01, 10, 11. Four tickets are chosen at random with replacement, the probability that sum of the numbers on the tickets is 23, is -

(A) 3/32 (B) 1/64

(C) 5/256 (D) 7/256

Sol. **[A]**

Total number of ways in which 4 tickets

can be drawn four times = $4^4 = 256$

Favourable number of ways of getting a sum of 23

= coffi. of x^{23} in $(x^{00} + x^{01} + x^{10} + x^{11})^4$

= coffi. of x^{23} in $[(1 + x)(1 + x^{10})]^4$

= coffi. of x^{23} in $(1 + x)^4 (1 + x^{10})^4$

= coffi. of x^{23} in $(1 + 4x + 6x^2 + 4x^3 + x^4)$
$\times(1 + 4x^{10} + 6x^{20} + 4x^{30} + x^{40}) = 24$

Required probability = $\frac{24}{256} = \frac{3}{32}$

Q.13 If the papers of 4 students can be checked by anyone of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teacher is -

(A) 2/7 (B) 6/49

(C) 32/343 (D) 12/49

Sol. **[B]**

$$P(A) = \frac{^7C_2(2^4-2)}{7\times7\times7\times7}$$

Q.14 An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is-

(A) 16/81 (B) 1/81

(C) 80/81 (D) 65/81

Sol. **[A]**

Let A = getting not less than 2 and not greater than 5

$\Rightarrow$ A = {2, 3, 4, 5}

$\Rightarrow P(A) = \frac{4}{6} = \frac{2}{3}$

But die is rolled four times,

Therefore the probability is

$$= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{16}{81}$$

Q.15 Let A, B, C be three mutually independent events. Consider the two statements S_1 and S_2

S_1 : A and $B \cup C$ are independent

S_2 : A and $B \cap C$ are independent

Then,

(A) Both S_1 and S_2 are true

(B) Only S_1 is true

(C) Only S_2 is true

(D) Neither S_1 nor S_2 is true

Sol. **[A]**

Q.16 There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is -

(A) 1/3 (B) 1/6 (C) 1/2 (D) 1/4

Sol. **[A]**

Let F means faulty and G means not faulty then required probability = $P(F_1F_2) + P(G_1G_2)$

$= P(F_1)\,P(F_2) + P(G_1)\,P(G_2)$

$= \frac{2}{4}.\frac{1}{3} + \frac{2}{4}.\frac{1}{3}$

$= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Part-B One or more than one correct answer type questions

Q.17 A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. The probability of occurrence of A is -

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) none

Sol. **[A, B]**

Given that $P(A\cap B) = \frac{1}{6}$ and $P(\overline{A} \cap \overline{B}) = \frac{1}{3}$

$\therefore P(A)\,P(B) = \frac{1}{6}$ and $P(\overline{A})\;P(\overline{B}) = \frac{1}{3}$

$\Rightarrow P(A)\,P(B) = \frac{1}{6}$ and $[1 - P(A)]\,[1 - P(B)] = \frac{1}{3}$

$\Rightarrow 1 - [P(A) + P(B)] + \frac{1}{6} = \frac{1}{3}$

$\Rightarrow P(A) + P(B) = \frac{5}{6}$ and $P(A)\,P(B) = \frac{1}{6}$

Solving we get

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$

or $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$

Q.18 A student appears for test I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests, I, II, III are p, q and 1/2, respectively. If the probability that the student is successful is 1/2, then -

(A) p = 1, q = 0

(B) p = 2/3, q = 1/2

(C) p = 3/5, q = 2/3

(D) there are infinitely many values of p and q.

Sol. **[A, B, C, D]**

Let A, B, C be the events that the student is successful in test I, II and III respectively then

P(The student is successful)

= P(A) P(B) {1 – P(C)} + P(A) {1 – P(B)} P(C) + P(A) P(B) P(C)

$= pq\left\{1-\frac{1}{2}\right\} + p(1-q)\ \frac{1}{2} + p.q.\ \frac{1}{2}$

$= \frac{pq}{2} + \frac{p}{2}$

$\therefore\ \frac{1}{2} = \frac{pq+p}{2} \Rightarrow pq + p = 1$

Hence p(1 + q) = 1

This satisfies by p = 1, q = 0

and $p = \frac{2}{3},\ q = \frac{1}{2}$

and $p = \frac{3}{5},\ q = \frac{2}{3}$

so there are many values of p and q

so option A,B,C,D are correct

Q.19 In throwing a die let A be the event coming up of an odd number', B be the event 'coming up of an even number', C be the event 'coming up of a number ≥ 4' and D be the event 'coming up of a number < 3', then –

(A) A and B are mutually exclusive and exhaustive

(B) A and C are mutually exclusive and exhaustive

(C) A , C and D form an exhaustive system

(D) B, C and D form an exhaustive system

Sol. **[A, C]**

A = {1, 3, 5}

B = {2, 4, 6}

C = {4, 5, 6}

D = {1, 2}

Q.20 For any two events A and B in a sample space -

(A) $P(A/B) \geq \frac{P(A)+P(B)-1}{P(B)}$, P(B) ≠ 0 is always true

(B) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$ does not hold

(C) $P(A \cup B) = 1 - P(\overline{A})\,P(\overline{B})$, if A and B are independent.

(D) $P(A \cup B) = 1 - P(\overline{A})\,P(\overline{B})$, if A and B are disjoint.

Sol. [A, C]

We know that

P

$$\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)+P(B)-P(A \cup B)}{P(B)}$$

since P(A∪B) < 1

∴ –P(A∪B) > – 1

⇒ P(A) + P(B) – P(A∪B) > P(A) + P(B) – 1

⇒

$$\frac{P(A)+P(B)-P(A \cup B)}{P(B)} > \frac{P(A)+P(B)-1}{P(B)}$$

$$\Rightarrow P\left(\frac{A}{B}\right) > \frac{P(A)+P(B)-1}{P(B)}$$

Hence (A) is correct

Choice (B) holds only for disjoint i.e.

P(A∩B) = 0

∴ P(A∪B) = P(A) + P(B) – P(A∩B)

= P(A) + P(B) – P(A) . P(B)

if A, B are independent

$= 1 - (1 - P(A))(1 - P(B)) = 1 - P(\overline{A})\,P(\overline{B})$

option C is correct

Θ if A, B are disjoint then

P(A∪B) = P(A) + P(B)

⇒ option A, C are correct

Part-C Assertion-Reason type questions

The following questions 21 to 24 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Q.21 **Assertion :** If P is chosen at random in the closed interval [0, 5], then the probability that the equation $x^2 + Px + \frac{1}{4}(P + 2) = 0$ has real root is $\frac{3}{5}$.

Reason : If discriminant ≥ 0 then roots of the quadratic equation are always real.

Sol. **[A]**

$4x^2 + 4Px^2 + (P + 2) = 0$

$D \geq 0$

$\Rightarrow 16P^2 - 16(P + 2) \geq 0$

$\Rightarrow P^2 - P - 2 \geq 0$

$P \in (-\infty, -1] \cup [2, \infty)$

$P \in [0, 5]$

$$P(A) = \frac{\int_2^5 dx}{\int_0^5 dx} = \frac{3}{5}$$

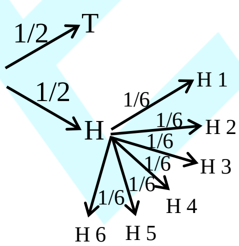
Q.22 **Assertion :** Since sample space of the experiment 'A coin is tossed if it turns up head, a die is thrown' is {(H, 1), (H, 2), (H, 3), (H, 4), (H, 4), (H, 5), (H, 6), T}.

$\therefore$ Prob. of the event {(H, 1), (H, 2), (H, 5)} is 3/7.

Reason : If all the sample points in the sample space of an experiment are pair wise mutually exclusive, equally likely and exhaustive, then probability of an event E is defined as

$$P(E) = \frac{\text{number of sample points favourable to the event E}}{\text{Total number of sample points in the sample space}}$$

Sol. **[D]**



every are not equally likely

Q.23 **Assertion :** If A and B are two independent events such that $P(A) \neq 0$, $P(B) \neq 0$ then A and B can not be mutually exclusive.

Reason : For independent events A and B, we have $P(A/B) = P(A)$ which is not so for mutually exclusive events.

Sol. **[A]**

Q.24 **Assertion :** If $\frac{1+4P}{4}, \frac{1-P}{4}, \frac{1-2P}{4}$ are probabilities of three pair-wise mutually exclusive events, then the possible values of P belong to the set $\left[-\frac{1}{4}, \frac{1}{2}\right]$.

Reason : If three events are pairwise mutually exclusive and exhaustive then sum of there probability is equal to 1.

Sol. **[B]**

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$0 \leq P(E_1) + P(E_2) \leq 1$

$0 \leq P(E_2) + P(E_3) \leq 1$

$0 \leq P(E_3) + P(E_1) \leq 1$

$$\left[\begin{array}{l} 0 \leq P(E_1) \leq 1 \\ 0 \leq P(E_2) \leq 1 \\ 0 \leq P(E_3) \leq 1 \end{array}\right.$$

Part-D Column Matching type questions

Match the items of column I to those of column II:

Q.25 The letters of the word PROBABILITY are written down at random in a row. Let E_1 denotes the event that two I′s are together and E_2 denotes the event that two B′s are together, then –

Column-I	**Column-II**
(A) $P(E_1) = P(E_2)$	(P) 2/55
(B) $P(E_1 \cap E_2)$	(Q) 2/11
(C) $P(E_1 \cup E_2)$	(R) 1/5
(D) $P(E_1/E_2)$	(S) 18/55

Sol. **[$A \to Q$, $B \to P$, $C \to S$, $D \to R$]**

word is PROBABILITY

E_1 is the event that two I's are together and E_2 is the event that two B's are together

(A) Here only two I's and two B's

so $n(E_1) = \frac{\lfloor 10}{\lfloor 2}$ and $n(S) = \frac{\lfloor 11}{\lfloor 2 \lfloor 2}$

$$P(E_1) = \frac{\lfloor 10}{\lfloor 2} \cdot \frac{\lfloor 2 \lfloor 2}{\lfloor 11} = \frac{2}{11} = P(E_2)$$

(B) when both I's and both B's are together

then $n(E) = \lfloor 9$, $n(S) = \frac{\lfloor 11}{\lfloor 2 \lfloor 2}$

$$P(E_1 \cap E_2) = \frac{\lfloor 9 \lfloor 2 \lfloor 2}{\lfloor 11} = \frac{2}{55}$$

(C) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{4}{11} - \frac{2}{55} = \frac{18}{55}$

(D) when E_1 happened then

$$P(E_1 \cap E_2) = P(E_2).\, P\left(\frac{E_1}{E_2}\right)$$

$$\Rightarrow \quad P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{\frac{2}{55}}{\frac{2}{11}} = \frac{11}{55} = \frac{1}{5}$$

Q.26 | **Column-I** | **Column-II**

(A) If the probability that units digit in square of an even integer is 4 is p,then the value of 5p is — (P) 1

(B) If A and B are independent events and $P(A \cap B) = \frac{1}{6}$, $P(\overline{A}) = \frac{2}{3}$, then $6P(B/A) =$ — (Q) 2

(C) Among 2 children, a child may equally be a boy or a girl if the probability that exactly one of them is a boy is p, then 6p = — (R) 3

(D) A boy has 20% chance of hitting at a target. Let p denote the probability of hitting the target for the first time at the n^{th} trial. If p satisfies the inequality $625p^2 - 175p + 12 < 0$, then value of n is — (S) 4

Sol. $\mathbf{A \to Q;\ B \to R;\ C \to R;\ D \to R}$

(A) {4, 6, 6, 4, 0}

$p = \frac{2}{5} \Rightarrow 5p = 2$

(B) $P(A)\,.\,P(B) = \frac{1}{6}$

$P(A) = 1 - P(\overline{A}) = \frac{1}{3}$

$$6P(B/A) = \frac{6P(B \cap A)}{P(A)} = \frac{6 \times \frac{1}{6}}{\frac{1}{3}} = 3$$

(C) $p = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

$6p = 3$

(D) $625p^2 - 175p + 12 < 0$

$\Rightarrow 625p^2 - 100p - 75p + 12 < 0$

$\frac{3}{25} < p < \frac{4}{25}$

$P(S) = \frac{1}{5}$

$P(F) = \frac{4}{5}$

$P = \left(\frac{4}{5}\right)^{n-1}\left(\frac{1}{5}\right)$

$$\frac{3}{25} < \left(\frac{4}{5}\right)^{n-1}\left(\frac{1}{5}\right) < \frac{4}{25}$$

Q.27 | **Column-I** | **Column-II**

(A) A pair of dice is thrown. If total of numbers turned up on both the dies is 8, then the probability that the number turn up on the second die is 5′ is — (P) 5/16

(B) A box contains 4 white and 3 black balls. Two balls are drawn successively and is found that second ball is white, then the probability that 1 st ball is also white is — (Q) 1/3

(C) A biased coin with probability p, 0 < p < 1 of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals (R) $\frac{1}{2}$

(D) A coin whose faces are marked 3 and 5 is tossed 4 times : what is the probability that the sum of the numbers thrown being less, than 15 ? (S) $\frac{1}{5}$

Sol. **A $\rightarrow$ S; B $\rightarrow$ R; C $\rightarrow$ Q; D $\rightarrow$ P**

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 A number is chosen randomly from one of the two sets,
A = {1801,1802..........,1899,1900}and
B = {1901,1902,.........,1999, 2000}.
If the number chosen represents a calendar year, find the probability that it has 53 Sundays.

Sol. $\frac{1}{2}\left[\left(\frac{25}{100}\right)\times\frac{2}{7}+\left(\frac{75}{100}\right)\times\frac{1}{7}\right]$

$+\frac{1}{2}\left[\left(\frac{24}{100}\right)\times\frac{2}{7}+\left(\frac{76}{100}\right)\times\frac{1}{7}\right]$

Q.2 A and B play for a prize 'A' is to throw a dice first and is to win if he throws six. If he fails B has to throw and to win he has to throw 6 or 5. If he fails, A is to throw again & win if he throws 6 or 5 or 4 and so on. Find the chance of each player.

Sol. $P(A) = A + \bar{A}\ \bar{B}A + \bar{A}\ \bar{B}\ \bar{A}\ \bar{B}A + ...$

$= \frac{1}{6}+\left(\frac{5}{6}\times\frac{4}{6}\times\frac{3}{6}\right)+\left(\frac{5}{6}\times\frac{4}{6}\times\frac{3}{6}\times\frac{2}{6}\times\frac{5}{6}\right)$

$P(A) = \frac{1}{6}+\frac{5}{18}+\frac{25}{324} = \frac{169}{324}$

$P(B) = \bar{A}B + \bar{A}\bar{B}\bar{A}B + ...$

$= \left(\frac{5}{6}\times\frac{2}{6}\right)+\left(\frac{5}{6}\times\frac{4}{6}\times\frac{3}{6}\times\frac{4}{6}\right)$

$+\left(\frac{5}{6}\times\frac{4}{6}\times\frac{3}{6}\times\frac{2}{6}\times\frac{1}{6}\times 1\right)$

$= \frac{5}{18}+\frac{5}{27}+\frac{5}{324}$

$= \frac{90+60+5}{324} = \frac{155}{324}$

Q.3 A and B throw alternately with a pair of dice. A win if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins show that his chance of winning is 30/61.

Sol. Probability for A through 6
n(E) = (1, 5), (2, 4), (3, 3), (4, 2) (5, 1) = 5
n(S) = 6 × 6 = 36

$P(A) = \frac{5}{36}$, $P(\bar{A}) = \frac{31}{36}$

probability for B through 7
n(E) = (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) = 6
n(S) = 36

$P(B) = \frac{1}{6}$, $P(\bar{B}) = \frac{5}{6}$

If A begins then probability

$P(A\text{ win}) = \frac{5}{36}+\frac{31}{36}.\frac{5}{6}.\frac{5}{36}+\frac{31}{36}.\frac{5}{6}.\frac{31}{36}.\frac{5}{6}.\frac{5}{36}+...$

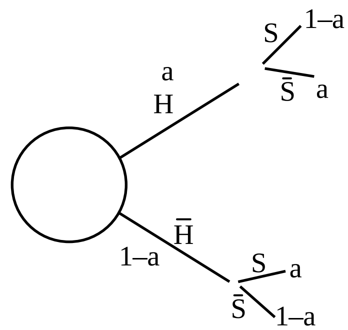
$= \frac{5}{36}\left(1+\frac{31}{36}.\frac{5}{6}+\left(\frac{31}{36}\right)^2\left(\frac{5}{6}\right)^2+......\right)$

$= \frac{5}{36}\left(\frac{1}{1-\frac{31}{36}.\frac{5}{6}}\right) = \frac{5}{36}\left(\frac{216}{216-155}\right)$

$= \frac{5}{36}.\frac{216}{61} = \frac{30}{61}$

Q.4 A certain drug, manufactured by a company is tested chemically for its toxic nature. Let the event " the drug is toxic" be denoted by H and the event " the chemical test reveals that the drug is toxic" be denoted by S. Let P (H) = a, P(S/H) = ($\bar{S}/\bar{H}$) = 1 – a. Then show that the probability that the drug is not toxic given that the chemical test reveal that it is toxic, is free from 'a'.

Sol.



$P(\bar{H}/S) = \frac{P(\bar{H}\cap S)}{P(S)} = \frac{(1-a)a}{a(1-a)+a(1-a)} = \frac{1}{2}$

Alternate :

$\frac{P(S\cap H)}{P(H)} = 1-a$

$P(S\cap H) = a(1 – a)$

$$P\left(\frac{\bar{S}}{H}\right) = 1 - a$$

$$P(\bar{S} \cap \bar{H}) = (1 - a)^2$$

Now,

$$\frac{P(\bar{H} \cap S)}{P(S)} = \frac{P(S) - P(H \cap S)}{P(S)}$$

$$= \frac{1 - (H \cap S)}{P(S)}$$

$P(S) = 2a(1 - a)$

$1 - P(S \cup H) = (1 - a)^2$

$1 - P(S) - P(H) + P(S \cap H)$

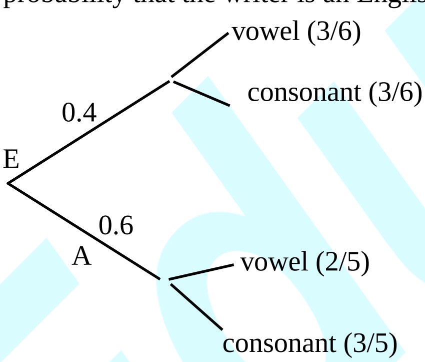
$= (1 - a)^2$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Q.5 In a Nigerian hotel, among the English speaking people 40% are English and 60% Americans.

The English and American spelling are "RIGOUR" and "RIGOR" respectively. An English speaking person in the hotel writes this word. A letter from this word is chosen at random and found to be a vowel. Find the probability that the writer is an Englishman.

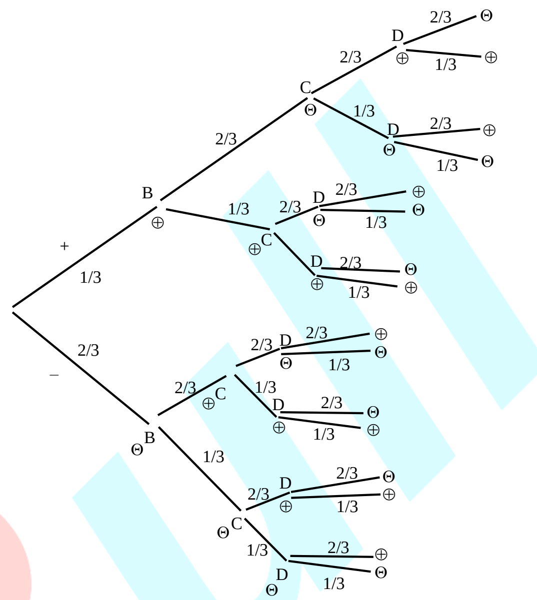
Sol.



$$P(E/V) = \frac{\frac{4}{10} \times \frac{1}{2}}{\frac{4}{10} \times \frac{1}{2} + \frac{6}{10} \times \frac{2}{5}} = \frac{2}{10} \times \frac{50}{22} = \frac{5}{11}$$

Q.6 A slip of paper is given to a person "A" who marks it with either a (+ ve) or a (-) ve sign, the probability of his writing a (+) ve sign being 1/3. "A" passes the slip to "B" who leave it alone or change the sign before passing it to "C. Similarly "C" passes on the slip to "D" and "D" passes on the slip to refree, who finds a plus sign on the slip. If it is known that B, C and D each change the sign with a probability of 2/3, then find the probability that "A" originally wrote a (+) sign.

Sol.



$$P(+/A) = \frac{\frac{4}{81} + \frac{4}{81} + \frac{4}{81} + \frac{1}{81}}{3 \times \frac{4}{81} + \frac{1}{81} + \frac{16}{81} + \frac{4}{81} + \frac{4}{81} + \frac{4}{81}} = \frac{13}{41}$$

Q.7 (a) Two natural number x and y are chosen at random. Find the probability that $x^2 + y^2$ is divisible by 10.

(b) Two numbers x and y are chosen at random from the set {1, 2, 3,............3n}. Find the probability $x^2 - y^2$ is divisible by 3.

Sol. Last digits

(a) $0^2 = 0$ $\quad 5^2 = 5$

$1^2 = 1$ $\quad 6^2 = 6$

$2^2 = 4$ $\quad 7^2 = 9$

$3^2 = 9$ $\quad 8^2 = 4$

$4^2 = 6$ $\quad 9^2 = 1$

$$\frac{1 + 1 + (2 \times 2) \times 2 + (2 \times 2) \times 2}{10 \times 10} \Rightarrow \frac{18}{100} = \frac{9}{50}$$

(b) $3k \rightarrow n$

$3k + 1 \rightarrow n$

$3k + 2 \rightarrow n$

$x^2 - y^2$

$\Rightarrow (x + y)(x - y)$

$$\Rightarrow \frac{{}^nC_2 + ({}^nC_1 \times {}^nC_1) + {}^nC_2 + {}^nC_2}{{}^{3n}C_2}$$

Q.8 A hunter knows that a deer is hidden in one of the two nearby bushes, the probability of its being hidden in bush-I being4/5. The hunter having a rifle containing 10 bullets decides to fire them all either at bush -I or II. It is known that each shot may hit one or two bushes independently of the other with probability 1/2. How many bullets must he fire on each of the two bushes to hit the animal with maximum probability.

Sol. $\frac{4}{5}\left[1-\left(\frac{1}{2}\right)^{x}\right]+\frac{1}{5}\left[1-\left(\frac{1}{2}\right)^{10-x}\right]$

$\Rightarrow 1-\frac{4}{5}\left(\frac{1}{2}\right)^{x}-\frac{1}{5}\left(\frac{1}{2}\right)^{10-x}$

$\Rightarrow 1-\frac{1}{5}\left[2^{2-x}+2^{x-10}\right]$

AM $\geq$ GM

AM = GM

when a = b

$\Rightarrow 2 - x = x - 10$

$\Rightarrow 2x = 12$

$x = 6$

$10 - x = 4$

For hitting only one bullet is reqd.

Q.9 An urn contains 10 white, 9 black, 8 red and 3 blue balls. Balls are drawn one by one at random from the urn until two blue balls are obtained at the 11^{th} draw. Find the probability of drawing 2 blue balls upto 11^{th} draw.

Sol. 11^{th} draw $\rightarrow$ 2^{nd} blue

$\frac{{}^{3}C_1 \times {}^{27}C_9}{{}^{30}C_{10}} \times \left(\frac{2}{20}\right)$

upto 11^{th} draw

$\frac{{}^{3}C_2 \times {}^{27}C_9}{{}^{30}C_{11}}$

Q.10 One ticket is selected at random from100 tickets numbered 00,01, 02...........98, 99.If 'X' and 'Y' denote the sum and product of the digits on tickets then, find P[X = 9 / Y = 0]

Sol. $P(X/Y) = P(X \cap Y) / P(Y) = \frac{2}{19}$

Q.11 A factory produces 10% defective values and another factory B produces 20% defective. A bag contains 4 values of factory A and 5 values of factory B. If two values are drawn at random from the bag, find the probability that at least one value is defective. Give your answer upto two places of decimal.

Sol. $\Rightarrow \frac{{}^{4}C_2}{{}^{9}C_2} \times \left(\frac{1}{10}\right)\left(\frac{1}{10}\right)$

$+ \frac{{}^{5}C_2}{{}^{9}C_2} \times \left(\frac{2}{10}\right)\left(\frac{1}{10}\right)$

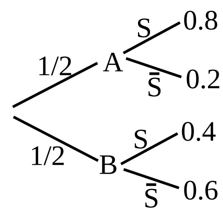
$+ \frac{{}^{4}C_1 \times {}^{5}C_1}{{}^{9}C_2} \times \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)$

$+ \frac{{}^{4}C_1 \times {}^{5}C_1}{{}^{9}C_2} \times \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)$

$+ \frac{{}^{4}C_1 \times {}^{5}C_1}{{}^{9}C_2} \times \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)$

Q.12 2 hunters A and B shot at a bear simultaneously. The bear was shot dead with only one hole in its hide. Probability of A shooting the bear 0.8 & that of B shooting the bear is 0.4. The hide was sold for Rs. 280/-. If this sum of money is divided between A & B in a fair way, then find their respective shares.

Sol.



Probability that bear was shot by A not by B

$= 0.8 \times 0.6$

Prob. that bear was shot by B not by A

$= 0.4 \times 0.2$

$\frac{P(A)}{P(B)} = \frac{6}{1}$

$\therefore$ A's share $= \frac{6}{7} \times 280 = 240$

B'S share $= \frac{1}{7} \times 280 = 40$

Q.13 A speaks the truth 2 out of 3 times, and B 4 times out of 5; A bag contains 6 balls of different colours. Which also includes one red ball. A ball is drawn randomly and both A and B tells that it is a red ball. Then find the probability that it is actually red.

Sol. **40/41**

Q.14 Odd in favour of A's speaking the truth are 1 : 2 and odds against B's speaking the truth are 1 : 3. A die is thrown. Both A and B assert that on the die 4 has turned up. Then find the probability of the truth of their assertion.

Sol. **15/17**

Q.15 In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidate decides to tick the answer at random, if he is allowed upto three chances to answer the questions, find the probability that he will get marks in the questions.

Sol. **[1/5]**

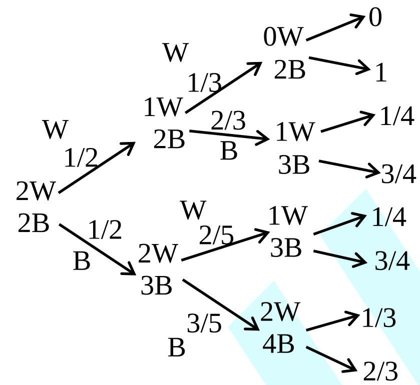
Total = $^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 15$

$$P(A) = \frac{1}{15} + \left(\frac{14}{15}\right)\times\frac{1}{14} + \frac{14}{15}\times\frac{13}{14}\times\frac{1}{13}$$

$$= \frac{1}{15}\times 3 = \frac{1}{5}$$

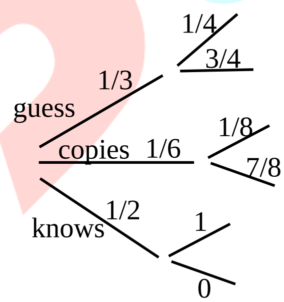
Q.16 An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white it is not replaced into the urn. Otherwise it is replaced along with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.

Sol. **[22/30]**



Q.17 In a test an examine either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knew the answer to the question given that he correctly answered it.

Sol. **[24/29]**



$$P\left(\frac{\text{knows}}{\text{correct}}\right) = \frac{1/2}{\frac{1}{2}+\frac{1}{12}+\frac{1}{48}} = \frac{24}{29}$$

Q.18 A lot contains 50 defective and 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as

A = (the first bulb is defective)

B = (the second bulb is non-defective)

C= (the two bulbs are both defective or both non defective)

Determine whether

(i) A, B, C are pairwise independent

(ii) A, B, C are independent

Sol. A, B, C are pairwise independent but A, B, C are dependent

Q.19 For the three events A, B and C, P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P (exactly one of the events C or A occurs) = p and P (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. What is the probability of at least one of the three events A, B & C occurring.

Sol. $[\frac{3p+2p^2}{2}]$

$P(A) + P(B) - 2P(A \cap B) = p$

$P(B) + P(C) - 2P(B \cap C) = p$

$P(A) + P(C) - 2P(A \cap C) = p$

$P(A \cap B \cap C) = p^2$

P(at least one event)

$= P(A) + P(B) + P(C) - P(A \cap B)$

$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3}{2}p + p^2$

Part-B Passage based objective questions

Passage - I (Q. 20 to 22)

Suppose we are given M urns, numbered 1 to M, among which we are to distribute n balls (n < M). Let P(A) denote the probability that each of the urns numbered 1 to n will contain exactly one ball. Then answer the following questions.

Q.20 If the balls are different and any number of balls can go to any urns, then P(A) is equal to -

(A) $\frac{M!}{n^M}$ (B) $\frac{n!}{M^n}$ (C) $\frac{n!}{{}^MP_n}$ (D) $\frac{1}{M^n}$

Sol. **[B]**

$$P(A) = \frac{n!}{\underbrace{(M \times M.....M)}_{n \text{ times}}} = \frac{n!}{M^n}$$

Q.21 If the balls are identical and any number of balls can go to any urns, then P(A) equals -

(A) $\frac{1}{M^n}$ (B) $\frac{1}{{}^{M+n-1}C_{M-1}}$

(C) $\frac{1}{{}^{M+n-1}C_{n-1}}$ (D) $\frac{1}{{}^{M+n-1}P_{M-1}}$

Sol. **[B]**

Q.22 If the balls are identical but atmost one ball can be put in any box, then P(A) is equal to –

(A) $\frac{1}{{}^MP_n}$ (B) $\frac{n!}{{}^nC_M}$ (C) $\frac{n!}{{}^MC_n}$ (D) $\frac{1!}{{}^MC_n}$

Sol. **[D]**

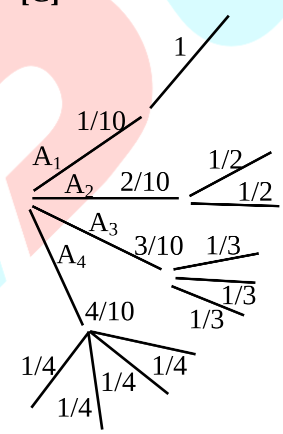
Passage II (Q. 23 to 25)

There are four boxes A_1, A_2, A_3 and A_4. Box A_i has i cards and on each card a number is printed, the numbers are from 1 to i. A box is selected randomly, the probability of selection of box A_i is $\frac{i}{10}$ and then a card is drawn. Let E_i represents the event that a card with number 'i' is drawn.

Q.23 $P(E_1)$ is equal to –

(A) $\frac{1}{5}$ (B) $\frac{1}{10}$ (C) $\frac{2}{5}$ (D) $\frac{1}{4}$

Sol. **[C]**



$$P(E_1) = \frac{1}{10} + \left(\frac{2}{10} \times \frac{1}{2}\right) + \left(\frac{3}{10} \times \frac{1}{3}\right) + \left(\frac{4}{10} \times \frac{1}{4}\right) = \frac{2}{5}$$

Q.24 $P(A_3/E_2)$ is equal to -

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Sol. **[B]**

$$P\left(\frac{A_3}{E_2}\right) = \frac{\frac{1}{10}}{\frac{1}{10} + \frac{1}{10} + \frac{1}{10}} = \frac{1}{3}$$

Q.25 Expectation of the number on the card is -

(A) 2 (B) 2.5 (C) 3 (D) 3.5

Sol. **[A]**

Passage III (Q. 26 to 28)

(i) If a pencil of length AB is having two specified mark P and Q in between AB, then probability of cutting the pencil between P and Q = $\dfrac{\text{length of PQ}}{\text{length of AB}}$

(ii) If in a Archery competition, the arrow has to hit a circle made upon a rectangular wooden board, then the probability of hitting of arrow inside the circle

$= \dfrac{\text{Area of circle}}{\text{Area of rectangular board}}$

On the basis of above information answer the following.

Q.26 If k $\in$ [0, 5], then the probability of the equation $x^2 + kx + \frac{1}{4}(k + 2) = 0$ to have real roots is -

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

Sol. **[C]**

$D \geq 0 \quad \Rightarrow k^2 - k - 2 \geq 0$

$\Rightarrow (k - 2)(k + 1) \geq 0$

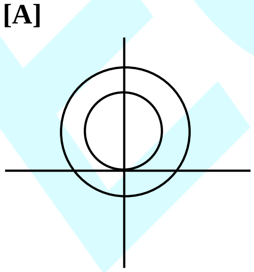
$k \in (-\infty, -1) \cup (2, \infty)$

$$\therefore P(A) = \frac{\int_2^5 dx}{\int_0^5 dx} = \frac{3}{5}$$

Q.27 There are two circles in the x-y plane whose equations are $x^2 + y^2 - 2y = 0$ & $x^2 + y^2 - 2y - 3 = 0$. If A point (x, y) is picked up randomly inside the larger circle then the probability of the point has been taken from the smaller circles is -

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Sol. **[A]**

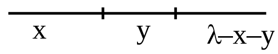


$$P(A) = \frac{\pi(1)^2}{\pi(2)^2} = \frac{1}{4}$$

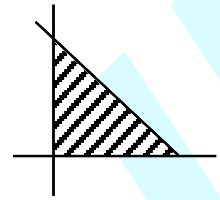
Q.28 If a stick of unit length is broken into three parts each of length x, y & z respectively, then probability of forming of triangle from broken length is -

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{5}$

Sol. **[B]**

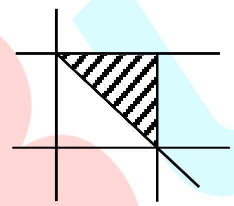


$$\left.\begin{array}{l} x > 0 \\ y > 0 \\ x + y < \lambda \end{array}\right]$$



Area = $\frac{1}{2}\lambda^2$

$x < \frac{\lambda}{2}$, $\quad y < \frac{\lambda}{2}$



$x + y > \frac{\lambda}{2}$

Area = $\frac{1}{2}\left(\frac{\lambda}{2}\right)^2 = \frac{\lambda^2}{8} \Rightarrow P(A) = \frac{1}{4}$

Passage IV (Q. 29 to 31)

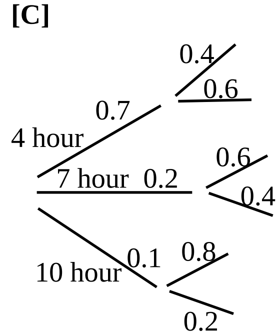
A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively.

On the basis of above information, answer the following question :

Q.29 The chance she will be successful, is -

(A) 0.28 (B) 0.38 (C) 0.48 (D) 0.58

Sol. **[C]**



Successful

= 0.28 + 0.12 + 0.08

= 0.48

Q.30 Given that she is successful, the chance she studied for 4 hours, is -

(A) $\frac{6}{12}$ (B) $\frac{7}{12}$ (C) $\frac{8}{12}$ (D) $\frac{9}{12}$

Sol. **[B]**

$$P\left(\frac{4}{S}\right) = \frac{0.28}{0.48} = \frac{7}{12}$$

Q.31 Given that she does not achieve success, the chance she studied for 4 hour, is -

(A) $\frac{18}{26}$ (B) $\frac{19}{26}$ (C) $\frac{20}{26}$ (D) $\frac{21}{26}$

Sol. **[D]**

$$P\left(\frac{4}{U}\right) = 0.42$$

0.42 + 0.08 + 0.02

$$= \frac{0.42}{0.52} = \frac{21}{26}$$

EXERCISE # 4

Old IIT-JEE questions

Q.1 **(i)** An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white ?

(ii) An unbiased die, with faces numbered, 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that, among the numbers, 1, 2, 3, 4, 5, 6, only three numbers appear in this list ? **[IIT-2001]**

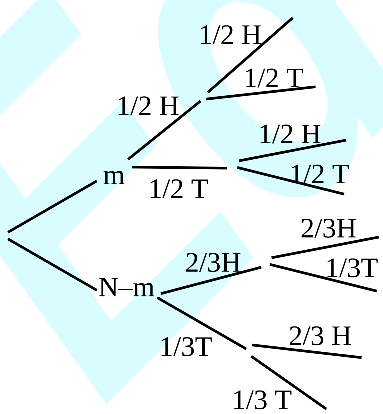
Sol. (ii) n-trial → n different balls

1, 2, 3, 4, 5, 6 → 6 diff. box

$$\frac{{}^{6}C_{3}(3^{n} - {}^{3}C_{1}[2^{n} - 2] - {}^{3}C_{2})}{6^{n}}$$

Q.2 A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **[IIT-2002]**

Sol.



$$P\left(\frac{m}{4}\right) = \frac{\frac{1}{4}\left(\frac{m}{N+m}\right)}{\frac{1}{4}\left(\frac{m}{N+m}\right) + \frac{2}{9}\left(\frac{N-m}{N+m}\right)}$$

$$= \frac{9m}{9m + 8N - 8m} = \frac{9m}{8N + m}$$

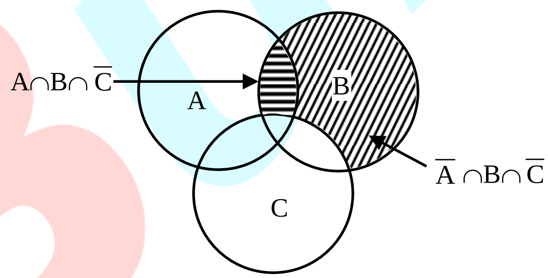
Q.3* Given that P (B) = 3/4, P $(A \cap B \cap \overline{C})$ = 1/3, P $(\overline{A} \cap B \cap \overline{C})$ = 1/3 then find probability of $B \cap C$, when $\overline{A}$, $\overline{B}$, $\overline{C}$ are negotiations of A, B, C respectively, is - **[IIT scr. 2003]**

(A) 2/3 (B) 1/12 (C) 1/15 (D) 1/4

Sol. **[B]**

Given that $P(B) = \frac{3}{4}$, $P(A \cap B \cap \overline{C}) = \frac{1}{3}$,

and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$



From diagram we get

$B \cap C = B - (A \cap B \cap \overline{C}) - (\overline{A} \cap B \cap \overline{C})$

$\Rightarrow P(B \cap C) = P(B) - P(A \cap B \cap \overline{C}) - P(\overline{A} \cap B \cap \overline{C})$

$\Rightarrow P(B \cap C) = \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{1}{12}$

Q.4 Two numbers are chosen, one by one (with out replacement) from the set of numbers A = {1, 2, 3, 4, 5, 6} The probability that minimum value of chosen number is less than 4 is - **[IIT scr. 2003]**

(A) 1/15 (B) 14/15 (C) 1/5 (D) 4/5

Sol. **[D]**

The minimum of two numbers will be less than 4.

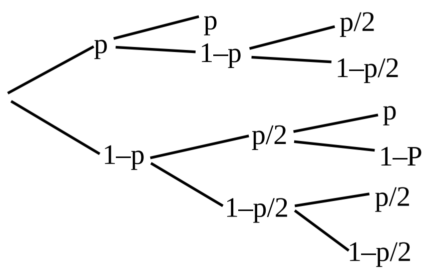
∴ P(at least one number less than 4)

⇒ 1 – P(both number greater than equal to 4)

$= 1 - \frac{3}{6} \cdot \frac{2}{5} = 1 - \frac{1}{5} = \frac{4}{5}$

Q.5 A student appears in an examination. To qualify he has to pass two out of three papers. The probability that he will pass the 1st paper is p. If he fails in one of the papers, then probability of his passing in the next paper is p/2. Otherwise it remains same. Find the probability that he will qualify. **[IIT 2003]**

Sol.



$\Rightarrow p^2 + \frac{p^2}{2}(1-p) \times 2$

$\Rightarrow 2p^2 - p^3$

Q.6 If A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are respectively $\frac{2}{3}, \frac{1}{2}, \frac{1}{3}$. If A has been hit then find the probability that B hits the target and C does not. **[IIT 2003]**

Sol. P(A has been hit)

$= 1 - P(\overline{B}\,\overline{C})$

$= 1 - \frac{1}{2} \times \frac{2}{3} = \frac{2}{3}$

$P\left(\frac{B \cap \overline{C}}{\text{A has been hit}}\right)$

$= \frac{P(B \cap \overline{C} \cap \text{A has been hit})}{P(\text{A has been hit})}$

$= \frac{P(B \cap \overline{C})}{P(\text{A has been hit})}$

$= \frac{\frac{1}{2} \times \frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$

Q.7 Three distinct numbers are chosen randomly from first 100 natural numbers, then the probability that all are divisible by 2 and 3 both is - **[IIT Scr.2004]**

(A) 4/33 (B) 4/35

(C) 4/25 (D) 4/1155

Sol. **[D]**

No. which are divisible by 6 are

6, 12, …….96 (Total 16 no.)

required probability $= \frac{{}^{16}C_3}{{}^{100}C_3}$

$= \frac{16.15.14}{100.99.98} = \frac{4}{1155}$

Q.8 Let A and B are two independent events and C be an event that exactly one of A & B occurs, prove that $P(A \cup B) . P(A' \cap B') \le P(C)$. **[IIT 2004]**

Sol. P(C) = P(A) P(B') + P(A')P(B)

Now

$P(A \cup B), P(A' \cap B')$

$\Rightarrow \{P(A) + P(B) - P(A \cap B)\} \times \{P(A') P(B')\}$

$\le [P(A) + P(B)] [P(A') P(B')]$

$\le P(A) P(A') P(B') + P(B) P(A') P(B')$

$\le P(A) P(B') + P(B) P(A')$

$\le P(C)$

Q.9 A bag contains 18 balls, 12 red and 6 white. Six balls are drawn one by one without replacement of which at least 4 are white. Find the probability that in the next two draws exactly one white ball is drawn. (Leave the answer in terms of ${}^nC_r$) **[IIT 2004]**

Sol. (4W . 2R) (1W 1R) + (5W . 1R) (1W . 1R)

Q.10 A fair dice is thrown until 1 comes, then probability that 1 comes in even number of trials is - **[IIT scr. 2005]**

(A) $\frac{5}{11}$ (B) $\frac{5}{6}$ (C) $\frac{6}{11}$ (D) $\frac{1}{6}$

Sol. **[A]**

T 1 + TTT1 + …….

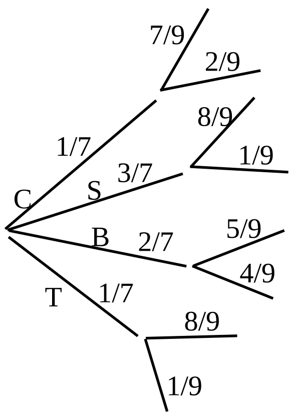
$\Rightarrow \left(\frac{5}{6}\right) \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \ldots\ldots.$

$\Rightarrow \frac{1}{6} \times \frac{5}{6} \times \frac{1}{1 - \frac{25}{36}}$

$= \frac{5}{11}$

Q.11 Ramesh goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}$, $\frac{2}{7}$ and $\frac{1}{7}$ respectively and probability that he is reaching office late if he takes car, scooter, bus or train is $\frac{2}{9}$, $\frac{1}{9}$, $\frac{4}{9}$ and $\frac{1}{9}$ respectively. Find the probability that he has traveled by car, if he reaches office in time. **[IIT 2005]**

Sol.



$$P\left(\frac{T}{C}\right) = \frac{\frac{1}{9}}{\frac{1}{9}+\frac{8}{21}+\frac{10}{63}+\frac{8}{63}} = \frac{1}{9}\times\frac{63}{49} = \frac{1}{7}$$

Passage (Q. 12 to 14)

Total n urns each containing (n+1) balls such that the i^{th} urn contains i white balls and (n + 1 – i) red balls.
Now u_i be the event of selecting i^{th} urn, i = 1, 2, 3 .... n and w denotes the event of getting a white ball.

Q.12 If P $(u_i) \propto i$, where i = 1, 2, 3 , ... n, then $\lim\limits_{n\to\infty}$ P(w) is equal to

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

Sol. **[A]**

x + 2x +……..+ nx = 1

$$\frac{xn(n+1)}{2} = 1$$

$$x = \frac{2}{n(n+1)}$$

$$P(w) = \sum_{i=1}^{n}\frac{i\times 2}{n(n+1)}\times\frac{i}{(n+1)}$$

$$= \frac{2}{n(n+1)^2}\times\sum_{i=1}^{n} i^2$$

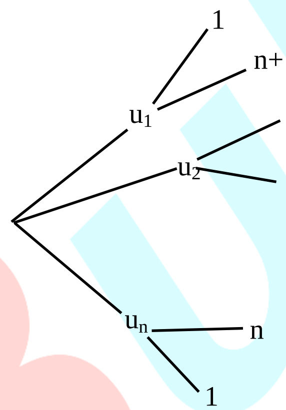
$$= \frac{2}{n(n+1)^2}\times\frac{n(n+1)(2n+1)}{6}$$

$$\lim_{n\to\infty} P(w) = \frac{2\times 2}{6} = \frac{2}{3}$$

Q.13 If P (u_i) = c, where c is a constant then P (u_n/w) is equal to -

(A) $\frac{2}{n+1}$ (B) $\frac{n}{n+1}$ (C) $\frac{1}{n+1}$ (D) $\frac{1}{2(n+1)}$

Sol. **[A]**



$$P\left(\frac{u_n}{w}\right) = \frac{c\times\frac{n}{n+1}}{c\times\frac{1}{n+1}+c\times\frac{2}{n+1}+c\times\frac{3}{n+1}......+c\times\frac{n}{n+1}}$$

$$= \frac{n2}{n(n+1)}$$

Q.14 If n is even and E denotes the event of choosing even numbered urn and also P(u_i) = $\frac{1}{n}$, then find the value of P (w/E)

(A) $\frac{n+2}{2(n+1)}$ (B) $\frac{n+2}{2n+1}$

(C) $\frac{n}{n+1}$ (D) $\frac{1}{n+1}$

Sol. **[A]**

$$\frac{P(w\cap E)}{P(E)} = \frac{\frac{1}{n}\left[\frac{2}{n+1}+\frac{4}{n+1}+......\frac{n}{n+1}\right]}{\left[\frac{1}{n}+\frac{1}{n}+........+\frac{1}{n}\right]}$$

$\frac{n}{2}$ times

Q.15 One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is - **[IIT-2007]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

Sol. **[C]**

$$P\left(\frac{\text{Indian adjacent}}{\text{American adjacent}}\right) = \frac{4!\times 2^5}{5!\times 2^4} = \frac{2}{5}$$

Q.16 Let $H_1, H_2, \ldots\ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$. **[IIT-2007]**

Assertion : $P(H_i \mid E) > P(E \mid H_i) \,.\, P(H_i)$ for $i = 1, 2, \ldots, n$.

Reason : $\sum_{i=1}^{n} P(H_i) = 1$

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Sol. **[D]**

Q.17 Let E^C denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^C \cap F^C | G)$ equals- **[IIT-2007]**

(A) $P(E^C) + P(F^C)$ (B) $P(E^C) - P(F^C)$

(C) $P(E^C) - P(F)$ (D) $P(E) - P(F^C)$

Q.18 An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is **[IIT 2008]**

(A) 2, 4 or 8 (B) 3, 6 or 9

(C) 4 or 8 (D) 5 or 10

Sol. **[D]**

Let $n(B) = x > 0$

Now given that A and B are independent events.

$P(A).P(B) = P(A \cap B)$

$\frac{4}{10} \cdot \frac{x}{10} = \frac{z}{10}$ where $z = n(A \cap B) \le \min^m (4, x)$

$\Rightarrow x = \frac{5z}{2}$ where z may be equal to 0, 1, 2, 4.

$\Rightarrow x = 5$ or 10.

Q.19 Consider the system of equations

$ax + by = 0$, $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$

Assertion : The probability that the system of equations has a unique solution is $\frac{3}{8}$

Reason : The probability that the system of equations has a solution is 1. **[IIT 2008]**

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Sol. **[B]**

Total possibilities $= 2^4 = 16$

Now favourable solution is possible if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

$\therefore$ cases possible are

$$\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 6$$

So probability $= \frac{6}{16} = \frac{3}{8}$

Passage (Q. 20 to 22)

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. **[IIT 2009]**

Q.20 The probability that $X = 3$ equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

Sol. **[A]**

$P(x = 3) \rightarrow$ probability of 3 tosses required

$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

Q.21 The probability that $X \ge 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

Sol. **[B]**

$P(x \ge 3) = 1 - \{P(x = 1) + P(x = 2)\}$

$= 1 - \left(\frac{1}{6} + \frac{5}{6} \times \frac{1}{6}\right)$

$$= 1 - \left(\frac{1}{6} + \frac{5}{36}\right) \quad = 1 - \frac{11}{36} = \frac{25}{36}$$

Q.22 The conditional probability that X ≥ 6 given X ≥ 3 equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

Sol. **[D]**

$$P\left(\frac{x \geq 6}{x > 3}\right) = \frac{P(x \geq 6)}{P(x > 3)}$$

P(x > 3) = 1 – {P(x = 1) + P(x = 2) + P(x = 3)}

$$= 1 - \left(\frac{1}{6} + \frac{5}{36} + \frac{25}{216}\right)$$

$$= 1 - \left\{\frac{36 + 30 + 25}{216}\right\}$$

$$= 1 - \frac{91}{216} = \frac{125}{216}$$

$$P\ (x \geq 6) = \left(\frac{5}{6}\right)^5 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \left(\frac{5}{6}\right)^7 \frac{1}{6} \ldots\ldots\ldots$$

$$= \left(\frac{5}{6}\right)^5 \frac{1}{6} \left\{1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots\ldots\right\}$$

$$= \left(\frac{5}{6}\right)^5 \frac{1}{6} \left\{\frac{1}{1 - \frac{5}{6}}\right\} = \frac{5^5}{6^5}$$

$$P\left(\frac{x \geq 6}{x > 3}\right) = \frac{P(x \geq 6)}{P(x > 3)}$$

$$P = \frac{5^5/6^5}{5^3/6^3} = \frac{5^2}{6^2} = \frac{25}{36}$$

Q.23 A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is 3/4. If the signal received at station B is green, then the probability that the original signal was green is **[IIT 2010]**

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Sol. **[C]**

Event (1) : original signal

OG : Original signal is green

OR : Original signal is red

Event (2) : Signal received by A.

AG : A received green

AR : A received Red

Event (3) : Signal received by B

BG : B received green

BR : B received Red

$$P\left(\frac{OG}{BG}\right) = \frac{P(OG).P\left(\frac{BG}{OG}\right)}{P(OG).P\left(\frac{BG}{OG}\right) + P(OR).P\left(\frac{BG}{OR}\right)}$$

$$= \frac{\frac{4}{5}\left[\frac{3}{4}.\frac{3}{4} + \frac{1}{4}.\frac{1}{4}\right]}{\frac{4}{5}\left[\frac{3}{4}.\frac{3}{4} + \frac{1}{4}.\frac{1}{4}\right] + \frac{1}{5}\left[\frac{1}{4}.\frac{3}{4} + \frac{3}{4}.\frac{1}{4}\right]} = \frac{20}{23}$$

Q.24 Let ω be a complex cube root of unity with ω ≠ 1. A fair die is thrown three times If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is **[IIT 2010]**

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Sol. **[C]**

$n(s) = 6^3$

r_1	r_2	r_3
1	1	x
	2	3,6
	3	2,5
	4	x
	5	3,6
	6	2,5
2	1	3,6
	2	x
	3	1,4
	4	3,6
	5	x
	6	1,4
3	1	2,5
	2	1,4
	3	x
	4	2,5
	5	1,4
	6	x

Similarly total number of elements in events set is 48

$$= \frac{48}{216} = \frac{12}{54} = \frac{2}{9}$$

Q.25 Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If $P(T)$ denotes the probability of occurrence of the event T, then -**[IIT 2011]**

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$

(B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$

(D) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

Sol.[A,D]

Passage (Q. 26 to 27)

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2. **[IIT 2011]**

Q.26 The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Sol. [B] $\boxed{3W\ 2R}$ $\boxed{1W}$

U_1 U_2

Required probability = $P(H)[P(W/H) \times P(W_2) + P(R/H)P(W_2)] + P(T)\,[P\left(\frac{both\,W}{T}\right) P(W_2) + P\left(\frac{both\,R}{T}\right) P(W_2) + P\left(\frac{R_1\,\&\,W_1}{T}\right) P(W_2)]$

=

$$\frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]+\frac{1}{2}\left[\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right]$$

$$=\frac{1}{2}\left[\frac{3}{5}+\frac{1}{5}\right]+\frac{1}{2}\left[\frac{3}{10}+\frac{1}{30}+\frac{2}{5}\right]=\frac{2}{5}+\frac{11}{30}=\frac{23}{30}$$

Q.27 Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

Sol. [D] Required probability

=

$$\frac{P(H)\left[P\left(\frac{W_1}{H}\right)P(W_2)+P\left(\frac{R_1}{H}\right)P(W_2)\right]}{P(H)\left[P\left(\frac{W_1}{H}\right)P(W_2)+P\left(\frac{R_1}{H}\right)P(W_2)\right]+P(T)\left[P\left(\frac{both\,W}{T}\right)P(W_2)+P\left(\frac{both\ R}{T}\right)P(W_2)+P\left(\frac{R_1\,\&\,W_1}{T}\right)P(W_2)\right]}$$

$$=\frac{\frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]}{\frac{23}{30}}=\frac{12}{13}$$

Q.28 A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true? **[IIT 2012]**

(A) $P\left[X_1^C \mid X\right]=\frac{3}{16}$

(B) P[Exactly two engines of the ship are functioning $\mid X] = \frac{7}{8}$

(C) $P\left[X \mid X_2\right]=\frac{5}{16}$

(D) $P\left[X \mid X_1\right]=\frac{7}{16}$

Sol. **[B, D]**

$P(X) = P\,(\overline{X}_1\cap X_2\cap X_3)+P(X_1\cap \overline{X}_2\cap X_3)\ +$

$P(X_1\cap X_2\cap \overline{X}_3)+P(X_1\cap X_2\cap X_3)$

$P(X) = \frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}+\frac{1}{2}.\frac{1}{4}.\frac{1}{4}$

$=\frac{1}{4}$

$$P\left(\frac{\overline{X}_1}{X}\right)=\frac{P(\overline{X}_1\cap X)}{P(X)}=\frac{P(\overline{X}_1\cap X_2\cap X_3)}{P(X)}$$

$$=\frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}}{\frac{1}{4}}=\frac{1}{8}$$

$$P\left(\frac{X}{X_2}\right)=\frac{P(X\cap X_2)}{P(X_2)}$$

$$=\frac{P(X_1).P(X_2).P(X_3)+P(X_1).P(X_2).P(\overline{X}_3)+P(\overline{X}_1).P(X_2).P(X_3)}{P(X_2)}$$

$= P(X_1).P(X_3) + P(X_1).P(\overline{X}_3) + P(\overline{X}_1).P(X_3)$

$= \frac{1}{2}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}+\frac{1}{2}.\frac{1}{4} = \frac{5}{8}$

$P\left(\frac{X}{X_1}\right) = P(X_2).P(X_3) + P(\overline{X}_2)\,P(X_3) +$

$P(X_2).P(\overline{X}_3)$

$= \frac{1}{4}.\frac{1}{4}+\frac{3}{4}.\frac{1}{4}+\frac{1}{4}.\frac{3}{4}$

$= \frac{7}{16}$

$P\left(\frac{\text{Exactly two engine are working}}{X}\right) =$

$$\frac{P\left(\frac{\text{Exactly two engine are working}}{X}\cap X\right)}{P(X)}$$

$=$

$$\frac{P(X_1)P(X_2)P(\overline{X}_3)+P(\overline{X}_1)P(X_2)P(X_3)+P(X_1)P(\overline{X}_2)P(X_3)}{P(X)}$$

$= \frac{\frac{7}{32}}{\frac{1}{4}} = \frac{7}{8}$

Q.29 Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is **[IIT 2012]**

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Sol. [A] required probability = 1 – P(D_4 has diff.)

$= 1 - \left(\frac{6.1.1.5+{}^3C_2.6.1.5.4+6.5.4.3}{6^4}\right)$

$= \frac{91}{216}$

Q.30 Let X and Y be two events such that $P(X|Y) = \frac{1}{2}$, $P(Y|X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct? **[IIT 2012]**

(A) $P(X \cup Y) = \frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P(X^C \cap Y) = \frac{1}{3}$

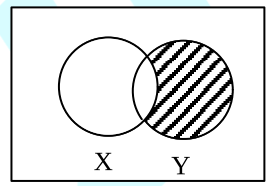
Sol. **[A, B]**

$P\left(\frac{X}{Y}\right) = \frac{P(X\cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$

$\frac{P(Y\cap X)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

$P(X \cap Y) = P(X).P(Y)$ True



$(X^C \cap Y)$

$P(X^C \cap Y) = P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6}$

$= \frac{1}{6}$

EXERCISE # 5

Q.1 m red socks and n blue socks (m > n) in a cupboard are well mixed up, where $m + n \le 101$. If two socks are taken out at random, the chance that they have the same colour is 1/2. Find the largest value of m.

Sol. 55

Q.2 A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number N (≥ 2) of ten and five paise coins. Five coins are taken out of the box at random, find the probability that the total value of these 5 coins is less than one rupee and fifty paise. **[IIT-1988]**

Sol. $1 - \frac{10(N+2)}{N+{}^7C_5}$

Q.3 Numbers are selected at random, one at a time, from the two digit numbers 00, 01, 02......99 with replacement. An event E occurs if the only if the product of the two digit of a selected numbers is 18. If four numbers are selected, find probability that the event E occurs at least 3 times. **[IIT-1993]**

Sol. [$\frac{97}{(25)^4}$]

Q.4 An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2,3, 4,...12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8 ? **[IIT-1994]**

Sol. **[0.2436]**

Q.5 If two events A and B are such that $P(A^C) = 0.3$, $P(B) = 0.4$ and $P(AB)^C = 0.5$, then $P\left(\frac{B}{A \cup B^c}\right)$ =.................... **[IIT 1994]**

Sol. [$\frac{1}{4}$]

Q.6 Sixteen players S_1, S_2 ... S_{16} play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two player of the pair. Assume that all the players are of equal strength

(a) Find the probability that the players S_1 is among the eight winners

(b) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winner **[IIT-97]**

Sol. [(a) $\frac{1}{2}$ (b) $\frac{8}{15}$]

Q.7* If the integers m and n are chosen one by one with replacement between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals - **[IIT-99]**

(A) 1/4 (B) 1/7 (C) 1/8 (D) 1/49

Sol. **[A]**

$7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$…….

Therefore, for 7^r, $r \in N$ the no. ends at unit place 7, 9, 3, 1, 7……

But $7^m + 7^n$ is divisible by 5 if it end at 5 or 0

For this m and n should be as follows

	m	**n**
1	4r	**4r + 2**
2	4r + 1	**4r + 3**
3	4r + 2	**4r**
4	**4r + 3**	**4r + 1**

For any given value of m there will be 25 values of n

Hence required probability = $\frac{100 \times 25}{100 \times 100} = \frac{1}{4}$

Q.8* The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects the student have a 75% chance of passing in atleast one, a 50% chance of passing in atleast two, and a 40% chance of passing in exactly two. Which of the following relations are true ? **[IIT-99]**

(A) p + m + c = 19/20 (B) p + m + c = 27/20

(C) pmc = 1/10 (D) pmc = 1/4

Sol. **[B, C]**

From question

$m + p + c - mp - mc - pc + mpc = \frac{3}{4}$ ….(i)

and $mp(1 - c) + mc(1 - p) + pc(1 - m) = \frac{2}{5}$

or $mp + pc + mc - 3mpc = \frac{2}{5}$ …..(ii)

Also $mp + pc + mc - 2mpc = \frac{1}{2}$

$\Rightarrow mpc = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

from (ii)

$\Rightarrow mp + pc + mc = \frac{2}{5} + \frac{3}{10} = \frac{7}{10}$

from (i)

$\Rightarrow m + p + c = \frac{3}{4} + \frac{7}{10} - \frac{1}{10} = \frac{27}{20}$

$\Rightarrow$ option B, C are correct

Q.9 Eight players P_1 , P_2 .... P_8 play a knock-out tournament. It is known that whenever the players P_i, and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final. **[IIT-99]**

Sol. [4/35]

Q.10 A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occur.

Prove that $p_1 = 1$, $p_2 = 1 - p^2$ and

$p_n = (1 - p)\, p_{n-1} + p(1 - p)\, p_{n-2}$

for all $n \geq 3$. **[IIT-2000]**

Sol.

Q.11 In a box, there are 8 alphabets cards with the letters : S, S, A, A, A, H, H, H. Find the probability that the word 'ASH' will form if .

(i) the three cards are drawn one by one & placed on the table in the same order that they are drawn.

(ii) the three cards are drawn simultaneously

Sol. [(i) $\frac{3}{56}$ (ii) $\frac{9}{28}$]

Q.12 To pass a test a child has to perform successfully in two consecutive tasks, one easy and one hard task. The easy task he can perform successfully with probability 'e' and the hard task he can perform successfully with probability 'h', where $h < e$. He is allowed 3 attempts, either in the order (Easy, Hard, Easy) (option A) or in the order (Hard, Easy, Hard) (option B) whatever may be the order, he must be successful twice in a row. Assuming that his attempts are independent, in what order he choses to take the tasks, in order to maximize his probability of passing the test.

Sol. **[**Option B**]**

Q.13 A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

Sol. **[120]**

Q.14 The probability that an archer hits the target when it is windy is 0.4; when it is not windy, her probability of hitting the target is 0.7. On any shot, the probability of a gust of wind is 0.3. Find the probability that

(a) She hit the target on first shot

(b) Hits the target exactly once in two shots

Sol. **[(a) 0.61 ; (b) 0.4758]**

Q.15 There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. the probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ (i = 1, 2, 3, 4). If we randomly select one of the urns & draw a ball, then the probability of ball being white is p/q where $p, q \in N$ are in their lowest form. Find (p + q).

Sol. **[2065]**

Q.16 The chance of one event happening is the square of the chance of a 2nd event, but odds against the first are the cubes of the odds against the 2nd. Find the chances of each. (assume that both events are neither sure nor impossible).

Sol. [$\frac{1}{9}, \frac{1}{3}$]

Q.17 A game is played with a special fair cubic die which has one red side, two blue sides, and three green sides. The result is the colour of the top side after the die has been rolled. If the die is rolled repeatedly, the probability that the second blue result occurs on or before the tenth roll, can be expressed in the form $\frac{3^p - 2^q}{3^r}$ where p, q, r are positive integers, find the value of $p^2 + q^2 + r^2$.

Sol. [283]

Q.18 An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

Sol. [$\frac{407}{576}$]

Q.19 3 Students {A, B, C} tackle a puzzle together and offers a solution upon which majority of the 3 agrees. Probability of A solving the puzzle correctly is p. Probability of B solving the puzzle correctly is also p. C is a dumb student who randomly supports the solution of either A or B. There is one more student D, whose probability of solving the puzzle correctly is once again, p. Out of the 3 member team {A, B, C} and one member team {D}, which one is more likely to solve the puzzle correctly.

Sol. [Both are equally likely]

Q.20 A uniform unbiased die is constructed in the shape of a regular tetrahedron with faces numbered 2, 2, 3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.

(i) exactly 6 on each of 3 successive throws.

(ii) more than 4 on at least one of the three successive throws.

Sol. **[(i)** $\frac{125}{16^3}$ **(ii)** $\frac{63}{64}$ **]**

Q.21 A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each.

Sol. [$\frac{^{24}C_2}{^{64}C_2}$ or $\frac{23}{168}$]

Q.22 A player tosses an unbiased coin and is to score two points for every head turned up and one point for every tail turned up. If P_n denotes the probability that his score is exactly n points, prove that $P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1})$ $n \geq 3$

Also compute P_1 and P_2 and hence deduce the pr that he scores exactly 4.

Sol. [$P_1 = \frac{1}{2}$; $P_2 = \frac{3}{4}$]

Q.23 Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p. Moreover, at the next stop either no passenger or exactly one passenger boards the bus. The probability of no passenger boarding the bus at the next stop being P_0. Find the probability that when the bus continues on its way after the stop, there will again be 'n' passengers in the bus.

Sol. [$(1 - p)^{n-1}$. $[p_0 (1 - p) + np(1 - p_0)]$]

Q.24 A student bunks the class of probability and equally likely to choose one of the four regions: (Chambal Garden (R-I); Gumanpura (R-II); Jawahar Nagar (R-III); Rajeev Nagar (R-IV) to reach away from the eye of teacher. If he chooses R-I he is successful with probability 1/6 and for R-II; R-III; R-IV this is 1/8, 1/10, 1/12 respectively. If the student is successful, then the probability that he chooses the region

Column-I	**Column-II**
(A) I	(P) 12/57
(B) II	(Q) 15/57

(C) III (R) 20/57

(D) IV (S) 10/57

(T) 9/57

Sol. **[A → R ; B → Q ; C → P ; D → S]**

Q.25 6 fair 6-sided dice are rolled. Then the probability that the sum of the values on the top faces of the dice is divisible by 7 can be expressed as $\frac{\lambda}{7776}$ then find the sum of the digits of λ.($\lambda \in N$)

Sol. **[4]**

Q.26 16 players take part in a tennis tournament. The order of the matches is chosen at random. There is always a player better than another one, the better wins. Find

(a) The probability that all the 4 best players reach the semifinals.

(b) The probability that the sixth best reaches the semifinals.

Sol. **[(a)** $\frac{64}{455}$ **(b)** $\frac{24}{91}$ **]**

Passage (Q. 27 to 29)

There are 16 players S_1, S_2, ... S_{16} playing in a knockout round tournament. They all are equally skilled. They are divided into 8 pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair, and they enter into the next round. The tournament proceed in the similar way.

On the basis of above information, answer the following questions -

Q.27 What is the probability that S_1 is among the 8 winners of 1st round ?

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Sol. **[D]**

Q.28 The probability that S_1 wins the tournament given that S_4 enters in semifinals is -

(A) $\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{16}$ (D) $\frac{1}{2}$

Sol. **[B]**

Q.29 The probability that S_1 wins the tournament given that S_2 enters into final is -

(A) $\frac{1}{20}$ (B) $\frac{1}{10}$ (C) $\frac{1}{2}$ (D) $\frac{1}{30}$

Sol. **[D]**

Q.30 Three shots are fired independently at a target in succession. The probabilities that the target is hit in the first shot is 1/2, in the second 2/3 and in the third shot is 3/4, In case of exactly one hit, the probability of destroying the target is 1/3 and in the case of exactly two hits, 7/11 and in the case of three hits is 1.0. Find the probability of destroying the target in three shots.

Sol. **[** $\frac{5}{8}$ **]**

Q.31 In a game of chance each player throws two unbiased dice and scores the difference between the larger and smaller number which arise. Two players compete and one or the other wins if and only if he scores atleast 4 more than his opponent. Find the probability that neither player wins.

Sol. **[** $\frac{74}{81}$ **]**

Q.32 A plane is landing. if the weather is favourable, the pilot landing the plane can see the runway. In this case the probability of a safe landing is p_1. if there is a low cloud ceiling, the pilot has to make a blind landing by instruments. The reliability (the probability of failure free functioning) of the instruments needed for a blind landing is P. If the blind landing instruments function normally, the plane makes a safe landing with the same probability p_1 as in the case of a visual landing. If the blind landing instruments fail, then the pilot may make a safe landing with probability $p_2 < p_1$. Compute the probability of a safe landing if it is known that in K percent of the cases there is a low cloud ceiling. Also find the probability that the pilot used the blind landing instrument, if the plane landed safely.

Sol. **[** $P(E) = \left(1 - \frac{K}{100}\right)p_1 + \frac{K}{100}[Pp_1 + (1 - P)p_2]$;

$$P(H_2/A) = \frac{\frac{K}{100}[Pp_1 + (1-P)p_2]}{\left(1-\frac{K}{100}\right)p_1 + \frac{K}{100}[Pp_1 + (1-P)p_2]}]$$

Q.33 A train consists of n carriages, each of which may have a defect with probability p. All the carriages are inspected, independently of one another, by two inspectors; the first detects defects (if any) with probability p_1, & the second with probability p_2. If none of the carriages is found to have a defect, the train departs. Find the probability of the event; " The train departs with atleast one defective carriage"

Sol. **[**$1 - [1- p(1 - p_1)(1- p_2)]^n$**]**

Q.34 During a power blackout, 100 persons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is know that the polygraph is 90% reliable when administered to a guilty person and 98% reliable when given to some one who is innocent, Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrong doing. If the probability that a given suspect is innocent given that the photograph says he is guilty is a/b where a and b are relatively prime find the value of (a + b).

Sol. **[**179**]**

Q.35 n people are asked a question successively in a random order & exactly 2 of the n people know the answer :

(a) if n > 5, find the probability that the first four of those asked do not know the answer.

(b) Show that the probability that the r^{th} person asked is the first person to know the answer is $\left[\frac{2(n-r)}{n(n-1)}\right]$, if 1 < r < n

Sol. **[(a)** $\frac{(n-4)(n-5)}{n(n-1)}$ **]**

Q.36 A box contains three coins two of them are fair and one two-headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.

(i) Find the probability that head appears twice.

(ii) If the same coin is tossed twice, find the probability that it is two headed coin

(iii) Find the probability that tail appears twice.

Sol. **[(i)** $\frac{1}{2}$ **(ii)** $\frac{1}{2}$ **(iii)** $\frac{1}{2}$ **]**

Q.37 The ratio of the number of trucks along a highway, on which a petrol pump is located, to the number of cars running along the same highway is 3 : 2. It is known that an average of one truck in thirty trucks and two cars in fifty cars stop at the petrol pump to be filled up with the fuel. If a vehicle stops at the petrol pump to be filled up with the fuel, find the probability that it is a car.

Sol. **[** $\frac{4}{9}$ **]**

Q.38 A batch of fifty radio sets was purchased from three different companies A, B and C. Eighteen of them were manufactured by A, twenty of them by B and the rest were manufactured by C. The companies A and C produce excellent quality radio sets with probability equal to 0.9; B produces the same with the probability equal to 0.6.

What is the probability of the event that the excellent quality radio set chosen at random is manufactured by the company B ?

Sol. **[** $\frac{4}{13}$ **]**

Q.39 Integers a, b, c and d not necessarily distinct, are chosen independently and at random from the set S = {0, 1, 2, 3 ……. 2006, 2007}. If the probability that | ad – bc | is even, is $\frac{p}{q}$ where p and q are relatively prime then find the value of (p + q).

Sol. **[**13**]**

Q.40 Suppose that there are 5 red points and 4 blue points on a circle. Let $\frac{m}{n}$ be the probability that a convex polygon whose vertices are among the 9 points has at least one blue vertex where m and n are relatively prime. Find (m + n).

Sol. **[458]**

Q.41 A doctor is called to see a sick child. The doctor knows (prior to the visit) that 90% of the sick children in that neighbourhood are sick with the flu, denoted by F, while 10% are sick with the measles, denoted by M. A well known symptom of measles is a rash, denoted by R. The probability of having a rash for a child sick with the measles is 0.95. However, occasionally children with the flu also develop a rash, with conditional probability 0.08.
Upon examination the child, the doctor finds a rash. What is the probability that the child has the measles ? If the probability can be expressed in the form of p/q where p, q $\in$ N and are in their lowest form, find (p + q).

Sol. **[262]**

Q.42 Two cards are randomly drawn from a well shuffled pack of 52 playing cards, without replacement. Let x be the first number and y be the second number. Suppose that Ace is denoted by the number 1; jack is denoted by the number 11; Queen is denoted by the number 12' King is denoted by the number 13.
Find the probability that x and y satisfy $\log_3(x + y) - \log_3 x - \log_3 y + 1 = 0$.

Sol. **[** $\frac{11}{663}$ **]**

Q.43 A hunter's chance of shooting an animal at a distance r is $\frac{a^2}{r^2}$ (r > a). He fires when r = 2a & if he misses he reloads & fires when r = 3a, 4a, … If he misses at a distance 'na', the animal escapes. Find the odds against the hunter.

Sol. **[n + 1: n –1]**

Q.44 A hotel packed breakfast for each of the three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese and fruit rolls. The preparer wrapped each of the nine rolls and once wrapped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. If the probability that each guest got one roll of each type is m/n where m and n are relatively prime integers, find the value of (m + n)

Sol. **[79]**

Q.45 There are two lots of identical articles with different amount of standard and defective articles. There are N articles in the first lot, n of which are defective and M articles in the second lot, m of which are defective. K articles are selected from the first lot and L articles from the second and a new lot results. Find the probability that an article selected at random from the new lot is defective.

Sol. **[** $\frac{KnM + LmN}{MN(K + L)}$ **]**

Q.46 With respect to a particular question on a multiple choice test (having 4 alternatives with only 1 correct) a student knows the answer and therefore can eliminate 3 of the 4 choices from consideration with probability 2/3, can eliminate 2 of the 4 choices from consideration with probability 1/6, can eliminate 1 choice from consideration with probability 1/9, and can eliminate none with probability 1/18. If the student knows the answer, he answers correctly, otherwise he guesses from among the choices not eliminated. If the answer given by the student was found correct, then the probability that he knew the answer is $\frac{a}{b}$ where a and b are relatively prime. Find the value of (a + b)

Sol. **[317]**

Q.47 A match between two players A and B is won by the player who first wins two games. A's chance of wining, drawing or losing any particular games are 1/2, 1/6 or 1/3 respectively. If the probability of A's winning the match can be expressed in the form p/q, find (p + q).

Sol. **[206]**

Q.48 A bag contains n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty, then the probability that each pair consists of one white and one red ball is -

(A) $\frac{2^n}{{}^{2n}C_n}$ (B) $\frac{2^{2n}}{{}^{3n}C_n}$

(C) $\frac{2^4}{{}^{2n}C_n}$ (D) $\frac{1}{{}^{2n}C_n}$

Sol. **[A]**

Let A_i (i = 1, 2,….n) be the event of getting one white and one red ball in i^{th} draw. Then required probability

$= P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n)$

$= P(A_1)\, P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1 \cap A_2}\right) \ldots\ldots$

$\ldots.\ P\left(\frac{A_n}{A_1 \cap A_2 \cap \ldots\ldots \cap A_{n-1}}\right)$ ….(1)

Now $P(A_1) = \frac{{}^nC_1 . {}^nC_1}{{}^{2n}C_2} = \frac{n^2}{{}^{2n}C_2}$

$P\left(\frac{A_2}{A_1}\right) = \frac{{}^{n-1}C_1 . {}^{n-1}C_1}{{}^{2n-2}C_2} = \frac{(n-1)^2}{{}^{2n-2}C_2}$

$P\left(\frac{A_3}{A_1 \cap A_2}\right) = \frac{{}^{n-2}C_1 . {}^{n-2}C_1}{{}^{2n-4}C_2} = \frac{(n-2)^2}{{}^{2n-4}C_2}$

M

P

$\left(\frac{A_{n-1}}{A_1 \cap A_2 \cap A_3 \ldots . \cap A_{n-2}}\right) = \frac{{}^2C_1 . {}^2C_1}{{}^4C_2} = \frac{2^2}{{}^4C_2}$

and $P\left(\frac{A_n}{A_1 \cap A_2 \cap \ldots . \cap A_{n-1}}\right) = \frac{1}{{}^2C_2}$

From (1) we have

required probability

$= \frac{n^2}{{}^{2n}C_2} \times \frac{(n-1)^2}{{}^{2n-2}C_2} \times \frac{(n-2)^2}{{}^{2n-4}C_2} \times \ldots . \times \frac{2^2}{{}^4C_2} \times \frac{1}{{}^2C_2}$

$= \frac{(\lfloor n)^2 . 2^n}{\lfloor 2n} = \frac{2^n}{{}^{2n}C_n}$

Q.49 In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than :

(A) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (B) $\frac{9}{\log_{10} 4 - \log_{10} 3}$

(C) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (D) $\frac{1}{\log_{10} 4 - \log_{10} 3}$

Sol. **[D]**

ANSWER KEY

EXERCISE # 1

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	B	A	B	A	B	A,B	B	C	A,B	A,C,D	B,C,D	B	B	B	C	C	A	A
Q.No	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	
Ans.	B	A	D	D	C	A	B	A	B	B	A	B	B	A	A	B	A	B	C	

40. False **41.** True **42.** True **43.** True **44.** 32/55 **45.** 2/5

EXERCISE # 2

(PART-A)

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	D	C	A	C	A	A	B	B	B	C	D	A	B	A	A	A

(PART-B)

Q.No	17	18	19	20
Ans.	A,B	A,B,C,D	A,C	A,C

(PART-C)

Q.No	21	22	23	24
Ans.	A	D	A	B

(PART-D)

25. $A \to Q$; $B \to P$; $C \to S$; $D \to R$ **26.** $A \to Q$; $B \to R$; $C \to R$; $D \to R$

27. $A \to S$; $B \to R$; $C \to Q$; $D \to P$

EXERCISE # 3

1. [249/1400] **2.** [P(A) = 169/324 ; P (B) = 155/324] **5.** [5/11] **6.** [13/41]

7. [(a) 9/50 ; (b) (5n – 3)/(3n – 3)] **8.** [(a) 6 on bush- I (b) 4 on bush - II] **9.**[${}^{3}C_{1}/{}^{27}C_{10}$) (2/20)]

10. [2/19] **11.** [0.024] **12.** 240 & 40 **13.** 40/41 **14.** 15/17

15. 1/5 **16.** 22/30 **17.** 24/29

18. A, B, C are pairwise independent but A, B, C are dependent **19.** $\frac{3p+2p^2}{2}$ **20.** B

21. B **22.** D **23.** C **24.** B **25.** A **26.** C **27.** A **28.** B

29. C **30.** B **31.** D

EXERCISE # 4

1. (i) $\frac{m}{m+n}$ (ii) $\frac{{}^6C_3[3^n-3(2^n-2)-3]}{6^n}$ **2.** $\frac{9m}{8N+m}$ **3.** B **4.** D

5. $2p^2-p^3$ **6.** $\frac{1}{2}$ **7.** D **9.** $\frac{{}^6C_4\,{}^{12}C_2.{}^2C_1\,{}^{10}C_1+{}^6C_5\,{}^{12}C_1.{}^{11}C_1}{{}^{12}C_2.{}^{18}C_6}$

10. A **11.** $\frac{1}{7}$ **12.** A **13.** A **14.** A **15.** C **16.** D

17. C **18.** D **19.** B **20.** A **21.** B **22.** A **23.** A

24. C **25.** A, D **26.** B **27.** D **28.** B, D **29.** A **30.** A, B

EXERCISE # 5

1. 55 **2.** $1-\frac{10(N+2)}{N+{}^7C_5}$ **3.** $\frac{97}{(25)^4}$ **4.** 0.2436 **5.** $\frac{1}{4}$

6. (a) $\frac{1}{2}$ (b) $\frac{8}{15}$ **7.** A **8.** B,C **9.** 4/35 **11.** (i) $\frac{3}{56}$ (ii) $\frac{9}{28}$

12. Option B **13.** 120 **14.** (a) 0.61(b) 0.4758 **15.** 2065

16. $\frac{1}{9},\frac{1}{3}$ **17.** 283 **18.** $\frac{407}{576}$ **19.** Both are equally likely

20. (i) $\frac{125}{16^3}$ (ii) $\frac{63}{64}$ **21.** $\frac{{}^{24}C_2}{{}^{64}C_2}$ or $\frac{23}{168}$ **22.** $P_1=\frac{1}{2}$; $P_2=\frac{3}{4}$ **23.** $(1-p)^{n-1}.\ [p_0(1-p)+np(1-p_0)]$

24. $A\to R$; $B\to Q$; $C\to P$; $D\to S$ **25.** 4 **26.** (a) $\frac{64}{455}$ (b) $\frac{24}{91}$

27. D **28.** B **29.** D **30.** $\frac{5}{8}$ **31.** $\frac{74}{81}$

32. $P(E)=\left(1-\frac{K}{100}\right)p_1+\frac{K}{100}[Pp_1+(1-P)p_2]$; $P(H_2/A)=\dfrac{\frac{K}{100}[Pp_1+(1-P)p_2]}{\left(1-\frac{K}{100}\right)p_1+\frac{K}{100}[Pp_1+(1-P)p_2]}$

33. $1-[1-p(1-p_1)(1-p_2)]^n$ **34.** 179 **35.** (a) $\frac{(n-4)(n-5)}{n(n-1)}$

36. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$ **37.** $\frac{4}{9}$ **38.** $\frac{4}{13}$ **39.** 13 **40.** 458

41. 262 **42.** $\frac{11}{663}$ **43.** n + 1: n –1 **44.** 79 **45.** $\frac{KnM+LmN}{MN(K+L)}$

46. 317 **47.** 206 **48.** A **49.** D