EXERCISE-I

Fundamental definite integration, Definite integration by substitution

1. The value of $\int_{-2}^{2}(ax^3+bx+c)$ depends on

(A) The value of a
(B) The value of b
(C) The value of c
(D) The values of a and b

2. $\int_{\pi/6}^{\pi/4}\text{cosec}\,2x\,dx =$

(A) $\log 3$ (B) $\log\sqrt{3}$
(C) $\log 9$ (D) None of these

3. $\int_0^{\pi/2}\sqrt{\cos\theta}\sin^3\theta\,d\theta =$

(A) $\frac{20}{21}$ (B) $\frac{8}{21}$
(C) $\frac{-20}{21}$ (D) $\frac{-8}{21}$

4. $\int_0^{\pi/4}\sec^7\theta\sin^3\theta\,d\theta =$

(A) $\frac{1}{12}$ (B) $\frac{3}{12}$
(C) $\frac{5}{12}$ (D) None of these

5. $\int_a^b\frac{\log x}{x}dx =$

(A) $\log\left(\frac{\log b}{\log a}\right)$ (B) $\log(a\,b)\log\left(\frac{b}{a}\right)$
(C) $\frac{1}{2}\log(a\,b)\log\left(\frac{b}{a}\right)$ (D) $\frac{1}{2}\log(a\,b)\log\left(\frac{a}{b}\right)$

6. The correct evaluation of $\int_0^{\pi/2}\sin x\sin 2x$ is

(A) $\frac{4}{3}$ (B) $\frac{1}{3}$
(C) $\frac{3}{4}$ (D) $\frac{2}{3}$

7. $\int_0^{\pi/2}\frac{dx}{2+\cos x} =$

(A) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\sqrt{3}\tan^{-1}\left(\sqrt{3}\right)$
(C) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $2\sqrt{3}\tan^{-1}\left(\sqrt{3}\right)$

8. $\int_0^1\frac{\tan^{-1}x}{1+x^2}dx =$

(A) $\frac{\pi^2}{8}$ (B) $\frac{\pi^2}{16}$
(C) $\frac{\pi^2}{4}$ (D) $\frac{\pi^2}{32}$

9. $\int_1^2\frac{\cos(\log x)}{x}dx =$

(A) $\sin(\log 3)$ (B) $\sin(\log 2)$
(C) $\cos(\log 3)$ (D) None of these

10. $\int_0^a\frac{x\,dx}{\sqrt{a^2+x^2}} =$

(A) $a(\sqrt{2}-1)$ (B) $a(1-\sqrt{2})$
(C) $a(1+\sqrt{2})$ (D) $2a\sqrt{3}$

11. The value of integral $\int_{1/\pi}^{2/\pi}\frac{\sin(1/x)}{x^2}dx =$

(A) 2 (B) -1
(C) 0 (D) 1

12. $\int_0^a\frac{x^4\,dx}{(a^2+x^2)^4} =$

(A) $\frac{1}{16a^3}\left(\frac{\pi}{4}-\frac{1}{3}\right)$ (B) $\frac{1}{16a^3}\left(\frac{\pi}{4}+\frac{1}{3}\right)$
(C) $\frac{1}{16}a^3\left(\frac{\pi}{4}-\frac{1}{3}\right)$ (D) $\frac{1}{16}a^3\left(\frac{\pi}{4}+\frac{1}{3}\right)$

13. $\int_0^{2\pi}e^{x/2}.\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx =$

(A) 1 (B) $2\sqrt{2}$
(C) 0 (D) None of these

14. $\int_0^1 \frac{e^{-x}}{1+e^{-x}}dx =$

(A) $\log\left(\frac{1+e}{e}\right)-\frac{1}{e}+1$ (B) $\log\left(\frac{1+e}{2e}\right)-\frac{1}{e}+1$

(C) $\log\left(\frac{1+e}{2e}\right)+\frac{1}{e}-1$ (D) None of these

15. $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx =$

(A) $\frac{1}{20}\log 3$ (B) $\log 3$

(C) $\frac{1}{20}\log 5$ (D) None of these

16. The value of the definite integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$ for $0<\alpha<\pi$ is equal to

(A) $\sin\alpha$ (B) $\tan^{-1}(\sin\alpha)$

(C) $\alpha\sin\alpha$ (D) $\frac{\alpha}{2}(\sin\alpha)^{-1}$

17. The value of the integral $\int_{-\pi}^{\pi}\sin mx\sin nx\,dx$ for $m\neq n$ $(m, n\in I)$, is

(A) 0 (B) π

(C) $\frac{\pi}{2}$ (D) 2π

18. The greater of $\int_0^{\pi/2}\frac{\sin x}{x}dx$ and $\frac{\pi}{2}$, is

(A) $\frac{\pi}{2}$ (B) $\int_0^{\pi/2}\frac{\sin x}{x}dx$

(C) Nothing can be said (D) None of these

19. The integral $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx =$

(A) π (B) 2π

(C) 3π (D) None of these

20. If $I_1=\int_e^{e^2}\frac{dx}{\log x}$ and $I_2=\int_1^2\frac{e^x}{x}dx$, then

(A) $I_1=I_2$ (B) $I_1>I_2$

(C) $I_1<I_2$ (D) None of these

21. $\int_0^{\pi/4}\frac{4\sin 2\theta\,d\theta}{\sin^4\theta+\cos^4\theta} =$

(A) $\pi/4$ (B) $\pi/2$

(C) π (D) None of these

22. $\int_0^1\frac{e^x(x-1)}{(x+1)^3}dx =$

(A) $\frac{e}{4}$ (B) $\frac{e}{4}-1$

(C) $\frac{e}{4}+1$ (D) None of these

23. If $x(x^4+1)\varphi(x)=1$, then $\int_1^2\varphi(x)dx =$

(A) $\frac{1}{4}\log\frac{32}{17}$ (B) $\frac{1}{2}\log\frac{32}{17}$

(C) $\frac{1}{4}\log\frac{16}{17}$ (D) None of these

24. $\int_{1/4}^{1/2}\frac{dx}{\sqrt{x-x^2}} =$

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

25. The value of $\int_0^2\frac{3^{\sqrt{x}}}{\sqrt{x}}dx$ is

(A) $\frac{2}{\log 3}.(3^{\sqrt{2}}-1)$ (B) 0

(C) $2.\frac{\sqrt{2}}{\log 3}$ (D) $\frac{3^{\sqrt{2}}}{\sqrt{2}}$

26. $\int_0^{\pi/4}\frac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx =$

(A) $\log_e\left(\frac{2}{3}\right)$ (B) $\log_e 3$

(C) $\frac{1}{2}\log_e\left(\frac{4}{3}\right)$ (D) $\log_e\left(\frac{4}{3}\right)$

27. $\int_0^{2/3}\frac{dx}{4+9x^2} =$

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{24}$

(C) $\frac{\pi}{4}$ (D) 0

28. The value of $\int_0^1\frac{x^4+1}{x^2+1}dx$ is

(A) $\frac{1}{6}(3\pi-4)$ (B) $\frac{1}{6}(3-4\pi)$

(C) $\frac{1}{6}(3\pi+4)$ (D) $\frac{1}{6}(3+4\pi)$

29. $\int_0^a x^2 \sin x^3 \, dx$ equals

(A) $(1 - \cos a^3)$ (B) $3(1 - \cos a^3)$

(C) $-\frac{1}{3}(1 - \cos a^3)$ (D) $\frac{1}{3}(1 - \cos a^3)$

30. $\int_0^{\pi/4} [\sqrt{\tan x} + \sqrt{\cot x}] dx$ equals

(A) $\sqrt{2}\pi$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{\sqrt{2}}$ (D) 2π

31. $\int_0^3 \frac{3x+1}{x^2+9} dx =$

(A) $\log(2\sqrt{2}) + \frac{\pi}{12}$ (B) $\log(2\sqrt{2}) + \frac{\pi}{2}$

(C) $\log(2\sqrt{2}) + \frac{\pi}{6}$ (D) $\log(2\sqrt{2}) + \frac{\pi}{3}$

32. The value of $\int_1^2 \frac{dx}{x(1+x^4)}$ is

(A) $\frac{1}{4}\log\frac{17}{32}$ (B) $\frac{1}{4}\log\frac{17}{2}$

(C) $\log\frac{17}{2}$ (D) $\frac{1}{4}\log\frac{32}{17}$

33. The value of $\int_2^3 \frac{x+1}{x^2(x-1)} dx$ is

(A) $2\log 2 - \frac{1}{6}$ (B) $\log\frac{16}{9} - \frac{1}{6}$

(C) $\log\frac{4}{3} - \frac{1}{6}$ (D) $\log\frac{16}{9} + \frac{1}{6}$

34. The value of $\int_1^e \log x \, dx$ is

(A) 0 (B) 1

(C) $e - 1$ (D) $e + 1$

35. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is

(A) 3 (B) 1

(C) 2 (D) 0

Properties of definite integration

36. If $f(x) = \begin{cases} 4x+3, & \text{if } 1 \le x \le 2 \\ 3x+5, & \text{if } 2 < x \le 4 \end{cases}$ then $\int_1^4 f(x)dx =$

(A) 80 (B) 20

(C) –20 (D) 37

37. $\int_0^{\pi/4} \log(1 + \tan\theta) d\theta =$

(A) $\frac{\pi}{4}\log 2$ (B) $\frac{\pi}{4}\log\frac{1}{2}$

(C) $\frac{\pi}{8}\log 2$ (D) $\frac{\pi}{8}\log\frac{1}{2}$

38. $\int_0^{2\pi} \frac{\sin 2\theta}{a - b\cos\theta} d\theta =$

(A) 1 (B) 2

(C) $\frac{\pi}{4}$ (D) 0

39. $\int_0^1 f(1-x) dx$ has the same value as the integral

(A) $\int_0^1 f(x) dx$ (B) $\int_0^1 f(-x) dx$

(C) $\int_0^1 f(x-1) dx$ (D) $\int_{-1}^1 f(x) dx$

40. The smallest interval [a, b] such that $\int_0^1 \frac{dx}{\sqrt{1+x^4}} \in [a, b]$ is given by

(A) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (B) [0, 1]

(C) $\left[\frac{1}{2}, 2\right]$ (D) $\left[\frac{3}{4}, 1\right]$

41. $\int_0^{1.5} [x^2] dx$, where $[\,.\,]$ denotes the greatest integer function, equals

(A) $2 + \sqrt{2}$ (B) $2 - \sqrt{2}$

(C) $-2 + \sqrt{2}$ (D) $-2 - \sqrt{2}$

42. $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx =$

(A) $\frac{\pi}{2} - 1$ (B) $\pi\left(\frac{\pi}{2} + 1\right)$

(C) $\frac{\pi}{2} + 1$ (D) $\pi\left(\frac{\pi}{2} - 1\right)$

43. $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx =$

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{2}$

(C) $\frac{3\pi^2}{2}$ (D) $\frac{\pi^2}{3}$

44. $\int_{-1}^{1} \sin^3 x \cos^2 x \, dx =$

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) 2

45. For any integer n, the integral

$\int_0^{\pi} e^{\sin^2 x} \cos^3 (2n+1)x \, dx =$

(A) -1 (B) 0

(C) 1 (D) π

46. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ is

(A) 1 (B) 0

(C) -1 (D) $\frac{1}{2}$

47. The value of $\int_0^{\pi} e^{\cos^2 x} \cos^5 3x \, dx$ is

(A) 1 (B) -1

(C) 0 (D) None of these

48. $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx =$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) 1

49. The value of $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx$ is

(A) 0 (B) $2\int_0^1 \frac{\sin x}{3-|x|} dx$

(C) $2\int_0^1 \frac{-x^2}{3-|x|} dx$ (D) $2\int_0^1 \frac{\sin x - x^2}{3-|x|} dx$

50. $\int_{-1}^{1} \sin^{11} x \, dx$ is equal to

(A) $\frac{10}{11}.\frac{8}{9}.\frac{6}{7}.\frac{4}{5}.\frac{2}{3}$ (B) $\frac{10}{11}.\frac{8}{9}.\frac{6}{7}.\frac{4}{5}.\frac{2}{3}.\frac{\pi}{2}$

(C) 1 (D) 0

51. The value of $\int_0^{2\pi} \cos^{99} x \, dx$ is

(A) 1 (B) -1

(C) 99 (D) 0

52. If $[x]$ denotes the greatest integer less than or equal to x, then the value of the integral $\int_0^2 x^2 [x] dx$ equals

(A) 5/3 (B) 7/3

(C) 8/3 (D) 4/3

53. $\int_0^{\pi} \cos^3 x \, dx =$

(A) -1 (B) 0

(C) 1 (D) π

54. $\int_0^{2\pi} |\sin x| dx =$

(A) 0 (B) 1

(C) 2 (D) 4

55. $\int_{-3}^{3} \frac{x^2 \sin 2x}{x^2+1} dx =$

(A) 0 (B) 1

(C) $2\log_e 3$ (D) None of these

56. $\int_{-a}^{a} \sin x \, f(\cos x) \, dx =$

(A) $2\int_0^a \sin x \, f(\cos x) \, dx$ (B) 0

(C) 1 (D) None of these

57. The value of $\int_0^{2\pi} |\sin^3 \theta| d\theta$ is

(A) 0 (B) $3/8$

(C) $8/3$ (D) π

58. $\int_{-1}^{2} |x| dx$

(A) 5/2 (B) 1/2

(C) 3/2 (D) 7/2

59. $\int_0^3 |2-x| dx$ equals

(A) 2/7 (B) 5/2

(C) 3/2 (D) $-3/2$

60. The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}} dx$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) π (D) 2π

61. The value of $\int_0^{\sqrt{2}} [x^2] dx$, where [.] is the greatest integer function

(A) $2-\sqrt{2}$ (B) $2+\sqrt{2}$

(C) $\sqrt{2}-1$ (D) $\sqrt{2}-2$

62. $\int_0^{1000} e^{x-[x]}dx$ is

(A) $e^{1000}-1$ (B) $\frac{e^{1000}-1}{e-1}$

(C) $1000(e-1)$ (D) $\frac{e-1}{1000}$

63. The value of the integral $\int_{\frac{1}{n}}^{\frac{an-1}{n}} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$ is

(A) $\frac{a}{2}$ (B) $\frac{na+2}{2n}$

(C) $\frac{na-2}{2n}$ (D) None of these

64. $\int_0^{\pi/2} \sin 2x \log \tan x\, dx$ is equal to

(A) π (B) $\pi/2$

(C) 0 (D) 2π

65. The integral $\int_{-1/2}^{1/2} \left\{[x]+\log\left(\frac{1+x}{1-x}\right)\right\}dx$ equal (where [.] is the greatest integer function)

(A) $-\frac{1}{2}$ (B) 0

(C) 1 (D) $2\log\frac{1}{2}$

66. The value of integral $\int_0^1 e^{x^2}dx$ lies in interval

(A) (0, 1) (B) (−1, 0)

(C) (1, e) (D) None of these

67. If $P=\int_0^{3\pi} f(\cos^2 x)dx$ and $Q=\int_0^{\pi} f(\cos^2 x)dx$, then

(A) $P-Q=0$ (B) $P-2Q=0$

(C) $P-3Q=0$ (D) $P-5Q=0$

68. Let a, b, c be non-zero real numbers such that $\int_0^3 (3ax^2+2bx+c)\,dx=\int_1^3 (3ax^2+2bx+c)\,dx$, then

(A) $a+b+c=3$ (B) $a+b+c=1$

(C) $a+b+c=0$ (D) $a+b+c=2$

69. $\int_{-\pi}^{\pi} (\cos px-\sin qx)^2\,dx$ is equal to (where p and q are integers)

(A) $-\pi$ (B) 0

(C) π (D) 2π

70. If $g(x)=\int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals

(A) $g(x)+g(\pi)$ (B) $g(x)-g(\pi)$

(C) $g(x)g(\pi)$ (D) $g(x)/g(\pi)$

Summation of series by definite Integration, Leibnitz's rule

71. $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2}dx=$

(A) $\pi\log\frac{1}{2}$ (B) $\pi\log 2$

(C) $2\pi\log\frac{1}{2}$ (D) $2\pi\log 2$

72. The points of intersection of $F_1(x)=\int_2^x (2t-5)\,dt$ and $F_2(x)=\int_0^x 2t\,dt$, are

(A) $\left(\frac{6}{5},\frac{36}{25}\right)$ (B) $\left(\frac{2}{3},\frac{4}{9}\right)$

(C) $\left(\frac{1}{3},\frac{1}{9}\right)$ (D) $\left(\frac{1}{5},\frac{1}{25}\right)$

73. $\int_0^{\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}=$

(A) $\frac{\pi}{2(a-b)}$ (B) $\frac{\pi}{2(b-a)}$

(C) $\frac{\pi}{(a+b)}$ (D) $\frac{\pi}{2(a+b)}$

74. $\int_0^{\infty} \frac{x^3\,dx}{(x^2+4)^2}=$

(A) 0 (B) ∞

(C) $\frac{1}{2}$ (D) None of these

75. $\int_0^{\pi/2} \sin^{2m} x\,dx=$

(A) $\frac{2\,m\,!}{(2^m.m\,!)^2}.\frac{\pi}{2}$ (B) $\frac{(2m)\,!}{(2^m.m\,!)^2}.\frac{\pi}{2}$

(C) $\frac{2m\,!}{2^m.(m\,!)^2}.\frac{\pi}{2}$ (D) None of these

76. The derivative of $F(x)=\int_{x^2}^{x^3}\frac{1}{\log t}dt,\ (x>0)$ is

(A) $\frac{1}{3\log x}-\frac{1}{2\log x}$ (B) $\frac{1}{3\log x}$

(C) $\frac{3x^2}{3\log x}$ (D) $(\log x)^{-1}.x(x-1)$

77. If $f(x)=\int_{x^2}^{x^2+1}e^{-t^2}dt$, then $f(x)$ increases in

(A) (2, 2) (B) No value of x

(C) $(0,\infty)$ (D) $(-\infty, 0)$

78. If $f(x)=\int_{x^2}^{x^4}\sin\sqrt{t}\,dt$, then $f'(x)$ equals

(A) $\sin x^2-\sin x$

(B) $4x^3\sin x^2-2x\sin x$

(C) $x^4\sin x^2-x\sin x$

(D) None of these

79. If $F(x)=\frac{1}{x^2}\int_4^x(4t^2-2F'(t))dt$, then $F'(4)$ equals

(A) 32 (B) $\frac{32}{3}$

(C) $\frac{32}{9}$ (D) None of these

80. The value of the integral $\sum_{k=1}^{n}\int_0^1 f(k-1+x)\,dx$ is

(A) $\int_0^1 f(x)\,dx$ (B) $\int_0^2 f(x)\,dx$

(C) $\int_0^n f(x)\,dx$ (D) $n\int_0^1 f(x)\,dx$

81. $\int_0^\infty \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$ is equal to

(A) $\pi\log 2$ (B) $-\pi\log 2$

(C) $(\pi/2)\log 2$ (D) $-(\pi/2)\log 2$

82. $\int_0^\infty \frac{x\ln x\,dx}{(1+x^2)^2}$ is equal to

(A) 0 (B) 1

(C) ∞ (D) None of these

83. If $f(t)=\int_{-t}^{t}\frac{dx}{1+x^2}$, then $f'(1)$ is

(A) Zero (B) 2/3

(C) -1 (D) 1

84. If $F(x)=\int_{x^2}^{x^3}\log t\,dt,\ (x>0)$, then $F'(x)=$

(A) $(9x^2-4x)\log x$ (B) $(4x-9x^2)\log x$

(C) $(9x^2+4x)\log x$ (D) None of these