POINT & STRAIGHT LINE

EXERCISE # 1

Question based on Coordinate system

Q.1 The cartesian coordinates of the points whose polar coordinates are $\left(-5, -\frac{\pi}{4}\right)$ equal to -

(A) $\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$ (B) $\left(\frac{5}{\sqrt{2}}, \frac{-5}{\sqrt{2}}\right)$

(C) $\left(\frac{-5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$ (D) $\left(\frac{-5}{\sqrt{2}}, \frac{-5}{\sqrt{2}}\right)$

Sol.[C] Polar coordinate $\left(-5, \frac{-\pi}{4}\right)$

Θ $r^2 = x^2 + y^2$

$\therefore 25 = x^2 + y^2 \Rightarrow x^2 + y^2 = 25$ ....(1)

Θ $\theta = \tan^{-1}\frac{y}{x}$

$\therefore \frac{-\pi}{4} = \tan^{-1}\frac{y}{x} \Rightarrow -\tan\frac{\pi}{4} = \frac{y}{x}$

$\Rightarrow x + y = 0$ ....(2)

from (1) & (2) we get

$2x^2 = 25 \Rightarrow x = \pm\frac{5}{\sqrt{2}}$

$\therefore \quad y = \mu\frac{5}{\sqrt{2}}$

$\therefore \quad \left(\frac{-5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$

Q.2 The polar form of the equation $x^2 + y^2 = ax$ is -

(A) $r = a \sin\theta$ (B) $r = a\cos\theta$

(C) $r = -a\sin\theta$ (D) None of these

Sol.[B] $x^2 + y^2 = ax$

put $x = r\cos\theta$

$y = r\sin\theta$

$r^2\cos^2\theta + r^2\sin^2\theta = a.r\cos\theta$

$r^2(\sin^2\theta + \cos^2\theta) = a.r\cos\theta$

$r = a\cos\theta$

Question based on Distance formula

Q.3 The abscissa of two points A, B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinate are the roots of $x^2 + 2px - q^2 = 0$ then the distance AB in terms of a, b, p, q is -

(A) $\sqrt{a^2 + b^2 + p^2 + q^2}$

(B) $\sqrt{a^2 + b^2 + 2p^2q^2}$

(C) $2\sqrt{a^2 + b^2 + p^2 + q^2}$

(D) $\frac{1}{2}\sqrt{a^2 + b^2 + p^2 + q^2}$

Sol.[C] A (x_1, y_1) ———— B (x_2, y_2)

$x^2 + 2ax - b^2 = 0$ ......(i)

since according to question,

x_1 & x_2 are roots of equation (i)

$\therefore \; x_1 + x_2 = -2a$

$x_1x_2 = -b^2$

Also y_1 & y_2 are roots of $x^2 + 2px - q^2 = 0$

$\therefore \; y_1 + y_2 = -2p$

$y_1y_2 = -q^2$

$\therefore AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

$= (-2a)^2 - 4(-b^2) = 4a^2 + 4b^2$

$= 4(a^2 + b^2)$

$(y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1y_2$

$= 4p^2 + 4q^2 = 4(p^2 + q^2)$

$\therefore \quad AB = \sqrt{4(a^2 + b^2) + 4(p^2 + q^2)}$

$= 2\sqrt{a^2 + b^2 + p^2 + q^2} = 2\sqrt{a^2 + b^2 + p^2 + q^2}$

Q.4 The distance between point (2, 15º) & (1, 75º) is -

(A) 1 (B) 3 (C) $2\sqrt{3}$ (D) $\sqrt{3}$

Sol.[D] (2, 15°) ———— (1, 75°)

Distance $= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$

$= \sqrt{4 + 1 - 4\cos 60°}$

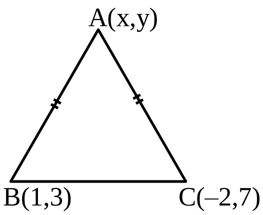
$= \sqrt{5 - 4.\frac{1}{2}} = \sqrt{3}$

Question based on Application of distance formula

Q.5 The coordinates of base BC of an isosceles triangle ABC are given by B (1, 3) and C (–2, 7) which of the following points can be the possible coordinates of the vertex A.

(A) (–7, 1/8) (B) (1, 6)
(C) (–1/2, 5) (D) (–5/6, 6)

Sol.[D]



Since AB = AC

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (x - 1)^2 + (y - 3)^2 = (x + 2)^2 + (y - 7)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 6y + 9$

$= x^2 + 4x + 4 + y^2 - 14y + 49$

$\Rightarrow -6x + 8y - 43 = 0$

$\Rightarrow 6x - 8y + 43 = 0$ …….(i)

only option (D) satisfies the equation (i)

hence (D) is correct option

Q.6 If vertices of a quadrilateral are A(0, 0), B(3, 4), C(7, 7) and D (4, 3), then quadrilateral ABCD is

(A) parallelogram (B) rectangle
(C) square (D) rhombus

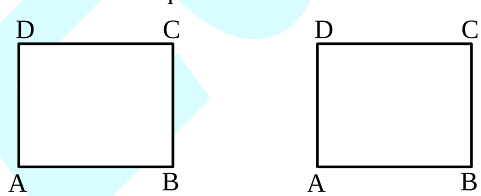
Sol.[D] A(0, 0), B(3, 4), C(7, 7), D(4, 3)

$AB = \sqrt{9+16} = 5$ $BC = \sqrt{16+9} = 5$

$CD = \sqrt{9+16} = 5$ $DA = \sqrt{16+9} = 5$

since AB = BC = CD = DA

Hence it is either square or rhombus



slope of AB = 4/3 = m_1 & slope of AD = 3/4 = m_2

$m_1 m_2 = 1 \neq -1$ Hence not a square

Now slope of AC = 7/7 = 1 = m_1

and slope of BD = –1/1 = –1 = m_2

$m_1 m_2 = -1$

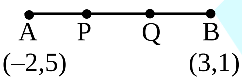
$\therefore$ it is rhombus, since we know that diagnoses of a rhombus cut at right angle.

Question based on Section formula

Q.7 P and Q are points on the line joining A(– 2, 5) and B(3, 1) such that AP = PQ = QB then the mid point of PQ is -

(A) (1/2, 3) (B) (– 1/2, 4)
(C) (2, 3) (D) (1, 4)

Sol.[A]



Since AP = PQ = QB (given)

Mid point of PQ will be the mid point of AB

$\therefore$ Mid point = $\left(\frac{3-2}{2}, \frac{5+1}{2}\right)$

$= \left(\frac{1}{2}, 3\right)$

Q.8 The line segment joining the points (1, 2) and (–2, 1) is divided by the line 3x + 4y = 7 in the ratio -

(A) 3 : 4 (B) 4 : 3
(C) 9 : 4 (D) 4 : 9

Sol.[D] Required ratio is

$= -\frac{3(1)+4(2)-7}{3(-2)+4(1)-7} = -\frac{3+8-7}{-6+4-7}$

$= -\frac{4}{-9} = \frac{4}{9}$

= 4 : 9

Q.9 The line segment joining the points (–3, –4) and (1, –2) is divided by y-axis in the ratio

(A) 1 : 3 (B) 2 : 3
(C) 3 : 1 (D) 3 : 2

Sol.[C] Required ratio is

$= -\frac{x_1}{x_2} = -\frac{-3}{1}$

$= \frac{3}{1} = 3 : 1$

Question based on Area of triangle

Q.10 If m_1 and m_2 are roots of the equation $x^2 + (\sqrt{3} + 2)x + (\sqrt{3} - 1) = 0$ then the area of the triangle formed by the lines $y = m_1x$, $y = m_2x$ and y = c is -

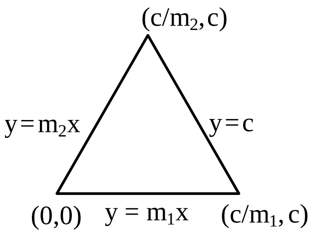
(A) $\left(\dfrac{\sqrt{33}-\sqrt{11}}{4}\right)c^2$ (B) $\left(\dfrac{\sqrt{33}+\sqrt{11}}{4}\right)c^2$

(C) $\left(\dfrac{\sqrt{33}+\sqrt{11}}{2}\right)c^2$ (D) $\left(\dfrac{\sqrt{33}-\sqrt{11}}{2}\right)c^2$

Sol.[B] m_1 & m_2 are roots of $x^2+(\sqrt{3}+2)x+(\sqrt{3}-1)=0$

$m_1 + m_2 = -(\sqrt{3}+2)$

$m_1m_2 = (\sqrt{3}-1)$



$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & \frac{c}{m_1} & \frac{c}{m_2} \\ 0 & c & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{c^2}{m_1}-\frac{c^2}{m_2}\right]$$

$$= \frac{1}{2}c^2\left[\frac{m_2-m_1}{m_1m_2}\right]$$

$$= \frac{c^2}{2}\left[\frac{\sqrt{(m_1+m_2)^2-4m_1m_2}}{m_1m_2}\right]$$

$$= \frac{c^2}{2}\left[\frac{\sqrt{(\sqrt{3}+2)^2-4(\sqrt{3}-1)}}{\sqrt{3}-1}\right]$$

$$= \frac{c^2}{2}\left[\frac{\sqrt{3+4+4\sqrt{3}-4\sqrt{3}+4}}{\sqrt{3}-1}\right]$$

$$= \frac{c^2}{2}\ \frac{\sqrt{11}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{c^2}{2}\cdot\frac{\sqrt{33}+\sqrt{11}}{2}$$

Question based on

Area of quadrilateral

Q.11 The points (0, 1), (–2, 3), (6, 7) and (8, 3) are-

(A) Collinear

(B) Vertices of parallelogram which is not a rectangle

(C) Vertices of rectangle which is not a square

(D) None of these

Sol.[D] A(0,1), B(–2, 3), C(6, 7), D(8, 3)

slope of AB = $\dfrac{2}{-2} = -1 = m_1$,

slope of BC = $\dfrac{4}{8} = \dfrac{1}{2} = m_2$

slope of CD = $\dfrac{-4}{2} = -2 = m_3$

$\therefore m_1 \ne m_2 \ne m_3$

$\therefore$ Not collinear

Now

$AB = \sqrt{4+4} = 2\sqrt{2}$, $BC = \sqrt{64+16} = 4\sqrt{5}$,

$CD = \sqrt{4+16} = 2\sqrt{5}$, $DA = \sqrt{64+4} = \sqrt{68}$

$AC = \sqrt{36+36} = \sqrt{72}$

$BD = \sqrt{100} = 10$

$AC \ne BD$

we can say that it is not rectangle and also not a square.

Q.12 The area of the pentagon whose vertices are (4, 1), (3, 6), (–5, 1), (–3, –3) and (–3, 0) is-

(A) 30 unit^2 (B) 60 unit^2

(C) 120 unit^2 (D) none of these

Sol.[A] Area of pentagon is given by

$$\frac{1}{2}\begin{vmatrix} 4 & 3 & -5 & -3 & -3 & 4 \\ 1 & 6 & 1 & -3 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\ [(24-3)+(3+30)+(15+3)+(-9)+(-3)]$$

$$= \frac{1}{2}\ [21+33+18-12] = \frac{1}{2}\times 60 = 30 \text{ sq. unit}$$

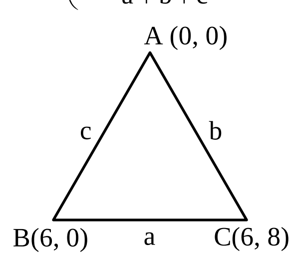
Question based on

Centers of triangle

Q.13 If the vertices of a triangle be (0, 0), (6, 0) and (6, 8), then its incentre will be-

(A) (2, 1) (B) (1, 2) (C) (4, 2) (D) (2, 4)

Sol.[C] Incentre = $\left(\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$



$a = BC = \sqrt{(8)^2} = 8$

$b = CA = \sqrt{36+64} = 10$

$c = AB = \sqrt{(6)^2} = 6$

$\therefore$ Incentre

$= \left(\frac{8(0)+10(6)+6(6)}{8+10+6}, \frac{8(0)+10(0)+6(8)}{8+10+6}\right)$

$= \left(\frac{0+60+36}{24}, \frac{0+0+48}{24}\right)$

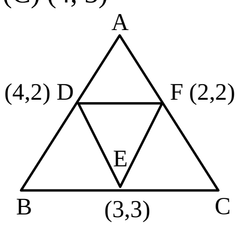
$= \left(\frac{96}{24}, \frac{48}{24}\right) = (4, 2)$

Q.14 The coordinates of the middle points of the sides of a triangle are (4, 2), (3, 3) and (2, 2) then the coordinates of its centroid are-

(A) $\left(3, \frac{7}{3}\right)$ (B) (3, 3)

(C) (4, 3) (D) None of these

Sol.[A]



catroid of ΔABC will be the same as centroid of ΔDEF

$\therefore$ centroid is $\left(\frac{4+3+2}{3}, \frac{2+3+2}{3}\right)$

$= \left(\frac{9}{3}, \frac{7}{3}\right) = \left(3, \frac{7}{3}\right)$

Q.15 The orthocentre of the triangle formed by the lines $4x - 7y + 10 = 0$, $x + y = 5$ and $7x + 4y = 15$, is-

(A) (1, 2) (B) (1, –2)

(C) (–1, –2) (D) (–1, 2)

Sol.[A] Given lines formed a right angled triangle and we know that in a right angled triangle , the orthocenter is that point where right angled formed. Here $4x - 7y + 10 = 0$ and $7x + 4y = 15$ are the line which formed right angled.

The point of intersection of these lines is the orthocentre

$4x - 7y + 10 = 0$ …... (1)

$7x + 4y - 15 = 0$ ……(2)

Multiply (1) by 7 and (2) by 4 we get

$28x - 49y + 70 = 0$

$28x + 16y - 60 = 0$

$- \quad - \quad +$

$-65y = -30$

$y = \frac{130}{65}$

$y = 2$

from (1) we get

$4x - 14 + 10 = 0 \Rightarrow 4x = 14 \Rightarrow x = 1$

$\therefore$ orthocentre is (1, 2)

Question based on Locus of a point

Q.16 The locus of the points of intersection of the lines $x \cos\alpha + y \sin\alpha = a$ and $x \sin\alpha - y \cos\alpha = b$ (where α is a variable) is –

(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 - y^2 = a^2 + b^2$

(C) $x^2 + y^2 = a^2 - b^2$ (D) None of these

Sol.[A] $x \cos \alpha + y \sin \alpha = a$ …… (1)

$x \sin \alpha - y \cos \alpha = b$ …… (2)

on squaring (1) & (2), then adding, we get

$x^2\cos^2\alpha + y^2\sin^2\alpha + 2xy\sin\alpha\cos\alpha + x^2\sin^2\alpha + y^2\cos^2\alpha - 2xy\sin\alpha\cos\alpha = a^2 + b^2$

$\Rightarrow x^2(\sin^2\alpha + \cos^2\alpha) + y^2(\sin^2\alpha + \cos^2\alpha) = a^2 + b^2$

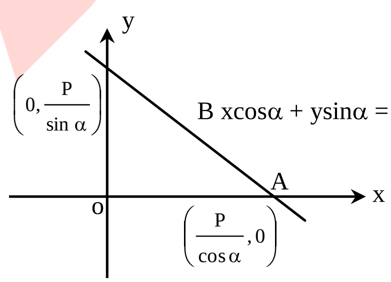
$\Rightarrow x^2 + y^2 = a^2 + b^2$

Q.17 The locus of the mid point of the portion intercept between the axes by the line $x \cos\alpha + y \sin\alpha = P$ where P is a constant is-

(A) $x^2 + y^2 = 4P^2$ (B) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{P^2}$

(C) $x^2 + y^2 = \frac{4}{P^2}$ (D) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{P^2}$

Sol.[B]



Let mid point of AB is (h, k)

Mid point of AB is given by :

$\left(\frac{P}{2\cos\alpha}, \frac{P}{2\sin\alpha}\right)$ $\therefore$ $h = \frac{P}{2\cos\alpha}$ & $k = \frac{P}{2\sin\alpha}$

$\Rightarrow \cos\alpha = \frac{P}{2h}$ and $\sin\alpha = \frac{P}{2k}$

on squaring and adding, we get

$\cos^2\alpha + \sin^2\alpha = \frac{P^2}{4h^2} + \frac{P^2}{4k^2}$

$\Rightarrow 1 = \frac{P^2}{4h^2} + \frac{P^2}{4k^2} \Rightarrow \frac{4}{P^2} = \frac{1}{h^2} + \frac{1}{k^2}$

$\therefore$ locus is $\Rightarrow \frac{4}{P^2} = \frac{1}{x^2} + \frac{1}{y^2}$

Q.18 The locus of a point which moves so that the algebraic sum of the perpendiculars let fall from it on two given straight lines is constant, is-

(A) a circle (B) a straight line

(C) a pair of lines (D) none of these

Sol.[B] Let two given lines are

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Let point is (h, k) from which perpendiculars are drawn to the given lines. Sum of perpendiculars is constant.

$$\frac{a_1h + b_1k + c_1}{\sqrt{a_1^2 + b_1^2}} + \frac{a_2h + b_2k + c_2}{\sqrt{a_2^2 + b_2^2}} = \text{const.}$$

$\Rightarrow \sqrt{a_2^2 + b_2^2}\,(a_1h + b_1k + c_1) + \left(\sqrt{a_1^2 + b_1^2}\right)$

$(a_2h + b_2k + c_2) = \text{const.}$

$\Rightarrow h\,(a_1\sqrt{a_2^2 + b_2^2} + a_2\sqrt{a_1^2 + b_1^2}) +$

$k\,(b_1\sqrt{a_2^2 + b_2^2} + b_2\sqrt{a_1^2 + b_1^2}) = \text{const.}$

$\Rightarrow hQ + kR = \text{const.}$

$\Rightarrow$ where $Q = a_1\sqrt{a_2^2 + b_2^2} + a_2\sqrt{a_1^2 + b_1^2} = \text{const.}$

and $R = b_1\sqrt{a_2^2 + b_2^2} + b_2\sqrt{a_1^2 + b_1^2} = \text{const.}$

$\Rightarrow$ locus is, $xQ + yR = \text{const.}$

which is a straight line.

Q.19 If P = (1, 0) and Q = (–1, 0) and R = (2, 0) are three given points, the locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is-

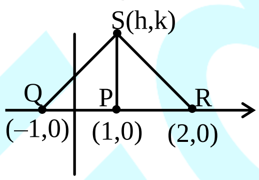
(A) a straight line parallel to the x-axis

(B) a circle passing through the origin

(C) a circle with the centre at the origin

(D) a straight line parallel to the y-axis

Sol. [D]



$SQ^2 + SR^2 = 2SP^2$

$(h + 1)^2 + k^2 + (h – 2)^2 + k^2 = 2\{(h – 1)^2 + k^2\}$

i.e., $– 2h + 5 – 2 = – 4h$

i.e., $2h = – 3$

Locus is $x = -\frac{3}{2}$

Question based on

Transformation of axes

Q.20 On shifting the origin to the point (2, –5) and keeping the axis parallel the new coordinates of the point (5, –3) will be-

(A) (–3, –2) (B) (3, 2)

(C) (–7, 8) (D) None of these

Sol.[B] Required point will be

(5 – 2, – 3 – (– 5)) = (3, – 3 + 5) = (3, 2)

Q.21 At what point the origin be shifted, if the coordinates of point (4, 5) becomes (–3, 9)

(A) (7, –4) (B) (–7, 4)

(C) (–7, –4) (D) None of these

Sol.[A] Let origin be shifted at (x_1, y_1)

Now (4, 5) becomes (– 3, 9)

$\Rightarrow (4 – x_1, 5 – y_1) = (– 3, 9)$

$\Rightarrow 4 – x_1 = – 3$ and $5 – y_1 = 9$

$\Rightarrow x_1 = 7$ and $y_1 = 5 – 9 = – 4 \Rightarrow (7 – 4)$

Q.22 Reflecting the point (2, –1) about Y axis coordinate axis are rotated at 45º angle in negative direction without shifting the origin . The new coordinates of the point are-

(A) $\left(\frac{-1}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)$ (B) $\left(\frac{-3}{\sqrt{2}}, -\sqrt{2}\right)$

(C) $\left(\frac{1}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ (D) None of these

Sol.[A] After reflection about y-axis, the point (2, –1) will be transformed to (–2, –1)

Now 45° fation in negative direction

Here $\alpha = – 45°$

$\therefore$ $x' = x\cos\alpha + y\sin\alpha$

and $y' = – x\sin\alpha + y\cos\alpha$

$\Rightarrow$ $x' = – 2\cos45° + \sin45°$

$= \frac{-2}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{-1}{\sqrt{2}}$

and $y' = – 2\sin 45° – \cos45° = \frac{-2}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= \frac{-3}{\sqrt{2}}$

$\therefore$ point is $\left(\frac{-1}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)$

Q.23 If the axes are rotated through an angle of 30º in the clockwise direction, the point $(4, –2\sqrt{3})$ in the new system was formerly

(A) $(2, \sqrt{3})$ (B) $(\sqrt{3}, –5)$

(C) $(\sqrt{3}, 2)$ (D) (2, 3)

Sol. **[B]**

Θ $\theta = – 30º$

$x = x' \cos\theta – y' \sin\theta$

$y = x' \sin\theta + y' \cos\theta$

$\therefore$ x = 4 cos 30° + 2$\sqrt{3}$ sin (–30°)

$= 4.\ \frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}$

$= 2\sqrt{3} - \sqrt{3}$

$= \sqrt{3}$

and y = 4 sin (–30°) – $2\sqrt{3}$ .cos 30°

$= 4.\left(-\frac{1}{2}\right) - 2\sqrt{3}\,.\,\frac{\sqrt{3}}{2}$

= –2 –3 = –5

$\therefore$ (x, y) ≡ ($\sqrt{3}$, –5)

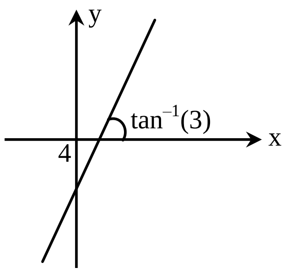
Question based on Different forms of a straight line

Q.24 The equation of a line which makes an angle of $\tan^{-1}$ (3) with the x-axis anticlockwise & cuts off an intercept of 4 units on negative direction of y-axis is -

(A) y = 3x + 4 (B) y = 3x – 4

(C) x = 3y + 4 (D) None of these

Sol.[B]



Here slope of line m = tanθ

⇒ m = tan $\tan^{-1}$(3)

here θ = $\tan^{-1}$(3)

⇒ m = 3 and c = – 4

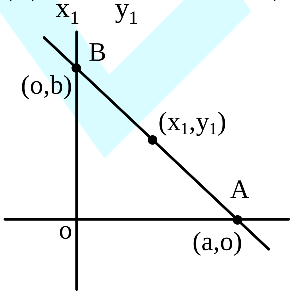
$\therefore$ equation of line y = mx + c = y = 3x – 4

Q.25 A line passes through (x_1, y_1). This point bisects the segment of the line between the axes. Its equation is-

(A) $\frac{x}{x_1}+\frac{y}{y_1}=2$ (B) $\frac{x}{x_1}+\frac{y}{y_1}=\frac{1}{2}$

(C) $\frac{x}{x_1}+\frac{y}{y_1}=1$ (D) None

Sol.[A]



Let equation of line is

$\frac{x}{a}+\frac{y}{b} = 1$ .... (1)

Since (x_1, y_1) is the mid point of AB

$\therefore$ $x_1 = \frac{a}{2}$ and $y_1 = \frac{b}{2}$

⇒ $a = 2x_1$ and $b = 2y_1$

from (1)

$\frac{x}{2x_1}+\frac{y}{2y_1} = 1$

⇒ $\frac{x}{x_1}+\frac{y}{y_1} = 2$

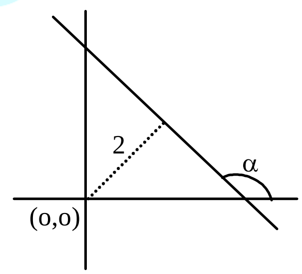
Q.26 The equation of the straight line on which the length of the perpendicular from the origin is 2 and the perpendicular makes an angle α with x-axis such that $\sin\alpha = \frac{1}{3}$ is -

(A) $2\sqrt{2}$ x – y = 6 (B) $2\sqrt{2}$ x + y = 6

(C) $3\sqrt{2}$ x + y = 6 (D) $2\sqrt{2}$ x – y = 5

Sol.[B] equation in perpendicular form in given by

x cos α + y sin α = p



Here sin α = $\frac{1}{3}$

$\therefore$ cos α = $\sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$

and p = 2 (given)

$\therefore$ Required equation of line is

$x\,.\,\frac{2\sqrt{2}}{3} + y\,.\,\frac{1}{3} = 2 \Rightarrow 2\sqrt{2}\,x + y = 6$

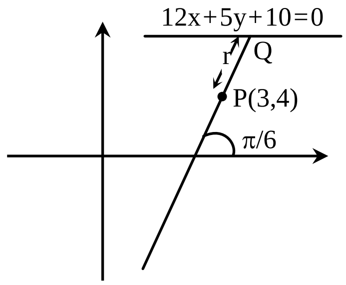
Question based on Distance form of a line

Q.27 If the straight line through the point P (3, 4) makes an angle $\frac{\pi}{6}$ with x-axis and meets the line 12x + 5y + 10 = 0 at Q. Then the length of PQ is -

(A) $\frac{132}{12\sqrt{3}+5}$ (B) $\frac{132}{12\sqrt{3}-5}$

(C) $\frac{132}{\sqrt{3}-5}$ (D) None of these

Sol.[A]



Co- ordinates of Q is given by

$x = 3 + r\cos\frac{\pi}{6}$

$y = 4 + r\sin\frac{\pi}{6}$

$\Rightarrow Q(3 + \frac{\sqrt{3}r}{2}, 4 + \frac{r}{2})$

point Q satisfies 12x + 5y + 10 = 0

$\Rightarrow 12\left(3+\frac{\sqrt{3}}{2}r\right)+5\left(4+\frac{r}{2}\right) + 10 = 0$

$\Rightarrow 36 + 20 + 6\sqrt{3}\,r + \frac{5r}{2} + 10 = 0$

$\Rightarrow 66 + 6\sqrt{3}\,r + \frac{5}{2}r = 0$

$\Rightarrow \ r = \frac{-66\times 2}{12\sqrt{3}+5}$

$\Rightarrow \ r = \frac{132}{12\sqrt{3}+5}$

Question based on

Angle between lines

Q.28 One side of square is x – y = 0 find side opposite to it if length of square is $2\sqrt{2}$

(A) x – y = ± 4 (B) x + y = ± 4

(C) x + y = 4 (D) x – y = 4

Sol.[A] One side of square is x – y – 5 = 0

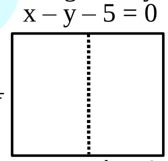
the side opposite to it, will be the parallel side

∴ equation of required side, let it be given by

x – y + λ = 0

Also given that

$4a = 2\sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$



where a is length of side of square

∴ distance between parallel side will be $\frac{1}{\sqrt{2}}$

∴ distance between the parabola sides

x – y – 5 = 0

and x – y + λ = 0 is given by

$\frac{|\lambda+5|}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow |\lambda + 5| = 1 \quad \Rightarrow \lambda + 5 = \pm 1$

$\Rightarrow \lambda = -5 \pm 1 \quad \Rightarrow \lambda = -4, -6$

∴ Required equation is

x – y – 4 = 0 and x – y – 6 = 0

Q.29 The equation of a line perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ and passing through the point where it meets x-axis is -

(A) $\frac{x}{a}+\frac{y}{b}+\frac{a}{b}=0$ (B) $\frac{x}{b}+\frac{y}{a}=\frac{a}{b}$

(C) $\frac{x}{b}+\frac{y}{a}=0$ (D) $\frac{x}{b}+\frac{y}{a}=\frac{b}{a}$

Sol.[B] equation perpendicular to line $\frac{x}{a}-\frac{y}{b} = 1$ is given by

$\frac{x}{b}+\frac{y}{a} + \lambda = 0$

it passes (a, 0) therefore

$\frac{a}{b} + 0 + \lambda = 0 \Rightarrow \lambda = -\frac{a}{b}$

∴ Required equation is

$\frac{x}{b}+\frac{y}{a}-\frac{a}{b} = 0 \Rightarrow \frac{x}{b}+\frac{y}{a}=\frac{a}{b}$

Q.30 The equation of the perpendicular bisector of the line segment joining points (1, 5) and (–3, 2) is -

(A) 4x + 3y – 29 = 0 (B) 4x + 3y – 13 = 0

(C) 8x + 6y – 13 = 0 (D) 8x + 6y + 13 = 0

Sol.[C] Slope of line segment is = $\frac{2-5}{-3-1} = \frac{-3}{-4} = \frac{3}{4}$

∴ Slope of perpendicular bisector is $-\frac{4}{3}$

Mid point of given line segment is = $\left(-1, \frac{7}{2}\right)$

∴ Equation of perpendicular bisector is given by

$y - \frac{7}{2} = -\frac{4}{3}(x + 1) \Rightarrow \frac{2y-7}{2} = \frac{-4x-4}{3}$

$\Rightarrow -8x - 8 = 6y - 21 \Rightarrow -8x - 6y + 21 - 8 = 0$

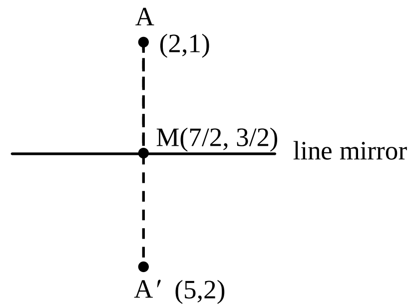
$\Rightarrow -8x - 6y + 13 = 0 \Rightarrow 8x + 6y - 13 = 0$

Q.31 The image of the point (2, 1) with respect to the line mirror be (5, 2). Then the equation of the mirror is -

(A) 3x + y – 12 = 0 (B) 3x – y + 12 = 0

(C) 3x + y + 12 = 0 (D) 3x – y – 12 = 0

Sol.[A]



line mirror will be the perpendicular line to the line joining A(2, 1) and A′(5, 2)

slope of AA′ line is $\frac{1}{3}$

$\therefore$ slope of line mirror is – 3

$\therefore$ equation of line mirror is

$$y - \frac{3}{2} = -3\left(x - \frac{7}{2}\right)$$

$$\Rightarrow \frac{2y-3}{2} = -3x + \frac{21}{2} = \frac{-6x+21}{2}$$

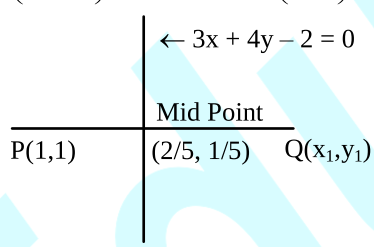
$$\Rightarrow 2y - 3 = -6x + 21 \Rightarrow 6x + 2y = 24 \Rightarrow 3x + y = 12$$

Q.32 Perpendicular bisector of segment PQ is $3x + 4y - 2 = 0$. If P is (1, 1) then point Q is -

(A) $\left(\frac{1}{5}, -\frac{3}{5}\right)$ (B) $\left(-\frac{1}{5}, -\frac{3}{5}\right)$

(C) $\left(-\frac{1}{5}, \frac{3}{5}\right)$ (D) $\left(\frac{1}{5}, \frac{3}{5}\right)$

Sol.[B]



Since slope of $3x + 4y - 2 = 0$ is $-3/4$

$\therefore$ slope of PQ is $\frac{4}{3}$

$\therefore$ equation of PQ is given by

$$y - 1 = \frac{4}{3}(x - 1)$$

$\Rightarrow 3y - 3 = 4x - 4$

$\Rightarrow 4x - 3y - 1 = 0$ ……(1)

and $3x + 4y - 2 = 0$ ……(2)

point of interscetion of (1) and (2) will be the mid point of PQ. On solving we get $\left(\frac{2}{5}, \frac{1}{5}\right)$

Let Q is (x_1, y_1) therefore

$$\frac{x_1+1}{2} = \frac{2}{5} \quad \text{and} \quad \frac{y_1+1}{2} = \frac{1}{5}$$

$\Rightarrow 5x_1 + 5 = 4$ and $5y_1 + 5 = 2$

$\Rightarrow x_1 = \frac{-1}{5}$ $\Rightarrow y_1 = -3/5$

$\therefore$ required point Q is $\left(-\frac{1}{5}, -\frac{3}{5}\right)$

Question based on: Distance of a point from a line

Q.33 The coordinates of a point on $x + y + 3 = 0$, whose distance from $x + 2y + 2 = 0$ is $\sqrt{5}$ is equal to -

(A) (9, 6) (B) (–9, 6)

(C) (–9, –6) (D) None of these

Sol.[B] Let required point is (x_1, y_1)

$\therefore x_1 + y_1 + 3 = 0$ …..(1)

and $\frac{x_1 + 2y_1 + 2}{\sqrt{1+4}} = \sqrt{5}$

$\Rightarrow x_1 + 2y_1 + 2 = \pm 5$

$\Rightarrow x_1 + 2y_1 - 3 = 0$ …..(2)

Also $x_1 + 2y_1 + 7 = 0$ …..(3)

from (1) & (2) $-y_1 + 6 = 0$

$\Rightarrow y_1 = +6$

$\therefore x_1 = -9$

point $\Rightarrow$ (– 9, 6)

from (1) & (3) $-y_1 - 4 = 0$

$y_1 = -4$

$\therefore x_1 = 1$

$\therefore$ point (1, – 4)

$\therefore$ Required point is (– 9, 6) & (1, – 4)

Question based on: Intersection of two lines

Q.34 The equation of the line through the point of intersection of the lines $2x + 3y - 7 = 0$ and $3x + 2y - 8 = 0$ which cuts equal intercepts on the axes is -

(A) $x + y = 3$ (B) $2x + 2y = 7$

(C) $x + y = 1$ (D) $3x + 3y = 8$

Sol.[A] Point of intersection of

$2x + 3y - 7 = 0$ ……(1)

$3x + 2y - 8 = 0$ …….(2)

$(1) \times 3 \Rightarrow 6x + 9y - 21 = 0$

$(2) \times 2 \Rightarrow 6x + 4y - 16 = 0$

– – +

$5y = 5$

$y = 1$

$\therefore x = \frac{7-3}{2} = 2$

$\therefore$ point of intersection is (2, 1) since line cuts equal intercept of the axes, therefore

Let equation of line is $\frac{x}{a}+\frac{y}{a}=1 \Rightarrow x + y = a$

since it passes through (2, 1)

$\therefore\ 2 + 1 = a \Rightarrow\ a = 3$

$\therefore$ required equation of line is

$$x + y = 3$$

Q.35 The point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ does not lies on the line -

(A) $x - y = 0$
(B) $(x + y)(a + b) = 2ab$
(C) $(\lambda x + my)(a + b) = (\lambda + m)\, ab.$
(D) $(\lambda x - my)(a - b) = (1 - m)\, ab$

Sol.[D] Point of intersection of lines

$\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ is findout as

$bx + ay = ab$ & $ax + by = ab$

$abx + a^2y = a^2b$

$abx + b^2y = ab^2$

– – –

$(a^2 - b^2)\, y = a^2b - ab^2$

$\Rightarrow (a^2 - b^2)\, y = ab\,(a - b)$

$\Rightarrow y = \frac{ab}{a+b}$

$\therefore abx = a^2b - a^2 \cdot \frac{ab}{a+b}$

$\Rightarrow abx = a^2b - \frac{a^3b}{a+b} = \frac{a^3b + a^2b^2 - a^3b}{a+b}$

$\Rightarrow x = \frac{ab}{a+b}$

$\therefore$ point of intersection $= \left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$

from options given, only (D) does not satisfy. Hence (D) is correct answer.

Q.36 For what value of λ, the three lines $2x - 5y + 3 = 0$, $5x - 9y + \lambda = 0$ & $x - 2y + 1 = 0$, are concurrent -

(A) 4 (B) 5 (C) 3 (D) 2

Sol.[A] Given lines are con-current if

$$\begin{vmatrix} 2 & -5 & 3 \\ 5 & -9 & \lambda \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-9 + 2\lambda) + 5(5 - \lambda) + 3(-10 + 9) = 0$

$\Rightarrow -18 + 4\lambda + 25 - 5\lambda - 3 = 0$

$\Rightarrow -l + 4 = 0$

$\Rightarrow \lambda = 4$

Question based on Angle bisector of two lines

Q.37 The equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ is -

(A) $21x + 77y - 101 = 0$
(B) $11x - 3y + 9 = 0$
(C) $11x - 3y - 9 = 0$
(D) none of these

Sol.[B] $\frac{3x-4y+7}{\sqrt{9+16}} = \pm\frac{12x+5y-2}{\sqrt{144+25}}$

$\Rightarrow \frac{3x-4y+7}{5} = \pm\frac{12x+5y-2}{13}$

$\Rightarrow 60x + 25y - 10 = 39x - 52y + 91$

$\Rightarrow 21x + 77y - 101 = 0$

and

$39x - 52y + 91 = -60x - 25y + 10$

$\Rightarrow 99x - 27y + 81 = 0$

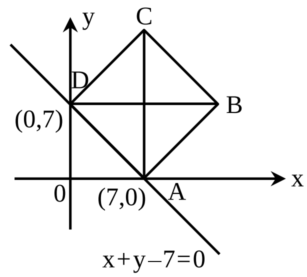
$\Rightarrow 11x - 3y + 9 = 0$

This is the required bisector

Q.38 On the portion of the straight line $x + y - 7 = 0$ which is intercepted between the axes a square is constructed on the side of the line away from the origin. Then the equation to the diagonals are -

(A) $-x + y = 7$; $x - y = 7$
(B) $x = -7$; $y = 7$
(C) $x = 7$; $y = -7$
(D) $x = 7$; $y = 7$

Sol.[D]



From diagram, it is clear that

$x = 7$ and $y = 7$ are the diagonals of the square.

Since diagonal BD makes 45° angle with $x + y - 7 = 0$

by using $y - y_1 = \frac{m \pm \tan\alpha}{1 \mu m \tan\alpha}(x - x_1)$

there $(x_1, y_1) \equiv (0, 7)$ and $m = -1$

$y - 7 = \frac{-1 \pm 1}{1 \mu (-1)(.1)}(x - o)$

$y - 7 = \frac{-0}{2}(x)$

$y - 7 = 0 \Rightarrow y = 7$

Similarly we get x = 7

$\therefore$ Diagonals are

x = 7, y = 7

Question based on **Family of lines**

Q.39 The equation of the line through the point of intersection of the lines x – y + 4 = 0 and y – 2x – 5 = 0 and passing through the point (3, 2) is -

(A) x – 4y + 5 = 0 (B) x + 4y – 11 = 0

(C) 2x – y – 4 = 0 (D) none of these

Sol.[B] Required equation of line is given by

$(x - y + 4) + \lambda (y - 2x - 5) = 0$

since it passes through (3, 2)

$\therefore (3 - 2 + 4) + \lambda(2 - 6 - 5) = 0$

$5 + \lambda(-9) = 0$

$\Rightarrow \lambda = \frac{5}{9}$

$\therefore$ equation will be

$(x - y + 4) + \frac{5}{9}(y - 2x - 5) = 0$

$\Rightarrow 9x - 9y + 36 + 5y - 10x - 25 = 0$

$\Rightarrow -x - 4y + 11 = 0$

$\Rightarrow x + 4y - 11 = 0$

Q.40 The equation of the line passing through the intersection of $x - \sqrt{3}\,y + \sqrt{3} - 1 = 0$ and x + y – 2 = 0 and making an angle of 15° with the first line is

(A) x – y = 0 (B) x – y + 1 = 0

(C) y = 1 (D) $\sqrt{3}\,x - y + 1 - \sqrt{3} = 0$

Sol.[A] The point of intersection of given lines

$x - \sqrt{3}\,y + (\sqrt{3} - 1) = 0$

$x + y - 2 = 0$

$-\quad -\quad +$

$-y(1 + \sqrt{3}) = -(1 + \sqrt{3})$

$y = 1$

$\therefore x = 1$

$\therefore$ point of intersection is (1, 1)

since line passes (1,1) and makes 15° angle with

$x - \sqrt{3}\,y + (\sqrt{3} - 1) = 0$

Slope of this line is equal to $\frac{1}{\sqrt{3}}$

i.e. it makes 30° angle with x-axis

Therefore desired line makes 30° + 15° = 45° angle with x-axis. Its slope m = tan 45° = 1

$\therefore$ Required line will be

$y - 1 = 1(x - 1 \Rightarrow y - 1 = x - 1 \Rightarrow x - y = 0$

Q.41 If a + b + c = 0 then the straight line 2ax + 3by + 4c = 0 passes through the fixed point-

(A) (2, 4/3) (B) (2,2)

(C) (4/3, 4/2) (D) none of these

Sol.[A] $a + b + c = 0$ …..(1)

$2ax + 3by + 4c = 0$ …..(2)

from (1), $c = -(a + b)$ …..(3)

from (2) $\frac{2ax}{4} + \frac{3by}{4} + c = 0$

$\Rightarrow \frac{ax}{2} + \frac{3b}{4}y + c = 0$ ….(4)

from (3) & (4) we get

$\frac{ax}{2} + \frac{3b}{4}y - a - b = 0$

$\Rightarrow a\left(\frac{x}{2} - 1\right) + b\left(\frac{3y}{4} - 1\right) = 0$

$\Rightarrow \frac{x}{2} - 1 = 0$ and $\frac{3y}{4} - 1 = 0$

$\Rightarrow \frac{x}{2} = 1$ and $\frac{3y}{4} = 1$

$\Rightarrow x = 2$ and $y = \frac{4}{3}$ $\therefore \left(2, \frac{4}{3}\right)$

Question based on **Homogeneous equation of 2nd degree**

Q.42 Find the separate equations of the straight lines whose joint equations is

$ab(x^2 - y^2) + (a^2 - b^2)xy = 0$

(A) bx + ay = 0 and ax – by = 0

(B) bx – ay = 0 and ax + by = 0

(C) bx – ay = 0 and ax – by = 0

(D) None of these

Sol.[A] $ab(x^2 - y^2) + (a^2 - b^2)xy = 0$

$\Rightarrow abx^2 + a^2xy - aby^2 - b^2xy = 0$

$\Rightarrow ax(bx + ay) - by(ay + bx) = 0$

$\Rightarrow (bx + ay)(ax - by) = 0$

$\therefore$ separate equations are

bx + ay = 0 and ax – by = 0

Q.43 The equation $ax^2 + by^2 + c(x + y) = 0$ represents a pair of straight lines if -

(A) c = 0 (B) a + b = 0

(C) Both (A) & (B) (D) none of these

Sol.[C] $ax^2 + by^2 + c(x + y) = 0$

it represents a pair of straight lines if

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\Rightarrow (a)(b)(0) + 2\left(\frac{c}{2}\right)\left(\frac{c}{2}\right)(0) - a\left(\frac{c}{2}\right)^2 - b\left(\frac{c}{2}\right)^2 - c(0)^2 = 0$$

$$\Rightarrow \frac{-ac^2}{4} - \frac{bc^2}{4} = 0 \Rightarrow -\frac{c^2}{4}(a + b) = 0$$

$\Rightarrow c^2(a + b) = 0$

$\Rightarrow c = 0$ & $a + b = 0$

Q.44 If lines $px^2 - qxy - y^2 = 0$ make angle 'α' and 'β' with x-axis then value of $\tan(\alpha + \beta)$ is -

(A) $\frac{-q}{1+p}$ (B) $\frac{q}{1+p}$ (C) $\frac{p}{1+q}$ (D) $-\frac{p}{1+q}$

Sol.[A] Since lines $px^2 - qxy - y^2 = 0$ makes angle 'α' and 'β' with x-axis

$\therefore$ $m_1 = \tan\alpha$ and $m_2 = \tan\beta$

$$\therefore m_1 + m_2 = \frac{-2\times(-q/2)}{-1} = -q$$

$m_1m_2 = -p$

$$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-q}{1+p}$$

➢ True or false type questions

Q.45 Only in case of acute angled triangle centroid divides line joining circumcentre and orthocentre in the ratio 2 : 1

Sol. **[False]**

Only in case of acute angle trianlge centroid divides line joining circum centre and orthocentre in the ratio 2 : 1.

Q.46 If $L_1 = 0$ and $L_2 = 0$ are parallel lines, then family of lines will be $L_1 + \lambda L_2 = 0$

Sol. **[False]**

If $L_1 = 0$ and $L_2 = 0$ are parallel lines then family of lines will be $L_1 + \lambda L_2 = 0$

Q.47 For a triangle there exists a unique point whose distance from all three sides is same and it is called incentre of triangle

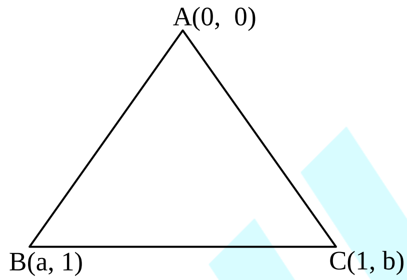
Sol. **[True]**

For a triangle there exists a unique point whose distance from all three sides is same and it is called in-centre of triangle

➢ Fill in the blanks type questions

Q.48 If a and b are real numbers between 0 and 1, such that the points (a, 1) (1, b) and (0, 0) form an equilateral triangle then 2(a + b) – ab is equal to ....

Sol.



$AB^2 = BC^2$

$a^2 + 1 = (a - 1)^2 + (1 - b)^2$

$\Rightarrow a^2 + 1 = a^2 + 1 - 2a + 1 + b^2 - 2b$

$2(a + b) + b^2 + 1$ ... (1)

$AB^2 + AC^2$

$a^2 + 1 = b^2 + 1$

$a^2 = b^2$; $a = b$

$\therefore ab = b^2$ ... (2)

from (1) and (2)

$2(a + b) = ab + 1 \Rightarrow 2(a + b) - ab = 1$

Q.49 The no. of points (p, q) such that $p, q \in \{1, 2, 3, 4\}$ and the equation $px^2 + qx + 1 = 0$ has real roots is...........

Sol. $px^2 + qx + 1 = 0$ has real roots

$D \ge 0$

$q^2 - 4p \ge 0$

$q^2 \ge 4p$

(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (4, 4)

Hence there are seven points.

Q.50 If α, β, γ are the real roots of the equation $x^3 - 3px^2 + 3qx - 1 = 0$ then centroid of the triangle with vertices $(\alpha, 1/\alpha)$, $(\beta, 1/\beta)$ and $(\gamma, 1/\gamma)$ is at the point ....

Sol. Centroid is $\left(\frac{\alpha+\beta+\gamma}{3}, \frac{\alpha\beta+\beta\gamma+\gamma\delta}{3\alpha\beta\gamma}\right)$

$\equiv \left(\frac{3p}{3}, \frac{3q}{3.1}\right) \equiv (p, q)$

Q.51 The integral values of α for which origin lies in the bisector of acute angle between lines $(\alpha^2 + 3)x + 4y + 3 = 0$ and $x + \alpha y + 1 = 0$ is ....

Sol. If origin lies in the bisector of acute angle between lines

$(\alpha^2 + 3) . x + 4y + 3 = 0$ and

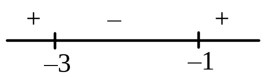
$x + \alpha y + 1 = 0$

then $a_1a_2 + b_1b_2 < 0$

$\Rightarrow (\alpha^2 + 3) . 1 + 4.\alpha < 0$

$\alpha^2 + 4\alpha + 3 < 0$

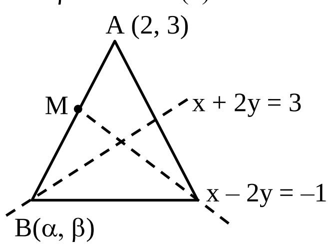
$(\alpha + 3)(\alpha + 1) < 0$



$\alpha = -2$ $\{\Theta \ \alpha \in I\}$

Q.52 In a triangle if vertex A is (2, 3) and angle bisector through B is x + 2y = 3 and median through C is x – 2y = – 1, then co-ordinate of vertex B is...............

Sol. Let the co-ordinates of B are (α, β)
also B lies on line
$x + 2y = 3$ so
$\alpha + 2\beta = 3$ ….(1)



Also, the point M lies on the median so

$$\left(\frac{\alpha+2}{2}\right) - 2\left(\frac{\beta+3}{2}\right) = -1$$

$\Rightarrow \alpha - 2\beta = 2$ ….(2)
from equation (1) and (2)

$$(\alpha, \beta) \equiv \left(\frac{5}{2}, \frac{1}{4}\right)$$

Q.53 The distance between lines whose combined equation are $x^2 + 2\sqrt{2}\ xy + 2y^2 + 4x + 4\sqrt{2}\ y + 1 = 0$ is

Sol. Reqd. distance $= 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$

$$= 2\sqrt{\frac{4-(1)(1)}{1(1+2)}} = 2\sqrt{\frac{3}{3}} = 2$$

Q.54 The slopes of two lines represented by $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ are m_1 and m_2, then $|m_1 - m_2|$ is equal to.......

Sol. Given equation is
$x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ ....(i)
and general equation of second degree
$ax^2 + 2hxy + by^2 = 0$ ....(ii)
Comparing (i) & (ii), we get
$a = \tan^2\theta + \cos^2\theta$
$h = -\tan\theta$
$b = \sin^2\theta$
Let separate lines of (ii) are $y = m_1x$ and $y = m_2x$
where $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$

$$\therefore\ m_1 + m_2 = -\frac{2h}{b} = \frac{2\tan\theta}{\sin^2\theta}$$

$$\text{and } m_1 m_2 = \frac{\tan^2\theta + \cos^2\theta}{\sin^2\theta}$$

$$\therefore\ m_1 - m_2 = \sqrt{(m_1+m_2)^2 - 4m_1m_2}$$

$$\Rightarrow = \sqrt{\frac{4\tan^2\theta}{\sin^4\theta} - \frac{4(\tan^2\theta + \cos^2\theta)}{\sin^2\theta}}$$

$$= \frac{2}{\sin^2\theta}\sqrt{\tan^2\theta - \sin^2\theta(\tan^2\theta + \cos^2\theta)}$$

$$\frac{2\sin\theta}{\sin^2\theta}(\sec^2\theta - \tan^2\theta - \cos^2\theta)$$

$$= \frac{2\sin\theta}{\sin^2\theta}\sqrt{1-\cos^2\theta} = \frac{2}{\sin\theta}.\sin\theta = 2$$

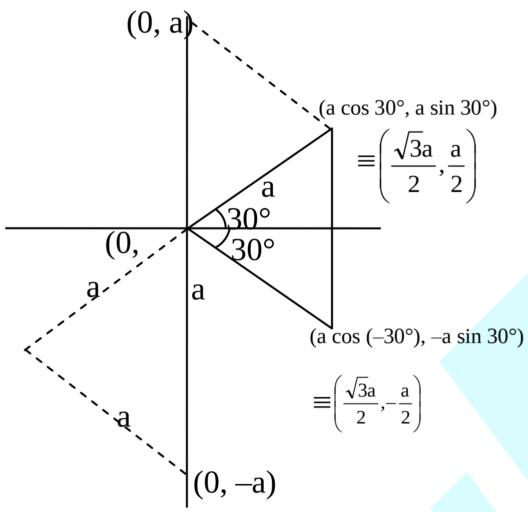
EXERCISE # 2

Part-A Only single correct answer type questions

Q.1 If one vertex of an equilateral triangle of side 'a' lies at the origin and the other lies on the line $x - \sqrt{3}\, y = 0$, the co-ordinates of the third vertex are

(A) (0, a) (B) $(\sqrt{3}\,a/2, -a/2)$
(C) (0, –a) (D) All of these

Sol.[D]

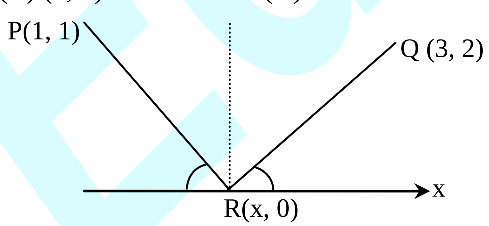


Hence, co-ordinates of the third vertex will be given by (0, a), (0, –a) and $\left(\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$

Q.2 Let P = (1, 1) and Q = (3, 2). The point R on the x-axis such that PR + RQ is the minimum is

(A) $\left(\frac{5}{3}, 0\right)$ (B) $\left(\frac{1}{3}, 0\right)$
(C) (3, 0) (D) none

Sol.[A]



$$\frac{2-0}{3-x} = -\frac{1-0}{1-x}$$

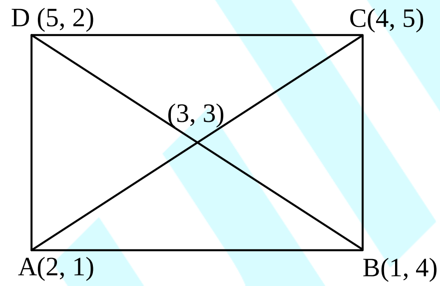
$$\Rightarrow \frac{2}{3-x} = \frac{1}{x-1} \Rightarrow 2x - 2 = 3 - x \Rightarrow 3x = 5$$

x = 5/3

∴ point R is (5/3, 0)

Q.3 The four points whose coordinates are (2,1), (1,4), (4,5), (5,2) form :
(A) a rectangle which is not a square
(B) a trapezium which is not a parallelogram
(C) a square
(D) a rhombus which is not a square

Sol.[C]



Slope of AB = $m_1 = \frac{3}{-1} = -3$ $\because$ AB ⊥ AD

Slope of AD = $m_2 = \frac{1}{3}$ and BC ⊥ BA

$\therefore m_1 m_2 = -3 \times \frac{1}{3} = -1$ AD ⊥ DC

CD ⊥ CB

and AB = BC = CD = DA

Since each angle is right angle and each side equal to same length. So formed figure is a square

Q.4 If the line segment joining (2, 3) and (–1, 2) is divided internally in the ratio 3 : 4 by the line x + 2y = k then k is

(A) $\frac{41}{7}$ (B) $\frac{5}{7}$ (C) $\frac{36}{7}$ (D) $\frac{31}{7}$

Sol.[A] x + 2y – k = 0 (given line)

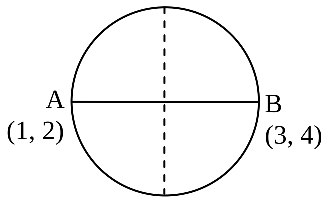
$$\Rightarrow -\frac{2+6-k}{-1+4-k} = \frac{3}{4} \Rightarrow -4(8-k) = 3(3-k)$$

$$\Rightarrow -32 + 4k = 9 - 3k \Rightarrow 7k = 32 + 9 \Rightarrow k = \frac{41}{7}$$

Q.5 Let A = (1, 2), B = (3, 4) and let C = (x, y) be a point such that (x–1) (x–3) + (y – 2) (y – 4) = 0. If ar (ΔABC) = 1 then maximum number of positions of C in the x-y plane is

(A) 2 (B) 4 (C) 8 (D) none

Sol.[B] (x –1) (x –3) + (y –2) (y –4) = 0 represents a circle whose diameters end points are (1, 2) and (3, 4)



$\because$ AB = $\sqrt{4+4}$ = 2 $\sqrt{2}$

$\therefore$ radius = $\frac{AB}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ = 1.41

Area of ΔABC = 1 (given)

$\Rightarrow \frac{1}{2}$ × base × height = 1

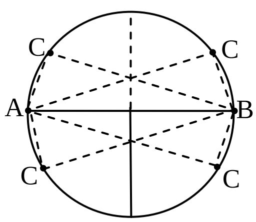
Let height of the triangle is h

$\therefore \frac{1}{2} \times 2\sqrt{2} \times h = 1$

$\Rightarrow h = \frac{1}{\sqrt{2}} = 0.707$

$\because$ h < r

Hence there are four positions possible.

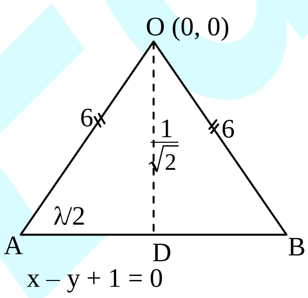


Q.6 The vertex O of an isosceles triangle OAB lies at the origin and the equation of the base AB is x – y + 1 = 0. If OA = OB = 6, the area of the triangle OAB

(A) $\sqrt{71}/2$ sq. units (B) $\sqrt{142}/2$ sq. units

(C) $2\sqrt{71}$ sq. units (D) $\sqrt{142}$ sq. units

Sol.[A]



Let length of AB is λ, therefore AD = λ/2

Length of perpendicular drawn from O (0, 0) to AB is given by OD = $\frac{0-0+1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

In ΔAOD,

$AO^2 = OD^2 + AD^2$

$36 = \frac{1}{2} + \frac{\lambda^2}{4}$

$\Rightarrow \frac{\lambda^2}{4} = \frac{71}{2} \Rightarrow \lambda^2 = 2 \times 71 \Rightarrow \lambda = \sqrt{2\times 71}$

$\therefore$ Area of ΔOAB

$= \frac{1}{2}$ × base × height

$= \frac{1}{2} \times \sqrt{2\times 71} \times \frac{1}{\sqrt{2}}$

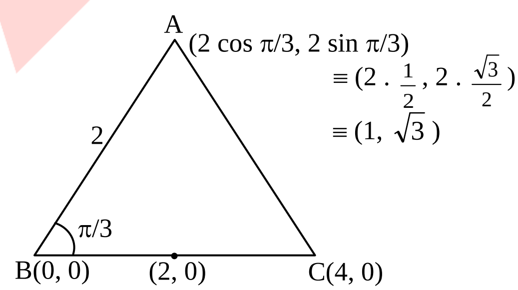
$= \frac{1}{2} \times \sqrt{2} \times \sqrt{71} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{71}}{2}$

Q.7 In the ΔABC, the coordinates of B are (0, 0) AB = 2, $\angle$ABC = π/3 and the middle point of BC has the coordinates (2, 0). The centroid of the triangle is –

(A) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{5}{3}, \frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{4+\sqrt{3}}{3}, \frac{1}{3}\right)$ (D) none of these

Sol.[B]



$\therefore$ centroid = $\frac{1+0+4}{3}, \frac{\sqrt{3}+0+0}{3}$

$= \left(\frac{5}{3}, \frac{1}{\sqrt{3}}\right)$

Q.8 The sides of a triangle are x + y = 1, 7y = x and $\sqrt{3}$ y + x = 0 Then the following is an interior point of the triangle -

(A) Circumcentre (B) Centroid

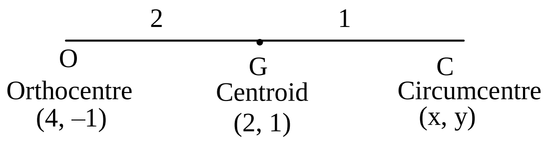
(C) Orthocentre (D) None of these

Sol.[B] The lines 7y = x and $\sqrt{3}$ y + x = 0 have angle between them is an obtuse angle. Therefore, orthocentre lies outside the given triangle. We know that in any triangle, the centroid and the incentre lie within the triangle. So centroid is the interior point of the triangle.

Q.9 If coordinates of orthocentre and centroid of a triangle are (4, –1) and (2, 1), then coordinates of a point which is equidistant from the vertices of the triangle is -

(A) (2, 2) (B) (3, 2)
(C) (2, 3) (D) (1, 2)

Sol. [D]



$\frac{2x+1(4)}{2+1} = 2$ and $\frac{2y+1(-1)}{2+1} = 1$

$\Rightarrow 2x + 4 = 6$ and $2y - 1 = 3$

$\Rightarrow 2x = +2$ $2y - 1 = 3$

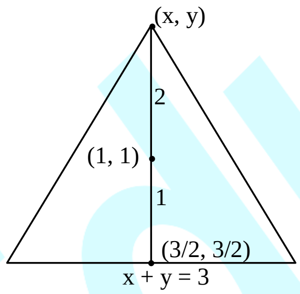
$\Rightarrow x = +1$ $2y = 4$ $y = 2$

(1, 2)

Q.10 One vertex of the equilateral triangle with circumcentre at (1, 1) and one side as x + y = 3 is-

(A) (2, 2) (B) (0, 0)
(C) (–2, –2) (D) None

Sol. [B]



$x - y = k$

(1, 1) pass k = 0

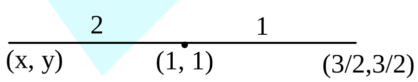
$x - y = 0$

$x + y = 3$ ...(1)

$x - y = 0$ ...(2)

$2x = 3$

$x = 3/2$



$\frac{2(3/2)+1(x)}{3} = 1$, $\frac{2(3/2)+1(y)}{3} = 1$

$3 + x = 3$, $3 + y = 3$

$\Rightarrow x = 0$, $y = 0$

$\therefore (0, 0)$

Q. 11 A point moves such that its distance from the point (4, 0) is half that of its distance from the line x = 16. The locus of this point is

(A) $3x^2 + 4y^2 = 192$ (B) $4x^2 + 3y^2 = 192$
(C) $x^2 + y^2 = 192$ (D) None of these

Sol. **[A]**

Let point is (x, y)

$\therefore \sqrt{(x-4)^2 + y^2} = \frac{1}{2} \cdot \frac{x-16}{\sqrt{1}}$

$\Rightarrow (x-4)^2 + y^2 = \frac{(x-16)^2}{4}$

$\Rightarrow 4\{x^2 + y^2 - 8x + 16\} = x^2 - 32x + 256$

$\Rightarrow 4x^2 + 4y^2 - 32x + 64 - x^2 + 32x - 256 = 0$

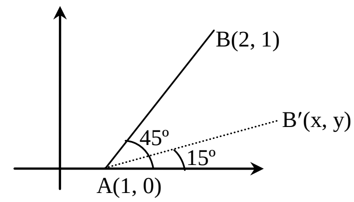
$\Rightarrow 3x^2 + 4y^2 = 192$

Q. 12 Let A = (1, 0) and B = (2, 1). The line AB turn about A through an angle π/6 in the clockwise sense, and the new position of B is B′. Then B′ has the coordinates

(A) $\left(\frac{3+\sqrt{3}}{2}, \frac{\sqrt{3}-1}{2}\right)$ (B) $\left(\frac{3-\sqrt{3}}{2}, \frac{\sqrt{3}+1}{2}\right)$

(C) $\left(\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$ (D) none of these

Sol. **[A]**



$r = \sqrt{2}$

$(x_1, y_1) = (1, 0)$

$x = x_1 + r\cos\theta = 1 + \sqrt{2} \cdot \cos 15°$

$y = y_1 + r\sin\theta = 0 + \sqrt{2}\ \sin 15°$

$\therefore x = 1 + \sqrt{2} \cdot \frac{\sqrt{3}+1}{2\sqrt{2}} = 1 + \frac{\sqrt{3}+1}{2} = \frac{3+\sqrt{3}}{2}$

$y = \sqrt{2}\ \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2}$

$\therefore B' = \left(\frac{3+\sqrt{3}}{2}, \frac{\sqrt{3}-1}{2}\right)$

Q. 13 The point (4, 1) undergoes the following three transformation successively :

(i) Reflection about the line y = x

(ii) Transformation through a distance 2 units along the positive direction of x-axis

(iii) Rotation through angle $\pi/4$ about the origin in the anticlockwise direction. The final position of the point is given by the coordinates :

(A) $\left(\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $(-2, 7\sqrt{2})$

(C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) $(\sqrt{2}, 7\sqrt{2})$

Sol. **[A]**

(i) (4, 1) after reflection about the line y = x is = (1, 4)

(ii) (1 + 2, 4) = (3, 4)

(iii) $x' = x\cos\theta + y\sin\theta$ &

$x' = 3\cos\frac{\pi}{4} + 4\sin\frac{\pi}{4}$ | $y' = -x\sin\theta + y\cos\theta$

$x' = 3.\frac{1}{\sqrt{2}} + 4.\frac{1}{\sqrt{2}}$ | $y' = -3\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4}$

$x' = \frac{7}{\sqrt{2}}$; $y' = \frac{1}{\sqrt{2}}$ | $y' = -3.\frac{1}{\sqrt{2}} + 4.\frac{1}{\sqrt{2}}$

$\therefore \left(\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **Ans.**

Q. 14 The image of the point A (1, 2) by the mirror y = x is the point B and the image of B by the line mirror y = 0 is the point (α, β). Then-

(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$

(C) $\alpha = 2, \beta = -1$ (D) none of these

Sol. **[C]**

Image of A(1, 2) by the mirror y = x is B′ (2, 1).

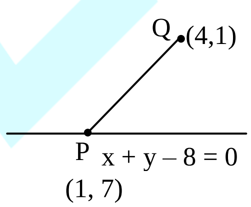
Image of B(2, 1) by the line mirror y = 0 is (2, –1)

Accordingly $(2, -1) = (\alpha, \beta) \Rightarrow \alpha = 2, \beta = -1$

Q.15 The distance of the line x + y – 8 = 0 from (4, 1) measured along the direction whose slope is –2 is

(A) $3\sqrt{5}$ (B) $6\sqrt{5}$ (C) $2\sqrt{5}$ (D) None

Sol. **[A]**



Equation of line passes through (4, 1) and whose slope is – 2 is

$y - 1 = -2(x - 4)$

$y - 1 = -2x + 8$

$2x + y - 9 = 0$

Therefore point of intersection of the lines

$x + y - 8 = 0$

and $2x + y - 9 = 0$

$- \quad - \quad +$

$-x + 1 = 0$

$\Rightarrow x = 1$

$\therefore y = 7$

$\therefore$ distance PQ $= \sqrt{9+36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

Q.16 In what direction a line be drawn through the point (1, 2) so that its point of intersection with the line x + y = 4 is at a distance $\frac{\sqrt{6}}{3}$ from the given point -

(A) 75º (B) 60º (C) 45º (D) 30º

Sol. **[A]** let the direction be θ

$\therefore \frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$

$\therefore \frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \frac{\sqrt{6}}{3}$

$\therefore x = 1 + \frac{\sqrt{6}}{3}\cos\theta$

$y = 2 + \frac{\sqrt{6}}{3}\sin\theta$

$\therefore \left(1 + \frac{\sqrt{6}}{3}\cos\theta\right) + \left(2 + \frac{\sqrt{6}}{3}\sin\theta\right) = 4$

$\Rightarrow \frac{\sqrt{6}}{3}(\sin\theta + \cos\theta) = 1$

squaring both sides

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{9}{6} = \frac{3}{2}$

$\Rightarrow \sin2\theta = \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow \sin2\theta = \frac{1}{2}$

$\Rightarrow \sin2\theta = \sin30^\circ$ or $\sin150^\circ$

$\Rightarrow 2\theta = 30^\circ$ or $2\theta = 150^\circ$

$\Rightarrow \theta = 15^\circ$ or $\theta = 75^\circ$

$\therefore \theta = 15^\circ$ or 75°

Q.17 P is a point on either of the two lines $y - \sqrt{3}\,|x| = 2$ at a distance of 5 units from their point of intersection. The coordinates of the foot of the perpendicular from P on the bisector of the angle between them are -

(A) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$ or $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ depending on which the point P is takes.

(B) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$

(C) $\left(0, \frac{4-5\sqrt{3}}{2}\right)$

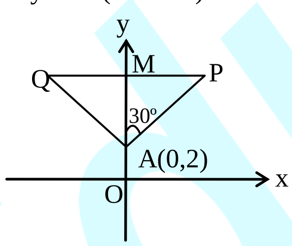
(D) $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

Sol. **[B]**

Since $|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

Therefore the equations of two lines are $y = \sqrt{3}\,x + 2,\ x \ge 0$ and $y = -\sqrt{3}\,x + 2 \quad x < 0$

Clearly, y-axis the only bisector of the angle between these two lines. there are two points P and Q on these lines at a distance of 5 units from A. clearly, M is foot of the perpendicular from P and Q on y-axis (bisector)



$AM = AP \cos 30^\circ = \frac{5\sqrt{3}}{2}$

Hence co-ordinates of M are $\left(0, 2 + \frac{5\sqrt{3}}{2}\right)$

$\equiv \left(0, \frac{4+5\sqrt{3}}{2}\right)$

Q.18 If the vertices of a quadrilateral is given by $(x^2 - 4)^2 + (y^2 - 9)^2 = 0$ then area of quadrilatural is-

(A) 36 (B) 24

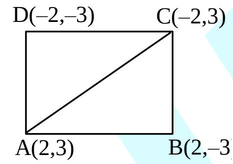
(C) 16 (D) 81

Sol. **[B]**

$(x^2 - 4)^2 + (y^2 - 9)^2 = 0$

$\Rightarrow x^2 - 4 = 0$ and $y^2 - 9 = 0 \Rightarrow x = \pm 2$ and $y = \pm 3$

$\therefore$ vertices are (2, 3), (2, –3), (–2, 3), (–2, –3)



Area of quadriateral ABCD is equal to the sum of the area of ΔABC & ΔACD

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 2 & 2 & -2 \\ 3 & -3 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 0 & 4 & -2 \\ 6 & -6 & 3 \\ 0 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}(24) = 12$

and area of $\Delta ACD = \frac{1}{2}\begin{vmatrix} 2 & -2 & -2 \\ 3 & 3 & -3 \\ 1 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 4 & 0 & -2 \\ 0 & 6 & -3 \\ 0 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2} \times 24 = 12$

$\therefore$ required area = 12 + 12 = 24 sq. units.

Q.19 The equation of the line through the point (–5, 4) such that its segment intercepted by the lines x + 2y + 1 = 0 and x + 2y – 1 = 0 is of length $\frac{2}{\sqrt{5}}$ is -

(A) 2x – y = 4 (B) 2x – y = – 14

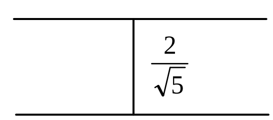
(C) 2x – y = 0 (D) none

Sol. **[B]**

Distance between two parallel lines

x + 2y + 1 = 0

x + 2y – 1 = 0



Apply $\frac{C_1 - C_2}{\sqrt{a^2 + b^2}}$

$$\frac{1+1}{\sqrt{1+4}} = \frac{2}{\sqrt{5}}$$

Therefore required line will be the $\perp^r$ line of the given two lines

$2x – y + \lambda = 0$

$\therefore$ it passes through (–5, 4)

$\therefore – 10 – 4 + \lambda = 0$

$\Rightarrow \lambda – 14 = 0$

$\Rightarrow \lambda = 14$

$\therefore$ Required line is

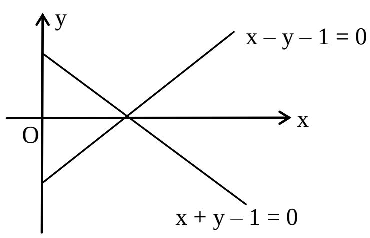
$2x – y + 14 = 0$

Q.20 If the point (cosθ, sinθ) does not fall in that angle between the lines y = | x – 1 | in which the origin lies then θ belongs to -

(A) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.   (B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $(0, \pi)$ (D) $\left[0, \frac{\pi}{2}\right]$

Sol. **[B]**



At point (0,0) ; $(x – y – 1)(x + y –1) > 0$

So, (cosθ, sinθ) should not lie in the region where origin lies

$(\cos\theta – \sin\theta –1) \times (\cos\theta + \sin\theta –1) < 0$

$\Rightarrow (\cos\theta – 1)^2 – \sin^2\theta < 0$

$\Rightarrow \cos^2\theta + 1 – 2\cos\theta – \sin^2\theta < 0$

$\Rightarrow 2\cos^2\theta – 2\cos\theta < 0$

$\Rightarrow 2\cos\theta(\cos\theta – 1) < 0$

$0 < \cos\theta < 1$

$\Rightarrow \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q.21 The straight line y = x – 2 rotates about a point where it cuts x-axis and becomes perpendicular on the straight line ax + by + c = 0 then its equation is -

(A) ax + by + 2a = 0   (B) ay – bx + 2b = 0

(C) ax + by + 2b = 0   (D) none of these

Sol. **[B]**

The line y = x – 2 cuts x-axis where its point of contact is (2, 0)

since line is perpendicular to ax + by + c = 0

$\therefore bx – ay + \lambda = 0$

It passes through (2, 0)

$\therefore 2b + \lambda = 0$

$\Rightarrow \lambda = – 2b$

Θ Required equation of line is

$bx – ay – 2b = 0$

$\Rightarrow ay – bx + 2b = 0$

Q.22 If $A\left(\frac{\sin\alpha}{3} - 1, \frac{\cos\alpha}{2} - 1\right)$ and B(1, 1), $-\pi \le \alpha \le \pi$ are two points on the same side of the line 3x – 2y + 1 = 0, then α belongs to the interval-

(A) $\left[-\pi, -\frac{3\pi}{4}\right) \cup \left(\frac{\pi}{4}, \pi\right]$ (B) $[-\pi, \pi]$

(C) ϕ (D) none of these

Sol. **[A]**

Since points A & B are same side of the line 3x – 2y + 1 = 0

$3\left(\frac{\sin\alpha}{3} - 1\right) - 2\left(\frac{\cos\alpha}{2} - 1\right) + 1 > 0$

$\Rightarrow 3\left(\frac{\sin\alpha - 3}{3}\right) - 2\left(\frac{\cos\alpha - 2}{2}\right) + 1 > 0$

$\Rightarrow \sin\alpha – 3 – \cos\alpha + 2 + 1 > 0$

$\Rightarrow (\sin\alpha – \cos\alpha) > 0$

$\Rightarrow \sin\alpha > \cos\alpha$

$\Rightarrow \sin^2\alpha + \cos^2\alpha – 2\sin\alpha\cos\alpha > 0$

$\Rightarrow \sin 2\alpha < 1$

$\Rightarrow \sin 2\alpha < \sin\frac{\pi}{2}$ or $\sin\frac{3\pi}{2}$

$\Rightarrow 2\alpha < \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$\Rightarrow \alpha < \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\therefore \alpha \in \left[-\pi, -\frac{3\pi}{4}\right) \cup \left(\frac{\pi}{4}, \pi\right]$

Q.23 Given four lines whose equations are x + 2y – 3 = 0, 2x + 3y – 4 = 0, 3x + 4y – 7 = 0 and 4x + 5y – 6 = 0 then

(A) they are all concurrent

(B) they are sides of a quadrilateral

(C) They are sides of trapezium

(D) none of these

Sol. **[D]**

Q.24 If a, b, c are in A.P., then $ax + by + c = 0$ represents -

(A) a single line

(B) a family of concurrent lines

(C) a family of parallel lines

(D) none of these

Sol. **[B]**

Since a, b, c are in A.P.

$\Rightarrow 2b = a + c$

$\therefore a - 2b + c = 0$ and $ax + by + c = 0$

$\Rightarrow$ it always passes through (1, –2)

therefore the line $ax + by + c = 0$ represents a family of concurrent lines.

Q.25 If the lines represented by $x^2 - 2pxy - y^2 = 0$ are rotated about the origin through an angle θ one in clockwise direction and other in anti-clockwise direction, then the equation of the bisectors of the angle between the lines in the new position is –

(A) $px^2 + 2xy - py^2 = 0$

(B) $px^2 + 2xy + py^2 = 0$

(C) $x^2 - 2pxy + y^2 = 0$

(D) None of these

Sol. **[A]**

Equation $x^2 - 2pxy - y^2 = 0$ represents two mutual perpendicular lines. If we rotate one line about origin through an angle θ in clockwise direction & other in anticlockwise direction therefore there is nothing change in any respect. so we have to find equation of angle bisector of the equation $x^2 - 2pxy - y^2 = 0$ which is given by

$$\frac{x^2 - y^2}{xy} = \frac{a-b}{h} \Rightarrow \frac{x^2 - y^2}{xy} = \frac{1+1}{-p}$$

$$\Rightarrow \frac{x^2 - y^2}{xy} = \frac{2}{-p} \Rightarrow -px^2 + py^2 = 2xy$$

$\Rightarrow -px^2 - 2xy + py^2 = 0 \Rightarrow px^2 + 2xy - py^2 = 0$

* The bisectors of the angles between the lines in new position are same as the bisectors of the angles between their old positions.

Q.26 If the equation $12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$ represent a pair of perpendicular straight line then -

(A) p = 12, q = 1 (B) p = 1, q = 12

(C) p = –1, q = 12 (D) p = 1, q = –12

Sol. **[A]**

equation $12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$ represents a pair of perpendicular straight line.

This means that coefficent of x^2+coeff. of y^2=0

$\Rightarrow 12 - p = 0 \Rightarrow p = 12$

since given equation represents two straight line

$\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\Rightarrow (12)(-p)(6) + 2\left(\frac{q}{2}\right)(-9)(7/2) - (12)\left(\frac{q}{2}\right)^2 - (-p)(-9)^2 - (6)(7/2)^2 = 0$$

$\Rightarrow -72p - \frac{63}{2}q - 3q^2 + 81p - \frac{147}{2} = 0$

$\Rightarrow -144p - 63q - 6q^2 + 162p - 147 = 0$

$\Rightarrow -144 \times 12 - 63q - 6q^2 + 162 \times 12 - 147 = 0$

$\Rightarrow -1728 - 63q - 6q^2 + 1944 - 147 = 0$

$\Rightarrow -6q^2 - 63q + 69 = 0 \Rightarrow 2q^2 + 21q - 23 = 0$

$\Rightarrow q = 1$ satisfy by the above equation

$\therefore p = 12, q = 1$

Q.27 One of the bisectors of the angle between the lines $a(x-1)^2 + 2h(x-1)(y-2) + b(y-2)^2 = 0$ is $x + 2y - 5 = 0$ The other bisector is -

(A) $2x - y = 0$ (B) $2x + y = 0$

(C) $2x + y - 4 = 0$ (D) $x - 2y + 3 = 0$

Sol. **[A]**

Equation $a(x-1)^2+2h(x-1)(y-2)+b(y-2)^2=0$ passes (1, 2) and it is given that $x + 2y - 5 = 0$ is one of the bisector therefore other bisector will be perpendicular to this one.

$\therefore 2x - y + \lambda = 0$

This passes (1, 2)

$\therefore 2 - 2 + \lambda = 0$

$\lambda = 0$

$\therefore 2x - y = 0$

Q.28 If the two pairs of lines $x^2 - 2mxy - y^2 = 0$ and $x^2 - 2nxy - y^2 = 0$ are such that one of them represents the bisectors of the angles between the other, then -

(A) mn + 1 = 0 (B) mn – 1 = 0

(C) 1/m + 1/n = 0 (D) 1/m – 1/n = 0

Sol. **[A]**

$x^2 - 2mxy - y^2 = 0$

equation of angle bisector

$$\frac{x^2 - y^2}{xy} = \frac{a-b}{h}$$

$$\Rightarrow \frac{x^2 - y^2}{xy} = \frac{1+1}{-m}$$

$\Rightarrow -mx^2 + my^2 = 2xy$

$\Rightarrow mx^2 + 2xy - my^2 = 0$

$\Rightarrow x^2 + \frac{2}{m}xy - y^2 = 0$ ....(i)

$x^2 - 2nxy - y^2 = 0$ ....(ii)

(1) & (2) are Identical

$\Rightarrow \frac{2}{m} = -2n$

$\Rightarrow mn + 1 = 0$

Q.29 The number of values of λ for which bisectors of the angle between the lines $ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$ are the same is -

(A) two (B) one (C) zero (D) infinite

Sol. **[D]**

$ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$

$\Rightarrow (a + \lambda)x^2 + 2hxy + (b + \lambda)y^2 = 0$

$\therefore$ angle bisector is given by

$$\frac{x^2 - y^2}{xy} = \frac{(a+\lambda)-(b+\lambda)}{h}$$

$$\Rightarrow \frac{x^2 - y^2}{xy} = \frac{a-b}{h}$$

$\therefore$ Bisector of angle between lines given by equation are the same.

$\therefore$ no. of values of λ is infinite.

Q.30 If the slope of one line is double the slope of another line and the combined equation of the pair of lines is $\frac{x^2}{a} + \frac{2xy}{h} + \frac{y^2}{b} = 0$ then $ab : h^2$ is equal to -

(A) 9 : 8 (B) 3 : 2

(C) 8 : 3 (D) none of these

Sol. **[A]**

Slope of one line = m

slope of another line = 2m

Given pair of lines is $\frac{x^2}{a} + \frac{2xy}{h} + \frac{y^2}{b} = 0$

$\therefore$ sum of slope of lines = $\frac{-2 \times \frac{1}{h}}{\frac{1}{b}}$

$= \frac{-2}{h} \times \frac{b}{1} = \frac{-2b}{h}$

Products of slopes = $\frac{1/a}{1/b} = \frac{b}{a}$

$\therefore 3m = -\frac{2b}{h} \Rightarrow m = -\frac{2b}{3h}$

and $2m^2 = \frac{b}{a}$

$\Rightarrow 2\left(\frac{-2b}{3h}\right)^2 = \frac{b}{a}$

$\Rightarrow 2.\ \frac{4b^2}{9h^2} = \frac{b}{a}$

$\Rightarrow 8ba = 9h^2$

$\Rightarrow \frac{ab}{h^2} = \frac{9}{8}$

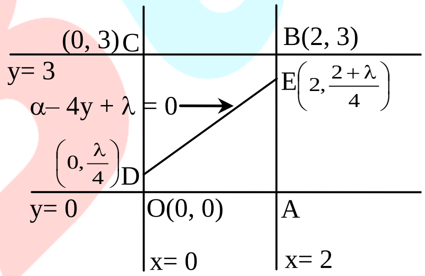
$\Rightarrow ab : h^2 = 9 : 8$

Q.31 The four sides of a quadrilateral are given by the equation $xy(x - 2)(y - 3) = 0$. The equation of the line parallel to $x - 4y = 0$ that divides the quadrilateral in two equal areas is -

(A) $x - 4y + 5 = 0$ (B) $x - 4y - 5 = 0$

(C) $4y = x + 1$ (D) $4y + 1 = x$

Sol. **[A]**



The line $x - 4y + \lambda = 0$ divide the quadrilateral in two equal areas if OD = BE

$\therefore OD^2 = BE^2$

$$\Rightarrow \left(\frac{\lambda}{4}\right)^2 = \left(3 - \frac{2+\lambda}{4}\right)^2$$

$\Rightarrow \frac{\lambda}{4} = 3 - \frac{2+\lambda}{4}$; $\Rightarrow \frac{\lambda}{4} = \frac{12-2-\lambda}{4}$

$\Rightarrow 2\lambda = 10$ $\Rightarrow \lambda = 5$

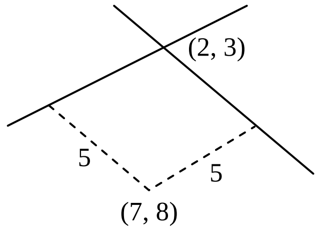
$\therefore$ reqd. line is $x - 4y + 5 = 0$

Q.32 The number of lines passing through (2, 3) each having distance equal to 5 units from the point (7, 8) is :

(A) two (B) zero

(C) one (D) infinite

Sol. **[A]**



$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = m(x - 2) \quad \Rightarrow y - 3 = mx - 2m$

$\Rightarrow mx - y + (3 - 2m) = 0$

It's distance from point (7, 8) is 5

$\therefore \frac{7m - 8 + (3 - 2m)}{\sqrt{m^2 + 1}} = 5$

$\Rightarrow 5m - 5 = 5\sqrt{m^2 + 1}$

$\Rightarrow 5(m - 1) = 5\sqrt{m^2 + 1}$

$\Rightarrow (m - 1)^2 = m^2 + 1$

$\Rightarrow m^2 - 2m + 1 = m^2 + 1$

$\Rightarrow m = 0$

$\therefore y - 3 = 0$

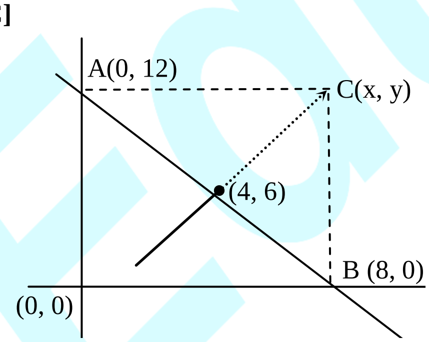
Part-B One or more than one correct answer type questions

Q.33 The line $3x + 2y = 24$ meets the y-axis at A and the x-axis at B. C is a point on the perpendicular bisector of AB such that the area of the triangle ABC is 91 sq. units. The coordinates of C are

(A) (29/2, –1) (B) (29/2, 13)

(C) (–13/2, –3/2) (D) (–13/2, 13)

Sol. [B,C]



Equation of perpendicular bisector of AB is $2x - 3y + \lambda = 0$

It passes through (4, 6)

$2(4) - 3(6) + \lambda = 0$

$\Rightarrow 8 - 18 + \lambda = 0$

$\Rightarrow \lambda = 10$

So equation is $2x - 3y + 10 = 0$ ... (1)

Area of $\Delta ABC = 91$

$\frac{1}{2}\begin{vmatrix} 0 & 8 & x \\ 12 & 0 & y \\ 1 & 1 & 1 \end{vmatrix} = \pm 91 \Rightarrow \begin{vmatrix} 0 & 8 & x \\ 12 & 0 & y \\ 1 & 1 & 1 \end{vmatrix} = \pm 182$

$\Rightarrow -8(12 - y) + x(12) = 182$

$\Rightarrow -96 + 8y + 12x = 182$

$\Rightarrow 12x + 8y - 278 = 0$

$\Rightarrow 6x + 4y - 139 = 0$ (+ve) ... (2)

By Eqn. (1) and (2)

$\Rightarrow 3(6x - 9y + 30 = 0)$

$6x + 4y - 139 = 0$

$- \quad - \quad +$

$-13y = -169$

$y = 13$

From (1), $2x - 39 + 10 = 0$

$\Rightarrow 2x = 29$ $\Rightarrow x = 29/2$

$\therefore$ C is (29/2, 13) **Ans.**

Also

$12x + 8y + 86 = 0$ (–ve)

$\Rightarrow 6x + 4y + 43 = 0$ ... (1)

$\Rightarrow 6x + 4y + 43 = 0$ ... (1)

$6x - 9y + 30 = 0$ ... (2)

$- \quad + \quad -$

$13y = -13$

$y = -1;\ 6x - 4 + 43 = 0;\ \Rightarrow 6x = -39$

$\Rightarrow x = -\frac{39}{6} = -\frac{13}{2}$

$\therefore \left(-\frac{13}{2}, -1\right)$

Q.34 Three vertices of a quadrilateral in order are (6, 1), (7, 2) and (–1, 0). If the area of the quadrilateral is 4 unit^2 then the locus of the fourth vertex has the equation

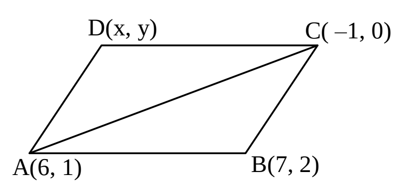
(A) $x - 7y = 1$

(B) $x - 7y + 15 = 0$

(C) $(x - 7y)^2 + 14(x - 7y) - 15 = 0$

(D) none of these

Sol.[A,B,C]



Area of ΔABC + Area of triangle ACD = 4 unit^2

$\Rightarrow \frac{1}{2}\begin{vmatrix} 6 & 7 & -1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} 6 & -1 & x \\ 1 & 0 & y \\ 1 & 1 & 1 \end{vmatrix} = \pm 4$

$$\Rightarrow \begin{vmatrix} -1 & 8 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 7 & -1-x & x \\ 1 & -y & y \\ 0 & 0 & 1 \end{vmatrix} = \pm 8$$

$\Rightarrow (-2 + 8) + (-7y) + (1 + x) = \pm 8$

$\Rightarrow 6 - 7y + 1 + x = 8$ or $6 - 7y + 1 + x = -8$

$\Rightarrow x - 7y - 1 = 0$ or $x - 7y + 15 = 0$

$\Rightarrow$ Hence locus is

$(x - 7y - 1)(x - 7y + 15) = 0$

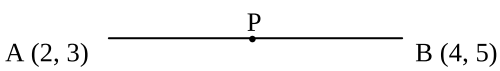
$\Rightarrow (x - 7y)^2 + 14(x - 7y) - 15 = 0$

Q. 35 If P is a point which is at a distance of 4 units and 3 units from x-axis and y-axis respectively then co-ordinate of P may be -

(A) (3, 4) (B) (–3, 4)

(C) (3, –4) (D) (–3, –4)

Sol. **[A, B, C, D]**



$\because AB = \sqrt{4+4} = 2\sqrt{2}$ and $AP = \sqrt{2}$ given

$\therefore$ P is the mid point of AB

$\therefore P = (3, 4)$

Q. 36 Let $x(y - 3) = 5$ where $x, y \in$ Integers then value of $x + y$ is equal to-

(A) 6 (B) 9 (C) –6 (D) –3

Sol. **[B, D]**

$x(y - 3) = 5$ where $x, y \in I$

This product is possible only when $x = 1$

& $(y - 3) = 5$ or $x = 5$ & $(y - 3) = 1$

$\Rightarrow x = 1$ & $y = 5 + 3 = 8$ or $x = 5$ and $y = 4$

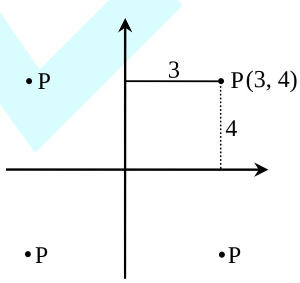
$\therefore x + y = 1 + 8 = 9$ or $x + y = 5 + 4 = 9$

Q. 37 If P is a point on the line joining points A (2, 3) & B (4, 5) such that $AP = \sqrt{2}$ then co-ordinates of P are-

(A) (3, 4) (B) (2, 4) (C) (1, 2) (D) (3, 2)

Sol. **[A, C]**

Distance from x- axis = y- coordinate of the point $\equiv$ 4 and distance from y- axis = x coordinate of the point $\equiv$ 3



$P \equiv (3, 4)$

$\equiv (-3, 4) \equiv (3, -4) \equiv (-3, -4)$

Q.38 Let A (2, α), B (3, 5), C (4, 5) are the vertices of Δ ABC whose area is 10(units)^2, then value of α is/are -

(A) 20 (B) 25

(C) –20 (D) –15

Sol. **[B, D]**

$$\frac{1}{2}\begin{vmatrix} 2 & 3 & 4 \\ \alpha & 5 & 5 \\ 1 & 1 & 1 \end{vmatrix} = \pm 10$$

$$= \begin{vmatrix} -1 & -1 & 4 \\ \alpha - 5 & 0 & 5 \\ 0 & 0 & 1 \end{vmatrix} = \pm 20$$

$\Rightarrow \alpha - 5 = \pm 20 \Rightarrow \alpha = 5 \pm 20 \Rightarrow \alpha = 25, -15$

Q.39 If area of ΔOPB = area of ΔOPA when O is origin, A ≡ (6, 0), B ≡ (0, 4) and P lies on line x + y = 1 then possible co-ordinate of P is/are-

(A) $\left(\frac{3}{5}, \frac{2}{5}\right)$ (B) (3, –2)

(C) (2, –1) (D) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Sol. **[A, B]**

Area of ΔOPB = Area of ΔOPA

Let P = (x, y) and A≡ (6, 0), B≡ (0, 4), O– (0, 0)

$$\therefore \frac{1}{2}\begin{vmatrix} 0 & x & 0 \\ 0 & y & 4 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 0 & x & 6 \\ 0 & y & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow 4x = -6y$

$\Rightarrow 2x + 3y = 0$ ... (1)

$x + y = 1$ ... (2)

$2x + 3y = 0$

$2x + 2y = 2$

– – –

$y = -2$ (3, –2)

$\therefore x = 3$

Also (2/3, 1/3) satisfies (2)

Hence $\left(\frac{2}{3}, \frac{1}{3}\right)$ and (3, –2)

Q.40 The combined equation of two sides of an equilateral triangle is $x^2 - 3y^2 - 2x + 1 = 0$. If the length of a side of the triangle is 4 then the equation of the third side is -

(A) $x = 2\sqrt{3} + 1$ (B) $y = 2\sqrt{3} + 1$

(C) $x + 2\sqrt{3} = 1$ (D) $x = 2\sqrt{3}$

Sol. **[A, C]**

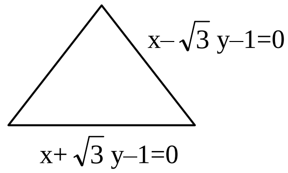
$x^2 - 3y^2 - 2x + 1 = 0$

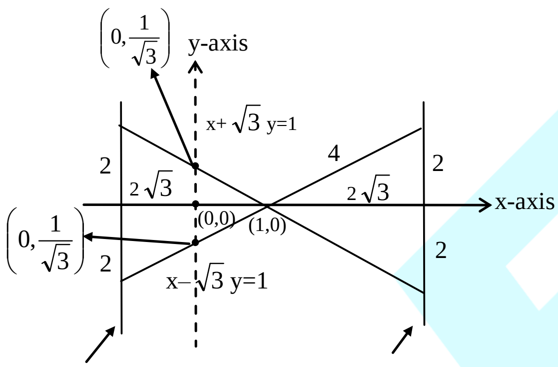
$\Rightarrow (x^2 - 2x + 1) - 3y^2 = 0$

$\Rightarrow (x - 1)^2 - (\sqrt{3}\,y)^2 = 0$

$\Rightarrow (x - 1 + \sqrt{3}\,y)(x - 1 - \sqrt{3}\,y) = 0$

$\Rightarrow x + \sqrt{3}\,y - 1 = 0$ & $x - \sqrt{3}\,y - 1 = 0$





$x = 1 - 2\sqrt{3}$

$= x + 2\sqrt{3} = 1$

Q.41 Two pairs of straight lines have the equations $y^2 + xy - 12x^2 = 0$ and $ax^2 + 2hxy + by^2 = 0$. One line will be common among them if-

(A) $a = -3(2h + 3b)$ (B) $a = 8(h - 2b)$

(C) $a = 2(b + h)$ (D) $a = -3(b + h)$

Sol. **[A,B]**

$y^2 + xy - 12x^2 = 0$

$\Rightarrow (y + 4x)(y - 3x) = 0$

$\therefore \frac{y}{x} = 3, -4$

The two pairs will have a line common if 3 or–4 is a root of $b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0$

$\therefore 9b + 6h + a = 0$ or $16b - 8h + a = 0$

$\Rightarrow a = -6h - 9b$ or $a = 8h - 16b$

$\Rightarrow a = -3(2h + 3b)$ or $a = 8(h - 2b)$

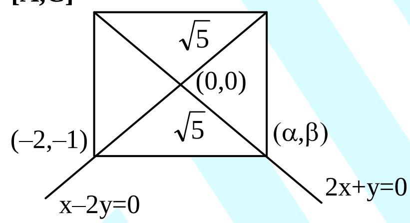
Therefore $a = -3(2h + 3b)$ and $a = 8(h - 2b)$

Q.42 The diagonals of a square are along the pair of lines whose equation is $2x^2 - 3xy - 2y^2 = 0$. If (2, 1) is a vertex of the square then another vertex consecutive to it can be -

(A) (1, –2) (B) (1, 4)

(C) (–1, 2) (D) (–1, –4)

Sol. **[A,C]**



The diagonals are $(2x + y)(x - 2y) = 0$

i.e $2x + y = 0$ and $x - 2y = 0$

If (α, β) is a consecutive vertex, then

$\alpha^2 + \beta^2 = 5$ and $2\alpha + \beta = 0$

on solving, we get $\alpha = 1, -1$ and $\beta = -2, 2$

Therefore another vertex can be

(1,–2), (1,2), (–1, –2), (–1, 2)

Q.43 The pairs of straight lines $ax^2 + 2hxy - ay^2 = 0$ and $hx^2 - 2axy - hy^2 = 0$ are such that-

(A) one pair bisects the angles between the other pair

(B) the lines of one pair are equally inclined to the lines of the other pair

(C) the lines of one pair are perpendicular to the lines of the other pair

(D) none of these

Sol. **[A, B]**

we have $ax^2 + 2hxy - ay^2 = 0$ ....(i)

and $hx^2 - 2axy - hy^2 = 0$ ....(ii)

equation of angle bisector of (i) is given by

$$\frac{x^2 - y^2}{xy} = \frac{a - b}{h}$$

$$\Rightarrow \frac{x^2 - y^2}{xy} = \frac{a + a}{h}$$

$\Rightarrow h(x^2 - y^2) = 2a(xy)$

$\Rightarrow hx^2 - 2axy - hy^2 = 0$

which is the equation (ii)

$\therefore$ one pair bisects the angle between the other pair.

Now, $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$

From (i), $\tan\theta = \dfrac{2\sqrt{h^2+a^2}}{0} = \infty$

$\theta = \tan^{-1}(\infty) = \dfrac{\pi}{2}$

From (ii), $\tan\theta = \dfrac{2\sqrt{a^2+h^2}}{0} = \infty$

$\tan\theta = \infty$

$\theta = \tan^{-1}(\infty) = \dfrac{\pi}{2}$

$\therefore$ The lines of one pair are equally inclined to the lines of the other pair.

Part-C Assertion-Reason type questions

The following questions 44 to 47 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Q. 44 **Assertion (A) :** In case of Isosceles triangle circum centre, centroid, orthocentre, Incentre are collinear.

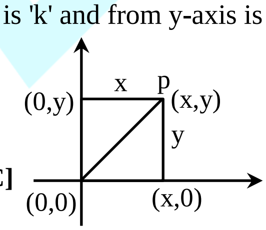
Reason (R) : In case of Isosceles triangle ABC where (AB = AC) perpendicular drawn from A will bisect $\angle A$, and will be perpendicular bisector of side BC

Sol. **[A]** Here [A] is correct answer.

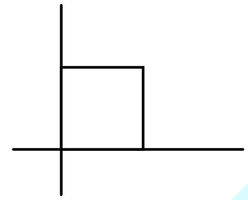
Q.45 **Assertion (A) :** A point 'P' moves such that sum of its distances from co-ordinates axis is 2 then area of region generated by movement of 'P' is 8 sq. units.

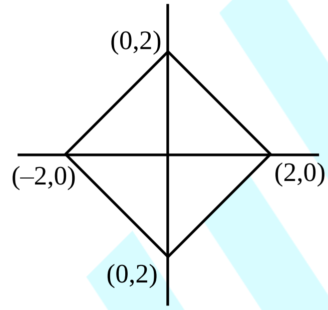
Reason (R) : Distance of point (h, k) from x-axis is 'k' and from y-axis is 'h'.

Sol. [C]



$|x| + |y| = 2$





$|x| + |y| = 2$

$\text{Area} = \dfrac{2c^2}{ab} = \dfrac{2\times(2)^2}{1\times1} = 8$

Hence option (C) is correct

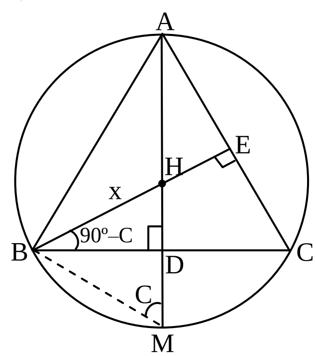
Q.46 **Assertion (A) :** Image of orthocentre about any sides of a Δ always lies on the circumcircle of this triangle.

Reason (R) : Circumcentre is always equidistant from vertices of triangle.

Sol. [B] To prove HD = DM

Let BH = x

where H is orthocentre



In ΔBHD, $\sin(90 - C) = \dfrac{HD}{x}$

$\Rightarrow HD = x \cos C$ ....(1)

Also $BD = x \cos (90 - C)$

$= x \sin C$

Now $\angle ACB = \angle AMB = \angle C$

In ΔBDM

$\tan C = \dfrac{BD}{DM} = \dfrac{x \sin C}{DM}$

$DM = x \cos C$ ...(2)

from (1) & (2)

HD = DM

Q.47 **Assertion (A) :** Each point on the line $y - x + 12 = 0$ is at same distance from the lines $3x + 4y - 12 = 0$ and $4x + 3y - 12 = 0$.

Reason (R) : Locus of points which is at equal distance from the two given lines is the angle bisectors of the two lines.

Sol. **[D]**

$y - x + 12 = 0$ is an angle bisector of given lines.Also this line is parallel to any of given lines.

Part-D Column Matching type questions

Q.48 Observe the following columns. Points given in the column -I are collinear then

Column-I	**Column -II**
(A) (a, b + c), (b, c + a), (c, a + b)	(P) if ad = bc
(B) (a, b), (c, d),(a + c, b + d)	(Q) if $\frac{1}{a} + \frac{1}{b} = 1$
(C) (a, 0), (0, b), (1, 1)	(R) if a = 1/2 , –1
(D) (a, 2 –2a), (–a + 1,, 2a) (– 4– a,6 – a)	(S) always

Sol. A→ S; B → P ; C → Q; D → R

(a, b + c), (b, c + a), (c, a + b) are collinear if

(a) $\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ 1 & 1 & 1 \end{vmatrix} = 0$

$c_1 \to c_1 - c_2$ & $c_2 \to c_2 - c_3$

$\Rightarrow \begin{vmatrix} a-b & b-c & c \\ b-a & c-b & a+b \\ 0 & 0 & 1 \end{vmatrix} = 0$

$\Rightarrow (a - b)(c - b) - (b - a)(b - c) = 0$

$\Rightarrow -(a - b)(b - c) + (a - b)(b - c) = 0$

Always

(b) (a, b), (c, d), (a + c, b – d) are collinear if

$\begin{vmatrix} a & c & a+c \\ b & d & b+d \\ 1 & 1 & 1 \end{vmatrix} = 0$

$c_1 \to c_1 - c_2$ & $c_2 \to c_2 - c_3$

$\Rightarrow \begin{vmatrix} a-c & -a & a+c \\ b-d & -b & b-d \\ 0 & 0 & 1 \end{vmatrix} = 0$

$(a - c)(-b) + a(b - d) = 0$

$\Rightarrow -ab + bc + ab - ad = 0 \Rightarrow bc = ad$

(c) (a, 0), (0, b), (1, 1) are collinear, if

$\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow a(b - 1) + (-b) = 0$

$\Rightarrow ab = a + b \Rightarrow \frac{1}{a} + \frac{1}{b} = 1$

(d) (a, 2–2a), (–a + 1, 2a), (– 4 – a, 6 – a) collinear, if

$\begin{vmatrix} a & -a+1 & -4-a \\ 2-2a & 2a & 6-a \\ 1 & 1 & 1 \end{vmatrix} = 0$

$c_1 \to c_1 - c_2$ and $c_2 \to c_2 - c_3$

$\Rightarrow \begin{vmatrix} 2a-1 & 5 & -4-a \\ 2-4a & 3a-6 & 6-a \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow (2a - 1)(3a - 6) - 5(2 - 4a) = 0$

$\Rightarrow 6a^2 - 12a - 3a + 6 - 10 + 20a = 0$

$\Rightarrow 6a^2 + 5a - 4 = 0$

$\Rightarrow 6a^2 + 8a - 3a - 4 = 0$

$\Rightarrow 2a(3a + 4) - 1(3a + 4) = 0$

$\Rightarrow (3a + 4)(2a - 1) = 0 \Rightarrow a = \frac{1}{2}$ or $\frac{-4}{3}$

Q.49 Let P(x, y) be any point on the locus then observe the following column

Column-I	**Column -II**
(A) The sum of the squares of distance from the coordinate axis is 25	(P) $x^2 + y^2 = 25$
(B) distances to the coordinate axes are in the ratio 2 : 3 respectively	(Q) $4x^2 - 9y^2 = 0$
(C) The square of whose distance from origin is 4 time its y-coordinate	(R) $x^2 + y^2 = 4y$
(D) Distance from P to (4, 0) is double the distance form P to the x-axis	(S) $x^2 - 3y^2 - 8x + 16 = 0$
	(T) $9x^2 + 4y^2 = 0$

Sol. A→ P; B → Q ; C → R; D → S

(A) $x^2 + y^2 = 25$

(B) $\frac{y}{x} = \frac{2}{3}$

$\Rightarrow 2x = 3y \Rightarrow 4x^2 = 9y^2$

$\Rightarrow 4x^2 - 9y^2 = 0$

(C) $x^2 + y^2 = 4y$

(D) $\sqrt{(x-4)^2+y^2} = 2y$

$\Rightarrow (x-4)^2 + y^2 = 4y^2$

$\Rightarrow x^2 - 8x + 16 - 3y^2 = 0$

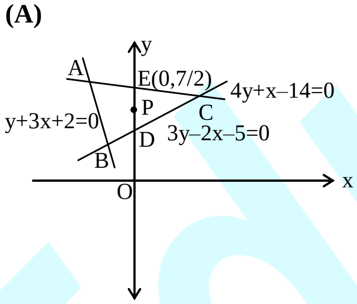
$\Rightarrow x^2 - 3y^2 - 8x + 16 = 0$

Q.50 The values of a,

Column-I	**Column-II**
(A) If (0, a) lies on or inside the triangle formed by the lines $y + 3x + 2 = 0$, $3y - 2x - 5 = 0$, $4y + x - 14 = 0$	(P) (–4, –3]
(B) If $(2a-5, a^2)$ is on the same side of the $x + y = 3$ as that of origin	(Q)$(-3,0)\cup(1/3,1)$
(C) If (a, 2) lies between the lines $x - y = 1$ and $2(x - y) + 5 = 0$	(R) (5/3, 7/2)
(D) Point $(a^2, a + 1)$ lies between the angles of the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ which contains origin if	(S) (5/2, 3)

Sol. **A→ R, S; B → P, Q; C → S; D → Q**

(A)



Let P(0, a) from above diagram, this point P(0,a) move on the line x = 0 (y–axis) for all a.

Now D & E are the intersection of $3y-2x-5= 0$, $x = 0$ and $4y + x - 14 = 0$, $x = 0$ respectively.

$\therefore D \equiv (0, 5/3)$ and $E \equiv (0,7/2)$

Thus the point on the line x = 0 whose y-co-ordinates lies between 5/3 and 7/2 lie on or inside the triangle ABC.

Hence $\frac{5}{3} \le a \le 7/2$

i.e. $a \in [5/3, 7/2]$

(B)

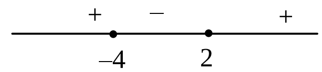
$2a - 5 + a^2 - 3 < 0$

$\Rightarrow a^2 + 2a - 8 < 0$

$\Rightarrow a^2 + 4a - 2a - 8 < 0$

$\Rightarrow a(a + 4) - 2(a + 4) < 0$

$\Rightarrow (a + 4)(a - 2) < 0$



$x \in (-4, 2)$

Therefore (A) & (B) are correct answers.

(C) $(x - y - 1)(x - y + 5/2)_{(a,2)} < 0$

$\Rightarrow (a - 3)(a + \frac{1}{2}) < 0$

$\Rightarrow -\frac{1}{2} < a < 3$

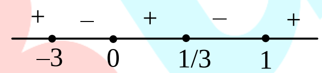
$\Rightarrow a \in \left(-\frac{1}{2}, 3\right)$

(D) $(3a^2 - a - 1 + 1)(a^2 + 2a + 2 - 5) < 0$

$(3a^2 - a)(a^2 + 2a - 3) < 0$

$a(3a - 1)(a^2 + 3a\ a - 3) < 0$

$a(3a - 1)(a + 3)(a - 1) < 0$



$(-3, 0) \cup \left(\frac{1}{3}, 1\right)$

Q.51 The equation of the line through the intersection of the line $2x - 3y = 0$ and $4x - 5y = 2$ and

Column-I	**Column-II**
(A) Through the point (2, 1)	(P) $2x - y = 4$
(B) ⊥ to line $x + 2y + 1 = 0$	(Q) $x + y - 5 = 0$, $x - y - 1 = 0$
(C) ‖ to line $3x - 4y + 5 = 0$	(R) $x - y - 1 = 0$
(D) Equally inclined to axes	(S) $3x - 4y - 1 = 0$

Sol. **A→ R; B → P ; C → S; D → Q**

Intersection of the line $2x-3y = 0$ and $4x-5y=2$

$4x - 6y = 0$

$4x - 5y = 2$

$- \quad + \quad -$

$-y = -2$

$y = 2$

$\therefore x = 3$

(3, 2) is the point of intersection.

(A) (3, 2) ____(2,1)

Slope $= \frac{1-2}{2-3} = \frac{-1}{-1} = 1$

$\therefore y - 2 = 1(x - 3)$

$\Rightarrow y - 2 = x - 3$

$\Rightarrow x - y - 1 = 0$

(B) $\perp^r$ line to $x + 2y + 1 = 0$ is $2x - y + \lambda = 0$

this passes (3, 2) $\Rightarrow 6 - 2 + \lambda = 0, \lambda = -4$

$\therefore$ equation is $2x - y - 4 = 0$

$\Rightarrow 2x - y = 4$

(C) || line to $3x - 4y + 5 = 0$ is $3x - 4y + \lambda = 0$

This passes (3, 2) $\Rightarrow 9 - 8 + \lambda = 0$

$\therefore$ equation is $3x - 4y - 1 = 0$

(D) Equally inclined to axes $\Rightarrow m = \pm 1$

$\therefore y - 2 = \pm 1(x - 3)$

$\Rightarrow y - 2 = x - 3$ or $y - 2 = -x + 3$

$\Rightarrow x - y - 1 = 0$ or $x + y - 5 = 0$

Q.52 Find the value of λ if the family of straight lines $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$ is

Column-I	**Column-II**
(A) || to y-axis	(P) $\lambda = -3/4$
(B) $\perp$ to $7x + y - 4 = 0$	(Q) $\lambda = -1/3$
(C) Passes through (1, 2)	(R) $\lambda = -17/41$
(D) || to x-axis	(S) $\lambda = 3$

Sol. **A $\rightarrow$ S; B $\rightarrow$ R ; C $\rightarrow$ P; D $\rightarrow$ Q**

$(2x + 3y + 4) + \lambda(6x - y + 12) = 0$

$\Rightarrow (2 + 6\lambda)x + (3 - \lambda)y + (4 + 12\lambda) = 0$

(A) || to y-axis $\Rightarrow$ slope is ∞

$\therefore -\dfrac{2+6\lambda}{3-\lambda} = \dfrac{1}{0}$

$\Rightarrow 3 - \lambda = 0$

$\Rightarrow \lambda = 3$

(B) $\perp^r$ to $7x + y - 4 = 0$

Slope of this one is -7

$\therefore -\dfrac{2+6\lambda}{3-\lambda} \times -7 = -1$

$-\dfrac{2+6\lambda}{3-\lambda} = \dfrac{1}{7}$

$14 + 42\lambda = -3 + \lambda$

$41\lambda = -17$

$\lambda = -17/41$

(C) passes through (1, 2)

$\Rightarrow (2 + 6\lambda) + 2(3 - \lambda) + 4 + 12\lambda = 0$

$\Rightarrow 2 + 6\lambda + 6 - 2\lambda + 4 + 12\lambda = 0$

$16\lambda = -12$

$\lambda = -12/16 = -3/4$

(D) || to x-axis $m = 0$

$-\dfrac{(2+6\lambda)}{3-\lambda} = 0$

$\Rightarrow 6\lambda = -2$

$\Rightarrow \lambda = -2/6 = -1/3$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 The coordinates of the points A, B and C are respectively (6, 3), (–3, 5) and (4, –2) and the coordinates of P are (x, y), prove that ar(PBC) : ar(ABC) = | x + y – 2 | : 7.

Sol. $\dfrac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC}$

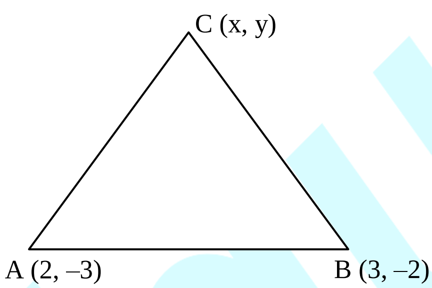
$$= \frac{\frac{1}{2}\left|\{x(5+2)-3(-2-y)+4(y-5)\}\right|}{\frac{1}{2}\left|\{6(5+2)-3(-2-3)+4(3-5)\}\right|}$$

$$= \frac{|7x+7y-14|}{|49|} = \frac{7(x+y-2)}{49}$$

$$= \frac{|x+y-2|}{7}$$

Q.2 The area of a triangle is 3/2 sq. units. Two of its vertices are the points A (2, –3) and B (3, –2), the centroid of the triangle lies on the line 3x – y – 8 = 0. Find the third vertex C.

Sol.



Centroid = $\left(\dfrac{x+2+3}{3}, \dfrac{y-3-2}{3}\right) = \left(\dfrac{x+5}{3}, \dfrac{y-5}{3}\right)$

Since centroid lies on 3x – y – 8 = 0

$$\therefore \frac{3(x+5)}{3} - \left(\frac{y-5}{3}\right) - 8 = 0$$

$\Rightarrow$ 3x + 15 – y + 5 – 24 = 0

$\Rightarrow$ 3x – y – 4 = 0 ... (1)

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 2 & 3 & x \\ -3 & -2 & y \\ 1 & 1 & 1 \end{vmatrix} = \pm\frac{3}{2}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} -1 & 3-x & x \\ -1 & -2-y & y \\ 0 & 0 & 1 \end{vmatrix} = \pm 3$$

(2 + y) + (3 – x) = ±3

If 3 is positive.

x – y = 2 ... (2)

from (1) and (2) , we get

3x – y – 4 = 0

x – y – 2 = 0

– + +

2x – 2 = 0

x = 1

$\therefore$ y = x – 2

y = 1 – 2 = –1

$\therefore$ (1, –1) **Ans.**

If 3 is negative.

(2 + y + (3 – x) = –3

x – y – 8 = 0 ... (3)

from (1) and (3)

3x – y – 4 = 0

x – y – 8 = 0

– + +

2x + 4 = 0

x = –2

$\therefore$ y = x – 8

y = –2 – 8

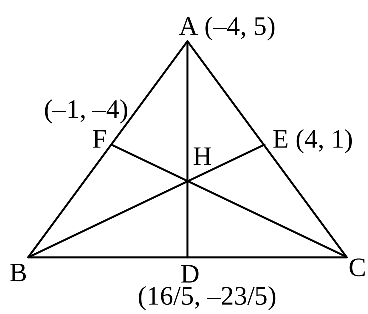
y = –10

$\therefore$ (–2, –10) **Ans.**

Q.3 The altitudes of a ΔABC are respectively AD, BE and CF. If the points A, D, E and F have the coordinates (–4, 5), $\left(\dfrac{16}{5}, \dfrac{-23}{5}\right)$, (4, 1) and (–1, –4) respectively, find the other vertices.

Sol. Slope of AD is $-\dfrac{4}{3}$ so that slope of BC is $\dfrac{3}{4}$ and is passes through D so that its equation is

3x – 4y – 28 = 0 .... (1)

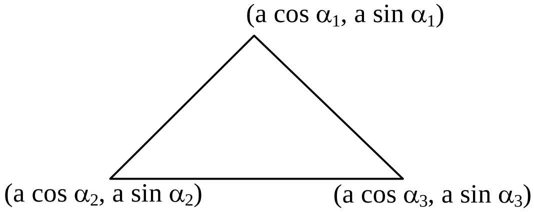


and equation of AB(AF) is 3x + y + 7 = 0 ...(2)

also equation of AC (AE) is $x + 2y - 6 = 0$... (3)
also (1) and (2) we get the coordinate of B is $(0, -7)$ and from (1) and (3) we get C $(8, -1)$

Q.4 Prove that the orthocentre of the triangle whose angular points are $(a\cos\alpha_r, a\sin\alpha_r)$;
$r = 1, 2, 3$ is the point $(a\Sigma\cos\alpha_r, a\Sigma\sin\alpha_r)$.

Sol. $(a\cos\alpha_1, a\sin\alpha_1)$, $(a\cos\alpha_2, a\sin\alpha_2)$, $(a\cos\alpha_3, a\sin\alpha_3)$ are the vertices of triangle.



clearly origin (0, 0) is its circumcentre (C) its centroid is given by

$$G \equiv \left\{\frac{a(\cos\alpha_1+\cos\alpha_2+\cos\alpha_3)}{3}, \frac{a(\sin\alpha_1+\sin\alpha_2+\sin\alpha_3)}{3}\right\}$$

we know that orthocentre, centroid and circumcentre are collinear and centroid divide these in 2 : 1 ratio.

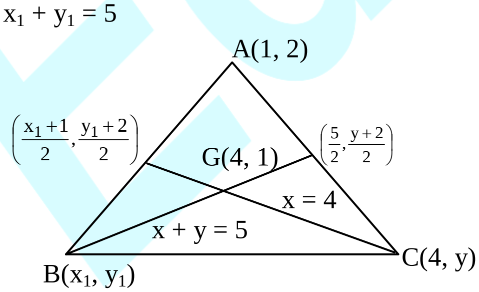
$$\left\{\frac{a(\cos\alpha_1+\cos\alpha_2+\cos\alpha_3)}{3}, \frac{a(\sin\alpha_1+\sin\alpha_2+\sin\alpha_3)}{3}\right\}$$

$$\frac{2(0)+1(x)}{3} = \frac{a(\cos\alpha_1+\cos\alpha_2+\cos\alpha_3)}{3}$$

$\Rightarrow x = a(\cos\alpha_1 + \cos\alpha_2 + \cos\alpha_3)$ and
$y = a(\sin\alpha_1 + \sin\alpha_2 + \sin\alpha_3)$
$\Rightarrow x = a\Sigma\cos\alpha_r$ and $y = a\Sigma\sin\alpha_r$
where $r = 1, 2, 3$

Q.5 In a triangle ABC, coordinates of A are (1, 2) and the equations to the medians through B and C are $x + y = 5$ and $x = 4$ respectively. Find the coordinates of B and C.

Sol. $\frac{x_1+1}{2} = 4$
$x_1 = 7$
$x_1 + y_1 = 5$



$y_1 = 5 - x_1$
$y_1 = 5 - 7 = -2$
B(7, –2) Ans.

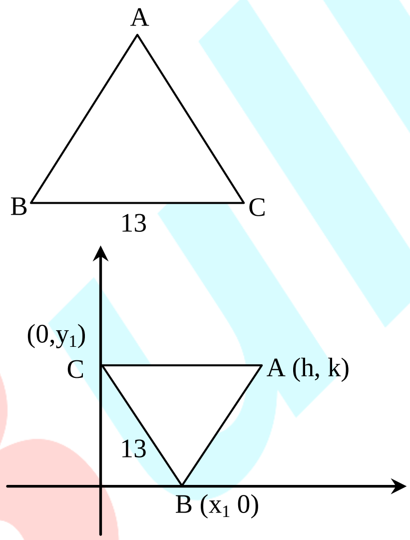
$$\frac{5}{2} + \frac{y+2}{2} = 5$$

$$\frac{y+2}{2} = \frac{5}{2}$$

$y = 3$
$\therefore$ C (4, 3)

Q.6 ABC is a variable triangle with a fixed centroid (5, 5). The side BC = 13 and B and C move on the x and y axes respectively. Find the equation of the locus of the vertex A.

Sol.



$$BC = \sqrt{x_1^2 + y_1^2} = 13$$

$\Rightarrow x_1^2 + y_1^2 = 169$ ... (i)

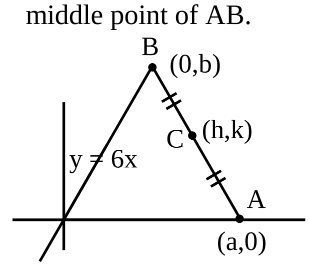
$$\text{centroid} = \left(\frac{h+x_1+0}{3}, \frac{k+y_1+0}{3}\right) = (5, 5)$$

$$\Rightarrow \frac{h+x_1}{3} = 5 \ \& \ \frac{k+y_1}{3} = 5$$

$\Rightarrow h + x_1 = 15$ & $k + y_1 = 15$
$\Rightarrow x_1 = 15 - h$ & $y_1 = 15 - k$
from (i)
$(15 - h)^2 + (15 - k)^2 = 169$
$\Rightarrow 225 + h^2 - 30h + 225 + k^2 - 30k = 169$
$\Rightarrow h^2 + k^2 - 30h - 30k + 281 = 0$
$\therefore$ locus is
$x^2 + y^2 - 30x - 30y + 281 = 0$

Q.7 A line AB of length 2λ moves with the end A always on the x-axis and the end B on the line $y = 6x$. Find the equation of the locus of the middle point of AB.

Sol.



$2h = b + a$ and $2k = 6a$

$\therefore$ a = k/3 and b = 2h – k/3

Θ $(a - b)^2 + 36b^2 = 4\lambda^2$

$$\therefore \left(\frac{2k}{3} - 2h\right)^2 + 36\left(\frac{k}{3}\right)^2 = 4\lambda^2$$

$$\left(\frac{k}{3} - h\right)^2 + k^2 = \lambda^2$$

$\therefore$ $9h^2 + 10k^2 - 6hk - \lambda^2 = 0$

$\therefore$ required locus is

$9x2 + 10y2 - 6xy - \lambda^2 = 0$

Q.8 Let a new distance d (P, Q) between the points P = (x_1, y_1) and Q = (x_2, y_2) be defined as

d (P, Q) = $|x_1 - x_2| + |y_1 - y_2|$. Let O = (0, 0) and A = (3, 2) be two fixed points. Let R = (x, y),

$x \geq 0$, $y \geq 0$ such that R is equidistant from the points O and A in the sense of the new distance. Prove that the locus of R consists of a line segment of finite length and an infinite ray.

Sol. Let P(x, y) be any point in the first quadrant. We have

d(P, 0) = |x – 0| + |y – 0| = |x| + |y| = x + y
(Θ x, y > 0)

d (P, A) = |x – 3| + |y –2| (given)

d (P, 0) = D(P, A) (given)

$\Rightarrow$ x + y = |x –3| + |y – 2| ... (i)

Case –I :

0 < x < 3, 0 < y < 2

In this case (i) becomes

x + y = 3 –x + 2 – y

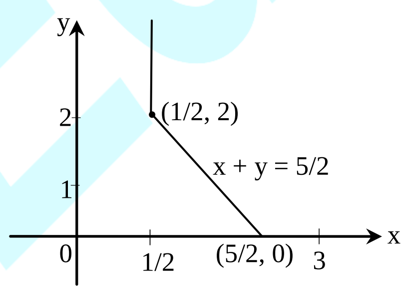
2x + 2y = 5

$\Rightarrow$ x + y = 5/2

Case- II :

0 < x < 3, y ≥ 2

Now, (i) becomes



$\Rightarrow$ x + y = 3 –x + y – 2 $\Rightarrow$ 2x = 1 $\Rightarrow$ x = ½

Case- III :

x ≥ 3, 0 < x < 2

Now (i) becomes x + y = x –3 + 2 – y

$\Rightarrow$ 2y = –1 or $y = -\frac{1}{2}$

Hence, no solution.

Case –IV :

x ≥ 3, y ≥ 2

In this case (i) changes to x + y = x – 3 + y – 2

$\Rightarrow$ 0 = –5

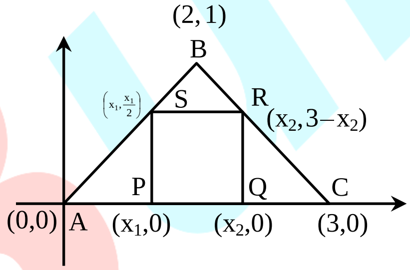
which is not possible. Hence the solution set is

{(x, y)| x = 1/2, y ≥ 2} ∪ {(x, y)}| x + y = 5/2, 0 < x < 3, 0 < y > 2}

Q.9 Find the coordinates of the vertices of a square inscribed in the triangle with vertices A(0,0),

B(2, 1), C(3, 0); given that two of its vertices are on the side AC.

Sol.



equation of AB = $y = \frac{1}{2}x_1$,

equation of BC : $y - 0 = \frac{1}{-1}(x - 3)$

y = 3 – x

Θ PQRS is a square

$\therefore$ PQ = SR $\Rightarrow$ $PQ^2 = SR^2$

$$(x_2 - x_1)^2 = (x_2 - x_1)^2 + (3 - x_2 - \frac{x_1}{2})^2$$

$$\Rightarrow x_1 + 2x_2 = 6 \Rightarrow x_2 = \frac{6 - x_1}{2}$$

$$\text{Now, } \frac{3 - x_2}{x_2 - x_1} \cdot \frac{\frac{x_1}{2}}{x_1 - x_2} = -1$$

$$\Rightarrow (3 - x_2)\frac{x_1}{2} = (x_1 - x_2)^2$$

$$\Rightarrow \left(3 - \frac{6 - x_1}{2}\right)\frac{x_1}{2} = \left(x_1 - \frac{6 - x_1}{2}\right)^2$$

$$\left(\frac{x_1}{2}\right)^2 = \left(\frac{3x_1 - 6}{2}\right)^2$$

$x_1 = 3x_1 - 6$	$x_1 = -3x_1 + 6$
$\Rightarrow 2x_1 = 6$	$4x_1 = 6$
$\Rightarrow x_1 = 3$	$x_1 = \frac{3}{2}$

(Impossible) $x_2 = 3 - \frac{3}{4} = \frac{9}{4}$

$\therefore$ vertices are : $\left(\frac{3}{2}, 0\right), \left(\frac{9}{4}, 0\right), \left(\frac{3}{2}, \frac{3}{4}\right), \left(\frac{9}{4}, \frac{3}{4}\right)$

Q.10 A straight line through A (–2, –3) cuts the lines $x + 3y = 9$ and $x + y + 1 = 0$ at B and C respectively. Find the equation of the line if AB . AC = 20.

Sol. Any point on this line is $(-2+r\cos\theta, -3+r\sin\theta)$

If $B \equiv (-2 + r_1\cos\theta, \; -3 + r_1\sin\theta)$

$c \equiv (-2 + r_2\cos\theta, -3 + r_2\sin\theta)$

then $AB = |r_1|$, $AC = |r_2|$

$\therefore -2 + r_1\cos\theta + 3(-3 + r_1\sin\theta) = 9$

$\Rightarrow r_1 = \frac{20}{\cos\theta + 3\sin\theta}$

and $-2 + r_2\cos\theta - 3 + r_2\sin\theta + 1 = 0$

$\Rightarrow r_2 = \frac{4}{\cos\theta + \sin\theta}$

$\therefore$ AB.AC = 20

$\therefore |r_1 r_2| = 20$

$\therefore r_1 r_2 = \pm 20$

$\therefore \frac{80}{(\cos\theta + 3\sin\theta)(\cos\theta + \sin\theta)} = \pm 20$

or $\cos^2\theta + 3\sin^2\theta + 4\sin\theta\cos\theta = 4$

(Negative sin is not possible)

or $1 + 3\tan^2\theta + 4\tan\theta = (1 + \tan^2\theta)$

or $\tan^2\theta - 4\tan\theta + 3 = 0 \;\therefore\; \tan\theta = 3, 1$

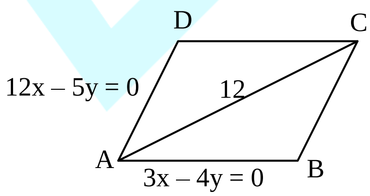
$\therefore$ Possible equation of the line is $y+3=3(x+2)$

or $y + 3 = 1(x + 1)$

Hence $3x - y + 3 = 0$ and $x - y - 2 = 0$

Q.11 Two sides of a rhombus, lying in the first quadrant, are given by $3x - 4y = 0$ & $12x - 5y = 0$. If the length of longer diagonal is 12, find the equations of the other two sides of the rhombus

Sol.



The lines AB & AD Intersect at A(0,0)

AC is the longer diagonal

AC is one of the bisectors of AB and AD

$\therefore \frac{3x - 4y}{5} = \pm \frac{12x - 5y}{13}$

$\Rightarrow 99x - 77y = 0$ and $21x + 27y = 0$

or $\frac{x}{7} = \frac{y}{9}$ it lies in Ist quadrant

$\tan\theta = 9/7$

The other bisector does not lie in Ist quadrant as it makes obtuse angle with x-axis.

$\tan\theta = -\frac{21}{27} = -7/9$

or $\frac{x}{7/\sqrt{130}} = \frac{y}{9/\sqrt{130}} = r = AC = 12$

$\therefore C = \left(\frac{84}{\sqrt{130}}, \frac{108}{\sqrt{130}}\right) = (x_1, y_1)$

Now write the equation of the lines through C which are parallel to given lines whose slopes are 3/4 and 12/5 by

$y - y_1 = m(x - x_1)$

Therefore $3x - 4y + \frac{180}{\sqrt{130}} = 0$ & $12x - 5y = \frac{468}{\sqrt{130}}$

Q.12 From a point (–2, 3) a ray of light is sent at an angle α ($\tan\alpha = 3$) to the axis of x. Upon reaching the x-axis the ray is reflected from it. Find the equation of the straight line which contains the reflected ray.

Sol. Incident ray $y - 3 = 3(x + 2)$, $\tan\alpha = 3$

surface is x-axis i.e. $y = 0$

Point P is (–3, 0) . Normal to surface is y-axis Incident ray makes and angle $90° - \alpha$ with normal y-axis and so will the reflected ray but in opposite sense.

Hence it will make an angle $-\alpha$ with x-axis where $\tan(-\alpha) = -\tan\alpha = -3$

Hence its equation is $y - 0 = -3(x + 3)$

or $y + 3x + 9 = 0$

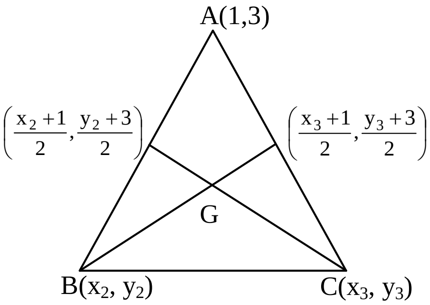
$\Rightarrow 3x + y + 9 = 0$

Q.13 Find the vertices of a ΔABC with A (1, 3) as a vertex and $x - 2y + 1 = 0$, $y - 1 = 0$ as the equations of two of its medians.

Sol. Let G is centroid,

solving $x - 2y + 1 = 0$ ....(1)

and $y - 1 = 0$ ....(2)



We get,

G(x, y) ≡ G(1, 1)

$$\frac{x_2 + x_3 + 1}{3} = 1 \qquad ....(3)$$

$$\frac{y_2 + y_3 + 3}{3} = 1 \qquad ....(4)$$

Mid point of AB, satisfies (1)

$$\left(\frac{x_2+1}{2}\right) - 2\left(\frac{y_2+3}{2}\right) + 1 = 0$$

$$x_2 - 2y_2 - 3 = 0 \qquad ....(5)$$

Similarly, mid point of AC satisfies (2)

$$\frac{y_3+3}{2} - 1 = 0$$

$y_3 = -1$

Put $y_3 = -1$ in (4),

$y_2 = 1$

Put $y_2 = 1$ in (5)

$x_2 = 5$

Put $x_2 = 5$ in (3)

$x_3 = -3$

so points are B(5, 1) & C(–3, –1)

Q.14 Two parallel straight line inclined at an angle of 135º to the axis of x meet the x-axis at the points A, B and y-axis at the points C and D respectively. Find the equation of the locus of point of intersection of AD and BC.

Sol. The lines makes an angle of 135º with x-axis and hence its slope is tan 135º i.e. –1

Let the two parallel lines with solpe–1 be x + y = a and x + y = b where a & b are parameters

Putting y = 0, we get the points A(a, 0), (B(b, 0)

putting x = 0, we get the points C(0,a), D (0, b)

equation to AD joining (a, 0)and (0, b) is

$$\frac{x}{a} + \frac{y}{b} = 1$$

equation to BC joining (b, 0) and (0, a) is

$$\frac{x}{b} + \frac{y}{a} = 1$$

In order to find the locus of the point of intersection of there lines, we have to eliminate the parameters a and b. subtracting, we get

$$x\left(\frac{1}{a} - \frac{1}{b}\right) + y\left(\frac{1}{b} - \frac{1}{a}\right) = 0$$

or x – y = 0

Q.15 The line 3x + 2y = 24 meets the y-axis at A and the x-axis at B. The perpendicular bisector of AB meets the line through (0, –1) parallel to x-axis at C. Find the area of the triangle ABC.

Sol. putting x = 0 in 3x + 2y = 24,

weget the point A(0,12)

putting y = 0 in 3x + 2y = 24, we get the point B(8, 0)

Mid point of AB is (4, 6)

Right bisector of AB is 2x – 3y + k = 0

where 2.4 – 3.6 + k = 0 ∴ k = 10

Hence its equation is

2x – 3y + 10 = 0 ....(i)

Any line parallel to x-axis is y = c. If its passes through (0, –1), then –1 = c and hence its equation is y = –1 ....(ii)

It meets (i) at c (–13/2, – 1)

Therefore three pointsA(0,12),B(8,0),C(–13/2,–1)

its Area

$$\frac{1}{2}\begin{vmatrix} 0 & 8 & -13/2 \\ 12 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-8\,(12 + 1) - \frac{13}{2}\,(12)\right]$$

$$= \frac{1}{2}\left[-104 - 78\right]$$

$$= \frac{1}{2} \times 182$$

= 91 sq. unuits

Q.16 If the line $\frac{x}{a} + \frac{y}{b} = 1$ moves in such a way that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ where c is a constant, prove that the foot of the perpendicular from the origin on the straight line describes the circle $x^2 + y^2 = c^2$.

Sol. Variable line is $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

Any line perpendicular to (i) and passing through the origin will be $\frac{x}{b} - \frac{y}{a} = 0$ ......(ii)

Now foot of the perpendicular from the origin to line (i) is the point of intersection of (i) and (ii) let it be P(α,β) then

$\frac{\alpha}{a} + \frac{\beta}{b} = 1$ ....(iii)

and $\frac{\alpha}{b} - \frac{\beta}{a} = 0$ ....(iv)

squaring and adding (iii) and (iv), we get

$$a^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right) + b^2\left(\frac{1}{b^2}+\frac{1}{a^2}\right) = 1$$

But $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ (given)

Here C is a constant and a,b are parameters (variables)

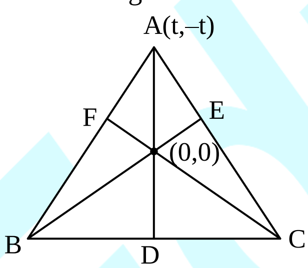
$\therefore (\alpha^2 + \beta^2).\frac{1}{c^2} = 1$

Hence the locus of P(α,β) is

$x^2 + y^2 = c^2$

Q.17 The equation of the altitude AD, BE, CF of a triangle ABC are x + y = 0, x – 4y = 0 and 2x – y = 0 respectively. The co-ordinates of A are (t, –t). Find coordinates of B and C. Prove that if t varies the locus of the centroid of the triangle ABC is x + 5y = 0.

Sol.



equation of Altitude AD is x + y = 0

equation of Altitude AD is x + 4y = 0

equation of Altitude AD is 2x – y = 0

Now find equation of AB which is ⊥r to CF and find equation of AC which is ⊥r to BE on solving we get B $\left(-\frac{2}{3}t, -\frac{1}{6}t\right)$ & and C $\left(\frac{t}{2}, t\right)$

If G be (x, y) then

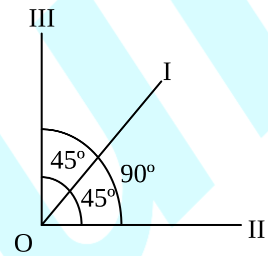
$$x = \frac{\sum x_1}{3} = \frac{5t}{18}$$

$$y = \frac{\sum y_1}{3} = -\frac{t}{18}$$

Now eliminating t, we get x + 5y = 0

Q.18 If the straight lines ax + by + p = 0 and x cos α + y sin α = p enclose an angle of π/4 between them and meet the line x sin α = y cos α in the same point, then show that $a^2 + b^2 = 2$.

Sol. Two of the lines are evidently perpendicular and the line ax + by + p = 0 makes an angle of 45º with one of them and hence it is bisector of the two lines.



∴ The bisectors of given lines are

$$\frac{x\cos\alpha + y\sin\alpha - p}{1} = \pm\frac{x\sin\alpha - y\cos\alpha}{1}$$

or x(cosα – sinα) + y(sinα + cosα) – p = 0

comparing with ax + by + p = 0

$$\frac{\cos\alpha - \sin\alpha}{a} = \frac{\sin\alpha + \cos\alpha}{b} = -1$$

– a = cosα – sinα ⇒ a = sinα – cosα

– b = sinα + cosα ⇒ b = – (sinα+cosα)

$\therefore a^2 + b^2 = (-\cos\alpha + \sin\alpha)^2 + (\sin\alpha + \cos\alpha)^2$

$= \sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha + \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha$

$= 2(\sin^2\alpha + \cos^2\alpha) = 2(1)$

= 2

$\therefore a^2 + b^2 = 2$

Q.19 Prove that all lines represented by the equation (2cosθ + 3sinθ) x + (3 cosθ – 5 sinθ)y – (5 cosθ – 2 sinθ) = 0 pass through a fixed point for all values of θ. Find the coordinates of this point and its reflection in the line x + y $= \sqrt{2}$.

Sol. The given equation can be written as

(2x + 3y – 5) cosθ + (3x – 5y + 2) sinθ = 0

or (2x + 3y – 5) + tanθ (3x – 5y + 2) = 0

This passes through the point of intersection of the lines. $2x + 3y - 5 = 0$ and $3x - 5y + 2 = 0$ for all values of θ. The co-ordinates of the points P of intersection are (1,1). Let Q(h, k) be the reflection of P(1, 1) in the line

$$x + y = \sqrt{2} \quad ....(i)$$

Then PQ is perpendicular to (i) and the mid point of PQ lies on (i)

$\therefore \dfrac{k-1}{h-1} = 1 \qquad \Rightarrow k = +h$

and $\dfrac{h+1}{2} + \dfrac{k+1}{2} = \sqrt{2}$

$\Rightarrow h = k = \sqrt{2} - 1$

Therefore (1, 1); $(\sqrt{2} - 1, \sqrt{2} - 1)$ are the required answers.

Q.20 A variable line through the point (6/5, 6/5) cuts the coordinate axes in the points A and B. If the point P divides AB internally in the ratio 2 : 1, show that the equation to the locus of P is $5xy = 2(2x + y)$.

Sol. Let The line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and since it passes through the point $\left(\dfrac{6}{5}, \dfrac{6}{5}\right) \therefore \dfrac{1}{a} + \dfrac{1}{b} = \dfrac{5}{6}$ ....(i)

It meets the axes at A(a, 0) and B(0, b).

Let (h, k) be the point which divides AB in the ratio 2 : 1 then

$h = \dfrac{2.0 + 1.a}{3} = \dfrac{a}{3}$, $k = \dfrac{2.b + 1.0}{3} = \dfrac{2b}{3}$

$\therefore$ a = 3h, b = 3k/2 putting in (i), weget

$\dfrac{1}{3h} + \dfrac{2}{3k} = \dfrac{5}{6}$

Generalising (h, k) the locus is

$\dfrac{y + 2x}{3xy} = \dfrac{5}{6}$ or $2(y + 2x) = 5xy$

Q.21 A variable straight line passes through a fixed point (h, k). Find the locus of the foot of the perpendicular on it drawn from the origin.

Sol. Equation of any straight line through (h, k) is $y - k = m(x - h)$ ....(i)

Hence m is the parameter.

Hence, equation of a line perpendicular to it and passing through the origin is

$$y - 0 = \frac{1}{m}(x - 0) ......(ii)$$

Let $p(\alpha,\beta)$be the foot of the perpendicular from the origin to line (i) clearly $P(\alpha,\beta)$ will be the point of intersection of lines (i) & (ii)

Since $P(\alpha,\beta)$ lies on line (i) & (ii)

$$\therefore \beta - k = m(\alpha - h) \quad .....(iii)$$

$$\text{and } \beta = -\frac{1}{m}\alpha \quad ....(iv)$$

we have to eliminate m,

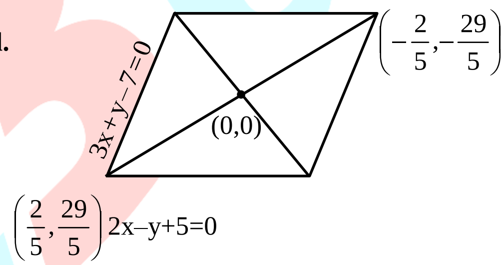
form (iv), $m = -\dfrac{\alpha}{\beta}$ and subtituting it in (iii) the required locus is given by

$y - k = -\dfrac{x}{y}(x - h)$

or $x^2 + y^2 = hx + ky$ or $x^2 + y^2 - hx - ky = 0$

Q.22 Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.

Sol.



$6x^2 - xy - y^2 + x + 12y - 35 = 0$

i.e., $(2x - y + 5)(3x + y - 7) = 0$

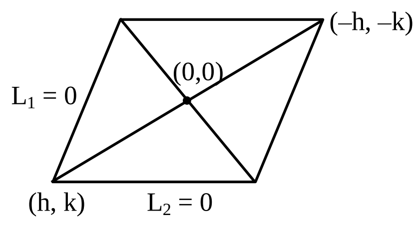
equation of other sides are

$(2x - y - 5)(3x + y + 7) = 0$

i.e., $6x^2 - xy - y^2 - x - 12y - 35 = 0$

Alternate - Let $L_1L_2 \equiv 6x^2 - xy - y^2 + x + 12y - 35 = 0$

Replacing (x, y) by (– x, – y) gives the solution



Q.23 Prove that the straight lines joining the origin to the points of intersection of the line $7x - y + 2 = 0$ and the curve $2x^2 + y^2 + x + y = 0$ are at right angles to one another.

Sol. $2x^2 + y^2 + (x + y)\dfrac{(7x - y)}{-2} = 0$

$\left(2-\frac{7}{2}\right)x^2 + \left(1+\frac{1}{2}\right)y^2 - 3xy = 0$

Here co-eff x^2 + co-eff y^2

$= 2 - \frac{7}{2} + 1 + \frac{1}{2} = 0$

Shows that the line joining origin to the intersection pts. of given curves subtend a right angle at the origin.

Q.24 Find the equations of the pair of lines both of which pass through the point (1, 2) and are parallel to the bisectors of the angles between the lines given by $x^2 + xy - 2y^2 + 4x - y + 3 = 0$.

Sol. $x^2 + xy - 2y^2 + 4x - y + 3 = 0$

i.e., $(x - y + 1)(x + 2y + 3) = 0$

equation of bisectors are

$$(x - y + 1)\sqrt{5} = \pm\sqrt{2}\,(x + 2y + 3)$$

i.e., $(\sqrt{5}-\sqrt{2})x - (\sqrt{5}+2\sqrt{2})y + \sqrt{5}-3\sqrt{2} = 0$ ....(1)

& $(\sqrt{5}+\sqrt{2})x - (\sqrt{5}-2\sqrt{2})y + \sqrt{5}+3\sqrt{2} = 0$ ....(2)

equation of line passes through (1, 2) and parallel to

(i) L_1 is ...... $(\sqrt{5}-\sqrt{2})x - (\sqrt{5}+2\sqrt{2})y + (\sqrt{5}+5\sqrt{2}) = 0$

(ii) L_2 is ...... $(\sqrt{5}+\sqrt{2})x - (\sqrt{5}-2\sqrt{2})y + (\sqrt{5}-5\sqrt{2}) = 0$

$\therefore\ L_1L_2 \equiv x^2 -.\ 6xy - y^2 + 10x + 10y - 15 = 0$

Q.25 A variable line L passing through the point B(2, 5) intersects the lines $2x^2 - 5xy + 2y^2 = 0$ at P and Q. Find the locus of the point R on L such that distances BP, BR and BQ are in harmonic progression.

Sol. We have, $2x^2 - 5xy + 2y^2 = 0$

$\Rightarrow (2x - y)(x - 2y) = 0$

$2x - y = 0$ and $x - 2y = 0$

So the equation of two lines are

$$2x - y = 0 \ \&\ x - 2y = 0 \qquad ....(i)$$

Let the equation of line L passing through B(2, 5) be

$\frac{x-2}{\cos\theta} = \frac{y-5}{\sin\theta}$

let $BP = r_1$, $BR = r_2$ and $BQ = r_3$

Then co-ordinates of P,R and Q are given by

$\frac{x-2}{\cos\theta} = \frac{y-5}{\sin\theta} = r_1$; $\frac{x-2}{\cos\theta} = \frac{y-5}{\sin\theta} = r_2$

and $\frac{x-2}{\cos\theta} = \frac{y-5}{\sin\theta} = r_3$ respectively.

$\therefore$ $P(2 + r_1\cos\theta, 5 + r_1\sin\theta)$, $R(2 + r_2\cos\theta, 5 + r_2\sin\theta)$ and $Q(2 + r_3\cos\theta, 5 + r_3\sin\theta)$ respectively

we have to find the locus of point R, so let its co-ordinate be (h, k) then $h = 2 + r_2\cos\theta$,

$k = 5 + r_2\sin\theta$

$\Rightarrow \frac{h-2}{r_2} = \cos\theta$, $\frac{k-5}{r_2} = \sin\theta$

$\because$ P & Q lie on $2x - y = 0$ & $x - 2y = 0$

$\Rightarrow 4 + 2r_1\cos\theta - 5 - r_1\sin\theta = 0$

and $2 + r_3\cos\theta - 10 - 2r_3\sin\theta = 0$

$\Rightarrow r_1(2\cos\theta - \sin\theta) = 1$ and $r_3(\cos\theta - 2\sin\theta) = 8$

$\Rightarrow \frac{r_1}{r_2}(2h - 4 - k + 5) = 1$

and $\frac{r_3}{r_2}(h - 2 - 2k + 10) = 8$

$\Rightarrow \frac{r_1}{r_2} = \frac{1}{2h-k+1}$ and $\frac{r_3}{r_2} = \frac{8}{h-2k+8}$

It is given that $BP = r_1$, $BR = r_2$ & $BQ = r_3$ are in H.P. therefore $\frac{2}{r_2} = \frac{1}{r_1} + \frac{1}{r_3}$

$\Rightarrow 2 = \frac{r_2}{r_1} + \frac{r_2}{r_3}$

$\Rightarrow 2 = 2h - k + 1 + \frac{h-2k+8}{8}$

$\Rightarrow 16 = 17h - 10k + 16$

$\Rightarrow 17h = 10k \Rightarrow 17h - 10k = 0$

$\therefore$ locus is $17x - 10y = 0$

Part-B Passage based objective questions

Passage-I (Q. 26 to 28)

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points E(3, 4) and F(1, 2) respectively and centroid of ΔABC is G(3, 2), then

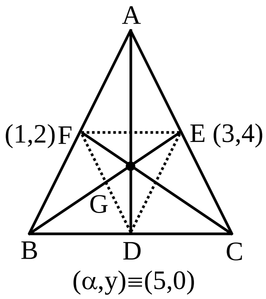
On the basis of above passage, answer the following questions :

Q.26 The equation of side AB is -

(A) $2x + y = 4$ (B) $x + y - 3 = 0$

(C) $4x - 2y = 0$ (D) None of these

Sol.[A]



Let co-ordinate of D is (x, y)

centroid of ΔABC will be the same as centroid of the ΔDEF

∴ centroid of ΔDEF is given by

$\frac{1+3+x}{3} = 3$ and $\frac{2+4+y}{3} = 2$

$\Rightarrow 4 + x = 9$ and $6 + y = 6$

$\Rightarrow x = 5$ and $y = 0$

∴ co-ordinate of D is (5, 0)

since side AB is parallel to DE

∴ slope of AB = slope of DE = $\frac{4}{-2} = -2$

∴ equation of AB which passes through (1,2) is

$y - 2 = -2(x - 1)$

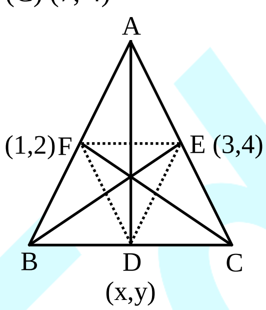
$\Rightarrow y - 2 = -2x + 2 \Rightarrow 2x + y - 4 = 0 \Rightarrow 2x + y = 4$

Q.27 Co-ordinate of D are -

(A) (7, –4) (B) (5, 0)

(C) (7, 4) (D) (–3, 0)

Sol.[B]



Let co-ordinate of point D is (x, y)

we know that centroid of ΔABC will be the same as that of centroid of ΔDEF

∴ centroid of ΔDEF is

$$\left(\frac{1+3+x}{3}, \frac{2+4+y}{3}\right) \equiv (3, 2)$$

$\Rightarrow \frac{4+x}{3} = 3$ and $\frac{6+y}{3} = 2$

$\Rightarrow 4 + x = 9$ and $6 + y = 6$

$\Rightarrow x = 5$ and $y = 0$

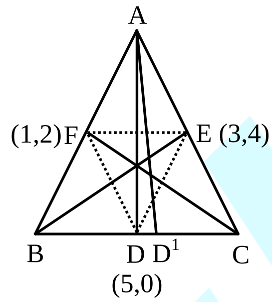
⇒ co-ordinate of D is (5, 0)

Q.28 Height of altitude drawn from point A is (in units) -

(A) $4\sqrt{2}$ (B) $3\sqrt{2}$

(C) $6\sqrt{2}$ (D) $2\sqrt{3}$

Sol.[C]



Let height of altitude = h

$$\text{Area of } \Delta DEF = \frac{1}{2}\begin{vmatrix} 1 & 3 & 5 \\ 2 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 6 \text{ sq.unit}$$

∴ Area of ΔABC = 4 × 6 = 24 sq.units

Base BC = 2 × FE = 2 × $2\sqrt{2}$ = $4\sqrt{2}$

∴ Area = $\frac{1}{2}$ × base × height (Altitude)

$24 = \frac{1}{2} \times 4\sqrt{2} \times h$

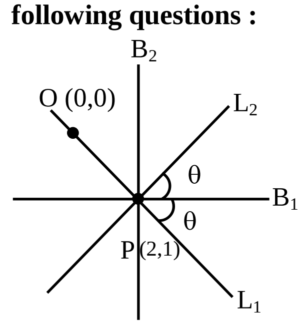
$h = \frac{12}{\sqrt{2}} = 6\sqrt{2}$

Passage-II (Q. 29 to 31)

Let $B_1 \equiv 3x + 4y - 10 = 0$ and $B_2 \equiv 4x - 3y - 5 = 0$ are the bisectors of angle between lines $L_1 = 0$ and $L_2 = 0$. If L_1 passes through origin and $L_2 = 0$ meets the curve $y^2 = 4ax$ at A and B and P is the point of intersection of $L_1 = 0$ and $L_2 = 0$ then

On the basis of above passage, answer the following questions :

Sol.



Point of intersection of

$B_1 \equiv 3x + 4y - 10 = 0$

and $B_2 \equiv 4x - 3y - 5 = 0$ is

P(2, 1)

Now L_1 passes through O(0, 0),

So slope of line $L_1 : m_1 = \frac{1-0}{2-0} = \frac{1}{2}$

Q.29 Equation of line $L_2 = 0$ is-

(A) $11x + 2y = 24$ (B) $11x - 2y = 20$

(C) $5x + 3y = 13$ (D) None of these

Sol. [B] $B_1 \equiv 3x + 4y - 10 = 0$ ....(1)

$B_2 \equiv 4x - 3y - 5 = 0$ .....(2)

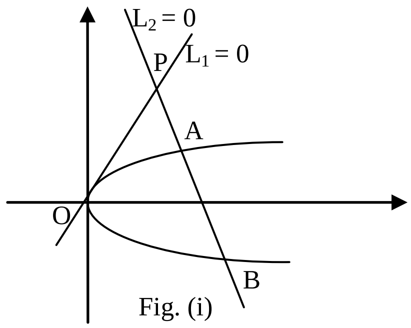


Fig. (i)

Solving (1) & (2)

We will get P

So from (1) & (2)

$P(a, b) \equiv (2, 1)$

Slope of L_1 is $= \frac{1-0}{2-0} = \frac{1}{2}$

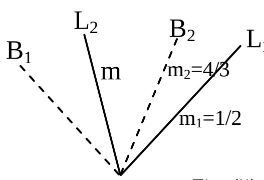


Fig. (ii)

Now B_2 is bisector of L_1 & L_2

So use $\frac{m - m_2}{1 + mm_2} = \frac{m_2 - m_1}{1 + m_1 m_2}$

we get m = 11/2

$\therefore \quad y - 1 = \frac{11}{2}(x - 2)$

$11x - 2y = 20$

Q30 If AB subtends 90º angle at origin, then a is equal to-

(A) $\frac{4}{11}$ (B) $\frac{3}{11}$

(C) $\frac{5}{11}$ (D) None of these

Sol. [C] Use homogenisation as

$\frac{11x - 2y}{20} = 1$

Put in $y^2 = 4ax \times 1$

We get

$y^2 = 4ax\ \frac{(11x - 2y)}{20}$

$\Rightarrow 5y^2 = 11ax^2 - 2axy$

$11ax^2 - 5y^2 - 2axy = 0$

for 90º

coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad 11a - 5 = 0$

$\Rightarrow \quad a = \frac{5}{11}$

Q.31 If $a = \frac{1}{4}$, then absolute value of product PA.PB is equal to-

(A) $\frac{2\sqrt{2}}{11}$ (B) $\frac{125}{121}$

(C) $\frac{3\sqrt{24}}{5}$ (D) None of these

Sol. [B] use $a = \frac{1}{4}$

$y^2 = 4 \times \frac{1}{4} \times x \Rightarrow y^2 = x$

using fig.(i)

Let any point on L_2 & $y^2 = 4ax$ is

$h = 2 + r\cos\theta$

$k = 1 + r\sin\theta$

& slope of L_2 is $\tan\theta = \frac{11}{2}$

$\Rightarrow \sin\theta = \frac{11}{\sqrt{125}}$

$\cos\theta = \frac{2}{\sqrt{125}}$

Put (h, k) is

$y^2 = x$

$(1 + r\sin\theta)^2 = 2 + r\cos\theta$

$r^2\sin^2\theta + r(2\sin\theta - \cos\theta) - 1 = 0$

Let r_1, r_2 are roots

$\Rightarrow \quad |r_1 r_2| = \frac{1}{\sin^2\theta} = \frac{125}{121}$

$\therefore$ PA.PB $= \frac{125}{121}$

Passage-III (Q. 32 to 34)

A (x_1, y_1), $B(x_2, y_2)$, $(y_1 < y_2)$ are two points on the line $x + y = 4$ from which perpendicular AQ and BP are drawn on line $4x + 3y = 10$ where P and Q are the feet of perpendiculars such that AQ = BP = 1. Now considering AB as diameter of a circle is drawn which meets the line $4x + 3y = 10$ at C and D such that C is closer to P.

On the basis of above passage, answer the following questions :

Q.32 The value of $\dfrac{y_1+y_2}{x_1+x_2}$ is equal to-

(A) 4 (B) –3

(C) –4 (D) None of these

Sol. **[B]**

Given AQ = BP = 1

So OA = OB

$\Rightarrow$ O is the mid point of AB.

so $\dfrac{y_1+y_2}{x_1+x_2} = \dfrac{\left(\dfrac{y_1+y_2}{2}\right)}{\left(\dfrac{x_1+x_2}{2}\right)} = \dfrac{6}{-2} = -3$

Q.33 The length PQ is equal to-

(A) $3\sqrt{14}$ (B) $4\sqrt{5}$

(C)14 (D) None of these

Sol. **[D]**

$$\tan\theta = \frac{-1-\left(\dfrac{-4}{3}\right)}{1+(-1)\left(\dfrac{-4}{3}\right)} = \left(\frac{1}{7}\right)$$

$\dfrac{MQ}{AQ} = \cot\theta = 7$

so MQ = 7

PQ = 2MQ = 14

Q.34 Length QD is equal to-

(A) $3\sqrt{2} + \sqrt{17}$ (B) $3\sqrt{2} - \sqrt{17}$

(C) $4\sqrt{2} + \sqrt{17}$ (D) $5\sqrt{2} - 7$

Sol. **[D]**

MD = MA

$= \sqrt{\left(\dfrac{14}{2}\right)^2 + 1^2}$

MD = $\sqrt{50} = 5\sqrt{2}$

and QD = MD – MQ

QD = $5\sqrt{2} - 7$

Passage-IV (Q. 35 to 37)

Let ABC be a triangle whose vertex A is (3, 4). $L_1 = 0$ and $L_2 = 0$ are the angle bisectors of angles B and C respectively where $L_1 \equiv x + 2y - 5 = 0$, $L_2 \equiv x - 2y - 3 = 0$.

On the basis of above passage, answer the following questions:

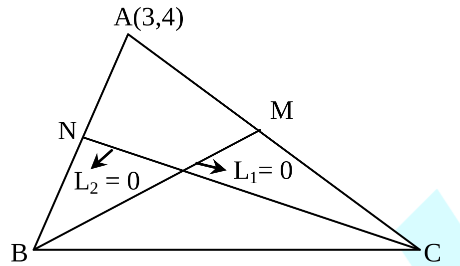
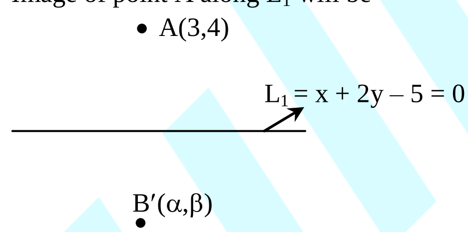


Image of A about L_1 & L_2 will lie on BC

$L_1 = x + 2y - 5 = 0$ & $L_2 = x - 2y - 3 = 0$

Image of point A along L_1 will be



then, $\left(\dfrac{\alpha+3}{2}, \dfrac{\beta+4}{2}\right)$ satisfies equation

$\therefore \dfrac{\alpha+3}{2} + \beta + 4 = 5$

$\alpha + 3 + 2\beta + 8 = 10$

$\alpha + 2\beta = -1$

...(1)

Slope of AB′ = $\dfrac{\beta-4}{\alpha-3}$

Slope of $L_1 = -\dfrac{1}{2}$

$\left(\dfrac{\beta-4}{\alpha-3}\right)\left(-\dfrac{1}{2}\right) = -1$

$\beta - 4 = 2\alpha - 6$

$2\alpha - \beta - 6 + 4 = 0$

$2\alpha - \beta = 2$ ....(2)

Solving equation (1) & (2)

$$\begin{aligned} 2\alpha + 4\beta &= -2 \\ 2\alpha - \beta &= 2 \\ - \quad + \quad & \quad - \\ \hline 5\beta &= -4 \end{aligned}$$

$\Rightarrow \beta = \dfrac{-4}{5}$

Putting value in (1)

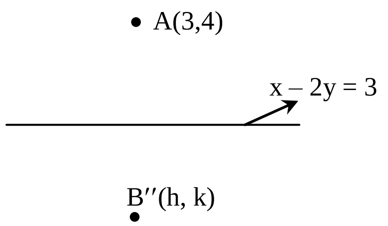
$\alpha + 2\beta = -1$

$\alpha - \dfrac{8}{5} = -1$

$\alpha = \dfrac{3}{5}$

$\therefore$ This point lie on 'BC'

Similarly image of point A along CN



$\left(\frac{h+3}{2}, \frac{k+4}{2}\right)$ will satisfy equation

$\therefore \frac{h+3}{2} - k - 4 = 3$

$h + 3 - 2k - 8 = 6$

$h - 2k - 5 = 6$

$h - 2k = 11$ ....(A)

Slope of AB′′ $\left(\frac{k-4}{h-3}\right)$

$\frac{k-4}{h-3} = -2$

$\Rightarrow 2h + k = 10$ ....(B)

Multiplying eq. (A) by (2)

$$\begin{array}{r} 2h - 4k = 22 \\ 2h + k = 10 \\ - \quad - \quad - \\ \hline -5k = 12 \end{array}$$

$\Rightarrow k = -\frac{12}{5}$

$\therefore h = \frac{31}{5}$

Q.35 Slope of side BC is-

(A) $\frac{1}{15}$ (B) $\frac{2}{15}$

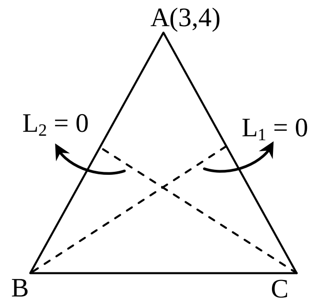
(C) $\frac{-2}{7}$ (D) None of these

Sol. Slope will be $\frac{y_2 - y_1}{x_2 - x_1} = -\frac{2}{7}$

Q.36 Co-ordinate of point B is-

(A) $\left(\frac{73}{5}, \frac{-24}{5}\right)$ (B) $\left(\frac{21}{5}, \frac{29}{10}\right)$

(C) $\left(\frac{110}{3}, \frac{203}{5}\right)$ (D) None of these



$L_1 = x + 2y = 5$

$10x + 20y = 50$

Equation of BC

point $\left(\frac{3}{5}, \frac{-4}{5}\right)$ passes through BC & slope of BC is $-\frac{2}{7}$.

$\therefore$ equation of BC

$y + \frac{4}{5} = -\frac{2}{7}\left(x - \frac{3}{5}\right)$

$\frac{5y+4}{5} = \frac{-2(5x-3)}{7 \times 5}$

$35y + 28 = -10x + 6$

$10x + 35y + 22 = 0$ ....(A_1)

solve with L_1 we get

$x = \frac{73}{5}, y = -\frac{24}{5}$

Q.37 Distance AI is equal to (where I = Incentre) -

(A) $\frac{\sqrt{51}}{2}$ (B) $\frac{\sqrt{53}}{2}$

(C) $\frac{\sqrt{55}}{2}$ (D) None of these

Sol. Solve L_1 & L_2 we get I $\left(4, \frac{1}{2}\right)$

$\therefore AI = \frac{\sqrt{53}}{2}$

EXERCISE # 4

Old IIT-JEE questions

Q.1 Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1,–1) and parallel to PS is- **[IIT-Screening-2000]**

(A) $2x - 9y - 7 = 0$ (B) $2x - 9y - 11 = 0$
(C) $2x + 9y - 11 = 0$ (D) $2x + 9y + 7 = 0$

Sol. **[D]**

S is the mid point of Q and R

$$\Rightarrow S \equiv \frac{7+6}{2}, \frac{3-1}{2} = \frac{13}{2}, 1$$

Now slope of PS = $m = \dfrac{2-1}{2-\dfrac{13}{2}}$

$$= -\frac{2}{9}$$

Now equation of the line passing through (1,–1) and parallel to PS is

$$y + 1 = -\frac{2}{9}(x - 1)$$

or $2x + 9y + 7 = 0$

Q.2 Find the number of integer value of m which makes the x coordinates of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ integer. **[IIT-Screening-2001]**

(A) 2 (B) 0 (C) 4 (D) 1

Sol. **[A]**

$3x + 4y = 9$
$y = mx + 1$ } solving for x,

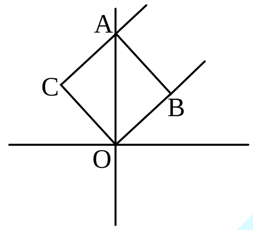
$$x = \frac{5}{3+4m}$$

Now, for x to be an integer,
$3 + 4m = \pm 5$ or ± 1
The integral values of m satisfying there conditions are – 2 and – 1 .
$\therefore$ number of integer values = 2

Q.3 Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$, $y = nx + 1$ is- **[IIT-Screening-2001]**

(A) $|m + n| / (m - n)^2$ (B) $2 / |m + n|$
(C) $1 / |m + n|$ (D) $1 / |m - n|$

Sol. **[D]**



Let OB : $y = mx$
CA : $y = mx + 1$
BA : $y = nx + 1$
OC : $y = nx$

The point of intersection B of OB and AB has x co-ordinate equal to $\dfrac{1}{m-n}$

Now, area of a parallelogram
OBAC = 2 × area of ΔOBA

$$= 2 \times \frac{1}{2} \times OA \times DB$$

$$= 2 \times \frac{1}{2} \times \frac{1}{m-n}$$

$$= \frac{1}{m-n} = \frac{1}{|m-n|}$$

depending upon whether m > r or n > m

$\therefore \dfrac{1}{|m-n|}$ is the correct answer.

Q.4 A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at the points P and Q respectively. Then the point O divides the segment PQ in the ratio- **[IIT-Screening-2002]**

(A) 1 : 2 (B) 3 : 4 (C) 2 : 1 (D) 4 : 3

Sol. **[B]**

The ratio will be same if we take the perpendicular line segment.

Now, distance of origin from $4x + 2y - 9 = 0$

is $\dfrac{|-9|}{\sqrt{4^2+2^2}} = \dfrac{9}{\sqrt{20}}$

and distance of origin from $2x + y + 6 = 0$ is

$$\frac{|6|}{\sqrt{2^2+1^2}} = \frac{6}{\sqrt{5}}$$

$$\therefore \text{Reqd. ratio} = \frac{9/\sqrt{20}}{6/\sqrt{5}} = \frac{3}{4} = 3 : 4$$

Q.5 Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is - **[IIT-Screening-2002]**

(A) $(\sqrt{3}/2)x + y = 0$ (B) $x + \sqrt{3}\,y = 0$

(C) $\sqrt{3}\,x + y = 0$ (D) $x + (\sqrt{3}/2)\,y = 0$

Sol. **[C]**

The line segment QR makes an angle of 60º with the positive direction of x-axis. Therefore the bisector of the angle PQR will make an angle of 60º with the negative direction of x-axis, It will therefore have angle of inclination of 120º and so its equation is

$y – 0 = \tan 120º (x – 0)$

$\Rightarrow y = -\sqrt{3}\,x$

$\Rightarrow y + \sqrt{3}\,x = 0$

Q.6 Let $0 < \alpha < \pi/2$ be a fixed angle. If P = (cos θ, sin θ) and Q=(cos (α–θ), sin (α–θ)) then Q is obtained from P by- **[IIT-Screening-2002]**

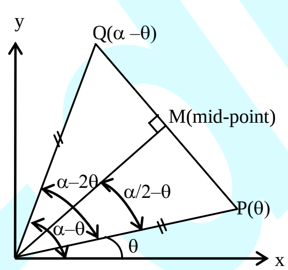
(A) clockwise rotation around origin through an angle α

(B) anticlockwise rotation around origin through an angle α

(C) reflection in the line through origin with slope tan α

(D) reflection in the line through origin with slope tan α/2

Sol. **[D]**



Clearly OP = OQ = 1 and ∠QOP = α – θ – θ = α – 2θ

The bisector of ∠QOP will be a perpendicular to PQ and also bisect it. Hence Q is reflection of P in the line OM which makes an angle ∠MOP + ∠POX with x-axis

i.e. $\frac{1}{2}(\alpha - 2\theta) + \theta = \frac{\alpha}{2}$

So that slope of OM is $\tan\frac{\alpha}{2}$

Q.7 A straight line L through the origin meets the lines x + y = 1 and x + y = 3 at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to 2x – y = 5 and 3x + y = 5 respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **[IIT-2002]**

Sol. Let the equation of straight line L be y = mx

$P \equiv \left(\frac{1}{m+1}, \frac{m}{m+1}\right) \Rightarrow Q \equiv \left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$

Now, equation of L_1 : $y - 2x = \frac{m-2}{m+1}$ ……..(i)

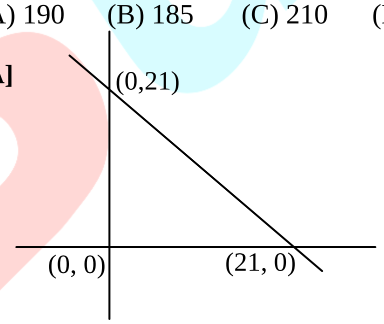
Equation of L_2 : $y + 3x = \frac{3m+9}{m+1}$ …....(ii)

By eliminating 'm' from equation (i) & (ii), weget locus of R as x – 3y + 5 = 0 which represents a straight line.

Q.8 No. of points with integer coordinates lie inside the triangle whose vertices are (0, 0), (0, 21), (21, 0) is: **[IIT Screening 2003]**

(A) 190 (B) 185 (C) 210 (D) 230

Sol. **[A]**



Equation of line joining (0, 21) and (21, 0) is

x + y =21

for interior points

x + y < 21 and x ≠ 0, y ≠ 0

Θ x + y < 21

⇒ x = 1, y = 19

2, 18

3, 17

⋮ ⋮

⋮ ⋮

⋮ ⋮

19 1

⇒ Σ 1 + 2 + 3 + ...+ 19

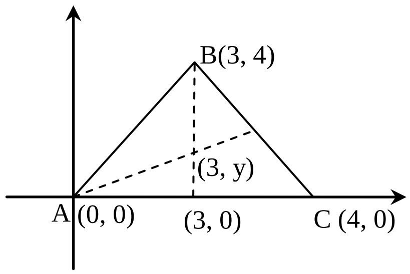
$\Rightarrow \frac{19}{2}[1 + 19] = \frac{19}{2}[20] = 190$

Q.9 Orthocentre of the triangle whose vertices are A (0, 0), B (3, 4) & C (4, 0) is **[IIT Scr. 2003]**

(A) $\left(3, \frac{3}{4}\right)$ (B) $\left(3, \frac{5}{4}\right)$

(C) (3, 12) (D) (2, 0)

Sol. **[A]**



y = mx where m = $\frac{1}{\text{slope of BC}}$

m = $\frac{1}{4}$

y = $\frac{1}{4}$ x

y = $\frac{3}{4}$

Θ x = 3

∴ orthocentre = $\left(3, \frac{3}{4}\right)$

Q.10 A pair of straight line $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ are forming a square. What is the centre of circle inscribed in the square **[IIT-Screening-2003]**

(A) (3, 2) (B) (7, 4) (C) (4, 7) (D) (0, 1)

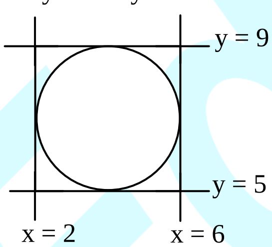
Sol. **[C]**

The lines given by $x^2 - 8x + 12 = 0$

are x = 2 & x = 6

the lines given by $y^2 - 14y + 45 = 0$

are y = 5 & y = 9



Centre of the required circle is the centre of the square.

∴ Required centre is

$$\left(\frac{2+6}{2}, \frac{5+9}{2}\right) = (4, 7)$$

Q.11 Area of the triangle formed by the line x + y = 3 and the angle bisector of the pair of lines $x^2 - y^2 + 2y = 1$ is – **[IIT-Screening-2004]**

(A) 1 (B) 3 (C) 2 (D) 4

Sol. **[C]**

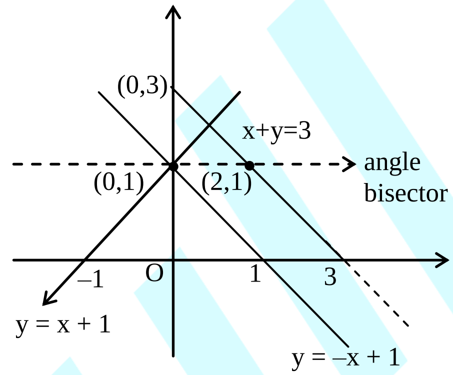
Here $x^2 - y^2 + 2y = 1$

$\Rightarrow x^2 - (y^2 - 2y + 1) = 0$

$\Rightarrow x^2 = (y - 1)^2$

$\Rightarrow x = y - 1$ or $x = - y + 1$

Which could be graphically as shown in figure



Which gives angle bisector as

y = 1 and x = 0

∴ Area of region bounded by

x + y = 3, x = 0 and y = 1

$= \frac{1}{2} \times 2 \times 2 = 2$

= 2 sq. unit.

Q.12 A line passes through the point P(h, k) is parallel to the x- axis. It forms a triangle with the lines y = x & x + y = 2 of area $4h^2$ then find the locus of P. **[IIT 2005]**

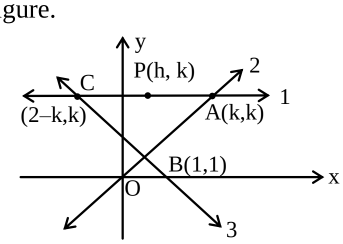
Sol. A line passing through P(h, k) and parallel to x-axis is y = k ……………(i)

The other two lines given are

y = x …….(ii)

and x + y = 2 ……..(iii)

Let ABC be the Δ formed by the points of intersection of the lines (i), (ii) and (iii) as shown in the figure.



Then A(k,k), B(1,1) and C(2–k, k)

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$$

Operating $C_1 - C_2$, weget

$$\frac{1}{2}\begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$$

$\Rightarrow \frac{1}{2}|(2-2k)(k-1)| = 4h^2$

$\Rightarrow (k-1)^2 = 4h^2 \Rightarrow k - 1 = \pm 2h$

$\Rightarrow k - 1 = 2h$ or $k - 1 = -2h$

$\Rightarrow k = 2h + 1$ or $k = -2h + 1$

$\therefore$ Locus of P(h, k) is $y = 2x + 1$ or $y = -2x + 1$

Q.13 If $f(x) = \min\{1, x^2, x^3\}$, then **[IIT-2006]**

(A) $f'(x) > 0 \ \forall \ x \in R$

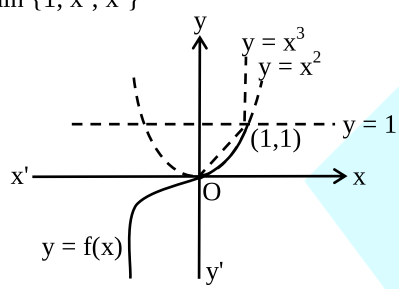
(B) f(x) is continuous $\forall \ x \in R$

(C) f(x) is not differentiable for two values of x

(D) f(x) is not differentiable but continuous $\forall \ x \in R$

Sol. **[B,D]**

$f(x) = \min\{1, x^2, x^3\}$



from graph f(x) is continuous every where but not differentiable at x = 1

Hence Option (B) & (D) are correct

Alternate : obviously $f(x) = \begin{cases} x^3, & x \le 1 \\ 1, & x > 1 \end{cases}$

$\therefore \lim_{x\to 1} f(x) = 1 = f(1)$ so f(x) is continuous at x = 1 and as such f(x) is continuous $\forall \ x \in R$

Further we note that $f'(1-0) = 3$ and $f'(1+0) = 0$

$\Rightarrow$ f(x) is not differentiable at x = 1

Also $f'(x)$ exists $\forall \ x \in R, x \ne 1$

Q.14 Given an isosceles triangle, whose one angle is 120° and in-radius is $\sqrt{3}$. So the area of triangle is - **[IIT-2006]**

(A) 4π (B) $12 + 7\sqrt{3}$

(C) $7 + 12\sqrt{3}$ (D) $12 - 7\sqrt{3}$

Sol. **[B]**

Area $= \frac{\sqrt{3}}{a} b^2 = \Delta$ ... (1)

By sine rule

$\frac{\sin 120^\circ}{a} = \frac{\sin 30^\circ}{b} \Rightarrow a = b\sqrt{3}$ ... (2)

Now $r = \frac{\Delta}{s} \Rightarrow \sqrt{3} = \frac{\Delta}{s}$ ... (3)

from (1), (2) and (3)

$\Delta = 12 + 7\sqrt{3}$

Q.15 Let O (0, 0), P(3, 4), Q (6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are **[IIT-2007]**

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$

(C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

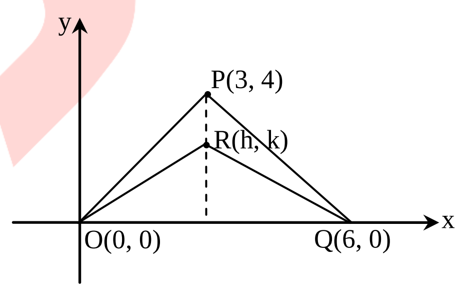
Sol. **[C]**

Area of $\Delta OPQ = \frac{1}{2}(6)(4) = 12$

Let co-ordinate of R be (h, k)

Area of $\Delta OQR = \frac{1}{3}$ area of ΔOPQ

$\Rightarrow \frac{1}{2}(6)(k) = \frac{1}{3}(12) \Rightarrow k = \frac{4}{3}$



Area of $\Delta OPR = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ h & k & 1 \\ 3 & 4 & 1 \end{vmatrix} = \frac{1}{2}(4h - 3k)$

$\Rightarrow \frac{1}{2}(4h - 3k) = \frac{1}{3}(12)$

$\Rightarrow 4h - 4 = 8$

$\Rightarrow h = 3$

$\therefore$ Thus, required point is $R\left(3, \frac{4}{3}\right)$

Q.16 Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[IIT-2007]**

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

because

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is True, Statement–2 is True; Statement–2 is not a correct explanation for Statement–1

(C) Statement–1 is True, Statement–2 is False

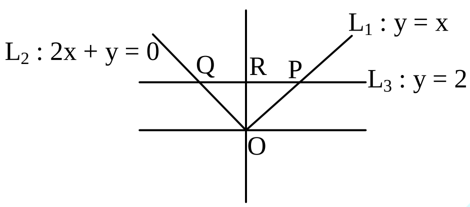
(D) Statement–1 is False, Statement–2 is True

Sol. **[C]**

P and Q are easily found as (–2, 2) and (1, – 2)

$\Rightarrow$ OP = $\sqrt{8}$ & OQ = $\sqrt{5}$

By bisector property,



$$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{8}}{\sqrt{5}}$$

$$= \frac{2\sqrt{2}}{\sqrt{5}}$$

$$= 2\sqrt{2} : \sqrt{5}$$

$\Rightarrow$ Statement(1) is clearly true.

The falsity of statement(2) follows from plane geometry.

Q. 17 Consider three points P = (– sin (β – α), – cos β), Q = (cos (β – α), sin β) and R = (cos (β – α + θ), sin (β – θ)), where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then - **[IIT 2008]**

(A) P lies on the line segment RQ

(B) Q lies on the line segment PR

(C) R lies on the line segment QP

(D) P, Q, R are non-collinear

Sol. **[D]**

P = (– sin (β – α), – cos β)

Q = (cos (β – α), sin β)

R = (cos (β – α + θ), sin (β – θ))

given $0 < \alpha, \beta, \theta < \frac{\pi}{4}$

Slope of QR

$$m_1(\text{let}) = \frac{\sin(\beta-\theta)-\sin\beta}{\cos(\beta-\alpha+\theta)-\cos(\beta-\alpha)}$$

$$= \frac{2\cos\left(\beta-\frac{\theta}{2}\right).\sin\left(\frac{-\theta}{2}\right)}{2\sin\left(\beta-\alpha+\frac{\theta}{2}\right).\sin\left(\frac{-\theta}{2}\right)}$$

$$m_1 = \frac{\cos\left(\beta-\frac{\theta}{2}\right)}{\sin\left(\beta-\alpha+\frac{\theta}{2}\right)}$$

Slope of PQ

$$m_2(\text{let}) = \frac{\sin\beta-(-\cos\beta)}{\cos(\beta-\alpha)-[-\sin(\beta-\alpha)]}$$

$$= \frac{\sin\beta+\cos\beta}{\sin(\beta-\alpha)+\cos(\beta-\alpha)}$$

$$= \frac{\cos\left(\beta-\frac{\pi}{4}\right)}{\sin\left(\beta-\alpha+\frac{\pi}{4}\right)}$$

Now for $m_1 = m_2$

$$\frac{\theta}{2} = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$$

But $\theta \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow \theta \neq \frac{\pi}{2}$

So points are ''non collinear''

Q.18 Consider the lines given by

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

Match the Statements/Expressions in Column-I with the Statements/Expressions in Column-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[IIT 2008]**

Column-I	**Column-II**
(A) L_1, L_2, L_3 are concurrent, if	(P) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(Q) $k = -\frac{6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(R) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(S) $k = 5$

Sol. **A → S; B → P,Q; C Q R; D → P,Q,S**

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

(A) For concurrent lines

$$\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix} = 0$$

$\Rightarrow k = 5$

(B) If L_2 is parallel to L_1

$$\frac{3}{k} = \frac{-1}{3} \Rightarrow k = -9$$

If $L_2 \,||\, L_3$

$$\frac{3}{k} = \frac{-5}{2} \Rightarrow k = \frac{-6}{5}$$

(C) To form a triangle

$k \neq 5, -9, \frac{-6}{5}$

(D) To not to form a triangle

$k = 5, -9, \frac{-6}{5}$

Q.19 A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}\,x + y = 1$. If L also intersects the x-axis, then the equation of L is **[IIT-2011]**

(A) $y + \sqrt{3}\,x + 2 - 3\sqrt{3} = 0$

(B) $y - \sqrt{3}\,x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}\,y - x + 3 + 2\sqrt{3} = 0$

(D) $\sqrt{3}\,y + x - 3 + 2\sqrt{3} = 0$

Sol. [B]

Let the slope of the line is m

$$\tan 60^\circ = \left|\frac{m + \sqrt{3}}{1 - \sqrt{3}m}\right|$$

$$\sqrt{3} = \left|\frac{m + \sqrt{3}}{1 - \sqrt{3}m}\right|$$

So, $m + \sqrt{3} = \pm \sqrt{3}\,(1 - \sqrt{3}\,m)$

$m + \sqrt{3} = \sqrt{3} - 3m$	$m + \sqrt{3} = -\sqrt{3} + 3m$
$m = 0$	$m = \sqrt{3}$
hence line	hence line
$y = -2$	$y + 2 = \sqrt{3}\,(x - 3)$
	$y - \sqrt{3}\,x + 2 + 3\sqrt{3} = 0$

as line intersects x-axis

So line will be, $y - \sqrt{3}\,x + 2 + 3\sqrt{3} = 0$

EXERCISE # 5

Q.1 The straight line 5x + 4y = 0 passes through the point of intersection of the straight lines x + 2y –10 = 0 and 2x + y + 5 = 0. **[T/F]** **[IIT 1983]**

Sol. Intersection of

$x + 2y - 10 = 0$ ....(1)

and $2x + y + 5 = 0$ ....(2)

be P(a, b)

Solving (1) & (2), we get

$x = -\frac{20}{3}$ and $y = \frac{25}{3}$

So, $P(a, b) \equiv \left(-\frac{20}{3}, \frac{25}{3}\right)$

Now if $5x + 4y = 0$ ....(3)

passes through P, then it should satisfy.

So, $5\left(-\frac{20}{3}\right) + 4\left(\frac{25}{3}\right)$

$-\frac{100}{3} + \frac{100}{3} = 0$

⇒ It P satisfies (3)

∴ 5x + 4y = 0 passes through point of intersection of x + 2y – 10 = 0 and 2x + y + 5 = 0

Q.2 Given the points A (0, 4) and B (0, –4), the equation of the locus of the point P(x, y) such that |AP – BP| = 6 is.............. **[IIT- 83]**

Sol. Let P ≡ (x, y) then by given condition

$\left|\sqrt{x^2 + (y-4)^2} - \sqrt{x^2 + (y+4)^2}\right| = 6$

$\Rightarrow \sqrt{x^2 + (y-4)^2} = 6 + \sqrt{x^2 + (y+4)^2}$

$\Rightarrow x^2 + y^2 - 8y + 16 = x^2 + y^2 + 8y + 16 + 36 + 12\sqrt{x^2 + y^2 + 8y + 16}$

$\Rightarrow (-4y - 9)^2 = 9(x^2 + y^2 + 8y + 16)$

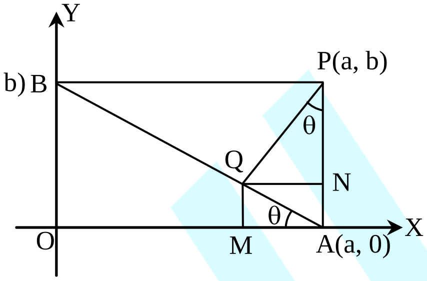
$\Rightarrow 9x^2 - 7y^2 + 63 = 0$

∴ locus is $9x^2 - 7y^2 + 63 = 0$

Q.3 The end A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle OAPB be completed, then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$. **[IIT 1983]**

Sol. Let the coordinates of A and B be (a, 0) and (0, b) on the axes so that its equation is $\frac{x}{a} + \frac{y}{b} = 1$

or $bx + ay - ab = 0$ ... (1)



By given condition AB = C

or $a^2 + b^2 = c^2$ ... (2)

Again if P be the vertex of rectangle then point P is (a, b). If (h, k) be the foot of perpendicular from P (a, b) on AB then $\frac{h-a}{1/a} = \frac{k-b}{1/b}$

$= -\frac{1+1-1}{\frac{1}{a^2} + \frac{1}{b^2}} = -\frac{a^2b^2}{a^2+b^2}$

$\therefore h = a - \frac{a^2b^2}{a^2+b^2} = \frac{a^3}{c^2}$ or $a = (c^2 h)^{1/3}$

similarly, $b = (c^2k)^{1/3}$

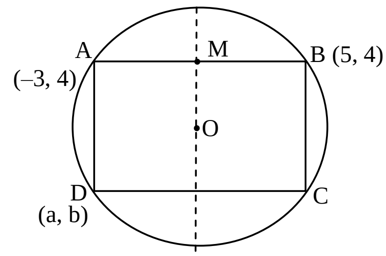
from (2), put a & b values, we get

$c^{4/3}(x^{2/3} + y^{2/3}) = c^2$

or $x^{2/3} + y^{2/3} = c^{2/3}$ (Proved)

Q.4 One of the diameter of the circle circumscribing the rectangle ABCD is 4y = x + 7. If A and B are the points (–3, 4) and (5, 4) respectively, then find the area of rectangle. **[IIT 1985]**

Sol. Let O be the centre of circle and M is the mid-point of AB



Then OM ⊥ AB ⇒ M (1, 4)

Since slope of AB = 0

Equation of straight line MO is x = 1 and equation of diameter is 4y = x + 7

⇒ centre is (1, 2)

Also O is the mid-point of BD

$\Rightarrow \left(\frac{a+5}{2}, \frac{b+4}{2}\right) = (1, 2)$

$\Rightarrow$ a = –3, b = 0

$\therefore$ AD = $\sqrt{(-3+3)^2+(4-0)^2}$ = 4

AB = $\sqrt{64+0}$ = 8

Thus, area of rectangle

= 8 × 4 = 32 sq. unit.

Q.5 Three lines px + qy + r = 0, qx + ry + p = 0 and rx + py + q = 0 are concurrent if **[IIT 1985]**

(A) p + q + r = 0

(B) $p^2 + q^2 + r^2 = pr + rp + pq$

(C) $p^3 + q^3 + r^3 = 3\,pqr$

(D) None of these

Sol. **[C]**

px + qy + r = 0, qx + rp + p = 0, rx + py + q = 0

are concurrent if

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} p+q+r & q & r \\ p+q+r & r & p \\ p+q+r & p & q \end{vmatrix} = 0$$

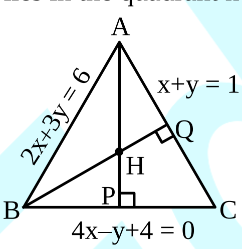
$\Rightarrow (p + q + r)\,(p^2 + q^2 + r^2 - pq - qr - rp) = 0$

$\Rightarrow p^3 + q^3 + r^3 - 3pqr = 0$

$\Rightarrow p^3 + q^3 + r^3 = 3pqr$

Q.6 The orthocentre of the triangle formed by the lines x + y = 1, 2x + 3y = 6 and 4x –y + 4 = 0 lies in the quadrant number............ **[IIT 1985]**

Sol.



Let AP and BQ be the altitudes.

equation of line passing through point A is

$(x + y - 1) + \lambda\,(2x + 3y - 6) = 0$

$\Rightarrow x\,(1 + 2\lambda) + y(1 + 3\lambda) - 1 + (-\,6\lambda) = 0$

But AP $\perp$ BC.

and $\left\{-\left(\dfrac{2\lambda+1}{3\lambda+1}\right)\right\} \times \{4\} = -\,1$

$\Rightarrow 8\lambda + 4 = 3\lambda + 1$

$\lambda = -\dfrac{3}{5}$

and equation of lines AP is

$x\left(1-\dfrac{6}{5}\right)+y\left(1-\dfrac{9}{5}\right) - 1 + 6 \times \dfrac{3}{5} = 0$

$\Rightarrow\ -x - 4y + 13 = 0$

x + 4y = 13 ……(1)

Similarly line BQ

$2x + 3y - 6 + \mu(4x - y + 4) = 0$

$\Rightarrow\ x(2 + 4\mu) + y(3 - \mu) - 6 + 4\mu = 0$

But BQ $\perp$ AC. so

$\left\{-\left(\dfrac{4\mu+2}{3-\mu}\right)\right\}$ (–1) = – 1

$\Rightarrow 4\mu + 2 = -\,3 + \mu \Rightarrow \mu = -\dfrac{5}{3}$

and equation of line BQ

$x\left(2-\dfrac{20}{3}\right)+y\left(3+\dfrac{5}{3}\right) - 6 - \dfrac{20}{3} = 0$

– 14x + 14y – 38 = 0

7x – 7y + 19 = 0

$x - y = -\dfrac{19}{7}$ ……(2)

Solving equation (1) & (2)

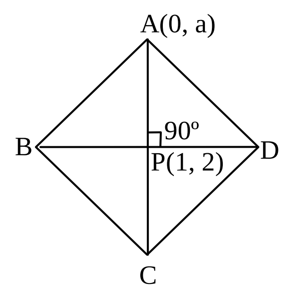
Coordinates of orthocenter are

$H \equiv \left(\dfrac{204}{35}, \dfrac{109}{35}\right)$

Which lies in 1st quadrant

Q.7 Two sides of a rhombus ABCD are parallel to the lines y = x + 2 and y = 7x + 3. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y- axis, find possible coordinates of A. **[IIT 1985]**

Sol. A being on y- axis may be chosen as (0, a). The diagonals intersect at P(1, 2). Again we know that diagonals will be parallel to the bisectors of the two sides.



i.e. $\dfrac{x-y+2}{\sqrt{2}} = \pm\,\dfrac{7x-y+3}{5\sqrt{2}}$

$\therefore$ d_1 is parallel to $x + 2y - \dfrac{7}{2} = 0$

and d_2 is parallel to $2x - y + \dfrac{13}{6} = 0$

The vertex A could be on any of the two diagonals. Hence slop of AP = 2 or $-\frac{1}{2}$

$\therefore \frac{2-a}{1-0} = 2 \text{ or } -\frac{1}{2}$ $\therefore a = 0, \frac{5}{2}$

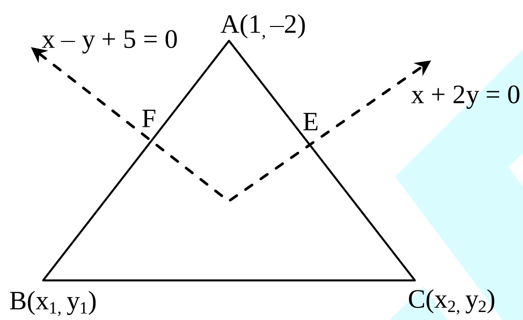
$\therefore$ A is (0, 0) or (0, 5/2).

Q.8 The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$, respectively. If the point A is (1,–2), find the equation of the line BC. **[IIT 1986]**

Sol. Let the coordinates of B and C be (x_1, y_1) & (x_2, y_2) respectively. Let m_1 and m_2 be the slopes of AB and AC respectively, then

m_1 = slope of AB = $\frac{y_1+2}{x_1-1}$

m_2 = slope of AC = $\frac{y_2+2}{x_2-1}$



Let F and E be the mid-point of AB and AC respectively. Then, the coordinate of E and F are

E $\left(\frac{x_2+1}{2}, \frac{y_2-2}{2}\right)$ and F $\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$

respectively.

Now F lies on $x - y + 5 = 0$

$\Rightarrow \frac{x_1+1}{2} - \frac{y_1-2}{2} = -5$

$\Rightarrow x_1 - y_1 + 13 = 0$ ... (1)

since AB is perpendicular to $x - y + 5 = 0$

$\therefore$ (slope of AB) (slope of $x - y + 5 = 0$) = –1

$\Rightarrow \frac{y_1+2}{x_1-1}(1) = -1$

$\Rightarrow y_1 + 2 = -x_1 + 1$

$\Rightarrow x_1 + y_1 + 1 = 0$ ...(2)

Solving (1) and (2), we get

$x_1 = -7, y_1 = 6$ $\therefore$ B is (–7, 6)

Now, E lies on $x + 2y = 0$

$\therefore \frac{x_2+1}{2} + (y_2 - 2) = 0$

$\Rightarrow x_2 + 2y_2 - 3 = 0$ ...(3)

Since AC is perpendicular to $x + 2y = 0$

(slope of AC). (slope of $x + 2y = 0$) = –1

$\Rightarrow \frac{y_2+2}{x_2-1}\left(-\frac{1}{2}\right) = -1$

$\Rightarrow 2x_2 - y_2 = 4$ ... (4)

Solving (3) and (4), we get

$x_2 = \frac{11}{5}$ and $y_2 = \frac{2}{5}$

$\therefore$ C is $\left(\frac{11}{5}, \frac{2}{5}\right)$

$\therefore$ Equation of BC is

$y - 6 = \frac{\frac{2}{5}-6}{\frac{11}{5}+7}(x+7)$

$\Rightarrow -23(y - 6) = 14(x + 7)$

$\Rightarrow 14x + 23y - 40 = 0$

Q.9 The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are vertices of **[IIT 1986]**

(A) an obtuse angled triangle

(B) an acute angled triangle

(C) a right angled triangle

(D) None of these

Sol.[D] Since for points (0, 8/3), (1, 3) and (82, 30)

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 8/3 & 1 \\ 1 & 3 & 1 \\ 82 & 30 & 1 \end{vmatrix} = 0$$

$\therefore$ points are collinear.

$\therefore$ option (D) is correct answer.

Q.10 All points lying inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy **[IIT 1986]**

(A) $3x + 2y \geq 0$ (B) $2x + y - 13 \geq 0$

(C) $2x - 3y - 12 \leq 0$ (D) $-2x + y \geq 0$

Sol. **[A, C]**

$3x + 2y \geq 0$ ... (1)

where (1, 3), (5, 0) and (–1, 2) satisfy (1)

again $2x + y - 13 \geq 0$ is not satisfied by (1, 3)

$\therefore$ (B) is false.

$2x - 3y - 12 \geq 0$

is satisfied for all points.

$\therefore$ (C) is true.

$-2x + y \geq 0$

is not satisfied by (5, 0)

$\therefore$ (D) is false.

Q.11 If P = (1, 0), Q = (–1, 0) and R = (2, 0) are three given points, then locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$, is- **[IIT 1988]**

(A) a straight line parallel to x-axis

(B) a circle passing through the origin

(C) a circle with the centre at the origin

(D) a straight line parallel to y-axis

Sol. **[D]**

Let S(x, y)

$\therefore SQ^2 + SR^2 = 2SP^2$

$\Rightarrow (x+1)^2 + y^2 + (x-2)^2 + y^2 = 2\{(x-1)^2 + y^2\}$

$\Rightarrow x^2 + 2x + 1 + y^2 + x^2 - 4x + 4 + y^2$

$= 2\{x^2 - 2x + 1 + y^2\} \Rightarrow 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

$\Rightarrow$ a straight line parallel to y- axis.

Q.12 The lines 2x + 3y + 19 = 0 and 9x + 6y –17 = 0 cut the coordinate axes in concyclic points. **[T/F]** **[IIT 1988]**

Sol. Let line 2x + 3y + 19 = 0 cuts axes in the points A and B, and line 9x + 6y – 17 = 0 cuts in the points C & D.

& $OA = \frac{19}{2}$, $OB = \frac{19}{3}$

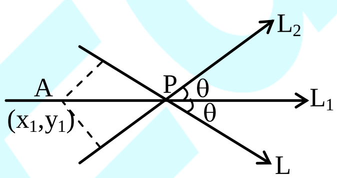
$OC = \frac{17}{9}$, $OD = \frac{17}{6}$

Clearly OA . OC = OB . OD

$\Rightarrow$ points are concyclic

Q.13 Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and make an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1. **[IIT 1988]**

Sol. Since, the required line L passes through the intersection of $L_1 = 0$ and $L_2 = 0$



So, the equation of the required line L is

$L_1 + \lambda L_2 = 0$

i.e. $(ax + by + c) + \lambda(\lambda x + my + n) = 0$ ......(i)

where λ is parameter.

since, L_1 is the angle bisector of L = 0 and $L_2 = 0$

$\therefore$ any point $A(x_1,y_1)$ on L_1 is equidistant from L = 0 and $L_2 = 0$ So, it must satisfy the equation of L_1 i.e. $ax_1 + by_1 + c_1 = 0$

$\Rightarrow \frac{|\lambda x_1 + my_1 + n|}{\sqrt{\lambda^2 + m^2}} = \frac{|(ax_1 + by_1 + c) + \lambda(\lambda x_1 + my_1 + n)|}{\sqrt{(a+\lambda\lambda)^2 + (b+\lambda m)^2}}$ ......(ii)

But $A(x_1,y_1)$ lies on L_1. So, it must satisfy the equation of L1 i.e. $ax_1+by_1+c = 0$ substitute $ax_1 + by_1 + c = 0$ in (ii), we get

$\frac{|lx_1 + my_1 + n|}{\sqrt{\lambda^2 + m^2}} = \frac{10 + \lambda(lx_1 + my_1 + n)}{\sqrt{(a+\lambda\lambda)^2 + (b+\lambda m)^2}}$

$\Rightarrow \lambda^2(\lambda^2 + m^2) = (a+\lambda\lambda)^2 + (b+\lambda m)^2$

$\therefore \lambda = -\frac{(a^2+b^2)}{2(a\lambda + bm)}$

Substituting the value of λ in (i) we get

$(ax + by + c) - \frac{(a^2+b^2)}{2(a\lambda + bm)}(\lambda x + my + n) = 0$

or $2(a\lambda+bm)(ax+by+c) - (a^2+b^2)(lx+my+n)=0$

as the required equation of line L.

Q.14 Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then **[IIT 1990]**

(A) $a^2 + b^2 = p^2 + q^2$

(B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

(C) $a^2 + p^2 = b^2 + q^2$

(D) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

Sol. **[B]**

Since the origin remains the same. So, length of the perpendicular from the origin on the line in its position $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{p} + \frac{y}{q} = 1$ are equal.

Therefore,

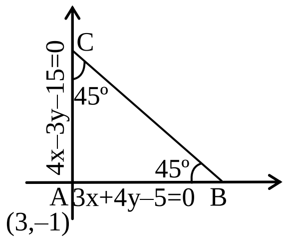
$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$$

$$\Rightarrow \sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \sqrt{\frac{1}{p^2}+\frac{1}{q^2}}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2}+\frac{1}{q^2}$$

Q.15 Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB = AC$. Determine the possible equations of the line BC passing through the point (1, 2). **[IIT 1990]**

Sol. Let m_1 and m_2 be the slopes of the lines $3x + 4y = 5$ and $4x - 3y = 15$ respectively.



Then, $m_1 = -\frac{3}{4}$ and $m_2 = \frac{4}{3}$ clearly $m_1 m_2 = -1$

$\therefore$ lines AB and AC are at right angles. Thus the triangle ABC is a right angled Isosceles triangle. Hence the line BC through (1, 2) will make an angle of 45° with the given lines. so the possible equations of BC are

$$y - 2 = \frac{m \pm \tan 45^\circ}{1 \mp m \tan 45^\circ}(x - 1)$$

where, m = slope of AB = –3/4

$$\Rightarrow y - 2 = \frac{-\frac{3}{4} \pm 1}{1 \mp (-3/4)}(x - 1)$$

$$\Rightarrow y - 2 = \frac{-3 \pm 4}{4 \pm 3}(x - 1)$$

$$\Rightarrow y - 2 = \frac{1}{7}(x - 1) \text{ and } y - 2 = -7(x - 1)$$

$$\Rightarrow x - 7y + 13 = 0 \text{ and } 7x + y - 9 = 0$$

Q.16 A line cuts the x-axis at A(7, 0) and the y-axis at B (0, –5). A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R. **[IIT 1990]**

Sol. The equation of the line AB is

$$\frac{x}{7} + \frac{y}{-5} = 1 \quad \text{..............(i)}$$

or $5x - 7y = 35$

Equation of line perpendicular to AB is

$$7x + 5y = \lambda \quad \text{...........(ii)}$$

It meet x-axis $P(\lambda/7, 0)$ and y-axis $Q(0, \lambda/5)$

The equation of lines AQ and BP are $\frac{x}{7} + \frac{5y}{\lambda} = 1$ and $\frac{7x}{\lambda} - \frac{y}{5} = 1$ respectively.

Let Q(h, k) be their point of intersection

then $\frac{h}{7} + \frac{5k}{\lambda} = 1$ and $\frac{7h}{\lambda} - \frac{k}{5} = 1$

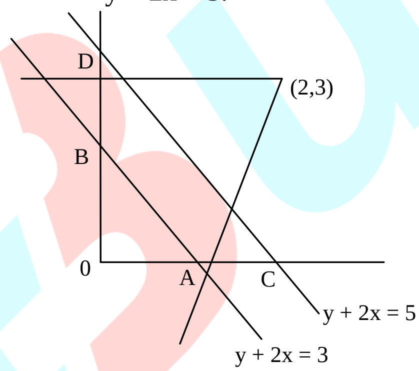
$$\Rightarrow \frac{1}{5k}\left(1 - \frac{h}{7}\right) = \frac{1}{7h}\left(1 + \frac{k}{5}\right) \text{ [on eliminating } \lambda \text{]}$$

$\Rightarrow h(7 - h) = k(5 + k)$

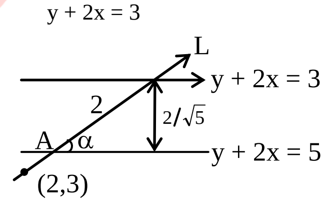
$\Rightarrow h^2 + k^2 - 7h + 5k = 0$

Hence, the locus is $x^2 + y^2 - 7x + 5y = 0$

Q.17 Find the equation of the line passing through the point (2, 3) and making intercept of length 2 units between the lines $y + 2x = 3$ and $y + 2x = 5$. **[IIT 1991]**



Sol.



Let L makes an angle α with the given parallel lines and inetercept AB is of 2 units.

As, distance between || lines

$$\Rightarrow \frac{5-3}{\sqrt{1+4}} = \frac{2}{\sqrt{5}}$$

$$\therefore \sin\alpha = \frac{2/\sqrt{5}}{2} = \frac{1}{\sqrt{5}}$$

$$\cos\alpha = \frac{2}{\sqrt{5}} \text{ and } \tan\alpha = \frac{1}{2}$$

$\Rightarrow$ equation of st. line passing through (2, 3) and making an angle α with $y + 2x = 5$ is

$$\frac{y-3}{x-2} = \tan(\theta \pm \alpha)$$

$$\Rightarrow \frac{y-3}{x-2} = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$$

and $\dfrac{y-3}{x-2} = \dfrac{\tan\theta - \tan\alpha}{1+\tan\theta\tan\alpha}$

$\Rightarrow \dfrac{y-3}{x-2} = -\dfrac{3}{4}$ and $\dfrac{y-3}{x-2} = \dfrac{1}{0}$

$\Rightarrow$ 3x + 4y = 18 and x = 2

$\Rightarrow$ x – 2 = 0 and 3x+ 4y = 18

Q.18 Show that all chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of the point. **[IIT 1991]**

Sol. The given curve is

$3x^2 - y^2 - 2x + 4y = 0$ ....(i)

Let y = mx + c be the chord of curne (i) which subtends right angle at origin. Then the combined equation of lines joining points of intersection of curve (i) and chord y = mx + c to the origin, can be obtained by the equation of the curve homogeneous i.e.

$$3x^2 - y^2 - 2x\left(\frac{y-mx}{c}\right) + 4y\left(\frac{y-mx}{c}\right) = 0$$

$\Rightarrow 3cx^2 - cy^2 - 2xy + 2mx^2 + 4y^2 - 4mny = 0$

$\Rightarrow (3c + 2m)x^2 - 2(1 + 2m)y + (4 - c)y^2 = 0$

As the lines represented are perpendicular to each other

$\therefore$ coefficient of x^2 + coefficient of y^2 = 0

$\Rightarrow$ 3c + 2m + 4 – c = 0 $\Rightarrow$ c + m + 2 = 0

on comparing with y = mx + c

$\Rightarrow$ y = mx + c passes through (1, –2)

$\Rightarrow$ (1, –2) is the reqd.

Q.19 Let the algebraic sum of the perpendicular distance from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero; then the line passes through a fixed point whose coordinates are .... **[IIT 1991]**

Sol. Let the equation of variable line is

L : ax + by + c = 0

and given

$$\left(\frac{2a+0+c}{\sqrt{a^2+b^2}}\right)+\left(\frac{0+2b+c}{\sqrt{a^2+b^2}}\right)+\left(\frac{a+b+c}{\sqrt{a^2+b^2}}\right) = 0$$

3a + 3b + 3c = 0

$\Rightarrow$ a + b + c = 0

$\Rightarrow$ a (1) + b (1) + c = 0

& line 'L' always passes through (1,1)

Q.20 If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is **[IIT 1992]**

(A) square

(B) circle

(C) straight line

(D) two intersecting lines

Sol. **[A]**

By the given conditions, we can take two perpendicular lines as x and y-axis, if (h, k) is any point on the locus then |h| + |k| = 1 therefore the locus is |x| + |y| = 1.

This consists of a square of side 1. Hence the required locus is a square.

Q.21 Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines

2x + 3y – 1 = 0

x + 2y – 3 = 0

5x –6y –1 = 0 **[IIT 1992]**

Sol. Solving the equations in pairs, we set the vertices of the triangle as $A\left(\frac{5}{4},\frac{7}{8}\right)$, $B\left(\frac{1}{3},\frac{1}{9}\right)$, C(–7, 5), if the given lines be BC, CA, AB respectively. Putting the coordinates in the opposite sides, we get the results +, –, – respectively. Hence we write the equations by multiplying by – sign so that all the results are +, +, +.

$\therefore$ BC : 2x + 3y – 1 = 0, CA : – x – 2y + 3 = 0

AB : –5x + 6y + 1 = 0

Now the points A, B, C are on the positive sides of the triangle whose revised forms of equations are written above. If the point (α, α^2) lies inside the triangle then the results obtained by putting the coordinates in the revised form of equations will be all positive.

$\therefore 2\alpha + 3\alpha^2 - 1$ = +ive

or (α + 1) (3α –1) = +ive ... (1)

$-\alpha - 2\alpha^2 + 3$ = +ive

(2α + 3) (α –1) = –ive ... (2)

$-5\alpha + 6\alpha^2 + 1$ = +ive

(3α –1) (2α –1) = +ive ... (3)

(1) $\Rightarrow$ α < – 1 or α > 1/3

(2) $\Rightarrow -\dfrac{3}{2} < \alpha < 1$

(3) $\Rightarrow \alpha < \dfrac{1}{3}$ or $\alpha > \dfrac{1}{2}$

$\therefore -\dfrac{3}{2} < \alpha < -1$ and $\dfrac{1}{2} < \alpha < 1$

$\therefore \alpha \in \left(-\dfrac{3}{2},-1\right) \cup \left(\dfrac{1}{2},1\right)$

Q.22 A line through A (–5, –4) meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at the points B, C, and D respectively. If $(15/AB)^2 + (10/AC)^2 = (6/AD)^2$, find the equation of the line. **[IIT 1993]**

Sol. Any line through A(–5,– 4) is

$y + 4 = \tan\theta\,(x + 5)$ ....(i)

or $\dfrac{x+5}{\cos\theta} = \dfrac{y+4}{\sin\theta} = r_1 = r_2 = r_3$

for B C D

B,C,D lie on the given lines respectively, where

$r_1 = AB$, $r_2 = AC$, $r_3 = AD$

$r_1 = -\dfrac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}$

for B which lies on $x + 3y + 2 = 0$

$AB = -\dfrac{1(-5) + 3(-4) + 2}{\cos\theta + 3\sin\theta}$

$= \dfrac{15}{\cos\theta + 3\sin\theta}$

or $\dfrac{15}{AB} = \cos\theta + 3\sin\theta$

similarly, $\dfrac{10}{AC} = 2\cos\theta + \sin\theta$

and $\dfrac{6}{AD} = \cos\theta - \sin\theta$

Hence from the given relation

$(\cos\theta + 3\sin\theta)^2 + (2\cos\theta + \sin\theta)^2 = (\cos\theta - \sin\theta)^2$

or $4\cos^2\theta + 12\sin\theta\cos\theta + 9\sin^2\theta = 0$

or $(2\cos\theta + 3\sin\theta)^2 = 0 \ \therefore\ \tan\theta = -2/3$

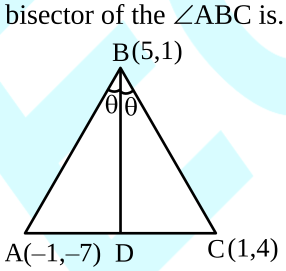
from (i), the equation of the line through A is

$$y + 4 = -\frac{2}{3}(x + 5)$$

or $2x + 3y + 22 = 0$

Q.23 The vertices of a triangle are A (–1, –7), B (5, 1) and C (1, 4). The equation of the bisector of the ∠ABC is........ **[IIT 1993]**

Sol.



$\dfrac{AD}{DC} = \dfrac{AB}{BC} = \dfrac{\sqrt{(5+1)^2 + (1+7)^2}}{\sqrt{(5-1)^2 + (1-4)^2}} = \dfrac{10}{5} = 2$

$\Rightarrow \dfrac{AD}{DC} = \dfrac{2}{1}$

Coordinates of point D $\equiv \left(\dfrac{2-1}{2+1}, \dfrac{8-7}{2+1}\right)$

$\equiv \left(\dfrac{1}{3}, \dfrac{1}{3}\right)$

& the equation of bisector (BD)

$y - 1 = \left(\dfrac{\frac{1}{3} - 1}{\frac{1}{3} - 5}\right)(x - 5)$

$\Rightarrow x - 7y + 2 = 0$

Q.24 The locus of a variable point whose distance from (–2, 0) is $\dfrac{2}{3}$ times its distance from the line $x = -\dfrac{9}{2}$ is - **[IIT 1994]**

(A) ellipse (B) parabola

(C) hyperbola (D) None of these

Sol. **[A]**

Let variable point is (h, k)

$\sqrt{(h^2+2)^2 + k^2} = \dfrac{2}{3}\left(\dfrac{h + 9/2}{\sqrt{1}}\right)$

$\Rightarrow \sqrt{(h^2+2)^2 + k^2} = \dfrac{2}{3}\left(h + \dfrac{9}{2}\right)$

$\Rightarrow \sqrt{(h^2+2)^2 + k^2} = \dfrac{2h+9}{3}$

on squaring

$\Rightarrow (h + 2)^2 + k^2 = \dfrac{1}{9}(2h + 9)^2$

$\Rightarrow 9\{h^2 + k^2 + 4h + 4\} = 4h^2 + 81 + 36h$

$\Rightarrow 9h^2 + 9k^2 + 36h + 36 - 4h^2 - 81 - 36h = 0$

$\Rightarrow 5h^2 + 9k^2 = 45$

$\Rightarrow \dfrac{h^2}{9} + \dfrac{k^2}{5} = 1$ which is an ellipse.

Q.25 The equations to a pair of opposite sides of parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$, the equations to its diagonals are **[IIT 1994]**

(A) $x + 4y = 13$, $y = 4x - 7$

(B) $4x + y = 13$, $4y = x - 7$

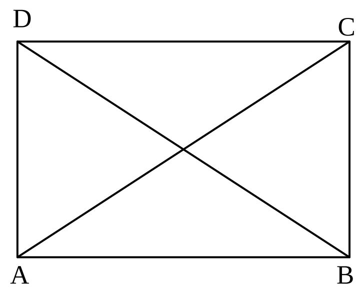
(C) $4x + y = 13$, $y = 4x - 7$

(D) $y - 4x = 13$, $y + 4x = 7$

Sol. **[C]**

$x^2 - 5x + 6 = 0 \Rightarrow x = 2, x = 3$

$y^2 - 6y + 5 = 0 \Rightarrow y = 1, y = 5$



$\therefore$ vertices of parallelogram ABCD is given by
A(2, 1), B(2, 5), C(3, 5), D(3, 1)
Equation of diagonal AC is

$$y - 1 = \frac{5-1}{3-2}(x - 2) \Rightarrow 4x - y = 7 \Rightarrow y = 4x - 7$$

Equation of diagonal BD is

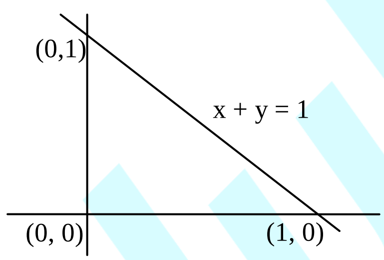
$$y - 5 = \frac{1-5}{3-2}(x - 2) \Rightarrow 4x + y = 13$$

$\therefore$ 4x + y = 13 and y = 4x – 7

Q.26 The orthocentre of the triangle formed by the lines xy = 0 and x + y = 1 is **[IIT 1995]**

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (C) (0, 0) (D) $\left(\frac{1}{4}, \frac{1}{4}\right)$

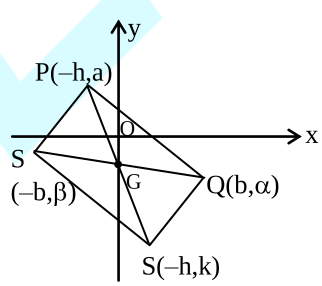
Sol. **[C]**



The formed triangle is right angled triangle.
$\therefore$ its orthocentre is the point where right angle is formed i.e. origin.
Therefore, orthocentre is (0, 0)

Q.27 A rectangle PQRS has its side PQ parallel to the line y = mx and vertices P, Q and S on the lines y = a , x = b and x = – b, respectively. Find the locus of the vertex R. **[IIT - 96]**

Sol.



Let the co-ordinate of Q be (b, α) and that of S be (–b, β) suppose PR and SQ meet in G. Since G is mid-point of SQ, its x-co-ordinate must be O. Let the co-ordinate of R be (h, k). since G is mid-point of PR, the x-coordinate of P must be –h, and as P lies on the line y = a, the co-ordinate of P are (–h, a). since PQ is parallel to y = mx slope of PQ = m

$$\Rightarrow \frac{\alpha - a}{b + h} = m \quad \text{..............(i)}$$

Again RQ $\perp$ PQ

Solpe of RQ $= -\frac{1}{m}$

$$\Rightarrow \frac{k - \alpha}{h - b} = -\frac{1}{m} \quad \text{..............(ii)}$$

From (i), we get

$$\alpha - a = m(b + h)$$

$$\Rightarrow \quad \alpha = a + m(b + h) \text{ ....(iii)}$$

and from (ii), we get

$$k - a = -\frac{1}{m}(h - b)$$

$$\Rightarrow \alpha = k + \frac{1}{m}(h - b)\text{...............(iv)}$$

From (iii) and (iv)

$$a + m(b + h) = k + \frac{1}{m}(h - b)$$

$$\Rightarrow am + m^2(b + h) = km + (h - b)$$

$$\Rightarrow m^2(b + h) + m(a - k) + (b - h) = 0$$

$$\Rightarrow (m^2 - 1)h - mk + b(m^2 + 1) + am = 0$$

$\therefore$ locus is
$(m^2 - 1)x - my + b(m^2 + 1) + am = 0$

Q.28 The diagonals of parallelogram PQRS are along the lines x + 3y = 4 and 6x – 2y = 7. Then PQRS must be a **[IIT-1998]**
(A) rectangle
(B) square
(C) cyclic quadrilateral
(D) rhombus

Sol. **[D]**

Θ slope of x + 3y = 4 is $-\frac{1}{3}$

and slope of 6x –2y = 7 is 3

Therefore, these two lines are perpendicular which shows that both diagonals are perpendicular. Hence PQRS must be a rhombus.

Q.29 If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) always rational points (s) ? **[IIT-1998]**

(A) Centroid (B) Incentre

(C) Circumcentre (D) Orthocentre

Sol. **[A, C, D]**

Since the coordinates of the centroid are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ the centroid is always a rational point also circumcentre and orthocentres are rational.

Q.30 If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) **[IIT-1999]**

(A) lie on a straight line

(B) lie on an ellipse

(C) lie on a circle

(D) are vertices of a triangle

Sol. **[A]**

x_1, x_2, x_3 are in G.P.

Let common ratio is r

$\therefore x_1, x_1r, x_1r^2$ are in G.P.

Similarly, y_1, y_2, y_3 are in G.P. with common ratio r

$\therefore y_1, y_1r, y_1r^2$ are in G.P.

$$\therefore \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x_1 & x_1r & x_1r^2 \\ y_1 & y_1r & y_1r^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}x_1y_1\begin{vmatrix} 1 & r & r^2 \\ 1 & r & r^2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$\therefore (x_1, y_1), (x_2, y_2), (x_3, y_3)$ lie on a straight line.

Q.31 Let PQR be a right angled isosceles triangle, right angled at P(2, 1). If the equation of the line QR is 2x + y = 3, then the equation representing the pair of lines PQ and PR is- **[IIT-99]**

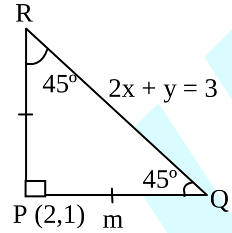
(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$

(B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$

(D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

Sol. **[B]**



Let m be the slope of PQ then

$$\tan 45^\circ = \left|\frac{m-(-2)}{1+m(-2)}\right|$$

$$1 = \left|\frac{m+2}{1-2m)}\right|$$

$$\Rightarrow \frac{m+2}{1-2m} = \pm 1$$

$\Rightarrow m + 2 = 1 - 2m$ or $-1 + 2m = m + 2$

$\Rightarrow m = -1/3$ or $m = 3$

As PR also makes ∠ 45° with RQ.

∴ The above two values of m are for PQ and PR

∴ Equation of PQ is

$$y - 1 = -\frac{1}{3}(x - 2)$$

$\Rightarrow 3y - 3 = -x + 2$

$x + 3y - 5 = 0$

and equation of PR is

$y - 1 = 3(x - 2)$

$\Rightarrow 3x - y - 5 = 0$

∴ combined equation of PQ and PR is

$(x + 3y - 5)(3x - y - 5) = 0$

$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

ANSWER KEY

EXERCISE # 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Ans.	C	B	C	D	A	D	A	D	C	B	D	A	C	A	A	A	B	B	D	B	A	A
Q.No.	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
Ans.	B	B	A	B	A	A	B	C	A	B	B	A	D	A	B	D	B	A	A	A	D	A

45. False **46.** False **47.** False **48.** 1 **49.** 7 **50.** (p, q) **51.** –2

52. (5/2, 1/4) **53.** 2 **54.** 2

EXERCISE # 2

(PART-A)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	D	A	C	A	B	A	B	B	D	B	A	A	C	C	A	A
Q.No.	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Ans.	B	B	B	D	B	A	D	B	A	A	A	A	D	A	A	A

(PART-B)

Q.No.	33	34	35	36	37	38	39	40	41	42	43
Ans.	B,C	A,B,C	A,B,C,D	B,D	A,C	B,D	A,B	A,C	A,B	A,C	A,B

(PART-C)

Q.No.	44	45	46	47
Ans.	A	C	B	D

(PART-D)

48. $A \to S; B \to P; C \to Q; D \to R$

49. $A \to P; B \to Q; C \to R; D \to S$

50 $A \to R, S; B \to P, Q; C \to S; D \to Q$

51 $A \to R; B \to P; C \to S; D \to Q$

52 $A \to S; B \to R; C \to P; D \to Q$

EXERCISE # 3

2. (1, –1) or (–2, –10)

3. (0, –7), (8, –1)

5. (4, 3), (7, – 2)

6. $x^2 + y^2 - 30x - 30y + 281 = 0$

7. $9x^2 + 10y^2 - 6xy - 9\lambda^2 = 0$

9. $\left(\frac{3}{2},\frac{3}{4}\right)\ \left(\frac{3}{2},0\right),\left(\frac{9}{4},\frac{3}{4}\right)\ \left(\frac{9}{4},0\right)$

10. $x - y = 1,\ 3x - y + 3 = 0$

11. $3x - 4y + \frac{180}{\sqrt{130}} = 0;\ 12x - 5y = \frac{468}{\sqrt{130}}$

12. $3x + y + 9 = 0$

13. B(5, 1), C(– 3, 4) **14.** x – y = 0 **15.** 91

17. C $\left(\frac{t}{2}, t\right)$, B$\left(\frac{-2}{3}t, \frac{-t}{6}\right)$ **19.** (1, 1), ($\sqrt{2}$ – 1, $\sqrt{2}$ – 1) **21.** $x^2 + y^2 - hx - ky = 0$

22. $6x^2 - xy - y^2 - x - 12y - 35 = 0$ **23.** x + 2y – 7 = 0, x – 4y = 1, x – y + 2 = 0

24. $x^2 - 6xy - y^2 + 10x + 10y - 15 = 0$ **25.** 17x – 10y = 0 **26.** (A) **27.** (B) **28.** (C)

29. (B) **30.** (C) **31.** (B) **32.** (B) **33.** (C) **34.** (D) **35.** (C)

36. (A) **37.** (B)

EXERCISE # 4

1. (D) **2.** (A) **3.** (D) **4.** (B) **5.** (C) **6.** (D) **7.** x – 3y + 5 = 0

8. (A) **9.** (A) **10.** (C) **11.** (C) **12.** $4x^2 = (y-1)^2$ **13.** (B,D)

14. (B) **15.** (C) **16.** (C) **17.** (D) **18.** A → S; B → P,Q; C → R; D → P,Q,S

19. (B)

EXERCISE # 5

1. True **2.** $\frac{y^2}{9} - \frac{x^2}{7} = 1$ **4.** 32 sq. units **5.** (A,B,C) **6.** 1st quadrant **7.** (0, 0) or (0, 5/2)

8. 14x + 23y – 40 = 0 **9.** (D) **10.** (A,C) **11.** (D) **12.** True

13. $(a^2 + b^2)(lx + my + n) - 2(al + bm)(ax + by + c) = 0$ **14.** (B) **15.** x –7y +13 = 0 or 7x + y –9 = 0

16. $x^2 + y^2 - 7x + 5y = 0$ **17.** 3x + 4y –18 = 0 or x – 2 = 0 **18.** (1, –2) **19.** (1, 1) **20.** (A)

21. $\alpha \in \left(-\frac{3}{2}, -1\right) \cup \left(\frac{1}{2}, 1\right)$ **22.** 2x + 3y + 22 = 0 **23.** x – 7y + 2 = 0 **24.** (A) **25.** (C)

26. (C) **27.** $(m^2 - 1)x - my + b(m^2 + 1) + am = 0$ **28.** (D) **29.** (A,C,D) **30.** (A) **31.** (B)