PERMUTATION & COMBINATION
EXERCISE # 1

Questions based on Fundamental principle of counting

Q.1 Find the total number of ways of answering 5 objective type questions, each question having 4 choices.

(A) 5^4 (B) $5^4 - 1$ (C) 4^5 (D) $4^5 - 1$

Sol. [C] total question = 5

each question having = 4 choices

Total number of ways of answering

is = $4 \times 4 \times 4 \times 4 \times 4 = 4^5$

Q.2 The number of positive integers satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$ is-

(A) Nine (B) Eight

(C) Five (D) None of these

Sol. **[A]**

${}^{(n+1)}C_{(n-2)} - {}^{(n+1)}C_{(n+1)} \le 100$

$$\Rightarrow \frac{(n+1)!}{(n-2)!\,(n+1-n+2)!} - \frac{(n+1)!}{(n-1)!\,(n+1-n+1)!} \le 100$$

$$\Rightarrow \frac{(n+1)!}{(n-2)!\,3!} - \frac{(n+1)!}{(n-1)!\,2!} \le 100$$

$$\Rightarrow (n+1)!\left[\frac{1}{3!\,(n-2)!} - \frac{1}{2!\,(n-1)!}\right] \le 100$$

$$\Rightarrow (n+1)!\left[\frac{(n-1)!-3(n-2)!}{6(n-1)!\,(n-2)!}\right] \le 100$$

$$\Rightarrow \frac{(n+1)[(n-1)(n-2)!-3(n-2)!]}{6(n-1)!\,(n-2)!} \le 100$$

$$\Rightarrow \frac{n(n+1)[(n-1)(n-2)!(n-1-3]}{6(n-1)!\,(n-2)!} \le 100$$

$$\Rightarrow \frac{n(n+1)[(n-1)(n-2)!(n-4]}{6(n-1)!\,(n-2)!} \le 100$$

$\Rightarrow n(n+1)(n-4) \le 600$

Put n = 1, 2, 3, 4, 5, 6, 7, 8, 9, ……

For n = 5 $\Rightarrow 600 \le 5 \times 6 \times 1$ Holds good

For n = 8 $\Rightarrow 600 \le 8 \times 9 \times 4 = 288$ Holds good

For n = 9 $\Rightarrow 600 \le 9 \times 10 \times 5 = 450$ Holds good

For n = 10 $\Rightarrow 600 \le 10 \times 11 \times 7 = 770$ does not holds

Hence, No of the integers must be nine to satisfy above equation.

$\therefore$ Option (A) is correct answer.

Q.3 If ${}^{m+n}P_2 = 90$ and ${}^{m-n}P_2 = 30$, then (m, n) is given by -

(A) (7, 3) (B) (16, 8) (C) (9, 2) (D) (8, 2)

Sol. [D] $\Rightarrow (m+n)(m+n-1) = 90 = 10 \times 9$

$\Rightarrow m + n = 10$ ……(1)

and $(m-n)(m-n-1) = 30 = 6 \times 5$

$\Rightarrow m - n = 6$ ……(2)

solving (1) & (2) we get

m = 8, n = 2

Questions based on Linear permutation

Q.4 Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Determine the number of words which have at least one letter repeated.

(A) 69762 (B) 69676 (C) 69760 (D) 69766

Sol. [C] All 5 letters word is = 100000

All five letters word which have no any letter repeated is = $10 \times 9 \times 8 \times 7 \times 6 = 30240$

Number of words which have at least one letter repeated is

= 100000 – 30240 = 69760

Q.5 There are m men and n monkeys (n > m). If a man have any number of monkeys. In how many ways may every monkey have a master?

(A) n^m (B) m^n (C) $m^n - n^m$ (D) mn

Sol. [B] Total ways

$$= \underbrace{m \times m \times m \ldots\ldots \times m}_{n \text{ times}} = m^n$$

Q.6 There are (n + 1) white and (n + 1) black balls each set numbered 1 to (n + 1). The number of ways in which the balls can be arranged in row so that the adjacent balls are of different colours is-

(A) (2n + 2)! (B) (2n + 2)! × 2

(C) (n + 1)! × 2 (D) $2\{(n + 1)!\}^2$

Sol. [D] ways to arranged white balls = $\lfloor\underline{n+1}$

ways to arranged black balls so that

no adjacent balls are same colour

is = $2\ {}^{n+1}C_{n+1}\ \lfloor\underline{n+1} = 2\lfloor\underline{n+1}$

Total ways = $2\left(\lfloor\underline{n+1}\right)^2$

Questions based on Permutation under various conditions

Q.7 How many numbers of four digits greater than 2300 can be formed with the digits 0, 1, 2, 3, 4, 5 and 6; no digit being repeated in any number ?

(A) 560 (B) 590

(C) 90 (D) 360

Sol. [A] When number started greater then 2 then

Total number = 4 × 6 × 5 × 4 = 480

when number started from 2 then

numbers = 1 × 4 × 5 × 4 = 80

Total number = 480 + 80 = 560

Q.8 The letters of the word SURITI are written in all possible orders and these words are written out as in a dictionary. Then find the rank of the words SURITI.

(A) 236 (B) 263 (C) 326 (D) 260

Sol. [A] Word SURITI

order of dictionary is IIRSTU

total words started with I = $\lfloor\underline{5}$ = 120

total words started with R = $\dfrac{\lfloor\underline{5}}{\lfloor\underline{2}}$ = 60

words started with SI = $\lfloor\underline{4}$ = 24

words started with SR = $\dfrac{\lfloor\underline{4}}{\lfloor\underline{2}}$ = 12

words started with ST = $\dfrac{\lfloor\underline{4}}{\lfloor\underline{2}}$ = 12

words started with SUI = $\lfloor\underline{3}$ = 6

word started with SURIIT = 1 = 1

word SURITI = 1 = 1

Rank = 120 + 60 + 24 + 12 + 12 + 6 + 1 + 1 = 236

Q.9 In how many ways four friends can put up in 8 hotels of a town if

(i) There is no restriction.

(A) 4096 (B) 4609 (C) 4926 (D) None

(ii) No two friends can stay together.

(A) 1680 (B) 1608 (C) 1660 (D) 1672

(iii) All the friends do not stay in same hotel?

(A) 4088 (B) 4808 (C) 4880 (D) None

Sol. (i) There is no restriction then

ways = 8^4 = 4096

(ii) No two friend stay together then

ways = 8 × 7 × 6 × 5 = 1680

(iii) All the friends do not stay in same hotel then ways

Total ways = 4096

All friends stay same hotel then ways = 8

Required ways = 4096 – 8 = 4088

Q.10 There are 12 balls numbered from 1 to 12. The number of ways in which they can be used to fill 8 places in a row so that the balls are with numbers in ascending or descending order is equal to-

(A) ${}^{12}C_8$ (B) ${}^{12}P_8$

(C) $2 \times {}^{12}P_8$ (D) $2 \times {}^{12}C_8$

Sol. **[D]**

Given : 12 balls numbered from 1 to 12 we have to arrange balls in a row that either they are in ascending order or descending order at eight places.

No. of ways = ${}^{12}C_8 + {}^{12}C_8$

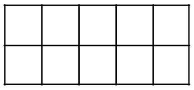
$= 2 \times {}^{12}C_8$

∴ Option (D) is correct answer.

Q.11 A double-decker bus has 5 empty seats in the upstair and 5 empty seats in the down stair. 10 people board the bus of which 2 are old people and 3 are children. The children refuse to take seats down stair while old people insist to stay downstair. In how many different arrangements can be 10 people take their seats in the bus?

(A) 144000 (B) 146000

(C) 146400 (D) None of these

Sol.[A] 

3 children upstairs = ${}^5C_3 \cdot 3!$

2 old people downstairs = ${}^5C_2 \cdot 2!$

rest 5 people = 5!

Total = ${}^5C_3 \cdot {}^5C_2 \cdot 3!\ 2!\ 5!$

$= 10 \times 10 \times 6 \times 2 \times 120$

$= 144000$

Questions based on Circular permutation

Q.12 20 persons were invited to a party. In how many ways can they and the host be seated at a circular table? In how many of these ways will two particular persons be seated on either side of the host.

(A) 20!, 2! 18! (B) 18!, 2! 20!

(C) 19!, 18! (D) 19!, 19!

Sol. [A] 20 persons and host seated at a round table are = $\lfloor 20$ ways

Again If two particular person sit on either side of the host then ways = $2\lfloor 18$

Q.13 The number of ways in which 6 red roses and 3 white roses (all roses different) can form a garland so that all the white roses come together is-

(A) 2170 (B) 2165

(C) 2160 (D) 2155

Sol. [C] Required ways = $\frac{15}{2} \cdot {}^6C_1 \lfloor 3 = 60 \times 6 \times 6 = 2160$

Q.14 The number of ways in which 4 boys and 4 girls can stand in a circle so that each boy and each girl is one after the other is-

(A) 3! . 4! (B) 4! . 4!

(C) 8! (D) 7!

Sol. [A] Required ways = $\lfloor 3 \lfloor 4$

Q.15 12 guests at a dinner party are to be seated along a circular table. Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must always, be placed next to one another, the number of ways in which the company can be placed, is-

(A) 20 . 10! (B) 22 . 10!

(C) 44 . 10! (D) None of these

Sol.[A] Master and mistress seated in $\lfloor 2$ ways two particular guests placed next to each other then ways = 2(5) = 10

remaining 10 guests seated in ways = $\lfloor 10$

So total ways = $2 \cdot 10\lfloor 10 = 20\lfloor 10$

Questions based on Combination

Q.16 A set contains (2n + 1) elements. The number of subsets of the set which contain at most n elements is-

(A) 2^n (B) 2^{n+1}

(C) 2^{n-1} (D) 2^{2n}

Sol. **[D]**

No. of sub sets = ${}^{(2n+1)}C_0 + {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 + {}^{(2n+1)}C_3 + {}^{(2n+1)}C_4 + \ldots + {}^{(2n+1)}C_n$

we know,

$2^{2n+1} = {}^{(2n+1)}C_0 + {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 + \ldots + {}^{(2n+1)}C_n + {}^{(2n+1)}C_{(n+1)} + {}^{(2n+1)}C_{(n+2)} + \ldots + {}^{(2n+1)}C_{2n} + {}^{(2n+1)}C_{(n+1)}$

$\Rightarrow 2^{(2n+1)} = 2[{}^{(2n+1)}C_0 + {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 + \ldots + {}^{(2n+1)}C_n]$

$\Rightarrow 2^{2n} = {}^{(2n+1)}C_0 + {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 + \ldots + {}^{(2n+1)}C_n$

$\therefore$ Option (D) is correct answer.

Q.17 There are 20 questions in a question paper. If no two students solve the same combination of questions but solve equal number of questions then the maximum number of students who appeared in the examination is-

(A) ${}^{20}C_9$ (B) ${}^{20}C_{11}$

(C) ${}^{20}C_{10}$ (D) None of these

Sol. **[C]**

No. of students who appeared in the examination = ${}^{20}C_r$

${}^{20}C_r$ will be maximum when

$r = \frac{20}{2} = 10$

$\therefore$ Maximum no. of students = $^{20}C_{10}$

$\therefore$ Option (C) is correct answer.

Q.18 A father with 7 children takes 4 at a time to the zoo, as after as he can with out taking the same four children together more than once. How after will he go? How after will each child go?

(A) 30, 10 (B) 35, 15

(C) 30, 20 (D) 35, 20

Sol. **[D]**

Total no. of ways making a group of 4 children out of 7 children = $^7C_4 = 35$

Hence, father will go 35 times

Add, No. of ways in which each child will go $^{7-1}C_4 = {^6C_3} = 20$

$\therefore$ Option (D) is correct answer.

Questions based on **Combination under various condition & its geometrical application & problems based on divisors**

Q.19 The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is-

(A) $^7P_2 \,.\, 2^5$ (B) $^7C_2 \,.\, 2^5$

(C) $^7C_2 \,.\, 5^2$ (D) None of these

Sol. [B] In seven digit number 2 occurs twice in each number so

number of numbers = $^7C_2 \,.\, 2^5$

Q.20 The number of triangles whose vertices are at the vertices of an octagon but none of sides happen to come from the sides of the octagon is-

(A) 24 (B) 52 (C) 48 (D) 16

Sol. [D] Total number of triangles are = $^8C_3 = 56$

Triangle whose one side is side of octagon are = 8 .(8 – 4) = 32

Triangles whose two sides are sides of octagon are = 4 × 2 = 8

Required number of triangles which none of side not the side of octagon are

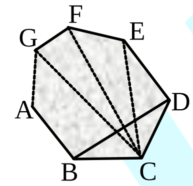
= 56 – 32 – 8 = 16

Q.21 In a polygon no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70 then the number of diagonals of the polygon is-

(A) 20 (B) 28 (C) 8 (D) None

Sol. **[A]**

Let n sided regular polygon no. of diagonals which have interior point of intersection



$$= \frac{\frac{n(n-3)}{2} + \frac{n}{2}}{2}$$

$$= \frac{n(n-3)}{2} + \frac{n}{4} = \frac{n(n-2)}{4}$$

No. of interior point of sections, n(n – 3) = 70

$\Rightarrow n^2 - 3n - 70 = 0 \Rightarrow n^2 - 10n + 7n - 70 = 0$

$\Rightarrow n(n - 10) + 7(n - 10) = 0$

$\Rightarrow (n - 10)(n + 7) = 0 \Rightarrow n = 10$

$\therefore$ No. of diagonals = 10 × 8/4 = 20

Q.22 The number of proper divisors of $2^p \,.\, 6^q \,.\, 15^r$ is-

(A) (p + q + 1) (q + r + 1) (r + 1)

(B) (p + q + 1) (q + r + 1) (r + 1) – 2

(C) (p + q) (q + r) r – 2

(D) None of these

Sol. **[B]**

Let $N = 2^p \,.\, 6^q \,.\, 15^r$

$= 2^p \,.\, 2^q \,.\, 3^q \,.\, 3^r \,.\, 5^r$

$= 2^{(p+q)} \,.\, 3^{(q+r)} \,.\, 5^r$

$\therefore$ No. of divisors of N excluding 1 and N.

= (p + q + 1) (q + r + 1) (r + 1) – 2

$\therefore$ Option (B) is correct answer.

Questions based on **Division into groups and person**

Q.23 The number of ways in which 19 different objects can be divided into two groups of 13 and 6 is -

(A) $^{19}C_{13} + {^{19}C_6}$ (B) $^{19}P_{13}$

(C) $^{19}C_{13}$ (D) None of these

Sol. **[C]**

Required no of ways

$= {^{19}C_{13}} \times {^6C_6} = {^{19}C_{13}}$

$\therefore$ Option (C) is correct answer.

Q.24 Number of ways in which a pack of 52 playing cards be distributed equally amount four players, so that each may have the ace, king, queen and jack of the same suit is-

(A) $\frac{36!}{(9!)^4}$ (B) $\frac{36!\,4!}{(9!)^4}$

(C) $\frac{36!}{(9!)^4\,4!}$ (D) None

Sol.[B] Total card = 52

4 ace + 4 king + 4 Queen + 4 Jack = 16 card

Remaining card = 52 – 16 = 36

Distribute equally between four players

$$\text{ways} = \frac{\lfloor 36}{\lfloor 9 \lfloor 9 \lfloor 9 \lfloor 9}$$

ways for distributing

ace, king, queen, jack of same suit = $\lfloor 4$

$$\text{Total ways} = \frac{\lfloor 36 \lfloor 4}{(\lfloor 9)^4}$$

Questions based on **Multinomial theorem**

Q.25 For the equation x + y + z + ω = 19, the number of positive integral solutions is equal to-

Sol.

x + y + z + w = 19

For positive integral solutions, it must be

$x \ge 1$; $y \ge 1$; $z \ge 1$; $w \ge 1$

Let $x_1 = x - 1 \Rightarrow x = x_1 + 1$ and $x_1 \ge 0$

$y_1 = y - 1 \Rightarrow y = y_1 + 1$ and $y_1 \ge 0$

$w_1 = w - 1 \Rightarrow w = w_1 + 1$ and $w_1 \ge 0$

$\Rightarrow x_1 + y_1 + z_1 + w_1 = 15$

$x_1 \ge 0, y_1 \ge 0, z_1 \ge 0, w_1 \ge 0$

It represents 15 identical things can be distributed among 4 persons.

Also, x + y + w = 19

no. of solutions = coefficient of x^{19} in

$(x + x^2 + x^3 + \ldots + x^{19})^4$

(A) the number of ways in which 15 identical things can be distributed among 4 persons

(B) the number of ways in which 19 identical things can be distributed among 4 persons

(C) coefficient of x^{19} in $(x^0 + x^1 + x^2 + \ldots + x^{19})^4$

(D) coefficient of x^{19} in $(x + x^2 + x^3 + \ldots + x^{19})^4$

Q.26 The number of ways of selecting n things out of 3n things, of which n are of one kind and alike, and n are of a second kind and alike and the rest are unlike, is.

(A) $(n + 2)^{2n}$ (B) $(n + 2)2^{n-1}$

(C) $(n + 2)2^{n+1}$ (D) None of these

Sol. Using multinomial theorem

No. of ways of selecting n things among 3n things

= coefficient of x^n in $(1 + x + x^2 + x^3 + \ldots + x^n)^2 \times \underbrace{(1+x)(1+x)\ldots(1+x)}_{\text{n times}}$

= coefficient of x^n in $\left(\frac{1-x^{n+1}}{1-x}\right)^2 \times (1 + x)^n$

= coefficient of x^n in $(1 - x)^{-2} \times (x + 1)^n$

= coefficient of x^n in $(1 + {}^2C_1\, x + {}^3C_2\, x^2 + {}^4C_3\, x^3 + {}^5C_4\, x^4 + {}^6C_5\, x^5 + \ldots + {}^{(n-3)}C_{(n-4)}\, x^{(n-4)} + {}^{(n-2)}C_{(n-3)}\, x^{(n-3)} + {}^{(n-1)}C_{(n-2)}\, x^{(n-2)} + {}^nC_{(n-1)}\, x^{(n-1)} + {}^{(n+1)}C_n\, x^n + \ldots) \times ({}^nC_0\, x^n + {}^nC_1\, x^{(n-1)} + {}^nC_2\, x^{(n-2)} + {}^nC_3\, x^{(n-3)} + {}^nC_4\, x^{(n-4)} + \ldots + {}^nC_{(n-4)}\, x^4 + {}^nC_{(n-3)}\, x^3 + {}^nC_{(n-2)} + {}^nC_{(n-1)} + {}^nC_n.\, x^0)$

$= {}^nC_0 + 2\,{}^nC_1 + 3\,{}^nC_2 + 4\,{}^nC_3 + 5\,{}^nC_4 + \ldots + (n-3)\,{}^nC_{(n-4)} + (n-2)\,{}^nC_{(n-3)} + (n-1)\,{}^nC_{(n-2)} + n\,{}^nC_{(n-1)} + (n+1)\,{}^nC_n.$

$= (n+2)\,{}^nC_0 + (n+2)\,{}^nC_1 + (n+2)\,{}^nC_2 + (n+2)\,{}^nC_3 + (n+2)\,{}^nC_4 + \ldots$ up to n/2 terms.

$= \frac{1}{2}\,(n + 2)\,[2^n]$

$= (n + 2)\,2^{(n-1)}$ Proved.

EXERCISE # 2

Part-A Only single correct answer type questions

Q.1 Sixteen men compete with one another in running, swimming and riding. How many prize lists could be made if there were altogether 6 prizes of different values, one for running, 2 for swimming and 3 for riding ?

(A) 12902400 (B) 12904200

(C) 12092400 (D) None of these

Sol. **[A]**

Since any person may get one or more prizes in different categories

No. of ways of getting one prize for running = 16

No. of ways of getting 2 prizes for swimming = 16 × 15 (Since one person cannot get similar kind of prizes at a time)

No. of ways of getting 3 prizes for riding

= 16 × 15 × 14

∴ Total no. of ways (using multiplication rule)

= 16 × 16 × 15 × 16 × 15 × 14

= $(16)^3 \times (15)^2 \times 14$ = 12902400 Ans.

Q.2 Find the number of ways in which 16 sovereigns can be distributed between four persons if each of the four persons is to receive not less than 3.

(A) 53 (B) 35 (C) 45 (D) 54

Sol. **[B]**

Following possibilities may a rise

S. No.	Parson				No.of ways
	P_1	P_2	P_3	P_4	
1.	3	3	3	7	$\frac{4!}{3!} = 4$
2.	3	3	4	6	$\frac{4!}{2!} = 12$
3.	3	3	5	5	$\frac{4!}{2! \times 2!} = 6$
4.	3	4	4	5	$\frac{4!}{2!} = 12$
5.	4	4	4	4	$\frac{4!}{4!} = 1$

∴ Total no. of ways of distributing 16 sovereigns among 4 persons such that each person have to get at least 3 sovereigns.

= 4 + 12 + 6 + 12 + 1 = 35 Ans.

Q.3 There are p points in space, no four of which are in the same plane with the exception of q points which are all in the same plane. The number of different planes determined by the points is-

(A) $\frac{p(p-1)(p-2)}{6} - \frac{q(q-1)(q-2)}{6}$

(B) $\frac{p(p-1)(p-2)}{6} - \frac{q(q-1)(q-2)}{6} + 1$

(C) $\frac{p(p-1)(p-2)(P-3)}{24} - \frac{q(q-1)(q-2)(q-3)}{24} + 1$

(D) None of these

Sol. **[B]**

Points pare in space such that no four points are in the same plane. i.e. Three points may or may not be at the same plane. Also q are points which are in the same plane then, No of ways of drawing planes taking three

at time = ${}^pC_3 - {}^qC_3 + 1$

$= \frac{p!}{3!(p-3)!} - \frac{q!}{3!(q-3)!+1} + 1$

$= \frac{p(p-1)(p-2)(p-3)!}{3! \times (p-3)!} - \frac{q(q-1)(q-2)(q-3)!}{3!(q-3)!} + 1$

$= \frac{p(p-1)(p-2)}{6} - \frac{q(q-1)(q-2)}{6} + 1$

Q.4 A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed replacing the elements of P. A subset Q of A is again chosen. The number of ways of choosing P and Q so that P ∩ Q contains exactly two elements is-

(A) $9.\,{}^nC_2$ (B) $3^{n-2}.\,{}^nC_2$

(C) 2. ^nC_n (D) None of these

Sol. **[B]**

The two elements P and Q such that $P \cap Q$ can be chosen out of n is $^nC_2$ ways. Let a general element be a_i and must satisfy one of the following possibilities :

(i) $a_i \in P$ and $a_i \in Q$ (ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$ (iv) $a_i \notin P$ and $a_i \notin Q$

Let $a_1, a_2 \in P \cap Q$

only choice (i) $a_i \in P$ and $a_i \in Q$ satisfy $a_1, a_2 \in P \cap Q$ and three choices (ii), (iii) and (iv) for each of remaining (n – 2) elements.

$\therefore$ No of ways of remaining elements = $3^{(n-2)}$

Hence, No. of way sin which $P \cap Q$ contains exactly two elements = $^nC_2 \times 3^{(n-2)}$

Q.5 How many 7 digit numbers can be written using three digits 1, 2 and 3 under the condition that the digits 2 occurs twice in each number ?

(A) $^7P_2 \times 2^5$ (B) $^7C_2 \times 2^5$

(C) $^7P_2 \times 2^3$ (D) $^7C_2 \times 2^3$

Sol. **[B]**

× × × × × × ×

7 places can be chosen by fixing digit 2 at two places

$= {}^7C_5$

$= {}^7C_2$

But two remaining digits 1 and 3 can occupy 5 places in 2^5 ways.

$\therefore$ Required no. of ways = $^7C_2 \times 2^5$

Q.6 At an election, a voter may vote for any number of candidates not greater than the number to be chosen. There are 10 candidates and 5 members are to be chosen. The number of ways in which a voter may vote for at least one candidate is given by-

(A) 637 (B) 638 (C) 639 (D) 640

Sol. **[A]**

Given : There are 10 candidates of which 5 are to be chosen.

$\therefore$ required no. of ways

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5$

$= 10 + \frac{10!}{2!\times 8!} + \frac{10!}{3!\times 7!} + \frac{10!}{4!\times 6!} + \frac{10!}{5!\times 5!}$

$= 10 + \frac{10\times 9}{2} + \frac{10\times 9\times 8}{6} + \frac{10\times 9\times 8\times 7}{24}$

$+ \frac{10\times 9\times 8\times 7\times 6}{120}$

= 10 + 45 + 120 + 210 + 252 = 637 Ans.

$\therefore$ Option (A) is correct answer.

Q.7 In an examination of 9 papers a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is-

(A) 255 (B) 256 (C) 193 (D) 319

Sol. **[B]**

A candidate to be unsuccessful if he failed at least in 5 papers.

$\therefore$ No. of ways to be failed

$= {}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$

$= \frac{9!}{4!\times 5!} + \frac{9!}{3!\times 6!} + \frac{9!}{2!\times 7!} + 9 + 1$

$= \frac{9\times 8\times 7\times 6}{24} + \frac{9\times 8\times 7}{6} + \frac{9\times 8}{2} + 10$

= 9 × 14 + 12 × 7 + 36 + 10

= 126 + 84 + 46

= 210 + 46

= 256

$\therefore$ Option (B) is correct answer.

Q.8 In a certain test there are n questions. In this test 2^{n-i} student gave wrong answers to atleast i questions where i = 1, 2, 3 ..... n . If the total no. of wrong answer given is 2047. Then n is equal to-

(A) 10 (B) 11 (C) 12 (D) 13

Sol. **[B]**

A student may give at least one answer wrong in this way

No. of wrong answer to be given by a student

= Either one or two or three or …. or all

$= 2^{(n-1)} + 2^{(n-2)} + 2^{(n-3)} + \ldots\ldots + 2 + 2^0$

If form a G.P. having n terms.

$\Rightarrow 1.\frac{(2^n - 1)}{(2-1)} = 2047 \Rightarrow (2^n - 1) = 2047$

$\Rightarrow 2^n = 2048 = (2)^{11} \Rightarrow n = 11$

Q.9 All possible two-factor products are formed from the numbers 1, 2, ----------100. The number of factors out of the total obtained which are multiple of 3 is-

(A) 2211 (B) 4950

(C) 2739 (D) None of these

Sol. **[C]**

Total – (not multiple of 3)

${}^{100}C_2 - {}^{67}C_2$

$= \frac{100\times99}{2} - \frac{67\times66}{2} = 2739$

Q.10 In a hall there are 10 bulbs and their 10 buttons. In how many ways this hall can be enlightened?

(A) 10^2 (B) 1023 (C) 2^{10} (D) 10!

Sol. **[B]**

Given, 10 Buttons, 10 Bulbs

Required no. of ways to Enlighten all bulbs using either one button or Two buttons or three buttons or …………… or ten buttons.

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + \ldots + {}^{10}C_{10}$

$= {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_2 + {}^{10}C_3 + \ldots + {}^{10}C_{10} - {}^{10}C_0$

$= 2^{10} - 1$

$= 1024 - 1$

$= 1023$

∴ Option (B) is correct answer.

Q.11 The number of ways in which n distinct objects can be put into two identical boxes so, that no box remains empty is-

(A) $2^n - 2$ (B) $2^n - 1$

(C) $2^{n-1} - 1$ (D) $n^2 - 2$

Sol. **[C]**

In following way n distinct things can be distributed.

First Box	Second Box	
1	$(n-1)$	$\longrightarrow {}^nC_1$
2	$(n-2)$	$\longrightarrow {}^nC_2$
3	$(n-3)$	$\longrightarrow {}^nC_3$
4	$(n-4)$	$\longrightarrow {}^nC_4$
5	$(n-5)$	$\longrightarrow {}^nC_5$
⋮	⋮	
$(n-1)$	1	$\longrightarrow {}^nC_{n-1}$

∴ No. of ways $= \frac{1}{2}\,[{}^nC_1 + {}^nC_2 + \ldots + {}^nC_{n-1}]$

$= \frac{1}{2}\,[2^n - 2]$

$= 2^{(n-1)} - 1$

∴ Option (C) is correct answer.

Q.12 The number of ways in which 5 different balls can be placed in three different boxes so that no box remains empty -

(A) 243 (B) 150

(C) 153 (D) None of these

Sol. **[B]**

Following possibilities may be created.

Box_1	Box_2	Box_3	No of ways
1	1	3	${}^5C_1 \times {}^4C_1 \times {}^3C_3 \times \frac{3!}{2!}$
1	2	2	${}^5C_1 \times {}^4C_2 \times {}^2C_2 \times \frac{3!}{2!}$

∴ required no of ways.

$= 5 \times 4 \times 3 + 5 \times 6 \times 3$

$= 60 + 90$

$= 150$

∴ Option (B) is correct answer.

OR

Following formula can also be used.

No of ways $= r^n - {}^rC_1\,(r-1)^n + {}^rC_2\,(r-2)^n$ …...

$= 3^5 - {}^3C_1\,(3-1)^5 + {}^3C_2\,(3-2)^5$ …

$= 81 \times 3 - 3 \times 32 + 3$

$= 243 - 96 + 3$

$= 246 - 96$

$= 150$ Ans.

Q.13 The number of ways in which 10 different balls can be placed in three different boxes so that at least one box remains empty -

(A) $3(2^{10}-2)$ (B) $2^{10}-1$

(C) $3(2^{10}-1)$ (D) None of these

Sol. **[C]**

No. of ways of choosing at least one box of 3 boxes $= {}^3C_1 = 3$

10 different balls can be put in three boxes.

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + \ldots + {}^{10}C_{10} = (2^{10} - 1)$

∴ using multiplication rule, we get

No. of required ways $= 3 \times (2^{10} - 1)$

Q.14 The number of permutations which can be formed out of the letters of the word 'SERIES' taking three letters together is-

(A) 120 (B) 60

(C) 42 (D) None of these

Sol. $S \rightarrow 2, E \rightarrow 2, R \rightarrow 1, I \rightarrow 1$

when three different letters then

Permu = ${}^4C_3 \lfloor 3 = 24$

when two same one different then

permutation = ${}^2C_1\ {}^3C_1 \frac{\lfloor 3}{2} = 18$

Total = 24 + 18 = 42

Q.15 The streets of a city are arranged like the lines of a chess board. There are m streets running North to South & 'n' streets running East to West. The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is-

(A) $\sqrt{m^2 + n^2}$ (B) $\sqrt{(m-1)^2.(n-1)^2}$

(C) $\frac{(m+n)!}{m!.n!}$ (D) $\frac{(m+n-2)!}{(m-1)!.(n-1)!}$

Sol.[D] This can be done he cross (m – 1) reads and (n – 1) reads so total reads = m + n – 2

Total ways = ${}^{m+n-2}C_{m-1} . {}^{n-1}C_{n-1}$

$$= \frac{\lfloor m+n-2}{\lfloor m-1 \lfloor n-1}$$

Q.16 5 Indian and 5 American couples meet at a party and shake hands. If no wife shakes hands with her husband and no Indian wife shakes hands with a male then find the number of hands shakes that take place in the party?

(A) 135 (B) 137

(C) 139 (D) None of these

Sol. **[A]**

Q.17 A box contains 6 balls which may be all of different colours or three each of two colours or two each of three different colours. The number of ways of selecting 3 balls from the box (if ball of same colour are identical), is-

(A) 60 (B) 31

(C) 30 (D) none

Sol. **[B]**

When all 6 balls are different colour then ways for selecting 3 balls = ${}^6C_3 = 20$

when 3 balls one colour and 3 are other then ways selecting 3 balls = ${}^2C_1\ {}^1C_1 + {}^2C_1 = 4$

when three group of 2 balls each in same colour then ways for selecting three balls

= ${}^3C_3 + {}^3C_1 . {}^2C_1 = 7$

Total ways = 20 + 4 + 7 = 31 ways

Q.18 Number of ways in which 2 Indians, 3 Americans, 3 Italians and 4 Frenchmen can be seated on a circle, if the people of the same nationality sit together is-

(A) $2 . (4!)^2 (3!)^2$ (B) $2 . (3!)^3 . 4!$

(C) $2 . (3!) (4!)^3$ (D) None of these

Sol.[B] If sit one Indian fix then ways for Indian = 2

ways for seated 3 Americans, 3 Italians and 4 Frenchmen = $\lfloor 3 \lfloor 3 \lfloor 3 \lfloor 4 = (\lfloor 3)^3 \lfloor 4$

Total ways = $2(\lfloor 3)^3 \lfloor 4$

Q.19 Number of ways in which all the letters of the word "ALASKA" can be arranged in a circle distinguishing between the clockwise and anticlockwise arrangement is-

(A) 60 (B) 40

(C) 20 (D) None of these

Sol.[C] ALASKA

Total word = 6

Here are 3 A's so in circle we put in the way

$= \frac{\lfloor 5}{\lfloor 3} = 20$

Q.20 Let P_n denotes the number of ways of selecting 3 people out of 'n' sitting in a row if no two of them are consecutive and Q_n is the corresponding figure when they are in a circle. If $P_n - Q_n = 6$, then 'n' is equal to-

(A) 8 (B) 9 (C) 10 (D) 12

Sol.[C] $P_n = {}^{n-2}C_3 \Rightarrow Q_n = {}^{n-4}C_2 \times \frac{n}{3} \Rightarrow P_n - Q_n = 6$

$\Rightarrow {}^{n-2}C_3 - {}^{n-4}C_2 \times \frac{n}{3} = 6 \Rightarrow n = 10$

Q.21 The number of ways selecting 8 books from a library which has 10 books each of Mathematics, Physics, Chemistry and English, if books of the same subject are alike, is-

(A) $^{13}C_4$ (B) $^{13}C_3$

(C) $^{11}C_4$ (D) $^{11}C_3$

Sol.[D] Coeff. of x^8 in $(x^0 + x^1 + \ldots\ldots + x^{10})^4$

$= \text{coeffi. of } x^8 \text{ in } \left(\frac{1-x^{11}}{1-x}\right)^4$

$= \text{coeffi of } x^8 \text{ in } (1 - x^{11})^4 (1 - x)^{-4}$

$= {}^{8+4-1}C_{4-1} = {}^{11}C_3$

Part-B **One or more than one correct answer type questions**

Q.22 There are n seats round a table numbered 1, 2, 3,.........,n. The number of ways in which m (≤n) persons can take seats is -

(A) ^nP_m (B) $^nC_m \times (m-1)!$

(C) $^{n-1}P_{m-1}$ (D) $^nC_m \times m!$

Sol. [A,D]

Required no. of ways

= (m persons can be seated in n seats in ^nC_m ways) × (m persons can be arranged in m! ways)

$= {}^nC_m \times m!$, $= {}^nP_m$

∴ Option (A) and (D) are correct answers.

Q.23 Kanchan has 10 friends among whom two are married to each other. She wishes to invite 5 of them for a party. If the married couple refuse to attend separately then the number of different ways in which she can invite five friends is -

(A) $^8C_5$ (B) $2 \times {}^8C_3$

(C) $^{10}C_5 - 2 \times {}^8C_4$ (D) None of these

Sol. [B,C]

Given : 10 friends among whom 1 married couple.

5 friends can be invited as = $^{10}C_5$

If only husband or wife invited among friends

no. of ways = $2 \times {}^{9-1}C_{5-1}$

$= 2 \times {}^8C_4$

∴ Required no. of ways in which husband and wife do not of attends separately = $^{10}C_5 - 2 \times {}^8C_4$

∴ Option (C) is correct answer.

Also, If married couple always included, then no. of ways = $^{10-2}C_{5-2} = {}^8C_3$

If married couple always exuded

no. of ways = $^{(10-2)}C_5 = {}^8C_5 = {}^8C_3$

∴ Required no. of ways = $^8C_3 \times 2$

∴ Option (B) is also correct answer.

Q.24 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and let $\vec{r}$ be a variable vector such that $\vec{r}\cdot\hat{i}$, $\vec{r}\cdot\hat{j}$ and $\vec{r}\cdot\hat{k}$ are positive integers. If $\vec{r}\cdot\vec{a} \le 12$, then the number of values of $\vec{r}$ is-

(A) $^{12}C_9 - 1$ (B) $^{12}C_3$

(C) $^{12}C_9$ (D) None of these

Sol. [B, C]

Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Let $\vec{r} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{r}.\vec{a} \le 12$

$\Rightarrow (a_1\hat{i} + b_1\hat{j} + c_1\hat{k})(\hat{i} + \hat{j} + \hat{k}) \le 12$

$\Rightarrow a_1 + b_1 + c_1 \le 12$

Since, a_1, b_1, c_1 are +ve integers.

∴ No. of values of $\vec{r} = {}^{12}C_3 = {}^{12}C_9$

∴ Option (B) and (C) are correct answer

Q.25 The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \le n$ (where n is a positive integer) is-

(A) $^{n+3}C_3$ (B) $^{n+4}C_4$

(C) $^{n+5}C_5$ (D) $^{n+4}C_n$

Sol.[B, D] $x_1 + x_2 + x_3 + x_4 + p = n$

non negative integral solution

$= {}^{n+5-1}C_{5-1} = {}^{n+4}C_4$ or $^{n+4}C_n$

Q.26 You are given 8 balls of different colour (black, white ....). The number of ways in which these balls can be arranged in a row so that the two balls of particular colour (say red and white) may never come together is-

(A) 8 ! – 2.7 ! (B) 6 . 7 !

(C) 2 . 6 ! . $^7C_2$ (D) 840

Sol.[A,B,C] Total balls = 8

ways to arranged 8 balls = $\lfloor 8$

Red and white are together then ways = $2\lfloor 7$

ways when red and white are not together

$= |\underline{8-2}\ |\underline{7}$

Again when we arrange 6 balls is $|\underline{6}$ ways

Then Red ball put in space in 7 ways and now white ball we can put only 6 ways

Total ways = $6.7\ .\ |\underline{6} = 6\ .\ |\underline{7}$

Again we put 6 balls in $|\underline{6}$ ways

Now seven space in between then

So we arrange 2 balls in space = ${}^7C_2\ |\underline{2}$

Total ways = $2.|\underline{6}\ {}^7C_2$

Total ways = $6.|\underline{7} = 6 \times 840 = 5040$

Q.27 Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3,........ n is-

(A) $\left(\frac{n-1}{2}\right)^2$ if n is even

(B) $\frac{n(n-2)}{4}$ if n is odd

(C) $\frac{(n-1)^2}{4}$ if n is odd

(D) $\frac{n(n-2)}{4}$ if n is even

Sol.[C,D]

a + c = 2b

a + c = even no.

no. of ways of selecting 3 nos. such that they are in A.P. equals to no. of ways of selecting 2 nos. such that their sum is even then third no. can be selected in one ways

even-even or 0 0 | n is even

n = 2k

n is odd $\ {}^kC_2 + {}^{k+1}C_2$ | even-even or odd odd

(2k + 1)

$= k^2$ | ${}^kC_2 + {}^kC_2$

$= \boxed{\left(\frac{n-1}{2}\right)^2}$ | $= k(k-1)$

$= \boxed{\frac{n(n-2)}{4}}$

Part-C Assertion-Reason type questions

The following questions consist of two statements each, printed as Statement-1 and Statement-2. While answering these questions you are to choose any one of the following four responses.

(A) If both Statement-1 and Statement-2 are true and the Statement-2 is correct explanation of the Statement-1.

(B) If both Statement-1 and Statement-2 are true but Statement-2 is not correct explanation of the Statement-1.

(C) If Statement-1 is true but the Statement-2 is false.

(D) If Statement-1 is false but Statement-2 is true.

Q.28 **Statement-1 :** The maximum number of points of intersection of 8 circles of unequal radii is 56.

Statement-2 : The maximum number of points into which 4 circles of unequal radii and 4 non coincident straight lines intersect, is 50.

Sol. [B] Maximum points of intersection of 8 circle

$= 2\ .\ {}^8C_2 = 2\ .\ \frac{8.7}{2} = 56$

Statement 1 is correct

intersection point of 4 circle and 4 straight lines

$= 2\ .\ {}^4C_2 + 2\ .\ {}^4C_1\ .\ {}^4C_1 + {}^4C_2$

= 12 + 32 + 6 = 50

Statement 2 is correct

But 2 is not correct explanation of 1

Q.29 **Statement-1 :** If there are six letters $L_1, L_2, L_3, L_4, L_5, L_6$ and their corresponding six envelopes $E_1, E_2, E_3, E_4, E_5, E_6$. Letters having odd value can be put into odd value envelopes and even value letters can be put into even value envelopes, so that no letter go into the right envelopes the number of arrangement will be equal to 4.

Statement-2 : If P_n number of ways in which n letter can be put in 'n' corresponding envelopes such that no letters goes to correct envelopes then $P_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} + ... + \frac{(-1)^n}{n!}\right)$

Sol. [A] n letters put in n envelopes then no letter goes to correct envelope then number of ways

$$P_n = \lfloor n \left(1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \ldots..(-1)^n \frac{1}{\lfloor n}\right)$$

So three letters can be put so that no letter go to correct envelope

$$= \lfloor 3 \left(1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3}\right)$$

$= (6 - 6 + 3 - 1) = 2$

and other three can be put in 2 ways

Total ways = 4

option A is correct

Q.30 **Statement-1 :** The maximum value of K such that $(50)^k$ divides 100 ! is 2.

Statement-2 : If P is any prime number, then power of P in n! is equal to

$$\left[\frac{n}{P}\right] + \left[\frac{n}{P^2}\right] + \left[\frac{n}{P^3}\right] + \ldots.$$

where [.] represents greatest integer function.

Sol. [D] $100! = 2^{\alpha}3^{\beta}5^{\gamma}7^{\delta}$……..

$$\alpha = \left[\frac{100}{2}\right] + \left[\frac{100}{4}\right] + \left[\frac{100}{8}\right] + \left[\frac{100}{16}\right] + \left[\frac{100}{32}\right] + \left[\frac{100}{64}\right] + \left[\frac{100}{128}\right] + \ldots..$$

$= 50 + 25 + 12 + 6 + 3 + 1$

$= 97$

$$\gamma = \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right]$$

$= 20 + 4$

$= 24$

$100! = 2^{97} . 3^{\beta} . 6^{24} . 7^{\delta}$ …..

$= 2^{97} . 3^{\beta} . (25)^{12} . 7^{\delta}$ …..

$= 2^{85} . 3^{\beta} (50)^{12}. 7^{\delta}$

$k = 12$

Q.31 **Statement-1 :** The number of ordered pairs (m, n); m, n ∈ {1, 2, 3, ... 20) such that $3^m + 7^n$ is a multiple of 10, is equal to 100.

Statement-2 : $3^m + 7^n$ has last digit zero, when m is of 4k + 2 type and n is of 4λ type where k, λ ∈ W.

Sol. [B] {1, 2, 3, …… 20}

3^1	3	A = {1, 5, 9, 13, 17}
3^2	9	B = {2, 6, 10, 14, 18}
3^3	7	C = {3, 7, 11, 15, 18}
3^4	1	D = {4, 8, 12, 16, 20}

7^1	7	A = {1, 5, 9, 13, 17}
7^2	9	B = {2, 6, 10, 14, 18}
7^3	3	C = {3, 7, 11, 15, 19}
7^4	1	D = {4, 8, 12, 16, 20}

$3^n + 7^n$

m	n		
A	A	$^5C_1 . {}^5C_1$	
B	D	$^5C_1 . {}^5C_1$	= 100
C	C	$^5C_1 . {}^5C_1$	
D	B	$^5C_1 . {}^5C_1$	

Part-D Column Matching type questions

Q.32 Consider the word "HONOLULU"

Column-I	**Column-II**
(A) Number of words that can be formed using the letters of the given word in which consonants & vowels are alternate is	(P) 26
(B) Number of words that can be formed without changing the order of vowels is	(Q) 144
(C) Number of ways in which 4 letters can be selected from the letters of the given word is	(R) 840
(D) Number of words in which two O's are together but U's are separated is	(S) 90

Sol. A → Q ; B → R ; C → P ; D → S

(A) HONOLULU

consonants and vowels are alternates

then words $= 2. \frac{\lfloor 4}{\lfloor 2}.\frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 144$

(B) Number of words we can formed without changing the order of vowels is

$$= \frac{\lfloor 8}{\lfloor 2 \lfloor 2 \lfloor 2 . \frac{\lfloor 4}{\lfloor 2 \lfloor 2}} = \frac{\lfloor 7}{\lfloor 3} = 7.6.5.4 = 840$$

(C) H → 1, O → 2, N → 1, L → 2, U → 2

(i) Four are different $= {}^5C_4 = 5$

(ii) Two same two different $= {}^3C_1 . {}^4C_2 = 18$

(iii) Two same two same $= {}^3C_2 = 3$

Total ways = 5 + 18 + 3 = 26

(D) Number of words when two O′s are together but two U′s are separate is

$$= \frac{\lfloor 7}{\lfloor 2 \lfloor 2} - \frac{\lfloor 6}{\lfloor 2} = 900$$

Q.33

Column-I	**Column-II**
(A) The total number of selections of fruits which	(P) Greater than 50

can be made from, 3 bananas, 4 apples and 2 oranges is

(B) If 7 points out of 12 are in the same straight line, then the number of triangles formed is — (Q) Greater than 100

(C) The number of ways of selecting 10 balls from unlimited number of red, black, white and green balls is — (R) Greater than 150

(D) The total number of proper divisors of 38808 is — (S) Greater than 200

Sol. A → P ; B → P,Q,R ; C → P,Q,R,S ; D → P

(A) Solution of x + y + z = 9

$= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$

(B) Total triangle $= {}^{12}C_3 - {}^{7}C_3$

$= 220 - 35 = 185$

(C) Coeffi of x^{10} in $(x^0 + x^1 + x^2 + \ldots\ldots)^4$

$= \text{coeffi of } x^{10} \text{ in } \left(\frac{1}{1-x}\right)^4$

$= \text{coeffi of } x^{10} \text{ in } (1-x)^{-4}$

$= {}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

(D) Factor of 38808 is

$= 2^3 . 3^2 . 7^2 . 11$

divisors = (3 + 1) (2 + 1) (2 + 1) (1 + 1) = 72

proper divisors = 72 – 2 = 70

Q.34 **Column-I** — **Column-II**

(A) Number of 4 letter words that can be formed using the letter of the words AABBCDEFG. — (P) $\frac{11!}{3!}$

(B) Number of ways of selecting 3 persons out of 12 sitting in a row, if no two selected persons were sitting together, is — (Q) 1206

(C) Number of solutions of the equation x + y + z = 20, where $1 \le x < y < z$ and $x, y, z \in I$, is — (R) 24

(D) Number of ways in which Indian team can bat, if Yuvraj wants to bat before Dhoni and Pathan wants to bat after Dhoni is (assume all the batsman bat) — (S) 120

Sol. A → Q; B → S; C → R; D → P

Q.35 **Column-I** — **Column-II**

(A) The number of five-digit numbers having the product of digits 20 is — (P) 77

(B) n dice are rolled. The number of possible outcomes in which at least one of the dice shows an even number is 189, then n^4 = — (Q) 81

(C) The number of integer between 1 & 1000 inclusive in which atleast two consecutive digits are equal is — (R) 50

(D) The value of $\frac{1}{15}\sum_{1\le i\le j\le 9}\sum i.j$ — (S) 181

Sol. A → R; B → Q; C → S; D → P

Q.36 No. of ways to arrange n balls into r different boxes is

Column 1

(A) When balls are distinct and empty boxes are not allowed.

(B) When balls are distinct and empty boxes are allowed.

(C) When balls are identical and empty boxes are not allowed.

(D) When balls are identical and empty boxes are allowed.

Column 2

(P) ${}^{n-1}C_{r-1}$

(Q) ${}^{n+r-1}C_{r-1}.n!$

(R) ${}^{n+r-1}C_{r-1}$

(S) ${}^{n-1}C_{r-1}.n!$

Sol. **(A)-S, (B)-Q, (C)-P, (D)-R**

We know, No of ways of n distinct objects are arranged into r groups $= {}^{(n+r-1)}P_n$ or $n!\ {}^{(n-1)}C_{(r-1)}$ according as blank group are or are not allowed.

Also, no of ways of n identical objects can be distributed into r groups $= {}^{(n+r-1)}C_{(r-1)}$ or ${}^{(n-1)}C_{(r-1)}$ according as blank groups are or are not allowed.

(i) No of ways when balls are distinct and empty boxes are not allowed $= n!\ {}^{(n-1)}C_{(r-1)}$

∴ (i) is matched with (D)

(ii) No of ways when balls are distinct and empty boxes are allowed $= {}^{(n+r-1)}P_n$

$$= \frac{(n+r-1)!}{(r-1)!} \times \frac{n!}{n!} = {}^{(n+r-1)}C_{(r-1)} \times n!$$

∴ (ii) is matched with (B).

(iii) No of ways when balls are identical and empty boxes are not allowed = ${}^{(n-1)}C_{(r-1)}$

$\therefore$ (iii) is matched with (A)

(iv) No of ways when balls are identical and empty boxes are allowed = ${}^{(n+r-1)}C_{(r-1)}$

$\therefore$ (iv) is matched with (C)

Q.37 The number of ways in which, 2n things of one sort, 2n things of another sort and 2n of a third sort can be divided between two person so that each get 3n things is -

(A) $3n^2 + 3n - 1$ (B) $3n^2 - 3n + 1$

(C) $3n^2 + 3n + 1$ (D) None of these

Sol. **[C]** Using multinomial theorem, we get

No of ways = coefficient of x^{3n} in $(1 + x + x^2 + x^3 + \ldots + x^{2n})^3$

= coefficient of x^{3n} in $\left(\frac{1-x^{2n+1}}{1-x}\right)^3$

= coefficient of x^{3n} in $(1 - x^{2n+1})^3 (1 - x)^{-3}$

= coefficient of x^{3n} in $[1 - x^{3(2n+1)} - 3x^{(2n+1)} (1-x)^{2n+1}] (1-x)^{-3}$

= coefficient of x^{3n} in $[1 - x^{3(2n+1)} - 3x^{(2n+1)} + 3x^{2(2n+1)}]$

$1 + {}^3C_1\, x + {}^4C_2\, x^2 + {}^5C_3\, x^3 + {}^6C_4\, x^4 + \ldots + {}^{(n+1)}C_{(n-1)}\, x^{(n-1)} + \ldots + {}^{(3n+2)}C_{3n}\, x^{3n} + \ldots..]$

$= {}^{(3n+2)}C_{3n} - 3\ {}^{(n+1)}C_{(n-1)}.$

$= \frac{(3n+2)}{3n! \times 2!} - 3\ \frac{(n+1)!}{2! \times (n-1)!}$

$= \frac{1}{2}[(3n+2)(3n+1) - 3n(n+1)]$

$= \frac{1}{2}[9n^2 + 6n + 2]$

$= \frac{1}{2}[6n^2 + 6n + 2]$

$= 3n^2 + 3n + 1$

Q.38 There are n straight lines in a plane, no two of which are parallel, and no three pass through the same point. Their points of intersection are joined. Then the number of fresh lines thus obtained is-

(A) $\frac{n(n-1)(n-2)}{8}$

(B) $\frac{n(n-1)(n-2)(n-3)}{6}$

(C) $\frac{n(n-1)(n-2)(n-3)}{8}$

(D) None of these

Sol. [C] Total points $\equiv {}^nC_2 \,.\, (1) = \frac{n(n-1)}{2}$ = P(say)

fresh lines = Total old lines

${}^PC_2 - (n)\ {}^{n-1}C_2 \rightarrow$ each old line each points on it

$= \frac{n(n-1)(n-2)(n-3)}{8}$

Q.39 In a conference 10 speakers are present. If S_1 wants to speak before S_2 and S_2 wants to speak after S_3, then the number of ways all the 10 speakers can give their speeches with the above restriction if the remaining seven speakers have no objection to speak at any number is-

(A) ${}^{10}C_3$ (B) ${}^{10}P_3$ (C) ${}^{10}P_8$ (D) $\frac{10!}{3}$

Sol.[D] S_1, S_2, S_3 speak in the way $S_3\ S_1\ S_2$

Total speakers = 10

Required number = $\frac{\lfloor 10}{\lfloor 3} = {}^{10}P_7$

Q.40 In how many ways can the letters of the word ARRANGE be arranged so that

(i) The two R' s are never together

(A) 900 (B) 872

(C) 960 (D) 980

(ii) The two A's are together but not the two R's are together.

(A) 240 (B) 260

(C) 300 (D) 340

(iii) Neither two A's nor the two R's are together.

(A) 660 (B) 600

(C) 640 (D) 690

Sol. (i) Total word = $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$ = 1260

two R's are together = $\frac{\lfloor 6}{\lfloor 2}$ = 360

Required ways = 1260 – 360 = 900

(ii) Two A's are together ways = $\frac{\lfloor 6}{\lfloor 2}$ = 360

and two A′s are together and two R′s are together then ways = $\lfloor 5$ = 120

Required ways = 360 – 120 = 240

(iii) Neither two A′s nor two R′s are together then total ways

= 1260 – (360 + 360 – 120) = 1260 – 600 = 660

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 The straight lines λ_1, λ_2 & λ_3 are parallel & lie in the same plane. A total of m points are taken on the line λ_1, n points on λ_2 & k points on λ_3. How many maximum number of triangles are there whose vertices are at these points?

Sol. ${}^{m+n+k}C_3 - ({}^mC_3 + {}^nC_3 + {}^kC_3)$

Q.2 Find the sum of the odd numbers of five digits that can be made with the digits 0, 1, 4, 5, 4.

Sol. **708894**

Q.3 All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once, and not divisible by 5 are arranged in the increasing order. Find the $(2004)^{th}$ number in this list.

Sol. 4316527

Q.4 In how many ways can a team of 6 horses be selected out of a stud of 16, so that there shall always be 3 out of ABCA′ B′ C′, but never AA′, BB′ or CC′ together.

Sol. 960

Q.5 Find the sum of all numbers greater than 10, 000 by using the digits 0, 2, 4, 6, 8 no digit being repeated in any number.

Sol. Given digits : 0, 2, 4, 6, 8

sum of all numbers having digits 0, 2, 4, 6, 8

$= (5-1)\,!\,(0+2+4+6+8) \times \frac{(10^5-1)}{9}$

$= 4\,! \times 20 \times \frac{(100000-1)}{9} = 4\,! \times 20 \times \frac{99999}{9}$

$= 24 \times 20 \times 11111 = 5333280$

sum of all numbers having digits 2, 4, 6, 8

$= (4-1)\,!\,(2+4+6+8) \times \frac{(10^4-1)}{9}$

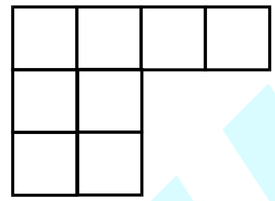
$= 3\,! \times 20 \times \frac{(10000-1)}{9} = 6 \times 20 \times \frac{9999}{9}$

$= 120 \times 1111 = 133320$

$\therefore$ sum of all numbers having five digits

$= 5333280 - 133320 = 5199960$ Ans.

Q.6 In how many different ways can the letters of the word KUMARI be placed in the 8 boxes of the given figure so that no row remains empty?



Sol. KUMARI have 6 different letter

No. of their arrangements in boxes so that each contains at least one letter

= Coefficient of x^6 in $6\,!\,({}^4C_1\,x + {}^4C_2\,x^2 + {}^4C_3\,x^3 + {}^4C_4\,x^4) \times ({}^2C_1\,x + {}^2C_2\,x^2)^2$

= coefficient of x^6 in $6\,!\,(4x + 6x^2 + 4x^3 + x^4) \times (2x + x^2)^2$

= coefficient of x^6 in $6\,!\,(4x + 6x^2 + 4x^3 + x^4) \times (4x^2 + x^4 + 4x^3)$

$= 6\,! \times (6 + 16 + 4) = 6\,! \times 26$

$= 720 \times 26 = 18720$ Ans.

Q.7 A badminton club has 10 couples as members. They meet to organise a mixed double match. If each wife refuse to partner as well as oppose her husband in the match then in how many different ways can the match be arranged ?

Sol. Given

C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8 C_9 C_{10}

H_1 H_2 H_3 H_4 H_5 H_6 H_7 H_8 H_9 H_{10}

W_1 W_2 W_3 W_4 W_5 W_6 W_7 W_8 W_9 W_{10}

C_1 , C_2 …. C_{10} Indicate couples.

H_1 , H_2 …. H_{10} Indicate husbands.

W_1 , W_2 …. W_{10} Indicate wife's.

No. of ways of selecting two husband out of 10 ${}^{10}C_2$

But their wives is to be excluded

Hence, Remaining wives can be selected as

No. of ways = ${}^8C_2$

But two different husband and wives can interchange their sides in 2! ways.

$\therefore$ Required no, of ways = ${}^{10}C_2 \times {}^8C_2 \times 2!$

$= \frac{10!}{2! \times 8!} \times \frac{8!}{2! \times 6!} \times 2!$

$= \frac{10 \times 9 \times 8!}{2! \times 8!} \times \frac{8 \times 7 \times 6!}{2! \times 6!} \times 2!$

$= 45 \times 56 = 2520$ Ans.

Q.8 There are 3 different books on Maths, 4 different on Physics and 5 different on English. How many different collection can be made such that each collection consists of:

(i) One book of each subject
(ii) At least one book of each subject
(iii) At least one book of English

Sol. 3 Maths books + 4 Physics books + 5 English books
(i) No. of ways of having one book of each subject
$= {}^3C_1 \times {}^4C_1 \times {}^5C_1$
$= 3 \times 4 \times 5 = 60$ Ans.
(ii) No. of ways of at least one book of each subject $= ({}^3C_1 + {}^3C_2 + {}^3C_3) \times ({}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4) \times ({}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5)$
$= (3 + 3 + 1) \times (4 + 6 + 4 + 1) \times (5 + 10 + 10 + 5 + 1)$
$= 7 \times 15 \times 31$
$= 31 \times 105$
$= 3255$ Ans.
(iii) No. of ways at least one book of English.
$= ({}^5C_1 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5) \times ({}^7C_0 + {}^7C_1 + {}^7C_1 + {}^7C_2 + {}^7C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7)$
$= (2^5 - 1) \times 2^7 = 128 \times 31 = 3968$ Ans

Q.9 The Indian cricket team with eleven players, the team manager, the physiotherapist and two umpires are to travel from the hotel where they are staying to the stadium where the test match is to be played. Four of them residing in the same town own cars, each a four seater which they will drive themselves. The bus which was to pick them up failed to arrive in time after leaving the opposite team at the stadium. In how many ways can they be seated in the cars ? In how many ways can they travel by these cars so as to reach in time, if the seating arrangement in each car is immaterial and all the cars reach the stadium by the same route.

Sol. $12!;\ \dfrac{11!\ 4!}{(3!)^4\ 2!}$

Q.10 To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, find the number of ways in which the selection can be made.

Sol. Given :
12 vacancies to be filled.
25 candidates = 20 general category + 5 scheduled caste
Three seats are reserved for 5C candidates
3 5 C candidates can be selected $= {}^5C_3$
9 vacancies to be filled from (20 General category + 2 5C category candidates)
$\therefore$ No. of selections $= {}^5C_3 \times {}^{22}C_9$
$= 4974200$ Ans.

Q.11 (A) Out of 18 points in a plane, no three are in the same straight line except five points which are collinear. How many
(i) straight lines
(ii) triangles can be formed by joining them ?

Sol. Given 18 points in a plane of which 5 are collinear.
(i) No. of straight lines $= {}^{18}C_2 - {}^5C_2 + 1$
$$= \frac{18!}{2! \times 16!} - \frac{5!}{2! \times 3!} + 1$$
$$= \frac{18 \times 17 \times 16!}{2! \times 16!} - \frac{5 \times 4 \times 3!}{2! \times 3!} + 1$$
$= 153 - 10 + 1$
$= 154 - 10 = 144$ Ans.
(ii) No. of triangles formed $= {}^{18}C_3 - {}^5C_3$
$$= \frac{18!}{3! \times 15!} - \frac{5!}{3! \times 2!}$$
$$= \frac{18 \times 17 \times 16 \times 15!}{3! \times 15!} - \frac{5 \times 4 \times 3!}{3! \times 2!}$$
$= 3 \times 17 \times 16 - 10 = 51 \times 16 - 10 = 806$ Ans.

Q.12 There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The number of games that the men played between themselves exceeded by 66 as compared to the number of games that the men played with the women. Find the number of participants & the total numbers of games played in the tournament.

Sol. 13, 156

Q.13 A firm of Chartered Accountants in Bombay has to send 10 clerks to 5 different companies, two clerks in each. Two of the companies are in Bombay and the others are outside. Two of the clerks prefer to work in Bombay while three others prefer to work outside. In how many ways can the assignment be made if the preferences are to be satisfied?

Sol. 5400

Q.14 Out of 3n consecutive numbers, the number of ways in which 3 numbers can be selected such that their sum is divisible by 3 is $\frac{3n^2 - 3n + 2}{2}.n$

Sol. In 3n consecutive no. there are
n nos. of the form 3k + 1 = A
n nos. of the form 3k + 2 = B
& n nos. of the form 3k = C
Three nos. divisible by 3 can come in following by manner
C C C or B B B or A A A or A B C
$^nC_3 + {}^nC_3 + {}^nC_3 + {}^nC_1 . {}^nC_1 . {}^nC_1$
$= 3\,{}^nC_3 + n^3$
$= \frac{n}{2}(3n^2 - 3n + 2)$

Q.15 In how many ways can you divide a pack of 52 cards equally among 4 players? In how many ways the cards can be divided in 4 sets, 3 of them having 17 cards each & the 4^{th} with 1 card.

Sol. $\frac{52!}{(13!)^4}$; $\frac{52!}{3!\,(17!)^3}$

Q.16 A train going from Cambridge to London stops at nine intermediate stations. 6 persons enter the train during the journey with 6 different tickets of the same class. How many different sets of ticket may they have had?

Sol. $^{45}C_6$

Q.17 A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time, or any combination of 1's and 2's. Find the total number of ways in which the person can go up the stair.

Sol. 89

Q.18 How many integers between 1000 and 9999 have exactly one pair of equal digit such as 4049 or 9902 but not 4449 or 4040?

Sol. 3888

Q.19 There are 20 books on Algebra & Calculus in our library. Prove that the greatest number of selections each of which consists of 5 books on each topic is possible only when there are 10 books on each topic in the library.

Q.20 Find the number of words of 5 letters that can be made with the letters of the word PROPOSITION.

Sol. PROPOSITION
Total letters = 11
P = 2
O = 3
I = 2
R = 1
S = 1
T = 1
N = 1
No. of permutations = coefficient of x^5 in 5 !
$(1 + x)^4 \times 5\,(1 + x + x^2/2)^2\,(1 + x + \frac{x^2}{2} + \frac{x^3}{6})$
= coefficient of x^5 in $5!\,(1 + x)^4 \times [\frac{13}{12}x^5 + \frac{31}{12}x^4 + \frac{25}{6}x^3 + \frac{9}{2}x^2 + 3x + 1]$
$= 5!\,[\frac{13}{12} + \frac{31}{12} \times {}^4C_1 + \frac{25}{6} + \frac{9}{2}\,{}^4C_3 + 3 \times {}^4C_4]$
$= 120\,[\frac{13}{12} + \frac{124}{12} + \frac{25}{6} \times 6 + \frac{9}{2} \times 4 + 3]$
$= 120\,[\frac{13}{12} + \frac{124}{12} + \frac{300}{12} + \frac{216}{12} + 3]$
= 10 [13 + 124 + 300 + 216 + 36] = 6890 Ans.

Part-B Passage based objective questions

PASSAGE - I (Q. 21 to 23)

There are 8 official 4 non-official members, out of these 12 members a committee of 5 members is to be formed, then answer the following questions.

Q.21 Number of committees consisting of 3 official and 2 non-official members are-

(A) 363 (B) 336

(C) 236 (D) 326

Sol. [B] Number of ways

$= {}^8C_3 \,.\, {}^4C_2 = 56 \times 6 = 336$

Q.22 Number of committees consisting of at least two non-official members, are-

(A) 456 (B) 546

(C) 654 (D) 466

Sol. [A] Number of ways

$= {}^8C_3 \,.\, {}^4C_2 + {}^8C_2\, {}^4C_3 + {}^8C_1\, {}^4C_4$

$= 56 \times 6 + 28 \times 4 + 8 \times 1 = 336 + 112 + 8 = 456$

Q.23 Number of committees in which a particular official member is never included, are-

(A) 264 (B) 642

(C) 266 (D) 462

Sol. [D] Number of ways $= {}^{11}C_5 = 462$

PASSAGE - 2 (Q. 24 to 26)

2 American men; 2 British men; 2 Chinese men and one each of Dutch, Egyptian, French and German persons are to be seated for a round table conference.

Q.24 If the number of ways in which they can be seated if exactly two pairs of persons of same nationality are together is p(6!), then find p.

(A) 60 (B) 62

(C) 64 (D) None of these

Sol. **[A]**

Q.25 If the number of ways in which only American pair is adjacent is equal to q(6!), then find q.

(A) 61 (B) 62

(C) 63 (D) 64

Sol. **[D]**

Q.26 If the number of ways in which no two people of the same nationality are together given by r(6!), find r.

(A) 241 (B) 242

(C) 243 (D) 247

Sol. **[D]**

PASSAGE - III (Q. 27 to 29)

Consider the letters of the word MATHEMATICS. There are eleven letters some of them are identical. Letters are classified as repeating and non-repeating letters. Set of repeating letters = {M, A, T}. Set of non-repeating letters = {H, E, I, C, S}

Q.27 Possible number of words taking all letters at a time such that atleast one repeating letter is at odd position in each word, is-

(A) $\dfrac{9!}{2!\,2!\,2!}$ (B) $\dfrac{11!}{2!\,2!\,2!}$

(C) $\dfrac{11!}{2!\,2!\,2!} - \dfrac{9!}{2!\,2!}$ (D) $\dfrac{9!}{2!\,2!}$

Sol.[B] Here M, A, T repenting letter and even places are 5 so in all word one letter taking always odd position

$\Rightarrow$ Number of ways $= \dfrac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2}$

Q.28 Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together, is-

(A) $7\,!,\ {}^8C_2$ (B) $\dfrac{11!}{2!\,2!\,2!} - \dfrac{10!}{2!\,2!}$

(C) $\dfrac{6!\,4!}{2!\,2!}$ (D) $\dfrac{9!}{2!\,2!\,2!}$

Sol. [A] Number of ways

$= \dfrac{\lfloor 9}{\lfloor 2} - \lfloor 8 = 7 \,.\, \dfrac{\lfloor 8}{\lfloor 2} = \lfloor 7 \dfrac{\lfloor 8}{\lfloor 2 \lfloor 6} = \lfloor 7 \,.\, {}^8C_2$

Q.29 Possible number of words in which no two vowels are together, is-

(A) $\dfrac{7!}{2!\,2!}.\,{}^8C_4.\dfrac{4!}{2!}$ (B) $\dfrac{7!}{2!}.\,{}^8C_4.\dfrac{4!}{2!}$

(C) $7!.\,{}^8C_4.\dfrac{4!}{2!}$ (D) $\dfrac{7!}{2!\,2!\,2!}.\,{}^8C_4.\dfrac{4!}{2!}$

Sol. [A] Number of ways

$$= \frac{\lfloor 7}{\lfloor 2 \lfloor 2} \cdot {}^{8}C_{4} \frac{\lfloor 4}{\lfloor 2}$$

EXERCISE # 4

Old IIT-JEE Questions

Q.1 Let T_n denote the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals- **[IIT-Scr-2001]**

(A) 5 (B) 7 (C) 6 (D) 4

Sol. **[B]**

n sided regular polygon have n vertices

$\therefore$ No of triangles = ${}^nC_3$

$T_n = {}^nC_3$

$T_{(n+1)} = {}^{(n+1)}C_3$

$T_{n+1} - T_n = 21$

${}^{(n+1)}C_3 - {}^nC_3 = 21$

$$\frac{(n+1)!}{3!\times(n-2)!} - \frac{n!}{3!\times(n-3)!} = 21$$

$$\frac{(n+1).n.(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 21$$

$\Rightarrow n(n-1)\,[n+1-n+2] = 6 \times 21$

$\Rightarrow n(n-1) \times 3 = 6 \times 21$

$\Rightarrow n^2 - n - 42 = 0$

$\Rightarrow n^2 - 7n + 6n - 42 = 0$

$\Rightarrow n(n-7)\,(n+6) = 0$

$\Rightarrow n = 7$

$\therefore$ Option (B) is correct answer.

Q.2 Using permutations or otherwise prove that $\dfrac{\lfloor n^2}{(\lfloor n)^n}$ is an integer. Here $n \in N$. **[IIT-2004]**

Sol. n^2 objects can be distributed in n identical groups. Since, order of group is important

$\therefore$ No of required arrangements

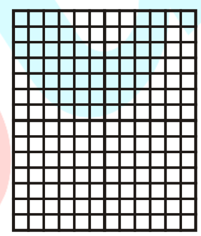
$$= {}^{n^2}C_n . {}^{(n^2-n)}C_n . {}^{(n^2-2n)}C_n \ldots . {}^nC_n$$

$$= \frac{(n^2)!}{n!\times(n^2-n)!} \times \frac{(n^2-n)!}{n!\times(n^2-2n)!} \times \frac{(n^2-2n)!}{n!\times(n^2-3n)!} \times \frac{(n^2-3n)!}{n!\times(n^2-4n)!} \times \ldots \times \frac{n!}{n!}$$

$$= \frac{(n^2)!}{(n!)^n} \Rightarrow \text{Integer}$$

Proved

Q.3 A rectangle has sides of (2m – 1) & (2n – 1) units as shown in the figure composed of squares having edge length one unit then no. of rectangles which have odd unit length- **[IIT-Scr2005]**



(A) $m^2 - n^2$ (B) $m(m+1)\,n(n+1)$

(C) 4^{m+n-2} (D) m^2n^2

Sol. **[D]**

Q.4 If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is- **[IIT-2006]**

(A) 224 (B) 225 (C) 252 (D) 256

Sol. **[B]**

Selection of r, s, t may be in following way so that L.C.M. of pand q would be $r^2\,t^4\,s^2$ as –

	p	q	
Selection of r	r^0	r^2	
	r^1	r^2	$\Rightarrow$ 5 ways
	r^2	r^0, r^1, r^2	

Selection of t	t^0	t^4	$\Rightarrow$ 9 ways
	t^1	t^4	
	t^2	t^4	
	t^3	t^4	
	t^4	t^0, t^1, t^2, t^3, t^4	

Selection of s	s^0	s^2	$\Rightarrow$ 5 ways
	s^1	s^2	

s^2 s^0, s^1, s^2

∴ Required no. of ways of getting cm of pand q
= 5 × 9 × 5 = 225

Q.5 The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is- **[IIT-2007]**

(A) 360 (B) 192 (C) 96 (D) 48

Sol. **[C]**
Required no of ways so that next word will be COCHIN
= (second place can be filled in $^4C_1$ ways) × (Remaining four alphabets can be arranged in 4! ways)
= 4 × 24 = 96

Q.6 Consider all possible permutation of the letters of the word 'ENDEANOEL' **[IIT- 2008]**

Column I	**Column II**
(A) The number of permutation containing the word ENDEA is	(P) 5!
(B) The number of permutations in which the letter E occurs in the first and the last positions is	(Q) 2 × 5!
(C) The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(R) 7 × 5!
(D) The number of permutations in which the letters A, E, O occur only in odd positions is	(S) 21 × 5!

Sol. **A → P ; B → S ; C → Q ; D → Q**

(A) No. of permutations containing word ENDEA = 5!

As consider ENDEA as one letter & remaining four

∴ total letters to be arranged = 5.

(B) ∴ Remaining 7 letters can be arranged in = 7!/2! = 21 × 5!

(C) D, I, N can occupy only four positions & remaining 5 alphabets can occupy

remaining places, in $\frac{4!}{2!}\times\frac{5!}{3!} = 2\times5!$

(D) odd positions are 5 so A, E,O can occupy these places in $\frac{5!}{3!}\times\frac{4!}{2!} = 2\times5!$.

Q.7 The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is **[IIT-2009]**

(A) 55 (B) 66 (C) 77 (D) 88

Sol. – – – – – – –
3 2 1 1 1 1 1 → 7!/5! = 42
2 2 2 1 1 1 1 → 7!/ 3!4! = 35 = 77

Q.8 Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to **[IIT-2010]**

(A) 25 (B) 34 (C) 42 (D) 41

Sol.[D] S = {1, 2, 3, 4}

Possible subsets No. of elements in Ways

Set A	Set B	Ways
0	0	= 1
1	0	= $^4C_1$ = 4
2	0	= $^4C_2$ = 6
1	1	= $^4C_2$ = 6
3	0	= $^4C_3$ = 4
2	1	= $^4C_2 . ^2C_1$ = 12
4	0	= $^4C_4$ = 1
3	1	= $\frac{4!}{3!\ 1!}$ = 4
2	2	= $\frac{4!}{2!\ 2!\ 2!}$ = 3

Total ⇒ 1 + 4 + 6 + 6 + 4 + 12 + 1 + 4 + 3 = 41

Q.9 The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is **[IIT-2012]**

(A) 75 (B) 150 (C) 210 (D) 243

Sol.[B]

G_1	G_2	G_3
1	1	3
1	2	2

$$\left(\frac{5!}{1!\,1!\,3!\,2!}+\frac{5!}{1!\,2!\,2!\,2!}\right)3!$$

= 150

Passage (Q. 10 to Q. 11)

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0. **[IIT-2012]**

Q.10 The value of b_6 is

(A) 7 (B) 8 (C) 9 (D) 11

Sol.[B] for b_n

first and last place are fixed by 1

so case (1) if only one zero is used such cases = ${}^{n-2}C_1$

case (2) if two zero are used the two zeros are such that no two zeros are consecutive = ${}^{n-3}C_2$

case (3) if three zeros are used then the positing of three zeros such that no two zeros are consecutive = ${}^{n-4}C_3$

So $b_n = {}^{n-2}C_1 + {}^{n-3}C_2 + {}^{n-4}C_3 + {}^{n-5}C_4 + {}^{n-6}C_5$ ....

for $b_6 = {}^{4}C_1 + {}^{3}C_2 + 1$

when no zero is used

= 8

Q.11 Which of the following is correct?

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$

(C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

Sol.[A] $b_n = 1 + {}^{n-2}C_1 + {}^{n-3}C_2 + {}^{n-4}C_3 + {}^{n-5}C_4 + {}^{n-6}C_5 + \ldots\ldots$

$c_n = 1 + {}^{n-3}C_1 + {}^{n-4}C_2 + {}^{n-5}C_3 + {}^{n-6}C_4 + \ldots\ldots$

$a_n = 1 + {}^{n-1}C_1 + {}^{n-2}C_2 + {}^{n-3}C_3 + {}^{n-4}C_4 + \ldots\ldots$

$a_{17} = 1 + {}^{16}C_1 + {}^{15}C_2 + {}^{14}C_3 + {}^{13}C_4 + {}^{12}C_5 + {}^{11}C_6 + {}^{10}C_7 + {}^{9}C_8 + {}^{8}C_9$

$a_{16} = 1 + {}^{15}C_1 + {}^{14}C_2 + {}^{13}C_3 + {}^{12}C_4 + {}^{11}C_5 + {}^{10}C_6 + {}^{9}C_7 + {}^{8}C_8$

$a_{15} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

$a_{17} = a_{16} + a_{15}$ so A is correct.

$c_{17} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7 + {}^{7}C_8$

$c_{16} = 1 + {}^{13}C_1 + {}^{12}C_2 + {}^{11}C_3 + {}^{10}C_4 + {}^{9}C_5 + {}^{8}C_6 + {}^{7}C_7$

$c_{15} = 1 + {}^{12}C_1 + {}^{11}C_2 + {}^{10}C_3 + {}^{9}C_4 + {}^{8}C_5 + {}^{7}C_6$

$c_{17} = c_{16} + c_{15}$

So, B is wrong.

$b_{17} = 1 + {}^{15}C_1 + {}^{14}C_2 + {}^{13}C_3 + {}^{12}C_4 + {}^{11}C_5 + {}^{10}C_6 + {}^{9}C_7 + {}^{8}C_8$

$b_{16} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

$c_{16} = 1 + {}^{13}C_1 + {}^{12}C_2 + {}^{11}C_3 + {}^{10}C_4 + {}^{9}C_5 + {}^{8}C_6 + {}^{7}C_7$

$b_{17} = b_{16} + c_{17}$ so C is wrong.

$a_{17} = 1 + {}^{16}C_1 + {}^{15}C_2 + {}^{14}C_3 + {}^{13}C_4 + {}^{12}C_5 + {}^{11}C_6 + {}^{10}C_7 + {}^{9}C_8 + {}^{8}C_7$

$c_{17} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

$b_{16} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

so D is wrong.

EXERCISE # 5

Q.1 The product of r consecutive integers is divisible by- **[IIT-1985]**

(A) r (B) $\sum_{k=1}^{r-1} k$

(C) r ! (D) None of these

Sol. [A,C]

Product of r consecutive integers

= m.(m + 1).(m + 2) …… (m + r – 1)

= m.(m + 1) (m + 2) – (m + 2). …… (m + r – 1) × $\frac{(m-1)!}{(m-1)!} \times \frac{r!}{r!}$

= $\frac{(m+r-1)!}{(m-1)! \times r!}$ × r !

= $^{(m+r-1)}C_r$ × r !

Which divisible by r and r !

Θ Option (A) and (C) are correct answers.

Q.2 An n-digit number is a positive number with exactly n digits. At least nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5, and 7. The smallest value of n for which this is possible is- **[IIT-1998]**

(A) 6 (B) 7

(C) 8 (D) 9

Sol. [B]

$3^n \ge 900$

$\Rightarrow 3^n \ge 9 \times 100 \Rightarrow 3^{(n-2)} \ge 100$

$\Rightarrow 3^{(n-2)} > 81 \Rightarrow 3^{(n-2)} > 3^4$

$\Rightarrow n > 6 \Rightarrow n = 7$

∴ Option (B) is correct answer.

Q.3 Number of divisors of the form 4n + 2 (n ≥ 0) of the integer 240 is- **[IIT-1998]**

(A) 4 (B) 8 (C) 10 (D) 3

Sol. **[A]**

240 = 4 × 60

= 4 × 4 × 15

= $2^4 \times 3^1 \times 5^1$

Total No. of divisors of 240 = (4 + 1) (1 + 1) (1 + 1)

= 5 × 2 × 2

= 20

No. of divisors of the form (4n + 2) ; n ≥ 0

✓ ✓ ✓ × × × × ✓ ×

2, 6, 10, 14, 18, 22, 26, 30, 34 ……….

Hence, required no. of divisors : 2, 6, 10, 30

Q.4 Find the total number of ways of selecting five letters from the letters of the word 'INDEPENDENT'. **[IIT 1998]**

Sol. INDEPENDENT

I → 1

N → 3

D → 2

E → 3

P → 1

T → 1

5 diff or 2 alike 3 diff. or 2 alike, 2 other alike 1 diff

$^6C_5 + {}^3C_2 . {}^5C_3 + {}^3C_2 . {}^4C_1$

6 + 30 + 12 = 48

or 3 alike 2 diff. or 3 alike 2 other alike

= $^2C_1 . {}^5C_2 + {}^2C_1 . {}^2C_1$ = 20 + 4 = 24

Total = 72

Q.5 Suppose that there are piles of red, blue, and green balls and that each pile contains at least eight balls.

(i) In how many ways can eight balls be selected ?

(ii) In how many ways can eight balls be selected if at least one ball of each colour is to be selected ?

Sol. (i) 45, (ii) 21

Q.6 The members of a chess club took part in a round robin competition in which each plays every one else once. All members scored the same number of points, except four juniors whose total score were 17.5. How many members were there in the club ? Assume that for each win a player scores 1 point, for draw 1/2 point and zero for losing.

Sol. 27

Q.7 In how many ways can 10 persons take seats in a row of 24 fixed seats so that no two persons take consecutive seats?

Sol. 286

Q.8 There are 12 seats in the first row of a theater, of which 4 are to be occupied. Find the number of ways of arranging 4 persons so that :

(i) No two persons sit side by side.

(ii) There should be at least 2 empty seats between any two persons.

(iii) Each person has exactly one neighbour.

Sol. (i) No. of ways so that no. two persons sit side by side = ${}^9P_4$

(ii)

    ✓   ✓   ✓   ✓   ✓   ✓

× × × × × × × × × × × ×

No. of ways so that three should beat least 2 empty seats between any two persons.

= person can be arranged in ether ticked position or Non-ticked position.

$= {}^6P_4 = \frac{6!}{2!} = \frac{6\times120}{2} = 360$ Ans.

(iii)

× × × × × × × × × × × ×

No. of ways of each person has exactly one neighbour.

$= 4! \times 8 + 4! \times 7 + 4! \times 6 + 4! \times 5 + 4! \times 4 + 4! \times 3 + 4! \times 2 + 4!$

$= 4!\,(8 + 7 + 6 + 5 + 4 + 3 + 2 + 1)$

$= 24 \times 36$

$= 864$ Ans

Q.9 In an election for the managing committee of a reputed club, the number of candidates contesting elections exceed the number of members to be elected by r(r > 0). If a voter can vote in 967 different ways to elect the managing committee by voting atleast 1 of them & can vote in 55 different ways to elect (r – 1) candidates by voting in the same manner. Find the number of candidates contesting the elections & the number of candidates losing the elections.

Sol. 10, 3

Q.10 How many 15 letter arrangements of 5A's, 5B's and 5C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters.

Sol. 2252

Q.11 Consider a 7 digit telephone number 336-7624 which has the property that the first three digit prefix, 336 equals the product of the last four digits. How many seven digit phone numbers beginning with 336 have this property, e.g. (336-7624)

Sol. 84

Q.12 In India–Pak one day International cricket match at Sharjah, India needs 14 runs to win just before the start of the final over. Find the number of ways in which India just manages to win the match (i.e. scores exactly 14 runs), assuming that all the runs are made off the bat & the batsman can not score more that 4 runs off any ball.

Sol. 1506

ANSWER KEY

EXERCISE # 1

1. (C) **2.** (B) **3.** (D) **4.** (C) **5.** (B) **6.** (D) **7.** (A) **8.** (A) **9.** (i) (A), (ii) (A), (iii) (A) **10.** (D)

11. (A) **12.** (A) **13.** (C) **14.** (A) **15.**(A) **16.** (D) **17.** (C) **18.** (D) **19.** (B) **20.** (D) **21.** (A) **22.** (B)

23. (C) **24.**(B) **25.** (A, D) **26.** (B)

EXERCISE # 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	B	B	B	B	A	B	B	C	B	C	B	C	C
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	
D	A	B	B	C	C	D	A,D	B,C	B,C	B,D	A,B,C	C,D	

28. (B) **29.** (A) **30.** (D) **31.** (B)

32. A $\rightarrow$ Q; B $\rightarrow$ R; C $\rightarrow$ P; D $\rightarrow$ S

33. A $\rightarrow$ P; B $\rightarrow$ P, Q, R; C $\rightarrow$ P, Q, R , S; D $\rightarrow$ P

34. A $\rightarrow$ Q; B $\rightarrow$ S; C $\rightarrow$ R; D $\rightarrow$ P

35. A $\rightarrow$ R; B $\rightarrow$ Q; C $\rightarrow$ S; D $\rightarrow$ P

36. A $\rightarrow$ S; B $\rightarrow$ Q; C $\rightarrow$ P; D $\rightarrow$ R

37. (C) **38.** (C) **39.** (D)

40. i. (A) ii. (A) iii (A)

EXERCISE # 3

1. $^{m+n+k}C_3 - (^mC_3 + {}^nC_3 + {}^kC_3)$

2. 708894

3. 4316527

4. 960

5. 5199960

6. 18720

7. 2520

8. (i) 60, (ii) 3255, (iii) 3968

9. 12!; $\frac{11!\ 4!}{(3!)^4\ 2!}$

10. 4974200

11. 144, 806

12. 13, 156

13. 5400

15. $\frac{52!}{(13!)^4}$; $\frac{52!}{3!\,(17!)^3}$

16. $^{45}C_6$

17. 89

18. 3888

20. 6890

21. (B)

22. (A)

23. (D)

24. (A)

25. (D)

26. (D)

27. (B)

28. (A)

29. (A)

EXERCISE # 4

1. (D) **3.** (D) **4.** (B) **5.** (C) **6.** **A** $\rightarrow$ P ; **B** $\rightarrow$ S ; **C** $\rightarrow$ Q; **D** $\rightarrow$ Q

7. (C) **8.** (D) **9.** (B) **10.** (B) **11.** (A)

EXERCISE # 5

1. A, B, C **2.** (B) **3.** (A) **4.** 72

5. (i) 45 (ii) 21 **6.** 27 **7.** 286 **8.** (i) ${}^{9}P_{4}$ (ii) 360 (iii) 864

9. 10, 3 **10.** 2252 **11.** 84 **12.** 1506