EXERCISE-I

Application in mechanics and rate measurer

1. If the law of motion in a straight line is $s = \frac{1}{2} v t$, then acceleration is

(A) Constant (B) Proportional to t

(C) Proportional to v (D) Proportional to s

2. A point moves in a straight line during the time $t = 0$ to $t = 3$ according to the law $s = 15t - 2t^2$. The average velocity is

(A) 3 (B) 9

(C) 15 (D) 27

3. The distance in seconds, described by a particle in t seconds is given by $s = ae^t + \frac{b}{e^t}$. Then acceleration of the particle at time t is

(A) Proportional to t (B) Proportional to s

(C) s (D) Constant

4. The equation of motion of a stone, thrown vertically upwards is $s = ut - 6.3t^2$, where the units of s and t are *cm* and *sec*. If the stone reaches at maximum height in 3 *sec*, then u =

(A) 18.9 cm / sec (B) 12.6 cm / sec

(C) 37.8 cm / sec (D) None of these

5. A particle moves in a straight line so that its velocity at any point is given by $v^2 = a + bx$, where $a, b \neq 0$ are constants. The acceleration is

(A) Zero (B) Uniform

(C) Non-uniform (D) Indeterminate

6. The volume V and depth x of water in a vessel are connected by the relation $V = 5x - \frac{x^2}{6}$ and the volume of water is increasing at the rate of $5 cm^3 / sec$, when $x = 2 cm$. The rate at which the depth of water is increasing, is

(A) $\frac{5}{18}$ cm / sec (B) $\frac{1}{4}$ cm / sec

(C) $\frac{5}{16}$ cm / sec (D) None of these

7. The equation of motion of a stone thrown vertically upward from the surface of a planet is given by $s = 10\,t - 3t^2$, and the units of s and t are *cm* and *sec* respectively. The stone will return to the surface of the planet after

(A) $\frac{10}{3}$ sec (B) $\frac{5}{3}$ sec

(C) $\frac{20}{3}$ sec (D) $\frac{5}{6}$ sec

8. A body moves according to the formula $v = 1 + t^2$, where v is the velocity at time t. The acceleration after 3 *sec* will be

(v in *cm/sec*)

(A) $24 cm / sec^2$ (B) $12 cm / sec^2$

(C) $6 cm / sec^2$ (D) None of these

9. The length of the side of a square sheet of metal is increasing at the rate of $4 cm / sec$. The rate at which the area of the sheet is increasing when the length of its side is 2 *cm*, is

(A) $16 cm^2 / sec$ (B) $8 cm^2 / sec$

(C) $32 cm^2 / sec$ (D) None of these

10. The equations of motion of two stones thrown vertically upwards simultaneously are $s = 19.6t - 4.9t^2$ and $s = 9.8t - 4.9t^2$ respectively and the maximum height attained by the first one is *h*. When the height of the first stone is maximum, the height of the second stone will be

(A) $h/3$ (B) $2h$
(C) h (D) 0

11. A ball thrown vertically upwards falls back on the ground after 6 *second*. Assuming that the equation of motion is of the form $s = ut - 4.9t^2$, where *s* is in *metre* and *t* is in *second*, find the velocity at $t = 0$

(A) $0\,m/s$ (B) 1 *m/s*
(C) 29.4 *m/s* (D) None of these

12. Radius of a circle is increasing uniformly at the rate of $3cm/sec$. The rate of increasing of area when radius is $10cm$, will be

(A) $\pi cm^2/s$ (B) $2\pi cm^2/s$
(C) $10\pi cm^2/s$ (D) None of these

13. The motion of stone thrown up vertically is given by $s = 13.8t - 4.9t^2$, where *s* is in *metre* and *t* is in *seconds*. Then its velocity at $t = 1$ *second* is

(A) 3 *m/s* (B) 5 *m/s*
(C) 4 *m/s* (D) None of these

14. A particle is moving in a straight line. Its displacement at time *t* is given by $s = -4t^2 + 2t$, then its velocity and acceleration at time $t = \frac{1}{2}$ *second* are

(A) – 2, – 8 (B) 2, 6
(C) – 2, 8 (D) 2, 8

15. If the distance travelled by a point in time *t is* $s = 180t - 16t^2$, then the rate of change in velocity is

(A) – 16 *t unit* (B) 48 *unit*
(C) – 32 *unit* (D) None of these

16. A man 2metre high walks at a uniform speed 5 metre/hour away from a lamp post 6 metre high. The rate at which the length of his shadow increases is

(A) 5 m/h (B) $\frac{5}{2}$ m/h
(C) $\frac{5}{3}$ m/h (D) $\frac{5}{4}$ m/h

17. A ladder 5 *m* in length is resting against vertical wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of $1.5\,m/sec$. The length of the highest point of the ladder when the foot of the ladder $4.0\,m$ away from the wall decreases at the rate of

(A) 2 m/sec (B) 3 m/sec
(C) 2.5 m/sec (D) 1.5 m/sec

18. If by dropping a stone in a quiet lake a wave moves in circle at a speed of 3.5 *cm/sec*, then the rate of increase of the enclosed circular region when the radius of the circular wave is 10 cm, is $\left(\pi = \frac{22}{7}\right)$

(A) 220 *sq. cm/sec* (B) 110 *sq. cm/sec*
(C) 35 *sq. cm/sec* (D) 350 *sq. cm/sec*

19. A ladder is resting with the wall at an angle of 30°. A man is ascending the ladder at the rate of 3 *ft/sec*. His rate of approaching the wall is

(A) 3 *ft/sec* (B) $\frac{3}{2}$ *ft/sec*
(C) $\frac{3}{4}$ *ft/sec* (D) $\frac{3}{\sqrt{2}}$ *ft/sec*

20. If the edge of a cube increases at the rate of 60 *cm per second*, at what rate the volume is increasing when the edge is 90 *cm*

(A) 486000 *cu cm per sec*
(B) 1458000 *cu cm per sec*
(C) 43740000 *cu cm per sec*
(D) None of these

21. A particle is moving along the curve $x = at^2 + bt + c$. If $ac = b^2$, then the particle would be moving with uniform

(A) Rotation (B) Velocity

(C) Acceleration (D) Retardation

22. The sides of an equilateral triangle are increasing at the rate of 2 *cm/sec*. The rate at which the area increases, when the side is 10 *cm* is

(A) $\sqrt{3}$ *sq. unit/sec* (B) 10 *sq. unit/sec*

(C) $10\sqrt{3}$ *sq. unit/sec* (D) $\frac{10}{\sqrt{3}}$ *sq. unit/sec*

23. The rate of change of the surface area of a sphere of radius *r* when the radius is increasing at the rate of 2 *cm/sec* is proportional to

(A) $\frac{1}{r}$ (B) $\frac{1}{r^2}$

(C) r (D) r^2

24. Moving along the *x*-axis are two points with $x = 10 + 6t; x = 3 + t^2$. The speed with which they are reaching from each other at the time of encounter is (*x* is in *cm* and *t* is in *seconds*)

(A) 16 *cm/sec* (B) 20 *cm/sec*

(C) 8 *cm/sec* (D) 12 *cm/sec*

25. The position of a point in time *'t'* is given by $x = a + bt - ct^2$, $y = at + bt^2$.

Its acceleration at time '*t*' is

(A) $b - c$ (B) $b + c$

(C) $2b - 2c$ (D) $2\sqrt{b^2 + c^2}$

26. Gas is being pumped into a spherical balloon at the rate of 30 ft^3/min. Then the rate at which the radius increases when it reaches the value 15 *ft* is

(A) $\frac{1}{30\pi}$ ft / min. (B) $\frac{1}{15\pi}$ ft / min.

(C) $\frac{1}{20}$ ft / min. (D) $\frac{1}{25}$ ft / min.

27. If the distance '*s*' *metre* traversed by a particle in *t seconds* is given by $s = t^3 - 3t^2$, then the velocity of the particle when the acceleration is zero, in *metre/sec* is

(A) 3 (B) – 2

(C) – 3 (D) 2

28. A particle moves in a straight line so that $s = \sqrt{t}$, then its acceleration is proportional to

(A) Velocity (B) $(\text{Velocity})^{3/2}$

(C) $(\text{Velocity})^3$ (D) $(\text{Velocity})^2$

29. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(A) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (B) $(2, -4)$

(C) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (D) $(2, 4)$

30. A spherical iron ball 10 *cm* in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 *cm*, then the rate at which the thickness of ice decreases, is

(A) $\frac{1}{54\pi}$ *cm/min* (B) $\frac{5}{6\pi}$ *cm/min*

(C) $\frac{1}{36\pi}$ *cm/min* (D) $\frac{1}{18\pi}$ *cm/min*

Increasing and Decreasing function

31. For which interval the given function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is decreasing

(A) $(-2, \infty)$

(B) $(-2, -1)$

(C) $(-\infty, -1)$

(D) $(-\infty, -2)$ and $(-1, \infty)$

32. $f(x)=x^3-27x+5$ is an increasing function, when

(A) $x<-3$ (B) $|x|>3$

(C) $x\le -3$ (D) $|x|<3$

33. The function $f(x)=x^2$ is increasing in the interval

(A) $(-1,1)$ (B) $(-\infty,\infty)$

(C) $(0,\infty)$ (D) $(-\infty,0)$

34. Function $f(x)=x^4-\frac{x^3}{3}$ is

(A) Increasing for $x>\frac{1}{4}$ and decreasing for $x<\frac{1}{4}$

(B) Increasing for every value of x

(C) Decreasing for every value of x

(D) None of these

35. For every value of x function $f(x)=e^x$ is

(A) Decreasing

(B) Increasing

(C) Neither increasing nor decreasing

(D) None of these

36. If $f'(x)$ is zero in the interval (a, b) then in this interval it is

(A) Increasing function

(B) Decreasing function

(C) Only for $a>0$ and $b>0$ is increasing function

(D) None of these

37. For the every value of x the function $f(x)=\frac{1}{5^x}$ is

(A) Decreasing

(B) Increasing

(C) Neither increasing nor decreasing

(D) Increasing for $x>0$ and decreasing for $x<0$

38. The interval for which the given function $f(x)=2x^3-3x^2-36x+7$ is decreasing, is

(A) $(-2, 3)$ (B) $(2, 3)$

(C) $(2,-3)$ (D) None of these

39. If $f(x)=\sin x-\frac{x}{2}$ is increasing function, then

(A) $0<x<\frac{\pi}{3}$ (B) $-\frac{\pi}{3}<x<0$

(C) $-\frac{\pi}{3}<x<\frac{\pi}{3}$ (D) $x=\frac{\pi}{2}$

40. If x tends 0 to π, then the given function $f(x)=x\sin x+\cos x+\cos^2 x$ is

(A) Increasing

(B) Decreasing

(C) Neither increasing nor decreasing

(D) None of these

41. The interval of the decreasing function $f(x)=x^3-x^2-x-4$ is

(A) $\left(\frac{1}{3},1\right)$

(B) $\left(-\frac{1}{3},1\right)$

(C) $\left(-\frac{1}{3},\frac{1}{3}\right)$

(D) $\left(-1,-\frac{1}{3}\right)$

42. The function $f(x)=x^3-3x^2-24x+5$ is an increasing function in the interval given below

(A) $(-\infty,-2)\cup(4,\infty)$

(B) $(-2,\infty)$

(C) $(-2, 4)$

(D) $(-\infty, 4)$

43. Which one is the correct statement about the function $f(x) = \sin 2x$

(A) $f(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(B) $f(x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$ and increasing in $\left(\frac{\pi}{2}, \pi\right)$

(C) $f(x)$ is increasing in $\left(0, \frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(D) The statements (A), (B) and (C) are all correct

44. The function *f* defined by $f(x) = (x+2)e^{-x}$ is

(A) Decreasing for all *x*

(B) Decreasing in $(-\infty, -1)$ and increasing in $(-1, \infty)$

(C) Increasing for all *x*

(D) Decreasing in $(-1, \infty)$ and increasing in $(-\infty, -1)$

45. If $f(x) = x^3 - 10x^2 + 200x - 10$, then

(A) $f(x)$ is decreasing in $]-\infty, 10]$ and increasing in $[10, \infty[$

(B) $f(x)$ is increasing in $]-\infty, 10]$ and decreasing in $[10, \infty[$

(C) $f(x)$ is increasing throughout real line

(D) $f(x)$ is decreasing throughout real line

46. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \le 1$, then in this interval

(A) Both $f(x)$ and $g(x)$ are increasing functions

(B) Both $f(x)$ and $g(x)$ are decreasing functions

(C) $f(x)$ is an increasing function

(D) $g(x)$ is an increasing function

47. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing, when

(A) $x < 2$ (B) $x > 2$

(C) $x > 1$ (D) $1 < x < 2$

48. $2x^3 + 18x^2 - 96x + 45 = 0$ is an increasing function when

(A) $x \le -8, x \ge 2$ (B) $x < -2, x \ge 8$

(C) $x \le -2, x \ge 8$ (D) $0 \le x \le -2$

49. The function $\frac{a\sin x + b\cos x}{c\sin x + d\cos x}$ is decreasing, if

(A) $ad - bc > 0$ (B) $ad - bc < 0$

(C) $ab - cd > 0$ (D) $ab - cd < 0$

50. The function $f(x) = 1 - e^{-x^2/2}$ is

(A) Decreasing for all *x*

(B) Increasing for all *x*

(C) Decreasing for $x < 0$ and increasing for $x > 0$

(D) Increasing for $x < 0$ and decreasing for $x > 0$

51. The function $f(x) = x^{1/x}$ is

(A) Increasing in $(1, \infty)$

(B) Decreasing in $(1, \infty)$

(C) Increasing in $(1, e)$, decreasing in (e, ∞)

(D) Decreasing in $(1, e)$, increasing in (e, ∞)

52. The function $f(x) = 1 - x^3 - x^5$ is decreasing for

(A) $1 \le x \le 5$ (B) $x \le 1$

(C) $x \ge 1$ (D) All values of *x*

53. The function x^x is increasing, when

(A) $x > \frac{1}{e}$ (B) $x < \frac{1}{e}$

(C) $x < 0$ (D) For all real *x*

54. $2x^3 - 6x + 5$ is an increasing function if

(A) $0 < x < 1$ (B) $-1 < x < 1$

(C) $x < -1$ or $x > 1$ (D) $-1 < x < -1/2$

55. The length of the longest interval, in which the function $3\sin x - 4\sin x$ is increasing, is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{2}$ (D) π

56. Let $f(x) = x^3 + bx^2 + cx + d, 0 < b^2 < c$. Then *f*

(A) Is bounded

(B) Has a local maxima

(C) Has a local minima

(D) Is strictly increasing

57. If $f(x) = x, -1 \le x \le 1$, then function $f(x)$ is

(A) Increasing (B) Decreasing

(C) Stationary (D) Discontinuous

58. For all $x \in (0,1)$

(A) $e^x < 1 + x$ (B) $\log_e(1+x) < x$

(C) $\sin x > x$ (D) $\log_e x > x$

59. The function $f(x) = 2x^3 - 3x^2 + 90x + 174$ is increasing in the interval

(A) $\frac{1}{2} < x < 1$ (B) $\frac{1}{2} < x < 2$

(C) $3 < x < \frac{59}{4}$ (D) $-\infty < x < \infty$

60. The function $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always an increasing function on the interval

(A) $(0, \pi)$ (B) $(0, \pi/2)$

(C) $(0, \pi/4)$ (D) $(0, 3\pi/4)$

Tangent and Normal

61. If $x = t^2$ and $y = 2t$, then equation of the normal at $t = 1$ is

(A) $x + y - 3 = 0$ (B) $x + y - 1 = 0$

(C) $x + y + 1 = 0$ (D) $x + y + 3 = 0$

62. The equation of the normal to the curve $y = \sin\frac{\pi x}{2}$ at (1, 1) is

(A) $y = 1$ (B) $x = 1$

(C) $y = x$ (D) $y - 1 = \frac{-2}{\pi}(x - 1)$

63. The equation of tangent to the curve $y = 2\cos x$ at $x = \frac{\pi}{4}$ is

(A) $y - \sqrt{2} = 2\sqrt{2}\left(x - \frac{\pi}{4}\right)$

(B) $y + \sqrt{2} = \sqrt{2}\left(x + \frac{\pi}{4}\right)$

(C) $y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$

(D) $y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

64. At which point the line $\frac{x}{a} + \frac{y}{b} = 1$, touches the curve $y = be^{-x/a}$

(A) (0, 0) (B) (0,*a*)

(C) (0, *b*) (D) (*b*, 0)

65. The angle between curves $y^2 = 4x$ and $x^2 + y^2 = 5$ at $(1, 2)$ is

(A) $\tan^{-1}(3)$ (B) $\tan^{-1}(2)$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

66. For the curve $by^2 = (x+a)^3$ the square of subtangent is proportional to

(A) $(\text{Subnormal})^{1/2}$ (B) Subnormal

(C) $(\text{Subnormal})^{3/2}$ (D) None of these

67. The tangent to the curve $y = ax^2 + bx$ at $(2, -8)$ is parallel to *x*-axis. Then

(A) $a = 2, b = -2$ (B) $a = 2,\ b = -4$

(C) $a = 2\ b = -8$ (D) $a = 4, b = -4$

68. The sum of intercepts on co-ordinate axes made by tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is

(A) a (B) 2a

(C) $2\sqrt{a}$ (D) None of these

69. Co-ordinates of a point on the curve $y = x\log x$ at which the normal is parallel to the line $2x - 2y = 3$ are

(A) (0,0) (B) (e, e)

(C) $(e^2, 2e^2)$ (D) $(e^{-2} - 2e^{-2})$

70. If normal to the curve $y = f(x)$ is parallel to *x*-axis, then correct statement is

(A) $\frac{dy}{dx} = 0$ (B) $\frac{dy}{dx} = 1$

(C) $\frac{dx}{dy} = 0$ (D) None of these

71. The abscissae of the points, where the tangent to curve $y = x^3 - 3x^2 - 9x + 5$ is parallel to *x*-axis, are

(A) 0 and 0 (B) $x = 1$ and -1

(C) $x = 1$ and -3 (D) $x = -1$ and 3

72. If the curve $y = a^x$ and $y = b^x$ intersect at angle α then, $\tan\alpha =$

(A) $\frac{a-b}{1+ab}$ (B) $\frac{\log a - \log b}{1 + \log a \log b}$

(C) $\frac{a+b}{1-ab}$ (D) $\frac{\log a + \log b}{1 - \log a \log b}$

73. The equation of tangent at $(-4, -4)$ on the curve $x^2 = -4y$ is

(A) $2x + y + 4 = 0$ (B) $2x - y - 12 = 0$

(C) $2x + y - 4 = 0$ (D) $2x - y + 4 = 0$

74. The point at which the tangent to the curve $y = 2x^2 - x + 1$ is parallel to $y = 3x + 9$ will be

(A) (2, 1) (B) (1, 2)

(C) (3, 9) (D) (–2, 1)

75. At what point on the curve $x^3 - 8a^2y = 0$, the slope of the normal is $\frac{-2}{3}$

(A) (a, a) (B) $(2a, -a)$

(C) $(2a, a)$ (D) None of these

76. The length of the normal at point *'t'* of the curve $x = a(t + \sin t),\ y = a(1 - \cos t)$ is

(A) $a\sin t$

(B) $2a\sin^3(t/2)\sec(t/2)$

(C) $2a\sin(t/2)\tan(t/2)$

(D) $2a\sin(t/2)$

77. The tangent drawn at the point (0, 1) on the curve $y = e^{2x}$ meets *x*-axis at the point

(A) $(1/2, 0)$ (B) $(-1/2,\ 0)$

(C) (2, 0) (D) $(0,\ 0)$

78. The equation of the tangent to the curve $(1 + x^2)y = 2 - x$, where it crosses the *x*-axis, is

(A) $x + 5y = 2$ (B) $x - 5y = 2$

(C) $5x - y = 2$ (D) $5x + y - 2 = 0$

79. The equation of the tangent to curve $y = be^{-x/a}$ at the point where it crosses *y*-axis is

(A) $ax + by = 1$ (B) $ax - by = 1$

(C) $\frac{x}{a} - \frac{y}{b} = 1$ (D) $\frac{x}{a} + \frac{y}{b} = 1$

80. The angle of intersection of curves $y = x^2$, $6y = 7 - x^3$ at (1, 1) is

(A) $\pi / 4$ (B) $\pi / 3$

(C) $\pi / 2$ (D) π

Maxima and Minima

81. The function $x\sqrt{1-x^2}, (x > 0)$ has

(A) A local maxima

(B) A local minima

(C) Neither a local maxima nor a local minima

(D) None of these

82. If two sides of a triangle be given, then the area of the triangle will be maximum if the angle between the given sides be

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

83. The function $x^5 - 5x^4 + 5x^3 - 1$ is

(A) Maximum at $x = 3$ and minimum at $x = 1$

(B) Minimum at $x = 1$

(C) Neither maximum nor minimum at $x = 0$

(D) Maximum at $x = 0$

84. The adjacent sides of a rectangle with given perimeter as 100 *cm* and enclosing maximum area are

(A) 10 *cm* and 40 *cm* (B) 20 *cm* and 30 *cm*

(C) 25 *cm* and 25 *cm* (D) 15 *cm* and 35 *cm*

85. The necessary condition to be maximum or minimum for the function is

(A) $f'(x) = 0$ and it is sufficient

(B) $f''(x) = 0$ and it is sufficient

(C) $f'(x) = 0$ but it is not sufficient

(D) $f'(x) = 0$ and $f''(x) = -ve$

86. The area of a rectangle will be maximum for the given perimeter, when rectangle is a

(A) Parallelogram (B) Trapezium

(C) Square (D) None of these

87. Of the given perimeter, the triangle having maximum area is

(A) Isosceles triangle

(B) Right angled triangle

(C) Equilateral

(D) None of these

88. The sufficient conditions for the function $f : R \to R$ is to be maximum at $x = a$, will be

(A) $f'(a) > 0$ and $f''(a) > 0$

(B) $f'(a) = 0$ and $f''(a) = 0$

(C) $f'(a) = 0$ and $f''(a) < 0$

(D) $f'(a) > 0$ and $f''(a) < 0$

89. 36 factorize into two factors in such a way that sum of factors is minimum, then the factors are

(A) 2, 18 (B) 9, 4

(C) 3, 12 (D) None of these

90. If $f(x) = 2x^3 - 3x^2 - 12x + 5$ and $x \in [-2, 4]$, then the maximum value of function is at the following value of *x*

(A) 2 (B) –1

(C) – 2 (D) 4

91. The value of *a* so that the sum of the squares of the roots of the equation $x^2 - (a-2)x - a + 1 = 0$ assume the least value, is

(A) 2 (B) 1

(C) 3 (D) 0

92. The largest term in the sequence $a_n = \dfrac{n^2}{n^3 + 200}$ is given by

(A) $\dfrac{529}{49}$ (B) $\dfrac{8}{89}$

(C) $\dfrac{49}{543}$ (D) None of these

93. The maximum value of *xy* subject to $x + y = 8$, is

(A) 8 (B) 16

(C) 20 (D) 24

94. A minimum value of $\int_0^x te^{-t^2}\,dt$ is

(A) 1 (B) 2

(C) 3 (D) 0

95. If sum of two numbers is 3, then maximum value of the product of first and the square of second is

(A) 4 (B) 3

(C) 2 (D) 1

96. The minimum value of the function $2\cos 2x - \cos 4x$ in $0 \le x \le \pi$ is

(A) 0 (B) 1

(C) $\dfrac{3}{2}$ (D) – 3

97. If $f(x) = 2x^3 - 21x^2 + 36x - 30$, then which one of the following is correct

(A) $f(x)$ has minimum at $x = 1$

(B) $f(x)$ has maximum at $x = 6$

(C) $f(x)$ has maximum at $x = 1$

(D) $f(x)$ has no maxima or minima

98. The maximum value of $2x^3 - 24x + 107$ in the interval [–3, 3] is

(A) 75 (B) 89

(C) 125 (D) 139

99. If the function $f(x) = x^4 - 62x^2 + ax + 9$ is maximum at $x = 1$, then the value of *a* is

(A) 120 (B) –120

(C) 52 (D) 128

100. The minimum value of the expression $7 - 20x + 11x^2$ is

(A) $\dfrac{177}{11}$ (B) $-\dfrac{177}{11}$

(C) $-\dfrac{23}{11}$ (D) $\dfrac{23}{11}$

101. The minimum value of $e^{(2x^2-2x+1)\sin^2 x}$ is

(A) *e* (B) 1/*e*

(C) 1 (D) 0

102. *x* and *y* be two variables such that $x > 0$ and $xy = 1$. Then the minimum value of $x + y$ is

(A) 2 (B) 3

(C) 4 (D) 0

103. What are the minimum and maximum values of the function $x^5 - 5x^4 + 5x^3 - 10$

(A) – 37, – 9

(B) 10, 0

(C) It has 2 min. and 1 max. values

(D) It has 2 max. and 1 min. values

104. Divide 20 into two parts such that the product of one part and the cube of the other is maximum. The two parts are

(A) (10, 10) (B) (5, 15)

(C) (13, 7) (D) None of these

105. The maximum and minimum values of $x^3 - 18x^2 + 96x$ in interval (0, 9) are

(A) 160, 0 (B) 60, 0

(C) 160, 128 (D) 120, 28

106. A cone of maximum volume is inscribed in a given sphere, then ratio of the height of the cone to diameter of the sphere is

(A) 2/3 (B) 3/4

(C) 1/3 (D) 1/4

107. The ratio of height of cone of maximum volume inscribed in a sphere to its radius is

(A) $\dfrac{3}{4}$ (B) $\dfrac{4}{3}$

(C) $\dfrac{1}{2}$ (D) $\dfrac{2}{3}$

108. The function $f(x)=x+\sin x$ has
(A) A minimum but no maximum
(B) A maximum but no minimum
(C) Neither maximum nor minimum
(D) Both maximum and minimum

109. The function $f(x)=ax+\frac{b}{x};a,b,x>0$ takes on the least value at *x* equal to
(A) *b* (B) $\sqrt{a}$
(C) $\sqrt{b}$ (D) $\sqrt{b/a}$

110. If $xy=c^2$, then minimum value of $ax+by$ is
(A) $c\sqrt{ab}$ (B) $2c\sqrt{ab}$
(C) $-c\sqrt{ab}$ (D) $-2c\sqrt{ab}$

111. The function $f(x)=x^{-x},(x\in R)$ attains a maximum value at *x* =
(A) 2 (B) 3
(C) 1/*e* (D) 1

112. If $ab=2a+3b, a>0, b>0$ then the minimum value of *ab* is
(A) 12 (B) 24
(C) $\frac{1}{4}$ (D) None of these

113. If *PQ* and *PR* are the two sides of a triangle, then the angle between them which gives maximum area of the triangle is
(A) π (B) $\pi/3$
(C) $\pi/4$ (D) $\pi/2$

114. The function $y=a(1-\cos x)$ is maximum when $x=$
(A) π (B) $\pi/2$
(C) $-\pi/2$ (D) $-\pi/6$

115. The minimum value of $\left(x^2+\frac{250}{x}\right)$ is
(A) 75 (B) 50
(C) 25 (D) 55

116. The minimum value of $x^2+\frac{1}{1+x^2}$ is at
(A) $x=0$ (B) $x=1$
(C) $x=4$ (D) $x=3$

117. If $x-2y=4$, the minimum value of xy is
(A) – 2 (B) 2
(C) 0 (D) – 3

118. The minimum value of $2x+3y$, when $xy=6$, is
(A) 12 (B) 9
(C) 8 (D) 6

119. The function $f(x)=|px-q|+r|x|$, $x\in(-\infty,\infty)$ where $p>0,q>0,r>0$ assumes its minimum value only at one point if
(A) $p\neq q$ (B) $q\neq r$
(C) $r\neq p$ (D) $p=q=r$

120. The minimum value of function $f(x)=3x^4-8x^3+12x^2-48x+25$ on [0, 3] is equal to
(A) 25 (B) – 39
(C) – 25 (D) 39

Rolle's theorem, Lagrange's mean value theorem

121. The function $f(x)=x(x+3)e^{-(1/2)x}$ satisfies all the conditions of Rolle's theorem in [–3, 0]. The value of *c* is
(A) 0 (B) –1
(C) – 2 (D) – 3

122. Rolle's theorem is true for the function $f(x)=x^2-4$ in the interval
(A) [–2, 0] (B) [–2, 2]
(C) $\left[0,\frac{1}{2}\right]$ (D) [0, 2]

123. For the function $x + \frac{1}{x}, x \in [1,3]$, the value of c for the mean value theorem is

(A) 1 (B) $\sqrt{3}$

(C) 2 (D) None of these

124. If from mean value theorem, $f'(x_1) = \frac{f(b) - f(a)}{b - a}$, then

(A) $a < x_1 \le b$ (B) $a \le x_1 < b$

(C) $a < x_1 < b$ (D) $a \le x_1 \le b$

125. Let $f(x)$ satisfy all the conditions of mean value theorem in [0, 2]. If $f(0) = 0$ and $|f'(x)| \le \frac{1}{2}$ for all x, in [0, 2] then

(A) $f(x) \le 2$

(B) $|f(x)| \le 1$

(C) $f(x) = 2x$

(D) $f(x) = 3$ for at least one x in [0, 2]

126. The function $f(x) = x^3 - 6x^2 + ax + b$ satisfy the conditions of Rolle's theorem in [1, 3]. The values of a and b are

(A) 11, – 6 (B) – 6, 11

(C) –11, 6 (D) 6, –11

127. Consider the function $f(x) = e^{-2x} \sin 2x$ over the interval $\left(0, \frac{\pi}{2}\right)$. A real number $c \in \left(0, \frac{\pi}{2}\right)$, as guaranteed by Rolle's theorem, such that $f'(c) = 0$ is

(A) $\pi / 8$ (B) $\pi / 6$

(C) $\pi / 4$ (D) $\pi / 3$

128. Let $f(x) = \sqrt{x-1} + \sqrt{x + 24 - 10\sqrt{x-1}}$; $1 < x < 26$ be real valued function. Then $f'(x)$ for $1 < x < 26$ is

(A) 0 (B) $\frac{1}{\sqrt{x-1}}$

(C) $2\sqrt{x-1} - 5$ (D) None of these

129. If $f(x)$ satisfies the conditions of Rolle's theorem in $[1, 2]$ and $f(x)$ is continuous in $[1, 2]$ then $\int_1^2 f'(x)dx$ is equal to

(A) 3 (B) 0

(C) 1 (D) 2

130. If the function $f(x) = x^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval $[1,3]$ and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$, then

(A) $a = -11$ (B) $a = -6$

(C) $a = 6$ (D) $a = 11$