Parabola
EXERCISE # 1

Question based on **Different forms of parabola**

Q.1 The equation of the parabola whose focus is (1, 1) and tangent at the vertex is x + y = 1 is

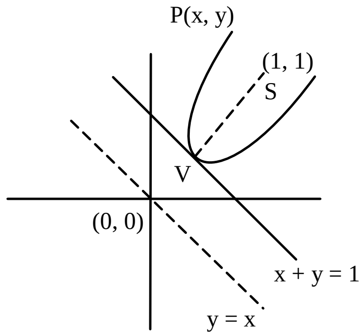
(A) $x^2 + y^2 - 2xy - 4x - 4y + 4 = 0$

(B) $x^2 + y^2 - 2xy + 4x + 4y + 4 = 0$

(C) $x^2 + y^2 - 2xy - 4x - 4y - 4 = 0$

(D) none of these

Sol. **[D]**



Distance between focus & vertex = a

$$a = \frac{1+1-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

SP = PM

Equation of directrix is y = x

i.e. x – y = 0

$SP^2 = PM^2$

$$(x-1)^2 + (y-1)^2 = \left(\frac{x-y}{\sqrt{2}}\right)^2$$

$$\Rightarrow x^2 + y^2 - 2x - 2y + 1 + 1 = \frac{(x-y)^2}{2}$$

$\Rightarrow 2(x^2 + y^2 - 2x - 2y + 2) = x^2 + y^2 - 2xy$

$\Rightarrow x^2 + y^2 + 2xy - 4x - 4y + 4 = 0$

$\Rightarrow x^2 + y^2 + 2xy - 4x - 4y + 4 = 0$

Q.2 The vertex, focus, directrix and length of the latus rectum of the parabola $y^2 - 4y - 2x - 8 = 0$ is

(A) A(6, 2), S(–11/2, 2)
Eq. of directrix x = –13/2, L. of L.R. = 2

(B) A(–6, 2), S(11/2, 2)
Eq. of directrix x = –13/2, L. of L.R. = 3

(C) A(–6, 2), S(–11/2, 2)
Eq. of directrix x = –13/2, L. of L.R. = 2

(D) None of these

Sol. **[C]**

Parabola is $y^2 - 4y - 2x - 8 = 0$

$\Rightarrow (y-2)^2 = 2x + 12$

$\Rightarrow (y-2)^2 = 2(x+6)$

$\therefore$ its vertex is x + 6 = 0 & y – 2 = 0

$\Rightarrow$ x = – 6, y = 2

$\therefore$ vertex = (– 6, 2)

$$\text{focus : } = \left(-6+\frac{1}{2},\ 2\right) = \left(-\frac{11}{2},\ 2\right)$$

Equation of directrix :

$$x = -6 - \frac{1}{2} \Rightarrow x = -\frac{13}{2}$$

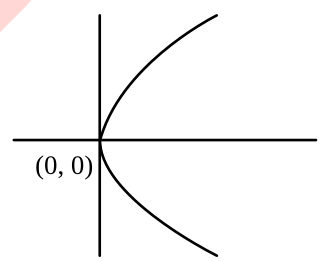
$$\text{Length of L.R. : } = 4a \Rightarrow 4.\frac{1}{2} = 2$$

Q.3 The equation of the parabola which passes through the point (4, 3) and having origin as its vertex and x-axis as its axis will be

(A) $9y^2 = 4x$ (B) $9y^2 + 4x = 0$

(C) $4y^2 + 9x = 0$ (D) $4y^2 - 9x = 0$

Sol. **[D]**



Let equation of parabola is $y^2 = 4ax$

Since it passes through (4, 3)

$\therefore$ 9 = 16a

$$\Rightarrow a = \frac{9}{16}$$

$\therefore$ equation of parabola is given by

$$y^2 = \frac{9}{16}.4x$$

$\Rightarrow 4y^2 = 9x$

$\Rightarrow 4y^2 - 9x = 0$

Q.4 If the vertex = (2, 0) and the extremities of the latus rectum are (3, 2) and (3, –2) then the equation of the parabola is

(A) $y^2 = 2x - 4$ (B) $x^2 = 4y - 8$

(C) $y^2 = 4x - 8$ (D) none of these

Sol. **[C]**

Vertex = (2, 0)

focus will be the mid point of extremities of latus rectum

$\therefore$ focus = (3, 0)

$\therefore$ distance between focus and vertex is

$a = 1$

since axis of parabola is x-axis

$\therefore$ its equation will be

$(y - 0)^2 = 4.1.(x - 2)$

$\Rightarrow y^2 = 4(x - 2)$

$\Rightarrow y^2 = 4x - 8$

Q.5 The equation of the parabola whose vertex and focus are on the positive side of the x-axis at distances a and b respectively from the origin is

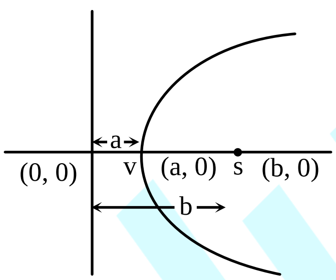
(A) $y^2 = 4(b - a)(x - a)$

(B) $y^2 = 4(a - b)(x - b)$

(C) $x^2 = 4(b - a)(y - a)$

(D) none of these

Sol. **[A]**



Let equation of parabola is

$(y - y_1)^2 = 4A(x - x_1)$

Here $A = (b - a)$

and vertex = $(x_1, y_1) \equiv (a, 0)$

$\therefore$ equation will be

$(y - 0)^2 = 4(b - a)(x - a)$

$\Rightarrow y^2 = 4(b - a)(x - a)$

$\Rightarrow y^2 = 4(b - a)(x - a)$

Question based on **Parametric form**

Q.6 Write the parametric equations of the parabola $(x + 1)^2 = 4(y - 1)$

(A) $x = 2t - 1, y = t^2 + 1$

(B) $x = 2t + 3, y = t^2 + 2$

(C) $x = 2t - 1, y = t^2 - 1$

(D) none of these

Sol. **[A]**

Given parabola is $(x + 1)^2 = 4(y - 1)$

Let $x + 1 = X$

and $y - 1 = Y$

$\therefore X^2 = 4Y$

Its parametric equation is

$X = 2at, Y = at^2$

$\Rightarrow x + 1 = 2at, y - 1 = at^2$

$\Rightarrow x = 2at - 1, y = at^2 + 1$

Here $4a = 4$

$\Rightarrow a = 1$

$\therefore$ parametric equation will be

$x = 2t - 1, y = t^2 + 1$.

Q.7 Which of the following are not parametric coordinates of any point on the parabola $y^2 = 4ax$

(A) $(at^2, 2at)$ (B) $(a, 2a)$

(C) $(a/m^2, 2a/m)$ (D) $(am^2, -2am)$

Sol. **[B]**

$y^2 = 4ax$

The parametric co-ordinate of the parabola is given by $(at^2, 2at)$ where t is parameter

also we know that $t = -m$, therefore $(am^2, -2am)$ is also parametric co-ordinates And $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ also the parametric co-ordinates.

But (a, 2a) is not a parametric co-ordinate of the parabola because there is no parameter. Hence option (B) is correct Answer.

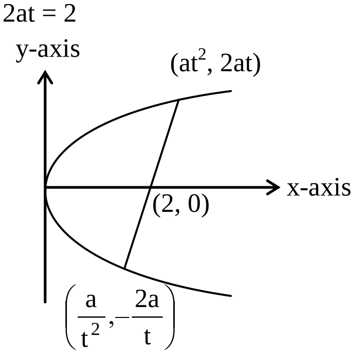
Question based on **Focal chord**

Q.8 The other extremity of the focal chord of the parabola $y^2 = 8x$ which is drawn at the point (1/2, 2) is

(A) (2, –4) (B) (2, 4) (C) (8, –8) (D) (8, 8)

Sol. **[C]**

Here $2at = 2$



$\Rightarrow 2 \times 2 \times t = 2$

$\Rightarrow t = \dfrac{1}{2}$

$\therefore$ other extremity is given by

$$\left(\frac{a}{t^2}, -\frac{2a}{t}\right) \equiv \left(\frac{2}{\left(\frac{1}{2}\right)^2}, -\frac{2\times 2}{\left(\frac{1}{2}\right)}\right)$$

$\equiv (8, -8)$.

Q.9 Length of focal chord drawn at point (8, 8) of parabola $y^2 = 8x$ is

(A) 25 (B) 18 (C) 25/4 (D) 25/2

Sol. **[D]**

Length of focal chord

$= a\left(t + \dfrac{1}{t}\right)^2$

Here 4a = 8

a = 2

and 2at = 8

$\Rightarrow 2 \times 2 \times t = 8$

$\Rightarrow t = 2$

$\therefore$ length of focal chord

$= a\left(t + \dfrac{1}{t}\right)^2$

$= 2\left(2 + \dfrac{1}{2}\right)^2$

$= 2\left(\dfrac{5}{2}\right)^2$

$= \dfrac{25}{2}$

Q.10 A circle described on any focal chord of parabola $y^2 = 4ax$ as its diameter touches

(A) Axis of Parabola

(B) directrix of Parabola

(C) Tangent drawn at vertex

(D) Latus Rectum

Sol. **[B]**

We know that a circle described on any focal chord of parabola $y^2 = 4ax$ as its diameter touches the directrix of the parabola.

Question based on **Tangent of parabola**

Q.11 The equation to the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(A) x + 2y + 4 = 0 (B) 2x + y – 4 = 0

(C) x – 2y – 4 = 0 (D) x – 2y + 4 = 0

Sol. **[D]**

Let equation of tangent to $y^2 = 4x$ is $y = mx + \dfrac{1}{m}$

This tangent is also touches $x^2 = -32y$

$\therefore x^2 = -32\left(mx + \dfrac{1}{m}\right)$

$mx^2 = -32(m^2x + 1)$

$\Rightarrow mx^2 + 32m^2x + 32 = 0$

D = 0

$(32m^2)^2 - 4 \cdot m \cdot 32 = 0$

$\Rightarrow 32m^3 = 4$

$\Rightarrow m^3 = \dfrac{1}{8} \Rightarrow m = \dfrac{1}{2}$

$\therefore$ Equation of line is $y = \dfrac{1}{2} \cdot x + \dfrac{1}{1/2}$

$\Rightarrow y = \dfrac{x}{2} + 2 \Rightarrow x - 2y + 4 = 0$

Q.12 If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then PA. PB is equal to (where P ≡ ($\sqrt{3}$, 0))

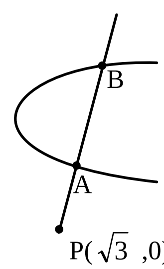
(A) $\dfrac{4(\sqrt{3}+2)}{3}$ (B) $\dfrac{4(2-\sqrt{3})}{3}$

(C) $2\sqrt{3}$ (D) $\dfrac{2(\sqrt{3}+2)}{3}$

Sol. **[A]**

Given parabola is $y^2 = x + 2$ and

given line is $y = \sqrt{3}x - 3$ and P ≡ ($\sqrt{3}$, 0)



AB makes an angle of 60° with the positive direction of x-axis. Co-ordinates of any point on this line may be taken as ($\sqrt{3}$ + r cos 60°, 0 + r sin 60°) i.e. $\left(\sqrt{3}+\frac{r}{2},\frac{r\sqrt{3}}{2}\right)$

If this point lies on $y^2 = x + 2$ then.

$\frac{3}{4}r^2 = \sqrt{3} + \frac{r}{2} + 2$ or $3r^2 = 4\sqrt{3} + 2r + 8$

or $3r^2 - 2r - 4(2 + \sqrt{3}) = 0$ ....(1)

Let r_1 and r_2 be the roots of equation (1)

then $r_1r_2 = -\frac{4(2+\sqrt{3})}{3}$

Now PA . PB = $|r_1|\ |r_2|$

$= |r_1 r_2| = \frac{4}{3}(2+\sqrt{3})$

Q.13 The equation of the tangents to the parabola $y^2 = 8x$ inclined at 45º to the x-axis and also the points of contact will be

(A) x – y – 2 = 0, (2, 4)

(B) x – y + 2 = 0, (2, 4)

(C) x – y + 2 = 0, (2, 3)

(D) none of these

Sol. **[B]**

Let equation of tangent is $y = mx + \frac{a}{m}$

Here m = tan 45° = 1

and a = 2

$\therefore\ y = x + \frac{2}{1}$

$\Rightarrow x - y + 2 = 0$

Now for point of contact

Put y = x + 2 in $y^2 = 8x$, we get

$(x + 2)^2 = 8x$

$\Rightarrow x^2 + 4x + 4 = 8x$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x - 2)^2 = 0$

$\Rightarrow x = 2$

$\therefore\ y = 4$

$\therefore$ point of contact is (2, 4)

Q.14 A tangent to the parabola $y^2 = 8x$ makes an angle of 45º with the straight line y = 3x + 5. The equation of the tangent and its point of contact are

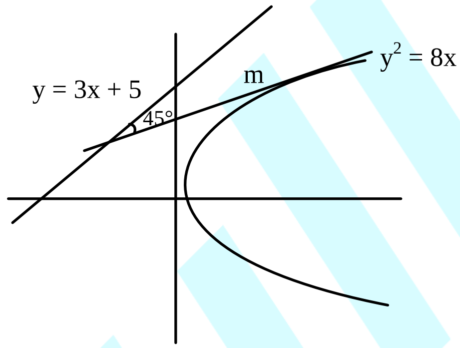
(A) 2x + y –1 = 0 at (1/2, –2),
x – 2y – 8 = 0 at (8, 8)

(B) 2x + y + 1 = 0 at (1, – 2),
x + 2y + 8 = 0 at (8, 8)

(C) 2x + y + 1 = 0 at (1/2, – 2),
x – 2y + 8 = 0 at (8, 8)

(D) None of these

Sol. **[C]**



Let equation of tangent is $y = mx + \frac{a}{m}$

Here a = 2

$\therefore\ y = mx + \frac{2}{m}$

$\tan\theta = \frac{m_1 - m_2}{1+m_1m_2}$

$\tan 45° = \left|\frac{m-3}{1+3m}\right|$

$1 + 3m = \pm (m - 3)$

$\Rightarrow 1 + 3m = m - 3$ or $1 + 3m = -m + 3$

$\Rightarrow 2m = -4$ or $4m = 2$

$\Rightarrow m = -2$ $\quad m = \frac{1}{2}$

$\therefore$ equation of tangent $y = -2x + \frac{2}{-2}$

$\Rightarrow 2x + y + 1 = 0$ or $y = \frac{1}{2}x + 4$

$\Rightarrow x - 2y + 8 = 0$

point of contact

$\left(\frac{a}{m^2},\frac{2a}{m}\right) \equiv \left(\frac{2}{(-2)^2},\frac{4}{-2}\right)$ or $\left(\frac{2}{(1/2)^2},\frac{4}{1/2}\right)$

$\equiv \left(\frac{1}{2},-2\right)$ or (8, 8)

Q.15 The equation of the line touching both the parabolas $y^2 = x$ and $x^2 = y$ is

(A) 4x + 4y + 1 = 0 (B) 4x + 4y – 1 = 0

(C) x + y + 1 = 0 (D) none of these

Sol. **[A]**

Let equation of tangent of $y^2 = x$ is

$y = mx + \frac{a}{m}$ where $a = \frac{1}{4}$

$y = mx + \frac{1}{4m}$ ...(i)

This is also touches $x^2 = y$ ...(ii)

$\therefore$ from (1) and (2)

$x^2 = mx + \frac{1}{4m}$

$\Rightarrow x^2 - mx - \frac{1}{4m} = 0$

D = 0

$m^2 + 4.\frac{1}{4m} = 0$

$m^3 = -1$

$m = -1$

$\therefore$ Raqd. equation

$y = -x - \frac{1}{4}$

$\Rightarrow 4y = -4x - 1 \Rightarrow 4x + 4y + 1 = 0$

Question based on

Normal of parabola

Q.16 The equations of the normals at the ends of the latus rectum of the parabola $y^2 = 4ax$ are given by

(A) $x^2 - y^2 - 6ax + 9a^2 = 0$

(B) $x^2 - y^2 - 6ax - 6ay + 9a^2 = 0$

(C) $x^2 - y^2 - 6ay + 9a^2 = 0$

(D) none of these

Sol. **[A]**

The ends of latus reaction of parabola $y^2 = 4ax$ is given by (a, 2a) and (a, – 2a)

Θ $y^2 = 4ax$

$\therefore 2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$

$\left(\frac{dy}{dx}\right)_{(a,2a)} = 1 \Rightarrow$ slope of tanent $\therefore$ slope of normal = – 1

& $\left(\frac{dy}{dx}\right)_{(a,-2a)} = -1 = m_2$ $\therefore$ slope of normal = 1

$\therefore$ y – 2a = – 1 (x – a)

$\Rightarrow$ x + y – 3a = 0 ...(1)

Also y + 2a = 1 (x – a) $\Rightarrow$ x – y – 3a = 0 ...(2)

from (1) and (2) $\Rightarrow$

(x + y – 3a) (x – y – 3a) = 0

$\Rightarrow x^2 - my - 3ax + xy - y^2 - 3ay - 3ax + 3ay + 9a^2 = 0$

$\Rightarrow x^2 - y^2 - 6ax + 9a^2 = 0$

Q.17 If a normal to the parabola $y^2 = 8x$ makes 45° angle with x-axis then its foot of the normal will be

(A) (2, 4) (B) (2, –4)

(C) (8, 8) (D) (8, –8)

Sol. **[B]**

Let foot of normal is $(am^2, -2am)$

normal is $y = mx - 2am - am^3$

Here a = 2 and m = tan 45° = 1

$\therefore$ foot of normal is

$(2(1)^2, -2(2)(1)) = (2, -4)$

Q.18 If the normal at the point (1, 2) on the parabola $y^2 = 4x$ meets the parabola again at the point $(t^2, 2t)$, then t is equal to

(A) 1 (B) –1

(C) 3 (D) – 3

Sol. **[D]**

Use $t_1 + t_2 = -\frac{2}{t_1}$ ...(1)

Here $2at_1 = 2 \Rightarrow 2.1.t_1 = 2$

$\Rightarrow t_1 = 1$ and $2at_2 = 2t$

$\Rightarrow 2.1.t_2 = 2.5 \Rightarrow t_2 = t$

From (1) $1 + t = -\frac{2}{1} = -2 \Rightarrow t = -3$.

Q.19 If P (–3, 2) is one end of the focal chord PQ of the parabola $y^2 + 4x + 4y = 0$, then the slope of the normal at Q is

(A) –1/2 (B) 2

(C) 1/2 (D) –2

Sol. **[A]**

As PQ is a focal chord, the normal at point Q will be parallel to the tangent at point 'P'.

Curve is : $(y + 2)^2 = -4(x - 1)$

using differentiation

$2(y + 2).\frac{dy}{dx} = -4$

$\left(\frac{dy}{dx}\right) = \left(\frac{-2}{y+2}\right)$

Slope of tangent at P(–3, 2)

$\left(\frac{dy}{dx}\right)_P = -\frac{1}{2}$ = Slope of normal at Q.

Q.20 The equation of the normal having slope m of the parabola $y^2 = x + a$ is

(A) $y = mx - 2am - am^3$

(B) $y = mx - am - am^3$

(C) $4y = 4mx + 4am - 2m - m^3$

(D) $4y = 4mx + 2am - am^3$

Sol. **[C]**

Given parabola is

$y^2 = x + a$

Let $y^2 = X$ where $X = x + a$

Equation of normal having slope m is given by

$y = mX - 2Am - Am^3$

Where $X = x + a$ and $A = \frac{1}{4}$

$\therefore y = m(x + a) - 2.\frac{1}{4}m - \frac{1}{4}m^3$

$\Rightarrow 4y = 4mx + 4ma - 2m - m^3$

$\Rightarrow 4y = 4mx + 4am - 2m - m^3$.

Q.21 The slopes of the three normals to the parabola $y^2 = 8x$ which pass through (18, 12) are

(A) 1, 2, 3 (B) 1, –2, 3

(C) 1, 2, –3 (D) –1, –2, 3

Sol. **[C]**

Let equation of normal is given by

$y = mx - 2am - am^3$

Here $4a = 8 \Rightarrow a = 2$

$\therefore y = mx - 4m - 2m^3$

since it passes through (18, 12)

Therefore

$12 = 18m - 4m - 2m^3$

$\Rightarrow 2m^3 - 14m + 12 = 0$

$\Rightarrow m^3 - 7m + 6 = 0$

Let m_1, m_2, m_3 are its roots

$\therefore \left.\begin{aligned} m_1 + m_2 + m_3 &= 0 \\ m_1m_2m_3 &= -6 \end{aligned}\right\}$ ...(1)

from (1) we can say that option (C) is the correct answer.

$\therefore$ slopes are 1, 2, – 3.

Q.22 The normal to the parabola $y^2 = 4ax$ at $(at_1^2, 2at_1)$ meets the curve again at $(at_2^2, 2at_2)$, then the number of points, where f(f(x)) is discontinuous is equal to where $f(x) = 1/(1 - x)$

(A) $t_1^2 + t_1t_2$ (B) $-t_1^2 - t_1t_2$

(C) $t_1^2 - t_1t_2$ (D) none of these

Sol. **[B]**

$f(x) = \frac{1}{1-x}$

$f(f(x)) = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{-x}$

& f [f(x)] is discontinuous at x = 1, 0

[Two points]

Now normal at 't_1' meets the parabola at 't_2'

then $t_2 = -t_1 - \frac{2}{t_1} \Rightarrow t_1t_2 + t_1^2 = -2$

$\Rightarrow -t_1^2 - t_1t_2 = 2$

Q.23 Number of normals can be drawn from point (1, 2) on the parabola $y^2 = 12x$ is

(A) 3 (B) 1

(C) 2 (D) None of these

Sol. **[A]**

$y^2 = 12x$ is the given parabola.

(1, 2) is the given point.

$S_1 = y_1^2 - 12x_1 = 4 - 12 < 0$

$\therefore$ point lies inside the parabola.

Therefore three normals are drawn.

Question based on Tangents from external point

Q.24 The tangents from the origin to the parabola $y^2 + 4 = 4x$ are inclined at

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Sol. **[D]**

Given parabola $y^2 + 4 = 4x$

$\Rightarrow y^2 - 4x + 4 = 0$

Equation of pair of tangents drawn from (0, 0) to the parabola

$SS_1 = T^2$

$(y^2 - 4x + 4)(4) = 4x^2$

$\Rightarrow x^2 - y^2 + 4x - 4 = 0$

this represent equation of a circle

Here a + b = 0

$\because$ a = 1, b = – 1

$\therefore$ tangents are perpendicular

Since coefficient of x^2 + coeff. of $y^2 = 0$

$\therefore$ Angle between them is $\frac{\pi}{2}$.

Q.25 The equation of the chord of contact of tangents drawn from the point (2, 3) to the parabola $y^2 + x = 0$ is

(A) $3y + x = 2$ (B) $6y - x = 2$

(C) $6y + x + 2 = 0$ (D) $3y - x = 2$

Sol. **[C]**

Given parabola is $y^2 + x = 0$

$\Rightarrow y^2 = -x$

Here $a = -\frac{1}{4}$

Equation of chord of contact of tangents drawn from (2, 3) is

$yy_1 = 2a(x + x_1) \Rightarrow y.3 = 2\left(-\frac{1}{4}\right)(x + 2)$

$\Rightarrow 6y = -(x + 2) \Rightarrow 6y + x + 2 = 0$

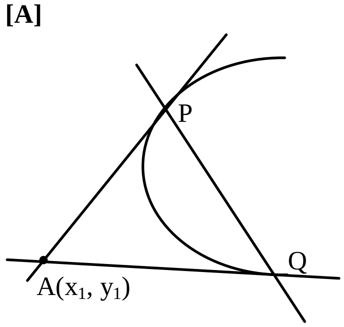
Question based on Chord of contact

Q.26 Line $x + y = 2$ meets parabola $y^2 = 8x$ at point P and Q. Point of intersection of tangents drawn at P and Q is

(A) (– 2, – 4) (B) (– 1, – 4)

(C) (– 2, – 3) (D) (– 3, – 2)

Sol. **[A]**



Let tangents meet at $A(x_1, y_1)$

Then equation of PQ

= chord of contact

$\Rightarrow yy_1 = 4(x + x_1)$

$\Rightarrow 4x - yy_1 + 4x_1 = 0$ …..(1)

also $x + y - 2 = 0$ ….(2)

comparing (1) & (2)

$\frac{4}{1} = \frac{-y_1}{1} = \frac{4x_1}{-2}$

& $(x_1, y_1) \equiv (-2, -4)$

Q.27 The chord of contact of the tangents to a parabola drawn from any point on its directrix passes through

(A) one extremity of LR (B) focus

(C) vertex (D) none of these

Sol. **[B]**

Let parabola is $y^2 = 4ax$ and point on directrix is $(-a, \beta)$ & equation of chord of contact

$y(\beta) = 2a(x - a)$

$\Rightarrow \beta ys = 2a(x - a)$

Which always passes through (a, 0), the focus of parabola.

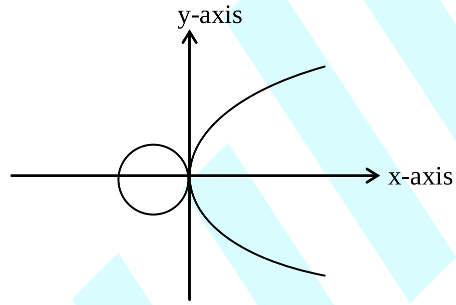
Q.28 The circle $x^2 + y^2 + 2\lambda x = 0$, $\lambda \in R$, touches the parabola $y^2 = 4x$ externally. Then

(A) $\lambda > 0$ (B) $\lambda < 0$

(C) $\lambda > 1$ (D) none of these

Sol. **[A]**

Centre of circle must on negative x-axis for that λ must be positive as centre of circle is $(-\lambda, 0)$.



Q.29 The equation of the director circle of the parabola $x^2 = 4ay$ is

(A) $x^2 + y^2 = a^2$ (B) $x^2 + y^2 = 2a^2$

(C) $x + a = 0$ (D) $y + a = 0$

Sol. **[D]**

We know that the director circle of a parabola is the directrix of that parabola.

Here given parabola is $x^2 = 4ay$

Its directrix is $y = -a$

$\Rightarrow y + a = 0$

This is the director circle of parabola

Question based on Chord with mid point

Q.30 If (a, b) be the mid point of a chord of the parabola $y^2 = 4x$ passing through its vertex then

(A) $a = 2b$ (B) $2a = b$

(C) $a^2 = 2b$ (D) $2a = b^2$

Sol. **[D]**

Equation of chord in terms of mid point

$T = S_1$

Here mid point is (a, b) & parabola is $y^2 = 4x$

$\therefore yy_1 = 2a'(x + x_1)$ Here $a' = 1$

$\therefore y.b = 2.1(x + a)$

$\Rightarrow by - 2(x + a) = 0$ is T and $S_1 = b^2 - 4a$

$\therefore$ from $T = S_1$

$by - 2(x + a) = b^2 - 4a$

Since it passes (0, 0)

Therefore $0 - 2(0 + a) = b^2 - 4a$

$\Rightarrow -2a = b^2 - 4a \Rightarrow b^2 = 2a$

Q.31 The mid-point of the line joining the common points of the line $2x - 3y + 8 = 0$ and $y^2 = 8x$ is

(A) (3, 2) (B) (5, 6)

(C) (4, –1) (D) (2, –3)

Sol. **[B]**

$y^2 = 4.2x$

$y^2 = 4.(3y - 8)$

$\Rightarrow y^2 = 12y - 32 \Rightarrow y^2 - 12y + 32 = 0$

$\Rightarrow y^2 - 8y - 4y + 32 = 0 \Rightarrow y(y - 8) - 4(y - 8) = 0$

$\Rightarrow (y - 4)(y - 8) = 0 \Rightarrow y = 4, 8$

$\therefore 2x = 3y - 8 \Rightarrow x = \frac{3y-8}{2}$

$\Rightarrow x = 2$ when $y = 4$ and $x = 8$ when $y = 8$

$\therefore$ points are (2, 4) and (8, 8)

$\therefore$ Mid point $= \left(\frac{2+8}{2}, \frac{4+8}{2}\right) = (5, 6)$

Q.32 If the tangent at the point P(2, 4) to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at Q and R, then the mid-point of QR is

(A) (2, 4) (B) (4, 2)

(C) (7, 9) (D) none of these

Sol. **[A]**

The equation of the tangent to $y^2 = 8x$ at P(2, 4) is given by

$4y = 4(x + 2)$

$\Rightarrow x - y + 2 = 0$ ...(i)

Let (x_1, y_1) be the mid-point of chord QR, then the equation of QR is

$yy_1 - 4(x + x_1) - 5 = y_1^2 - 8x_1 - 5$

$\Rightarrow 4x - yy_1 - 4x_1 + y^2 = 0$ ...(ii)

Clearly equations (i) and (ii) represents the same line, therefore

$\frac{4}{1} = -\frac{y_1}{-1} = \frac{-4x_1 + y_1^2}{2}$

$\Rightarrow y = 4$

and $8 = -4x_1 + y_1^2$

$\Rightarrow y_1 = 4$ and $x_1 = 2$

$\therefore$ reqd. mid point is (2, 4).

EXERCISE # 2

Part-A **Only single correct answer type questions**

Q.1 If t_1 and t_2 be the ends of a focal chord of the parabola $y^2 = 4ax$, then the equation $t_1x^2 + ax + t_2 = 0$ has

(A) imaginary roots

(B) both roots positive

(C) one positive and one negative roots

(D) both roots negative

Sol. **[C]**

$\therefore$ t_1 and t_2 be the ends of a focal chord of $y^2 = 4ax$

$\therefore t_1t_2 = -1$

$t_1x_2 + ax + t_2 = 0$ (given)

its discriminant $D = B^2 - 4AC$

$D = a^2 - 4t_1t_2 \Rightarrow D = a^2 - 4(-1)$

$D = a^2 + 4 \Rightarrow D = (a)^2 + (2)^2$

D = positive $\Rightarrow D > 0$

Let roots of the given equation is α & β.

$$\therefore \alpha\beta = \frac{t_2}{t_1} = \frac{t_1t_2}{t_1^2} = -\frac{1}{t_1^2} = -\text{ve}$$

$\Rightarrow$ one root must be negative

$\Rightarrow$ one positive and one negative roots.

Q.2 The length of a focal chord of the parabola $y^2 = 4ax$ at a distance b from the vertex is c. Then

(A) $2a^2 = bc$ (B) $a^3 = b^2c$

(C) $ac = b^2$ (D) $b^2c = 4a^3$

Sol. **[D]**

Let the ends of the focal chord be $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ then $t_1t_2 = -1$

$\Rightarrow c^2 = [a(t_1^2 - t_2^2)]^2 + \{2a(t_1 - t_2)\}^2$

The equation of the focal chord is

$$y - 2at_1 = \frac{2at_1 - 2at_2}{at_1^2 - at_2^2}(x - at_1^2)$$

$$\text{or } y - 2at_1 = \frac{2}{t_1 + t_2}(x - at_1^2)$$

$$\therefore b = \frac{\frac{2}{t_1+t_2}(-at_1^2) + 2at_1}{\sqrt{1 + \frac{4}{(t_1+t_2)^2}}} = \frac{2at_1t_2}{\sqrt{4 + (t_1+t_2)^2}}$$

$$\therefore b = \frac{-2a}{\sqrt{2 + t_1^2 + t_2^2}}$$

Also, $c^2 = a^2(t_1 - t_2)^2 \{(t_1 + t_2)^2 + 4\}$

$= a^2 (t_1^2 + t_2^2 + 2)(t_1^2 + t_2^2 + 2)$

$\therefore c = a(2 + t_1^2 + t_2^2)$

$$\therefore \frac{c}{a} = 2 + t_1^2 + t_2^2$$

$$\text{Hence, } b^2 = \frac{4a^2}{2 + t_1^2 + t_2^2}$$

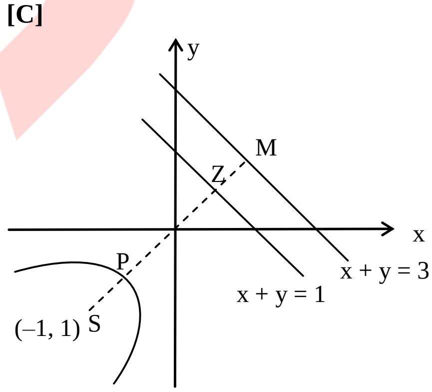
$$b^2 = \frac{4a^2}{c/a} \Rightarrow b^2 = \frac{4a^3}{c} \Rightarrow b^2c = 4a^3$$

Q.3 The shortest distance between line x + y = 3 and parabola whose directrix is x + y = 1 and focus at (–1, –1) is

(A) $\frac{3}{2\sqrt{2}}$ (B) $\frac{3}{\sqrt{2}} + \sqrt{2}$

(C) $\frac{3}{2\sqrt{2}} + \sqrt{2}$ (D) None of these

Sol. **[C]**



$\therefore$ SP = PZ

$\therefore$ SZ = 2PZ

$\therefore$ $\perp$r distance form (–1, –1) to $x + y \neq 1$ is

$$2PZ = \frac{|-3|}{\sqrt{2}} \Rightarrow PZ = \frac{3}{2\sqrt{2}}$$

And distacne between parallel lines x + y = 1 & x + y = 3 is

$$\frac{3-1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$\therefore$ The shortest distance will be PZ + ZM

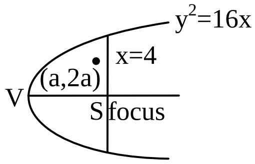
$$= \frac{3}{2\sqrt{2}} + \sqrt{2}$$

Q.4 The point (a, 2a) is an interior point of the region bounded by the parabola $y^2 = 16x$ and the double ordinate through the focus. Then a belongs to the open interval

(A) $a < 4$ (B) $0 < a < 4$

(C) $0 < a < 2$ (D) $a > 4$

Sol. **[B]**



(a, 2a) is an interior point of $y^2 = 16x$ if

$(2a^2) - 16a < 0$

i.e. $a^2 - 4a < 0$

v(0, 0) and (a, 2a) are on the same side of $x - 4 = 0$

Therefore $a - 4 < 0$

i.e. $a < 4$

Now, $a^2 - 4a < 0$

$\Rightarrow 0 < a < 4$

Q.5 The vertex of the parabola $y^2 = 8x$ is at the centre of a circle and the parabola cuts the circle at the ends of its latus rectum. Then the equation of the circle is

(A) $x^2 + y^2 = 4$ (B) $x^2 + y^2 = 20$

(C) $x^2 + y^2 = 80$ (D) None of these

Sol. **[B]**

Vertex = (0, 0)

The ends of latus rectum are (2,4), (2, –4)

$\therefore$ Centre = (0, 0)

radius $= \sqrt{2^2 + 4^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$\therefore$ equation of the circle is given by

$(x - 0)^2 + (y - 0)^2 = (\sqrt{20})^2$

$\Rightarrow x^2 + y^2 = 20$

Q.6 The co-ordinates of the point on the parabola $y^2 = 8x$, which is at minimum distance from the circle $x^2 + (y + 6)^2 = 1$ are

(A) (2, 4) (B) (–2, 4)

(C) (2, –4) (D) none of these

Sol. **[C]**

Given parabola is $y^2 = 8x$ ....(i)

Here a = 2

Given circle is $x^2 + (y + 6)^2 = 1$ ....(ii)

its centre is A (0, – 6)

Let $P(2t^2, 4t)$ be a point on parabola (i)

Equation of normal to parabola (i) at P is

$y - 4t = -t(x - 2t^2)$ ....(iii)

For P to be at minimum distance from circle (ii), normal (iii) should pass through centre A(0, 6) of the circle.;

$\therefore -6 - 4t = -t(0 - 2t^2)$

$\Rightarrow t^3 + 2t + 3 = 0$

$\Rightarrow (t + 1)(t^2 - t + 3) = 0$

$\Rightarrow t = -1$

$\therefore P \equiv (2, -4)$

Q.7 The equation of common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is

(A) $a^{1/3}x + b^{1/3}y + (ab) = 0$

(B) $a^{1/3}x + b^{1/3}y + (ab)^{2/3} = 0$

(C) $b^{1/3}x + a^{1/3}y + (ab)^{1/3} = 0$

(D) $b^{1/3}x - a^{1/3}y - (ab)^{1/3} = 0$

Sol. **[B]**

Any tangent to the parabola $y^2 = 4ax$ is

$y = mx + \dfrac{a}{m}$ ....(i)

If it is a tangent to the parabola $x^2 = 4by$ then it will cut it in two coincident points.

on eliminating y, we get

$x^2 = 4b\left[mx + \dfrac{a}{m}\right]$

$x^2 - 4bmx - 4b.\dfrac{a}{m} = 0$ ....(ii)

$\therefore \Delta = 0$

$\Rightarrow 16b^2m^2 + \dfrac{16ab}{m} = 0$

$m = -\left(\dfrac{a}{b}\right)^{1/3}$

on putting the value of m in (i), the equation of the common tangent is

$y = -\left(\dfrac{a}{b}\right)^{1/3} x + \dfrac{a}{-\left(\dfrac{a}{b}\right)^{1/3}}$

$y = \dfrac{-a^{1/3}x}{b^{1/3}} - b^{1/3}a^{2/3}$

$b^{\frac{1}{3}}y = -a^{1/3}x - b^{2/3}a^{2/3}$

$\Rightarrow a^{1/3}x + b^{1/3}y + a^{2/3}b^{2/3} = 0$

Q.8 Two parabolas $y^2 = 4a(x - \lambda_1)$ and $x^2 = 4a(y - \lambda_2)$ always touch each other, λ_1 and λ_2 being variable parameters. Then, their points of contact lie on a

(A) straight line (B) circle
(C) parabola (D) hyperbola

Sol. **[D]**

Let $P(x_1, y_1)$ be the point of contact of the two parabolas. Tangnts p to the two parobolas are

$yy_1 = 2a(x + x_1) - 4a\lambda_1$

$\Rightarrow 2ax - yy_1 = 2a(2\lambda_1 - x_1)$ ....(i)

and $xx_1 = 2a(y + y_1) - 4a\lambda_2$

$\Rightarrow xx_1 - 2ay = 2a(y_1 - 2\lambda_2)$ .....(ii)

Clearly (i) and (ii) reprents the same line

$\therefore \frac{2a}{x_1} = \frac{y_1}{2a}$

$\Rightarrow x_1y_1 = 4a^2$

Hence the locus of (x_1, y_1) is

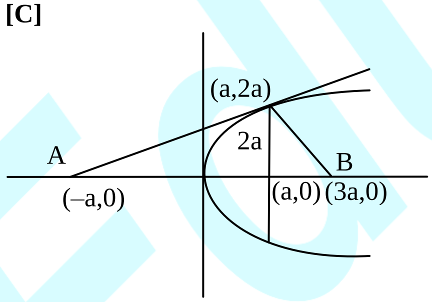
$xy = 4a^2$

which is a hyperbola.

Q.9 The area of the triangle formed by the tangent and the normal to the parabola $y^2 = 4ax$, both drawn at the same end of the latus rectum, and the axis of the parabola is

(A) $2\sqrt{2}a^2$ (B) $2a^2$
(C) $4a^2$ (D) none of these

Sol. **[C]**



equation of tangent drawn at (a, 2a) to $y^2 = 4ax$

its slope $m = 2y\frac{dy}{dx} = 4a$

$m = \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2a} = 1$

$y - 2a = (x - a)$

$x - y + a = 0$ ....(i)

Equation of normal

$\perp r$ to $x - y + a = 0$

$x + y + \lambda = 0 \Rightarrow a + 2a + \lambda = 0$

$\Rightarrow \lambda = -3a$

$x + y - 3a = 0$

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 4a \times 2a$

$= 4a^2$

Q.10 The condition that the parabolas $y^2 = 4c(x - d)$ and $y^2 = 4ax$ have a common normal other than x-axis $(a > c > 0)$ is

(A) $2a < 2c + d$ (B) $2c < 2a + d$
(C) $2d < 2a + c$ (D) $2d < 2c + a$

Sol. **[A]**

The two parabolas are given as

$y^2 = 4ax$ ....(i)

and $y^2 = 4c(x - d)$.........(ii)

Equation to any normal to (i) is

$y = mx - 2am - am^3$ ....(iii)

and equation to any normal to (ii) is

$y = m(x - d) - 2cm - cm^3$ ....(iv)

If there is any common normal then (iii) and (iv) must be idenfical. As the coefficients of x and y are equal, so the constant term will also be equal. therefore

$-2am - am^3 = -dm - 2mc - cm^3$

$\Rightarrow m[m^2(c - a) - 2a + d + 2c] = 0$

so either $m = 0$ or $m^2 = \frac{2a - d - 2c}{c - a}$

if m = 0, the common normal is the x-axis.

If $m^2 = \frac{2a - d - 2c}{c - a}$

then $m = \sqrt{\frac{2(a - c) - d}{c - a}} = \sqrt{-2 - \frac{d}{c - a}}$

If the volume of m is real and not zero then

$-2 - \frac{d}{c - a} > 0, -\frac{d}{c - a} > 2$ or $\frac{d}{a - c} > 2$

$\Rightarrow 2a < 2c + d$

Q.11 If the normal at three points $(ap^2, 2ap)$, $(aq^2, 2aq)$ and $(ar^2, 2ar)$ are concurrent then the common root of equations $px^2 + qx + r = 0$ and $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ is

(A) p (B) q
(C) r (D) 1

Sol. **[D]**

Equation of the three normals are

$px + y - 2(p^3 + 2p) = 0$

$qx + y - 2(q^3 + 2q) = 0$

$rx + y - 2(r^3 + 2r) = 0$

since there lines are con-current

$$\therefore \begin{vmatrix} p & 1 & p^3+2p \\ q & 1 & q^3+2q \\ r & 1 & r^3+2r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p & 1 & p^3 \\ q & 1 & q^3 \\ r & 1 & r^3 \end{vmatrix} = 0 \qquad [C_3 \to C_3 - 2C_1]$$

$$\Rightarrow \begin{vmatrix} 1 & p & p^3 \\ 1 & q & q^3 \\ 1 & r & r^3 \end{vmatrix} = 0$$

$\Rightarrow (p + q + r)(p - q)(q - r)(r - p) = 0$

$\Rightarrow p + q + r = 0$

$\Rightarrow$ 1 is a root of equation $px^2 + qx + r = 0$

Also 1 is a root of second equation. Hence 1 is the common root.

Q.12 If two of the three feet of normals drawn from a point to the parabola $y^2 = 4x$ be (1, 2) and (1, –2) then the third foot is

(A) $(2, 2\sqrt{2})$ (B) $(2, -2\sqrt{2})$

(C) (0, 0) (D) none of these

Sol. **[C]**

We know that the sum of ordinates of their feet is always is zero

let the third foot is (α,β)

then $2 + (-2) + \beta = 0$

$\Rightarrow \beta = 0$

Also, we know that centroid of the triangle formed by joining their feet lies on the axis of the parabola.

Here axis of the parabola $y^2 = 4x$ is x-axis

$\therefore$ Centroid lies on x-axis

Centroid will be $\left(\frac{1+1+\alpha}{3}, \frac{2-2+\beta}{3}\right)$

$\therefore (\alpha, \beta)$ lie on $y^2 = 4x$

$\therefore \beta^2 = 4\alpha,\ \therefore 0 = 4\alpha,\ \therefore \alpha = 0$

$\therefore (\alpha,\beta) \equiv (0, 0)$

Q.13 Let the line $\lambda x + my = 1$ cut the parabola $y^2 = 4ax$ in the points A and B. Normals at A and B meet at point C. Normal from C other than these two meet the parabola at D then the coordinate of D are

(A) (a, 2a) (B) $\left(\frac{4am}{\lambda^2}, \frac{4a}{\lambda}\right)$

(C) $\left(\frac{2am^2}{\lambda^2}, \frac{2a}{\lambda}\right)$ (D) $\left(\frac{4am^2}{\lambda^2}, \frac{4am}{\lambda}\right)$

Sol. **[D]**

Let $A(at_2, 2at_1)$ and $B(at_2^2, 2at_2)$ be the points on the point on the parabola where it met by $\lambda x + my = 1$, then $\lambda at_1^2 + 2mat_1 = 0$ and $\lambda at_2^2 + 2mat_2 - 1 = 0$

$\Rightarrow t_1, t_2$ are the roots of the equation $\lambda at^2 + 2mat - 1 = 0$

$\Rightarrow t_1 + t_2 = -\frac{2m}{\lambda}$ and $t_1 t_2 = -\frac{1}{\lambda a}$

The normals at A and B intersect at C and Normal form C Meet parabola at D where D is

$D \equiv (2a + a(t_1^2 + t_2^2 + t_1t_2)), -at_1t_2(t_1 + t_2))$

Now, $t_1^2 + t_2^2 + t_1t_2 = (t_1 + t_2)^2 - t_1t_2$

$$= \frac{4m^2}{\lambda^2} + \frac{1}{\lambda a}$$

$\therefore$ co-ordinate of D is

$$\left(2a + \frac{4am^2}{\lambda_2} + \frac{1}{\lambda}, \frac{-2m}{\lambda^2}\right) \equiv \left(\frac{4am^2}{\lambda^2}, \frac{4am}{\lambda}\right)$$

Q.14 The equation of normal in terms of slope m to the parabola $(y - 2)^2 = 4(x - 3)$ is -

(A) $y = mx - 2m - m^3$

(B) $y = mx - 5m - m^3 + 2$

(C) $y - 2 = mx - 2m - m^3$

(D) none of these

Sol. **[B]**

we know eqution of normal interms of m to the parabola $y^2 = 4ax$ is given by

$y = mx - 2am - am^3$

Here given parabola is

$(y - 2)^2 = 4(x - 3)$

Let $y - 2 = Y$ and $x - 3 = X$

$\therefore Y^2 = 4X$

equation of normal will be

$Y = mx - 2am - am^3$

Here a = 1

$\therefore Y = mx - 2m - m^3$

$\therefore y - 2 = m(x - 3) - 2m - m^3$

$y - 2 = mx - 3m - 2m - m^3$

$\Rightarrow y = mx - 5m - m^3 + 2$

Q.15 The point on the line $x - y + 2 = 0$ from which the tangent to the parabola $y^2 = 8x$ is perpendicular to the given line is (a, b), then the line ax + by + c = 0 is

(A) parallel to x-axis

(B) parallel to y-axis

(C) equally inclined to the axes

(D) none of these

Sol. **[B]**

Given line is $x - y + 2 = 0$ ....(i)

Let P(a, b) be a point on line (i)

$\therefore$ $a - b + 2 = 0$

Equation of the tangent to parabola $y^2 = 8x$ at $(2t^2, 4t)$ is

$yt = x + 2t^2$ ....(iii)

line (iii) is perpendicular to (i)

$\therefore \frac{1}{t} = -1$ or $t = -1$

$\therefore$ Equation of (iii) becomes

$x + y + 2 = 0$ ....(iv)

since line (iv) passes through point P(a, b)

$\therefore$ $a + b + 2 = 0$ ....(v)

from (ii) and (v)

$a = -2$, $b = 0$

slope of line ax + by + c = 0 is $-\frac{a}{b} = \frac{2}{0}$

(undefined)

Hence line ax + by + c = 0 is parallel to y-axis.

Q.16 If two tangents drawn from the point (α, β) to the parabola $y^2 = 4x$ be such that the slope of one tangent is double of the other then

(A) $\beta = \frac{2}{9}\alpha^2$ (B) $\alpha = \frac{2}{9}\beta^2$

(C) $2\alpha = 9\beta^2$ (D) none of these

Sol. **[B]**

Any tangent to the parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$

it passes through (α, β) if

$\beta = m\alpha + \frac{1}{m}$ or $\alpha m^2 - \beta m + 1 = 0$

it will have root m_1, $2m_1$ if

$m_1 + 2m_1 = \frac{\beta}{\alpha}$

$m_1 \cdot 2m_1 = \frac{1}{\alpha} \Rightarrow 2.\left(\frac{\beta}{3\alpha}\right)^2 = \frac{1}{\alpha}$

$\Rightarrow \frac{2\beta^2}{9\alpha^2} = \frac{1}{\alpha} \Rightarrow 2\beta^2 = 9\alpha \Rightarrow \alpha = \frac{2}{9}\beta^2$

Q.17 If line 3x + y = 8 meets parabola $(y-2)^2 = 4(x-1)$ at A and B, then the point of intersection of tangents drawn at A and B lies on line

(A) $x = -1$ (B) $x = -1/2$

(C) $x = 0$ (D) None of these

Sol. **[C]**

Given line is 3x + y = 8

and given parabola is

$(y-2)^2 = 4(x-1)$

its vertex is = (1, 2)

and its focus is = (1 + 1, 2) = (2, 2)

Since given line 3x + y = 8 passes through focus. therefore given line is the focal chord of parabola Therefore A and B are end points of focal chord. the tangents drawn at A and B meets its directrix at right angle.

Therefore points of intersection lie on its directrix is

$x + 1 - 1 = 0 \Rightarrow x = 0$

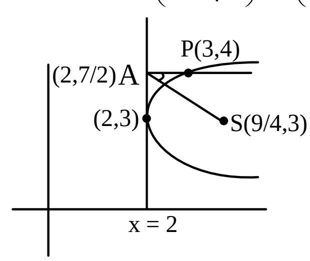
Q.18 Let tangent at P(3, 4) to the parabola $(y-3)^2 = (x-2)$ meets line x = 2 at A and if S be the focus of parabola then ∠SAP is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) None

Sol. **[B]**

Vertex of the parabola is (2, 3)

its focus S = $\left(2+\frac{1}{4},3\right) = \left(\frac{9}{4},3\right)$



Equation of tangent we have of find

$(y-3)^2 = x - 2$

differentiate

$2(y-3)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2(y-3)}$

$\left(\frac{dy}{dx}\right)_{(3,4)} = \frac{1}{2}$

$\therefore$ equation ot tangents is

$y - 4 = \frac{1}{2}(x - 3) \Rightarrow y = 4 + \frac{1}{2}(2 - 3)$

$\Rightarrow y = 4 - \frac{1}{2} = 7/2$

$\therefore$ Slope of SA $= \frac{3 - 7/2}{\frac{9}{4} - 2} = \frac{-1/2}{1/4} = -2 = m_1$

Slope of PA $= \frac{4 - 7/2}{3 - 2} = \frac{1}{2} = m_2$

since $m_1 m_2 = -1$

$\therefore \angle SAP = \pi/2$

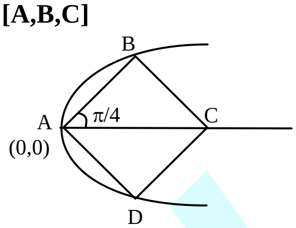
Part-B One or more than one correct answer type questions

Q.19 A square has one vertex at the vertex of the parabola $y^2 = 4ax$ and one of the diagonal through this vertex lies along the axis of the parabola. If the ends of the other diagonal lie on the parabola, the coordinates of the vertices of the squares are

(A) (4a, 4a) (B) (4a, –4a)

(C) (8a, 0) (D) $(4\sqrt{2}a, 0)$

Sol. **[A,B,C]**



AC is one diagonal along x-axis then the other diagonal is BD where both B and D lie on parabola. Also slope of AB is $\tan\frac{\pi}{4} = 1$

If B is $(at^2, 2at)$ then the slope of AB

$= \frac{2at}{at^2} = \frac{2}{t} = 1$

$\therefore t = 2$

$\therefore$ B is (4a, 4a) and hence D is (4a, – 4a)

Clearly C is (8a, 0)

Q.20 Let the equations of a circle and a parabola be $x^2 + y^2 - 4x - 6 = 0$ and $y^2 = 9x$ respectively. Then

(A) (1, –1) is a point on the common chord

(B) the equation of the common chord is y + 1 = 0

(C) the length of the common chord is 6

(D) none of these

Sol. **[A,C]**

$x^2 + 9x - 4x - 6 = 0$

$\Rightarrow x^2 + 5x - 6 = 0$

$\Rightarrow x^2 + 6x - x - 6 = 0$

$\Rightarrow x(x + 6) - 1(x + 6) = 0$

$\Rightarrow (x + 6)(x - 1) = 0$

$\Rightarrow x = 1, -6$

when x = 1, y = ±3 x = – 6x

A(1, 3) & β(1, –3) are common points

Length of common chord AB

$AB = \sqrt{(6)^2} = 6$

Q.21 The equation of a common tangent to the parabola $y^2 = 2x$ and the circle $x^2 + y^2 + 4x = 0$ is

(A) $2\sqrt{6}x + y = 12$

(B) $x + 2\sqrt{6}y + 12 = 0$

(C) $x - 2\sqrt{6}y + 12 = 0$

(D) $2\sqrt{6}x - y = 12$

Sol. **[B,C]**

Let equation of tangent of $y^2 = 2x$ is

$y = mx + \frac{1}{2m}$

This is also a tangent to $x^2 + y^2 + 4x = 0$

$\therefore x^2 + \left(mx + \frac{1}{2m}\right)^2 + 4x = 0$

$\Rightarrow x^2 + m^2x^2 + \frac{1}{4m^2} + 5x = 0$

$\Rightarrow x^2(1 + m^2) + 5x + \frac{1}{4m^2} = 0$

D = 0

$25 - \frac{4(1+m^2)}{4m^2} = 0$

$\Rightarrow 100m^2 = 4(1 + m^2)$

$\Rightarrow 25m^2 - m^2 = 1$

$\Rightarrow 24m^2 = 1$

$\Rightarrow m^2 = \frac{1}{24}$

$\Rightarrow m = \pm\frac{1}{\sqrt{24}}$

$\Rightarrow m = \pm\frac{1}{2\sqrt{6}}$

$\therefore y = \pm\frac{1}{2\sqrt{6}}x \pm \frac{1}{2} \times 2\sqrt{6}$

$\Rightarrow 2\sqrt{6}\ y = \pm x \pm 12$

$\Rightarrow x - 2\sqrt{6}\ y + 12 = 0$ or $-x - 2\sqrt{6}\ y - 12 = 0$

$\Rightarrow x - 2\sqrt{6}\ y + 12 = 0$ or $x + 2\sqrt{6}\ y + 12 = 0$

Q.22 The slope of a tangent to the parabola $y^2 = 9x$ which passes through the point (4, 10) is

(A) 9/4 (B) 1/4 (C) 3/4 (D) 1/3

Sol. **[A,B]**

Let m be the slope of tangent to $y^2 = 9x$

Here $4a = 9 \Rightarrow a = 9/4$

$\therefore$ equation of tangent is

$y = mx + \frac{9}{4m}$

it passes through (4, 10)

$\Rightarrow 10 = 4m + \frac{9}{4m}$

$\Rightarrow 16m^2 - 40m + 9 = 0$

$\Rightarrow m = \frac{40 \pm \sqrt{1600 - 576}}{32}$

$\Rightarrow m = \frac{40 \pm 32}{32}$

$\Rightarrow m = \frac{72}{32}$ or $\frac{8}{32}$

$\Rightarrow m = \frac{9}{4}$ or $\frac{1}{4}$

Therefore slope is

$\frac{9}{4}$ and $\frac{1}{4}$

Q.23 A tangent to the parabola $y^2 = 4ax$ is inclined at an angle $\pi/3$ with the axis of the parabola. The point of contact is

(A) $\left(\frac{a}{3}, \frac{-2a}{\sqrt{3}}\right)$ (B) $(3a, -2\sqrt{3}\ a)$

(C) $(3a, 2\sqrt{3}\ a)$ (D) $\left(\frac{a}{3}, \frac{2a}{\sqrt{3}}\right)$

Sol. **[A,C,D]**

If the point of contact is $(at^2, 2at)$ then the tangent is $y.\ 2at = 2a(x + at^2)$

$\therefore m = \frac{1}{t}$

form the questions

$\frac{1}{t} = \tan\left(\pm\frac{\pi}{3}\right) = \pm\sqrt{3} \Rightarrow t = \pm\frac{1}{\sqrt{3}}$

$\therefore$ point of contact

$= \left(\frac{a}{3}, \pm\frac{2a}{\sqrt{3}}\right)$

$= \left(\frac{a}{3}, \frac{2a}{\sqrt{3}}\right)$ and $\left(\frac{a}{3}, -\frac{2a}{\sqrt{3}}\right)$

Q.24 AB, AC are tangents to a parabola $y^2 = 4ax$. If $\lambda_1, \lambda_2, \lambda_3$ are the length of perpendicualrs from A, B, C on any tangent to the parabola, then

(A) $\lambda_1, \lambda_2, \lambda_3$ are in GP

(B) $\lambda_2, \lambda_1, \lambda_3$ are in GP

(C) $\lambda_3, \lambda_1, \lambda_2$ are in GP

(D) $\lambda_3, \lambda_2, \lambda_1$ are in GP

Sol. **[B,C]**

Let the co-ordinates of B and C be $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ respectively. Then the co-ordinates of A are $(at_1t_2, (t_1+t_2)$

The equation of any tangent to $y^2 = 4ax$ is

$ty = x + at^2$

$\therefore \lambda_1 = \frac{at_1t_2 - a(t_1+t_2)t + at^2}{\sqrt{1+t^2}}$

$\lambda_2 = \frac{at_1^2 - 2at_1 + at^2}{\sqrt{1+t_2}}$

$\lambda_3 = \frac{at_2^2 - 2at_2 + at^2}{\sqrt{1+t^2}}$

Clearly $\lambda_2\lambda_3 = \lambda_1^2$

Therefore $\lambda_2, \lambda_1, \lambda_3$ are in G.P.

Part-C Assertion-Reason type questions

The following questions 25 to 27 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Q.25 **Assertion (A) :** The latus rectum is the shortest focal chord in a parabola $y^2 = 4ax$.

Reason (R) : As the length of focal chord of the parabola $y^2 = 4ax$ is $a(t + 1/t)^2$ which is minimum when $t = \pm 1$.

Sol. **[A]**

For 't_1' and 't_2' being the ends of focal chord, we know

$$t_1 t_2 = -1$$

& length of chord with the ends

$P(at^2, 2at)$ and

$$Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$$

$$PQ = \sqrt{\left(at^2 - \frac{a}{t^2}\right)^2 + \left(2at + \frac{2a}{t}\right)^2}$$

$$= a\left(t + \frac{1}{t}\right)\sqrt{\left(t - \frac{1}{t}\right)^2 + 4}$$

$$PQ = a\left(t + \frac{1}{t}\right)^2$$

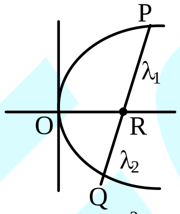
Which is minimum when $t = \pm 1$.

That is when PQ becomes latus rectum.

Q.26 **Assertion (A) :** If 4 and 3 are length of two focal segment of a focal chord of parabola then latus rectum of this parabola will be $\frac{48}{7}$.

Reason (R) : If λ_1 and λ_2 are lengths of segments of a focal chord of a parabola, then its latus rectum is $\frac{2\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$.

Sol. **[C]**



Parabola : $y^2 = 4ax$

Let PQ be the focal chord

$P(at^2, 2at)$

$$Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$$

then $\lambda_1 = a + at^2$

$$\lambda_2 = a + \frac{a}{t^2}$$

so $$\frac{2\lambda_1\lambda_2}{\lambda_1 + \lambda_2} = 2\left(\frac{a^2.(1+t^2).\left(1 + \frac{1}{t^2}\right)}{a\left(1 + t^2 + 1 + \frac{1}{t^2}\right)}\right)$$

$$= \frac{2a.\left(\frac{t^2+1}{t}\right)^2}{\left(t + \frac{1}{t}\right)^2} = 2a$$

& Reason is false

Latus Rectum is $\frac{4\lambda_1\lambda_2}{\lambda_1\lambda_2}$

Q.27 **Assertion (A) :** Through $(h, h + 1)$ there cannot be more than one normal to the parabola $y^2 = 4x$, if $h < 2$.

Reason (R) : The point $(h, h + 1)$ lies outside the parabola for all $h \neq 1$.

Sol. **[C]**

$y^2 = 4x$, $(h, h + 1)$

$y^2 - 4x = 0$

$\Rightarrow (h + 1)^2 - 4h$

$\Rightarrow h^2 - 2h + 1$

$\Rightarrow (h - 1)^2$

$\Rightarrow (h - 1)^2 > 0$

$\Rightarrow$ The point $(h, h + 1)$ lies outside the parabola for all $h \neq 1$.

Equation of normal is given by in terms of m

$y = mx - 2am - am^3$

$\Rightarrow h + 1 = mh - 20\ m - m^3$ $[\Theta\ a = 1]$

$\Theta\ y^2 = 4x$

$$\therefore 2y\frac{dy}{dx} = 4$$

$$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{2}{y} = \frac{2}{h+1} = \text{slope of tangent}$$

$$\therefore \text{sope of normal } m = -\frac{h+1}{2}$$

$h + 1 = m\,h - 2m - m^3$

$$\Rightarrow (h + 1) = -\frac{h(h+1)}{2} + \frac{2(h+1)}{2} - \frac{(h+1)^3}{8}$$

$\Rightarrow 8(h + 1) = -4h^2 - 4h + 8h + 8 - (h + 1)^3$

$\Rightarrow (h + 1)\ [8 + (x + 1)^2] = 4h - 4h^2 + 8$

$\Rightarrow (h + 1)\ [8 + h^2 + 2h + 1] = 4h - 4h^2 + 8$

$\Rightarrow 8h + h^3 + 2h^2 + h + 8 + h^2 + 1 = 4h + 4h^2 - 8 = 0$

$\Rightarrow h^3 + 7h^2 + 7h + 1 = 0$

$\Rightarrow (h^3 + 1) + 7h\ (h + 1) = 0$

$\Rightarrow (h + 1)\ (h^2 - h + 1) + 7h\ (h + 1) = 0$

$\Rightarrow (h + 1)\,[h^2 - h + 1 + 7h] = 0$

$\Rightarrow (h + 1)\,[h^2 + 6h + 1] = 0$

Part-D Column Matching type questions

Q.28 In column I different equations of parabolas are given and in column II their one of the parameters is given match them.

Column I	**Column II**
(A) Vertex of parabola $x^2 - 6x - 4y + 3 = 0$	(P) $\left(\frac{5}{4}, 2\right)$
(B) Focus of parabola $y^2 - 4y - 3x - 2 = 0$	(Q) $\left(\frac{-5}{4}, 2\right)$
(C) Mid point of vertex & Focus of parabola $(y - 2)^2 = 2(x - 1)$	(R) $\left(3, -\frac{3}{2}\right)$
(D) Point at which normal drawn on parabola $y^2 = 4x$, makes equal angle with axis	(S) (1, 2)

Sol. **A → R; B → Q; C → P, D → S**

(A) $x^2 - 6x - 4y + 3 = 0$

$(x - 3)^2 = 4y + 6$

$(x - 3)^2 = 4\left(y + \frac{3}{2}\right)$

so vertex $\equiv \left(3, \frac{-3}{2}\right)$

(B) $y^2 - 4y - 3x - 2 = 0$

$(y - 2)^2 = 3x + 6$

$(y - 2)^2 = 3(x + 2)$

& focus $\equiv \left(-2 + \frac{3}{4}, 2\right) \equiv \left(\frac{-5}{4}, 2\right)$

(C) $(y - 2)^2 = 2(x - 1)$

vertex ≡ (1, 2)

focus $\equiv \left(\frac{1}{2} + 1, 2\right)$

& mid point $\equiv \left(\frac{1 + \frac{3}{2}}{2}, \frac{2 + 2}{2}\right)$

$\equiv \left(\frac{5}{4}, 2\right)$

(D) $y^2 = 4x$

Normal : $y = mx - 2am - am^3$

at the point $(am^2, -2am)$

But a = 1

& Point is $(m^2, -2m)$

the normal makes equal angle with axes

So $m = \pm 1$

& points are (1, –2) and (1, 2)

Q.29

Column I	**Column II**
(A) The length of the latus ractum of $y^2 + 2x + 2by + \alpha = 0$, equals	(P) 2
(B) If the tangents from point (0, 2) to $y^2 = 4ax$ are inclined at an angle $3\pi/4$, then a =	(Q) – 2
(C) A tangent at point A of the circle $2(x^2 + y^2) - 3x + 4y = 0$ pass through the point P (2, 1). Then PA is equal to	(R) –1
(D) One of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m =	(S) 1

Sol. **A → P; B → P,Q; C → P, D → R,S**

(A) $y^2 + 2x + 2by + \alpha = 0$

$(y + b)^2 + 2x + \alpha - b^2 = 0$

$\Rightarrow (y + b)^2 = -2\left(x + \frac{\alpha}{2} - \frac{b^2}{2}\right)$

so length of L.R. = 2

(B) Let the equation of tangent

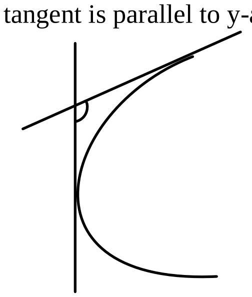
$y = mx + \frac{a}{m}$

If it passes through (0, 2),

$2 = \frac{a}{m} \Rightarrow m = \frac{a}{2}$.

also the coefficient of $m^2 = 0$

$\Rightarrow$ one tangent is parallel to y-axis.



& $m = \pm 1$

& $a = \pm 2$

(C) PA = $\sqrt{S_1}$ = length of tangent

where $S = x^2 + y^2 - \frac{3}{2}x + 2y$

point P ≡ (2, 1)

& $S_1 = 4 + 1 - \frac{3}{2} \times 2 + 2 = 4$

& PA = 2

(D) Bisector is $x = \pm y$

⇒ using in $my^2 + (1 - m^2)xy - mx^2 = 0$

we get $(1 - m^2) = 0 \Rightarrow m = \pm 1$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Find the latus rectum, the vertex, the focus, equations of the axis, the directrix and the tangent at the vertex of the parabola $y^2 - 8y - 2x + 9 = 0$.

Sol. Given equation of parabola is

$$y^2 - 8y - 2x + 9 = 0$$

$\Rightarrow (y-4)^2 = 2x - 9 + 16$

$\Rightarrow (y-4)^2 = 2x + 7$

$\Rightarrow (y-4)^2 = 2(x + 7/2)$

$\therefore$ its length of latus rectum = 2

For vertex, $y - 4 = 0$ & $x + \frac{7}{2} = 0$

$\Rightarrow y = 4$ & $x = -7/2 \Rightarrow$ vertex is $(-7/2, 4)$

$\therefore$ its focus is given by

Here $4a = 2 \therefore a = \frac{1}{2}$

$(a + x, y)$

$= \left(\frac{1}{2} - \frac{7}{2}, 4\right) = (-3, 4)$

$\therefore$ its euqation of axis is given by

$$y - 4 = 0$$
$$y = 4$$

$\therefore$ its directrix is $x + \frac{7}{2} = -\frac{1}{2}$

$\Rightarrow x = -\frac{7}{2} - \frac{1}{2} \Rightarrow x = -4$

$\therefore$ its tangent at vertex is given by

$x + \frac{7}{2} = 0 \Rightarrow x = -7/2$

Q.2 Two parabolas C and D intersect at the two different points, where C is $y = x^2 - 3$ and D is $y = kx^2$. The intersection at which the x-value is positive is designated point A, and x = a at this intersection. The tangent line λ at A to the curve D intersects curve C at point B, other than A. If x-value of point B is 1, then find value of 'a'.

Sol. The given parabolas intersect at the points whose x-values are given by $kx^2 = x^2 - 3$

since $x = a(>0)$ satisfies this relation , we get

$ka^2 = a^2 - 3 \Rightarrow k = \frac{a^2-3}{a^2}$

Also, the co-ordinate of A, the point of intersection of C & D is $(a, a^2 - 3)$

Equation of the tangent λ at A to the curve.

$D : y = k\,x^2 = \frac{a^2-3}{a^2}x^2$ is

$\frac{1}{2}(y + a^2 - 3) = \frac{a^2-3}{a^2}\,xa$

$\Rightarrow y = \frac{2(a^2-3)}{a}x - a^2 + 3$

The line λ-meets the curve $C : y = x^2 - 3$ at the points whose x-values are given by

$x^2 - 3 = \frac{2(a^2-3)}{a}x + a^2 + 3$

$\Rightarrow ax^2 + (6 - 2a^2)x + a^3 - 6a$

$\Rightarrow (x - a)\,[ax + 6 - a^2] = 0$

$\Rightarrow x = a$ (which represents point A)

or $x = \frac{a^2-6}{a}$ (which represent point B), Since the x-values of B is 1, we have

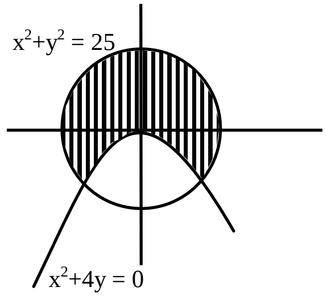
$$\frac{a^2-6}{a} = 1 \Rightarrow a^2 - a - 6 = 0$$

$\Rightarrow (a-3)(a+2) = 0$

since $a > 0$, $\therefore a = 3$

Q.3 Find the number of points with integral coordinates $(2a, a-1)$ that fall in the interior of the larger segment of the circle $x^2 + y^2 = 25$ cut off by the parabola $x^2 + 4y = 0$. Also find their coordinates.

Sol. The region is shaded. So point $(2a, a - 1)$ lies inside the circle but outside the parabola.



& $(2a)^2 + (a-1)^2 - 25 < 0$

$5a^2 - 2a - 24 < 0$

$5a^2 + 10a - 12a - 24 < 0$

$5a(a+2) - 12(a+2) < 0$

$(a + 2)(5a - 12) < 0$

$$\begin{array}{ccccc} + & & - & & + \\ & -2 & & 12/5 & \end{array}$$

& $a \in \{-1, 0, 1, 2\}$ ……(1)

Now $(2a)^2 + 4(a - 1) > 0$

$4a^2 + 4a - 4 > 0$

$4(a^2 + a - 1) > 0$

$$\Rightarrow \quad 4\left[a - \left(\frac{-1+\sqrt{5}}{2}\right)\right]\left[a + \frac{1+\sqrt{5}}{2}\right] > 0$$

$$\begin{array}{ccccc} + & & - & & + \\ & -\left(\frac{1+\sqrt{5}}{2}\right) & & \left(\frac{\sqrt{5}-1}{2}\right) & \end{array}$$

& $a \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$

& possible values of a = 1, 2

$\Rightarrow$ coordinates = (2, 0) and (4, 1)

Q.4 Prove that in the parabola $y^2 = 4ax$, the length of the chord passing through the vertex and inclined to the x-axis at an angle θ is $(4a \cos\theta)/\sin^2\theta$.

Sol. $\therefore x = \lambda\cos\theta$; $y = \lambda\sin\theta$

putting $y^2 = 4ax$

$\lambda^2 \sin^2\theta = 4a.\lambda \cos\theta$

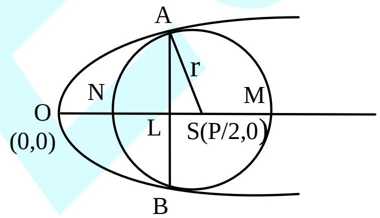
$$\therefore \lambda = \frac{4a\cos\theta}{\sin^2\theta}$$

Q.5 From the focus of the parabola $y^2 = 2px$ as centre a circle is described so that a common chord of the curves is equidistant from the vertex and the focus of the parabola. Write the equation of the circle.

Sol. $y^2 = 2px$, its focus S is (P/2, 0)

circle with centre (P/2,0) is

$(x - P/2)^2 + y^2 = r^2$



We have to find the radius

AB is the common chord, (P/4, 0) is mid-point

LS = P/4 = OL = x

$AL^2 = y^2$ where $y^2 = 2P.\frac{P}{4} = \frac{P^2}{2}$

$\therefore r^2 = AL^2 + SL^2$

$$= \frac{P^2}{2} + \frac{P^2}{16} = \frac{9P^2}{16}$$

$$\therefore r = \frac{3P}{4}$$

Hence the circle is

$$\left(x - \frac{P}{2}\right)^2 + y^2 = \frac{9P^2}{16}$$

Q.6 Find the equation of common tangents to the circle $x^2 + y^2 = 1$ and the parabola $y^2 = 4x$, if any such tangent exists.

Sol. $y = mx + \frac{a}{m}$ is equation of tangent to $y^2 = 4ax$

$\therefore y = mx + \frac{1}{m}$ is equation of tangent to $y^2 = 4x$

$$x^2 + \left(mx + \frac{1}{m}\right)^2 = 1$$

$$\Rightarrow x^2 + m^2x^2 + \frac{1}{m^2} + 2x - 1 = 0$$

$$\Rightarrow x^2(1 + m^2) + 2x + \left(\frac{1}{m^2} - 1\right) = 0$$

D = 0

$$\Rightarrow 4 - 4(1 + m^2)\left(\frac{1}{m^2} - 1\right) = 0$$

$$\Rightarrow 1 = \frac{1 - m^4}{m^2}$$

$\Rightarrow 1 - m^4 = m^2$

$\Rightarrow m^4 + m^2 - 1 = 0$

$$\therefore m^2 = \frac{-1 \pm \sqrt{1+4}}{2.1} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\therefore m = \pm\sqrt{\frac{-1 \pm \sqrt{5}}{2}} = \sqrt{\frac{\sqrt{5}-1}{2}} = \sqrt{\frac{4}{2(\sqrt{5}+1)}}$$

$$= \pm\sqrt{\frac{2}{\sqrt{5}+1}}$$

$$\therefore y = \pm\sqrt{\frac{2}{\sqrt{5}+1}}\, x \pm \sqrt{\frac{\sqrt{5}+1}{2}}$$

$$\Rightarrow \pm\sqrt{\frac{1}{2}(1+\sqrt{5})y} + \frac{1}{2}\left(1+\sqrt{5}\right) = 0$$

Q.7 A variable tangent to the parabola $y^2 = 4ax$ meets the circle $x^2 + y^2 = r^2$ at P and Q. Prove that the locus of the mid point of PQ is $x(x^2 + y^2) + ay^2 = 0$.

Q.8 Find the equation of the normal to the parabola $y^2 = 4x$ at the point (4, 4). Also find the point on this normal from which the other two normals drawn to the parabola will be at right angles.

Sol. Equation of normal to $y^2 = 4ax$ is given by

$$y = mx - 2am - am^3$$

Here $y^2 = 4x$ $\quad \therefore 4a = 4$

$\therefore a = 1$

and $2y\dfrac{dy}{dx} = 4 \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{y}$

$$\left(\frac{dy}{dy}\right)_{(4,4)} = m' = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ slope of normal m = – 2

$\therefore$ equation of normal is

$y = -2x - 2.1(-2) - (-2)^3$

$\Rightarrow y = -2x + 4 + 8, \Rightarrow y = -2x + 12$

$\Rightarrow 2x + y = 12$

Q.9 A circle cuts a parabola in four points. Prove that the common chords are in pairs equally inclined to the axis of the parabola.

Sol. Let the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(i)

and parabola $y^2 = 4ax$ ....(ii)

form (i) & (ii) we get

$$\frac{y^4}{16a^2} + y^2 + \frac{gy^2}{2a} + 2fy + c = 0$$

$\Rightarrow y^4 + (16a^2 + 8ag)y^2 + 32a^2fy + 16a^2c = 0$

Let four points are P, Q, R, S.

$\therefore y_1 + y_2 + y_3 + y_4 = 0$

$\therefore$ co-ordinate of $P \equiv \left(\dfrac{y_1^2}{4a}, y_1\right)$, $Q \equiv \left(\dfrac{y_2^2}{4a}, y_2\right)$

$R \equiv \left(\dfrac{y_3^2}{4a}, y_3\right)$ $S \equiv \left(\dfrac{y_4^2}{4a}, y_4\right)$

The line joining P & Q is

$y(y_1 + y_2) - 4ax - y_1y_2 = 0$

its slope $m_1 = \dfrac{4a}{y_1 + y_2}$

Similarly slope of joining R & S is

$$m_2 = -\frac{4a}{y_1 + y_2} = -m_1$$

Hence these lines are equally inclined to the axes.

Q.10 The normal at a point P to the parabola $y^2 = 4ax$ meets its axis at G. Q is another point on the parabola such that QG is perpendicular to the axis of the parabola. Prove that $QG^2 - PG^2$ = constant.

Sol. P is $(at^2, 2at)$ and G is $(2a + at^2, 0)$

$\therefore PG^2 = 4a^2 + 4a^2t^2$ ....(i)

Q is a point on the parabola such that QG is perpendicular to axis so that its ordinate is QG and abscissac is same as of G.

Hence the point Q is $(2a + at^2, QG)$

But Q lies on the parabola $y^2 = 4ax$

$\therefore QG^2 = 4a(2a + at^2)$

$= 8a^2 + 4a^2t^2$ ....(ii)

$\therefore QG^2 - PG^2 = 4a^2$ = constant

Hence $QG^2 - PG^2$ = constant

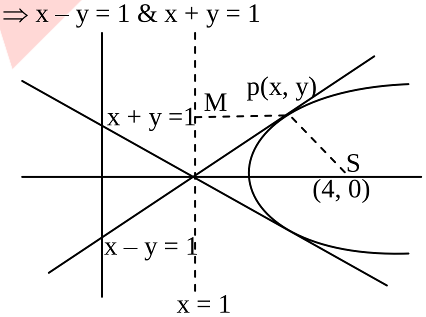
Q.11 Find the equation of the parabola whose axis is along x-axis and which touches the pair of lines $x^2 - y^2 - 2x + 1 = 0$, focus being at (4, 0).

Sol. $(x - 1)^2 = y^2$

$\Rightarrow y = \pm(x - 1)$

$\Rightarrow y = x - 1$ & $y = -(x - 1)$

$\Rightarrow x - y = 1$ & $x + y = 1$



SP = PM $\quad \Rightarrow SP^2 = PM^2$

$\Rightarrow (x - 4)^2 + y^2 = (x - 1)^2$

$\Rightarrow x^2 + y^2 - 8x + 16 = x^2 + 1 - 2x$

$\Rightarrow y^2 - 6x + 15 = 0 \Rightarrow y^2 = 6x - 15$

Q.12 If $a^2 > 8b^2$, prove that a point can be found such that the two tangents from it to the parabola $y^2 = 4ax$ are normals to the parabola $x^2 = 4by$.

Sol. Let $y = mx + \dfrac{a}{m}$ be a tangent to the parabola $y^2 = 4ax$, this can be written as

$$x = \frac{y}{m} - \frac{a}{m^2}$$

or $x = \left(\frac{1}{m}\right) y - \frac{a}{m^2}$

This will be a normal to the parabola $x^2 = 4by$

if $\frac{-a}{m^2} = \frac{-2b}{m} - \frac{b}{m^3}$

$\Rightarrow am = 2bm^2 + b$

$\Rightarrow 2bm^2 - am + b = 0$

Since m is real, therefore

$D \geq 0$

$a^2 - 8b^2 \geq 0 \Rightarrow a^2 \geq 8b^2$

Since it is given that $a^2 > 8b^2$ therefore such a point can be found.

Q.13 The tangents from the point T to the parabola $y^2 = 4ax$ touch at P and Q. If the chord of contact PQ is a normal to the parabola at P, prove that TP is bisected by the directrix of the parabola.

Sol. Let P be t_1 and Q be t_2 so that the point T is $[at_1 t_2, a(t_1 + t_2)]$ ....(i)

Again PQ is normal at P

$\therefore t_1 + t_2 = \frac{-2}{t_1}$

if (h, k) be the mid-point of TP, then

$2h = at_1^2 + at_1 t_2 = at_1(t_1 + t_2)$ ....(ii)

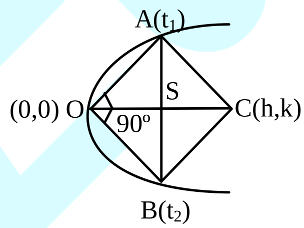
$= at_1\left(-\frac{2}{t_1}\right)$

$= -2a$ by (ii)

$\therefore$ h = –a. if clearly lies on the directrix x = – a whatever be the value of k.

Q.14 If from the vertex of the parabola $y^2 = 4ax$ a pair of chords be drawn at right angles to one another and with these chords as adjacent sides a rectangle be drawn. Prove that the locus of the vertex of the farther angle of the rectangle is the parabola $y^2 = 4a(x - 8a)$.

Sol.



$OA \perp OB \Rightarrow t_1 t_2 = -4$

If C is (h, k) then the diagonals of a rectangle bisect each other

$h + 0 = a(t_1^2 + t_2^2)$

$k + 0 = 2a(t_1 + t_2)$

$\therefore h = a[(t_1 + t_2)^2 - 2t_1t_2]$

$\Rightarrow h = a\left[\frac{k^2}{4a^2} + 8\right] \Rightarrow (h - 8a) = \frac{k^2}{4a}$

$\therefore$ locus is $y^2 = 4a(x - 8a)$

Part-B Passage based objective questions

Passage-I (Q. 15 to 17)

The coordinates of the vertex of the parabola $f(x) = 2x^2 + px + q$ are (– 3, 1)

Q.15 The value of p is

(A) 12 (B) – 12

(C) 19 (D) – 19

Sol. [A] $f(x) = 2x^2 + px + q$ (given parabola)

Its vertex = (–3, 1)

$2x^2 + px + q = f(x) = y$ (let)

$x^2 + \frac{p}{2}x + \frac{q}{2} - \frac{y}{2} = 0$

$\Rightarrow \left(x + \frac{p}{4}\right)^2 = \frac{y}{2} - \frac{q}{2} + \frac{p^2}{16}$

$\Rightarrow \left(x + \frac{p}{4}\right)^2 = \frac{1}{2}\left(y - q + \frac{p^2}{8}\right)$

Its vertex is given by

$x + \frac{p}{4} = 0$

$\Rightarrow x = -\frac{p}{4} = -3$ (given)

$\Rightarrow p = 12$

Q.16 The value of q is

(A) – 19 (B) 19

(C) – 12 (D) none of these

Sol. [B] In above question, we get

$y - q + \frac{p^2}{8} = 0$

$\Rightarrow y = q - \frac{p^2}{8} = 1$ (given)

$\Rightarrow q = 1 + \frac{p^2}{8} = 1 + \frac{144}{8} = 1 + 18 = 19$

$\therefore q = 19.$

Q.17 The parabola

(A) touches the x-axis

(B) intersecting the x-axis in two real and distinct points

(C) lies completely above the x-axis

(D) lies completely below the x-axis

Sol. [C] $\therefore$ $y = 2x^2 + px + q$

$y = 2x^2 + 12x + 19$

Since its vertex is (–3, 1)

Which show that the parabola lies completely above the x-axis.

Passage-II (Q. 18 to 20)

y = x is tangent to the parabola $y = ax^2 + c$

Q.18 If a = 2, then the value of c is

(A) $\frac{1}{8}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1

Sol. [A] y = x is tangent to $y = ax^2 + c$

$\Rightarrow x = ax^2 + c$

$\Rightarrow ax^2 - x + c = 0$

D = 0

$\Rightarrow 1 - 4\,ac = 0$

$\Rightarrow 4ac = 1$

It a = 2, then

$c = \frac{1}{8}$.

Q.19 If (1, 1) is point of contact then a is –

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Sol. [A] (1, 1) is point of contact, therefore

$1 = a + c$ ...(i)

and $4ac = 1 \Rightarrow c = \frac{1}{4a}$ ...(ii)

from (1) and (2), we get

$1 = a + \frac{1}{4a}$

$\Rightarrow 4a = 4a^2 + 1 \Rightarrow 4a^2 - 4a + 1 = 0$

$\Rightarrow (2a - 1)^2 = 0 \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$.

Q.20 If c = 2, then point of contact is

(A) (2, 2) (B) (4, 4)

(C) (6, 6) (D) (3, 3)

Sol. [B] If c = 2, then $y = ax^2 + 2$

$\Rightarrow ax^2 - x + 2 = 0$

$\Rightarrow 1 - 4a.2 = 0 \Rightarrow a = \frac{1}{8}$

$\therefore y = \frac{x^2}{8} + 2,$

$x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0 \Rightarrow x = 4$.

EXERCISE # 4

Old IIT-JEE questions

Q.1 Consider a circle with centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is **[IIT-95]**

(A) (p/2, p) (B) (p/2, –p)
(C) (–p/2, p) (D) (–p/2, –p)

Sol. **[A,B]**

$\therefore$ 4a = 2p

Θ a = p/2

centre (a, 0) = (p/2, 0)

equation of circle is $(x-p/2)^2 + y^2 = r^2$

it touches directrix x + a = 0 or x + p/2 = 0

$\therefore$ condition of tangency gives p = r

$$\therefore \left(x - \frac{p}{2}\right)^2 + y^2 = p^2$$

and $y^2 = 2px$

on solving above equations

we get the points as

(p/2, p) and (p/2, – p)

Q.2 If x + y = k is normal to $y^2 = 12x$, then k is- **[IIT-Screening -2000]**

(A) 3 (B) 9 (C) –9 (D) –3

Sol. **[B]**

y = mx + c is a normal to $y^2 = 4ax$ if

$c = -2am - am^2$

Here 4a = 12 $\therefore$ a = 3

y = – x + k $\therefore$ m = – 1, c = k

$\therefore$ k = – 6(–1) – 3 (–1)

$\therefore$ k = 6 + 3

= 9

Q.3 If the line x – 1 = 0 is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of k is- **[IIT-Screening -2000]**

(A) 1/8 (B) 8 (C) 4 (D) 1/4

Sol. **[C]**

Parabola is $y^2 - kx + 8 = 0$

$$y^2 = k\left(x - \frac{8}{k}\right)$$

$\Rightarrow y^2 = 4AX$

where 4A = k, Y = y, $X = x - \frac{8}{k}$

its directrix is X = – A

or $x - \frac{8}{k} = -\frac{k}{4}$

or $x = \frac{8}{k} - \frac{k}{4}$

comparing with x = 1, we get $1 = \frac{32 - k^2}{4k}$

or $k^2 + 4k - 32 = 0$

$\Rightarrow$ (k + 8) (k – 4) = 0

$\therefore$ k = 4

and k = – 8 is also true but not given in any of four choices.

Therefore k = 4

Q.4 Let C_1 and C_2 be, respectively, the parabolas $x^2 = y - 1$ and $y^2 = x - 1$. Let P be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1 be the reflections of P and Q, respectively, with respect to the line y = x. Prove that P_1 lies on C_2, Q_1 lies on C_1 and PQ > min {PP_1, QQ_1}. Hence or otherwise determine points P_0 and Q_0 on the parabola C_1 and C_2 respectively such that $P_0Q_0 \le$ PQ for all pairs (P, Q) with P on C_1 and Q on C_2. **[IIT - 2000]**

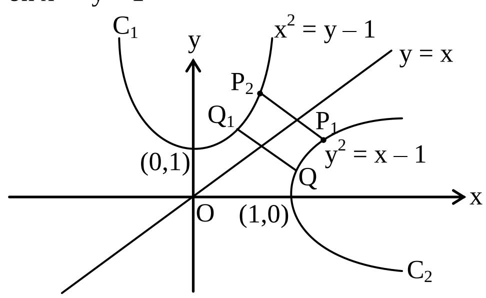
Sol. Let co-ordinates of P be $(t, t^2 + 1)$

Reflection of P in y = x is P_1 (t^2+1, t)

Which clearly lies on $y^2 = x - 1$

Similarly, let co-ordinates of Q b $(s^2 + 1, s)$

its reflection of y = x is Q_1 $(s, s^2 + 1)$ which lies on $x^2 = y - 1$



We have,

$$PQ_1^2 = (t - s)^2 + (t^2 - s^2)^2 = P_1Q^2$$

$\Rightarrow$ $PQ_1 = P_1Q$

Also $PP_1 \parallel QQ_1$ [Θboth $\perp$ to y = x]

Thus PP_1 QQ_1 is an Isosceles trapezium.

Also P lies on PQ_1 and Q lies on P_1Q we have PQ $\ge$ min of {PP_1, QQ_1}.

Let us take min {PP_1,QQ_1} = PP_1

$\therefore PQ^2 \ge PP_1^2 = (t^2 + 1 - t)^2 + (t^2 + 1 - t)^2$

$= 2(t^2 + 1 - t)^2 = f(t)$ (say)

we have, $f'(t) = 4(t^2 + 1 - t)(2t - 1)$

$$= 4\left[\left(t-\frac{1}{2}\right)^2 + \frac{3}{4}\right][2t - 1]$$

Now, $f'(t) = 0$

$\Rightarrow t = 1/2$

Also $f'(t) < 0$ for $t < 1/2$

and $f'(t) > 0$ for $t < \frac{1}{2}$

Thus f(t) is least when $t = \frac{1}{2}$

corresponding to $t = \frac{1}{2}$, point P_0 on C_1 is $\left(\frac{1}{2}, \frac{5}{4}\right)$

and P_1 (which we taken as Q_0) and C_2 are (5/4, 1/2)

Note that $P_0Q_0 \le PQ$ for all pairs of (P, Q) with P on C_1 and Q on C_2.

Q.5 Above x-axis, the equation of the common tangents to the circle $(x - 3)^2 + y^2 = 9$ and parabola $y^2 = 4x$ is- **[IIT-Screening -2001]**

(A) $\sqrt{3}\,y = 3x + 1$ (B) $\sqrt{3}\,y = -(x + 3)$

(C) $\sqrt{3}\,y = x + 3$ (D) $\sqrt{3}\,y = -(3x + 1)$

Sol. **[C]**

Any tangent to the parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ (a = 1)

or $m^2x - my + 1 = 0$ ....(i)

Apply p = r, the condition of tangency with given circle (3, 0), 3

$\Rightarrow \frac{3m^2+1}{\sqrt{m^4+m^2}} = 3$

$\Rightarrow (3m^2 + 1)^2 = 9(m^4 + m^2)$

or $3m^2 = 1$ $\therefore m = \pm\frac{1}{\sqrt{3}}$

Since the tangent touches the parabola above x-axis, it will make an acute angle with x-axis so that $\tan\theta = m = +$ ve

Hence we choose $m = \frac{1}{\sqrt{3}}$

from (i) put $m = \frac{1}{\sqrt{3}}$, we get

$x - \sqrt{3}y + 3 = 0$

$\Rightarrow \sqrt{3}y = x + 3$

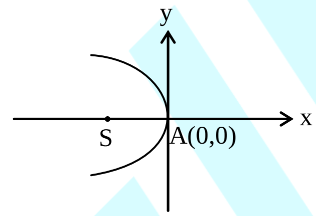
Q.6 The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is- **[IIT-Screening -2001]**

(A) $x = -1$ (B) $x = 1$ (C) $x = -\frac{3}{2}$ (D) $x = \frac{3}{2}$

Sol. **[D]**

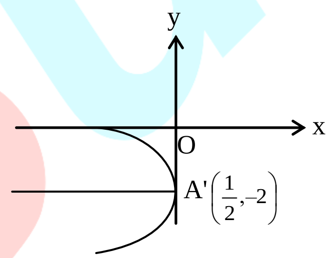
$y^2 + 4y + 4x + 2 = 0$

$\Rightarrow (y + 2)^2 + 4x - 2 = 0$



$\Rightarrow (y + 2)^2 = -4\left(x - \frac{1}{2}\right)$

replace $y + 2 = Y$ & $x - \frac{1}{2} = X$



We have $Y^2 = -4x$

This is a parabola with directrix at x = 1

$\Rightarrow x - \frac{1}{2} = 1 \Rightarrow x = 3/2$

Q.7 The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix- **[IIT-Screening -2002]**

(A) $x = -a$ (B) $x = -a/2$

(C) $x = 0$ (D) $x = a/2$

Sol. **[C]**

Let P(h, k) be the mid-point of the line segment joining the focus (a, 0) and a general point Q (x, y) on the parabola, then

$h = \frac{x+a}{2}$, $k = \frac{y}{2}$

$\Rightarrow x = 2h - a, y = 2k$

Put these value in $y^2 = 4ax$, we get

$4k^2 = 4a(2h - a)$

$\Rightarrow 4k^2 = 8ah - 4a^2$

$\Rightarrow k^2 = 2ah - a^2$

So locus of P(h, k) is

$y^2 = 2ax - a^2$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

its directrix is

$x - \frac{a}{2} + \frac{a}{2} = 0$

$\Rightarrow x = 0$

Q.8 If focal chord of $y^2 = 16x$ touches $(x-6)^2 + y^2 = 2$, then slope of such chord is **[IIT-2003]**

(A) (1, –1) (B) $\left(2, -\frac{1}{2}\right)$

(C) $\left(\frac{1}{2}, -2\right)$ (D) (2, –2)

Sol. **[A]**

Here the focal chord to $y^2 = 16x$ is tangent to the given circle $(x-6)^2 + y^2 = 2$

$\Rightarrow$ focus of parabola as (a, 0) i.e. (4, 0)

Now, tangents are drawn form (4, 0) to the circle $(x-6)^2 + y^2 = 2$

Since, DA is tangent to circle

= $\tan\theta$ = slope of tangent

$= \frac{AC}{AP} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

or $\frac{BC}{BP} = -1$

$\therefore$ slope of focal chord as tangent to circle = ±1

$\therefore$ (1, –1)

Q.9 Normals with slopes m_1, m_2 & m_3 are drawn from a point P, not on the axes, to the parabola $y^2 = 4x$. If the locus of P under the condition $m_1m_2 = \alpha$ is a part of the parabola, determine the value of α. **[IIT - 2003]**

Sol. We know equation of normal to $y^2 = 4ax$ is given by $y = mx - 2am - am^3$

Thus equation of normal to $y^2 = 4x$ is

$y = mx - 2m - m^3$ (Here a =1)

Let it pass through (h, k)

$\Rightarrow k = mh - 2m - m^3$

or $m^3 + m(2-h) + k = 0$ ....(i)

Here $m_1 + m_2 + m_3 = 0$

$m_1m_2 + m_2m_3 + m_3m_1 = 2 - h$

$m_1m_2\, m_3 = -k$ where $m_1m_2 = \alpha$(given)

$\Rightarrow m_3 = -\frac{k}{\alpha}$ It must satisfy (i)

$\Rightarrow -\frac{k^3}{\alpha^3} - \frac{k}{\alpha}(2-h) + k = 0$ ....(i)

Here $m_1 + m_2 + m_3 = 0$

$m_1m_2 + m_2m_3 + m_3m_1 = 2 - h$

$m_1m_2m_3 = -k$ where $m_1\, m_2 = \alpha$ (given)

$\Rightarrow m_3 = \frac{-k}{\alpha}$ it must satisfy (i)

$\Rightarrow -\frac{k^3}{\alpha^3} - \frac{k}{\alpha}(2-h) + k = 0 \Rightarrow k^2 = \alpha^2 h - 2\alpha^2 + \alpha^3$

comparing with $y^2 = 4x$

$\Rightarrow \alpha^2 = 4$ and $-2\alpha^2 + \alpha^3 = 0 \Rightarrow \alpha = 2$

Q.10 Angle between the tangents drawn from (1, 4) to the parabola $y^2 = 4x$ is– **[IIT-Screening -2004]**

(A) $\pi/2$ (B) $\pi/3$ (C) $\pi/6$ (D) $\pi/4$

Sol. **[B]**

Any tangent to parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ($\because$ a = 1)

it passes through (1, 4)

$\therefore m^2 - 4m + 1 = 0$

$\therefore \tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$

$= \frac{\sqrt{16-4}}{1+1} = \sqrt{3}$

$\therefore \theta = \frac{\pi}{3}$

Q.11 A tangent is drawn at any point P on the parabola $y^2 - 2y - 4x + 5 = 0$, which meets the directrix at Q. Find the locus of point R which divides QP externally in $\frac{1}{2}$: 1. **[IIT - 2004]**

Sol. Here, $y^2 - 2y = 4x - 5$

or $(y-1)^2 = 4(x-1)$

whose parametric co-ordinates are

$x - 1 = t^2$ and $y - 1 = 2t$

or $P(1 + t^2, 1 + 2t)$

$\therefore$ Equation of tangent at P is

$t(y-1) = x - 1 + t^2$, which meet the directrix x = 0 at Q

$\Rightarrow y = 1 + t - \frac{1}{t}$ or $Q = \left(0, 1 + t - \frac{1}{t}\right)$

Let R(h, k) which divides QP externally in the ratio $\frac{1}{2}$: 1 or Q is mid-point of RP.

$\Rightarrow 0 = \frac{h + t^2 + 1}{2}$ or $t^2 = -(h+1)$ ......(i)

and $1 + t - \frac{1}{t} = \frac{k + 2t + 1}{2}$ or $t = \frac{2}{1-k}$ ....(ii)

$\therefore$ From equation (i) & (ii)

$$\frac{4}{(1-k)^2} + (h+1) = 0$$

or $(k-1)^2 (h+1) + 4 = 0$

$\therefore$ locus is $(y-1)^2 (x+1) + 4 = 0$

$\Rightarrow 4 + (x+1)(1-y)^2 = 0$

Q.12 A tangent at any point P (1, 7) the parabola $y = x^2 + 6$, which is touching to the circle $x^2 + y^2 + 16x + 12y + c = 0$ at point Q, then Q is **[IIT-Screening -2005]**

(A) (–6, –7) (B) (–10, –15)
(C) (–9, –7) (D) (–6, –3)

Sol. **[A]**

$y = x^2 + 6$

$m = \frac{dy}{dx} = 2x$

$\therefore \left(\frac{dy}{dx}\right)_{P(1,7)} = 2$

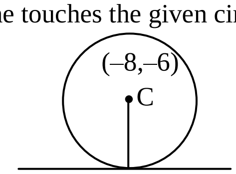
$\therefore m = 2$

equation of tanget $y - 7 = 2(x-1)$

$\Rightarrow y - 7 = 2x + 2$

$\Rightarrow 2x - y + 5 = 0$ ....(i)

This line touches the given circle



equation of CQ which is perpendicular to

$2x - y + 5 = 0$

$x + 2y + \lambda = 0$

This passes (–8, – 6)

$\therefore -8 - 12 + \lambda = 0 \Rightarrow \lambda = 20$

$\therefore x + 2y + 20 = 0$ ....(ii)

from (i) & (ii), we find point of intersection i.e. Q is

$$\begin{array}{r} 2x - y + 5 = 0 \\ 2x + 4y + 40 = 0 \\ - \quad - \quad - \\ \hline -5y = 35 \\ y = -7 \end{array}$$

$\therefore x = -\frac{12}{2} = -6$ $\therefore (-6, -7)$

Q.13 The axis of parabola is along the line y = x and the distance of vertex from origin is $\sqrt{2}$ and that from its focus is $2\sqrt{2}$. If vertex and focus both lie in the first quadrant, so the equation of parabola is **[IIT- 2006]**

(A) $(x-y)^2 = 16(x+y-2)$

(B) $(x-y)^2 = 4(x+y-2)$

(C) $(x-y)^2 = (x+y-2)$

(D) $(x-y)^2 = (x-y-2)$

Sol. **[A]**

The directrix must be x + y = 0

By definition equation of parabola

$$\frac{x+y}{\sqrt{2}} = \sqrt{(x-2)^2 + (y-2)^2}$$

$\Rightarrow x + y = \sqrt{2} \cdot \sqrt{(x-2)^2 + (y-2)^2}$

$\Rightarrow (x+y)^2 = 2\{x^2 + y^2 - 4x - 4y + 8\}$

$\Rightarrow x^2 + y^2 + 2xy = 2x^2 + 2y^2 - 8x - 8y + 16$

$\Rightarrow x^2 + y^2 - 2xy - 8x - 8y + 16 = 0$

$\Rightarrow x^2 + y^2 - 2xy = 8x + 8y - 16$

$\Rightarrow (x-y)^2 = 8(x+y-2)$

Q.14 The equation(s) of common tangent(s) to the parabola $y = x^2$ and $y = -(x-2)^2$ **[IIT-2006]**

(A) $y = -4(x-1)$ (B) $y = 0$
(C) $y = 4(x-1)$ (D) $y = -30x - 50$

Sol. **[C]**

Let the common tangent by y = mx + c

then both the quadratic equations

$mx + c = x^2$ and $mx + c = c - (x-2)^2$ must have equal roots giving easily

$m^2 = -4c$, $(m-4)^2 = c + 4$

on eliminating c, we get

$2m^2 - 8m = 0$

$\Rightarrow 2m(m-4) = 0$

$\Rightarrow m = 0, 4$

$\therefore c = 0$, when $m = 0$

and $C = -4$ when $m = 4$

$\therefore y = 0$ and $y = 4(x-1)$

Q.15 Three normals drawn at P,Q and R on the parabola $y^2 = 4x$ intersect at (3, 0). Then **[IIT-2006]**

Column 1	**Column 2**
(A) Radius of circumcircle of Δ PQR	(P) 5/2
(B) Area of Δ PQR	(Q) (5/2,0)
(C) Centroid of Δ PQR	(R) (2/3, 0)
(D) Circumcentre of Δ PQR	(S) 2

Sol. **A → P, B → S, C → R, D → Q**

Any normal is $y + tx = 2t + t^3$

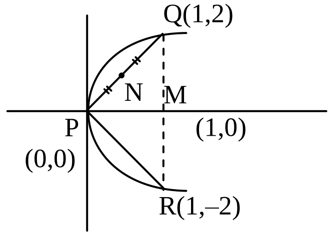
If passes through [3, 0] then $t^3 - t = 0$

$\Rightarrow t = 0, 1, -1$

$\Rightarrow$ P, Q , R are (0, 0),(1, 2) and (1, –2)

centroid is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{0+1+1}{3}, \frac{0+2-2}{3}\right) = \left(\frac{2}{3}, 0\right)$



Now area of ΔPQR

$= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times QR \times MP$

$= \frac{1}{2} \times 4 \times 1$

= 2 sq. units

Now circum centre is point intersection of right bisectors PM is one of them (y = 0)

Now slope PQ = 2 and N is $\left(\frac{1}{2}, 1\right)$

$\Rightarrow$ Equation of another right bisector is –

$y - 1 = -\frac{1}{2}\left(x - \frac{1}{2}\right)$

$\Rightarrow 2x + 4y = 5$

$\Rightarrow$ Point of intersection of two bisectors is (5/2,1)

$\Rightarrow$ circumcentre is $\left(\frac{5}{2}, 1\right)$ and its distance from vertex P is circum radius which is 5/2.

Q.16 Consider the two curves **[IIT-2008]**

$C_1 : y^2 = 4x$

$C_2 : x^2 + y^2 - 6x + 1 = 0$

Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

Sol. **[B]**

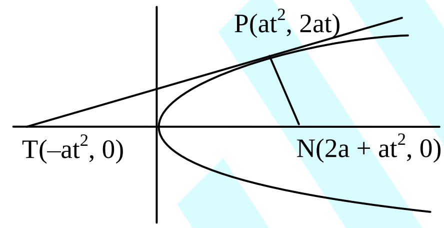
On solving the given two curves C_1 and C_2 we get the points of tangency (1, ± 2).

Q.17 The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose : **[IIT-2009]**

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is x = 0

(C) latus rectum is $\frac{2a}{3}$ (D) focus is (a, 0)

Sol. **[A, D]**



$ty = x + at^2$

$y = -tx + 2at + at^3$

$h = \frac{at^2 - at^2 + 2a + at^2}{3}$, $k = \frac{2at + 0 + 0}{3}$

$3h = a(2 + t^2)$, $t = \frac{3k}{2a}$

$\Rightarrow 3h = a\left(2 + \frac{9k^2}{4a^2}\right)$

$\Rightarrow 12\,ah = 8a^2 + 9k^2$

$\Rightarrow 9y^2 = 12ax - 8a^2$

$\Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2}{3}a\right)$

vertex $\left(\frac{2a}{3}, 0\right)$ directrix $x = \frac{a}{3}$

Latus rectum $\frac{4a}{3}$, Focus (a, 0)

Q.18 Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be - **[IIT-2010]**

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Sol. **[C, D]**

Let $A\left(t_1^2, 2t_1\right)$ $B\left(t_2^2, 2t_2\right)$

Slope $= \frac{2(t_2 - t_1)}{t_2^2 - t_1^2} = \frac{2}{t_1 + t_2}$

Equation of circle will be

$(x - t_1^2)(x - t_2^2) + (y - 2t_1)(y - 2t_2) = 0$

$x^2 + y^2 - x(t_1^2 + t_2^2) - 2y(t_1 + t_2) + t_1^2 t_2^2 + 4t_1t_2 = 0$

As it touches x axis so

$t_1^2 t_2^2 + 4t_1t_2 = \dfrac{(t_1^2 + t_2^2)^2}{4}$

$4t_1^2 t_2^2 + 16\, t_1t_2 = t_1^4 + t_2^4 + 2t_1^2 t_2^2$

$\left(t_1^2 - t_2^2\right)^2 = 16\, t_1t_2$ ...(1)

AB is diameter so

$(t_1^2 - t_2^2)^2 + 4\,(t_1 - t_2)^2 = 4r^2$ ...(2)

From (1) and (2)

$4\, t_1\, t_2 + (t_1 - t_2)^2 = r^2$

$(t_1 + t_2)^2 = r^2$

$t_1 + t_2 = \pm\, r$

$\therefore$ Slope $= \pm\, \dfrac{2}{r}$

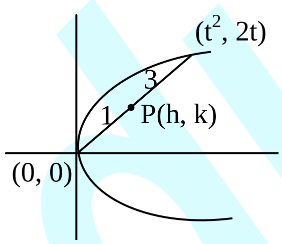
Q. 19 Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in the ratio $1 : 3$. Then the locus of P is **[IIT-2011]**

(A) $x^2 = y$ (B) $y^2 = 2x$

(C) $y^2 = x$ (D) $x^2 = 2y$

Sol. **[C]**

$h = \dfrac{t^2}{4}$, $k = \dfrac{2t}{4}$



$t^2 = 4h$, $t = 2k$

so $4k^2 = 4h$

$\therefore k^2 = h$

hence required locus is $y^2 = x$

Q.20 Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then L is given by **[IIT-2011]**

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$

(C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

Sol. **[A. B, D]**

$y = mx - 2m - m^3$

It passes through (9, 6)

$6 = 9m - 2m - m^3$

$m^3 - 7m + 6 = 0$

$(m - 1)(m - 2)(m + 3) = 0$

$\therefore m = -3, 1, 2$

Hence equations will be

$y = x - 3$, $y = 2x - 12$ and $y = -3x + 33$

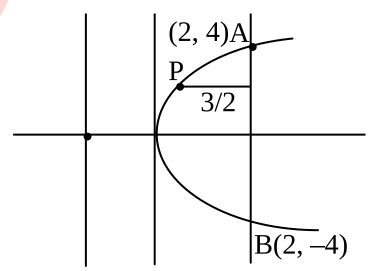
Q.21 Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\dfrac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\dfrac{\Delta_1}{\Delta_2}$ is **[IIT-2011]**

Sol. **[2]**

It is a property that area of triangle formed by joining three points lying on parabola is twice the area of triangle formed by tangents at these points

Alternate : $y^2 = 8x$

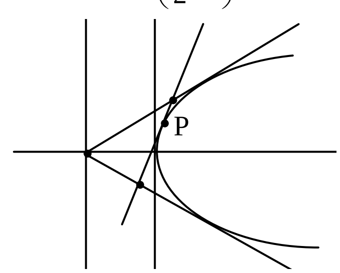
$P\left(\dfrac{1}{2}, 2\right)$



$\Delta_1 = \dfrac{1}{2}\,|\text{Base} \times \text{Height}| = \dfrac{1}{2} \times \dfrac{3}{2} \times 8 = 6$

Also

Equation of tangent at $P\left(\dfrac{1}{2}, 2\right)$



$y\,(2) = 4.\left(x + \dfrac{1}{2}\right)$

$y = 2x + 1$ ....(1)

Tangent at A : $y = x + 2$

Tangent at B : $-y = +x + 2 \Rightarrow y = -x - 2$
Point of intersection
L(–2, 0), M (1, 3), N (–1, –1)

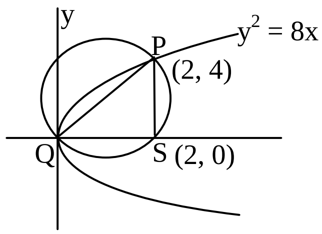
$$\Delta_2 = \left|\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & 1 \end{vmatrix}\right| = \left|\frac{1}{2}[-2(4) + (-1 + 3)]\right|$$

$$= \left|\frac{1}{2}[-8+3-1]\right| = 3$$

So, $\frac{\Delta_1}{\Delta_2} = \frac{6}{3} = 2$

Q.22 Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is **[IIT-2012]**

Sol. [4] $(x-1)^2 + (y-2)^2 = (\sqrt{5})^2$



$x^2 + 8x - 2x - 4.2\sqrt{2x} = 0$

$x^2 + 6x - 8\sqrt{2x} = 0$

$x^{3/2} + 6x^{1/2} - 8\sqrt{2} = 0$

$x^{1/2} = t$

$t^3 + 6t - 8\sqrt{2} = 0$

$(t - \sqrt{2})(t^2 - \sqrt{2}\,t + 4) = 0$

$t = \sqrt{2}$ $\quad x = 2$

$y = 4$

P(2, 4) Q(0, 0) S(2, 0)

Area $\Delta PQS = \frac{1}{2} \times 2 \times 4 = 4$

EXERCISE # 5

Q.1 If $(a^2, a-2)$ be a point interior to the region of the parabola $y^2 = 2x$ bounded by the chord joining the points (2, 2) and (8, –4) then find the set of all possible real values of a.

Sol. If $P(a^2, a-2)$ lies inside the parabola, then

$(a-2)^2 - 2a^2 < 0$

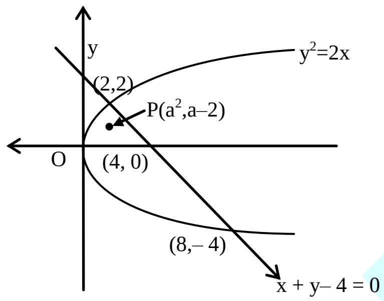
$\Rightarrow a^2 - 4a + 4 - 2a^2 < 0$

$\Rightarrow -a^2 - 4a + 4 < 0$

$\Rightarrow a^2 + 4a - 4 > 0$

$\Rightarrow (a+2)^2 - \left(2\sqrt{2}\right)^2 > 0$

$\Rightarrow a + 2 < -2\sqrt{2}$ or $a + 2 > 2\sqrt{2}$ ....(i)



$\Rightarrow a < -2 - 2\sqrt{2}$ or $a > 2\sqrt{2} - 2$

Since point $P(a^2, a-2)$ and the originO (0, 0) are on the same side of the chord joining (2, 2) and (8, – 4), therefore $(0 + 0 - 4)(a^2 + a - 2 - 4) > 0$

$\Rightarrow a^2 + a - 6 < 0$

$\Rightarrow (a+3)(a-2) < 0$

$\Rightarrow -3 < a < 2$ ....(ii)

Also, $-4 < a - 2 < 2$ and $0 < a^2 < 8$

$\Rightarrow -2 < a < 4$ and $-2\sqrt{2} < a < 2\sqrt{2}$

$\Rightarrow -2 < a < 2\sqrt{2}$ ....(iii)

From (i), (ii) & (iii) we get

$-2 < a < -2 + \sqrt{2}$

Hence $a \in (-2, -2 + \sqrt{2})$

Q.2 Three normals are drawn from the point (c, 0) to the curve $y^2 = x$. Show that c must be greater than 1/2. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other. **[IIT 1991]**

Sol. We know that normal for $y^2 = 4ax$ is given by

$y = mx - 2am - am^3$, here a = 1/4

$\therefore$ normal for $y^2 = x$

$\Rightarrow y = mx - \frac{m}{2} - \frac{m^3}{4}$

Since normal passes through (c, 0)

$\therefore mc - \frac{m}{2} - \frac{m^3}{4} = 0$

$\Rightarrow m\left(c - \frac{1}{2} - \frac{m^2}{4}\right) = 0$

$\Rightarrow m = 0$ or $m^2 = 4\left(c - \frac{1}{2}\right)$

m = 0 shows normal is y = 0

Also, $m^2 \geq 0$

$\Rightarrow c - \frac{1}{2} \geq 0$

$\Rightarrow c \geq \frac{1}{2}$

at $c = \frac{1}{2} \Rightarrow m = 0$

$\therefore$ for normals other than x-axis

$\Rightarrow c > \frac{1}{2}$

Now for other normals to be perpendicularer to each other, we must have $m_1m_2 = -1$

or $\frac{m^2}{4} + \left(\frac{1}{2} - c\right) = -1$ has $m_1m_2 = -1$

$\Rightarrow \frac{\left(\frac{1}{2} - c\right)}{1/4} = -1$

$\Rightarrow \frac{1}{2} - c = -\frac{1}{4} \Rightarrow -c = -\frac{1}{4} - \frac{1}{2}$

$\Rightarrow c = \frac{3}{4}$

Q.3 Through the vertex O of parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ. **[IIT 1994]**

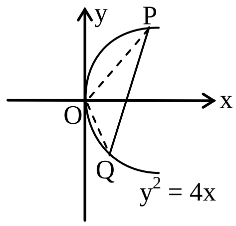
Sol. Let the equation of chord OP be y = mx

Then equation of chord will be $y = -\frac{1}{m}x$

P is point of intersection of y = mx and $y^2 = 4x$

solving the two, we get $P\left(\frac{y}{m^2}, \frac{y}{m}\right)$

Q is point of intersection of $y = -\frac{1}{mn}$ and $y^2 = 4x$



Solving the two, we get

Q $(4m^2, -4m)$

Now, equation of PQ is

$$y + 4m = \frac{\frac{4}{m} + 4m}{\frac{4}{m^2} - 4m^2}(x - 4m^2)$$

$$\Rightarrow y + 4m = \frac{m}{1 - m^2}(x - 4m^2)$$

$\Rightarrow (1 - m^2)y + 4m - 4m^3 = mx - 4m^3$

$\Rightarrow mx - (1 - m^2)y - 4m = 0$

This line meets x-axis, where y = 0

i.e x = 4 ⇒ OL = 4 which is constant as independent of m.

Again let (h, k) be the mid-point of PQ, then

$$h = \frac{4m^2 + \frac{4}{m^2}}{2} \text{ and } k = \frac{\frac{4}{m} - 4m}{2}$$

$$\Rightarrow h = 2\left(m^2 + \frac{1}{m^2}\right) \text{ and } k = 2\left(\frac{1}{m} - m\right)$$

$$\Rightarrow h = 2\left\{\left(m - \frac{1}{m}\right)^2 + 2\right\} \text{ and } k = 2\left(\frac{1}{m} - m\right)$$

Eliminating m, we get

$2h = k^2 + 8$ or $k^2 = 2h - 8$ or $k^2 = 2(h - 4)$

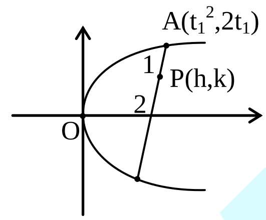
∴ locus is

$y^2 = 2(x - 4)$

Q.4 Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is a parabola. Find the vertex of this parabola. **[IIT 1995]**

Sol. Let $A(t_1^2, 2t_1)$ and $B(t_2^2, 2t_2)$ be co-ordinates of the end point of a chord of the parabola $y^2 = 4x$ having slope 2.

Now, slope of AB is $m = \frac{2t_2 - 2t_1}{t_2^2 - t_1^2}$



$$\therefore m = \frac{2(t_2 - t_1)}{(t_2 - t_1)(t_2 + t_1)} = \frac{2}{t_2 + t_1}$$

But m = 2, $\Rightarrow 2 = \frac{2}{t_2 + t_1}$

$\Rightarrow t_1 + t_2 = 1$ ....(i)

Let P(h, k) be a point on AB such that it divides AB inernally in the ratio 1 : 2,

Then $h = \frac{2t_1^2 + t_2^2}{2+1}$ and $k = \frac{2(2t_1) + 2t_2}{2+1}$

$\Rightarrow 3h = 2t_1^2 + t_2^2$ ....(ii)

$\Rightarrow 3k = 4t_1 + 2t_2$ ....(iii)

substituting value of t_1 from (i) in (iii)

$3k = 4(1 - t^2) + 2t_2 \Rightarrow 3k = 4 - 2t_2$

$\Rightarrow 2t_2 = 4 - 3k \Rightarrow t_2 = 2 - \frac{3k}{2}$ ....(iv)

Substituting $t_1 = 1 - t_2$ in (ii),we get

$3h = 2(1 - t_2)^2 + t_2^2 = 2(1 - 2t_2 + t_2^2) + t_2^2$

$= 2 - 4t_2 + 2t_2^2 + t_2^2 = 3t_2^2 - 4t_2 + 2$

$$= 3\left(t_2^2 - \frac{4}{3}t^2 + \frac{2}{3}\right) = 3\left(t_2 - \frac{2}{3}\right)^2 + \frac{2}{3}$$

$$\Rightarrow 3h - \frac{2}{3} = 3\left(t_2 - \frac{2}{3}\right)^2$$

$$\Rightarrow 3\left(h - \frac{2}{9}\right) = 3\left[2 - \frac{3k}{2} - \frac{2}{3}\right]^2 \quad \text{.....form (iv)}$$

$$\Rightarrow 3\left(h - \frac{2}{9}\right) = 3\left[\frac{4}{3} - \frac{3k}{2}\right]^2$$

$$\Rightarrow \left(h - \frac{2}{9}\right) = \frac{9}{4}\left[k - \frac{8}{9}\right]^2$$

$$\Rightarrow \left(k - \frac{8}{9}\right)^2 = \frac{4}{9}\left(h - \frac{2}{9}\right)$$

∴ locus is

$$\left(y - \frac{8}{9}\right)^2 = \frac{4}{9}\left(x - \frac{2}{9}\right)$$

This repersents a parabola whose vertex is $x - \frac{2}{9} = 0$ & $y - \frac{8}{9} = 0$

$$\Rightarrow x = \frac{2}{9} \ \& \ y = \frac{8}{9}$$

vertex is $\left(\frac{2}{9}, \frac{8}{9}\right)$

Q.5 A ray of light is coming along the line y = b from the positive direction of x-axis & strikes a concave mirror whose intersection with the xy plane is a parabola $y^2 = 4ax$. Find the equation of the reflected ray & show that it passes through the focus of the parabola. Both a & b are positive. **[IIT 1995]**

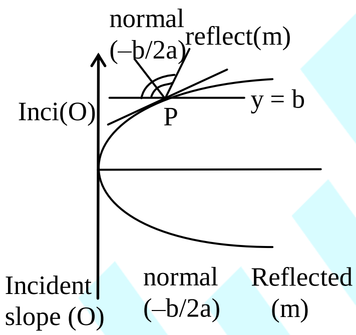
Sol. y = b meets the parabola $y^2 = 4ax$ at $P\left(\frac{b^2}{4a}, b\right)$

diffrentiate $y^2 = 4ax$ w.r.t x

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{4a}{2y}$$

$$\therefore \text{ slope of normal } = \frac{-2y}{4a} = \frac{-y}{2a} = \frac{-b}{2a}$$



Since the lines are equally inclined to normal
Therefore

$$\frac{0-(-b/2a)}{1+0(-b/2a)} = \frac{-m-(-b/2a)}{1+m(-b/2a)}$$

or $\frac{b}{2a} = -\frac{2am+b}{2a-mb}$

or $2ab - mb^2 = -4a^2m - 2ab$

or $4ab = (b^2-4a^2)m$

or $m = \frac{4ab}{b^2-4a^2}$

Hence equation of reflected ray through P is

$$y - b = \frac{4ab}{b^2-4a^2}\left(x - \frac{b^2}{4a}\right)$$

It is satisfied by the point (a, 0) i.e. focus of the parabola.

Q.6 (a) Points A, B and C lie on the parabola $y^2 = 4ax$. The tangents to the parabola at A, B & C taken in pairs, intersect at points P, Q & R. Determine the ratio of the areas of the triangles ABC & PQR.

(b) From a point A common tangents drawn to the circle $x^2 + y^2 = (a^2/2)$ and parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle & the chord of contact of the parabola. **[IIT 1996]**

Sol.

(a) Let the three point on the parabola be $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$ and $C(at_3^2, 2at_3)$

Equation of the tangent to the parabola at $(at^2, 2at)$ is

$$ty = x + at^2$$

Therefore equations of tangent at A and B are

$$t_1y = x + at_1^2 \quad \text{.............(i)}$$

$$t_2y = x + at_2^2 \quad \text{.............(ii)}$$

solving (i) & (ii) simultfaneously, we get

$$t_1y = t_2y - at_2^2 + at_1^2$$

$$t_1y - t_2y = at_1^2 - at_2^2$$

$$y(t_1 - t_2) = a(t_1 - t_2)(t_1 + t_2)$$

$\Rightarrow y = a(t_1+t_2)$ $(\Theta\ t_1 \neq t_2)$

and $t_1a(t_1+t_2) = x + at_1^2$

$\Rightarrow x = at_1^2 + at_1t_2 - at_1^2$

$\Rightarrow x = at_1t_2$

Therefore, co-ordinate of P are $(at_1t_2, a(t_1 + t_2))$ similarly, the co-ordinates of Q and R are respetively $[at_2t_3, a(t_2+t_3)]$ and $[at_1t_2, a(t_1+t_3)]$

Let Δ_1 = Area of the ΔABC

$$= \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}$$

applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$, weget

$$\Delta_1 = \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ a(t_2^2 - t_1^2) & 2a(t_2 - t_1) & 0 \\ a(t_3^2 - t_2^2) & 2a(t_3 - t_2) & 0 \end{vmatrix}$$

$\Delta_1 = a^2|(t_2 - t_1)(t_3 - t_2)(t_1 - t_3)|$

Next let Δ_2 = area of the ΔPQR

$$= \frac{1}{2}\begin{vmatrix} at_1t_2 & a(t_1+t_2) & 1 \\ at_2t_3 & a(t_2+t_3) & 1 \\ at_3t_1 & a(t_3+t_1) & 1 \end{vmatrix}$$

$$= \frac{1}{2} a.a. \begin{vmatrix} t_1t_2 & t_1+t_2 & 1 \\ t_2t_3 & t_2+t_3 & 1 \\ t_3t_1 & t_3+t_1 & 1 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$, weget

$$\frac{a^2}{2} \begin{vmatrix} t_1t_2 & t_1+t_2 & 1 \\ t_2(t_3-t_1) & t_3-t_2 & 0 \\ t_3(t_1-t_2) & t_1-t_2 & 0 \end{vmatrix}$$

$$= \frac{a^2}{2} \quad |(t_3-t_1)\,(t_1-t_2)(t_2-t_3)|$$

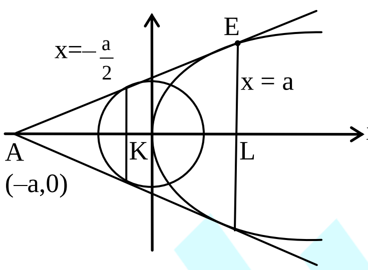
$$\therefore \frac{\Delta_1}{\Delta_2} = \frac{a^2\,|(t_2-t_1)(t_3-t_2)(t_1-t_3)|}{\frac{a^2}{2}|(t_3-t_1)(t_1-t_2)(t_2-t_3)|} = 2$$

$$\therefore \frac{\Delta_1}{\Delta_2} = 2 : 1$$

(b) Equation of any tangent to the parabola

$y^2 = 4ax$ is $y = mx + \frac{a}{m}$

This line will touch the circle $x^2 + y^2 = \frac{a^2}{2}$



If $\left(\frac{a}{m}\right)^2 = \frac{a^2}{2}(m^2 + 1)$

$\Rightarrow \frac{1}{m^2} = \frac{1}{2}(m^2 + 1) \Rightarrow 2 = m^4 + m^2$

$\Rightarrow (m^2 - 1)\,(m^2 + 2) = 0$

$\Rightarrow m^2 - 1 = 0$, $m^2 = -2$ (Not possible)

$\therefore m = \pm 1$

Therefore, two common tangents are

$y = x + a$ and $y = -x - a$

There two intersect at $A(-a,0)$

The chord of contact of $A(-a, 0)$ for the circle $x^2 + y^2 = \frac{a^2}{2}$ is $(-a)x + 0.y = a^2/2$ or $x = -a/2$

and chord of contact of $A(-a, 0)$ for the parabola $y^2 = 4ax$ is $0.y = 2a(x - a)$ or $x = a$

Again lenght of BC = 2Bk $= 2\sqrt{OB^2 - OK^2}$

$$= 2\sqrt{\frac{a^2}{2} - \frac{a^2}{4}} = 2\sqrt{\frac{a^2}{4}} = 2.\ \frac{a}{2} = a$$

and we know that DE is the latus rectum of parabola so its lenght is 4a.

Thus area of the trapezium BCDE

$$= \frac{1}{2}(BC + DE)\,(KL)$$

$$= \frac{1}{2}(a + 4a)\left(\frac{3a}{2}\right) = \frac{15a^2}{4}$$

Q.7 Find the locus of the point of intersection of those normals to the parabola $x^2 = 8y$ which are at right angles to each other. **[IIT 1997]**

Sol. Any normal to $y^2 = 4ax$ is

$$y = mx - 2am - am^3$$

$\therefore$ Any normal to $x^2 = 4ay$ is obtained from above by interchanging x and y.

$\therefore$ Normal to $x^2 = 8y$ (Here a = 2) is

$$x = my - 4m - 2m^3$$

If it passes through (h, k) then

$$h = mk - 4m - 2m^3$$

or $2m^3 + 0.m^2 + m(4 - k) + h = 0$ ....(i)

Above shows that three normal will pass through (h, k). If two of them are perpendicular then $m_2m_3 = -1$. But $m_1m_2m_3 = -\frac{h}{2}$

$\therefore -m_1 = -\frac{h}{2}$ or $m_1 = \frac{h}{2}$

since m_1 is root of (i), It will satisfy it.

$$\therefore 2\left(\frac{h^3}{8}\right) + \frac{h}{2}(4 - k) + h = 0$$

or $h^2 + 2(4 - k) + 4 = 0$

or $h^2 - 2k + 12 = 0$

$\therefore$ locus is given by

$x^2 - 2y + 12 = 0$

ANSWERKEY

EXERCISE # 1

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	A	C	D	C	A	A	B	C	D	B	D	A	B	C	A	A
Ques.	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Ans.	B	D	A	C	C	B	B	D	C	A	B	A	D	D	B	A

EXERCISE # 2

(PART-A)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	C	D	C	B	B	C	B	D	C	A	D	C	D	B	B	B	C	B

(PART-B)

Ques.	19	20	21	22	23	24
Ans.	A,B,C	A,C	B,C	A,B	A,D	B,C

(PART-C)

Ques.	25	26	27
Ans.	A	C	B

(PART-D)

28. A $\rightarrow$ R, B $\rightarrow$ Q, C $\rightarrow$ P, D $\rightarrow$ S

29. A $\rightarrow$ P, B $\rightarrow$ P,Q, C $\rightarrow$ P, D $\rightarrow$ R, S

EXERCISE # 3

1. 2, (–7/2, 4), (–3, 4), y = 4, x = –4 and x = –7/2

2. 3

3. Two ; (2, 0); (4, 1)

5. $\left(x - \frac{p}{2}\right)^2 + y^2 = \frac{9p^2}{16}$

6. $x \pm \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ y + \frac{1}{2}\ (1 + \sqrt{5}) = 0$

8. $y + 2x = 12$; $(7, -2)$

11. $y^2 = 6x - 15$

15. (A) **16.** (B) **17.** (C)

18. (A) **19.** (A) **20.** (B)

EXERCISE # 4

1. (A,B) **2.** (B) **3.** (C) **4.** $P_0 = (1/2, 5/4)$, $Q_0 = (5/4, 1/2)$ **5.** (C) **6.** (D)

7. (C) **8.** (A) **9.** $\alpha = 2$ **10.** (B) **11.** $4 + (x + 1)(1 - y)^2 = 0$ **12.** (A)

13. (A) **14.** (B,C) **15.** $A \to P$, $B \to S$, $C \to R$, $D \to Q$ **16.** (B) **17.** (A, D)

18. (C, D) **19.** (C) **20.** (A,B,D) **21.** 2 **22.** 4

EXERCISE # 5

1. $(-2 + 2\sqrt{2}, 2)$

2. $c = \frac{3}{4}$

3. $y^2 = 2(x - 4)$

4. $\frac{4}{9}\left(x - \frac{2}{9}\right) = \left(y - \frac{8}{9}\right)^2$, vertex $\left(\frac{2}{9}, \frac{8}{9}\right)$

5. $4abx + (4a^2 - b^2)\,y - 4a^2 b = 0$

6. (a) 2 : 1 (b) $\frac{15a^2}{4}$

7. $x^2 + 2(4 - y) + 4 = 0$