EXERCISE-I

Derivative at a point, Standard differentiation

1. $\frac{d}{dx}\left(\tan^{-1}\frac{\cos x}{1+\sin x}\right)=$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) -1 (D) 1

2. $\frac{d}{dx}[\cos(1-x^2)^2]=$

(A) $-2x(1-x^2)\sin(1-x^2)^2$

(B) $-4x(1-x^2)\sin(1-x^2)^2$

(C) $4x(1-x^2)\sin(1-x^2)^2$

(D) $-2(1-x^2)\sin(1-x^2)^2$

3. $\frac{d}{dx}\left(x^2\sin\frac{1}{x}\right)=$

(A) $\cos\left(\frac{1}{x}\right)+2x\sin\left(\frac{1}{x}\right)$

(B) $2x\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)$

(C) $\cos\left(\frac{1}{x}\right)-2x\sin\left(\frac{1}{x}\right)$

(D) None of these

4. If $y=\cos(\sin x^2)$, then at $x=\sqrt{\frac{\pi}{2}}, \frac{dy}{dx}=$

(A) -2 (B) 2

(C) $-2\sqrt{\frac{\pi}{2}}$ (D) 0

5. If $y=\sin^{-1}(x\sqrt{1-x}+\sqrt{x}\sqrt{1-x^2})$, then $\frac{dy}{dx}=$

(A) $\frac{-2x}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$

(B) $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

(C) $\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$

(D) None of these

6. If $y=\sqrt{(1-x)(1+x)}$, then

(A) $(1-x^2)\frac{dy}{dx}-xy=0$

(B) $(1-x^2)\frac{dy}{dx}+xy=0$

(C) $(1-x^2)\frac{dy}{dx}-2xy=0$

(D) $(1-x^2)\frac{dy}{dx}+2xy=0$

7. $\frac{d}{dx}\left(\frac{\cot^2 x-1}{\cot^2 x+1}\right)=$

(A) $-\sin 2x$ (B) $2\sin 2x$

(C) $2\cos 2x$ (D) $-2\sin 2x$

8. If $f(x)=x\tan^{-1}x$, then $f'(1)=$

(A) $1+\frac{\pi}{4}$ (B) $\frac{1}{2}+\frac{\pi}{4}$

(C) $\frac{1}{2}-\frac{\pi}{4}$ (D) 2

9. If $y=\log_{10}x+\log_x 10+\log_x x+\log_{10}10$, then $\frac{dy}{dx}=$

(A) $\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)^2}$

(B) $\frac{1}{x\log_e 10}-\frac{1}{x\log_{10} e}$

(C) $\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)^2}$

(D) None of these

10. If $y=b\cos\log\left(\frac{x}{n}\right)^n$, then $\frac{dy}{dx}=$

(A) $-n\ b\sin\log\left(\frac{x}{n}\right)^n$

(B) $n\ b\sin\log\left(\frac{x}{n}\right)^n$

(C) $-n\,b\sin\log\left(\frac{x}{n}\right)^n$

(D) None of these

11. $\frac{d}{dx}\left(\frac{\log x}{\sin x}\right) =$

(A) $\frac{\frac{\sin x}{x} - \log x.\cos x}{\sin x}$

(B) $\frac{\frac{\sin x}{x} - \log x.\cos x}{\sin^2 x}$

(C) $\frac{\sin x - \log x.\cos x}{\sin^2 x}$

(D) $\frac{\frac{\sin x}{x} - \log x}{\sin^2 x}$

12. If $y = \frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$, then $\frac{dy}{dx} =$

(A) $2x + \frac{2x^3}{\sqrt{x^4-1}}$ (B) $2x + \frac{x^3}{\sqrt{x^4-1}}$

(C) $x + \frac{2x^3}{\sqrt{x^4-1}}$ (D) None of these

13. If $y = \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$, then $\frac{dy}{dx} =$

(A) $\frac{ay}{x\sqrt{a^2-x^2}}$ (B) $\frac{ay}{\sqrt{a^2-x^2}}$

(C) $\frac{ay}{x\sqrt{x^2-a^2}}$ (D) None of these

14. If $y = (x\cot^3 x)^{3/2}$, then $dy/dx =$

(A) $\frac{3}{2}(x\cot^3 x)^{1/2}[\cot^3 x - 3x\cot^2 x\,\text{cosec}^2 x]$

(B) $\frac{3}{2}(x\cot^3 x)^{1/2}[\cot^2 x - 3x\cot^2 x\,\text{cosec}^2 x]$

(C) $\frac{3}{2}(x\cot^3 x)^{1/3}[\cot^3 x - 3x\,\text{cosec}^2 x]$

(D) $\frac{3}{2}(x\cot^3 x)^{3/2}[\cot^3 x - 3x\,\text{cosec}^2 x]$

15. $\frac{d}{dx}\{\cos(\sin x^2)\} =$

(A) $\sin(\sin x^2).\cos x^2.2x$

(B) $-\sin(\sin x^2).\cos x^2.2x$

(C) $-\sin(\sin x^2).\cos^2 x.2x$

(D) None of these

16. The differential coefficient of the given function $\log_e\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$ with respect to x is

(A) $\text{cosec}\, x$ (B) $\tan x$

(C) $\cos x$ (D) $\sec x$

17. $\frac{d}{dx}\left[\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right] =$

(A) $\sec x$ (B) $\text{cosec}\, x$

(C) $\text{cosec}\frac{x}{2}$ (D) $\sec\frac{x}{2}$

18. $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right] =$

(A) $-\frac{1}{2}$ (B) 0

(C) $\frac{1}{2}$ (D) 1

19. If $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $\frac{1}{2(1+\cos x)}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) None of these

20. $\frac{d}{dx}e^{x\sin x} =$

(A) $e^{x\sin x}(x\cos x + \sin x)$

(B) $e^{x\sin x}(\cos x + x\sin x)$

(C) $e^{x\sin x}(\cos x + \sin x)$

(D) None of these

21. $\frac{d}{dx}\left\{\log\left(\frac{e^x}{1+e^x}\right)\right\} =$

(A) $\frac{1}{1-e^x}$ (B) $-\frac{1}{1+e^x}$

(C) $-\frac{1}{1-e^x}$ (D) None of these

22. $\frac{d}{dx}\left[\frac{2}{\pi}\sin x^0\right] =$

(A) $\frac{\pi}{180}\cos x^0$ (B) $\frac{1}{90}\cos x^0$

(C) $\frac{\pi}{90}\cos x^0$ (D) $\frac{2}{90}\cos x^0$

23. $\frac{d}{dx}\left[\log\sqrt{\sin\sqrt{e^x}}\right] =$

(A) $\frac{1}{4}e^{x/2}\cot(e^{x/2})$ (B) $e^{x/2}\cot(e^{x/2})$

(C) $\frac{1}{4}e^x\cot(e^x)$ (D) $\frac{1}{2}e^{x/2}\cot(e^{x/2})$

24. If $f(x) = |x|$, then $f'(0) =$

(A) 0 (B) 1

(C) x (D) None of these

25. At $x = \sqrt{\frac{\pi}{2}}, \frac{d}{dx}\cos(\sin x^2) =$

(A) –1 (B) 1

(C) 0 (D) None of these

26. $\frac{d}{dx}\left[\log\left(x+\frac{1}{x}\right)\right] =$

(A) $\left(x+\frac{1}{x}\right)$ (B) $\frac{\left(1+\frac{1}{x^2}\right)}{\left(1+\frac{1}{x}\right)}$

(C) $\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)}$ (D) $\left(1+\frac{1}{x}\right)$

27. If $y = \sin^{-1}\sqrt{x}$, then $\frac{dy}{dx} =$

(A) $\frac{2}{\sqrt{x}\sqrt{1-x}}$ (B) $\frac{-2}{\sqrt{x}\sqrt{1-x}}$

(C) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ (D) $\frac{1}{\sqrt{1-x}}$

28. If $y = \sin^{-1}\sqrt{(1-x)} + \cos^{-1}\sqrt{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\sqrt{x(1-x)}}$ (B) $\frac{-1}{\sqrt{x(1-x)}}$

(C) $\frac{1}{\sqrt{x(1+x)}}$ (D) None of these

29. If $y = x^n \log x + x(\log x)^n$, then $\frac{dy}{dx} =$

(A) $x^{n-1}(1+n\log x) + (\log x)^{n-1}[n+\log x]$

(B) $x^{n-2}(1+n\log x) + (\log x)^{n-1}[n+\log x]$

(C) $x^{n-1}(1+n\log x) + (\log x)^{n-1}[n-\log x]$

(D) None of these

30. If $y\sqrt{x^2+1} = \log\left\{\sqrt{x^2+1}-x\right\}$, then $(x^2+1)\frac{dy}{dx} + xy + 1 =$

(A) 0 (B) 1

(C) 2 (D) None of these

31. If $y = 3^{x^2}$, then $\frac{dy}{dx}$ is equal to

(A) $(x^2)3^{x^2-1}$ (B) $3x^2.2x$

(C) $3^{x^2}.2x.\log 3$ (D) $(x^2-1).3$

32. The first derivative of the function $(\sin 2x\cos 2x\cos 3x + \log_2 2^{x+3})$ with respect to x at $x = \pi$ is

(A) 2 (B) –1

(C) $-2+2^{\pi}\log_e 2$ (D) $-2+\log_e 2$

33. The values of x, at which the first derivative of the function $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ *w.r.t.* x is $\frac{3}{4}$, are

(A) ± 2 (B) $\pm\frac{1}{2}$

(C) $\pm\frac{\sqrt{3}}{2}$ (D) $\pm\frac{2}{\sqrt{3}}$

34. If $y=\frac{(1-x)^2}{x^2}$, then $\frac{dy}{dx}$ is

(A) $\frac{2}{x^2}+\frac{2}{x^3}$ (B) $-\frac{2}{x^2}+\frac{2}{x^3}$

(C) $-\frac{2}{x^2}-\frac{2}{x^3}$ (D) $-\frac{2}{x^3}+\frac{2}{x^2}$

35. If $pv=81$, then $\frac{dp}{dv}$ is at $v=9$ equal to

(A) 1 (B) –1

(C) 2 (D) None of these

36. If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ is

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\frac{-x}{\sqrt{1+x^2}}$

(C) $\frac{x}{\sqrt{1-x^2}}$ (D) None of these

37. The differential coefficient of the function $|x-1|+|x-3|$ at the point $x=2$ is

(A) -2 (B) 0

(C) 2 (D) Undefined

38. If $f(x)$ is a differentiable function, then $\lim\limits_{x\to a}\frac{af(x)-xf(a)}{x-a}$ is

(A) $af'(a)-f(a)$ (B) $af(a)-f'(a)$

(C) $af'(a)+f(a)$ (D) $af(a)+f'(a)$

39. If $x=\exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$, then $\frac{dy}{dx}$ equals

(A) $2x[1+\tan(\log x)]+x\sec^2(\log x)$

(B) $x[1+\tan(\log x)]+\sec^2(\log x)$

(C) $2x[1+\tan(\log x)]+x^2\sec^2(\log x)$

(D) $2x[1+\tan(\log x)]+\sec^2(\log x)$

40. If $f(x)=\sqrt{ax}+\frac{a^2}{\sqrt{ax}}$, then $f'(a)=$

(A) – 1 (B) 1

(C) 0 (D) a

Differentiation, Logarithmic differentiation & Differentiation of infinite series

41. If $x^3+8xy+y^3=64$, then $\frac{dy}{dx}=$

(A) $-\frac{3x^2+8y}{8x+3y^2}$ (B) $\frac{3x^2+8y}{8x+3y^2}$

(C) $\frac{3x+8y^2}{8x^2+3y}$ (D) None of these

42. If $ax^2+2hxy+by^2+2gx+2fy+c=0$, then $\frac{dy}{dx}=$

(A) $-\frac{ax+hy+g}{hx-by+f}$ (B) $\frac{ax+hy+g}{hx-by+f}$

(C) $\frac{ax-hy-g}{hx-by-f}$ (D) None of these

43. If $y=f\left(\frac{5x+1}{10x^2-3}\right)$ and $f'(x)=\cos x$, then $\frac{dy}{dx}=$

(A) $\cos\left(\frac{5x+1}{10x^2-3}\right)\frac{dy}{dx}\left(\frac{5x+1}{10x^2-3}\right)$

(B) $\frac{5x+1}{10x^2-3}\cos\left(\frac{5x+1}{10x^2-3}\right)$

(C) $\cos\left(\frac{5x+1}{10x^2-3}\right)$

(D) None of these

44. If $f(x) = \frac{1}{1-x}$, then the derivative of the composite function $f[f\{f(x)\}]$ is equal to

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) 2

45. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$, then $g'(f(c))$ equals

(A) $f'(c)$ (B) $\frac{1}{f'(c)}$

(C) $f(c)$ (D) None of these

46. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^3}$. Then $g'(x)$ is equal to

(A) $\frac{1}{1+(g(x))^3}$ (B) $\frac{1}{1+(f(x))^3}$

(C) $1+(g(x))^3$ (D) $1+(f(x))^3$

47. Let f and g be differentiable functions satisfying $g'(a) = 2$, $g(a) = b$ and $fog = I$ (identity function). Then $f'(b)$ is equal to

(A) $\frac{1}{2}$ (B) 2

(C) $\frac{2}{3}$ (D) None of these

48. The differential coefficient of $f[\log(x)]$ when $f(x) = \log x$ is

(A) $x \log x$ (B) $\frac{x}{\log x}$

(C) $\frac{1}{x \log x}$ (D) $\frac{\log x}{x}$

49. The derivative of $F[f\{\varphi(x)\}]$ is

(A) $F'[f\{\varphi(x)\}]$

(B) $F[f\{\varphi(x)\}]f'\{\varphi(x)\}$

(C) $F'[f\{\varphi(x)\}]f'\{\varphi(x)\}$

(D) $F'[f\{\varphi(x)\}]f'\{\varphi(x)\}\varphi'(x)$

50. Let $f(x) = e^x$, $g(x) = \sin^{-1} x$ and $h(x) = f(g(x))$, then $h'(x)/h(x) =$

(A) $e^{\sin^{-1}x}$ (B) $1/\sqrt{1-x^2}$

(C) $\sin^{-1} x$ (D) $1/(1-x^2)$

51. If $\cos x = \frac{1}{\sqrt{1+t^2}}$ and $\sin y = \frac{t}{\sqrt{1+t^2}}$, then $\frac{dy}{dx} =$

(A) -1 (B) $\frac{1-t}{1+t^2}$

(C) $\frac{1}{1+t^2}$ (D) 1

52. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{dy}{dx} =$

(A) $\cos\theta$ (B) $\tan\theta$

(C) $\sec\theta$ (D) $\operatorname{cosec}\theta$

53. If $x = a\cos^4\theta, y = a\sin^4\theta$, then $\frac{dy}{dx}$, at $\theta = \frac{3\pi}{4}$, is

(A) -1 (B) 1

(C) $-a^2$ (D) a^2

54. If $x = \sin^{-1}(3t - 4t^3)$ and $y = \cos^{-1}\sqrt{(1-t^2)}$, then $\frac{dy}{dx}$ is equal to

(A) 1/2 (B) 2/5

(C) 3/2 (D) 1/3

55. If $x = a\left(t - \frac{1}{t}\right), y = a\left(t + \frac{1}{t}\right)$ then $\frac{dy}{dx} =$

(A) $\frac{y}{x}$ (B) $\frac{-y}{x}$

(C) $\frac{x}{y}$ (D) $\frac{-x}{y}$

56. If $x = \sin t \cos 2t$ and $y = \cos t \sin 2t$, then at $t = \frac{\pi}{4}$, the value of $\frac{dy}{dx}$ is equal to

(A) -2 (B) 2

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

57. If $\ln(x+y) = 2xy$, then $y'(0) =$

(A) 1 (B) -1

(C) 2 (D) 0

58. If $y = x^x$, then $\frac{dy}{dx} =$

(A) $x^x \log ex$ (B) $x^x\left(1+\frac{1}{x}\right)$

(C) $(1+\log x)$ (D) $x^x \log x$

59. The first derivative of the function $\left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right)+x^x\right]$ with respect to x at $x = 1$ is

(A) $\frac{3}{4}$ (B) 0

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

60. If $y = \sqrt{\frac{1+x}{1-x}}$, then $\frac{dy}{dx} =$

(A) $\frac{2}{(1+x)^{1/2}(1-x)^{3/2}}$

(B) $\frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$

(C) $\frac{1}{2(1+x)^{1/2}(1-x)^{3/2}}$

(D) $\frac{1}{(1+x)^{3/2}(1-x)^{1/2}}$

61. $\frac{d}{dx}(x^{\log_e x}) =$

(A) $2x^{(\log_e x-1)}.\log_e x$ (B) $x^{(\log_e x-1)}$

(C) $\frac{2}{x}\log_e x$ (D) $x^{(\log_e x-1)}.\log_e x$

62. If $x^y = y^x$, then $\frac{dy}{dx} =$

(A) $\frac{y(x\log_e y+y)}{x(y\log_e x+x)}$ (B) $\frac{y(x\log_e y-y)}{x(y\log_e x-x)}$

(C) $\frac{x(x\log_e y-y)}{y(y\log_e x-x)}$ (D) $\frac{x(x\log_e y+y)}{y(y\log_e x+x)}$

63. If $y = x^{(x^x)}$, then $\frac{dy}{dx} =$

(A) $y[x^x(\log ex).\log x + x^x]$

(B) $y[x^x(\log ex).\log x + x]$

(C) $y[x^x(\log ex).\log x + x^{x-1}]$

(D) $y[x^x(\log_e x).\log x + x^{x-1}]$

64. If $y = x^{\sin x}$, then $\frac{dy}{dx} =$

(A) $\frac{x\cos x.\log x+\sin x}{x}.x^{\sin x}$

(B) $\frac{y[x\cos x.\log x+\cos x]}{x}$

(C) $y[x\sin x.\log x+\cos x]$

(D) None of these

65. $\frac{d}{dx}\{(\sin x)^x\} =$

(A) $\left[\frac{x\cos x+\sin x\log\sin x}{\sin x}\right]$

(B) $(\sin x)^x\left[\frac{x\cos x+\sin x\log\sin x}{\sin x}\right]$

(C) $(\sin x)^x\left[\frac{x\sin x+\sin x\log\sin x}{\sin x}\right]$

(D) None of these

66. If $y = \frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$, then $\frac{dy}{dx} =$

(A) $y\left[\frac{1}{2x}+\frac{4}{2x+3}-\frac{1}{2(x+1)}\right]$

(B) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{2(x+1)}\right]$

(C) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{x+1}\right]$

(D) None of these

67. $\frac{d}{dx}\{(\sin x)^{\log x}\} =$

(A) $(\sin x)^{\log x}\left[\frac{1}{x}\log\sin x+\cot x\right]$

(B) $(\sin x)^{\log x}\left[\frac{1}{x}\log\sin x+\cot x\log x\right]$

(C) $(\sin x)^{\log x}\left[\frac{1}{x}\log\sin x+\log x\right]$

(D) None of these

68. If $y = (\tan x)^{\cot x}$, then $\frac{dy}{dx} \setminus =$

(A) $y \operatorname{cosec}^2 x(1 - \log \tan x)$

(B) $y \operatorname{cosec}^2 x(1 + \log \tan x)$

(C) $y \operatorname{cosec}^2 x(\log \tan x)$

(D) None of these

69. If $y = x^2 + x^{\log x}$, then $\frac{dy}{dx} =$

(A) $\frac{x^2 + \log x.x^{\log x}}{x}$

(B) $x^2 + \log x.x^{\log x}$

(C) $\frac{2(x^2 + \log x.x^{\log x})}{x}$

(D) None of these

70. If $y = x^2 + \cfrac{1}{x^2 + \cfrac{1}{x^2 + \cfrac{1}{x^2 + ...\infty}}}$, then

$\frac{dy}{dx} =$

(A) $\frac{2xy}{2y - x^2}$ (B) $\frac{xy}{y + x^2}$

(C) $\frac{xy}{y - x^2}$ (D) $\frac{2xy}{2 + \frac{x^2}{y}}$

Differentiation by substitution, Higher order derivatives

71. If $y = \sin^{-1} \frac{\sqrt{(1+x)} + \sqrt{(1-x)}}{2}$,

then $\frac{dy}{dx} =$

(A) $\frac{1}{\sqrt{(1 - x^2)}}$ (B) $-\frac{1}{\sqrt{(1 - x^2)}}$

(C) $-\frac{1}{2\sqrt{(1 - x^2)}}$ (D) None of these

72. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$ is

(A) 8 (B) 1

(C) 4 (D) 5

73. If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to

(A) -1 (B) 1

(C) $\log 2$ (D) $-\log 2$

74. Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to

(A) $2/7$ (B) $1/2$

(C) 2 (D) $7/2$

75. $\frac{d}{dx}\left[\sin^2 \cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ equals

(A) -1 (B) $\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) 1

76. If $y = \tan^{-1}\left(\frac{x}{1 + \sqrt{1 - x^2}}\right)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{2\sqrt{1 - x^2}}$ (B) $1 - \sqrt{1 - x^2}$

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{1 - x^2}}$

77. Differential coefficient of $\cos^{-1}(\sqrt{x})$ with respect to $\sqrt{(1-x)}$ is

(A) $\sqrt{x}$ (B) $-\sqrt{x}$

(C) $\frac{1}{\sqrt{x}}$ (D) $-\frac{1}{\sqrt{x}}$

78. If $y = \tan^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right)$, then $\frac{dy}{dx} =$

(A) $-\frac{1}{\sqrt{1 - x^2}}$ (B) $\frac{x}{\sqrt{1 - x^2}}$

(C) $\frac{1}{\sqrt{1 - x^2}}$ (D) $\frac{\sqrt{1 - x^2}}{x}$

79. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ equals

(A) $\frac{2}{1-x^2}$ (B) $\frac{1}{1+x^2}$

(C) $\pm\frac{2}{1+x^2}$ (D) $-\frac{2}{1+x^2}$

80. The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is

(A) $\sqrt{1-x^2}$ (B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{2\sqrt{1-x^2}}$ (D) x

81. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ *w.r.t.* $\cot^{-1}\left(\frac{1-3x^2}{3x-x^2}\right)$ is

(A) 1 (B) $\frac{3}{2}$

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

82. The differential of e^{x^3} with respect to $\log x$ is

(A) e^{x^3} (B) $3x^2e^{x^3}$

(C) $3x^3e^{x^3}$ (D) $3x^2e^{x^3}+3x^2$

83. The 2^{nd} derivative of $a\sin^3 t$ with respect to $a\cos^3 t$ at $t=\frac{\pi}{4}$ is

(A) $\frac{4\sqrt{2}}{3a}$ (B) 2

(C) $\frac{1}{12a}$ (D) None of these

84. The differential coefficient of $f(\sin x)$ with respect to *x*, where $f(x)=\log x$, is

(A) $\tan x$ (B) $\cot x$

(C) $f(\cos x)$ (D) $1/x$

85. If $y=\sin x\sin 3x$, then $y_n =$

(A) $\frac{1}{2}\left[\cos\left(2x+n\frac{\pi}{2}\right)-\cos\left(4x+n\frac{\pi}{2}\right)\right]$

(B) $\frac{1}{2}\left[2^n\cos\left(2x+n\frac{\pi}{2}\right)-4^n\cos\left(4x+n\frac{\pi}{2}\right)\right]$

(C) $\frac{1}{2}\left[4^n\cos\left(4x+n\frac{\pi}{2}\right)-2^n\cos\left(2x+n\frac{\pi}{2}\right)\right]$

(D) None of these

86. n^{th} derivative of x^{n+1} is

(A) $(n+1)!x$ (B) $(n+1)!$

(C) $n!x$ (D) $n!$

87. If $y=a_0+a_1x+a_2x^2+.....+a_nx^n$, then $y_n =$

(A) $n!$ (B) $n!a_nx$

(C) $n!a_n$ (D) None of these

88. If $y=A\cos nx+B\sin nx$, then $\frac{d^2y}{dx^2}=$

(A) n^2y (B) $-y$

(C) $-n^2y$ (D) None of these

89. $\frac{d^n}{dx^n}(e^{2x}+e^{-2x})=$

(A) $e^{2x}+(-1)^ne^{-2x}$

(B) $2^n(e^{2x}-e^{-2x})$

(C) $2^n[e^{2x}+(-1)^ne^{-2x}]$

(D) None of these

90. If $x=\log p$ and $y=\frac{1}{p}$, then

(A) $\frac{d^2y}{dx^2}-2p=0$ (B) $\frac{d^2y}{dx^2}+y=0$

(C) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ (D) $\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$

91. A curve is given by the equations $x = a\cos\theta + \frac{1}{2}b\cos 2\theta$, $y = a\sin\theta + \frac{1}{2}b\sin 2\theta$, then the points for which $\frac{d^2y}{dx^2} = 0$, is given by

(A) $\sin\theta = \frac{2a^2 + b^2}{5ab}$

(B) $\tan\theta = \frac{3a^2 + 2b^2}{4ab}$

(C) $\cos\theta = \frac{-\left(a^2 + 2b^2\right)}{3ab}$

(D) $\cos\theta = \frac{\left(a^2 - 2b^2\right)}{3ab}$

92. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(A) n^2y (B) $-n^2y$

(C) $-y$ (D) $2x^2y$

93. $f(x)$ and $g(x)$ are two differentiable function on $[0,2]$ such that $f''(x) - g''(x) = 0, f'(1) = 2, g'(1) = 4$, $f(2) = 3$, $g(2) = 9$, then $f(x) - g(x)$ at $x = 3/2$ is

(A) 0 (B) 2

(C) 10 (D) -5

94. If $y = ae^x + be^{-x} + c$ where a, b, c are parameters then $y''' =$

(A) y (B) y'

(C) 0 (D) y''

95. If $y = a\cos(\log x) + b\sin(\log x)$ where a, b are parameters then $x^2y'' + xy' =$

(A) y (B) $-y$

(C) $2y$ (D) $-2y$

96. If $u = x^2 + y^2$ and $x = s + 3t, y = 2s - t$, then $\frac{d^2u}{ds^2} =$

(A) 12 (B) 32

(C) 36 (D) 10

97. $\frac{d^n}{dx^n}(\log x) =$

(A) $\frac{(n-1)!}{x^n}$ (B) $\frac{n!}{x^n}$

(C) $\frac{(n-2)!}{x^n}$ (D) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

98. The n^{th} derivative of xe^x vanishes when

(A) $x = 0$ (B) $x = -1$

(C) $x = -n$ (D) $x = n$

99. $\frac{d^2}{dx^2}(2\cos x\cos 3x) =$

(A) $2^2(\cos 2x + 2^2\cos 4x)$

(B) $2^2(\cos 2x - 2^2\cos 4x)$

(C) $2^2(-\cos 2x + 2^2\cos 4x)$

(D) $-2^2(\cos 2x + 2^2\cos 4x)$

100. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - .....$, then $\frac{d^2y}{dx^2} =$

(A) x (B) $-x$

(C) $-y$ (D) y

Partial differentiation

101. If $u = \tan^{-1}\frac{y}{x}$, then by Euler's Theorem the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) $\tan u$ (B) $\sin u$

(C) 0 (D) $\cos 2u$

102. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) $\sin 2u$ (B) $\cos 2u$

(C) $\tan 2u$ (D) $\sec 2u$

103. If $F(u) = f(x, y, z)$ be a homogeneous function of degree n in x, y, z then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$

(A) nu (B) $n F(u)$

(C) $\frac{n F(u)}{F'(u)}$ (D) None of these

104. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)(x + y + z) =$

(A) 0 (B) 1

(C) 2 (D) 3

105. If $z = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(A) $\frac{1}{2}\sin z$ (B) $\frac{1}{2}\tan z$

(C) 0 (D) None of these

106. If $u = \log_e(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} =$

(A) 0 (B) $2u$

(C) $1/u$ (D) u

107. If $x^x y^y z^z = c$, then $\frac{\partial z}{\partial x} =$

(A) $\frac{1 + \log x}{1 + \log z}$ (B) $-\frac{1 + \log x}{1 + \log z}$

(C) $-\frac{1 + \log y}{1 + \log z}$ (D) None of these

108. If $u = xy^2 \tan^{-1}\left(\frac{y}{x}\right)$, then $xu_x + yu_y =$

(A) $2u$ (B) u

(C) $3u$ (D) $u/3$

109. If $z^2 = \frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$

(A) $\frac{z}{6}$ (B) $\frac{z}{3}$

(C) $\frac{z}{2}$ (D) $\frac{z}{12}$

110. If $u = \tan^{-1}(x + y)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) $\sin 2u$ (B) $\frac{1}{2}\sin 2u$

(C) $2\tan u$ (D) $\sec^2 u$