HINTS & SOLUTIONS

EXERCISE - 1

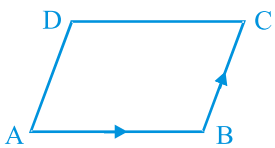
Single Choice

6. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= 2\hat{i} - 2\hat{j} + 4\hat{k}$

$\overrightarrow{BD} = -\overrightarrow{AB} + \overrightarrow{BC}$

$= -4\hat{i} + 2\hat{j}$



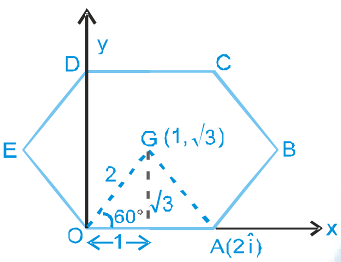
Let Angle between $\overrightarrow{AC}$ & $\overrightarrow{BD}$ is θ

$\therefore \quad \dfrac{\overrightarrow{AC}.\overrightarrow{BD}}{|\overrightarrow{AC}|\,|\overrightarrow{BD}|} = \cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{-12}{4\sqrt{6}\sqrt{5}} = -\sqrt{\dfrac{3}{10}}$.

$\Rightarrow$ Acute angle between diagonals $= \cos^{-1}\sqrt{\dfrac{3}{10}}$

13.



$G \equiv (\hat{i} + \sqrt{3}\hat{j})$

Let Position vector of P is $\vec{p}$

$\Rightarrow \quad \overrightarrow{GP} \,||\, \hat{k}$

then $\quad \vec{p} - (\hat{i} + \sqrt{3}\hat{j}) = \lambda\hat{k}$

$\Rightarrow \quad \vec{p} = \hat{i} + \sqrt{3}\,\hat{j} + \lambda\hat{k}$

also $\left|\overrightarrow{OP}\right| = 3$

$\Rightarrow \quad \sqrt{1+3+\lambda^2} = 3$

$\Rightarrow \quad \lambda^2 = 5$

$\Rightarrow \quad \lambda = \pm\sqrt{5} \qquad \Rightarrow \quad \vec{p} = \hat{i} + \sqrt{3}\,\hat{j} \pm \sqrt{5}\,\hat{k}$

For positive Z-axis $\vec{p} = \hat{i} + \sqrt{3}\,\hat{j} + \sqrt{5}\,\hat{k}$

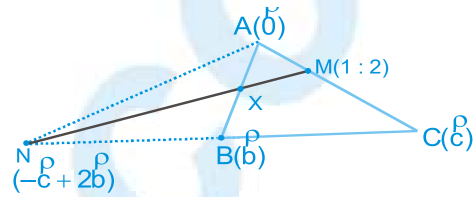
So $\quad \overrightarrow{AP} = \vec{p} - 2\hat{i} = -\hat{i} + \sqrt{3}\,\hat{j} + \sqrt{5}\,\hat{k}$

17. Position vector of M $\equiv \dfrac{\vec{c}}{3}$

Position vector of N $\equiv (-\vec{c} + 2\vec{b})$

$\therefore$ equation of line BC is $\vec{r} = \vec{b} + \lambda(\vec{b} - \vec{c})$

$\therefore$ equation of line AB is is $\vec{r} = \vec{0} + \mu\vec{b}$



$\therefore$ equation of line MN is $\vec{r} = \dfrac{\vec{c}}{3} + t\left(\dfrac{4\vec{c}}{3} - 2\right)$

$\Rightarrow \quad \mu = -2t, \quad 0 = \dfrac{1}{3} + \dfrac{4}{3}t$

which gives $\mu = \dfrac{1}{2} \quad \Rightarrow$ Position vector of X is $\dfrac{\vec{b}}{2}$.

18. $\vec{a} = \hat{i} + \hat{j}$ & $\vec{b} = 2\hat{i} - \hat{k}$

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a} \quad \Rightarrow \quad (\vec{r} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \quad \vec{r} = \vec{b} + \lambda\vec{a}$ .....(i)

similarly $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$

$\Rightarrow \quad \vec{r} = \vec{a} + \mu\vec{b}$ .....(ii)

Putting the vector $\vec{a}$ & $\vec{b}$ in (i) & (ii) and equating

we get $2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j}) = \hat{i} + \hat{j} + \mu(2\hat{i} - \hat{k})$

$\Rightarrow \quad 2 + \lambda = 1 + 2\mu, \ \lambda = 1, \mu = 1$

$\therefore$ Point of intersection is $3\hat{i} + \hat{j} - \hat{k}$.

20. Equation of plane containing L_1 and parallel to

L_2 is $\begin{vmatrix} x-2 & y-1 & z+1 \\ 1 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix} = 0$

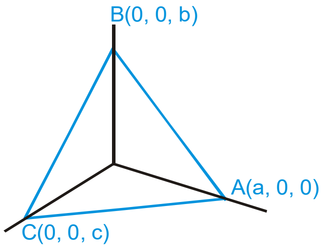
$\Rightarrow \quad 2x - 3y - z = 2$

distance from origin $= \dfrac{2}{\sqrt{14}} = \sqrt{\dfrac{2}{7}}$

21. Let the equation of plane be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

as (α, β, γ) is centroid



22. L. H. S $=\left(\lambda(\vec a+\vec b)\times\lambda^2\vec b\right).\lambda\vec c$

$=\lambda^4\left((\vec a+\vec b)\times\vec b\right).\vec c=\lambda^4[a\ b\ c]$

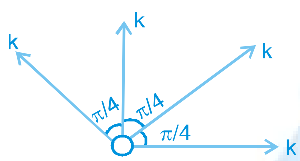
R.H.S. $=(\vec a\times(\vec b+\vec c)).\vec b=[\vec a\ \vec c\ \vec b]$

$\Rightarrow\ \lambda^4[a\ b\ c]=-[a\ b\ c]$

$\Rightarrow\ \lambda^4=-1$ which is not possible.

23. These forces can be written in terms of vector as

$k\hat i,\ \frac{k}{\sqrt2}\hat i+\frac{k}{\sqrt2}\hat j,\ k\hat j$ and $-\frac{k}{\sqrt2}\hat i+\frac{k}{\sqrt2}\hat j$



Resultant $=k\hat i+(k+\sqrt2k)\hat j$

magnitude $=\sqrt{k^2+(k+\sqrt2k)^2}=k\sqrt{4+2\sqrt2}$

24. Equation of plane is $\vec r.\hat n=\frac{q}{|\vec n|}$

for intercept on x-axis take dot product with $\hat i$

$\Rightarrow$ intercept on x-axis $=\frac{q}{\hat i.\vec n}$

25. $\vec c.\vec a=\vec a.(\vec a\times\vec b)\quad\Rightarrow\quad\vec c.\vec a=0=\vec c.\vec b\quad=\vec a.\vec b$

Also $|\vec a\times\vec b|=|\vec c|$

$|\vec a||\vec b|\sin 90°=|\vec c|$

$|\vec a|^2=|\vec a|\ \Rightarrow\ |\vec a|=|\vec b|=|\vec c|=1$

$|3\vec a+4\vec b+12\vec c|=\sqrt{9a^2+16b^2+144c^2}=13$

$\{\because|\vec a|=|\vec b|=|\vec c|=1\}$

28. From P(f, g, h) the foot of perpendicular on plane yz = (0, g, h),

similarly from P(f, g, h) perpendicular to z x = (f,0,h)

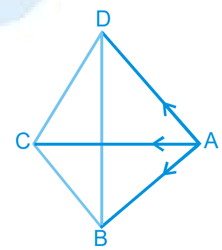
Equation of plane is

$$\begin{vmatrix}x & y & z\\ f & 0 & h\\ 0 & g & h\end{vmatrix}=0\ \Rightarrow\ \frac{x}{f}+\frac{y}{g}-\frac{z}{h}=0$$

30. $\overrightarrow{AD}=-2\hat i+2\hat j-\hat k$

$\overrightarrow{AC}=\hat i+2\hat j+2\hat k$

$\overrightarrow{AB}=3\hat j+4\hat k$



$$\vec n_1=\overrightarrow{AD}\times\overrightarrow{AC}=\begin{vmatrix}\hat i & \hat j & \hat k\\ -2 & 2 & -1\\ 1 & 2 & 2\end{vmatrix}=6\hat i+3\hat j-6\hat k$$

$$=3\left(2\hat i+\hat j-2\hat k\right)$$

$$\vec n_2=\overrightarrow{AC}\times\overrightarrow{AB}=\begin{vmatrix}\hat i & \hat j & \hat k\\ 1 & 2 & 2\\ 0 & 3 & 4\end{vmatrix}=2\hat i-4\hat j+3\hat k$$

$$|\vec n_1\times\vec n_2|=3\begin{vmatrix}\hat i & \hat j & \hat k\\ 2 & 1 & -2\\ 2 & -4 & 3\end{vmatrix}=3(-5\hat i-10\hat j-10\hat k)$$

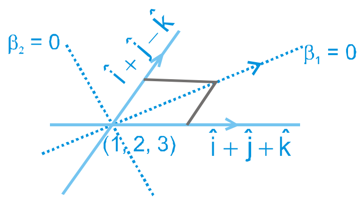
$$\sin\theta=\frac{5}{\sqrt{29}}\qquad\left(\sin\theta=\frac{|\vec n_1\times\vec n_2|}{|\vec n_1|\,|\vec n_2|}\right)$$

33. Dr's of bisector

$$\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}+\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}=\lambda(\hat{i}+\hat{j})$$

Hence Dr's are $\lambda, \lambda, 0$ $(\lambda \in R)$

Equation of bisector

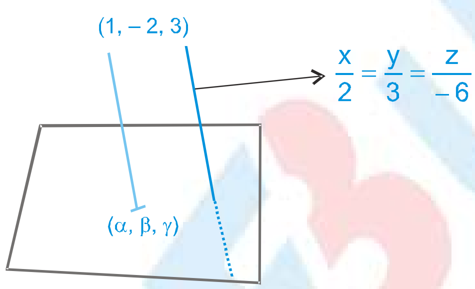


$$\frac{x-1}{\lambda}=\frac{y-2}{\lambda}=\frac{z-3}{0}$$

$$\frac{x-1}{2}=\frac{y-2}{2}; z-3=0$$

34. $\alpha-1=2\lambda \Rightarrow \alpha=2\lambda+1$

$\beta+2=3\lambda \Rightarrow \beta=3\lambda-2$



$\gamma-3=-6\lambda \Rightarrow \gamma=-6\lambda+3$

$2\lambda+1-3\lambda+2-6\lambda+3=5$

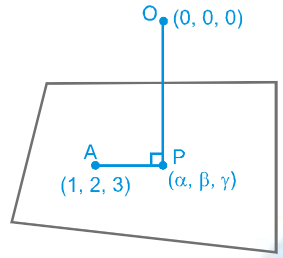
$7\lambda=1 \Rightarrow \lambda=1/7$

$\therefore$ Point on the plane is $\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

Distance $=\sqrt{(\alpha-1)^2+(\beta+2)^2+(\gamma-3)^2}$

$=\lambda\sqrt{4+9+36}=\frac{1}{7}.7=1$

36. $OP \perp AP$



$\alpha(\alpha-1)+\beta(\beta-2)+\gamma(\gamma-3)=0$

$\therefore$ Locus of $P(\alpha, \beta, \gamma)$ is

$x^2+y^2+z^2-x-2y-3z=0$

51. $a(x-2)+b(y-3)+6(z-1)=0$ .....(i)

$2a-2b-3c=0$

$4a+0.b+6c=0$

$$\frac{a}{-12-0}=\frac{b}{-12-12}=\frac{c}{0+8}$$

$$\frac{a}{3}=\frac{b}{6}=\frac{c}{-2}=\lambda \text{ (let)}$$

Put these values of a, b, c in (i)

$3(x-2)+6(y-3)-2(z-1)=0$

$3x+6y-2z-22=0$

$$d=\left|\frac{-15-24-16-22}{\sqrt{9+36+4}}\right|=\left|\frac{77}{7}\right|=11$$

54. Let the tetrahedron cut x-axis, y-axis and z-axis at a, b & c respectively.

volume $=\frac{1}{6}[a\hat{i}\ b\hat{j}\ c\hat{k}]$ (Given)

Then $\frac{1}{6}(abc)=64K^3$ .....(i)

Let centroid be (x_1, y_1, z_1)

$\therefore x_1=\frac{a}{4},\ y_1=\frac{b}{4},\ z_1=\frac{c}{4}$

put in (i) wet get

$x_1y_1z_1=6K^3$

$\therefore$ Locus is $xyz=6K^3$

The required locus is $xyz=6K^3$

58. $\vec{r}.\vec{n} = d$ ....**(i)**

$\vec{r} = \vec{r}_0 + t\vec{n}$ ....**(ii)**

from **(i)** and **(ii)**

$(\vec{r}_0 + t\vec{n}).\vec{n} = d \Rightarrow t = \dfrac{d - \vec{r}_0.\vec{n}}{\vec{n}^2}$

substitute the value of 't' in **(ii)**

$r = \vec{r}_0 + \left(\dfrac{d - \vec{r}_0.\vec{n}}{\vec{n}^2}\right)\vec{n}$

59. $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{b})$

$\vec{a} \times (\vec{b} - 2\vec{c}) = 0 \quad \Rightarrow \quad \vec{b} - 2\vec{c} = \alpha\vec{a}$

squaring $b^2 + 4c^2 - 4\vec{b}\cdot\vec{c} = \alpha^2 a^2$

$16 + 4 - 4.4.1.\dfrac{1}{4} = \alpha^2 \Rightarrow \alpha = \pm 4$

$\vec{b} = 2\vec{c} \pm 4\vec{a}$

$|\bullet| + |\mu| = 6$

60. $(a - b)\vec{x} + (b - c)\vec{y} + (c - a)(\vec{x} \times \vec{y}) = 0$

As $\vec{x}, \vec{y}$ & $(\vec{x} \times \vec{y})$ are non zero, non coplanar vectors, then

$a - b = b - c = c - a = 0$

$\Rightarrow a = b = c$

Hence ΔABC is an equilateral triangle.

Hence, acute angled triangle.

63. $\vec{c}$ is along the vector $\vec{a} \times (\vec{a} \times \vec{b})$

$= (\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b}$

$= (-1)(i + j - k) - 3(i - j + k) = -4i + 2j - 2k$

$\vec{c} = \dfrac{-2i + j - k}{\sqrt{6}}$

$\vec{d} = \dfrac{(\vec{a} \times \vec{c})}{|\vec{a} \times \vec{c}|}$;

$\vec{a} \times \vec{c} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -2 & 1 & -1 \end{vmatrix} = -j(-3) + k.3 = 3(j + k)$

$\vec{d} = \dfrac{j + k}{\sqrt{2}}$

65. Equation of plane containing

$\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z+2}{1}$ and point (0, 7, – 7) is

$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ -1 & -4 & 5 \end{vmatrix} = 0$

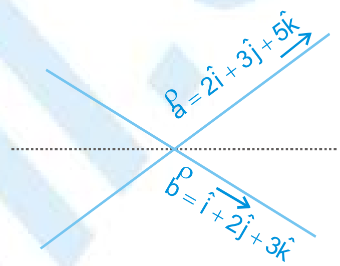
By solving we get

$x + y + z = 0$

68. $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{5}$ .....**(i)**

$\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ .....**(ii)**

$\hat{a} + \hat{b} = \dfrac{2\hat{i} + 3\hat{j} + 5\hat{k}}{\sqrt{38}} + \dfrac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$



$\Rightarrow$ (A) and (B) will be incorrect

Let the dr's of line $\perp$ to (1) and (2) be a, b, c

$\Rightarrow 2a + 3b + 5c = 0$ ... **(iii)**

and $a + 2b + 3c = 0$ ... **(iv)**

$\therefore \dfrac{a}{9 - 10} = \dfrac{b}{5 - 6} = \dfrac{c}{4 - 3}$

$\Rightarrow \dfrac{a}{-1} = \dfrac{b}{-1} = \dfrac{c}{1} \quad \Rightarrow \dfrac{a}{1} = \dfrac{b}{1} = \dfrac{c}{-1}$

$\therefore$ equation of line passing through (0, 0, 0) and is $\perp$r to the lines **(i)** and **(ii)** is

$\dfrac{x}{1} = \dfrac{y}{1} = \dfrac{z}{-1}$

70. $[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

$= \begin{vmatrix} 1 & \cos\theta & \cos\theta \\ \cos\theta & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}$

EXERCISE - 2

Part # I : Multiple Choice

5. $\vec{a}, \vec{b}, \vec{c}$ are unit vector mutually perpendicular to each other then angle between $\vec{a}+\vec{b}+\vec{c}$ & $\vec{a}$ is given by

$$\cos\theta = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2}\,|\vec{a}|}$$

$$= \frac{|\vec{a}|}{\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2}}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\sqrt{2}$$

7. $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + \sqrt{2}\hat{k})$

$\cos\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 30°$, $\cos\beta = \frac{\sqrt{3}}{2} \Rightarrow \beta = 30°$,

$\cos\gamma = \frac{\sqrt{2}}{2} \Rightarrow \gamma = 45°$

By putting the values check options

8. $\vec{r}_1 = \vec{a} - \vec{b} + \vec{c}$ .....**(i)**

$\vec{r}_2 = \vec{b} + \vec{c} - \vec{a}$ .....**(ii)**

$\vec{r}_3 = \vec{c} + \vec{a} + \vec{b}$ .....**(iii)**

$\vec{r} = 2\vec{a} - 3\vec{b} + 4\vec{c}$

If $\vec{r} = \lambda_1\vec{r}_1 + \lambda_2\vec{r}_2 + \lambda_3\vec{r}_3$

then $2\vec{a} - 3\vec{b} + 4\vec{c}$

$= (\lambda_1 - \lambda_2 + \lambda_3)\vec{a} + (\lambda_2 - \lambda_1 + \lambda_3)\vec{b} + (\lambda_1 + \lambda_2 + \lambda_3)\vec{c}$

$\Rightarrow \lambda_1 + \lambda_3 - \lambda_2 = 2$ .....**(iv)**

$\lambda_2 + \lambda_3 - \lambda_1 = -3$ .....**(v)**

$\lambda_1 + \lambda_2 + \lambda_3 = 4$ .....**(vi)**

Solving **(iv) (v)** & **(vi)** we get

$\lambda_2 = 1$; $\lambda_1 = 7/2$; $\lambda_3 = -1/2$

Now check options

9. $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ & $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$

$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{c} \times \vec{d} - \vec{b} \times \vec{d}$

$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d}$

$\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$

11. $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$

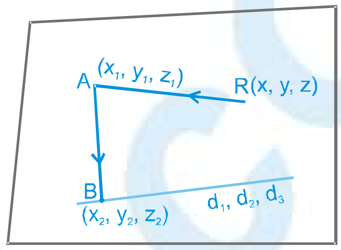
$(\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

But $\vec{b}.\vec{c} \neq 0$, $\vec{a}.\vec{b} \neq 0$

$\Rightarrow \vec{a}$ & $\vec{c}$ must be parallel.

14. Vectors $\overrightarrow{AR}$, $\overrightarrow{AB}$ & $\vec{C}$ are coplanar

Equation of the required plane



$\vec{C} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

or
$$\begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

16 . Let vector is $\vec{r} = \lambda_1\hat{a} + \lambda_2\hat{b} + \lambda_3(\hat{a} \times \hat{b})$ also

$$\cos\theta = \frac{\vec{r}.\hat{a}}{|\vec{r}||\hat{a}|} = \frac{\vec{r}.\hat{b}}{|\vec{r}||\hat{b}|} = \frac{\vec{r}(\hat{a} \times \hat{b})}{|\vec{r}||\hat{a} \times \hat{b}|}$$

$\Rightarrow \vec{r}.\hat{a} = \vec{r}.\hat{b} = \vec{r}.(\hat{a} \times \hat{b})$

$[\ |\hat{a} \times \hat{b}| = |\hat{a}|\ |\hat{b}|\ \sin 90° = 1]$

$\Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = \lambda$ (let)

$\therefore \vec{r} = \lambda(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$

$$7|\vec{r}| = \left|\lambda\sqrt{\hat{a}^2 + \hat{b}^2 + \left(\hat{a} \times \hat{b}\right)^2 + 2\hat{a}.\hat{b} + 2\hat{b}.\left(\hat{a} \times \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right).\hat{a}}\right| = 1$$

$\Rightarrow \left|\lambda\sqrt{1+1+1}\right| = 1 \quad \Rightarrow \lambda = \pm\frac{1}{\sqrt{3}}$

$\therefore \vec{r} = \pm\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$

17. Let $\vec a = x\hat i + y\hat j + z\hat k$

it makes equal angle with

$\frac{1}{3}(\hat i - 2\hat j + 2\hat k), \frac{1}{5}(-4\hat i - 3\hat k), \hat j$ then

$$\frac{x-2y+2z}{3} = \frac{-4x-3z}{5} = y$$

$4x + 5y + 3z = 0$ ...(i)

$x - 5y + 2z = 0$ ...(ii)

from (i) & (ii)

$x = -z$ & $x = -5y$

$$\vec a = x\left(\hat i - \frac{1}{5}\hat j - \hat k\right).$$

18. $\vec a \times (\vec b \times \vec c) = \frac{\vec b + \vec c}{\sqrt 2}$

$\Rightarrow (\vec a.\vec c)\vec b - (\vec a.\vec b)\vec c = \frac{\vec b}{\sqrt 2} + \frac{\vec c}{\sqrt 2}$

$\Rightarrow \left(\vec a.\vec c - \frac{1}{\sqrt 2}\right)\vec b - \left(\vec a.\vec b + \frac{1}{\sqrt 2}\right)\vec c = 0$

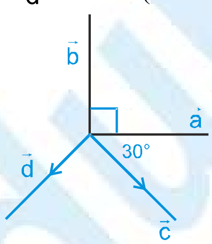
$\therefore \vec a.\vec b = \frac{-1}{\sqrt 2}$ & $\vec a.\vec c = \frac{1}{\sqrt 2}$

angle between $\vec a$ & $\vec b = \cos^{-1}\left(-\frac{1}{\sqrt 2}\right) = \frac{3\pi}{4}$

19. If $\lambda = -1$ then $\vec a \perp \vec b$, $\vec c \perp \vec d$ and angle between

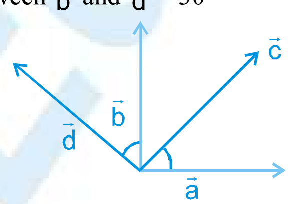
$\vec a \times \vec b$, $\vec c \times \vec d$ is π

$\angle$ between $\vec b$ and $\vec d = 360° - (90° + 90° + 30°) = 150°$

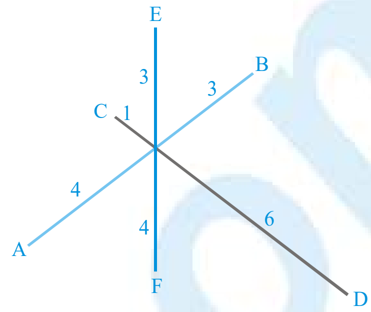


If $(\vec a \times \vec b)\,.\,(\vec c \times \vec d) = 1$, then following figure is possible

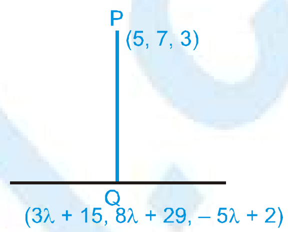
then $\angle$ between $\vec b$ and $\vec d = 30°$



23. $\frac{3\vec a + 4\vec b}{7} = \frac{6\vec c + \vec d}{7} = \frac{4\vec e + 3\vec f}{7} = \frac{\vec x}{7}$



27. d.r's of line are 3, 8, – 5



d.r's of PQ are $3\lambda + 10, 8\lambda + 22, -5\lambda + 2$

$\therefore$ both are perpendidcular

$\therefore (3\lambda + 10)3 + (8\lambda + 22)8 + (-5\lambda + 2)(-5) = 0$

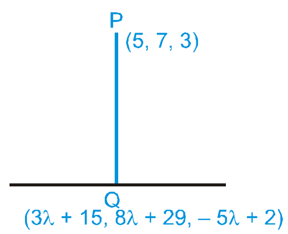
i.e. $\lambda = -2$

$\therefore$ foot is (9, 13, 15), PQ = 14

Since (5, 7, 3), (9, 13, 15) lies on the plane

$9x - 4y - z - 14 = 0$ and $3 \times 9 + 8(-4) + (-5)(-1) = 0$

$\therefore$ equation of the required plane is $9x - 4y - z - 14 = 0$



29. Let any point on line $\frac{x-1}{2} = \frac{y+1}{-3} = z = \lambda$

be $(1 + 2\lambda, -1 - 3\lambda, \lambda)$

$$4\sqrt{14} = \sqrt{(1 + 2\lambda - 1)^2 + (-1 - 3\lambda + 1)^2 + \lambda^2}$$

$$4\sqrt{14} = \sqrt{4\lambda^2 + 9\lambda^2 + \lambda^2}$$

$\Rightarrow |\lambda| = 4 \Rightarrow \lambda = \pm 4$

$\therefore$ Points (9, – 13, 4) and (– 7, 11, – 4)

30. Let $\vec{r} = xi + yj + zk$

then $[\vec{r}\,\vec{b}\,\vec{c}] = 0 \Rightarrow \begin{vmatrix} x & y & z \\ 1 & 2 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 0,$

$-3x + y - z = 0$ ....**(1)**

$\frac{\vec{r}.\vec{a}}{|\vec{a}|} = \pm\frac{\sqrt{2}}{3} \Rightarrow \frac{2x - y + 3}{\sqrt{6}} = \pm\sqrt{\frac{2}{3}}$

$2x - y + z = \pm 2$ ...**(2)**

from **(1)** and **(2)** $x = \mp 2$; $y - z = \mp 6$

there fore $\vec{r} = \mp 2i + yj + (y \pm 6)k$

(A) & (C) are answer

31. The vector parallel to line of intersection of planes is

$$\lambda\begin{vmatrix} i & j & k \\ 6 & 4 & -5 \\ 1 & -5 & 2 \end{vmatrix} = -\lambda(17\hat{i} + 17\hat{j} + 34\hat{k})$$

$= \lambda'(\hat{i} + \hat{j} + 2\hat{k})$ (λ' is scalar)

Now angle between the lines

$$\cos\theta = \frac{\lambda'(\hat{i} + \hat{j} + 2\hat{k}).(2\hat{i} - \hat{j} + \hat{k})}{\lambda'\sqrt{6} \times \sqrt{6}} = \frac{1}{2}$$

$\Rightarrow \theta = \frac{\pi}{3}$

33. any such vector $= \lambda\,(\hat{a} + \hat{b})$

$$= \lambda\left(\frac{7\hat{i} - 4\hat{j} - 4\hat{k}}{9} + \frac{-2\hat{i} - \hat{j} + 2\hat{k}}{3}\right)$$

$$= \frac{\lambda}{9}\left[7\hat{i} - 4\hat{j} - 4\hat{k} + 3(-2\hat{i} - \hat{j} + 2\hat{k})\right]$$

$$= \frac{\lambda}{9}\left[\hat{i} - 7\hat{j} + 2\hat{k}\right]$$

$|\vec{c}| = 5\sqrt{6} \Rightarrow \left|\frac{\lambda}{9}\sqrt{1 + 49 + 4}\right| = 5\sqrt{6}$

$\Rightarrow \left|\frac{\lambda}{9}\sqrt{54}\right| = 5\sqrt{6}$

$\Rightarrow \lambda = \pm\frac{9 \times 5\sqrt{6}}{\sqrt{54}} = \pm 15$

$\Rightarrow \vec{c} = \pm\frac{15}{9}(\hat{i} - 7\hat{j} + 2\hat{k}) = \pm\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$

35. **(A)** $\vec{a} \times [\vec{a} \times (\vec{a} \times \vec{b})]$

$= \vec{a} \times [(\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b}] = 0 - (\vec{a})^2(\vec{a} \times \vec{b})$. False

(B) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

$\left.\begin{matrix}\vec{v}.\vec{a} = 0 \\ \vec{v}.\vec{b} = 0 \\ \vec{v}.\vec{c} = 0\end{matrix}\right\} \Rightarrow \vec{v}.(\vec{a} + \vec{b} + \vec{c}) = 0$

But $\vec{a} + \vec{b} + \vec{c} \neq 0 \Rightarrow \vec{v} = 0$. i.e. null vector which is true

(C) $\vec{a} \times \vec{b}$ & $\vec{c} \times \vec{d}$ are perpendicular

so $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) \neq 0$. False

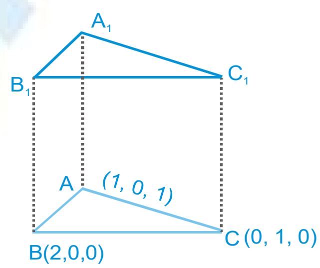
(D) $a' = \frac{\vec{b} \times \vec{c}}{[a\,b\,c]}, b' = \frac{\vec{c} \times \vec{a}}{[a\,b\,c]}, c' = \frac{\vec{a} \times \vec{b}}{[a\,b\,c]}$

is valid only if $\vec{a}, \vec{b}, \vec{c}$ are non coplanar, hence false.

36. Volume of prism = Area of base ABC × height

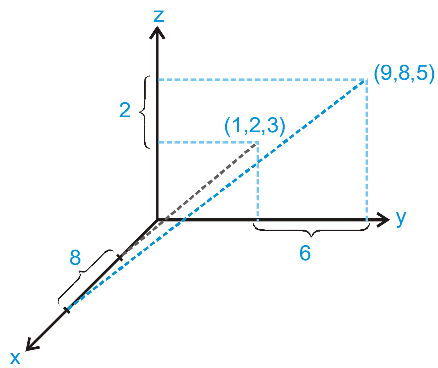
or $3 = \frac{\sqrt{6}}{2} \times h$

$\Rightarrow h = \sqrt{6}$



Required point A_1 should be just above point A i.e. line AA_1 is normal to plane ABC and $AA_1 = \sqrt{6}$

41.



Hence, edge length of the parallelopiped

$|x_2 - x_1| = 8$

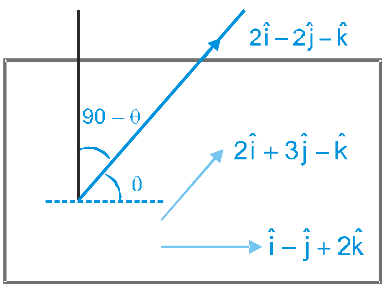
$|y_2 - y_1| = 6$

$|z_2 - z_1| = 2$

42. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(6-1) - \hat{j}(4+1) + \hat{k}(-2-3)$

$= 5\hat{i} - 5\hat{j} - 5\hat{k}$

$\cos(90-\theta) = \left|\dfrac{10+10-5}{5\sqrt{3}.\,3}\right|$



$\sin\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = \cot^{-1}(\sqrt{2})$

43. Equation of bisector of plane

$\dfrac{2x-y+2z+3}{\sqrt{2^2+1^2+2^2}} = \pm\dfrac{3x-2y+6z+8}{\sqrt{9+4+36}}$

$\Rightarrow \dfrac{2x-y+2z+3}{3} = \pm\dfrac{(3x-2y+6z+8)}{7}$

$\Rightarrow 14x-7y+14z+21 = \pm(9x-6y+18z+24)$

$\Rightarrow 5x-y-4z=3$ and

$23x-13y+32z+45=0$

47. Let normal vector n_1 perpendicular to plane determining $\hat{i}, \hat{j}+\hat{k}$ is

$n_1 = \hat{i}\times(\hat{i}+\hat{j}) = \hat{k}$

similarly $n_2 = (\hat{i}-\hat{j})\times(\hat{i}-\hat{k}) = \hat{i}+\hat{j}+\hat{k}$

Now vector parallel to intersection of plane $= \vec{n}_2 \times \vec{n}_1$

$= \vec{k}\times(\hat{i}+\hat{j}+\hat{k}) = -(\hat{j}-\hat{i}) \Rightarrow \dfrac{x-0}{1-0} = \dfrac{y-0}{1-0} = \dfrac{z-0}{1-0}$

Angle between $\lambda(-\hat{j}+\hat{i})$ and $(\hat{i}-2\hat{j}+2\hat{k})$

$\cos\theta = \dfrac{\lambda(-\hat{j}+\hat{i}).(\hat{i}-2\hat{j}+2\hat{k})}{\lambda\sqrt{2}\times 3} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = \dfrac{\pi}{4}$ or $\theta = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

Part # II : Assertion & Reason

2. **Statement-I** Equation of plane is

$(\vec{r}-\vec{a}).(\vec{b}\times\vec{c}) = 0$ ....(1)

$\bar{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ satisfies above equation

Hence True

Statement-II is also true & explain statement I

3. **Statement - I**

$A(\vec{a})$ & $B(\vec{b})$

$\overrightarrow{PA}.\overrightarrow{PB} \le 0$, then locus of P is sphere having diameter $|(\vec{a}-\vec{b})|$

volume $= \dfrac{4}{3}\pi\left|\dfrac{\vec{a}-\vec{b}}{2}\right|^3 = \dfrac{\pi}{6}|\vec{a}-\vec{b}|^2 \cdot |\vec{a}-\vec{b}|$

$= \dfrac{\pi}{6}(\vec{a}^2 + \vec{b}^2 - 2\vec{a}.\vec{b})|\vec{a}-\vec{b}|$

Hence true.

Statement - II : Diameter of sphere subtend acute angle at point P then point P moves out side the sphere having radius r.

5. $[\vec{d}\ \vec{b}\ \vec{c}]\vec{a} + [\vec{d}\ \vec{c}\ \vec{a}]\vec{b} + [\vec{d}\ \vec{a}\ \vec{b}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

$= ([\vec{b}\ \vec{c}\ \vec{d}]\vec{a} - [\vec{b}\ \vec{c}\ \vec{a}]\vec{d}) + ([\vec{d}\ \vec{a}\ \vec{b}]\vec{c} - [\vec{d}\ \vec{a}\ \vec{c}]\vec{b})$

$= (\vec{b}\times\vec{c})\times(\vec{a}\times\vec{d}) + (\vec{d}\times\vec{a})\times(\vec{c}\times\vec{b})$

$= (\vec{d}\times\vec{a})\times(\vec{b}\times\vec{c}) - (\vec{d}\times\vec{a})\times(\vec{b}\times\vec{c}) = \vec{0}$

8. Let the coordinates of A, B, C, D be A(1, 0, 0), B(1, 1, 0), C(0, 1, 0) and D(0, 0, 0)

so that coordinates of A_1, B_1, C_1 are

$A_1(1,0,1)$, $B_1(1,1,1)$, $C_1(0, 1, 1)$ & $D_1(0, 0, 1)$

The coordinates of midpoint of B_1A_1 is

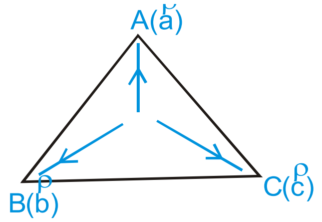
$P\left(1, \dfrac{1}{2}, 1\right)$ and that of B_1C_1 is $Q\left(\dfrac{1}{2}, 1, 1\right)$

Equation of the plane PBQ is $2x + 2y + z = 4$

Its distance from D(0, 0, 0) is $\frac{4}{3}$

So Statement-1 is false and Statement-2 is clearly true.

10.



$$I \equiv \frac{a\vec{a} + b\vec{b} + c\vec{c}}{a+b+c}$$

13. plane P_1 is $\perp$ to $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + \hat{j} + \hat{k}$

and plane P_2 is $\perp$ to $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -2 \\ 2 & -1 & -3 \end{vmatrix} = -2\hat{i} - \hat{j} - \hat{k}$

$\Rightarrow$ $\vec{a} \,||\, \vec{b}$ $\Rightarrow$ P_1 & P_2 are parallel

also L is parallel to $\vec{c} = \hat{i} - \hat{j} - \hat{k}$

also $\vec{a}.\vec{c} = 0$ & $\vec{b}.\vec{c} = 0$

but it is not essential that if P_1 & P_2 are parallel to L then P_1 & P_2 must be parallel.

So Statement-II is not a correct explanation of Statement-I.

14. Statement - I

$\vec{a} = \hat{i}, \vec{b} = \hat{j}$ & $\vec{c} = \hat{i} + \hat{j}$

$\vec{c} = \vec{a} + \vec{b}$ linearly dependent

$\vec{a}$ & $\vec{b}$ are linearly independent

Hence true.

Statement - II :

$\vec{a}$ & $\vec{b}$ are linearly dependent

$\vec{a} = t\vec{b}$

then $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ which is linearly dependent.

EXERCISE - 3

Part # I : Matrix Match Type

1. **(A)** If P is a point inside the triangle such that area($\Delta PAB + \Delta PBC + \Delta PCA$) $=$ area (ΔABC)

Then P is centroid.

(B) $\vec{V} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$

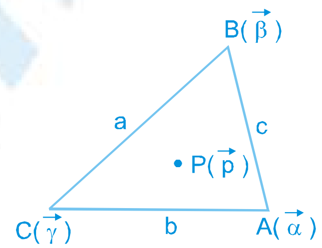
$0 = \vec{a} - \vec{p} + \vec{b} - \vec{p} + \vec{c} - \vec{p}$

$\vec{p} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$ which is centroid.

(C) $\vec{P} = (BC)\overrightarrow{PA} + (CA)\overrightarrow{PB} + (AB)\overrightarrow{PC} = 0$

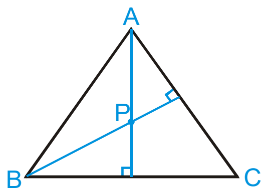
$a(\vec{\alpha} - \vec{p}) + b(\vec{\beta} - \vec{p}) + c(\vec{\gamma} - \vec{p}) = 0$

$\Rightarrow \vec{p} = \frac{a\vec{\alpha} + b\vec{\beta} + c\vec{\gamma}}{a+b+c}$



which is incentre.

(D) From fig.



$\overrightarrow{PA} \cdot \overrightarrow{CB} = 0$

$\overrightarrow{PB}.\overrightarrow{AC} = 0$

$\Rightarrow$ P is orthocentre.

2. **(A)** Vector parallel to line of intersection of the plane is $(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k}) = \hat{k} - \hat{j} + \hat{i}$

equation of line whose dr's, are (1, – 1, 1) and passing through (0, 0, 0) is

$x = -y = z$

(B) Similarly $(\hat{i} \times \hat{j}) = \hat{k}$.

Hence dr's = (0, 0, 1)

and passing through the point (2, 3, 0)

$\therefore$ Equation of line $\frac{x-2}{0} = \frac{y-3}{0} = \frac{z}{1}$

(C) Similarly $\hat{i} \times (\hat{j} + \hat{k}) = \hat{k} - \hat{j}$

dr's = (0, –1, 1)

Equation of line $\frac{x-2}{0} = \frac{y-2007}{-1} = \frac{z+2004}{1}$

because $x = 2$ & $y + z = 3$

so $y = 2007$, $z = -2004$ satisfy above equation

(D) $x = 2, x + y + z = 3$

$y + z = 1$

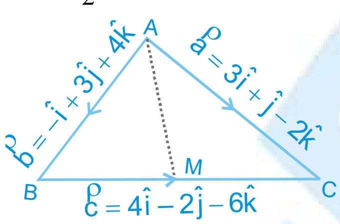
same as part C

we get $\frac{x-2}{0} = \frac{y}{-1} = \frac{z-1}{1}$

3. (A).

here $\vec{a} = \vec{b} + \vec{c}$

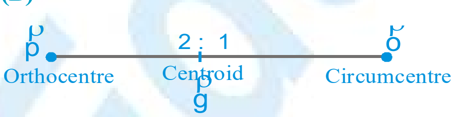
$\overrightarrow{AM} = \frac{1}{2}(\vec{a} + \vec{b})$



$= \frac{1}{2}[2\hat{i} + 4\hat{j} + 2\hat{k}] = \hat{i} + 2\hat{j} + \hat{k}$

$\Rightarrow \lambda = \sqrt{6}$

(B)



(C) Area $= |\vec{a} \times \vec{b}| = |(\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})|$

$= |\vec{p} \times \vec{q} + 4\vec{q} \times \vec{p}| = |3\vec{p} \times \vec{q}| = 3 \times \frac{1}{2} = \frac{3}{2}$

(D) $\vec{u} + \vec{v} + \vec{w} = 0$

$\Rightarrow |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}.\vec{v}) + 2(\vec{v}.\vec{w}) + 2(\vec{w}.\vec{u}) = 0$

$\Rightarrow 9 + 16 + 25 + 2 \quad [\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}] = 0$

$\Rightarrow \sqrt{|\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}|} = 5$

Part # II : Comprehension

Comprehension # 2

1. Equation of the second plane is $-x + 2y - 3z + 5 = 0$

$2(-1) + 3.2 + (-4)(-3) > 0$

$\therefore$ O lies in obtuse angle.

$(2 \times 1 + 3(-2) - 4 \times 3 + 7)(-1 + 2(-2) - 3 \times 3 + 5)$

$= (2 - 6 - 12 + 7)(-1 - 4 - 9 + 5) > 0$

$\therefore$ P lies in obtuse angle.

2. $1 \times 2 + 2 \times 1 - 3 \times 3 < 0$

$\therefore$ O lies in acute angle.

Also

$(2 + 2(-1) - 3(2) + 5)(2 \times 2 - 1 + 3 \times 2 + 1) = (-1)(10) < 0$

$\therefore$ P lies in obtuse angle.

3. $1 - 4 - 9 < 0$

$\therefore$ O lies in acute angle.

Further

$(1 + 4 - 6 + 2)(1 - 4 + 6 + 7) > 0$

$\therefore$ The point P lies in acute angle.

Comprehension # 5

1. We have : $\vec{a}' = \lambda(\vec{b} \times \vec{c})$, $\vec{b}' = \lambda(\vec{c} \times \vec{a})$ and

$\vec{c}' = \lambda(\vec{a} \times \vec{b})$, where $\lambda = \frac{1}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$\vec{b} \times \vec{b}' = \vec{b} \times \lambda(\vec{c} \times \vec{a}) = \lambda\{\vec{b} \times (\vec{c} \times \vec{a})\}$

$= \lambda\{(\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a}\}$

and

$\vec{c} \times \vec{c}' = \vec{c} \times \lambda(\vec{a} \times \vec{b}) = \lambda\{\vec{c} \times (\vec{a} \times \vec{b})\}$

$= \lambda\{(\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}\}$

$\therefore \vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$

$= \lambda\{(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}\} + \lambda\{(\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a}\}$
$+ \lambda\{(\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}\}$

$= \lambda[(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} + (\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a} + (\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}]$

$= \lambda[(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} + (\vec{a}.\vec{b})\vec{c} - (\vec{b}.\vec{c})\vec{a} + (\vec{b}.\vec{c})\vec{a} - (\vec{a}.\vec{c})\vec{b}]$

$= \lambda\vec{0} = \vec{0}$

2. $\vec{a}' \times \vec{b}' = \dfrac{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})}{[\vec{a}\vec{b}\vec{c}]^2} = \dfrac{\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$

$\therefore \quad \vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}' = \dfrac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$

so $\lambda = 1$

3. $(a' \times b') \times (b' \times c') = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]^2}$

$\left[\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]^2}\ \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]^2}\ \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]^2}\right] = \dfrac{[\vec{a}\vec{b}\vec{c}]^2}{[\vec{a}\vec{b}\vec{c}]^6} = [\vec{a}\vec{b}\vec{c}]^{-4}$

$\therefore \quad n = -4$

Comprehension # 6

A (2, 1, 0), B (1, 0, 1)

C (3, 0, 1) and D(0, 0, 2)

1. Equation of plane ABC

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & 1 & -1 \\ 2 & 0 & 0 \end{vmatrix} = 0 \quad \Rightarrow \quad y + z = 1$$

2. Equation of $L = 2\hat{k} + \lambda(\overrightarrow{AB} \times \overrightarrow{AC})$

so $L = 2\hat{k} + \lambda(\hat{j} + \hat{k})$

3. Equation of plane ABC

$y + z - 1 = 0$

distance from (0, 0, 2) is $= \dfrac{2-1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

Comprehension # 7

Vector $\vec{p} = \hat{i} + \hat{j} + \hat{k}$, $\vec{q} = 2\hat{i} + 4\hat{j} - \hat{k}$,

$\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$

1. **(A)** $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} = 13 - 7 - 2 = 4 \neq 0$

Hence non coplanar; so linearly independent

(B) In triangle, let length of sides of triangle are a, b, c then triangle is formed if sum of two sides is greater than the third side. Check yourself.

(C) $(\vec{q} - \vec{r}).\vec{p}$

$= (i + 3j - 4k).(i + j + k) = 1 + 3 - 4 = 0$

Hence true.

2. $((\vec{p}\times\vec{q})\times\vec{r}) = u\vec{p} + v\vec{q} + w\vec{r}$

$(\vec{p}.\vec{r})\vec{q} - (\vec{q}.\vec{r})\vec{p} = u\vec{p} + v\vec{q} + w\vec{r}$

By solving $\vec{p}.\vec{r}$ & $\vec{q}.\vec{r}$, we get

$5\vec{q} - 3\vec{p} + 0\vec{r} = u\vec{p} + v\vec{q} + w\vec{r}$

compare

$u + v + w = 5 - 3 + 0 = 2.$

3. $\vec{s}$ is unit vector

$(\vec{p}.\vec{s})(\vec{q} \times \vec{r}) + (\vec{q}.\vec{s})(\vec{r} \times \vec{p}) + \vec{r}.\vec{s}(\vec{p} \times \vec{q})$

$\vec{q} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} = 13\hat{i} - 7\hat{j} - 2\hat{k}$

$\vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -2\hat{i} + 2\hat{j}$

$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 4 & -1 \end{vmatrix} = -5\hat{i} + 3\hat{j} + 2\hat{k}$

Let $\vec{s} = \hat{i}$

Putting the value we get

$13\hat{i} - 7\hat{j} - 2\hat{k} + 2(-2\hat{i} + 2\hat{j}) + (-5\hat{i} + 3\hat{j} + 2\hat{k})$

$= 13\hat{i} - 7\hat{j} - 2\hat{k} - 4\hat{i} + 4\hat{j} - 5\hat{i} + 3\hat{j} + 2\hat{k}$

$= 4\hat{i} + 0\hat{j} + 0\hat{k} = 4\hat{i}$

Magnitude = 4.

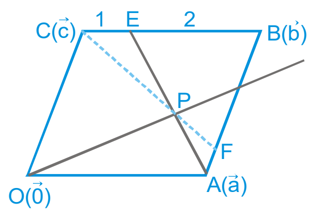
Comprehension # 8

$$E = \frac{2\vec{c} + \vec{b}}{3}$$

equation of OP $\vec{r} = \lambda\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{c}}{|\vec{c}|}\right)$ ...(1)

Let P divide EA in $\mu : 1$

$$P\left[\frac{\mu\vec{a} + \frac{2\vec{c}+\vec{b}}{3}}{\mu+1}\right]$$



P lies on (1)

$$\frac{\mu\vec{a} + \frac{2\vec{c}+\vec{b}}{3}}{\mu+1} = \lambda\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{c}}{|\vec{c}|}\right)$$

$$\vec{a} + \vec{c} = \vec{b}$$

$$\frac{\mu\vec{a} + \frac{3\vec{c}+\vec{a}}{3}}{\mu+1} = \lambda\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{c}}{|\vec{c}|}\right)$$

Comparing coefficient of $\vec{a}$ and $\vec{c}$

$$\frac{\mu + \frac{1}{3}}{\mu = 1} = \frac{\lambda}{|\vec{a}|} \quad ...(2)$$

and $\frac{1}{\mu+1} = \frac{\lambda}{|\vec{c}|}$ ...(3)

divided (2) by (3) $\mu + \frac{1}{3} = \frac{|\vec{c}|}{|\vec{a}|}$

$$\mu = \frac{|\vec{c}|}{|\vec{a}|} - \frac{1}{3}$$

Put in (3) $\frac{1}{\frac{|c|}{|a|} + \frac{2}{3}} = \frac{\lambda}{|\vec{c}|}$

$$\lambda = \frac{3|\vec{a}||\vec{c}|}{c|\vec{c}| + 2|\vec{a}|}$$

So position vector of P

$$\vec{p} = \frac{3|\vec{a}||\vec{c}|}{3|\vec{c}| + 2|\vec{a}|}\left(\frac{a}{|\vec{a}|} + \frac{c}{|\vec{c}|}\right)$$

Now for solution of 4

equation of AB,

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) = \vec{a} + \lambda(\vec{c})$ ...(4)

equation of CP, $\vec{r} = \vec{c} + \mu$

$$\left(\frac{3|c|\vec{a}}{3|\vec{c}| + 2|\vec{a}|} + \frac{3|a|\vec{c}}{3|c| + 2|\vec{a}|} - \vec{c}\right)$$

$$\vec{r} = \vec{c} + \mu\left[\frac{3|c|\vec{a} + 3|a|c - 3|c|c - 2|a|\vec{c}}{3|c| + 2|a|}\right]$$

$$r = \vec{c} + \mu\left[\frac{3|c|\vec{a} + |a|\vec{c} - 3|c|\vec{c}}{3|c| + 2|a|}\right] \quad ..(5)$$

Comparing (4) and (5)

$$\lambda = 1 + \frac{\mu|a| - 3\mu|c|}{3|c| + 2|a|}$$

$$\lambda = \frac{3|\vec{c}| + 2|\vec{a}| + \mu|a| - 3\mu|\vec{c}|}{3|\vec{c}| + 2|\vec{a}|} \quad ...(6)$$

$$\mu = \frac{3|\vec{c}| + 2|\vec{a}|}{3|\vec{c}|}$$

Put value of μ in equation (6)

$$\lambda = 1 + \frac{\mu\,(|\vec{a}| - 3|\vec{c}|)}{3|\vec{c}| + 2|\vec{a}|}$$

$$\lambda = 1 + \frac{|a| - 3|\vec{c}|}{3|c|} = \frac{1|a|}{3|c|}$$

So position vector of F is = $\vec{a} + \frac{1|a|}{3|c|}\vec{c}$

Solution – 5

$$\vec{A}\vec{F} = \text{p.v. of F} - \text{p.v. of A} = \vec{a} + \frac{1}{3}\frac{|a|}{|c|}\vec{c} - \vec{a}$$

$$= \frac{1|a|}{3|c|}\vec{c}$$

EXERCISE - 4

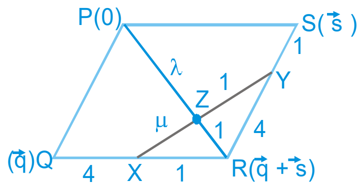
Subjective Type

1. $\overrightarrow{QX} = 4\overrightarrow{XR}$

$\overrightarrow{RY} = 4\overrightarrow{YS}$

Let $\vec{p}$ be origin

& $R(\vec{q}+\vec{s})$



from figure

$$\text{P.V. of } X = \frac{4(\vec{q}+\vec{s})+\vec{q}}{5} = \frac{5\vec{q}+4\vec{s}}{5}$$

$$\text{P.V. of } Y = \frac{4\vec{s}+\vec{q}+\vec{s}}{5} = \frac{5\vec{s}+\vec{q}}{5}$$

Now Let Z divides PR in ratio $\lambda : 1$

Now Let Z divides XY in ratio $\mu : 1$

$$\text{P.V. of } Z = \frac{\lambda(\vec{q}+\vec{s})}{\lambda+1} \quad \text{(from PR)}$$

$$\text{P.V. of } Z = \frac{\dfrac{\mu(5\vec{s}+\vec{q})}{5}+\dfrac{5\vec{q}+4\vec{s}}{5}}{\mu+1} \quad \text{(from XY)}$$

equating both Z then we get

$$\frac{\lambda}{\lambda+1} = \frac{\mu+5}{5(\mu+1)} \quad \text{.......(i)}$$

$$\frac{\lambda}{\lambda+1} = \frac{5\mu+4}{5(\mu+1)} \quad \text{......(ii)}$$

from (i) & (ii), $\mu = \frac{1}{4}$ & $\lambda = \frac{21}{4}$

$$\text{So P.V. of } Z = \frac{\frac{21}{4}}{\frac{21}{4}+1}(\vec{q}+\vec{s})$$

$$= \frac{21}{25}(\vec{q}+\vec{s}) = \frac{21}{25}\overrightarrow{PR}$$

2. PVs of vertex P,Q,R,S are (Let) $\vec{0}, \vec{a}, \vec{b}+\vec{a}, \vec{b}$

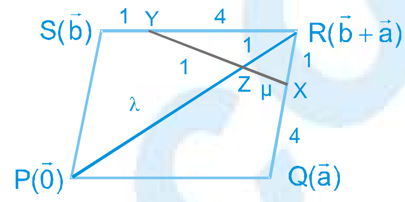
using section rule PVs of

$$X \equiv \frac{4(\vec{b}+\vec{a})+\vec{a}}{5} \quad \text{and} \quad Y \equiv \frac{(\vec{b}+\vec{a})+4\vec{b}}{5}$$

again Let $\frac{PZ}{ZR} = \lambda$ and $\frac{XZ}{YZ} = \mu$

PVs of point Z may be given as

$$\frac{\lambda(\vec{b}+\vec{a})+\vec{0}}{\lambda+1} \;\&\text{ also as }\; \frac{\mu\left(\vec{b}+\dfrac{\vec{a}}{5}\right)+1\left(\vec{a}+\dfrac{4\vec{b}}{5}\right)}{\mu+1}$$



Equating both answers and coefficient of $\vec{a}$ & $\vec{b}$

(they are representing non collinear vectors $\overrightarrow{PQ}$ & $\overrightarrow{PS}$)

$$\frac{\lambda}{\lambda+1} = \frac{\mu+\left(\frac{1}{5}\right)}{\mu+1} \quad \text{and} \quad \frac{\lambda}{\lambda+1} = \frac{\left(\frac{4\mu}{5}\right)+1}{\mu+1}$$

Solving these equations gives $\lambda = \frac{21}{4}$

3. After rotation equation of plane is new position will be

$\bullet x + my + a'z = 0$ ....(1)

Let angle between (1) and $\bullet x + my = 0$

is θ, then

$$\cos\theta = \frac{l^2+m^2}{\sqrt{l^2+m^2}\sqrt{l^2+m^2+a'^2}}$$

Solving we get

$a'^2 = (\bullet^2 + m^2)\tan^2\theta$

$\Rightarrow a' = \pm\sqrt{(l^2+m^2)}\tan\theta$

Equation is $lx + my \pm z\sqrt{(l^2+m^2)}\tan\theta = 0$

4. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r$ (Let) ... (1)

$\Rightarrow$ $(2r+1,\ 3r+2,\ 4r+3)$ represents any point on (1)

$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$ ... (2)

To find point of intersection of (1) and (2)

$$\frac{2r+1-4}{5} = \frac{3r+2-1}{2} = \frac{4r+3}{1}$$

$\Rightarrow \quad \frac{2r-3}{5} = \frac{3r+1}{2} = \frac{4r+3}{1}$

$\Rightarrow \quad 4r - 6 = 15r + 5$

$\Rightarrow \quad 11r = -11 \quad \Rightarrow \quad r = -1$

$\therefore$ point of intersection of **(1)** and **(2)** is $(-1, -1, -1)$

$\vec r = (\hat i + \hat j - \hat k) + \lambda(3\hat i - \hat j)$ ... **(1)**

$\vec r = (4\hat i - \hat k) + \mu(2\hat i + 3\hat k)$ ... **(2)**

For their point of intersection

$3\lambda + 1 = 4 + 2\mu \quad \Rightarrow 3\lambda - 2\mu - 3 = 0$ ... **(3)**

$1 - \lambda = 0 \quad \Rightarrow \lambda = 1$ ... **(4)**

and $-1 = -1 + 3\mu \quad \mu = 0$

$\therefore$ point of intersection is $(4, 0, -1)$

$\therefore$ required distance

$= \sqrt{(4+1)^2 + 1 + 0} = \sqrt{25+1} = \sqrt{26}$

5. $\left|(\vec a.\vec d)(\vec b \times \vec c) + (\vec b.\vec d)(\vec c \times \vec a) + (\vec c.\vec d)(\vec a \times \vec b)\right|$

$\left|(\vec a.\vec d)(\vec b \times \vec c) - (\vec c.\vec d)(\vec b \times \vec a) + (\vec b.\vec d)(\vec c \times \vec a)\right|$

$\left|\vec b \times [(\vec a.\vec d)\vec c - (\vec c.\vec d)\vec a] + (\vec b.\vec d)(\vec c \times \vec a)\right|$

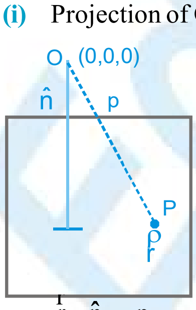
$\left|\vec b \times \{(\vec a \times \vec c) \times \vec d\} + (\vec b.\vec d)(\vec c \times \vec a)\right|$

$= \left|(\vec b.\vec d)(\vec a \times \vec c) - \{\vec b.(\vec a \times \vec c)\}\vec d - (\vec b.\vec d)(\vec a \times \vec c)\right|$

$= \left|[\vec b\ \vec a\ \vec c]\vec d\right| = [\vec b\ \vec a\ \vec c]\ |\vec d| \quad \because \ |\vec d| = 1$

$= \left[\vec b \quad \vec a \quad \vec c\right]$ Proved.

6. **(i)** Projection of OP on $\hat n$



$\vec r \, . \, \hat n = p$

(ii) $\vec r \, . \, \vec a - p + \lambda(\vec r \, . \, \vec b - q) = 0$

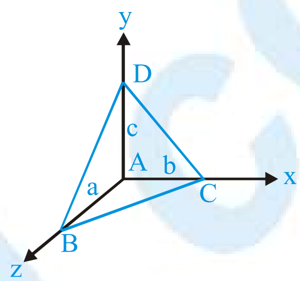
$\vec r = \vec 0$

$\therefore \quad -p - \lambda q = 0 \quad \lambda = -\frac{p}{q}$

$\vec r \, . \, \vec a - p - \frac{p}{q}(\vec r \, . \, \vec b - q) = 0$

$\vec r \, . (\vec a q - p\vec b) = 0$

7.



Area of $\Delta ABC \Rightarrow \frac{1}{2}ab = x$ ...**(i)**

Area of $\Delta ABC \Rightarrow \frac{1}{2}bc = y$ ...**(ii)**

Area of $\Delta ACD \Rightarrow \frac{1}{2}ac = z$ ....**(iii)**

Area of $\Delta BCD = \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}$

$= \frac{1}{2} \times 2\sqrt{x^2 + y^2 + z^2}$

$= \sqrt{x^2 + y^2 + z^2}$

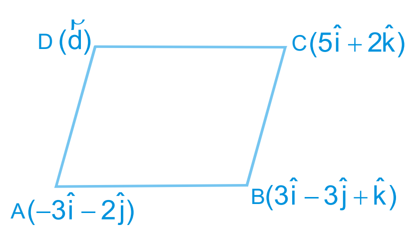
8. **(a)** $(3\hat i - 3\hat j + \hat k + \vec d) \equiv 2\hat i - 2\hat j + 2\hat k$

$\Rightarrow \quad \vec d = -\hat i + \hat j + \hat k$

(b) $\overrightarrow{AB} = 6\hat i - \hat j + \hat k$

$\overrightarrow{AC} = 8\hat i + 2\hat j + 2\hat k$

$\Rightarrow \quad |\overrightarrow{AC}| = \sqrt{64+4+4} = \sqrt{72}$



Required vector is $\frac{\sqrt{72}}{\sqrt{38}}(6\hat{i}-\hat{j}+\hat{k})$

$= \frac{6}{\sqrt{19}}(6\hat{i}-\hat{j}+\hat{k})$

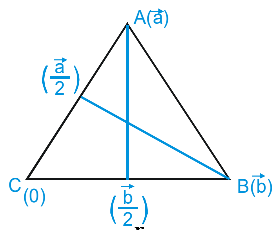
(c) $\overrightarrow{BD} = -4\hat{i}+4\hat{j}$

$\cos\theta = \frac{\overrightarrow{AC}.\overrightarrow{BD}}{|\overrightarrow{AC}||\overrightarrow{BD}|} = \frac{-32+8}{\sqrt{72}\sqrt{32}} = \frac{-24}{6\sqrt{2}\cdot4\sqrt{2}}$

$= -\frac{1}{2}$

$\Rightarrow \theta = \frac{2\pi}{3}$

9. Let origin be C



Given $\left|\vec{a}-\frac{\vec{b}}{2}\right| = \left|\vec{b}-\frac{\vec{a}}{2}\right|$ (medians are equal)

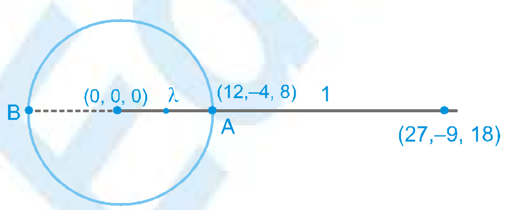
$\Rightarrow \vec{a}^2 + \frac{\vec{b}^2}{4} - \vec{a}.\vec{b} = \vec{b}^2 + \frac{\vec{a}^2}{4} - \vec{a}.\vec{b}$

$\frac{3\vec{a}^2}{4} = \frac{3}{4}\vec{b}^2 \quad \Rightarrow |\vec{a}| = |\vec{b}|$

10. $A\left(\frac{27\lambda+12}{\lambda+1}, \frac{-9\lambda-4}{\lambda+1}, \frac{18\lambda+8}{\lambda+1}\right)$

Which lies on the sphere

$\therefore \left(\frac{27\lambda+12}{\lambda+1}\right)^2 + \left(\frac{-9\lambda-4}{\lambda+1}\right)^2 + \left(\frac{18\lambda+8}{\lambda+1}\right)^2 = 504$



Solving above we get $9\lambda^2 = 4 \quad \lambda = \pm\frac{2}{3}$

11. Let point on line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ .....**(1)**

are $(3+2\lambda, 3+\lambda, \lambda)$

Equation of line which pass through origin is

$\frac{x-0}{3+2\lambda} = \frac{y-0}{3+\lambda} = \frac{z-0}{\lambda}$ .....**(2)**

Angle between **(1)** & **(2)**

$\cos\frac{\pi}{3} = \frac{(3+2\lambda)2+(3+\lambda)1+\lambda\times1}{\sqrt{(3+2\lambda)^2+(3+\lambda)^2+\lambda^2}\sqrt{2^2+1^2+1}}$

Solving we get

$\lambda^2+3\lambda+2=0$

$\Rightarrow \lambda = -1, -2$

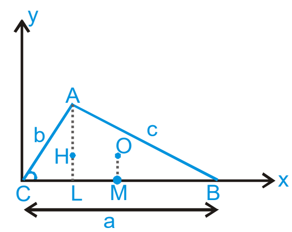
Putting the value of λ in equation **(2)**

$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ or $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$

12. M is mid point of CB, also OM = R cosA

$\Rightarrow$ PV's of circumcentre O is $\equiv \left(\frac{a}{2}\hat{i} + R\cos A\hat{j}\right)$

again CL = bcosC and HL = 2RcosB cosC



$\Rightarrow$ PV's of orthocentre H is

$\equiv (b\cos C\,\hat{i} + 2R\cos B\cos C\,\hat{j})$

Distance between points O & H

$\equiv \left|\left(\frac{a}{2}-b\cos C\right)\hat{i} + \left(R\cos A - 2R\cos B\cos C\right)\hat{j}\right|$

$= \sqrt{(R\sin A - 2R\sin B\cos C)^2 + (R\cos A - 2R\cos B\cos C)^2}$

$= \sqrt{\begin{matrix}\sin^2 A + 4\sin^2 B\cos^2 C - 4\sin A\sin B\cos C + \cos^2 A \\ +4\cos^2 B\cos^2 C - 4\cos A\cos B\cos C\end{matrix}}$

$= R\sqrt{1+4\cos^2 C - 4\cos C(\sin A\sin B + \cos A\cos B)}$

$= R\sqrt{1+4\cos^2 C - 4\cos C\cos(A-B)}$

$= R\sqrt{1+4\cos^2 C + 4\cos(A+B)\cos(A-B)}$

$= R\sqrt{1+4\cos^2 C + 4\cos^2 A - 4\sin^2 B}$

$= R\sqrt{1-8\cos A\ \cos B\ \cos C}$

13. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$[\vec{a}\ \vec{b}\ \vec{c}]$ is written as $\begin{vmatrix} \vec{a}.\hat{i} & \vec{a}.\hat{j} & \vec{a}.\hat{k} \\ \vec{b}.\hat{i} & \vec{b}.\hat{j} & \vec{b}.\hat{k} \\ \vec{c}.\hat{i} & \vec{c}.\hat{j} & \vec{c}.\hat{k} \end{vmatrix}$

Now $\{(n\vec{a}+\vec{b})\times(n\vec{b}+\vec{c})\}.(n\vec{c}+\vec{a})$

$= \{n^2(\vec{a}\times\vec{b}) + n(\vec{a}\times\vec{c}) + \vec{b}\times\vec{c}\}.(n\vec{c}+\vec{a})$

$= n^3[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{c}\ \vec{a}]$

$= (n^3+1)\ [\vec{a}\ \vec{b}\ \vec{c}]$

14. $\vec{w} + (\vec{w}\times\vec{u}) = \vec{v}$ ... **(1)**

Dot **(1)** with $\vec{v}$

$\vec{w}\,.\,\vec{v} + [v\ w\ u] = 1$ ... **(2)**

Dot **(1)** with $\vec{u}$

$\vec{w}.\vec{u} + 0 = \vec{v}.\vec{u}$ ... **(3)**

cross **(1)** with $\vec{u}$

$\vec{u}\times\vec{w} + (\vec{u}.\vec{u})\vec{w} - (\vec{u}.\vec{w})\vec{u} = \vec{u}\times\vec{v}$

Using **(3)** we get

$\vec{u}\times\vec{w} + \vec{w} - (\vec{v}.\vec{u})\vec{u} = \vec{u}\times\vec{v}$

$[v\,u\,w] + (\vec{v}.\vec{w}) - (\vec{v}.\vec{u})^2 = 0$

Using **(2)** we get

$[v\,u\,w] + 1 - [v\ w\ u] - (\vec{u}.\vec{v})^2 = 0$

$2[u\ v\ w] = 1 - (\vec{u}.\vec{v})^2$

$[u\ v\ w]_{max} = \frac{1}{2}$

when $\vec{u}.\vec{v} = 0 \Rightarrow \vec{u}\perp\vec{v}$

15. Angular point OABC are (0, 0, 0), (0, 0, 2), (0, 4, 0) & (6, 0, 0)

Let centre of sphere be (r, r, r)

Equation of plane passing ABC is

$$\frac{x}{6}+\frac{y}{4}+\frac{z}{2}=1$$

$$r = \left|\frac{\frac{r}{6}+\frac{r}{4}+\frac{r}{2}-1}{\sqrt{\frac{1}{6^2}+\frac{1}{4^2}+\frac{1}{2^2}}}\right|$$

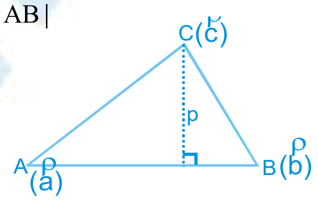
$7r = \pm(11r-12)$

$r = \frac{2}{3}$, $r = 3$ (not satisfied)

16. (a) Let $\perp$ distance of $\vec{c}$ from line joining $\vec{a}$ and $\vec{b}$ is p.

Now $\Delta = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\left|\overrightarrow{AB}\right|\times p$

$\Rightarrow p = \frac{|\overrightarrow{AB}\times\overrightarrow{AC}|}{|\overrightarrow{AB}|}$



$= \frac{|(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})|}{|\vec{b}-\vec{a}|} = \frac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{|\vec{b}-\vec{a}|}$

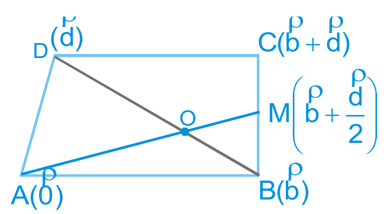
(b) Equation of line AM is

$\vec{r} = \lambda\left(\vec{b}+\frac{\vec{d}}{2}\right)$

Equation of line BD is

$\vec{r} = \vec{b} + \mu(\vec{d}-\vec{b})$

to obtain point of intersection



$\lambda\left(\vec{b}+\frac{\vec{d}}{2}\right) = \vec{b} + \mu(\vec{d}-\vec{b})$

$\Rightarrow \lambda = 1-\mu$ & $\frac{\lambda}{2} = \mu$

$\Rightarrow \quad \lambda = 1 - \frac{\lambda}{2} \qquad$ or $\qquad \lambda = \frac{2}{3}$

hence point O is $\frac{2}{3}\left(\vec{b} + \frac{\vec{d}}{2}\right)$

Area OMCD = Area OMC + Area OCD

$= \frac{1}{2}\left|\frac{1}{3}\left(\vec{b} + \frac{\vec{d}}{2}\right) \times \left(\frac{\vec{b}}{3} + \frac{2\vec{d}}{3}\right)\right| + \frac{1}{2}\left|\left(\frac{\vec{b}}{3} + \frac{2\vec{d}}{3}\right) \times \left(\frac{-2}{3}\vec{b} + \frac{2}{3}\vec{d}\right)\right|$

$= \frac{1}{2}\left|\frac{1}{9}\left(\vec{b} \times 2\vec{d} + \frac{\vec{d}}{2} \times \vec{b}\right)\right| + \frac{1}{2}\left|\frac{1}{9}(\vec{b} \times 2\vec{d} - 4\vec{d} \times \vec{b})\right|$

$= \frac{1}{18}\left|\frac{3}{2}\vec{b} \times \vec{d}\right| + \frac{1}{18}|6\vec{b} \times \vec{d}| = \frac{1}{18} \times \frac{15}{2}|\vec{b} \times \vec{d}|$

$= \frac{15}{18 \times 2} \times 12 = 5$ sq. units

$= \frac{1}{18}\left|\frac{3}{2}\vec{b} \times \vec{d}\right| + \frac{1}{18}|6\vec{b} \times \vec{d}| = \frac{1}{18} \times \frac{15}{2}|\vec{b} \times \vec{d}|$

$= \frac{15}{18 \times 2} \times 12 = 5$ sq. units

17. Let $|\vec{u}| = \lambda$

$\vec{u} = \frac{\lambda}{2}(\hat{i} + \sqrt{3}\,\hat{j})$

Given $\left|\frac{\lambda}{2}(\hat{i} + \sqrt{3}\,\hat{j}) - \hat{i}\right|^2 = \lambda\left|\frac{\lambda}{2}(\hat{i} + \sqrt{3}\,\hat{j}) - 2\hat{i}\right|$

$\left(\left(\frac{\lambda}{2} - 1\right)^2 + \frac{3\lambda^2}{4}\right)^2 = \lambda^2$

$\left(\left(\frac{\lambda - 4}{2}\right)^2 + \frac{3\lambda^2}{4}\right)$

$(4\lambda^2 - 4\lambda + 4)^2 = 16\lambda^2(\lambda^2 - 2\lambda + 4)$

$(\lambda^2 - \lambda + 1)^2 = \lambda^2(\lambda^2 - 2\lambda + 4)$

solving we get $\lambda = \frac{-2 \pm \sqrt{4 + 4}}{2} = -1 \pm \sqrt{2}$

But $\lambda > 0$

$\Rightarrow \quad \lambda = \sqrt{2} - 1$

$\therefore \quad a = 2, \; b = 1$

18. For linearly dependent vectors

$\bullet(i - 2j + 3k) + m(-2i + 3j - 4k) + n(i - j + xk) = 0$

$\bullet - 2m + n = 0, -2\bullet + 3m - n = 0$

$3\bullet - 4m + nx = 0$

$\therefore \quad \begin{vmatrix} 1 & -2 & 1 \\ -2 & 3 & -1 \\ 3 & -4 & x \end{vmatrix} = 0$ is $x = 1$

20. (i) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$\Rightarrow \quad 10\vec{b} - 3\vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$

$p = 0, \; q = +10, \; r = -3$

$[\vec{a}, \vec{b}, \vec{c}$ are non coplanar$]$

(ii) $(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}) \cdot \vec{d}$

$= \{((\vec{a} \times \vec{b}) \cdot \vec{c})\vec{a} - ((\vec{a} \times \vec{b}) \cdot \vec{a})\vec{c}\} \cdot \vec{d}$

$= [\vec{a} \; \vec{b} \; \vec{c}] \; \vec{a} \cdot \vec{d} - 0 = 20 \times (-5) = -100$

21. $\pm\hat{i}$

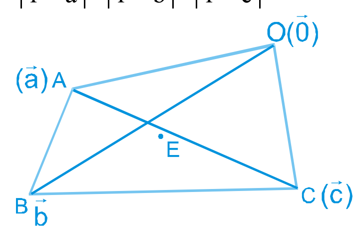
22. vectors $\vec{a}, \vec{b}$ & $\vec{c}$ are non coplanar so are the vectors $\vec{a} \times \vec{b}, \; \vec{b} \times \vec{c}$

Let position vector of circumcentre

$\vec{r} \equiv x(\vec{a} \times \vec{b}) + y(\vec{b} \times \vec{c}) + z(\vec{c} \times \vec{a})$

also $\quad OE = AE = EB = EC$

$\Rightarrow \quad |\vec{r}| = |\vec{r} - \vec{a}| = |\vec{r} - \vec{b}| = |\vec{r} - \vec{c}|$



or $\quad \vec{r}^2 = \vec{r}^2 + \vec{a}^2 - 2\vec{r} \cdot \vec{a}$

$= \vec{r}^2 + \vec{b}^2 - 2\vec{r} \cdot \vec{b} = \vec{r}^2 + \vec{c}^2 - 2\vec{r} \cdot \vec{c}^2$

$\Rightarrow \quad 2\vec{r} \cdot \vec{a} = \vec{a}^2, \quad 2\vec{r} \cdot \vec{b} = \vec{b}^2, \quad 2\vec{r} \cdot \vec{c} = \vec{c}^2$

or $\quad 2y[\vec{a} \; \vec{b} \; \vec{c}] = \vec{a}^2 \quad \Rightarrow \quad y = \frac{\vec{a}^2}{2[\vec{a} \; \vec{b} \; \vec{c}]}$

23. $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$

$\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$

$\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$

$\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are non coplanar

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$\Rightarrow$ $(a - b)(b - c)(c - a) \neq 0$ $\Rightarrow$ $a \neq b \neq c$

If α_1, β_1 & γ_1 are coplanar

$$\text{Then} \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & b_1 & b_1^2 \\ 1 & c_1 & c_1^2 \end{vmatrix} = 0$$

$\Rightarrow$ $a_1 = b_1 = c_1$

$$\text{Given} \begin{vmatrix} (a_1 - a)^2 & (a_1 - b)^2 & (a_1 - c)^2 \\ (b_1 - a)^2 & (b_1 - b)^2 & (b_1 - c)^2 \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c)^2 \end{vmatrix} = 0$$

$\Rightarrow$ $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$, we get

$$(a_1 - b_1)(b_1 - c_1)\begin{vmatrix} a_1 + b_1 - 2a & a_1 + b_1 - 2b & a_1 + b_1 - 2c \\ b_1 + c_1 - 2a & b_1 + c_1 - 2b & b_1 + c_1 - 2c \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c)^2 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow (a_1 - b_1)(b_1 - c_1)\begin{vmatrix} a_1 - c_1 & a_1 - c_1 & a_1 - c_1 \\ b_1 + c_1 - 2a & b_1 + c_1 - 2b & b_1 + c_1 - 2c \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c^2) \end{vmatrix} = 0$$

$\Rightarrow$ $(a_1 - b_1)(b_1 - c_1)(c_1 - a_1)$

$$\begin{vmatrix} 1 & 1 & 1 \\ b_1 + c_1 - 2a & b_1 + c_1 - 2b & b_1 + c_1 - 2c \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c)^2 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$\Rightarrow$ $(a_1 - b_1)(b_1 - c_1)(c_1 - a_1)$

$$\underbrace{\begin{vmatrix} 0 & 0 & 1 \\ 2(b - a) & 2(c - b) & b_1 + c_1 - 2c \\ a^2 - b^2 - 2c_1(a - b) & b^2 - c^2 - 2c_1(b - c) & (c_1 - c)^2 \end{vmatrix}}_{\Delta} = 0$$

$(a_1 - b_1)(b_1 - c_1)(c_1 - a_1)\,\Delta = 0$

$\Rightarrow$ $(a_1 - b_1)(b_1 - c_1)(c_1 - c_1) = 0$ $[\Delta \neq 0]$

$\Rightarrow$ $a_1 = b_1 = c_1$

$\Rightarrow$ $\vec{\alpha}_1, \vec{\beta}_1, \vec{\gamma}_1$ are coplanar

24. $\ell + m + n = 0$ ......**(1)**

$\ell^2 + m^2 = n^2$ ......**(2)**

Put $n = -(\ell + m)$ in (2)

$\ell^2 + m^2 = \ell^2 + m^2 + 2\ell m$

$\Rightarrow$ $\ell m = 0$

(i) if $\ell = 0$; $m \neq 0$ then from (1) $m = -n$

$\therefore$ $\frac{1}{0} = \frac{m}{1} = \frac{n}{-1}$

$\therefore$ direction cosine are : $0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

(ii) if $\ell \neq 0$; $m = 0$, then from (1), $\ell = -n$

$\therefore$ $\frac{1}{1} = \frac{m}{0} = \frac{n}{-1}$

$\therefore$ direction cosine are : $\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}$

Let θ be the angle between the lines

$\therefore$ $\cos\theta = 0 + 0 + \frac{1}{2}$

$\cos\theta = \frac{1}{2}$ $\Rightarrow$ $\theta = \frac{\pi}{3}$

25. $|\vec{r} + b\vec{s}|$ is minimum

Let $f(b) = \sqrt{\vec{r}^2 + b^2\vec{s}^2 + 2\vec{r}.b\vec{s}}$

for maxima & minima

$$f'(b) = \frac{2b\vec{s}^2 + 2\vec{r}.\vec{s}}{\sqrt{\vec{r}^2 + b^2\vec{s}^2 + 2b\vec{r}.\vec{s}}} = 0$$

$$b = -\frac{\vec{r}.\vec{s}}{\vec{s}^2}$$

$$|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2 = b^2\vec{s}^2 + \vec{r}^2 + b^2\vec{s}^2 + 2b\,\vec{r}\cdot\vec{s}$$

$$= 2b^2\vec{s}^2 + \vec{r}^2 - 2b^2\vec{s}^2 = |\vec{r}|^2$$

26. Angle between two vectors

$$= \frac{1\times1+(-1)(1)+(1)(-1)}{\sqrt{3}\,\sqrt{3}} = -\frac{1}{3}$$

Hence obtuse angle between them.

Vector along acute angle bisector

$$=\lambda\left[\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}} - \frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}\right]$$

$$\frac{2\lambda}{\sqrt{3}}\left[-\hat{j}+\hat{k}\right] = t(\hat{j}-\hat{k})$$

hence equation of acute angle bisector

$$= (\hat{i}+2\hat{j}+3\hat{k}) + t(\hat{j}-\hat{k})$$

27. Line : $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$

Plane : $x - y + z + 2 = 0$

The vector perpendicular to required plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 1 & -1 & 2 \end{vmatrix} = 2\hat{i} + 3\hat{j} + \hat{k}$$

Now equation of plane passing through $(1, -2, 0)$

and perpendicular to $2\hat{i} + 3\hat{j} + \hat{k}$

$$(x-1)\,2 + (y+2)\,3 + (z-0)\,1 = 0$$

$\Rightarrow$ $2x + 3y + z + 4 = 0$

28. $L_1 : \frac{x}{0} = \frac{y}{b} = \frac{z-c}{-c} = r$

$L_2 : \frac{x}{a} = \frac{y}{0} = \frac{z+c}{c} = \bullet$

Dr's of AB are $\quad -a\bullet,\ br,\ -cr - c\bullet + 2c$

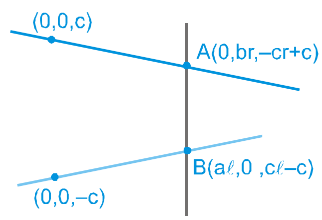
AB is perpendicular to both the lines

$\therefore$ $0(-a\bullet) + b.\,br + (-c)\,(-cr - c\bullet + 2c) = 0$

$(b^2 + c^2)\,r + c^2\bullet = 2c^2$ .....**(1)**

and $a(-a\bullet) + 0(br) + c\,(-cr - c\bullet + 2c) = 0$

$-(a^2 + c^2)\bullet - c^2 r + 2c^2 = 0$



$(a^2 + c^2)\bullet + c^2 r = 2c^2$ .....**(2)**

from (1) & (2)

$$l = \frac{2b^2c^2}{a^2b^2 + b^2c^2 + c^2a^2}, \quad r = \frac{2a^2c^2}{a^2b^2 + b^2c^2 + c^2a^2}$$

$$A\left(0, \frac{2a^2bc^2}{a^2b^2+b^2c^2+c^2a^2}, c\left(\frac{a^2b^2+b^2c^2-c^2a^2}{a^2b^2+b^2c^2+c^2a^2}\right)\right)$$

$$B\left(\frac{2ab^2c^2}{a^2b^2+b^2c^2+c^2a^2}, 0, c\left(\frac{b^2c^2-a^2b^2-c^2a^2}{a^2b^2+b^2c^2+c^2a^2}\right)\right)$$

$$4d^2 = \frac{4a^2b^4c^4}{(a^2b^2+b^2c^2+c^2a^2)^2} + \frac{4a^4b^2c^4}{(a^2b^2+b^2c^2+c^2a^2)^2} + \frac{4c^2(a^4b^4)}{(a^2b^2+b^2c^2+c^2a^2)^2}$$

$$\frac{1}{d^2} = \frac{(a^2b^2+b^2c^2+c^2a^2)^2}{a^2b^4c^4 + a^4b^2c^4 + a^4b^4c^2} = \frac{a^2b^2+b^2c^2+c^2a^2}{a^2b^2c^2}$$

$$\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

29. Given $\overrightarrow{OP}_{n-1} + \overrightarrow{OP}_{n+1} = \frac{3}{2}\overrightarrow{OP}_n \quad n = 2,3$

(a) Let P_1 & P_2 be $\left(t_1, \frac{1}{t_1}\right)$ & $\left(t_2, \frac{1}{t_2}\right)$

for $\quad n = 2$

$$\overrightarrow{OP}_1 + \overrightarrow{OP}_3 = \frac{3}{2}\overrightarrow{OP}_2$$

$$\Rightarrow \overrightarrow{OP}_3 = \frac{3}{2}\,(t_2\hat{i} + \frac{1}{t_2}\hat{j}) - t_1 i - \frac{1}{t_1}\hat{j}$$

or $\quad \overrightarrow{OP}_3 = \left(\frac{3}{2}t_2 - t_1\right)\hat{i} + \left(\frac{3}{2t_2} - \frac{1}{t_1}\right)\hat{j}$

Point $\quad P_3 = \left(\frac{3t_2 - 2t_1}{2}, \frac{3t_1 - 2t_2}{2t_1t_2}\right)$

which does not lie on $xy = 1$

(b) Let P_1 & P_3 on circle $x^2 + y^2 = 1$ are $(\cos\alpha, \sin\alpha)$, $(\cos\beta, \sin\beta)$

For $n = 2$, $\overrightarrow{OP_1} + \overrightarrow{OP_3} = \frac{3}{2}\overrightarrow{OP_2}$

$$\overrightarrow{OP_2} = \frac{2}{3}\{(\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}) + (\cos\beta\,\hat{i} + \sin\beta\,\hat{j})\}$$

$$\overrightarrow{OP_2} = \frac{2}{3}\{(\cos\alpha + \cos\beta)\hat{i} + (\sin\alpha + \sin\beta)\hat{j}\}$$

As P_2 lies on the circle then

$$|\overrightarrow{OP_2}| = 1$$

$$\frac{4}{9}\{(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2\} = 1$$

$$2 + 2\cos(\alpha - \beta) = \frac{9}{4}$$

$$\Rightarrow \cos(\alpha - \beta) = \frac{1}{8}$$

$$\overrightarrow{OP_4} = \frac{3}{2}\overrightarrow{OP_3} - \frac{2}{3}\left(\overrightarrow{OP_1} + \overrightarrow{OP_3}\right)$$

$$= \frac{5}{6}\overrightarrow{OP_3} - \frac{2}{3}\overrightarrow{OP_1}$$

$$= \left(\frac{5}{6}\cos\alpha - \frac{2}{3}\cos\beta\right)\hat{i} + \left(\frac{5}{6}\sin\alpha - \frac{2}{3}\sin\beta\right)\hat{j}$$

$$|\overrightarrow{OP_4}|^2 = \frac{25}{36} + \frac{4}{9} - 2.\frac{5}{6}.\frac{2}{3}\cos(\alpha - \beta) = 1$$

$\Rightarrow P_4$ lies on $x^2 + y^2 = 1$

30. $3\hat{i} + 3\hat{k}$

31. a **(i)** $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$ $\overrightarrow{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$\overrightarrow{AD} = 2\hat{i} + 2\hat{j}$

$$V = \frac{1}{6}\begin{vmatrix} 3 & -1 & -1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = 6 \text{ cubic unit}$$

a **(ii)** Equation of line AB is

$$\vec{r} = \hat{j} + 2\hat{k} + \lambda\ (3\hat{i} - \hat{j} - \hat{k})$$

Equation of Line CD is

$$\vec{r} = 4\hat{i} + 3\hat{j} + 6\hat{k} + \mu(-2\hat{i} - 4\hat{k})$$

$$\text{Shortest distance} = \frac{(a_2 - a_1).(b_1 \times b_2)}{|b_1 \times b_2|}$$

$$= \frac{[(4\hat{i} + 3\hat{j} + 6\hat{k}) - (\hat{j} + 2\hat{k})].[(3\hat{i} - \hat{j} - \hat{k}) \times (-2\hat{i} - 4\hat{k})]}{|(3\hat{i} - \hat{j} - \hat{k}) \times (2\hat{i} - 4\hat{k})|}$$

$$= \frac{[4\hat{i} + 2\hat{j} + 4\hat{k}]\cdot[4\hat{i} + 14\hat{j} - 2\hat{k}]}{|4\hat{i} + 14\hat{j} - 2\hat{k}|}$$

$$= \frac{16 + 28 - 8}{\sqrt{16 + 196 + 4}} = \frac{36}{\sqrt{216}} = \frac{26}{2\sqrt{54}} = \frac{18}{3\sqrt{6}} = \sqrt{6}$$

(b) $\overrightarrow{AD} = -2\hat{i} + 2\hat{j} - \hat{k}$, $\overrightarrow{AC} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ vector perpendicular to the face ADC is

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & -1 \\ 1 & 2 & 2 \end{vmatrix} = 6\hat{i} + 3\hat{j} - 6\hat{k}$$

$$\overrightarrow{AB} = 3\hat{j} + 4\hat{k}$$

$\therefore$ A vector perpendicular to the face ABC is

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 4 \\ 1 & 2 & 2 \end{vmatrix} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

$\therefore$ acute angle between the two faces is given by

$$\cos\theta = \left|\frac{-12 + 12 + 18}{\sqrt{36 + 9 + 36}\sqrt{4 + 16 + 9}}\right| = \frac{2}{\sqrt{29}}$$

$\therefore \tan\theta = \frac{5}{2}$ $\therefore \theta = \tan^{-1}\frac{5}{2}$

32. $\overrightarrow{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$

after rotation of $\overrightarrow{OP}$, let new vector is $\overrightarrow{OP}'$

Now $\overrightarrow{OP}, \hat{i}, \overrightarrow{OP}'$ will be coplanar

So $$\overrightarrow{OP}' = |\overrightarrow{OP}|\frac{(\overrightarrow{OP} \times \hat{i}) \times \overrightarrow{OP}}{|(\overrightarrow{OP} \times \hat{i}) \times \overrightarrow{OP}|} \quad [\because |\overrightarrow{OP}| = |\overrightarrow{OP}'|]$$

But $(\overrightarrow{OP} \times \hat{i}) \times \overrightarrow{OP} = 8\hat{i} - 2\hat{j} - 2\hat{k}$

$$\Rightarrow \overrightarrow{OP}' = \frac{3(8\hat{i} - 2\hat{j} - 2\hat{k})}{2 \times 3\sqrt{2}}$$

or $$\overrightarrow{OP}' = \frac{4}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$$

33. $\vec{a}\times\vec{b}-\vec{c}\times\vec{b}+\vec{c}\times\vec{a}-\vec{c}\times\vec{c}$

$(\vec{a}-\vec{c})\times\vec{b}+\vec{c}\times(\vec{a}-\vec{c})=0$

$(\vec{a}-\vec{c})\times(\vec{b}-\vec{c})=0$

$\overrightarrow{CA}\times\overrightarrow{CB}=0$ $\quad\therefore\quad$ $\overrightarrow{BC}$ is || to $\overrightarrow{AC}$

$\overrightarrow{BC}=\pm 14\left(\dfrac{2\hat{i}-3\hat{j}+6\hat{k}}{7}\right)=\pm(4\hat{i}-6\hat{j}+12\hat{k})$

34. O (0,0), A(1,0) & B (–1, 0)

Let P (x,y)

$\overrightarrow{PA}=(1-x)\hat{i}-y\hat{j}$

$\overrightarrow{PB}=-(1+x)\hat{i}-y\hat{j}$

$\overrightarrow{PA}\cdot\overrightarrow{PB}+3\,\overrightarrow{OA}.\overrightarrow{OB}=0$

$\Rightarrow$ $(x^2-1)+y^2-3=0$

$x^2+y^2=4$ ... **(1)**

$|\overrightarrow{PA}||\overrightarrow{PB}|=\sqrt{(x-1)^2+y^2}\ \sqrt{(x+1)^2+y^2}$

$=\sqrt{5-2x}\cdot\sqrt{5+2x}$

$=\sqrt{25-4x^2}$, $x\in(-2,2)$ (from **(1)**)

so M = 5, m = 3

$\Rightarrow$ $M^2+m^2=25+9=34$

35. Let the plane is

$(2x+3y-z)+1+\lambda(x+y-2z+3)=0$ ....**(1)**

$(2+\lambda)x+(3+\lambda)y-(1+2\lambda)z+1+3\lambda=0$

$3(2+\lambda)-(3+\lambda)+2(1+2\lambda)=0$

$6\lambda+5=0 \Rightarrow \lambda=-5/6$

Putting value of λ in **(1)**

$7x+13y+4z-9=0$

Now image of (1, 1, 1) in plane π is

$\dfrac{x-1}{7}=\dfrac{y-1}{13}=\dfrac{z-1}{4}=-2\left(\dfrac{7+13+4-9}{49+169+16}\right)$

$\Rightarrow\ \dfrac{x-1}{7}=\dfrac{y-1}{13}=\dfrac{z-1}{4}=-\dfrac{15}{117}$

$x=\dfrac{12}{117},\ y=\dfrac{-78}{117},\ z=\dfrac{57}{117}$

36. $\lambda=-2\pm\sqrt{29}$

37. Equation of plane passing through (1, 1, 1) is

$a(x-1)+b(y-1)+c(z-1)=0$ ... **(1)**

$\because$ it passes through (1, – 1, 1) and (– 7, – 3, – 5)

$\therefore$ $a.0-2.b+0.c=0 \Rightarrow b=0$

and $-8a-4b-6c=0$

$4a+2b+3c=0 \Rightarrow b=0$

$\therefore$ $4a+3c=0 \Rightarrow c=-\dfrac{4a}{3}$

$\therefore$ dr's of normal to the plane are $1, 0-\dfrac{4}{3}$

and dr's of the normal to the x-z plane are 0, 1, 0

$\therefore$ $\cos\theta=\left|\dfrac{0+0+0}{\sqrt{\Sigma a^2}\sqrt{\Sigma a_1^2}}\right|=0$ $\quad\therefore\quad \theta=\dfrac{\pi}{2}$

38. $\vec{x}\times\vec{a}+(\vec{x}.\vec{b})\vec{a}=\vec{c}$ ....**(i)**

taking cross product with $\vec{b}$:

$(\vec{x}\times\vec{a})\times\vec{b}+(\vec{x}.\vec{b})(\vec{a}\times\vec{b})=\vec{c}\times\vec{b}$

$(\vec{x}.\vec{b})\vec{a}-(\vec{a}.\vec{b})\vec{x}+(\vec{x}.\vec{b})(\vec{a}\times\vec{b})=\vec{c}\times\vec{b}$ ....**(ii)**

Now taking dot product with $\vec{a}$ in **(i)**

$(\vec{x}.\vec{b})a^2=\vec{a}.\vec{c}$

$\vec{x}.\vec{b}=\dfrac{\vec{a}.\vec{c}}{a^2}$

$\dfrac{(\vec{a}.\vec{c})}{a^2}\vec{a}-(\vec{a}.\vec{b})\vec{x}+\dfrac{\vec{a}.\vec{c}}{a^2}(\vec{a}\times\vec{b})=\vec{c}\times\vec{b}$

$\dfrac{1}{(\vec{a}.\vec{b})}\left[\dfrac{(\vec{a}.\vec{c})}{a^2}\vec{a}+\dfrac{\vec{a}.\vec{c}}{a^2}\left(\vec{a}\times\vec{b}\right)-\vec{c}\times\vec{b}\right]=\vec{x}$

$\vec{x}=\dfrac{1}{(\vec{a}.\vec{b})}\left[\dfrac{\vec{a}.\vec{c}}{a^2}\left(\vec{a}-\vec{b}\times\vec{a}\right)+\vec{b}\times\vec{c}\right]$

39. $SD=\dfrac{(\hat{i}-\hat{j}+2\hat{k}-4\hat{i}+\hat{j})\cdot[(\hat{i}+2\hat{j}-3\hat{k})\times(2\hat{i}+4\hat{j}-5\hat{k})]}{|(\hat{i}+2\hat{j}-3\hat{k})\times(2\hat{i}+4\hat{j}-5\hat{k})|}$

$=\left|\dfrac{(-3\hat{i}+2\hat{k}).(2\hat{i}-\hat{j})}{|2\hat{i}-\hat{j}|}\right|=\dfrac{6}{\sqrt{5}}$

40. $x = \dfrac{\dfrac{\vec{a}\times\vec{b}}{\gamma} - \vec{a}\times\dfrac{\vec{a}\times\vec{b}}{\gamma}}{\left(\dfrac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$;

$y = \dfrac{\vec{a}\times\vec{b}}{\gamma}$; $\quad z = \dfrac{\dfrac{\vec{a}\times\vec{b}}{\gamma} + \vec{b}\times\dfrac{\vec{a}\times\vec{b}}{\gamma}}{\left(\dfrac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$

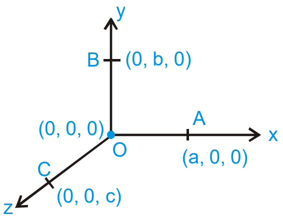
41. Let the required point be $P(\alpha, \beta, \gamma)$

$OP = PA = PB = PC$

$\therefore \quad OP^2 = PA^2 = PB^2 = PC^2$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha - a)^2 + \beta^2 + \gamma^2 = \alpha^2 + (\beta - b)^2 +$

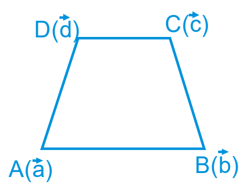
$y^2 = \alpha^2 + \beta^2 + (\gamma - c)^2$



$\therefore \quad \alpha = \dfrac{a}{2}$; $\beta = \dfrac{b}{2}$; $\gamma = \dfrac{c}{2}$

$\therefore$ required point is $\left(\dfrac{a}{2}, \dfrac{b}{2}, \dfrac{c}{2}\right)$

42.



In cyclic quadrilateral

tanA + tan C = 0

$\Rightarrow \dfrac{|\overrightarrow{AB}\times\overrightarrow{AD}|}{\overrightarrow{AB}.\overrightarrow{AD}} + \dfrac{|\overrightarrow{CB}\times\overrightarrow{CD}|}{\overrightarrow{CB}.\overrightarrow{CD}} = 0$

$\Rightarrow \dfrac{|(\vec{b}-\vec{a})\times(\vec{d}-\vec{a})|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})} + \dfrac{|(\vec{b}-\vec{c})\times(\vec{d}-\vec{c})|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})} = 0$

$\Rightarrow \dfrac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})} + \dfrac{|\vec{b}\times\vec{c}+\vec{c}\times\vec{d}+\vec{d}\times\vec{b}|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})} = 0$

43. $\therefore \quad 3.1 - 2.4 + 5\times 1 = 0$, line is parallel to the plane

$\therefore$ reflection of line will also have same direction ratios

i.e. 3, 4, 5

Also mirror image of (1, 2, 3) will be on required line.

$\dfrac{x-1}{1} = \dfrac{y-2}{-2} = \dfrac{z-3}{1} = -2\left(\dfrac{1-4+3-6}{1^2+1^2+(-2)^2}\right)$

$(x, y, z) = (3, -2, 5)$

$\therefore$ equation of straight line $\dfrac{x-3}{3} = \dfrac{y+2}{4} = \dfrac{z-5}{5}$

44. Planes are $x - 2y + z = 1$ ....(i)

$x + 2y - 2z = 5$ ....(ii)

$2x + 2y + z = -6$ ....(iii)

Add (i) + (ii) + (iii)

$4x + 2y = 0 \quad \Rightarrow \quad y = -2x$ ....(iv)

From equations (iii) – (i)

$x + 4y = -7$ ....(v)

from (iv) and (v) we get

$x = 1, y = -2$

Put in (i) we get $z = -4$

So point of intersection is $(1, -2, -4)$

45. $2r + 1 - (3r + 2) + 2(4r + 3) + 2 = 0$

$7r + 7 = 0 \quad \Rightarrow \quad r = -1$

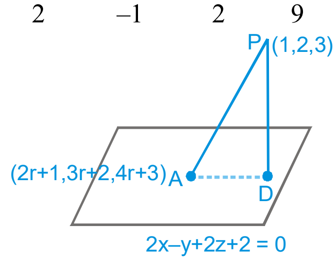
$\therefore \quad A(-1, -1, -1)$

required line will be projection of given line in the plane

foot of $\perp$ of P will be on D

$\dfrac{x-1}{2} = \dfrac{y-2}{-1} = \dfrac{z-3}{2} = -\left(\dfrac{2.1-2+2.3+2}{2^2+(-1)^2+2^2}\right)$

$\dfrac{x-1}{2} = \dfrac{y-2}{-1} = \dfrac{z-3}{2} = \dfrac{-8}{9}$



$x = \dfrac{-7}{9}$; $y = \dfrac{26}{9}$; $z = \dfrac{11}{9}$

$\dfrac{x+1}{2/9} = \dfrac{y+1}{35/9} = \dfrac{z+1}{20/9}$

$\dfrac{x+1}{2} = \dfrac{y+1}{35} = \dfrac{z+1}{20}$

46. $\vec x + \vec c \times \vec y = \vec a$ ......**(i)**

$\vec y + \vec c \times \vec x = \vec b$ .....**(ii)**

$\Rightarrow$ $\vec y = \vec b - \vec c \times \vec x$ put in **(i)**

$\vec x + \vec c \times \vec b - \vec c \times (\vec c \times \vec x) = \vec a$

$\vec x - (\vec c.\vec x)\,\vec c + (\vec c.\vec c)\vec x = \vec a - \vec c \times \vec b$

$(1 + c^2)\,\vec x = \vec a - \vec c \times \vec b + (\vec c.\vec x)\,\vec c$ ......**(iii)**

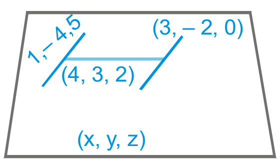
Taking both side dot product with $\vec c$ in equation **(i)**

We get $\vec x.\vec c = \vec a.\vec c$, (put in **(iii)**)

$$\vec x = \frac{\vec a + (\vec a.\vec c)\vec c + \vec b \times \vec c}{1 + \vec c^2}$$

Putting in **(ii)**, we get $\vec y = \dfrac{\vec b + (\vec c.\vec b)\vec c + \vec a \times \vec c}{1 + (\vec c)^2}$

47. $\dfrac{x-4}{1} = \dfrac{y-3}{-4} = \dfrac{z-2}{5}$ .... **(1)**



$\dfrac{x-3}{1} = \dfrac{y+2}{1/\lambda} = \dfrac{z-0}{1/\mu}$ .... **(2)**

Equation of the plane is

$$\begin{vmatrix} x-3 & y+2 & z \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

$(x-3)(25+8) - (y+2)(5-2) + z(-4-5) = 0$

$33x - 99 - 3y - 6 - 9z = 0$

$33x - 3y - 9z - 105 = 0$

$11x - y - 3z = 35$

48. $\vec a = \sqrt{3}\,\hat i - \hat j$, $\vec b = \dfrac{1}{2}\hat i + \dfrac{\sqrt{3}}{2}\hat j$

$\Rightarrow$ $\vec a.\vec b = 0$

$\vec x.\vec y = 0$ (given)

$(\vec a + (q^2 - 3)\vec b).(-p\vec a + q\vec b) = 0$

$\Rightarrow$ $p = \dfrac{q(q^2-3)}{4} = f(q)$

for monotonocity

$p' = 3q^2 - 3$

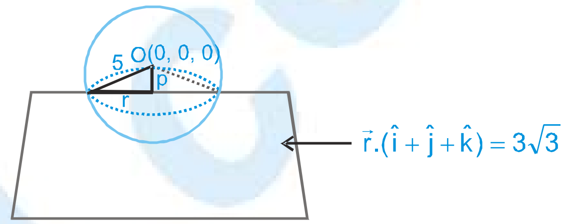
if $p' < 0$ then $f(q)$ is decreasing

$\Rightarrow$ $(q-1)(q+1) < 0$

$\Rightarrow$ $-1 < q < 1$

Decreasing for $q \in (-1, 1)$, $q \neq 0$

49.



$x + y + z - 3\sqrt{3} = 0$

$p = \left|\dfrac{-3\sqrt{3}}{\sqrt{3}}\right| = 3$

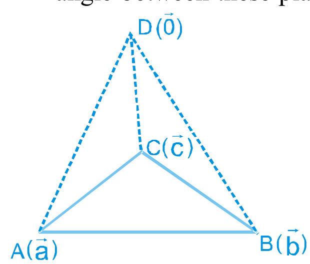
$\Rightarrow$ $r = 4$

50. **(a)** Since tetrahedron is regular AB = BC = AC = DC and angle between two adjcant side = $\pi/3$ consider planes ABD and DBC

vector, normal to plane ABD is = $\vec a \times \vec b$

vector, normal to plane DBC is = $\vec b \times \vec c$

angle between these planes is angle between



vectors $(\vec a \times \vec b)$ & $(\vec b \times \vec c)$

$$\Rightarrow \cos\theta = \frac{(\vec a \times \vec b).(\vec b \times \vec c)}{|\vec a \times \vec b||\vec b \times \vec c|} = \frac{-\frac{1}{4}|\vec b|^2|\vec a||\vec c|}{\frac{3}{4}|\vec a||\vec b|^2|\vec c|} = -\frac{1}{3}$$

Since acute angle is required $\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

(b) circum-radius ≡ distance of circum centre from any of the vertex

$\equiv$ distance of $\dfrac{\vec{a}+\vec{b}+\vec{c}}{4}$ from vertex D $(\vec{0})$ [tetrahedron is regular]

Circumradius

$$=\frac{1}{4}\left|\vec{a}+\vec{b}+\vec{c}\right|=\frac{1}{4}\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})}$$

$$=\frac{1}{4}\sqrt{k^2+k^2+k^2+2\left(\frac{k^2}{2}+\frac{k^2}{2}+\frac{k^2}{2}\right)}$$

$$=\frac{1}{4}\sqrt{6k^2}=\sqrt{\frac{3}{8}}\,k$$

$$\frac{r}{R}=\frac{1}{3}\quad\Rightarrow\quad r=\frac{R}{3}=\frac{k}{\sqrt{24}}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

6. We have,

$\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$

$\Rightarrow$ $\hat{n}\perp\vec{u}$ and $\hat{n}\perp\vec{v}$

$\Rightarrow$ $\hat{n}=\pm\dfrac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|}$

Now, $\vec{u}\times\vec{v}=(\hat{i}+\hat{j})\times(\hat{i}-\hat{j})=-2\hat{k}$

$\therefore$ $\hat{n}=\pm\hat{k}$

Hence, $|\vec{w}.\hat{n}|=|(\hat{i}+2\hat{j}+3\hat{k}).(\pm\hat{k})|=3$

7. We have,

$\vec{F}$ = Total force = $7\hat{i}+2\hat{j}-4\hat{k}$

$\vec{d}$ = Displacement vector = $4\hat{i}+2\hat{j}-2\hat{k}$

$\Rightarrow$ Work done = $\vec{F}.\vec{d}=(28+4+8)$ units

= 40 units

8. Let D be the mid-point of BC. Then,

$\overrightarrow{AD}=\dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$

$\Rightarrow$ $|\overrightarrow{AD}|=4\hat{i}+\hat{j}+4\hat{k}$

$\Rightarrow$ $|\overrightarrow{AD}|=\sqrt{16+1+16}=\sqrt{33}$

Hence, required length = $\sqrt{33}$ units.

9. We have,

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow$ $|\vec{a}+\vec{b}+\vec{c}|=\vec{0}$ $\Rightarrow$ $|\vec{a}+\vec{b}+\vec{c}|^2=0$

$\Rightarrow$ $|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow$ $1+4+9+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow$ $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-7$

11. We have,

$(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w})$

$=\vec{u}.(\vec{u}\times\vec{v})-\vec{u}.(\vec{u}\times\vec{w})\times\vec{u}+(\vec{v}\times\vec{w})$

$+\vec{v}.(\vec{u}\times\vec{v})-\vec{v}.(\vec{u}\times\vec{w})+\vec{v}.(\vec{v}\times\vec{w})$

$-\vec{w}.(\vec{u}\times\vec{v})+\vec{w}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{v}\times\vec{w})$

$= \vec{u}.(\vec{v}\times\vec{w}) - \vec{v}.(\vec{u}\times\vec{w}) - \vec{w}.(\vec{u}\times\vec{v})$

$= [\vec{u}\,\vec{v}\,\vec{w}] - [\vec{v}\,\vec{u}\,\vec{w}] - [\vec{w}\,\vec{u}\,\vec{v}]$

$= [\vec{u}\,\vec{v}\,\vec{w}] + [\vec{u}\,\vec{v}\,\vec{w}] - [\vec{u}\,\vec{v}\,\vec{w}]$

$= [\vec{u}\,\vec{v}\,\vec{w}] = \vec{u}.(\vec{v}\times\vec{w})$

12. It is given that

$\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$

$\Rightarrow$ $\vec{a}+2\vec{b}=\lambda\vec{c}$ and $\vec{b}+3\vec{c}=\mu\vec{a}$ for some scalar λ and μ.

$\Rightarrow$ $\vec{b}+3\vec{c}=\mu(\lambda\vec{c}-2\vec{b})$

$\Rightarrow$ $(2\mu+1)\vec{b}+(3-\mu\lambda)\vec{c}=\vec{0}$

$\Rightarrow$ $2\mu+1=0$ and $3-\mu\lambda=0$

$\Rightarrow$ $\mu=-\frac{1}{2}$, $\lambda=-6$ $\left[\begin{matrix}\because \vec{b} \text{ and } \vec{c} \\ \text{are non-collinear}\end{matrix}\right]$

$\therefore$ $\vec{a}+2\vec{b}=\lambda\vec{c}$

$\Rightarrow$ $\vec{a}+2\vec{b}=-6\vec{c}$ $\Rightarrow$ $\vec{a}+2\vec{b}+6\vec{c}=\vec{0}$

14. Let $\vec{\alpha}=\vec{a}+2\vec{b}+3\vec{c}$, $\vec{\beta}=\lambda\vec{b}+4\vec{c}$ and $\vec{\gamma}=(2\lambda-1)\vec{c}$.

Then, $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = \begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & (2\lambda-1)\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow$ $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=\lambda(2\lambda-1)\,[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow$ $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=0$, if $\lambda=0, \frac{1}{2}$ $\quad [\because [\vec{a}\,\vec{b}\,\vec{c}]\neq 0]$

Hence, $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are non-coplanar for all values of λ except two values 0 and $\frac{1}{2}$.

16. $(a\times b)\times c = 1/3|b|\,|c|\,a$

$\Rightarrow$ $(a.c)b-(b.c)a = 1/3|b||c|\,a$

$\Rightarrow$ $(a.c)\,b = \left\{(b.c)+\frac{1}{3}|b||c|\right\}a$

$\Rightarrow$ $(a.c)b = |b|\,|c|\left\{\cos\theta+\frac{1}{3}\right\}a$

As a and b are not parallel, a.c=0 and $\cos\theta+\frac{1}{3}=0$

$\Rightarrow$ $\cos\theta=-\frac{1}{3}$. Hence $\sin\theta=\frac{2\sqrt{2}}{3}$

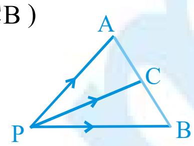
17. $\overrightarrow{PA}+\overrightarrow{PB}=(\overrightarrow{PA}+\overrightarrow{AC})+(\overrightarrow{PB}+\overrightarrow{BC})-(\overrightarrow{AC}+\overrightarrow{BC})$

$=\overrightarrow{PC}+\overrightarrow{PC}-(\overrightarrow{AC}-\overrightarrow{CB})$

$=2\overrightarrow{PC}-0$

$(\because \overrightarrow{AC}=\overrightarrow{CB})$

$\therefore$ $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$



21. $[abc]=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}=\begin{vmatrix}1 & 0 & 0\\ x & 1 & 1\\ y & x & 1+x\end{vmatrix}=\begin{vmatrix}1 & 1\\ x & 1+x\end{vmatrix}=1$

22. $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

$\Rightarrow$ $(\vec{b}.\vec{c})\vec{a}=(\vec{a}.\vec{b})\vec{c}$

So that $\vec{a}$ is parallel to $\vec{c}$

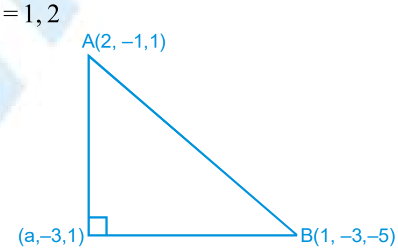
24. $AC\perp BC$

$\therefore$ dr's of AC and BC will be (2–a,2,0) and (1–a,0,–6)

So that $(2-a)(1-a)+2\times 0+0\times(-6)=0$

$\Rightarrow$ $a^2-3a+2=0$

$\therefore$ $a=1, 2$



29. $[3\vec{u}\;\; p\vec{v}\;\; p\vec{w}]-[p\vec{v}\;\;\vec{w}\;\; q\vec{u}]-[2\vec{w}\;\; q\vec{v}\;\; q\vec{u}]=0$

$3p^2[\vec{u}\;\vec{v}\;\vec{w}]-pq[\vec{v}\;\vec{w}\;\vec{u}]-2q^2[\vec{w}\;\vec{v}\;\vec{u}]=0$

$(3p^2-pq+2q^2).[\vec{u}\;\vec{v}\;\vec{w}]=0$

$3p^2-pq+2q^2=0$

has exactly one solution

$p=q=0$

30. $(\vec{a}\times\vec{b})+\vec{c}=0$

$(\vec{a}\times\vec{b})=-\vec{c}$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{b})=-\vec{a}\times\vec{c}$

$\Rightarrow (\vec{a}.\vec{b})\vec{a}-|\vec{a}|^2\,\vec{b}=-\vec{a}\times\vec{c}$

$\Rightarrow 3(\hat{j}-\hat{k})-2\vec{b}=-(-2\hat{i}-\hat{j}-\hat{k})$

$(\vec{a}\times\vec{c}=-2\hat{i}-\hat{j}-\hat{k})$

$\Rightarrow 2\vec{b}=(-2\hat{i}+2\hat{j}-4\hat{k})$

$\Rightarrow \vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

31. Give $\vec{a} \perp \vec{b}$, $\vec{a} \perp \vec{c}$ & $\vec{b} \perp \vec{c}$

so $\vec{a}.\vec{c} = 0$ & $\vec{b}.\vec{c} = 0$

$\Rightarrow \lambda - 1 + 2\mu = 0$ & $2\lambda + 4 + \mu = 0$

$\Rightarrow \lambda = -3$ & $\mu = 2$

32. a.b. $\neq 0$

a.d = 0

b × c = b × d

a × (b × c) = a × (b × d)

(a.c)b – (a.b)c – (a.c)b – (a.b)d {a.d=0}

$\Rightarrow$ (a.b)d = (a.b)c (a.c)b (divide by a.b)

$$\boxed{d = c - \frac{(a.c)}{(a.b)}b}$$

33. $\vec{a}.\vec{b} = 0$ and |a| = |b| = 1

$(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b}) = (\vec{a} \times \vec{b}) \times \vec{a} + (\vec{a} \times \vec{b}) \times 2\vec{b}$

$= -\left[\vec{a} \times (\vec{a} \times \vec{b}) + 2\vec{b} \times (\vec{a} \times \vec{b})\right]$

$= -\left[(\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} + 2(\vec{b}.\vec{b})\vec{a} - 2(\vec{b}.\vec{a})\vec{b}\right]$

$= -\left[0 - \vec{b} + 2\vec{a} + 0\right] = \left[\vec{b} - 2\vec{a}\right]$

$\therefore (2\vec{a} - \vec{b}) . \left[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right]$

$= (2\vec{a} - \vec{b}) . (\vec{b} - 2\vec{a})$

$= -4a^2 - b^2 + 4\vec{a}.\vec{b} = -5$

34. $\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$

p(qr – 1) – (r – 1) + (1 – q) = 0

pqr – p – r + 1 + 1 – q = 0

pqr – (p + r + q) + 2 = 0

pqr – (p + r + q) = –2

35. Let

$a + 3b = \lambda\vec{c}$

add $6\vec{c}$ both side

$\vec{a} + 3\vec{b} + 6\vec{c} = (\lambda + 6)\vec{c}$

Let

$\vec{b} + 2\vec{c} = \mu\vec{a}$

$3b + 6\vec{c} = 3\mu\vec{a}$

add $\vec{a}$ both side

$\vec{a} + 3\vec{b} + 6\vec{c} = (3\mu + 1)\vec{a}$

Hence $(\lambda + 6)\vec{c} = (3\mu + 1)\vec{a}$

But given $\vec{a}$ and $\vec{c}$ are non coliner

Hence $\lambda + 6 = 3\mu + 1 = 0$

so $\vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$

36. $\vec{c}.\vec{d} = 0$

$\Rightarrow (\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0$

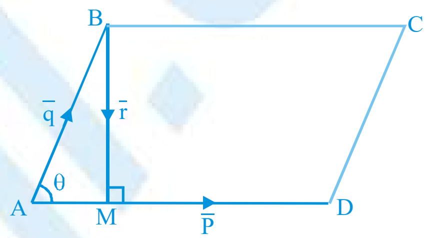
$\Rightarrow 5 - 8 + 6\hat{a}.\hat{b} = 0$

$\Rightarrow \hat{a}.\hat{b} = 1/2$

$\Rightarrow \cos\theta = 1/2$

$\Rightarrow \theta = \frac{\pi}{3}$

37.



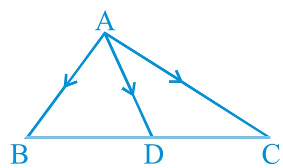
$\bar{q} + \bar{r} = \overline{AM}$

$\Rightarrow \bar{r} = -\bar{q} + \overline{AM}$

$\Rightarrow \bar{r} = -\bar{q} + \frac{\bar{p}.\bar{q}}{|\bar{p}|^2}\vec{p}$

$\Rightarrow \bar{r} = -\bar{q} + \left(\frac{\bar{p}.\bar{q}}{\bar{p}.\bar{p}}\right)\bar{p}$

38.



$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = 4\hat{j} - \hat{j} + 4\hat{k}$

$|\overrightarrow{AD}| = \sqrt{33}$

43. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix}=0$

$k^2+3k=0 \Rightarrow k(k+3)=0 \Rightarrow k=0$ or -3

45. Let $\vec{n}_1$ and $\vec{n}_2$ be the vectors normal to the faces OAB and ABC. Then,

$$\vec{n}_1=\overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}=5\hat{i}-\hat{j}-3\hat{k}$$

and, $$\vec{n}_2=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix}=\hat{i}-5\hat{j}-3\hat{k}$$

If θ is the angle between the faces OAB and ABC, then

$$\cos\theta=\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$$\Rightarrow \cos\theta=\frac{5+5+9}{\sqrt{25+1+9}\sqrt{1+25+9}}=\frac{19}{35}$$

$$\Rightarrow \theta=\cos^{-1}\left(\frac{19}{35}\right)$$

46. $\bullet_1-am_1=0$ and $cm_1-n_1=0 \Rightarrow \frac{l_1}{a}=\frac{m_1}{1}=\frac{n_1}{c}$

Also $\bullet_2-a'm_2=0$ and $c'm_2-n_2=0$

$$\Rightarrow \frac{l_2}{a'}=\frac{m_2}{1}=\frac{n_2}{c'}$$

$\therefore$ $\bullet_1\bullet_2+m_1m_2+n_1n_2=aa'+cc'+1=0$

47. Here, $\bullet=\cos\theta$, $m=\cos\beta$, $n=\cos\theta$, (✈ $\bullet=n$)

Now, $\bullet^2+m^2+n^2=1 \Rightarrow 2\cos^2\theta+\cos^2\beta=1$

$\Rightarrow$ Given, $\sin^2\beta=3\sin^2\theta \Rightarrow 2\cos^2\theta=3\sin^2\theta$

$5\cos^2\theta=3$, $\therefore \cos^2\theta=\frac{3}{5}$

48. Given plane are $2x+y+2z-8=0$

or $4x+2y+4z-16=0$ ..... (i)

and $4x+2y+4z+5=0$ ..... (ii)

Distance between two parallel planes

$$=\left|\frac{-16-5}{\sqrt{4^2+2^2+4^2}}\right|=\frac{21}{6}=\frac{7}{2}$$

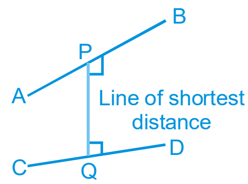
49. Let the two lines be AB and CD having equation

$\frac{x}{1}=\frac{y+a}{1}=\frac{z}{1}=\lambda$ and $\frac{x+a}{2}=\frac{y}{1}=\frac{z}{1}=\mu$

then $P\equiv(\lambda,\lambda-a,\lambda)$ and $Q=(2\mu-a,\mu,\mu)$

So according to question,

$$\frac{\lambda-2\mu+a}{2}=\frac{\lambda-a-\mu}{1}=\frac{\lambda-\mu}{2}$$



$\Rightarrow \mu=a$ and $\lambda=3a$

$\therefore$ $P\equiv(3a,2a,3a)$

and $Q\equiv(a,a,0)$

50. We have, $\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s$

and $\frac{x-0}{1/2}=\frac{y-1}{1}=\frac{z-2}{-1}=t$

Since, lines are coplanar then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ 1/2 & 1 & -1 \end{vmatrix}=0$$

On solving, $\lambda=-2$

52. Angle between line and normal to plane is

$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{1\times2-2\times1+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$, where θ is the angle between line and plane

$$\Rightarrow \sin\theta=\frac{1\times2+2\times(-1)+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}} \Rightarrow \frac{1}{3}=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$$

$$\Rightarrow \frac{1}{3}=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} \Rightarrow \lambda=\frac{5}{3}$$

53. The lines are $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ and $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$

Since, $a_1a_2+b_1b_2+c_1c_2=6-24+18=0$

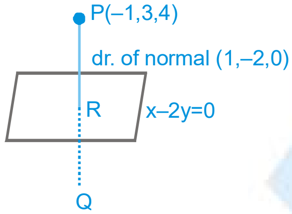
$\Rightarrow\ \theta=90°$

58. Equation of line PQ is

$$\frac{x+1}{1}=\frac{y-3}{-2}=\frac{z-4}{0}=\lambda$$

For some suitable value of λ, co-ordinates of point $Q(\lambda-1,3-2\lambda,4)$

R is the mid point of P and Q.



$\therefore\ R\equiv\left(\frac{\lambda-2}{2},\frac{6-2\lambda}{2},4\right)$

$R\equiv\left(\frac{\lambda}{2}-1,\ 3-\lambda,\ 4\right)$

It satisfies $x-2y=0$

$\Rightarrow\ \lambda=\frac{14}{5}$

$\therefore\ Q=\left(\frac{2}{5},\frac{1}{5},4\right)$

59. If direction cosines of L be ●, m, n then

$2\bullet+3m+n=0$

$\bullet+3m+2n=0$

Solving, we get, $\frac{1}{3}=\frac{m}{-3}=\frac{n}{3}$

$\therefore\ \bullet:m:n=\frac{1}{\sqrt3}:-\frac{1}{\sqrt3}:\frac{1}{\sqrt3}\Rightarrow\cos\alpha=\frac{1}{\sqrt3}$

60. $\bullet=\cos\frac{\pi}{4},\ m=\cos\frac{\pi}{4}$

we know that $\bullet^2+m^2+n^2=1$

$\frac{1}{2}+\frac{1}{2}+n^2=1\Rightarrow n=0$

Hence angle with positive direction of z-axis is $\frac{\pi}{2}$

64. Line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ .... **(1)**

Plane $x+3y-\alpha z+\beta=0$ .... **(2)**

Point (2, 1, –2) put in **(2)**

$2+3+2\alpha+\beta=0$

$\Rightarrow\ 2\alpha+\beta=-5$

Now $a_1a_2+b_1b_2+c_1c_2=0$

$3-15-2\alpha=0$

$-12-2\alpha=0$

$\alpha=-6$

$-12+\beta=-5$

$\beta=7$

$\alpha=-6,\ \beta=7$

65. Proj. of a vector ($\vec r$) on x–axis $=|\vec r|\,\bullet$

on y–axis $=|\vec r|\,m$

on z–axis $=|\vec r|\,n$

$6=7\bullet,\ \Rightarrow\ \bullet=\frac{6}{7}$ similarly $m=-\frac{3}{7},\ n=\frac{2}{7}$

66. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ ...**(i)**

$\alpha=45°,\ \beta=120°$

Put in equation **(i)**

$\Rightarrow\ \frac{1}{2}+\frac{1}{4}+\cos^2\gamma=1$

$\Rightarrow\ \cos^2\gamma=\frac{1}{4}$

$\Rightarrow\ \gamma=60°$

67. Mirror image of B(1, 3, 4) in plane x–y+z = 5

$$\frac{x-1}{1}=\frac{y-3}{-1}=\frac{z-4}{1}=-2\frac{(1-3+4-5)}{1+1+1}=2$$

$\Rightarrow\ x=3,\ y=1,\ z=6$

$\therefore$ mirror image of B (1, 3, 4) is A (3, 1, 6)

statement-1 is correct

statement-2 is true but it is not the correct explanation.

68. $\frac{x}{1}=\frac{y-1}{2}=\frac{z-3}{\lambda}$ equation of line

equation of plane $x+2y+3z=4$

$$\sin\theta=\frac{1+4+3\lambda}{\sqrt{14}\sqrt{1+4+\lambda^2}}$$

$\Rightarrow\ \lambda=\frac{2}{3}$

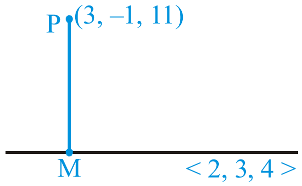
69. $1(1-1)+2(0-6)+3(7-3)$

$= 0 - 12 + 12 = 0$

mid point AB (1, 3, 5)

lies on $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

70.



M(2r, 3r + 2, 4r + 3)

Dr's of PM $< 2r - 3, 3r + 3, 4r - 8 >$

$2(2r-3) + 3(3r+3) + 4(4r-8) = 0$

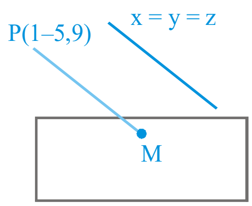
$29r - 29 = 0$

$r = 1$

M(2, 5, 7)

Distance PM $= \sqrt{1+36+16} = \sqrt{53}$

71.



eq^n. of a line ∥ to x = y = z and passing through (1, –5, 9) is

$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = r$

Let is meets plane at M(r+1, r–5, r+9)

Put in equation of plane

$x - y + z = 5$

$r + 1 - r + 5 + r + 9 = 5$

$r = -10$

Hence M (–9, –15, –1)

Distance PM $= \sqrt{100+100+100} = 10\sqrt{3}$

72. Equation of plane parallel to

$x - 2y + 2z - 5 = 0$ is $x - 2y + 2z = k$

or $\frac{x}{3} - \frac{2}{3}y + \frac{2}{3}z = \frac{K}{3}$

$\left|\frac{K}{3}\right| = 1$

$\Rightarrow K = \pm 3$

$\therefore$ Equation of required plane is

$x - 2y + 2z \pm 3 = 0$

73. $\begin{vmatrix} 3-1 & K+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2 & K+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 2K - 9 = 0$

$\Rightarrow K = \frac{9}{2}$

74. $4x + 2y + 4z + 5 = 0$

$4x + 2y + 4z - 16 = 0$

$\Rightarrow d = \left|\frac{21}{\sqrt{36}}\right| = \frac{7}{2}$

75. $\Rightarrow (\vec{a} - \vec{b}).(\vec{c} \times \vec{d}) = 0$

$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow (1 + 2k) + (1 + k^2) - (2 - k) = 0$

$\Rightarrow k^2 + 3k = 0 \begin{cases} 0 \\ -3 \end{cases}$

82. $\ell(3) + m(-2) - (-4) = 9$

$3\ell - 2m = 5$ .....**(i)**

$3\ell - 2m - 3 = 0$

$2\ell - m = 3$ .....**(ii)**

$4\ell - 2m = 6$ .....**(iii)**

(iii) – (i)

$\ell = 1$

$m = -1$ $\quad \ell^2 + m^2 = 2$

83. $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=\frac{\sqrt{3}}{2}\vec{b}+\frac{\sqrt{3}}{2}\vec{c}$

$\Rightarrow$ $\vec{a}.\vec{c}=\frac{\sqrt{3}}{2}$ and $\vec{a}.\vec{b}=-\frac{\sqrt{3}}{2}$

$\Rightarrow$ Angle between $\vec{a}$ & $\vec{c}$ $=30^{o}$

$\vec{a}$ & $\vec{c}$ $=150^{o}=\frac{5\pi}{6}$

84. Equation of line : $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

Any point is $(\lambda+1,\lambda-5,\lambda+9)$

It lies on plane

$\Rightarrow$ $(\lambda+1)-(\lambda-5)+(\lambda+9)=5$

$\Rightarrow$ $\lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow$ $\lambda=-10$

$\therefore$ Point is $(-9,-15,-1)$, another is $(1,-5,9)$

Distance $=\sqrt{100+100+100}=10\sqrt{3}$

Part # II : IIT-JEE ADVANCED

1. **(b)** Given that $\vec{a},\vec{b},\vec{c},\vec{d}$ are vectors such that

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=0$ .....**(1)**

P_1 is the plane determined by vectors $\vec{a}$ and $\vec{b}$

$\therefore$ Normal vectors $\vec{n}_1$ to P_1 will be given by

$\vec{n}_1=\vec{a}\times\vec{b}$

Similarly P_2 is the plane determined by vectors $\vec{c}$ and $\vec{d}$

$\therefore$ Normal vector $\vec{n}_2$ to P_2 will be given by

$\vec{n}_2=\vec{c}\times\vec{d}$

Substituting the values of $\vec{n}_1$ and $\vec{n}_2$ in equation **(1)** we get $\vec{n}_1\times\vec{n}_2=0$

$\Rightarrow$ $\vec{n}_1||\vec{n}_2$

and hence the planes will also be parallel to each other.

Thus angle between the planes = 0.

3. **(a)** $\hat{a},\hat{b},\hat{c}$ are unit vectors.

$\therefore$ $\hat{a}\cdot\hat{a}=\hat{b}\cdot\hat{b}=\hat{c}\cdot\hat{c}=1$

Now, $x=|\hat{a}-\hat{b}|^2+|\hat{b}-\hat{c}|^2+|\hat{c}-\hat{a}|^2$

$=\hat{a}.\hat{a}+\hat{b}.\hat{b}-2\hat{a}.\hat{b}+\hat{b}.\hat{b}+\hat{c}.\hat{c}-$

$2\hat{b}.\hat{c}+\hat{c}.\hat{c}+\hat{a}.\hat{a}-2\hat{c}.\hat{a}$

$=6-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})$ ......**(1)**

Also $|\hat{a}+\hat{b}+\hat{c}|\geq 0$

$\Rightarrow$ $\hat{a}.\hat{a}+\hat{b}.\hat{b}+\hat{c}.\hat{c}+2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\geq 0$

$\Rightarrow$ $3+2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\geq 0$

$\Rightarrow$ $-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\leq 3$

$\Rightarrow$ $6-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\leq 9$ ... **(2)**

From **(1)** and **(2)**, $x\leq 9$

$\therefore$ x does not exceed 9

5. Given data is insufficient to uniquely determine the three vectors as there are only 6 equations involving 9 variables.

$\therefore$ We can obtain infinitely many set of three vectors, $\vec{v}_1, \vec{v}_2, \vec{v}_3$, satisfying these conditions.

From the given data, we get

$\vec{v}_1.\vec{v}_1 = 4 \Rightarrow |\vec{v}_1| = 2$

$\vec{v}_2.\vec{v}_2 = 2 \Rightarrow |\vec{v}_2| = \sqrt{2}$

$\vec{v}_3.\vec{v}_3 = 29 \Rightarrow |\vec{v}_3| = \sqrt{29}$

Also $\vec{v}_1.\vec{v}_2 = -2$

$\Rightarrow |\vec{v}_1||\vec{v}_2|\cos\theta = -2$

[where θ is the angle between $\vec{v}_1$ and $\vec{v}_2$]

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} \Rightarrow \theta = 135°$

Now since any two vectors are always coplanar, let us suppose that $\vec{v}_1$ and $\vec{v}_2$ are in x–y plane. Let $\vec{v}_1$ is along the positive direction of x-axis then $\vec{v}_1 = 2\hat{i}$. $[\because |\vec{v}_1| = 2]$

As $\vec{v}_2$ makes an angle 135° with $\vec{v}_1$ and ies in x–y plane, also keeping in mind $|\vec{v}_2| = \sqrt{2}$ we obtain

$\vec{v}_2 = -\hat{i} \pm \hat{j}$

Again let, $\vec{v}_3 = \alpha\hat{i} + \beta\hat{i} + \gamma\hat{k}$

$\because \vec{v}_3.\vec{v}_1 = 6 \Rightarrow 2\alpha = 6 \Rightarrow \alpha = 3$

and $\vec{v}_3.\vec{v}_2 = -5 \Rightarrow -\alpha \pm \beta = -5 \Rightarrow \beta = \pm 2$

Also $|\vec{v}_3| = \sqrt{29} \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 29$

$\Rightarrow \gamma = \pm 4$

Hence $\vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$

Thus, $\vec{v}_1 = 2\hat{i}; \vec{v}_2 = -\hat{i} \pm \hat{j}; \vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$

are some possible answers.

6. $\vec{A}(t)$ is parallel to $\vec{B}(t)$ for some $t \in [0,1]$ if and only if

$\frac{f_1(t)}{g_1(t)} = \frac{f_2(t)}{g_2(t)}$ for some $t \in [0,1]$

or $f_1(t).g_2(t) = f_2(t).g_1(t)$ for some $t \in [0,1]$

Let $h(t) = f_1(t).g_2(t) - f_2(t).g_1(t)$

$h(0) = f_1(0).g_2(0) - f_2(0).g_1(0)$

$= 2 \times 2 - 3 \times 3 = -5 < 0$

$h(1) = f_1(1).g_2(1) - f_2(1).g_1(1)$

$= 6 \times 6 - 2 \times 2 = 32 > 0$

Since h is a continuous function, and $h(0).h(1) < 0$

$\Rightarrow$ there is some $t \in [0,1]$ for which $h(t) = 0$

i.e., $\vec{A}(t)$ and $\vec{B}(t)$ are parallel vectors for this t.

8. Given that, $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

where $a_r, b_r, c_r, r = 1,2,3$ are all non negative real numbers.

Also $\sum_{r=1}^{3} (a_r + b_r + c_r) = 3L$

To prove $V \le L^3$ Where V is vol. of parallelopiped formed by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$

$\therefore$ We have $V = [\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$\Rightarrow V = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)$ ....(1)

Now we know that AM ≥ GM

$\therefore \frac{(a_1+b_1+c_1)+(a_2+b_2+c_2)+(a_3+b_3+c_3)}{3}$

$\ge [(a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)]^{1/3}$

$\Rightarrow \frac{3L}{3} \ge [(a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)]^{1/3}$

$\Rightarrow L^3 \ge (a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)$

$= a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 + 24$ more such terms