HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

PART 1

1. Let $x \in A \Rightarrow x \in A \cup B$, [$\because A \subseteq A \cup B$]

$\Rightarrow x \in A \cap B$, [$\because A \cup B = A \cap B$]

$\Rightarrow x \in A$ and $x \in B \Rightarrow x \in B$, $\therefore A \subseteq B$

Similarly, $x \in B \Rightarrow x \in A$, $B \subseteq A$

Now $A \subseteq B$, $B \subseteq A \Rightarrow A = B$.

6. $A \cap (A \cup B)' = A \cap (A' \cap B')$

($\because (A \cup B)' = A' \cap B'$)

$= (A \cap A') \cap B'$ (by associative law)

$= \phi \cap B'$ ($\because A \cap A' = \phi$)

$= \phi$

8. From De' morgan's law, $(A \cap B)' = A' \cup B'$.

10. $B' = \{1, 2, 3, 4, 5, 8, 9, 10\}$

$\therefore A \cap B' = \{1, 2, 5\} \cap \{1, 2, 3, 4, 5, 8, 9, 10\} = \{1, 2, 5\} = A$

13. $x^2 = 16 \Rightarrow x = \pm 4$

$2x = 6 \quad x = 3$

No common value of x

18. $n(A^c \cap B^c) = n[(\{A \cup B)^c]$

$= n(U) - n(A \cup B) = n(U) - [n(A) + n(B) - n(A \cap B)]$

$= 700 - [200 + 300 - 100] = 300.$

20. $4^n - 3n - 1 = (3 + 1)^n - 3n - 1$

$= 3^n + {}^nC_1 3^{n-1} + {}^nC_2 3^{n-2} + ... + {}^nC_{n-1}\, 3 + {}^nC_n - 3n - 1$

$= {}^nC_2 3^2 + {}^nC_3 . 3^3 + ... + {}^nC_n 3^n, ({}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}$ etc)

$= 9[{}^nC_2 + {}^nC_3(C) + ... + {}^nC_n 3^{n-1}]$

$\therefore$ X contains elements, which are multiples of 9, and clearly Y contains all multiples of 9.

$\therefore X \subseteq Y$ i.e. $X \cup Y = Y$.

21. $A \cap B \subseteq A$. Hence $A \cup (A \cap B) = A$.

22. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

25. Null set is the subset of all given sets.

28. $A = [x : x \in R, -1 < x < 1]$

$B = [x : x \in R : x - 1 \le -1$ or $x - 1 \ge 1] = [x : x \in R : x \le 0$ or $x \ge 2]$

$\therefore A \cup B = R - D$, where $D = [x : x \in R, 1 \le x < 2]$

31. Since $\frac{1}{y} \ne 0, \frac{1}{y} \ne 2, \frac{1}{y} \ne \frac{-2}{3}$, [$\because y \in N$]

$\therefore \frac{1}{y}$ can be 1, [$\because$ y can be 1]

PART 2

37. Since x </ x, therefore R is not reflexive. Also x < y does not imply that y < x, So R is not symmetric. Let xRy and yRz. Then, x < y and

$y < z \Rightarrow x < z$ i.e., xRz. Hence R is transitive.

38. For any $x \in R$, we have $x - x + = \sqrt{2}$ an irrational number.

$\Rightarrow$ xRx for all x. So, R is reflexive.

39. R_4 is not a relation from X to Y, because $(7, 9) \in R_4$ but $(7, 9) \notin X \times Y$.

40. Here $\alpha R \beta \Leftrightarrow \alpha \perp \beta \therefore \alpha \perp \beta \Leftrightarrow \beta \perp \alpha$

Hence, R is symmetric.

43. $A = \{2, 3\}, B = \{2, 4\}, C = \{4, 5\}$

$B \cap C = \{4\}$

$\therefore A \times (B \cap C) = \{(2, 4), (3, 4)\}$.

45. Number of relation from A to B $= 2^{12}$

47. $R_1 \rightarrow$ Domain = {1, 3, 5}

Range = {3, 5, 7} so R_1 is a relation

$R_2 \rightarrow$ Domain = {1,2, 3, 4, 5}

Range = {1, 3, 5} so R_2 is a relation

$R_3 \rightarrow$ Domain = {1, 3, 5}

Range = {1, 3, 5, 7} so R_3 is a relation

$R_4 \rightarrow$ Domain = {1, 2, 7} X so R_4 is not a relation

55. We have (a, b) R (a, b) for all $(a, b) \in N \times N$

Since a + b = b + a. Hence, R is reflexive.

R is symmetric for we have (a,b)R(c,d) $\Rightarrow$ a+d=b+c

$\Rightarrow$ $d + a = b + c$ $\Rightarrow$ $c + b = d + a$ $\Rightarrow$ (c,d) R(a,b)

Hence R is symmetric

Then by definition of R, we have

$a + d = b + c$ and $c + f = d + e$,

hence by addition, we get

$a + d + c + f = b + c + d + e$ or $a + f = b + e$

Hence, (a, b) R (e, f)

Thus, (a, b) R(c, d) and (c, d) R(e, f) $\Rightarrow$ (a, b) R(e,f).

Hence R is transitive.

56. For (a, b), (c, d) $\in N \times N$

(a, b) R(c, d) $\Rightarrow$ $ad(b + c) = bc(a + d)$

Reflexive : Since $ab(b + a) = ba(a + b)$ $\forall$ $ab \in N$,

$\therefore$ (a, b)R(a, b), $\therefore$ R is reflexive.

Symmetric : For (a, b), (c, d) $\in N \times N$, let (a,b)R(c,d)

$\therefore$ $ad(b + c) = bc(a + d)$ $\Rightarrow$ $bc(a + d) = ad(b + c)$

$\Rightarrow$ $cb(d + a) = da(c + b)$ $\Rightarrow$ (c, d)R(a, b)

$\therefore$ R is symmetric

Transitive : For (a, b), (c, d), (e, f) $\in N \times N$,

Let (a, b)R(c, d), (c, d)R(e, f)

$\therefore$ $ad(b + c) = bc(a + d)$, $cf(d + e) = de(c + f)$

$\Rightarrow$ $adb + adc = bca + bcd$ ... **(i)**

and $cfd + cfe = dec + def$ ... **(ii)**

(i) $\times ef$ + **(ii)** $\times ab$ gives,

$adbef + adcef + cfdab + cefab$

$= bcaef + bcdef + decab + defab$

$\Rightarrow$ $adcf(b + e) = bcde(a + f)$

$\Rightarrow$ $af(b + e) = be(a + f)$ $\Rightarrow$ (a, b)R(e, f).

$\therefore$ R is transitive. Hence R is an equivalence relation

59. For Reflexive R $\in$ {(1,1) (2,2) (3,3)}

For symmetric (1, 2) $\in$ R but (2, 1) R Not symmetric

for transitive (1, 2), (2, 3) $\in$ R $\Rightarrow$ (1, 3) $\in$ R

so transitive

61. For any a $\in$ N, we find that a is divisible by a, therefore R is reflexive but R is not symmetric, because aRb does not imply that bRa.

66. $2x + x = 41$ $\Rightarrow$ x= $\therefore$ R is not reflexive

$2x + y = 41$ $2y + x = 41$ $\therefore$ R is not symmetric

$2x + y = 41$ and $2y + z = 41$ $\Rightarrow$ $4x - z = 41 \Rightarrow$ (x, z) R

$\therefore$ R is not transitive

67. Here $\bullet_1 R \bullet_2$, $\bullet_1$ is parallel to $\bullet_2$ and $\bullet_2$ is parallel to $\bullet_1$, so it is symmetric.

Clearly, it is also reflexive and transitive. Hence it is equivalence relation.

71. Let (a, b) $\in$ R

Then, (a, b) $\in$ R $\Rightarrow$ (b,a) $\in R^{-1}$, [by def or R^{-1}]

$\Rightarrow$ (b, a) $\in$ R, [$\because$ $R = R^{-1}$], So R is symmetric.

74. A = {1, 2, 3}

$|a^2 - b^2| \le 5$ $\quad -5 \le a^2 - b^2 \le 5$

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

Domain of R = {1, 2, 3}

Range of R = {1, 2, 3}

76. (A) (q, q) R_1 (B) (p, p) R_2 (C) (q, p) R_3

Not reflexive not reflexive not symmetric

79. R is reflexive if it contains (1, 1) (2, 2) (3, 3)

$\because$ (1, 2) $\in$ R, (2, 3) $\in$ R

$\therefore$ R is symmetric if (2, 1), (3, 2) $\in$ R

Now, R = {(1, 1), (2, 2),(3, 3),(2, 1),(3, 2),(2, 3), (1, 2)}

R will be transitive if (3, 1) ; (1, 3) $\in$ R. Thus, R becomes and equivalence relation by adding (1, 1) (2, 2) (3, 3) (2, 1), (3, 2), (1, 3), (1, 2). Hence, the total number of ordered pairs is 7.

80. Obviously, the relation is not reflexive and transitive but it is symmetric, because

$$x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$$

81. Clearly, the relation is symmetric but it is neither reflexive nor transitive.

83. We have, R = {(1,3); (1,5); (2,3); (2,5); (3,5); (4,5)}

R^{-1} = {(3, 1); (5, 1); (3, 2); (5, 2); (5, 3); (5, 4)}

Hence ROR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}

85. Given R, and S are relations on set A.

$\therefore$ $R \subseteq A \times A$ and $S \subseteq A \times A \Rightarrow R \cap S \subseteq A \times A$

$\Rightarrow$ $R \cap S$ is also a relation on A.

Reflexivity : Let a be an arbitrary element of A. Then a $\in$ A $\Rightarrow$ (a, a) $\in$ R [$\because$ R and S are reflexive]

and (a, a) $\in$ S

$\Rightarrow$ (a, a) $\in R \cap S$

Thus, (a, a) $\in R \cap S$ for all a $\in$ A.

So, $R \cap S$ is a reflexive relation on A.

Symmetry : Let $a, b \in A$ such that $(a, b) \in R \cap S$.

Then, $(a, b) \in R \cap S \Rightarrow (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow$ $(b, a) \in R$ and $(b, a) \in S$

[$\because$ R and S are symmetric]

$\Rightarrow$ $(b, a) \in R \cap S$

Thus, $(a, b) \in R \cap S \Rightarrow (b, a) \in R \cap S$ for all

$(a, b) \in R \cap S$.

So, $R \cap S$ is symmetric on A.

Transitivity : Let $a,b,c \in A$ such that $(a,b) \in R \cap S$ and $(b, c) \in R \cap S$. Then $(a, b) \in R \cap S$ and

$(b, c) \in R \cap S$

$\Rightarrow$ $\{((a, b) \in R \text{ and } (a, b) \in S)\}$

and $\{((b, c) \in R \text{ and } (b, c) \in S)\}$

$\Rightarrow$ $\{(a, b) \in R, (b, c) \in R\}$ and $\{(a,b) \in S, (b, c) \in S\}$

$\Rightarrow$ $(a, c) \in R$ and $(a, c) \in S$ $\Rightarrow$ $(a, c) \in R \cap S$

Thus, $(a, b) \in R \cap S$ and $(b, c) \in R \cap S$

$\Rightarrow$ $(a, c) \in R \cap S$. So $R \cap S$ is transitive on A

Hence, R is an equivalence relation on A.

Part # II : Comprehension

Comprehension-1

1. (D) **2.** (B) **3.** (C)

Comprehension-2

1. (B) **2.** (C) **3.** (A)

EXERCISE - 2

Subjective Type

1. {a, e, i} **2.** (i) Finite (ii) Infinite

3. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

4. $\{x : x = n^2 - 1, n \in N, n \le 6\}$

5. {T, R, I, G, O, N, M, E, Y} **6.** $n = 3, m = 6$

7. (i) False (ii) True

9. {9, 21}

13. (i) 6 (ii) 9 **14.** 43 **15.** 22

16. Domain = {1, 2, 3, 4, 5, 6},
Range = {5, 6, 7, 8, 9, 10}

17. {(4, 3), (4, 4), (6, 3), (6, 4)}

EXERCISE - 3

Part # I : AIEEE/JEE-MAIN

1. **(B)**

$(1, 1) \notin R$ $\therefore$ not reflexive

$(2, 3) \in R$ and $(3, 2) \notin R$ not symmetric

$(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$

$\therefore$ not transitive

2. **(A)**

$(a, a) \in R$ for each $a \in A$ $\therefore$ reflexive

$(6, 12) \in R$ but $(12, 6)$ R $\therefore$ not symmetric

$(a, b) \in R$ and $(b, c) \in R$ $\Rightarrow$ $(a, c) \in R$

$\Rightarrow$ transitive

3. **(A)**

$(x, x) \in R$ $\forall$ $x \in W$ $\Rightarrow$ R is reflexive

Let $(x, y) \in R$, then $(y, x) \in R$

[$\because$ x, y have at least one letter in common]

$\Rightarrow$ R is symmetric.

But R is not transitive

eg. (TALL) R (LIGHT) and (LIGHT) R (HIGH) but (TALL) (HIGH)

5. **(A)**

(1) S is not reflexive so not equivalence as $x \ne x + 1$

(2) $(x, y) \in T \Rightarrow x - y$ is an integer

(i) $x - x$ is an integer $\Rightarrow$ reflexive

(ii) $x - y$ = integer $\Rightarrow$ $y - x$ = integer $\therefore$ T is symmetric

(iii) $x - y = m$, $y - z = n$

$\Rightarrow x - y + y - z = m + n$

$x - z = m + n$ $\Rightarrow$ Transitive

so T is equivalence relation

6. **(B)**

We have, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C$ [$\because (A \cup C) \cap C = C$]

$\Rightarrow (A \cap B) \cup (B \cap C) = C$ ...(i) [$\because A \cap C = A \cap B$]

Again, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$

$\Rightarrow B = (A \cap B) \cup (C \cap B)$ $\Rightarrow (A \cap B) \cup (C \cap B) = B$

$\Rightarrow (A \cap B) \cup (B \cap C) = B$ ...(ii)

From (i) and (ii), we get $B = C$

8. **(C)**

Statement - 1 :

(i) $x - x$ is an integer $x \in R$ so A is reflexive relation.

(ii) $y - x \in I$ $\Rightarrow$ $x - y \in I$ so A is symmetric relation.

(iii) $y - x \in I$ and $z - y \in I$ $\Rightarrow$ $y - x + z - y \in I$

$\Rightarrow$ $z - x \in I$ so A is transitive relation.

Therefore A is equivalence relation.

Statement - 2 :

(i) $x = \alpha x$ when $\alpha = 1$ $\Rightarrow$ B is reflexive relation

(ii) for $x = 0$ and $y = 2$, we have $0 = \alpha(2)$ for $\alpha = 0$

But $2 = \alpha(0)$ for no α

so B is not symmetric so not equivalence.

9. (B)

for reflexive

$(A , A) \in R$

$\Rightarrow$ $A = P^{-1} A P$

which is true for $P = I$

$\therefore$ reflexive

for symmetry

As $(A, B) \in R$ for matrix P

$A = P^{-1} BP$

$\Rightarrow$ $PA = PP^{-1} BP$ $\Rightarrow$ $PAP^{-1} = IBPP^{-1}$

$\Rightarrow$ $PAP^{-1} = IBI$ $\Rightarrow$ $PAP^{-1} = B$

$\Rightarrow$ $B = PAP^{-1}$

$\therefore$ $(B, A) \in R$ for matrix P^{-1}

$\therefore$ R is symmetric

for transitivity

$A = P^{-1} BP$

and $B = P^{-1}CP$

$\Rightarrow$ $A = P^{-1} (P^{-1} CP)P$

$\Rightarrow$ $A = (P^{-1})^2 CP^2$

$\Rightarrow$ $A = (P^2)^{-1} C(P^2)$

$\therefore$ $(A, C) \in R$ for matrix P^2

$\therefore$ R is transitive

so R is equivalence

10. (B)

Every element has 3 options. Either set Y or set Z or none

so number of ordered pairs = 3^5

11. (C)

$n(A) = 2$

$n(B) = 4$

$n(A \times B) = 8$

${}^8C_3 + {}^8C_4 + \ldots\ldots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$

$= 256 - 1 - 8 - 28 = 219$

Part # II : CBSE

1. (i) Reflexive

$\because$ $a + b = b + a$ $\Rightarrow$ $(a, b) R(a, b)$

Hence R is reflexive.

(ii) Symmetric

$(a, b) R(c, d)$ $\Rightarrow$ $a + d = b + c$

$\Rightarrow$ $b + c = a + d$ $\Rightarrow$ $c + b = d + a$

$\Rightarrow$ $(c, d) R(a, b)$

Hence R is symmetric

(iii) $(a, b) R(c, d)$ and $(c, d) R(e, f)$

$\Rightarrow$ $a + d = b + c$ and $c + f = d + e$

$\Rightarrow$ $a + d + c + f = b + c + d + e$

$\Rightarrow$ $a + f = b + e$ $\Rightarrow$ $(a, b) R(e, f)$

Hence R is transitive

Therefore, relation R is an equivalence relation.

2. **Reflexive :** Let $a \in A$

$\therefore$ $|a - a| = 0$ is an even number

$\Rightarrow$ $(a, a) \in R$

$\therefore$ R is reflexive

Symmetric : Let $a, b \in R$

Let $(a, b) \in R$

$\Rightarrow$ $|a - b|$ is even $\Rightarrow$ $|-(b - a)|$ is even

$\Rightarrow$ $|b - a|$ is even

$\Rightarrow$ $(b, a) \in R$

$\therefore$ R is symmetric

Transitive : Let $a, b, c \in R$

Let $(a, b) \in R$ and $(b , c) \in R$

$\Rightarrow$ $|a - b|$ is even and $|b - c|$ is even

$\Rightarrow$ $a - b$ is even and $b - c$ is even

$\Rightarrow$ $(a - b) + (b - c)$ is even

$\Rightarrow$ $|a - c|$ is even $\Rightarrow$ $(a, c) \in R$

$\therefore$ R is transitive.

Hence R is an equivalence relation as R is reflexive, symmetric and transitive.

3. **Reflexive :** $\because$ $a - a$ is divisible by 5 for all $a \in Z$

$\therefore$ R is reflexive

Symmetric : $(a, b) \in R$ $\Rightarrow$ $a - b$ is divisible by 5

$\Rightarrow$ $b - a$ is divisible by 5

$\Rightarrow$ $b - a \in R$

$\therefore$ R is symmetric

Transitive : (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ a – b and b – c are both divisible by 5

$\Rightarrow$ a – b + b – c is divisible by 5

$\Rightarrow$ a – c is divisible by 5

$\Rightarrow$ (a, c) $\in$ R

$\therefore$ R is transitive

Since R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation.

4.(i) As $a \le a^3$ is not true for all a $\in$ R

$\therefore$ R is not reflexive

For example, if $a = \frac{1}{3}$ then $a > a^3$

i.e. $a \le a^3$ is not true

(ii) If (a, b) $\in$ R then need not imply that (b, a) $\in$ R

$\therefore$ R is not symmetric

For example, if (1, 2) $\in$ R but (2, 1) $\notin$ R,

As $1 \le 2^3$ but $2 \not\le 1^3$

(iii) If (a, b) $\in$ R and (b, c) $\in$ R, then need not imply that (a, c) $\in$ R

$\therefore$ R is not transitive

For example (100, 5) $\in$ R and (5, 2) $\in$ R but (100, 2) $\notin$ R

As $100 \le 5^3$ and $5 \le 2^3$ but $100 \ge 2^3$.

5. **Reflexive :** For all a $\in$ A

|a – a| = 0 is divisible by 4 $\Rightarrow$ (a, a) $\in$ S

$\therefore$ S is reflexive

Symmetric : Let a, b $\in$ A, (a , b) $\in$ S

$\Rightarrow$ |a – b| is divisible by 4

$\Rightarrow$ |b – a| is divisibe by 4 $\Rightarrow$ (b, a) $\in$ S

$\therefore$ S is symmetric

Transitive : Let a, b, c $\in$ A, (a, b) $\in$ S, (b, c) $\in$ S

$\Rightarrow$ |a – b| is divisible by 4 and |b – c| is divisible by 4

$\Rightarrow$ (a – b) and (b – c) is divisible by 4

$\Rightarrow$ (a – b) + (b – c) = a – c is divisible by 4

$\Rightarrow$ |a – c| is divisible by 4 $\Rightarrow$ (a, c) $\in$ S

$\therefore$ S is transitive

Since S is reflexive, symmetric and transitive.

Hence, S is an equivalence relation.

The set of all elements of A, related to 1 is {1, 5, 9}

6. **Reflexive**

$\because$ a + b = b + a $\Rightarrow$ (a, b) R(a, b)

Hence R is reflexive.

(ii) Symmetric

(a, b) R(c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ b + c = a + d $\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R(a, b)

Hence R is symmetric

(iii) (a, b) R(c, d) and (c, d) R(e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e $\Rightarrow$ (a, b) R(e, f)

Hence R is transitive

Therefore, relation R is an equivalence relation.

7. **Reflexive :** For every a $\in$ X, since f(a) = f(a)

$\therefore$ (a, a) $\in$ R $\therefore$ R is reflexive

Symmetric : Let (a, b) $\in$ R for every a, b $\in$ X

$\Rightarrow$ f(a) = f(b) $\Rightarrow$ f(b) = f(a) $\Rightarrow$ (b, a) $\in$ R

$\therefore$ R is symmetric

Transitive : Let a, b, c $\in$ X. Let (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ f(a) = f(b) and f(b) = f(c)

$\Rightarrow$ f(a) = f(c) $\Rightarrow$ (a, c) $\in$ R

$\therefore$ R is transitive.

Since R is reflexive, symmetric and transitive.

Therefore R is an equivalence relation.

8. In the case of transitive relation

(a, b), (b, c) $\in$ R

$\Rightarrow$ (a, c) $\in$ R

Here (1, 2) and (2, 1) $\in$ R but (1, 1) $\notin$ R.

So, it is not transitive.