MATRICES
EXERCISE # 1

Questions based on **Types of Matrices, Addition, subtraction**

Q.1 If A = $\begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then | 2A– 3B | equals -

(A) 77 (B) – 53 (C) 53 (D) – 77

Sol.[B] $|2A - 3B| = \begin{vmatrix} 4 & -5 \\ -13 & 3 \end{vmatrix} = 12 - 65 = -53$

Q.2 The minimum number of zeros in a upper triangular matrix will be-

(A) $\dfrac{n(n-1)}{2}$ (B) $\dfrac{n(n+1)}{2}$

(C) $\dfrac{2n(n-1)}{2}$ (D) None of these

Sol. **[A]**

Minimum no. of zero in a triangular matrix is given by $\dfrac{n(n-1)}{2}$ where n is order of matrix.

∴ Option (A) is correct answer.

Q.3 The total number of matrices formed with the help of 6 different numbers are -

(A) 6 (B) 6! (C) 2(6!) (D) 4 (6!)

Sol. **[D]**

Taking any six digits as 1, 2, 3, 4, 5, 6

No. of matrices = they are either in 6 × 1 or 1 × 6 or 2 × 3 or 3 × 2 matrix form = 4 × (6!)

∴ Option (D) is correct answer.

Q.4 How many matrices can be obtained by using one or more numbers from four given numbers-

(A) 76 (B) 148 (C) 124 (D) None

Sol. **[B]**

Taking any four digits as 1, 2, 3, 4

No. of matrices = taking either one digit or two digits or three digits or four digits

Taking one digit, no. of matrices = $^4C_1$

Taking two digits, no. of matrices

$= {}^4C_2 \times 2! \times 2^*$

(*because they are either in row matrix or column matrix)

Taking three digits, no. of matrices

$= {}^4C_3 \times 3! \times 2^*$

(*because they are either in row matrix or column matrix)

Taking four digits, no. of matrices

$= {}^4C_4 \times 4! \times 3^{@}$

($^{@}$ because they are either 3 × 1 or 1 × 3 or 2 × 2 matrices).

∴ Total no. of matrices

$= {}^4C_1 + {}^4C_2 \times 2! \times 2 + {}^4C_3 \times 3! \times 2 + {}^4C_4 \times 4! \times 3$

$= 4 + \dfrac{4!}{2! \times 2!} \times 2! \times 2 + \dfrac{4!}{1! \times 3!} \times 3! \times 2 + 4! \times 3$

= 4 + 24 + 48 + 72 = 4 + 144 = 148

∴ Option (B) is correct answer.

Questions based on **Multiplication of matrices**

Q.5 If A and B are matrices of order m × n and n × n respectively, then which of the following are defined -

(A) AB, BA (B) AB, A^2

(C) A^2, B^2 (D) AB, B^2

Sol.[D] A × B $\qquad$ B^2

$m \times \boxed{n \qquad n} \times n \qquad n \times \boxed{n \qquad n} \times n$

Q.6 The root of the equation

$$[x \ \ 1 \ \ 2] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ -1 \\ 1 \end{bmatrix} = 0 \text{ is-}$$

(A) 1/3 (B) –1/3 (C) 0 (D) 1

Sol.[A] $[3 \quad x+2 \quad x+1] \begin{bmatrix} x \\ -1 \\ 1 \end{bmatrix} = 0$

3x – (x + 2) + (x + 1) = 0

3x – 1 = 0 ⇒ x = 1/3

Q.7 If A = $\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$ f(A) equals -

(A) $\begin{bmatrix} 14 & -1 \\ 0 & 9 \end{bmatrix}$ (B) $\begin{bmatrix} -14 & 1 \\ 0 & 9 \end{bmatrix}$

(C) $\begin{bmatrix} 14 & 1 \\ 0 & -9 \end{bmatrix}$ (D) None of these

Sol.[A] $2\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} - 3\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$

$2\begin{bmatrix} 4 & 1 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} -6 & 3 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 0 & 18 \end{bmatrix} - \begin{bmatrix} -6 & 3 \\ 0 & 9 \end{bmatrix}$

$= \begin{bmatrix} 14 & -1 \\ 0 & 9 \end{bmatrix}$

Q.8 If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then A^5 equals-

(A) 5A (B) 10A (C) 16A (D) 32 A

Sol. **[C]**

$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^5 = 32\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^5 = 16\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 16A$

$\therefore$ Option (C) is correct answer.

Q.9 If $A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ then A^n equal to-

(A) $\begin{bmatrix} 1 & k^n \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & nk \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} k^n & 1 \\ 0 & 1 \end{bmatrix}$ (D) None of these

Sol. **[B]**

$A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

Since, $A^2 = \begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 3k \\ 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 4k \\ 0 & 1 \end{bmatrix}$

Similarly, $A^n = \begin{bmatrix} 1 & nk \\ 0 & 1 \end{bmatrix}$

$\therefore$ Option (B) is correct answer.

Q.10 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $A^2 - KA - I_2 = 0$, then value of K is -

(A) 4 (B) 2 (C) 1 (D) – 4

Sol. **[A]**

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$; $A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+4 & 2+6 \\ 2+6 & 4+9 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$

$A^2 - kA - I_2 = 0$

$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - k\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 5-k-1 & 8-2k-0 \\ 8-2k-0 & 13-3k-1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 4-k & 8-2k \\ 8-2k & 12-3k \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow k = 4$

$\therefore$ Option (A) is correct answer.

Q.11 If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ then $(A - 2I)(A - 3I)$ equals-

(A) $A^2 + 6I$ (B) I

(C) Zero matrix (D) None of these

Sol. **[C]**

$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$; $(A-2I)(A-3I) = A^2 - 5AI + 6I^2$

$A^2 = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 16-2 & 8+2 \\ -4-1 & -2+1 \end{bmatrix} = \begin{bmatrix} 14 & 10 \\ -5 & -1 \end{bmatrix}$

$\Rightarrow A^2 - 5AI + 6I^2 = A^2 - 5A + 6I$

$= \begin{bmatrix} 14 & 10 \\ -5 & -1 \end{bmatrix} - 5\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 14-20+6 & 10-10+0 \\ -5+5+0 & -1-5+6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

= Null matrix

$\therefore$ Option (C) is correct answer.

Q.12 If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(aI_2 + bA)^2 = A$, then-

(A) $a = b = \sqrt{2}$ (B) $a = b = 1/\sqrt{2}$

(C) $a = b = \sqrt{3}$ (D) $a = b = 1/\sqrt{3}$

Sol. **[B]**

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I_2$$

$(aI_2 + bA)^2 = A$

$\Rightarrow a^2 I_2^2 + b^2 A^2 + 2abAI_2 = A$

$\Rightarrow a^2I_2 + b^2(-I_2) + 2abAI_2 = A$

$\Rightarrow a^2I_2 - b^2I_2 + (2ab - 1) A = 0$

$$\Rightarrow a^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - b^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (2ab - 1) A = 0$$

$$\Rightarrow \begin{bmatrix} a^2 & 0 \\ 0 & a^2 \end{bmatrix} + \begin{bmatrix} -b^2 & 0 \\ 0 & -b^2 \end{bmatrix}$$

$$+ (2ab - 1) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} a^2 & 0 \\ 0 & a^2 \end{bmatrix} + \begin{bmatrix} -b^2 & 0 \\ 0 & -b^2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & (2ab-1) \\ (1-2ab) & 0 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} a^2-b^2+0 & 0+0+2ab-1 \\ 0+0+1-2ab & a^2-b^2+0 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} a^2-b^2 & 2ab-1 \\ 1-2ab & a^2-b^2 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow a^2 - b^2 = 0$ and $2ab - 1 = 0$

$\Rightarrow a = \pm b$ and $2ab - 1 = 0$

$\Rightarrow a = b$ and $2a^2 - 1 = 0$

$\Rightarrow a = b$ and $a = \pm \dfrac{1}{\sqrt{2}}$

Q.13 If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then which of following statement is true -

(A) $A_\alpha . A_\beta = A_{\alpha\beta}$ & $(A_\alpha)^n = \begin{bmatrix} \cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha \end{bmatrix}$

(B) $A_\alpha . A_\beta = A_{\alpha\beta}$ & $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

(C) $A_\alpha . A_\beta = A_{\alpha+\beta}$ & $(A_\alpha)^n = \begin{bmatrix} \cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha \end{bmatrix}$

(D) $A_\alpha . A_\beta = A_{\alpha+\beta}$ & $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

Sol. **[D]**

$$A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A_\alpha . A_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = A_{(\alpha+\beta)}$$

Also, $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

$\therefore$ Option (D) is correct answer.

Questions based on

Transpose of Matrices

Q.14 If A, B are 3×2 order matrices and C is a 2×3 order matrix, then which of the following matrices not defined -

(A) $A^T + B$ (B) $B + C^T$

(C) $A^T + C$ (D) $A^T + B^T$

Sol. **[A]**

$A_{3\times 2} \Rightarrow A^T{}_{2\times 3}$

$B_{3\times 2} \Rightarrow B^T{}_{2\times 3}$

$C_{2\times 3} \Rightarrow C^T{}_{3\times 2}$

$A^T_{2\times 3} + B_{3\times 2}$ Not possible

$B_{3\times 2} + C^T_{3\times 2}$ possible

$A^T_{2\times 3} + C_{2\times 3}$ possible

$A^T_{2\times3} + B^T_{2\times3}$ possible

∴ Option (A) is correct answer.

Q.15 If A is skew symmetric matrix & C is column matrix then C′A C =

(A) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (B) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

(C) [1] (D) [0]

Sol. **[D]**

Let A = $\begin{bmatrix}0 & 1/2 & 1\\-\frac{1}{2} & 0 & -3\\-1 & 3 & 7\end{bmatrix}_{3\times3}$ = –A′

C = $\begin{bmatrix}1\\0\\0\end{bmatrix}$ ⇒ C′ = $[1\ 0\ 0]_{1\times3}$

C′A = $[1\ \ 0\ \ 0]_{1\times3}$. $\begin{bmatrix}0 & 1/2 & 1\\-\frac{1}{2} & 0 & -3\\-1 & 3 & 7\end{bmatrix}_{3\times3}$

⇒ C′A = $[0\ \ 1/2\ \ 1]_{1\times3}$

C′AC = $[0\ \ 1/2\ 1]_{1\times3}$ $\begin{bmatrix}1\\0\\0\end{bmatrix}_{3\times1}$

= $[0 + 0 + 0]_{1\times1}$

= [0]

∴ Option (D) is correct answer.

Questions based on Symmetric & Skew symmetric Matrices & determinant of matrices

Q.16 For any square matrix A, A + A^T will be symmetric matrix then A – A^T will be-

(A) unit matrix

(B) symmetric matrix

(C) skew symmetric matrix

(D) null matrix

Sol. **[C]**

Every square matrix can be uniquely expressed as the sum of symmetric and skew symmetric matrix. i.e. A = $\frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

where $\frac{1}{2}(A + A^T)$ is symmetric part of A

$\frac{1}{2}(A - A^T)$ is skew symmetric part of A.

∴ Option (C) is correct answer

Q.17 If A is square matrix then A + A^T will be-

(A) inverse matrix

(B) skew symmetric matrix

(C) symmetric matrix

(D) unit matrix

Sol. **[C]**

Option (C) is correct answer.

Q.18 Matrix A and transpose matrix A^T then AA^T will be -

(A) Symmetric (B) Inverse matrix

(C) Skew symmetric (D) None of these

Sol. **[A]**

If A is square matrix, then A + A^T, AA^T, A^TA are symmetric matrices.

∴ Option (A) is correct answer.

Q.19 If A is symmetric as well as skew symmetric matrix, then-

(A) A is a diagonal matrix

(B) A is a null matrix

(C) A is a unit matrix

(D) A is a triangular matrix

Sol.[B] $a_{ij} = a_{ji}$ $a_{ij} = -a_{ij}$ and also

$a_{ij} = -a_{ji}$ $a_{ij} = 0$ $a_{ij} = 0$

(i ≠ j) (i = j)

Q.20 If A is a square matrix of order 3, then correct statement is -

(A) det (–A) = – det A

(B) det (–A) = 0

(C) det (A+ I) = I + det A

(D) det 2A = 2 det A

Sol. **[A]**

Option (A) is correct answer.

Questions based on Adjoint of matrix

Q.21 If A = $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 4\\0 & 0 & 2\end{bmatrix}$, then the value of adj (adj A) is -

(A) $|A|^2$ (B) – 2A (C) 2A (D) A^2

Sol.[B] adj(adj A) = $|A|^{n-2}$.A

adj(adj A) = (–2) A = –2A

Q.22 If $A = \begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix}$ then A (adj A) =

(A) –14 I (B) –10A (C) 8 I (D) –1.14 I

Sol. **[A]**

$A = \begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix}$; $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

= Matrix formed by co-factors of above matrix.

Then $C_{11} = -4$; $C_{12} = -1$

$C_{21} = -2$; $C_{22} = 3$

$\therefore C = \begin{bmatrix} -4 & -1 \\ -2 & 3 \end{bmatrix}$

$\Rightarrow$ adj $A = C^1 = \begin{bmatrix} -4 & -2 \\ -1 & 3 \end{bmatrix}$

$\therefore$ A (adj A) = $\begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} -4 & -2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -12-2 & -6+6 \\ -4+4 & -2-12 \end{bmatrix}$

$= \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} = -14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= –14I

$\therefore$ Option (A) is correct answer.

Q.23 If k is a scalar and I is a unit matrix of order 3, then adj (kI) equals-

(A) k^3 I (B) k^2I (C) – k^3 I (D) – k^2 I

Sol. **[B]**

We know,

Adj.(kA) = $k^{(n-1)}$ (adj A)

(where, n is order of matrix A)

$\therefore$ adj (kI) = $k^{(3-1)}$ (adj I)

= k^2I (Θ adj I = I)

$\therefore$ Option (B) is correct answer.

Q.24 The adjoint matrix $\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$ is equals -

(A) $\begin{bmatrix} d_1d_2 & 0 & 0 \\ 0 & d_2d_3 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

(B) $\begin{bmatrix} d_2d_3 & 0 & 0 \\ 0 & d_1d_3 & 0 \\ 0 & 0 & d_1d_2 \end{bmatrix}$

(C) $\begin{bmatrix} d_1d_3 & 0 & 0 \\ 0 & d_2d_3 & 0 \\ 0 & 0 & d_1d_2 \end{bmatrix}$

(D) $\begin{bmatrix} d_1^{-1} & 0 & 0 \\ 0 & d_2d_3 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

Sol. **[B]**

Let $A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

Co-factors can be found as

$C_{11} = \begin{vmatrix} d_2 & 0 \\ 0 & d_3 \end{vmatrix} = d_2d_3$; $C_{12} = -\begin{vmatrix} 0 & 0 \\ 0 & d_3 \end{vmatrix} = 0$;

$C_{13} = \begin{vmatrix} 0 & d_2 \\ 0 & 0 \end{vmatrix} = 0$

$C_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & d_3 \end{vmatrix} = 0$; $C_{22} = \begin{vmatrix} d_1 & 0 \\ 0 & d_3 \end{vmatrix} = d_1d_3$;

$C_{23} = -\begin{vmatrix} d_1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$C_{31} = \begin{vmatrix} 0 & 0 \\ d_2 & 0 \end{vmatrix} = 0$; $C_{32} = -\begin{vmatrix} d_1 & 0 \\ 0 & 0 \end{vmatrix} = 0$;

$C_{33} = \begin{vmatrix} d_1 & 0 \\ 0 & d_2 \end{vmatrix} = d_1d_2$

$\therefore C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} =$

$\begin{bmatrix} d_2d_3 & 0 & 0 \\ 0 & d_1d_3 & 0 \\ 0 & 0 & d_1d_2 \end{bmatrix}$

adj $A = C^T = \begin{bmatrix} d_2d_3 & 0 & 0 \\ 0 & d_1d_3 & 0 \\ 0 & 0 & d_1d_2 \end{bmatrix}$

$\therefore$ Option (B) is correct answer.

Q.25 If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ then |A | | Adj A| is equal to-

(A) a^3 (B) a^6 (C) a^9 (D) a^{27}

Sol. **[C]**

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

= matrix formed by co-factors of above matrix.

$C_{11} = \begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} = a^2$; $C_{12} = -\begin{vmatrix} 0 & 0 \\ 0 & a \end{vmatrix} = 0$;

$C_{13} = \begin{vmatrix} 0 & a \\ 0 & 0 \end{vmatrix} = 0$

$C_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & a \end{vmatrix} = 0$; $C_{22} = \begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} = a^2$;

$C_{23} = -\begin{vmatrix} a & 0 \\ 0 & 0 \end{vmatrix} = 0$

$C_{31} = \begin{vmatrix} 0 & 0 \\ a & 0 \end{vmatrix} = 0$; $C_{32} = -\begin{vmatrix} a & 0 \\ 0 & 0 \end{vmatrix} = 0$;

$C_{33} = \begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} = a^2$

$$\Rightarrow C = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

$$\Rightarrow \text{adj } A = C^T = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

$\Rightarrow |A| = a^3$ and $|\text{adj } A| = a^6$

$\Rightarrow |A| \, . \, |\text{adj} \, . \, A| = a^3.a^6 = a^9$

$\therefore$ Option (C) is correct answer.

Questions based on Inverse of a matrix

Q.26 Matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is not invertible, if -

(A) $\lambda = -15$ (B) $\lambda = -17$

(C) $\lambda = -16$ (D) $\lambda = -18$

Sol.[B] $A = \begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

$|A| = 0, \quad \lambda = -17$

Q.27 If $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X is a matrix such that A = BX, then X equals -

(A) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (B) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (D) None of these

Sol.[B] BX = A

$B^{-1}BX = B^{-1}A$

$X = B^{-1}A$

$X = \frac{\text{adj}(B)}{|B|}.A$

$X = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$

Q.28 If for a matrix A, $A^3 = I$ then A^{-1} equals -

(A) A^2 (B) A

(3) A^3 (D) None of these

Sol. **[A]**

$A^3 = I$

Taking A^{-1} both sides, we get

$A^{-1} A A^2 = A^{-1} I$

$\Rightarrow A^{-1} = A^2$

$\therefore$ Option (A) is correct answer.

Questions based on Special case of matrices

Q.29 Matrix $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is -

(A) Involutary (B) idempotent

(C) nilpotent (D) orthogonal

Sol.[A] $A^2 = I$ so involutary

Q.30 Matrix $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is -

(A) Involutary (B) idempotent

(C) nilpotent (D) orthogonal

Sol.[C] $A^2 = 0 \qquad A \neq 0$

Q.31 If A is an idempotent matrix and I is identity matrix of the same order, then the value of n, $n \in N$ such that $(A + I)^n = I + 127\,A$ is-

(A) n = 7 (B) n = 8 (C) n = 9 (D) n = 3

Sol.[A] Idempotent Matrix $A^2 = A$

$^nC_1 + {}^nC_2 + \ldots\ldots\ldots + {}^nC_n = 127$

$2^n = 128$

$n = 7$

Questions based on System of equation

Q.32 For the equation $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$,

(A) there is only one solution

(B) there exists infinitely many solution

(C) there is no solution

(D) None of these

Sol.[A] $\Delta = 3$, $\Delta_1 = 0$, $\Delta_2 = -3$, $\Delta_3 = 3$

$x = 0$, $y = -1$ $z = +1$

Q.33 The equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$ have -

(A) Unique solution

(B) Infinite many solutions

(C) Inconsistent

(D) None of these

Sol.[A] $\Delta = 3$, $\Delta_1 = 0$, $\Delta_2 = -3$, $\Delta_3 = 3$

$x = 0$, $y = -1$ $z = +1$

➤ Fill in the blanks type questions

Q.34 Let A be a square matrix which satisfies the equation $A^2 = A$, then $(I + A)^4 =$................

Sol. $A^2 = A$

$(I + A)^4 = (I + A)^2 . (I + A)^2$

$= (I^2 + A^2 + 2AI)\,(I^2 + A^2 + 2AI)$

$= (I + A + 2A)\,(I + A + 2A)$

$= (I + 3A)\,(I + 3A)$

$= (I + 3A)^2$

$= I^2 + 9A^2 + 6AI$

$= I + 15\,A$

Q.35 If $A = \begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is a singular matrix then

$\lambda =$ ....

Sol. $A = \begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$

For singular matrix, $|A| = 0$

$\Rightarrow \begin{vmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$

$\Rightarrow 1(40 - 40) - 3\,(20 - 24) + (\lambda + 2)\,(10 - 12) = 0$

$\Rightarrow 0 - 3\,(-4) + (\lambda + 2)\,(-2) = 0$

$\Rightarrow (\lambda + 2)\,(-2) = -12$

$\Rightarrow \lambda + 2 = 6$

$\Rightarrow \lambda = 4$

EXERCISE # 2

Part-A **Only single correct answer type questions**

Q.1 If A and B are square matrices of order 3×3 and $|A| = -1$, $|B| = 3$, then $|3AB|$ equals-

(A) 81 (B) – 81 (C) – 27 (D) – 9

Sol. **[B]**

$|3AB| = 27|AB|$

$= 27\,|A|\,|B|$

$= 27\,(-1) \times 3$

$= -81$

$\therefore$ Option (B) is correct answer.

Q.2 If AB = C; then A, B, C are-

(A) $A_{2\times3}$, $B_{3\times2}$, $C_{2\times3}$

(B) $A_{3\times2}$, $B_{2\times3}$, $C_{3\times3}$

(C) $A_{3\times2}$, $B_{2\times3}$, $C_{3\times2}$

(D) $A_{3\times3}$, $B_{2\times3}$, $C_{3\times3}$

Sol. **[B]**

If AB = C then

Option (B) is correct answer because

$A_{3\times2}.\ B_{2\times3} = C_{3\times3}$

Q.3 If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, I is the unit matrix of the order two & a, b are arbitrary constant then $(aI + bA)^2 =$

(A) $a^2I + ab\,A$ (B) $a^2I + 2abA$

(C) $a^2I + b^2\,A$ (D) None of these

Sol. **[B]**

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(aI + bA)^2 = a^2I^2 + b^2A^2 + 2ab\ IA$

$I^2 = I$ and $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = Null matrix.

$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 1+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = A$

$\therefore (aI + bA)^2 = a^2I + + b.\ 0 + 2abA$

$= a^2I + 2abA$

$\therefore$ Option (B) is correct answer.

Q.4 Which one of the following statements, is true-

(A) Non singular square matrix does not have a unique inverse

(B) Determinant of a non- singular matrix is zero

(C) If $A' = A$, then A is a square matrix

(D) If $|A| \neq 0$, then $|A.\ \text{adj}\ A| = |A|^{(n-1)}$, where $A = [a_{ij}]_{n\times n}$

Sol. **[C]**

If $A' = A$

i.e. order of matrices must be same

i.e. A must be square matrix.

$\therefore$ Option (C) is correct answer

Q.5 the same order such that AB = 0, then -

(A) adj A = 0 or adj B = 0

(B) adj A = 0 and adj B = 0

(C) $|A| = 0$ or $|B| = 0$

(D) None of these

Sol. **[D]**

If AB = 0. It does not mean that A = 0 or B = 0. Again product of two non-zero matrix may be zero matrix.

$\therefore$ Option (D) is correct answer.

Q.6 If $A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$ and AB = I, then B =

(A) $\cos^2\dfrac{\theta}{2}.\ A$ (B) $\cos^2\dfrac{\theta}{2}.\ A^T$

(C) $\cos^2\dfrac{\theta}{2}.\ I$ (D) None of these

Sol. **[B]**

$$A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$$

AB = I

$\Rightarrow B = A^{-1} I$

$= A^{-1} = \frac{\text{adj.}(A)}{|A|}$

Let $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

= matrix formed by cofactors of above matrix.

$C_{11} = 1$; $C_{12} = \tan\theta/2$;

$C_{21} = -\tan\frac{\theta}{2}$; $C_{22} = 1$

$$C = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\Rightarrow \text{adj }(A) = C^T = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$= 1 + \tan^2\frac{\theta}{2} = \sec^2\frac{\theta}{2}$$

$$\therefore B = A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \times \frac{1}{\sec^2\frac{\theta}{2}}$$

$$\Rightarrow B = \cos^2\frac{\theta}{2} \times \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$= \cos^2\theta/2 \times A^T$

$\therefore$ Option (B) is correct answer.

Q.7 If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then $(A^{-1})^3$ is equal to –

(A) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ (B) $\frac{1}{27}\begin{bmatrix} -1 & 26 \\ 0 & 27 \end{bmatrix}$

(C) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & -27 \end{bmatrix}$ (D) $\frac{1}{27}\begin{bmatrix} -1 & -26 \\ 0 & -27 \end{bmatrix}$

Sol. **[A]**

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

= Matrix formed by cofactors of matrix A.

$\therefore C_{11} = 1$; $C_{12} = 0$; $C_{21} = -2$; $C_{22} = 3$

$$\Rightarrow C = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$$

$$\Rightarrow C^T = \text{adj }(A) = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$\therefore A^{-1} = \frac{\text{adj }(A)}{|A|} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$(A^{-1})^2 = \frac{1}{9}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1-0 & -2-6 \\ 0+0 & 0+9 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}$$

$$(A^{-1})^3 = \frac{1}{27}\begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$= \frac{1}{27}\begin{bmatrix} 1-0 & -2-24 \\ 0+0 & 0+27 \end{bmatrix}$$

$$\Rightarrow (A^{-1})^3 = \frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$$

$\therefore$ Option (A) is correct answer.

Q.8 The solution of the equation

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ is-}$$

(A) x =1, y =1, z =1 (B) x = – 1, y = 0, z = 2

(C) x = –1, y =2, z =2 (D) x = 0, y = –1, z =2

Sol. **[B]**

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x+0+z \\ -x+y+0 \\ 0-y+z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow x + z = 1$

$-x + y = 1$

$-y + z = 2$

$$\Rightarrow \begin{array}{r} y+z=2 \\ -y+z=2 \\ \hline 2z=4 \end{array}$$

$\Rightarrow z = 2$

$\Rightarrow x = -1$ and $y = 0$

$\therefore$ Option (B) is correct answer.

Q.9 Let three matrices

$A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$,

then $\text{tr}(A) + \text{tr}\left(\frac{ABC}{2}\right) + \text{tr}\left(\frac{A(BC)^2}{4}\right) + \text{tr}\left(\frac{A(BC)^3}{8}\right) + \ldots\ldots\ldots\infty =$

(A) 6 (B) 9 (C) 12 (D) None

Sol.[A] BC = I

$\text{tr}(A) + \text{tr}(A/2) + \text{tr}(A/4) + \ldots\ldots\ \infty$

$3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \ldots\ldots\ \infty$

$= \frac{3}{1-\frac{1}{2}} = 6$

Q.10 Let the matrix A and B be defined as

$A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$, then the value of Det $(2A^9B^{-1})$ is

(A) 2 (B) 1 (C) – 1 (D) –2

Sol.[D] Det $(2A^9B^{-1}) = 2^2\ |A|^9\ |B^{-1}|$

$= 2^2|A|^9\ |B|^{-1}$

$= 2^2(-1) \times \frac{1}{2} = -2$

Q.11 Consider the matrices $A = \begin{bmatrix} 4 & 6 & -1 \\ 3 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$,

$B = \begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$. Out of the given matrix products

(i) $(AB)^T C$ (ii) $C^T C(AB)^T$

(iii) C^TAB (iv) $A^T ABB^TC$

(A) exactly one is defined

(B) exactly two are defined

(C) exactly three are defined

(D) all four are defined

Sol.[C] (A) $(AB)^T.C = (2 \times 3) . (3 \times 1) = 2 \times 1$ possible

(B) $C^TC(AB)^T = (1 \times 3).(3 \times 1).(2 \times 3)$

$= (1 \times 1).(2 \times 3)$ not possible.

(C) $C^TAB = (1 \times 3).(3 \times 2) = (1 \times 2)$ possible

(D) $A^T AB B^TC = (A^T \times A). (B.B^T) (C)$

$= (3 \times 3) . (3 \times 3) . (3 \times 1)$

$= (3 \times 3) . (3 \times 1) = 3 \times 1$

possible

Q.12 **Statement-I:** If A is an invertible 3×3 matrix and B is a 3×4 matrix, then A^{-1} B is defined.

Statement-II: It is never true that A + B, A – B and AB are all defined

Statement-III: Every matrix none of whose entries are zero is invertible

Statement-IV: Every invertible matrix is square and has no two rows the same

(A) TFFF (B) TTFF (C) TFFT (D) TTTF

Sol.[C] Theoretical point.

Q.13 P is an orthogonal matrix and A is a periodic matrix with period 4 and $Q = PAP^T$ then $X = P^TQ^{2005}P$ will be equal to

(A) A (B) A^2 (C) A^3 (D) A^4

Sol.[A] P is an orthogonal matrix

$P.P^T = I$	$X = P^TQ^{2005}P$
$Q = PAP^T$	$X = P^T.PA^{2005}P^T.P.$
$Q^2 = (PA.P^T)\ (P.AP^T)$	$X = A^{2005}$
$Q^2 = PA^2P^T$	period is 4
M	X = A
$Q^{2005} = PA^{2005}P^T$	

Q.14 If the value of a fourth order determinant is 3, then the value of the determinant formed by the co-factors, is

(A) 27 (B) 64 (C) 1 (D) 81

Sol.[A] 27

Part-B One or more than one correct answer type questions

Q.15 If D_1 & D_2 are two 3×3 diagonal matrices, then-

(A) D_1D_2 is a diagonal matrix

(B) $D_1D_2 = D_2D_1$

(C) $D_1^2 + D_2^2$ is a diagonal matrix

(D) none of these

Sol. **[A, B, C]**

Let $D_1 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix}_{3\times3}$

$D_2 = \begin{bmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix}_{3\times3}$

$D_1D_2 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix}\begin{bmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix}$

$= \begin{bmatrix} a_1a_2+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+b_1b_2+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+c_1c_2 \end{bmatrix}$

$= \begin{bmatrix} a_1a_2 & 0 & 0 \\ 0 & b_1b_2 & 0 \\ 0 & 0 & c_1c_2 \end{bmatrix}$ = diagonal matrix

$D_2D_1 = \begin{bmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix}\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix}$

$= \begin{bmatrix} a_1a_2 & 0 & 0 \\ 0 & b_1b_2 & 0 \\ 0 & 0 & c_1c_2 \end{bmatrix}$ = diagonal matrix.

$\Rightarrow D_1D_2 = D_2D_1$

$D_1^2 + D_2^2 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix}\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix}$

$+ \begin{bmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix}\begin{bmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix}$

$= \begin{bmatrix} a_1^2 & 0 & 0 \\ 0 & b_1^2 & 0 \\ 0 & 0 & c_1^2 \end{bmatrix} + \begin{bmatrix} a_2^2 & 0 & 0 \\ 0 & b_2^2 & 0 \\ 0 & 0 & c_2^2 \end{bmatrix}$

$= \begin{bmatrix} a_1^2+a_2^2 & 0 & 0 \\ 0 & b_1^2+b_2^2 & 0 \\ 0 & 0 & c_1^2+c_2^2 \end{bmatrix}$

= diagonal matrix.

Hence, options (A), (B) and (C) are correct answers.

Q.16 If AB = A and BA = B, then -

(A) $A^2B = A^2$ (B) $B^2A = B$

(C) ABA = A (D) BAB = B

Sol.[A,B,C,D]

Q.17 If $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ then $\lim\limits_{n\to\infty} \frac{1}{n} A^n$ is -

(A) $\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ (D) does not exist

Sol. **[A]**

$A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$

$A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & a+a \\ 0+0 & 0+1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$

$A^3 = A^2.\, A = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & a+2a \\ 0+0 & 0+1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$

Similarly, $A^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$

$\therefore \lim\limits_{n\to\infty} \frac{1}{n} A^n = \lim\limits_{n\to\infty} \frac{1}{n}\begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$

$$= \lim_{n\to\infty}\begin{pmatrix} 1/n & a \\ 0 & 1/n \end{pmatrix} = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$$

$\therefore$ Option (A) is correct answer.

Q.18 Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ then -

(A) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \ \forall\ n \in N$

(B) $\lim\limits_{n\to\infty} \dfrac{1}{n} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

(C) $\lim\limits_{n\to\infty} \dfrac{1}{n^2} A^{-n} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) none of these

Sol. **[A, B, C]**

$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

= matrix formed by cofactors of A.

$C_{11} = 1; C_{12} = -1; C_{21} = 0; C_{22} = 1$

$\therefore C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\therefore$ adj $A = C^T = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$|A| = 1 - 0 = 1$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}}{1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$A^{-2} = A^{-1}.\ A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ -1-1 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$A^{-3} = A^{-2}.\ A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ -2-1 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

Similarly, $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

$$\lim_{n\to\infty} \frac{1}{n} A^{-n} = \lim_{n\to\infty} \frac{1}{n}\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$$

$$= \lim_{n\to\infty}\begin{bmatrix} 1/n & 0 \\ -1 & 1/n \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

Also, $\lim\limits_{n\to\infty} \dfrac{1}{n^2} A^{-n} = \lim\limits_{n\to\infty} \dfrac{1}{n^2}\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

$$= \lim_{n\to\infty}\begin{bmatrix} 1/n^2 & 0 \\ -\frac{1}{n} & 1/n^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ Options (A), (B) and (C) are correct answers.

Q.19 If A and B are square matrices of the same order such that $A^2 = A$, $B^2 = B$, $AB = BA = 0$, then

(A) $AB^2 = 0$

(B) $(A + B)^2 = A + B$

(C) $(A - B)^2 = A - B$

(D) none of these

Sol. **[A, B]**

Given $A^2 = A$, $B^2 = B$

$AB = BA = 0$

$\Rightarrow AB^2 = AB = 0$

$(A + B)^2 = A^2 + B^2 + BA + AB$

$= A + B$

$\therefore$ Option (A) and (B) are correct answers.

Q.20 If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then -

(A) $a + d = 0$ (B) $k = -|A|$

(C) $k = |A|$ (D) none of these

Sol. **[A, C]**

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^2 + k = 0 \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} + k = 0$$

$$\Rightarrow k + \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = 0$$

$$\Rightarrow k.I + \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} a^2+bc+k & ab+bd \\ ac+cd & bc+d^2+k \end{bmatrix} = 0$$

$\Rightarrow a^2 + bc + k = 0;\ ab + bd = 0$

$ac + cd = 0$ and $bc + d^2 + k = 0$

$\Rightarrow b(a + d) = 0$ and $c(a + d) = 0$

$\Rightarrow bc\ (a + d)^2 = 0$

$\Rightarrow bc \neq 0$ and $a + d = 0$

Also, $bc + d^2 + k = 0 \Rightarrow bc + d\,(-a) + k = 0$

$$\Rightarrow k = ad - bc = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = |A|$$

Hence, Option (A) and (C) are correct answer.

Q.21 If $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then -

(A) $|A| = 2$

(B) A is non-singular

(C) Adj. A = $\begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$

(D) A is skew symmetric matrix

Sol. **[B, C]**

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix} = \frac{\text{adj}\,A}{|A|}$$

Here, $|A| \neq 0$ i.e. A must be non-singular.

Q.22 Let A = $\begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$,

then A^{-1} exists if -

(A) $x \neq 0$ (B) $\lambda \neq 0$

(C) $3x + \lambda \neq 0,\ \lambda \neq 0$ (D) $x \neq 0,\ \lambda \neq 0$

Sol. **[C]**

$$A = \begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$$

$A^{-1} = \dfrac{\text{adj}\,A}{|A|}$ exists if $|A| \neq 0$

$$\therefore |A| = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$

$= (x + \lambda)\,(x^2 + \lambda^2 + 2\lambda x - x^2) - x(x^2 + \lambda x - x^2)$

$+ x\,(x^2 - x^2 - \lambda x)$

$= (x + \lambda)\,(\lambda^2 + 2\lambda x) - x\,(\lambda x) + x(-\lambda x)$

$= x\lambda^2 + 2\lambda x^2 + \lambda^3 + 2\lambda^2 x - \lambda x^2 - \lambda x^2$

$= \lambda^3 + 3x\lambda^2 = \lambda^2\,(\lambda + 3x) \neq 0$

$\Rightarrow \lambda \neq 0$ and $3x + \lambda \neq 0$

$\therefore$ Option (C) is correct answer.

Q.23 A square matrix A with elements from the set of real numbers is said to be orthogonal if $A' = A^{-1}$. If A is an orthogonal matrix, then -

(A) A' is orthogonal

(B) A^{-1} is orthogonal

(C) Adj. A = A'

(D) $|A^{-1}| = 1$

Sol. **[A, B]**

If A is orthogonal, then $AA' = I$

$\Rightarrow A' = A^{-1}$

Since, A is orthogonal, then A' is also orthogonal matrix i.e. A^{-1} must be orthogonal matrix.

Q.24 Let A, B and C be 2 × 2 matrices with entries from the set of real numbers. Define * as follows $A * B = \dfrac{AB + BA}{2}$, then

(A) A * B = B * A

(B) A *A = A^2

(C) A * (B + C) = A * B + A * C

(D) A * I = A

Sol.[A,B,C,D]

Part-C Assertion-Reason type questions

The following questions 25 to 27 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) **If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.**

(C) **If Assertion is true but the Reason is false.**

(D) **If Assertion is false but Reason is true**

Q.25 **Assertion :** There are only finitely many 2 ×2 matrices which commute with the matrix $\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

Reason : If A is non-singular then it commutes with I, Adj A and A^{-1}.

Sol. **[D]**

Since, matrices holds good for commutative law whether it is 2 × 2 or 3 × 3 or higher orders.

∴ Assertion is not correct. While reason is correct.

∴ Option (D) is correct answer

Q.26 **Assertion :** If A is a skew symmetric of order 3 then its determinant should be zero.

Reason : If A is square matrix then

$$\det A = \det A' = \det (-A')$$

Sol. **[C]**

Since, A is skew symmetric matrix

Let $A = \begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{vmatrix}$

$= 0\,(0 + f^2) + h(0 + gf) - g(hf - 0)$

$= hgf - hgf = 0$

Hence assertion is correct.

Reason :

Since, A is square matrix (other than skew symmetric)

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}_{3\times 3}$

$A' = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix}_{3\times 3}$

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = 1\,(-1-0) -2(1-0) + 3(\,2-0)$

$= -1 -2 + 6$

$= 3$

$|A'| = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 2 \\ 3 & 0 & 1 \end{vmatrix} = 1\,(-1-0)-1(2-6) + 0\,(0 + 3)$

$= -1 + 4 = 3$

$|-A'| = -\,|A'| = -3$

Since, $|A| = |A'| \ne |-A'|$

∴ Reason is not correct.

∴ Option (C) is correct answer.

Q.27 **Assertion :** If a, b, c are distinct and x, y, z are not all zero given that ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz = 0 then $a + b + c \ne 0$

Reason : $a^2 + b^2 + c^2 > ab + bc + ca$ if a, b, c are distinct

Sol. **[D]**

$ax + by + cz = 0$

$bx + cy + az = 0$

$cx + ay + bz = 0$

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a\,(bc - a^2) - b(b^2 - ca) + c\,(ab - c^2)$

$= abc - a^3 - b^3 + abc + abc - c^3$

$= 3abc - a^3 - b^3 - c^3$

$= (ab + bc + ca - a^2 - b^2 - c^2)\,(a + b + c) = 0$

$\Rightarrow$ Either $(a + b + c) = 0$

or $(ab + bc + ca - a^2 - b^2 - c^2) = 0$

or $(a + b + c) = 0$ and

$(ab + bc + ca - a^2 - b^2 - c^2) = 0$

Also, from the property of A.M .≥ G.M.

$\frac{a^2 + b^2}{2} \ge ab\ ;\ \frac{b^2 + c^2}{2} \ge bc\ ;\ \frac{c^2 + a^2}{2} \ge ca$

Adding above three terms, we get

$$\frac{2(a^2+b^2+c^2)}{2} \geq (ab + bc + ca)$$

$\Rightarrow (a^2 + b^2 + c^2) \geq (ab + bc + ca)$

$\therefore$ Assertion is false but reason is true.

$\therefore$ Option (D) is correct answer.

Part-D Column Matching type questions

Q.28 Let $a_k = {}^nC_k$ for $0 \leq k \leq n$ and $A_k = \begin{bmatrix} a_{k-1} & 0 \\ 0 & a_k \end{bmatrix}$

and $B = \sum_{k=1}^{n-1} A_k.A_{k+1} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$,

Column-I	**Column-II**
(A) a	(P) $\frac{2n}{n+1}({}^{2n}C_n)$
(B) a – b	(Q) 0
(C) a + b	(R) ${}^{2n}C_{n+1}$
(D) $\frac{a}{b}$	(S) 1

Sol. A → R ; B → Q ; C → P ; D → S

$a_k = {}^nC_k$

$$A_k.A_{k+1} = \begin{bmatrix} a_{k-1} & 0 \\ 0 & a_k \end{bmatrix}\begin{bmatrix} a_k & 0 \\ 0 & a_{k+1} \end{bmatrix}$$

$$A_k.A_{k+1} = \begin{bmatrix} a_k a_{k-1} & 0 \\ 0 & a_k.a_{k+1} \end{bmatrix}$$

$$\sum_{k=1}^{n-1} A_k.A_{k+1} =$$

$$\begin{bmatrix} C_0C_1 + C_1C_2 + ... + C_{n-1}C_{n-2} & 0 \\ 0 & C_1C_2 + C_2C_3 + ... + C_{n-1}C_n \end{bmatrix}$$

$a = C_0C_1 + C_1C_2 + ... + C_{n-1}\,C_{n-2} = {}^{2n}C_{n-1} - n$

$b = aC_2 + ... + C_{n-1}\,C_n = {}^{2n}C_{n-1} - n$

(1) $a = b = {}^{2n}C_{n-1} - n$

(2) $a/b = 1$

(3) $a + b = 2 \,.\, {}^{2n}C_{n-1} - 2n$

$= 2[{}^{2n}C_{n+1} - n\,]$

(4) $a - b = 0$

Q.29 Using n distinct real numbers matrices each having distinct elements are to be used in making matrices of all possible order then possible arrangements are

Column-I	**Column-II**
(A) n = 4	(P) 2880 possible matrices
(B) n = 3	(Q) 240 possible matrices
(C) n = 6	(R) 12 possible matrices
(D) n = 5	(S) 72 possible matrices

Sol. **A → S, B → R, C → P, D → Q**

(A) n = 4

${}^4C_3 \times 3 \times 4! = 72$

(B) n = 3

${}^3C_3 \times 2 \times 3! = 12$

(C) n = 6

${}^6C_6 \times 4 \times 6! = 2880$

(D) n = 5

${}^5C_5 \times 2 \times 5\,! = 240$

Q.30 Let $A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} \lambda - 2c & -2c \\ c & \lambda \end{bmatrix}$,

$C = \begin{bmatrix} \sqrt{1-\lambda\mu} & \lambda \\ \mu & -\sqrt{1-\lambda\mu} \end{bmatrix}$, be matrices then match the following:

Column-I	**Column-II**
(A) AX = XA for X =	(P) A
(B) X^2 = I for X =	(Q) C
(C) X^2 = – I for X =	(R) none of A,B,C
(D) X^2 = X for X =	(S) B

Sol. **[A → S; B → Q; C → P; D → R]**

(A) A × A = A × A

$$AB = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} \lambda - 2c & -2c \\ c & \lambda \end{bmatrix}$$

$$AB = \begin{bmatrix} \lambda & -2c + 2\lambda \\ -\lambda + c & 2c - \lambda \end{bmatrix}$$

$$BA = \begin{bmatrix} \lambda - 2c & -2c \\ c & \lambda \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

$$BA = \begin{bmatrix} \lambda & 2\lambda - 2c \\ c - \lambda & 2c - \lambda \end{bmatrix}$$

(B) $X^2 = I$ X = c

$$X^2 = \begin{bmatrix} \sqrt{1-\lambda\mu} & \lambda \\ \mu & -\sqrt{1-\lambda\mu} \end{bmatrix}\begin{bmatrix} \sqrt{1-\lambda\mu} & \lambda \\ \mu & -\sqrt{1-\lambda\mu} \end{bmatrix}$$

$$X^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(C) $X^2 = -I$

$$A \times A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

(D) $X^2 = X$. None for A, B, C.

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Let A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ where $a \neq 0$. Show that for $n \geq 0$, $A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{(a-1)} \\ 0 & 1 \end{bmatrix}$.

Sol. Let A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a^2 + 0 & ab + b \\ 0 + 0 & 0 + 1 \end{bmatrix}$

$= \begin{bmatrix} a^2 & ab + b \\ 0 & 1 \end{bmatrix}$

$A^3 = A^2 . A$

$= \begin{bmatrix} a^2 & ab + b \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a^3 + 0 & ba^2 + ab + b \\ 0 + 0 & 0 + 1 \end{bmatrix}$

$= \begin{bmatrix} a^3 & b + ab + ba^2 \\ 0 & 1 \end{bmatrix}$

$A^4 = A^3 . A = \begin{bmatrix} a^3 & b + ab + ba^2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a^4 + 0 & ba^3 + b + ab + a^2 b \\ 0 + 0 & 0 + 1 \end{bmatrix}$

$= \begin{bmatrix} a^4 & b + ab + a^2 b + ba^3 \\ 0 & 1 \end{bmatrix}$

M

M

Similarly,

$A^n = \begin{bmatrix} a^4 & b + ab + a^2 b + ba^3 + ... + ba^{n-1} \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a^4 & b(1 + a + a^2 + ... + a^{n-1}) \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^n = \begin{bmatrix} a^n & b\frac{(a^{n-1} - 1)}{(a-1)} \\ 0 & 1 \end{bmatrix}$

Hence proved.

Q.2 Suppose a matrix A satisfies $A^2 - 5A + 7I = 0$. If $A^8 = aA + bI$, find a.

Sol.[1265]

Q.3 Find the value of adj (P^{-1}) in terms of P where P is a non-singular matrix and hence show that

adj $(Q^{-1} BP^{-1})$ = PAQ,

given that adj B = A and |P| = |Q| = 1

Sol. adj (P^{-1}) = adj $\left(\frac{\text{adj } P}{|P|}\right)$

= adj adj P

= $|P|^{(n-2)}$ P (Assume order of P is n) = P

Also, adj $(Q^{-1} BP^{-1})$ = (adj P^{-1}) adj Q^{-1}B

$= \left(\text{adj}\left(\frac{\text{adj } P}{|P|}\right)\right)$ adj B adj Q^{-1}

$\Rightarrow |P|^{n-2}$. P.A. adj $\left(\frac{\text{adj } Q}{|Q|}\right)$

= PA. $|Q|^{n-2}$Q

= PAQ. Hence proved.

Q.4 Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix, then for $n \geq 2$, find the value of A^n.

Sol. $A^2 = 2A - I$ ... (1)

Multiplying by A in (1), we get

$A^3 = 2A^2 - IA$

$= 2(2A - I) - IA$

$= 4A - 2I - A$

$A^3 = 3A - 2I$ ...(2)

Multiplying by A in (2), we get

$A^4 = 3A^2 - 2AI$

$= 3(2A - I) - 2AI$

$= 6A - 3I - 2A$

$A^4 = 4A - 3I \quad \ldots (3)$

Multiplying by A in (3), we get

$A^5 = 4A^2 - 3AI$

$= 4(2A - I) - 3A$

$= 8A - 4I - 3A$

$= 5A - 4I \quad \ldots (4)$

Multiplying by A in (4), we get

$A^6 = 5A^2 - 4AI$

$= 5A^2 - 4A$

$= 5(2A - I) - 4A$

$= 10A - 5I - 4A$

$= 6A - 5I$

M

M

Similarly, $A^{(n-1)} = (n-1)A - (n-2)I$

Now, multiplying by A in above equation, we get

$A^n = (n-1)A^2 - (n-2)AI$

$= (n-1)(2A - I) - (n-2)A$

$= (n-1)2A - (n-1)I - (n-2)I$

$= A(2n - 2 - n + 2) - (n-1)I$

$= A.\ n - (n-1)I$

$\Rightarrow A^n = nA - (n-1)I$

Q.5 Discuss for all values of λ, the system of equations :

$x + y + 4z = 6$

$x + 2y + 2z = 6$

$\lambda x + y + z = 6$

as regards existence and nature of solutions

Sol. $x + y + 4z = 6$

$x + 2y + 2z = 6$

$\lambda x + y + z = 6$

$Ax = B$

$$\begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ \lambda & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ \lambda & 1 & 1 \end{bmatrix};\ B = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$$

$$C = [A : B] = \begin{bmatrix} 1 & 1 & 4 & 6 \\ 1 & 2 & 2 & 6 \\ \lambda & 1 & 1 & 6 \end{bmatrix}$$

$R_3 \to 2R_3 - R_2$

$$C = \begin{bmatrix} 1 & 1 & 4 & 6 \\ 1 & 2 & 2 & 6 \\ (2\lambda - 1) & 0 & 0 & 6 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$C = \begin{bmatrix} 1 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ (2\lambda - 1) & 0 & 0 & 6 \end{bmatrix}$$

If rank of A = rank of C = 3

Then there will be unique solution and

$(2\lambda - 1) \neq 0 \Rightarrow \lambda \neq 1/2$

Q.6 Using matrix method find the values of λ and μ so that the system of equations.

$2x - 3y + 5z = 12$

$3x + y + \lambda z = \mu$

$x - 7y + 8z = 17$

(a) unique solution

(b) infinite solution (c) no solution.

Sol. $2x - 3y + 5z = 12$

$3x + y + \lambda z = \mu$

$x - 7y + 8z = 17$

AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 1 & \lambda \\ 1 & -7 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ \mu \\ 17 \end{bmatrix}$$

$$C = [A : B] = \begin{bmatrix} 2 & -3 & 5 & 12 \\ 3 & 1 & \lambda & \mu \\ 1 & -7 & 8 & 17 \end{bmatrix}$$

$R_1 \to R_2 - R_1$

$$C = \begin{bmatrix} 1 & 4 & (\lambda-5) & (\mu-12) \\ 3 & 1 & \lambda & \mu \\ 1 & -7 & 8 & 17 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$$C = \begin{bmatrix} 1 & 4 & (\lambda-5) & (\mu-12) \\ 0 & -11 & (15-2\lambda) & (36-2\mu) \\ 1 & -7 & 8 & 17 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$C = \begin{bmatrix} 1 & 4 & (\lambda-5) & (\mu-12) \\ 0 & -11 & (15-2\lambda) & (36-2\mu) \\ 0 & -11 & (13-\lambda) & (29-\mu) \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$C = \begin{bmatrix} 1 & 4 & (\lambda-5) & (\mu-12) \\ 0 & -11 & (15-2\lambda) & (36-2\mu) \\ 0 & 0 & (\lambda-2) & (7-\mu) \end{bmatrix}$$

(a) for unique solution

Rank of A = rank of C = r = n

where, r = Rank

n = order of matrix

$\therefore \lambda \neq 2$

(b) For infinite solution

Rank of A = Rank of C = r < n

i.e. $\lambda = 2$ and $\mu = 7$

(c) For no solution

Rank of A $\neq$ Rank of C

$\therefore \lambda = 2$ and $\mu \neq 7$

Q.7 Given $A = \begin{bmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{bmatrix}$ For what values of α does A^{-1} exists. Find A^{-1} & prove that $A^{-1} = A^2 - 6A + 11I$ when $\alpha = 1$.

Sol. $|A| \neq 0$

$\begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix} \neq 0 \Rightarrow \alpha(-5\alpha + 6) \neq 0 \Rightarrow \alpha \neq 0$ and $\alpha \neq 6/5$

Now $\alpha = 1$ then $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \Rightarrow |A| = 1$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Now $A^{-1} = A^2 - 6A + 11I$

$I = A^3 - 6A^2 + 11.A$

Put value of A

Then R.H.S = L.H.S

Q.8 Solve the following systems of linear equations by matrix method.

(i) $2x - y + 3z = 8$
$-x + 2y + z = 4$
$3x + y - 4z = 0$

(ii) $x + y + z = 9$
$2x + 5y + 7z = 52$
$2x + y - z = 0$

Sol. $AX = B \Rightarrow X = A^{-1}B$

$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}$ $\quad A^{-1} = \frac{\text{adj}(A)}{|A|} = -\frac{1}{38}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$

Q.9 If the following system of equations $(a - t)x + by + cz = 0$, $bx + (c - t)y + az = 0$ and $cx + ay + (b - t)z = 0$ has non-trivial solutions for different values of t then show that we can express product of these values of t in the form of determinant

Sol. homogenous equation so for non trivial solution

$$\begin{vmatrix} a-t & b & c \\ b & c-t & a \\ c & a & b-t \end{vmatrix} = 0$$

$(a - t) [(c - t) (b - t) - a^2] - b[b(b - t) - ac] + c[ab - c(c - t)] = 0$

$-t^3 + t^2(a + b + c) - t(ab + bc + ca - a^2 - b^2 - c^2) - a^3 - b^3 - c^3 + 3abc = 0$

product of roots $= a^3 + b^3 + c^3 - 3abc$

$$= -\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Q.10 Matrices A and B satisfy $AB = B^{-1}$ where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$. Find

(i) without finding B^{-1}, the value of k for which $KA - 2B^{-1} + I = 0$

(ii) without finding A^{-1}, then matrix X satisfying $A^{-1} XA = B$

(iii) The matrix A, using A^{-1}

Sol. (i) $kA - 2B^{-1} + I = 0$

$k(AB) - 2(B^{-1}B) + B = 0$

$k B^{-1} - 2I + B = 0$

$kI - 2B + B^2 = 0$

$kI = 2B - B^2$ compare $k = 2$

(ii) $A^{-1} XA = B$

$XA = AB$

$X = AB A^{-1}$

$X = B^{-1}A^{-1}$

$X = (AB)^{-1}$

$X = (B^{-1})^{-1} = B$

(iii) $AB = B^{-1}$

$AB^2 = I$

$AA^{-1} = I$

Then $A^{-1} = B^2$

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$$

$AA^{-1} = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Compare

$a = -\dfrac{1}{2}$ $b = \dfrac{1}{2}$

$c = -1$ $d = \dfrac{1}{2}$

Q.11 If A is a skew symmetric matrix and I + A is non singular, then prove that the matrix $B = (I - A) (I + A)^{-1}$ is an orthogonal matrix.

Use this to find a matrix B given $A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$

Sol. A is skew symmetric matrix $A^T = -A$

$BB^T = (I - A) (I + A)^{-1} [(I - A) (I + A)^{-1}]^T$

$BB^T = (I - A) (I + A)^{-1} [(I + A)^T]^{-1} (I - A)^T$

$BB^T = (I - A) (I + A)^{-1} (I - A)^{-1} (I + A)$

$BB^T = (I - A) [(I - A) (I + A)]^{-1} (I + A)$ $[(I - A) (I + A) = (I + A) (I - A)]$

$BB^T = (I - A) [(I + A) (I - A)]^{-1} (I + A)$

$BB^T = (I - A) (I - A)^{-1} (I + A)^{-1} (I + A)$

$BB^T = I \times I = I$

Q.12 Given $A = \begin{bmatrix} -1 & 0 \\ k & -2 \end{bmatrix}$, where k is only integer show that $A^2 + 3A + 2I = 0$ and use this result to find out matrices B and C such that $A = B^3 + C^3$

Sol. $A^2 + 3A + 2I = 0$

$A^3 + 3A^2 + 2A = 0$

$A^3 + 3A^2 + 3A + I^3 = A + I^3$

$(A + I)^3 - I^3 = A$

$A = (A + I)^3 + (-I)^3$

$B = A + I$

$C = -I$

Q.13 Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $P = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Such that AP = P and a + d = 5050. Find the value of (ad – bc)

Sol. AP = P

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

$ap + bq = p$ ....(1)

$cp + dq = q$ ....(2)

both equation for value of p/q

1 – (a + d) + ad = bc

ad – bc = (a + d) –1

ad – bc = 5050 – 1 = 5049

Part-B Passage based objective questions

Passage I (Question 14 to 16)

Let A and B are two matrices of same order i.e. 3 × 3 where

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & K & 5 \\ 4 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

On the basis of above information, answer the following questions-

Q.14 If matrix 2A + 3 B is singular, then the value of K is -

(A) –3 (B) $-\dfrac{40}{17}$

(C) $-\dfrac{42}{13}$ (D) None of these

Sol. **[B]**

$$2A + 3B = 2\begin{bmatrix} 1 & -3 & 2 \\ 2 & k & 5 \\ 4 & 2 & 1 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 4 \\ 4 & 2k & 10 \\ 8 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 3 & 9 \\ 12 & 6 & 12 \\ 9 & 9 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6 & -6+3 & 4+9 \\ 4+12 & 2k+6 & 10+12 \\ 8+9 & 4+9 & 2+15 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 & 13 \\ 16 & 2k+6 & 22 \\ 17 & 13 & 17 \end{bmatrix}$$

$$\therefore |2A + 3B| = \begin{vmatrix} 8 & -3 & 13 \\ 16 & 2k+6 & 22 \\ 17 & 13 & 17 \end{vmatrix} = 0$$

{Because 2A + 3B is a singular matrix)

$\Rightarrow$ 8[34k + 102 –286] + 3 [16 × 17 – 17 × 22] + 13 [16 × 13 –34k –102] = 0

$\Rightarrow$ 8 [34K – 184] + 3 × 17 × (–6) + 13(208 – 102) –13 × 34 K = 0

$\Rightarrow$ 34K (8 –13) = 8 × 184 + 17 × 18 – 13 × 106

$\Rightarrow$ 34K (–5) = 1472 + 306 –1378

= 1778 –1378

$$\Rightarrow K = \frac{400}{-34\times 5} = -\frac{40}{17}$$

$\therefore$ Option (B) is correct answer.

Q.15 If K = 2 then tr(AB) + tr(BA) is equal to -

(A) 66 (B) 42 (C) 84 (D) 63

Sol. **[C]**

For k = 2

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 2 & 5 \\ 4 & 2 & 1 \end{bmatrix};\ B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 2 & 5 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-12+6 & 1-6+6 & 3-12+10 \\ 4+8+15 & 2+4+15 & 6+8+25 \\ 8+8+3 & 4+4+3 & 12+8+5 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 1 & 1 \\ 27 & 21 & 39 \\ 19 & 11 & 25 \end{bmatrix}$$

$\therefore$ trace of (AB) = tr. (AB)

= sum of diagonal element

= – 4 + 21 + 25

= –4 + 46

= 42

$$\text{Also, } BA = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & -3 & 2 \\ 2 & 2 & 5 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2+12 & -6+2+6 & 4+5+3 \\ 4+4+16 & -12+4+8 & 8+10+4 \\ 3+6+20 & -9+6+10 & 6+15+5 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 2 & 12 \\ 24 & 0 & 22 \\ 29 & 7 & 26 \end{bmatrix}$$

$\therefore$ trace of (BA) = tr. (BA)

= sum of diagonal elements

= 16 + 0 + 26 = 42

$\therefore$ tr. (AB) + tr. (BA) = 42 + 42

= 84

$\therefore$ Option (C) is correct answer.

Q.16 If C = A – B and tr (C) = 0 then K is equal to -

(A) 5 (B) –5 (C) 7 (D) –7

Sol. **[C]**

$$C = A - B = \begin{bmatrix} 1 & -3 & 2 \\ 2 & k & 5 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 & -1 \\ -2 & k-2 & 1 \\ 1 & -1 & -4 \end{bmatrix}$$

$\therefore$ trace of C = tr. (C) = 0

$\Rightarrow -1 + k - 2 - 4 = 0$

$\Rightarrow k - 7 = 0$

$\Rightarrow k = 7$

$\therefore$ Option (C) is correct answer.

Passage II (Question 17 to 19)

If A and B are square matrix of order 3 given by

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

On the basis of above information, answer the following questions-

Q.17 adj (adj A) is equal to -

(A) 100 A (B) 10 A

(C) 1000 A (D) None of these

Sol. **[B]**

We know,

adj (adj A) = $|A|^{n-2}$ A

where, n is order of matrix

$\therefore$ adj (adj A) = $(10)^{3-2}$ A = 10A

$\therefore$ Option (B) is correct answer.

Q.18 |adj (adj A)| is equal to -

(A) 10^2 (B) 100^3

(C) 10^4 (D) None of these

Sol. **[C]**

We know,

$|\text{adj (adj A)}| = |A|^{(n-1)^2} = (10)^{(3-1)^2}$

$= 10^4$

$\therefore$ Option (C) is correct answer.

Q.19 | adj (AB) | is equal to -

(A) 100 (B) 1000

(C) 10^4 (D) None of these

Sol. **[D]**

$$AB = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8+28 & 2+10+32 & 3+12+36 \\ 4+12+35 & 8+15+40 & 12+18+45 \\ 2+16+42 & 4+20+48 & 6+24+54 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 44 & 51 \\ 51 & 63 & 75 \\ 60 & 72 & 84 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 27 & 44 & 51 \\ 51 & 63 & 75 \\ 60 & 72 & 84 \end{vmatrix}$$

= 27 (–108) – 44(–216) + 51 (–108)

= 44 × 216 – 108 × 78

= 1180

$\therefore$ Option (D) is correct answer.

Passage III (Question 20 to 22)

$$\text{If } A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \ \& \ B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

$B_n = \text{adj}(B_{n-1})$, $n \in N$ and I is an identity matrix of order 3 then answer the following questions

Q.20 Det. $(A_0 + A_0^2B_0^2 + A_0^3 + A_0^4B_0^4 + \ldots..10 \text{ terms}) =$

(A) 1000 (B) – 800

(C) 0 (D) –8000

Sol.[C] |A| = 0

|B| = 1

Det $(A_0 + A_0^2\,B_0^2 + A_0^3 + \ldots\ldots..) = 0$

Q.21 $B_1 + B_2 + \ldots\ldots.+ B_{49} =$

(A) B_0 (B) $7B_0$

(C) $49B_0$ (D) 491

Sol.[C] $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$$\text{adj }(B_0) = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

$B_1 = \text{adj } B_0 = B_0$

$B_2 = \text{adj }(\text{adj } B_0) = B_0$

M

$B_{49} = \text{adj }(\ldots\ldots\text{ adj }(B_0)) = B_0.$

Then $B_1 + B_2 + \ldots\ldots + B_{49} = 49B_0$

Q.22 For a variable matrix X the equation $A_0X = C$,

where $C = \begin{bmatrix} -4 \\ 1 \\ 4 \end{bmatrix}$ will have

(A) unique solution

(B) infinite solution

(C) finitely many solution

(D) no. solution

Sol.[D] $|A_0| = 0$

$$(\text{adj } A)\ B = \begin{bmatrix} -1 & 2 & -4 \\ 1 & -2 & 4 \\ -1 & 2 & -4 \end{bmatrix}\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} \neq$$

0

No solution.

EXERCISE # 4

➤ Old IIT-JEE questions

Q.1 If A and B are square matrices of equal order, then which one is correct among the following? **[1995S]**

(A) A + B = B + A (B) A + B = A – B
(C) A – B = B – A (D) AB = BA

Sol. **[A]**

Q.2 If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ and $A^2 = B$, then- **[IIT Scr. 2003]**

(A) Statement is not true for any real value of α
(B) $\alpha = 1$
(C) $\alpha = -1$
(D) $\alpha = 4$

Sol. **[A]**

$A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2+0 & 0+0 \\ \alpha+1 & 0+1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = B$

$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 = 1$ and $\alpha = 4$

$\Rightarrow \alpha = \pm 1$ and $\alpha = 4$

They are not satisfied simultaneously.

$\therefore$ Option (A) is correct answer.

Q.3 If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c are real positive numbers, abc = 1 and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$. **[IIT 2003]**

Sol. $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

a b c = 1 and $A^TA = I$

$\Theta\ A^T = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

$A^TA = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & b^2+c^2+a^2 & ab+bc+ca \\ ab+bc+ca & bc+ca+ab & c^2+a^2+b^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow a^2 + b^2 + c^2 = 1$ and $ab + bc + ca = 0$

Since $(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

$= (a^3 + b^3 + c^3 - 3abc)$

We know,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= a^2 + b^2 + c^2 + 2 \times 0 \quad = 1$

$\therefore (a + b + c) = 1$

$\therefore (a^3 + b^3 + c^3 - 3abc) =$

$(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

$= (1 - 0)(1) = 1$

$\Rightarrow a^3 + b^3 + c^3 - 3 \times 1 = 1$

$\Rightarrow a^3 + b^3 + c^3 = 4$

Q.4 $\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} = A$ & $|A^3| = 125$, then α is – **[IIT Scr. 2004]**

(A) 0 (B) ± 2 (C) ± 3 (D) ± 5

Sol. **[C]**

Given $\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} = A$ and $|A^3| = 125$

$$A^2 = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2+4 & 2\alpha+2\alpha \\ 2\alpha+2\alpha & 4+\alpha^2 \end{bmatrix} = \begin{bmatrix} \alpha^2+4 & 4\alpha \\ 4\alpha & \alpha^2+4 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \alpha^2+4 & 4\alpha \\ 4\alpha & \alpha^2+4 \end{bmatrix}\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^3+4\alpha+8\alpha & 2\alpha^2+8+4\alpha^2 \\ 4\alpha^2+2\alpha^2+8 & 8\alpha+\alpha^3+4\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^3+12\alpha & 6\alpha^2+8 \\ 6\alpha^2+8 & \alpha^3+12\alpha \end{bmatrix}$$

$$\Rightarrow |A^3| = \begin{vmatrix} \alpha^3+12\alpha & 6\alpha^2+8 \\ 6\alpha^2+8 & \alpha^3+12\alpha \end{vmatrix} = 125$$

$\Rightarrow (\alpha^3 + 12\alpha)^2 - (6\alpha^2 + 8)^2 = 125$

$\Rightarrow \alpha^6+144\alpha^2 + 24\alpha^4 - (36\alpha^4 + 64 + 96\alpha^2) = 125$

$\Rightarrow \alpha^6 - 12\alpha^4 + 48\alpha^2 = 189$

above equation satisfy $\alpha = \pm 3$

$\therefore$ Option (C) is correct answer.

Q.5 If M is a 3 × 3 matrix, where det M = 1 and $MM^T = I$, where 'I' is an identity matrix, prove that det (M– I) = 0 **[IIT 2004]**

Sol. Det (M – I) = Det (M – MM^T) = Det M. Det(1 – M^T) = Det (M) Det $(I - M)^T$

Then Det (M – I) = –Det (M – I)

Det (M – I) = –Det (M – I)

2 Det (M – I) = 0 ⇒ Det (M – I) = 0

Q.6 $A = \begin{bmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix},$

$V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$

If there is a vector matrix X, such that AX = U has infinitely many solutions, then prove that BX = V cannot have a unique solution. If a + d ≠ 0. Then prove that BX = V has no solution. **[IIT 2004]**

Sol. Given

$$A = \begin{bmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix},$$

$$u = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, v = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

Ax = U

$\Rightarrow x = A^{-1}U$

$A^{-1} = \dfrac{\text{adj}A}{|A|}$

For infinitely many solutions, |A| = 0

$$\Rightarrow |A| = \begin{vmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{vmatrix} = a(cd - db) - 0 + 1(d - c) = 0$$

$\Rightarrow abc - adb + d - c = 0$

$\Rightarrow c(ab - 1) + d(1 - ab) = 0$

$\Rightarrow (ab - 1)(c - d) = 0$

$\Rightarrow ab = 1$ and $c = d$

$$\text{Also, } |A_1| = \begin{vmatrix} a & 0 & f \\ 1 & c & g \\ 1 & d & h \end{vmatrix} = 0$$

$\Rightarrow a(ch - dg) - 0 + f(d - c) = 0$

$\Rightarrow ac(h - g) - 0 + f(0) = 0$

$\Rightarrow h = g$

$$|A_2| = \begin{vmatrix} a & f & 1 \\ 1 & g & b \\ 1 & h & b \end{vmatrix}$$

$= a(gb - hb) - f(b - b) + 1(h - g)$

$= ab(g - h) - f(0) + (h - g) = 0$

$\Rightarrow (g - h)(ab - 1) = 0$

$\Rightarrow g - h = 0$

$\Rightarrow g = h$

$$|A_3| = \begin{vmatrix} f & 0 & 1 \\ g & c & b \\ h & d & b \end{vmatrix}$$

$= f(cb - db) - 0 + 1(gd - hc) = 0$

$\Rightarrow$ f(0) + c(g – h) = 0 $\quad\Rightarrow$ g = h

$\therefore$ ab = 1, c = d and g = h

We can take, Bx = V

$\Rightarrow X = B^{-1}V$

$$B^{-1} = \frac{\text{adj}B}{|B|}$$

$$\therefore |B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix}$$

= a (dh – gc) – 1(0 – fc) + 1 (0 – fd)

= ac (h – g) + fc – fd

= ac (h – g) + f(c – d)

= ac (0) + f(0) = 0

$\Rightarrow$ |B| = 0

$\therefore$ Bx = V

can't have a unique solution.

$$\text{Also, } |B_1| = \begin{vmatrix} a & 1 & a^2 \\ 0 & d & 0 \\ f & g & 0 \end{vmatrix} = a(0) - 1(0) + a^2(-fd)$$

$= -a^2 fd$

$= -a^2fc \neq 0$

$\therefore$ Bx = V has no solution.

Q.7 Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{6}[A^2 + CA + dI]$, find ordered pair (c,d) ?

[IIT Scr. 2005]

(A) (6, 11) (B) (–6, –11)

(C) (–6 , 11) (D) (6, –11)

Sol. **[C]**

Given

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \ \& \ I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1-2 & 0+1+4 \\ 0+0+0 & 0-2-8 & 0-2+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

= matrix formed by cofactors of A.;

$C_{11} = 4 + 2 = 6$; $C_{12} = 0$; $C_{13} = 0$

$C_{21} = 0$; $C_{22} = 4$; $C_{23} = 2$

$C_{31} = 0$; $C_{32} = -1$; $C_{33} = 1$

$$C = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$C^T = \text{adj}(A) = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{vmatrix} = 1(4+2) - 0 + 0 = 6$$

$$\therefore A^{-1} = \frac{1}{6}(A^2 + CA + dI)$$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix} \times \frac{1}{6} = \frac{1}{6}(A^2 + CA + dI)$$

$$\Rightarrow \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} +$$

$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix} + \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+c+d & 0+0+0 & 0+0+0 \\ 0+0+0 & -1+c+d & 5+c \\ 0+0+0 & -10-2c+0 & 14+4c+d \end{bmatrix}$$

$\Rightarrow c + d + 1 = 6 \quad \Rightarrow c + d = 5$

$\Rightarrow d = 11$

$\Rightarrow 5 + c = -1 \quad \Rightarrow c = -6$

Hence ordered pair (–6, 11)

$\therefore$ Option (C) is correct answer.

Q.8 Let a matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ & $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$Q = PAP^T$ where P^T is transpose of matrix P. Find $P^T Q^{2005} P$ is - **[IIT Scr. 2005]**

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(B) $\frac{1}{4}\begin{bmatrix} 1+2005\sqrt{3} & 6015 \\ 2005 & 1-2005\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 1+2005\sqrt{3} & 2005 \\ 2005 & 1-2005\sqrt{3} \end{bmatrix}$

(D) $\begin{bmatrix} 2005 & 2005 \\ 0 & 1 \end{bmatrix}$

Sol. **[A]**

Given

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ & $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$Q = PAP^T$

$$\text{Since, } PP^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{4}+\frac{1}{4} & \frac{-\sqrt{3}}{4}+\frac{\sqrt{3}}{4} \\ \frac{-\sqrt{3}}{4}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow P^T = P^{-1}$$

$\therefore P^T Q^{2005} P$

$$= P^T \left[\underbrace{(PAP^{-1})(PAP^{-1})(PAP^{-1})\ldots(PAP^{-1})}_{2005 \text{ times}} \right] P$$

$= P^T [PA^{2005} P^{-1}] P = A^{2005}$

$$\text{Since, } A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

....

....

By symmetry, $A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$\therefore P^T Q^{2005} P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

Passage (Q.9 to Q.11) **[IIT-2006]**

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1, U_2 and U_3 are column matrices satisfying

$A\,\underline{U}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $A\underline{U}_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $A\underline{U}_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, and U is 3 × 3 matrix whose columns are U_1, U_2, U_3 then answer the following questions.

Q.9 The value of |U| is –

(A) 3 (B) –3 (C) 3/2 (D) 2

Sol.[A] $U_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ $U_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$ $U_3 = \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$ same as

compare

compare

$a_1 = 1$ $a_2 = 2$ $a_3 = 2$

$b_1 = -2$ $b_2 = -1$ $b_3 = -1$

$c_1 = 1$ $c_2 = -4$ $c_3 = -3$

So $U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$

$|U| = 3$

Q.10 Sum of the elements of U^{-1} is–

(A) –1 (B) 0 (C) 5 (D) 3

Sol.[B] $U^{-1} = \dfrac{\text{adj}U}{|U|} = \dfrac{1}{3}\begin{bmatrix} -7 & -14 & 0 \\ 5 & 1 & -3 \\ 9 & 0 & 3 \end{bmatrix}$ sum of all element = 0

Q.11 Find the value of $[3\ \ 2\ \ 0]\ U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is -

(A) 5 (B) 5/2 (C) 4 (D) 3/2

Sol.[A] $[3\ 2\ 0]\begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = 5$

Q.12 Consider the system of equations

$x - 2y + 3z = -1$

$-x + y - 2z = k$

$x - 3y + 4z = 1$

Statement-1 : The system of equations has no solution for $k \neq 3$.

Statement-2 : The determinant

$\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$. **[IIT 2008]**

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

Sol. **[A]**

since $\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$

$\therefore$ for having either $\Delta_x \neq 0$ or $\Delta_y \neq 0$ or $\Delta_z \neq 0$

no solution

$\therefore\ \Delta_x = \begin{vmatrix} -1 & -2 & 3 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} \neq 0$

$\Rightarrow\ 3 - k \neq 0 \Rightarrow\ k \neq 3$

Now again $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0 \Rightarrow k \neq 3$

Passage (Q.13 to Q.15)

Let A be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0. **[IIT 2009]**

Q.13 The number of matrices in set A is

(A) 12 (B) 6

(C) 9 (D) 3

Sol. **[A]**

$\begin{bmatrix} A & D & E \\ D & B & F \\ E & F & C \end{bmatrix}$

case (1) : any two of D, E, F are 1

${}^3C_2$. 3

Case (2) : any two of D, E, F are 0

${}^3C_2$

So number of matrices ${}^3C_2$. 3 + ${}^3C_2$ = 12

Q.14 The number of matrices A for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

(A) less than 4

(B) at least 4 but less than 7

(C) at least 7 but less than 10

(D) at least 10

Sol. **[B]**

$$A = \begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix}$$

$|A| \neq 0$

$|A| = abc - af^2 - cd^2 + 2def - e^2 b \neq 0$

if case (1) $af = 1 \Rightarrow d = 1$ or $e = 1$

case (2) $cd = 1 \Rightarrow e = 1$ or $f = 1$

case (3) $eb = 1 \Rightarrow d = 1$ or $e = 1$

So total six cases occurs.

Q.15 The number of matrices A for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is

(A) 0 (B) More than 2

(C) 2 (D) 1

Sol. **[B]**

All possible cases are

$$\text{(I)}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{(II)}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{(III)}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\text{(IV)}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{(V)}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{(VI)}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\text{(VII)}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{(VIII)}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{(IX)}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\text{(X)}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{(XI)}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \text{(XII)}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Case V, VII, XI, XII Satisfies for which $|A| = 0$ & at least one of Δ_1 or Δ_2 or Δ_3 are non zero

Q.16 Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det(adj B) = 10^6, then [k] is equal to

[Note : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k] **[IIT 2010]**

Sol. **[4]**

$$\det(A) = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$$

$= (2k - 1)\,[-1 + 4k^2] - 2\sqrt{k}\,[-2\sqrt{k} - 4k\sqrt{k}\,] + 2\sqrt{k}\,[4k\sqrt{k} + 2\sqrt{k}\,]$

$\det(A) = (2k - 1)(4k^2 - 1) + 4k(2k + 1) + 4k(2k + 1)$

$= (2k - 1)(4k^2 - 1) + 8k(2k + 1)$

$\det(B) = 0$

$\det(\text{adj } A) = (\det A)^2 = 10^6$ $\quad\det A = 10^3$

$8k^3 + 1 - 2k - 4k^2 + 16k^2 + 8k = 10^3$

$8k^3 + 12k^2 + 6k - 999 = 0$

$k = 2 \rightarrow 64 + 48 + 12 - 999 < 0$

$k = 3 \rightarrow 8(27) + 109 + 18 - 999 < 0$

$k = 4 \rightarrow 8(64) + 12(16) + 24 - 999$

$512 + 192 + 24 - 999 < 0$

$k = 5 \rightarrow 8(125) + 12(25) + 6(5) - 999 > 0$

so $[k] = 4$

Passage (Q. 17 to Q. 19)

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0,1,2,\ldots\ldots,p-1\}\right\}$$

[IIT 2010]

Q.17 The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is -

(A) $(p-1)^2$ (B) $2(p-1)$

(C) $(p-1)^2+1$ (D) $2p-1$

Sol.[D] $A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$ $a, b, c \in \{0, 1, 2, p-1\}$

If A is skew symmetric matrix then a = 0, b = – c

$|A| = -b^2$

Thus p divides |A| only when b = 0

again if A is symmetric matrix then b = c and |A| $= a^2 - b^2$.

Thus p divides |A| if either p Divides (a – b) or p divides (a + b).

Divides (a – b) only when a = b.

i.e. $a = b \in \{0, 1, 2, ..., (p-1)\}$

i.e. p choice, p divides (a + b)

p choices including a = b = 0 includes in (i)

Total choice are (p + p – 1) = 2p – 1

Q18 The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is **[Note :** The trace of a matrix is the sum of its diagonal entries.**]**

(A) $(p-1)(p^2-p+1)$ (B) $p^3-(p-1)^2$

(C) $(p-1)^2$ (D) $(p-1)(p^2-2)$

Sol.[C] Trace of A = 2a will be divisible by iff a = 0.

$|A| = a^2 - bc$ for $(a^2 - bc)$ to be divisible by p. There are exactly (p – 1) ordered pair (b, c). For any value of a required number is $(p-1)^2$

Q.19 The number of A in T_p such that det (A) is not divisible by p is -

(A) $2p^2$ (B) p^3-5p

(C) p^3-3p (D) p^3-p^2

Sol.[D] The number of matrices for which p does not divide.

$Tr(A) = (p-1)p^2$ of there $(p-1)^2$ are such that p divides |A|. The number of matrices p divide Tr(A) and p does not divides |A| are $(p-1)^2$.

Required number $= (p-1)p^2 - (p-1)^2 + (p-1)^2$

$= p^3 - p^2$

Passage (Q. 20 to 22)

Let *a*, *b* and *c* be three real numbers satisfying

$$[a\ \ b\ \ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ \ 0\ \ 0] \quad ....(E)$$

[IIT 2011]

Q.20 If the point *P*(*a*, *b*, *c*), with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12 (C) 7 (D) 6

Sol. [D] $[a\ b\ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ 0\ 0]$

$a + 8b + 7c = 0$

$9a + 2b + 3c = 0$

$7a + 7b + 7c = 0$

On solving above equation

$(a, b, c) \equiv \left(-\frac{\lambda}{7}, -\frac{6\lambda}{7}, \lambda\right)$

$\therefore$ (*a*, *b*, *c*) lies on the plane $2x + y + z = 1$

So $-\frac{2\lambda}{7} - \frac{6\lambda}{7} + \lambda = 0$

on solving $\lambda = -7$

So $7a + b + c = 6$

Q.21 Let ω be a solution of $x^3 - 1 = 0$ with Im(ω) > 0. If $a = 2$ with *b* and *c* satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(A) –2 (B) 2 (C) 3 (D) –3

Sol. [A] $\therefore$ $(a, b, c) \equiv \left(-\frac{\lambda}{7}, -\frac{6\lambda}{7}, \lambda\right)$

$\therefore$ $a = 2$ is given so $\lambda = -14$

So $(a, b, c) \equiv (2, 12, -14)$

So $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = -2$

Q.22 Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

Sol. [B] $\therefore\ (a, b, c) \equiv \left(-\frac{\lambda}{7}, -\frac{6\lambda}{7}, \lambda\right)$

$\therefore\ b = 6$ so $\lambda = -7$.

So $(a, b, c) \equiv (1, 6, -7)$

So the equation $ax^2 + bx + c = 0$

$x^2 + 6x - 7 = 0$

So $\alpha = 1, B = -7$

$$S = \sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n = \sum\left(\frac{1}{1}-\frac{1}{7}\right)^n$$

$$= \sum\left(\frac{6}{7}\right)^n = 1 + \frac{6}{7} + \left(\frac{6}{7}\right)^2 + \ldots.\ \infty$$

$$= \frac{1}{1-\frac{6}{7}} = 7$$

Q.23 Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is : **[IIT 2011]**

(A) 2 (B) 6 (C) 4 (D) 8

Sol. [A] $\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$

$(1 - \omega c) - a(\omega - \omega^2 c) + b(\omega^2 - \omega^2) \neq 0$

$1 - \omega c - a\omega + ac\omega^2 \neq 0$

$(1 - \omega c) - a\omega(1 - \omega c) \neq 0$

$(1 - \omega c)(1 - a\omega) \neq 0$

$c \neq \omega^2$ & $a \neq \omega^2$ & $b = \omega$ or ω^2

$(a, b, c) \equiv (\omega, \omega, \omega)$ or $(\omega, \omega^2, \omega)$

Q.24 Let M be a 3× 3 matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}, M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$ & $M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}$.

Then the sum of the diagonal entries of M is. **[IIT 2011]**

Sol. [9] Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\Theta\ M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix} \Rightarrow b = -1, e = 2, h = 3$

$\therefore\ M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix} \Rightarrow a = 0, d = 3, g = 2$

$M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix} \Rightarrow c = 1, f = -5, i = 7$

So $a + e + i = 0 + 2 + 7 = 9$

Q.25 Let $P = [a_{ij}]$ be a 3 × 3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \leq i, j \leq 3$, If the determinant of P is 2, then the determinant of the matrix Q is **[IIT 2012]**

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

Sol. [D] Let $P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

So $Q = \begin{bmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{bmatrix}$

Now $\det(Q) = 2^2 2^3 2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33} \end{vmatrix}$

$= 2^9 \times 2 \times 2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= 2^{12}\det(P)$

$= 2^{12} \times 2 = 2^{13}$

Q.26 If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that **[IIT 2012]**

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) PX = X

(C) PX = 2X (D) PX = – X

Sol. [D] $P^T = 2P + I$

$\Rightarrow P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I$

$\Rightarrow P = -I$

$PX = -IX$

$PX = -X$

Q.27 If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are) **[IIT 2012]**

(A) –2 (B) –1 (C) 1 (D) 2

Sol. **[A, D]**

$\text{adj}(P) = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$|\text{adj } P| = |P|^{n-1} = |P|^2$

$|\text{adj } P| = 4 = |P|^2$

$|P| = 2$ or -2

EXERCISE # 5

Q.1 Find the number of 2 × 2 matrix satisfying

(i) a_{ij} is 1 or –1

(ii) $a_{11}^2 + a_{12}^2 = a_{21}^2 + a_{22}^2 = 2$

(iii) $a_{11}\,a_{21} + a_{12}\,a_{22} = 0$

Sol. **[8]**

Q.2 Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Such that AB = B and a + d = 5050. Find the value of (ad – bc).

Sol. **[5049]**

Q.3 If the matrix A is involuntary, show that $\frac{1}{2}(I + A)$ and $\frac{1}{2}(I - A)$ are idempotent and $\frac{1}{2}(I + A).\ \frac{1}{2}(I - A) = O$.

Sol.

Q.4 Let X be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \subseteq N$ then find the minimum value of $\sum (\cos^x \theta + \sin^x \theta)$, $\theta \in R$.

Sol. **[2]**

Q.5 $A = \begin{pmatrix} 3 & a & -1 \\ 2 & 5 & c \\ b & 8 & 2 \end{pmatrix}$ is Symmetric and

$B = \begin{pmatrix} d & 3 & a \\ b-a & e & -2b-c \\ -2 & 6 & -f \end{pmatrix}$ is Skew Symmetric, then find AB. Is AB a symmetric, skew symmetric or neither of them. Justify your answer.

Sol. AB is neither symmetric nor skew symmetric

Q.6 A is a square matrix of order n.

λ = maximum number of distinct entries if A is a triangular matrix.

m = maximum number of distinct entries if A is a diagonal matrix.

p = minimum number of zeroes if A is a triangular matrix.

if $\lambda + 5 = p = 2m$, find the order of the matrix.

Sol. **[4]**

Q.7 If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ then solve the following matrix equation.

(a) AX = B – I (b) (B – I)X = IC

(c) CX = A

Sol. **[**(a) $X = \begin{bmatrix} -3 & -3 \\ 5/2 & 2 \end{bmatrix}$; (b) $X = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$;

(c) no solution**]**

Q.8 Let A be a 3 × 3 matrix such that $a_{11} = a_{33} = 2$ and all the other $a_{ij} = 1$. Let $A^{-1} = xA^2 + yA + zI$ then find the value of (x + y + z) where I is a unit matrix of order 3.

Sol. **[1]**

Q.9 $A_{3\times3}$ is a matrix such that |A| = a, B = (adj A) such that |B| = b. Find the value of $(ab^2 + a^2b + 1)S$ where $\frac{1}{2}S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} + \ldots$ up to ∞, and a = 3.

Sol. **[225]**

Q.10 Given $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$. I is a unit matrix of order 2. Find all possible matrix X in the following cases.

(i) AX = A (ii) XA = I

(iii) XB = O but BX ≠ O

Sol. **[**(i) $X = \begin{bmatrix} a & b \\ 2-2a & 1-2b \end{bmatrix}$ for a,b ∈ R (ii) X does not exist (iii) $X = \begin{bmatrix} a & -3a \\ c & -3c \end{bmatrix}$ for a, c ∈ R and 3a + c ≠ 0, 3b + d ≠ 0**]**

Q.11 If A is an orthogonal matrix and B = AP where P is a non singular matrix then show that the matrix PB^{-1} is also orthogonal.

Sol.

Q.12 Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. Find P such that $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Sol. [$\begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$]

Q.13 Determine the values of a and b for which the system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$

(i) has a unique solution

(ii) has no solution and

(iii) has infinitely many solutions

Sol. [(i) $a \neq -3$, $b \in R$ (ii) $a = -3$ and $b \neq 1/3$ (iii) $a = -3$, $b = 1/3$]

Q.14 Given the matrix $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ and X be the solution set of the equation $A^x = A$, where $x \in N - \{1\}$. Evaluate $\prod \left(\frac{x^3+1}{x^3-1}\right)$ where the continued product extends $\forall x \in X$.

Sol. [3/2]

Q.15 Consider the two matrices A and B where $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$; $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$. If n(A) denotes the number of elements in A such that n(XY) = 0, when the two matrices X and Y are not conformable for multiplication. If C = (AB)(B'A); D = (B'A)(AB) then, find the value of $\left(\frac{n(C)\left(|D|^2 + n(D)\right)}{n(A) - n(B)}\right)$.

Sol. [650]

Q.16 If $\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$ then find a + n.

Sol. [200]

Passage (Q.17 to Q.20)

Matrix A is called orthogonal matrix if $AA^T = I = A^TA$. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ be an orthogonal matrix. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$.

Then $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

i.e $\vec{a}, \vec{b}$ & $\vec{c}$ forms mutually perpendicular triad of unit vectors. if abc = p and $Q = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$, where Q is an orthogonal matrix. Then.

On the basis of above information, answer the following questions :

Q.17 The values of a + b + c is -

(A) 2 (B) p (C) 2p (D) ± 1

Sol. [D]

Q.18 The values of ab + bc + ca is -

(A) 0 (B) p (C) 2p (D) 3p

Sol. [A]

Q.19 The value of $a^3 + b^3 + c^3$ is -

(A) p (B) 2p (C) 3p (D) none

Sol. [D]

Q.20 The equation whose roots are a, b, c is -

(A) $x^3 - 2x^2 + p = 0$

(B) $x^3 - px^2 + px + p = 0$

(C) $x^3 - 2x^2 + 2px + p = 0$

(D) $x^3 \pm x^2 - p = 0$

Sol. [D]

Q.21 $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 4 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ then rank of matrix A is

(A) 2 (B) 3 (C) 1 (D) None

Sol. [B]

Q.22 Rank of matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ is-

(A) 2 (B) 3 (C) 4 (D) None

Sol. **[B]**

Q.23 For all values of λ, find the rank of the matrix $\begin{bmatrix} 1 & 4 & 5 \\ \lambda & 8 & 8\lambda - 6 \\ 1+\lambda^2 & 8\lambda + 4 & 2\lambda + 21 \end{bmatrix}$

Sol. **[**for λ = 2; rank =1, $\lambda \neq 2$ but λ = – 1, rank = 2 and $\lambda \neq 2, -1$; rank = 3**]**

Q.24 Find the rank of the following matrices:

(i) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

Sol. **[(i) 2 ; (ii) 3]**

Q.25 Consider the matrices A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ and let P be any orthogonal matrix and $Q = PAP^T$ and $R = P^TQ^KP$ also $S = PBP^T$ and $T = P^TS^KP$

Column-I

(A) If we vary K from 1 to n then the first row first column elements at R will form

(B) if we vary K from 1 to n then 2nd row 2nd column elements at R will form

(C) If we vary K from 1 to n then the first row first column elements of T will form

(D) If we vary K from 3 to n then the first row 2nd column elements of T will represent the sum of

Column-II

(P) G.P. with common ratio a

(Q) A.P. with common difference 2

(R) G.P. with common ratio b

(S) A.P. with common difference –2

Sol. **[(A) A $\to$ Q; B $\to$ S; C $\to$ P; D $\to$ P]**

ANSWER KEY

EXERCISE # 1

1. B **2.** A **3.** D **4.** B **5.** D **6.** A **7.** A **8.** C **9.** B **10.** A **11.** C **12.** B
13. D **14.** A **15.** D **16.** C **17.** C **18.** A **19.** B **20.** A **21.** B **22.** A **23.** B **24.** B
25. C **26.** B **27.** B **28.** A **29.** A **30.** C **31.** A **32.** A **33.** A **34.** I + 15 A **35.** 4

EXERCISE # 2

(PART – A)

1. B **2.** B **3.** B **4.** C **5.** D **6.** A **7.** B **8.** A **9.** B **10. A** **11.** D **12.** C **13.** C **14.** A

(PART – B)

15. A, B, C **16.** A, B, C, D **17.** A **18.** A, B. C **19.** A, B **20.** A, C **21.** B, C **22.** C **23.** A,B **24.** A, B, C, D

PART – C)

25. D **26.** C **27.** D

(PART – D)

28. A $\to$ R; B $\to$ Q; C $\to$ P; D $\to$ S **29.** A $\to$ S; B $\to$ R; C $\to$ P; D $\to$ Q **30.** A $\to$ S; B $\to$ Q; C $\to$ P; D $\to$ R

EXERCISE # 3

2. 1265 **3.** adj(P)
6. (a) unique solution $\lambda \neq 2$, (b) $\lambda = 2$ and $\mu = 7$, Infinite no. of solutions, (c) $\lambda = 2$and $\mu \neq 7$, no solution

8. (i) x = 2; y = 2; z = 2 (ii) x = 1; y = 3; z = 5 **10.** (i) k = 2 (ii) X = B (iii) $A = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -4 & 2 \end{bmatrix}$

11. $\frac{1}{13}\begin{bmatrix} -12 & -5 \\ 5 & -12 \end{bmatrix}$ **12.** B = A + I, C = – I **13.** 5049 **14.** B **15.** C **16.** D

17. B **18.** B **19.** D **20.** C **21.** C **22.** D

EXERCISE # 4

1. A **2.** A **3.** 4 **4.** C **7.** C **8.** A **9.** A **10.** B **11.** A **12.** A **13.** A **14.** B **15.** B
16. 4 **17.** D **18.** C **19.** D **20.** D **21.** A **22.** B **23.** A **24.** 9 **25.** D **26.** D **27.** A, D

EXERCISE # 5

1. 8 **2.** 5049 **4.** 2 **5.** AB is neither symmetric nor skew symmetric

6. 4 **7.** (a) $X = \begin{bmatrix} -3 & -3 \\ 5/2 & 2 \end{bmatrix}$ (b) $X = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ (c) no solution **8.** 1 **9.** 225

10. (i) $X = \begin{bmatrix} a & b \\ 2-2a & 1-2b \end{bmatrix}$ for a,b $\in$ R (ii) X does not exist (iii) $X = \begin{bmatrix} a & -3a \\ c & -3c \end{bmatrix}$ for a, c $\in$ R and 3a + c $\neq$ 0, 3b + d $\neq$ 0

12. $\begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$ **13.** (i) $a \neq -3$, $b \in R$ (ii) a = – 3 and b $\neq$ 1/3 (iii) a = – 3, b = 1/3

14. 3/2 **15.** 650 **16.** 200 **17.** (D) **18.** (A) **19.** (D) **20.** (D) **21.** (B) **22.** (B)
23. for $\lambda = 2$; rank =1, $\lambda \neq 2$ but $\lambda = -1$, rank = 2 and $\lambda \neq 2, -1$; rank = 3

24. (i) 2 (ii) 3 **25.** (A) A $\rightarrow$ Q; B $\rightarrow$ S; C $\rightarrow$ P; D $\rightarrow$ P