EXERCISE-I

Expansion of determinants

1. $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} =$

(A) $1+a^2+b^2+c^2$ (B) $1-a^2+b^2+c^2$

(C) $1+a^2+b^2-c^2$ (D) $1+a^2-b^2+c^2$

2. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix}$ is

(A) 20 (B) 10

(C) 0 (D) 250

3. If $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0$,then the value of k is

(A) – 1 (B) 0

(C) 1 (D) None of these

4. If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$,

then $D_1 + D_2 + D_3 + D_4 + D_5 =$

(A) 0 (B) 25

(C) 625 (D) None of these

5. If A, B, C be the angles of a triangle, then $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} =$

(A) 1 (B) 0

(C) $\cos A \cos B \cos C$ (D) $\cos A + \cos B \cos C$

6. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} =$

(A) $3\sqrt{3}i$ (B) $-3\sqrt{3}i$

(C) $i\sqrt{3}$ (D) 3

7. $\begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix} =$

(A) 0 (B) abc

(C) $1/abc$ (D) None of these

8. The determinant $\begin{vmatrix} a & b & a-b \\ b & c & b-c \\ 2 & 1 & 0 \end{vmatrix}$ is equal to zero if a, b, c are in

(A) G. P. (B) A. P.

(C) H. P. (D) None of these

9. If $\begin{vmatrix} x+1 & 1 & 1 \\ 2 & x+2 & 2 \\ 3 & 3 & x+3 \end{vmatrix} = 0$, then x is

(A) 0, – 6 (B) 0, 6

(C) 6 (D) – 6

10. The determinant $\begin{vmatrix} 4+x^2 & -6 & -2 \\ -6 & 9+x^2 & 3 \\ -2 & 3 & 1+x^2 \end{vmatrix}$ is not divisible by

(A) x (B) x^3

(C) $14+x^2$ (D) x^5

11. If ω is a cube root of unity and $\Delta = \begin{vmatrix} 1 & 2\omega \\ \omega & \omega^2 \end{vmatrix}$, then Δ^2 is equal to

(A) $-\omega$ (B) ω

(C) 1 (D) ω^2

Minor & Co-factor and their properties

12. The cofactor of the element '4' in the determinant $\begin{vmatrix} 1 & 3 & 5 & 1 \\ 2 & 3 & 4 & 2 \\ 8 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$ is

(A) 4 (B) 10

(C) – 10 (D) – 4

13. If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and A_1, B_1, C_1 denote the co-factors of a_1, b_1, c_1 respectively, then the value of the determinant $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ is

(A) Δ (B) Δ^2

(C) Δ^3 (D) 0

14. If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_1, B_1, C_1 etc. be the co-factors of a_1, b_1, c_1 etc., then which of the following relations is incorrect

(A) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$

(B) $a_2A_2 + b_2B_2 + c_2C_2 = \Delta$

(C) $a_3A_3 + b_3B_3 + c_3C_3 = \Delta$

(D) $a_1A_2 + b_1B_2 + c_1C_2 = \Delta$

Properties of determinants

15. The roots of the equation $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ are

(A) 0, – 3 (B) 0, 0, – 3

(C) 0, 0, 0, – 3 (D) None of these

16. One of the roots of the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$ is

(A) $-(a+b)$ (B) $-(b+c)$

(C) $-a$ (D) $-(a+b+c)$

17. $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} =$

(A) 2 (B) – 2

(C) $x^2 - 2$ (D) None of these

18. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} =$

(A) $a^3 + b^3 + c^3 - 3abc$

(B) $a^3 + b^3 + c^3 + 3abc$

(C) $(a+b+c)(a-b)(b-c)(c-a)$

(D) None of these

19. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} =$

(A) $-2abc$ (B) abc

(C) 0 (D) $a^2 + b^2 + c^2$

20. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

(A) $3abc + a^3 + b^3 + c^3$ (B) $3abc - a^3 - b^3 - c^3$

(C) $abc - a^3 + b^3 + c^3$ (D) $abc + a^3 - b^3 - c^3$

21. $\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix} =$

(A) $abc(a+b+c)$ (B) $3a^2b^2c^2$

(C) 0 (D) None of these

22. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} =$

(A) abc (B) $1/abc$

(C) $ab+bc+ca$ (D) 0

23. $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} =$

(A) abc (B) $4abc$

(C) $4a^2b^2c^2$ (D) $a^2b^2c^2$

24. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$, if a, b, c are in

(A) A. P. (B) G. P.

(C) H. P. (D) None of these

25. The value of the determinant $\begin{vmatrix} 31 & 37 & 92 \\ 31 & 58 & 71 \\ 31 & 105 & 24 \end{vmatrix}$ is

(A) – 2 (B) 0

(C) 81 (D) None of these

26. The value of the determinant

$\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b+c-a & c+a-b & a+b-c \end{vmatrix}$ is

(A) abc (B) $a+b+c$

(C) $ab+bc+ca$ (D) None of these

27. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix} =$

(A) Δ (B) $k\Delta$

(C) $3k\Delta$ (D) $k^3\Delta$

28. $\begin{vmatrix} a-1 & a & bc \\ b-1 & b & ca \\ c-1 & c & ab \end{vmatrix} =$

(A) 0 (B) $(a-b)(b-c)(c-a)$

(C) $a^3+b^3+c^3-3abc$ (D) None of these

29. $\begin{vmatrix} a_1 & ma_1 & b_1 \\ a_2 & ma_2 & b_2 \\ a_3 & ma_3 & b_3 \end{vmatrix} =$

(A) 0 (B) $ma_1a_2a_3$

(C) $ma_1a_2b_3$ (D) $mb_1a_2a_3$

30. The value of $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$ is equal to

(A) 0 (B) 679

(C) 779 (D) 1000

31. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax-12$, then the value of A is

(A) 12 (B) 24

(C) –12 (D) – 24

32. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$

(A) 0 (B) $12\cos^2 x - 10\sin^2 x$

(C) $12\sin^2 x - 10\cos^2 x - 2$ (D) $10\sin 2x$

33. The roots of the equation

$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ are

(A) 1, 2 (B) – 1, 2

(C) 1, – 2 (D) –1, – 2

34. $\begin{vmatrix} bc & bc'+b'c & b'c' \\ ca & ca'+c'a & c'a' \\ ab & ab'+a'b & a'b' \end{vmatrix}$ is equal to

(A) $(ab-a'b')(bc-b'c')(ca-c'a')$

(B) $(ab+a'b')(bc+b'c')(ca+c'a')$

(C) $(ab'-a'b)(bc'-b'c)(ca'-c'a)$

(D) $(ab'+a'b)(bc'+b'c)(ca'+c'a)$

35. The roots of the determinant equation (in x)

$$\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = 0$$

(A) $x = a, b$ (B) $x = -a, -b$
(C) $x = -a, b$ (D) $x = a, -b$

36. $2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2-bc & b^2-ac & c^2-ab \end{vmatrix} =$

(A) 0 (B) 1
(C) 2 (D) $3abc$

37. The value of $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ is equal to

(A) $9a^2(a+b)$ (B) $9b^2(a+b)$
(C) $a^2(a+b)$ (D) $b^2(a+b)$

38. If a, b, c are different and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, then

(A) $a+b+c=0$ (B) $abc=1$
(C) $a+b+c=1$ (C) $ab+bc+ca=0$

39. If $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = Ka^2b^2c^2$, then $K =$

(A) – 4 (B) 2
(C) 4 (D) 8

40. $\begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bd \\ 1 & 1+ae & 1+be \end{vmatrix} =$

(A) 1 (B) 0
(C) 3 (D) $a+b+c$

41. At what value of x, will $\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$

(A) $x=0$ (B) $x=1$
(C) $x=-1$ (D) None of these

42. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is

(A) 3ω (B) $3\omega(\omega-1)$
(C) $3\omega^2$ (D) $3\omega(1-\omega)$

43. If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k\,abc(a+b+c)^3$, then the value of k is

(A) – 1 (B) 1
(C) 2 (D) –2

44. The value of $\begin{vmatrix} 41 & 42 & 43 \\ 44 & 45 & 46 \\ 47 & 48 & 49 \end{vmatrix} =$

(A) 2 (B) 4
(C) 0 (D) 1

45. The value of $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$ is

(A) 441 × 446 × 451 (B) 0
(C) – 1 (D) 1

46. If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then the value of $abc(ab+bc+ca)$ is

(A) $a+b+c$ (B) 0
(C) $a^2+b^2+c^2$ (D) $a^2-b^2+c^2$

47. If $a^2+b^2+c^2=-2$ and

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

then $f(x)$ is a polynomial of degree

(A) 3 (B) 2
(C) 1 (D) 0

48. The value of the determinant $\begin{vmatrix} 0 & b^3-a^3 & c^3-a^3 \\ a^3-b^3 & 0 & c^3-b^3 \\ a^3-c^3 & b^3-c^3 & 0 \end{vmatrix}$ is equal to

(A) $a^3+b^3+c^3$ (B) $a^3-b^3-c^3$

(C) 0 (D) $-a^3+b^3+c^3$

49. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P. and $a_i > 0$ for each i, then the value of the determinant $\Delta = \begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$ is equal to

(A) 1 (B) 2

(C) 0 (D) None of these

Differentiation, Integration, Summation of determinant, Multiplication of determinants

50. If $\Delta_1 = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}$, then $\Delta_2\Delta_1$ is equal to

(A) ac (B) bd

(C) $(b-a)(d-c)$ (D) None of these

51. If $\Delta(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$,

then the value of $\frac{d^n}{dx^n}[\Delta(x)]$ at $x=0$ is

(A) – 1 (B) 0

(C) 1 (D) Dependent of a

52. The value of $\sum_{n=1}^{N} U_n$,

if $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N \end{vmatrix}$ is

(A) 0 (B) 1

(C) – 1 (D) None of these

53. If $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$,

then the value of $\sum_{r=1}^{n} D_r =$

(A) 1 (B) – 1

(C) 0 (D) None of these

54. If $a_i^2 + b_i^2 + c_i^2 = 1,\ (i = 1,2,3)$ and $a_i a_j + b_i b_j + c_i c_j = 0\ \ (i \neq j, i, j = 1,2,3)$

then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is

(A) 0 (B) 1/2

(C) 1 (D) 2

55. If $\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix} = A_0 + A_1x + A_2x^2 + A_3x^3$

then A_1 is equal to

(A) abc (B) 0

(C) 1 (D) None of these

Cramer's Rule

56. The number of solutions of the equations $x+4y-z=0,\ 3x-4y-z=0, x-3y+z=0$ is

(A) 0 (B) 1

(C) 2 (D) Infinite

57. The value of a for which the system of equations $a^3x+(a+1)^3y+(a+2)^3z=0$, $ax+(a+1)y+(a+2)z=0,\ \ x+y+z=0$, has a non zero solution is

(A) – 1 (B) 0

(C) 1 (D) None of these

58. If $a_1x+b_1y+c_1z=0, a_2x+b_2y+c_2z=0$

$a_3x+b_3y+c_3z=0$ and $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$,

then the given system has

(A) One trivial and one non-trivial solution
(B) No solution
(C) One solution
(D) Infinite solution

59. The value of k for which the set of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ has a non trivial solution over the set of rationals is
(A) 15 (B) 31/2
(C) 16 (D) 33/2

60. If the system of equation
$3x - 2y + z = 0$, $\lambda x - 14y + 15z = 0$,
$x + 2y + 3z = 0$
have a non-trivial solution, then $\lambda =$
(A) 5 (B) – 5
(C) – 29 (D) 29

61. The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if
(A) $k \neq 0$ (B) $-1 < k < 1$
(C) $-2 < k < 2$ (D) $k = 0$

62. The system of equations $x_1 - x_2 + x_3 = 2$, $3x_1 - x_2 + 2x_3 = -6$ and $3x_1 + x_2 + x_3 = -18$ has
(A) No solution
(B) Exactly one solution
(C) Infinite solutions
(D) None of these

63. The number of values of k for which the system of equations $(k+1)x + 8y = 4k$, $kx + (k+3)y = 3k - 1$ has infinitely many solutions, is
(A) 0 (B) 1
(C) 2 (D) Infinite

64. The existence of the unique solution of the system $x + y + z = \lambda$, $5x - y + \mu z = 10$, $2x + 3y - z = 6$ depends on
(A) μ only (B) λ only
(C) λ and μ both (D) Neither λ nor μ