SOLVED EXAMPLES

Ex. 1 Express set $A = \{x : x \in N \text{ and } x = 2n \text{ for } n \in N\}$ in roster form.

Sol. $A = \{2, 4, 6, ...\}$

Ex. 2 Express set $B = \{x^2 : x < 4, x \in W\}$ in roster form.

Sol. $B = \{0, 1, 4, 9\}$

Ex. 3 Express set $A = \{2, 5, 10, 17, 26\}$ in set builder form.

Sol. $A = \{x : x = n^2 + 1, n \in N, 1 \le n \le 5\}$

Ex. 4 The set $A = [x : x \in R, x^2 = 16 \text{ and } 2x = 6]$ equal-

(A) ϕ **(B)** $[14, 3, 4]$ **(C)** $[3]$ **(D)** $[4]$

Sol. $x^2 = 16 \Rightarrow x = \pm 4$

$2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations.

Thus, $A = \phi$

Hence (A) is the correct answer

Ex. 5 Examine whether the following statements are true or false :

(i) $\{a, b\} \not\subseteq \{b, c, a\}$

(ii) $\{a, e\} \not\subseteq \{x : x \text{ is a vowel in the English alphabet}\}$

(iii) $\{1, 2, 3\} \subseteq \{1, 3, 5\}$

(iv) $\{a\} \in \{a, b, c\}$

Sol. **(i)** False as $\{a, b\}$ is subset of $\{b, c, a\}$

(ii) True as a, e are vowels

(iii) False as element 2 is not in the set $\{1, 3, 5\}$

(iv) False as $a \in \{a, b, c\}$ and $\{a\} \subseteq \{a, b, c\}$

Ex. 6 Two finite sets of have m and n elements respectively the total number of elements in power set of first set is 56 more thatn the total number of elements in power set of the second set find the value of m and n respectively.

Sol. Number of elements in power set of 1st set $= 2^m$

Number of elements in power set of 2nd set $= 2^n$

Given $2^m = 2^n + 56$

$\Rightarrow 2^m - 2^n = 56 \Rightarrow 2^n(2^{m-n} - 1) = 2^3(2^3 - 1)$

$\Rightarrow n = 3$ and $m = 6$

Ex. 7 Find power set of set $A = \{1, 2\}$.

Sol. $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Ex. 8 Let $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$ then find $A \cup B$.

Sol. $A \cup B = \{2, 4, 6, 8, 10, 12\}$

Ex. 9 Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$. Find $A - B$ and $B - A$.

Sol. $A - B = \{x : x \in A \text{ and } x \notin B\} = \{1, 3, 5\}$

Similarly $B - A = \{8\}$

Ex. 10 Prove that if $A \cup B = C$ and $A \cap B = \phi$ then $A = C - B$.

Sol. Let $x \in A$

$\Rightarrow x \in A \cup B \quad \Rightarrow x \in C \quad (\because A \cup B = C)$

Now $A \cap B = \phi \quad \Rightarrow x \notin B \quad (\because x \in A)$

$\Rightarrow x \in C - B \quad (\because x \in C \text{ and } x \notin B)$

$\Rightarrow A \subseteq C - B$

Let $x \in C - B \quad \Rightarrow \quad x \in C$ and $x \notin B$

$\Rightarrow x \in A \cup B$ and $x \notin B \quad \Rightarrow \quad x \in A \quad \Rightarrow \quad C - B \subseteq A$

$\therefore A = C - B$

Ex. 11 In a group of 40 students, 26 take tea, 18 take coffee and 8 take neither of the two. How many take both tea and coffee

Sol. $n(U) = 40, \quad n(T) = 26, \quad n(c) = 18$

$n(T' \cap C') = 8 \quad \Rightarrow \quad n(T \cup C)' = 8$

$\Rightarrow \quad n(U) - n(T \cup C) = 8$

$\Rightarrow \quad n(T \cup C) = 32$

$\Rightarrow \quad n(T) + n(c) - n(T \cap C) = 32$

$\Rightarrow \quad n(T \cap C) = 12$

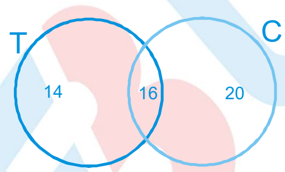
Ex. In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find

(i) How many drink tea and coffee both ? **(ii)** How many drink coffee but not tea ?

Sol. T : people drinking tea

C : people drinking coffee

(i) $n(T) = n(T - C) + n(T \cap C) \quad \Rightarrow \quad 30 = 14 + n(T \cap C) \quad \Rightarrow \quad n(T \cap C) = 16$



(ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

Ex. 12 If $A = \{1, 2\}$ and $B = \{3, 4\}$, then find $A \times B$.

Sol. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

Ex. 13 Let $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$ be two sets and let R be a relation from A to B defined by the phrase "$(x, y) \in R \Rightarrow x > y$". Find relation R and its domain and range.

Sol. Under relation R, we have 3R2, 5R2, 5R4, 7R4 and 7R6

i.e. $R = \{(3,2), (5,2), (5,4), (7,2), (7,4), (7,6)\}$

$\therefore$ Dom (R) $= \{3, 5, 7\}$ and range (R) $= \{2, 4, 6\}$

Ex. 14 Let A be the set of first ten natural numbers and let R be a relation on A defined by $(x, y) \in R \Leftrightarrow x + 2y = 10$, i.e. $R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y = 10\}$. Express R and R^{-1} as sets of ordered pairs. Determine also **(i)** domain of R and R^{-1} **(ii)** range of R and R^{-1}

Sol. We have $(x, y) \in R \Leftrightarrow x + 2y = 10 \Leftrightarrow y = \frac{10-x}{2}, x, y \in A$

where $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Now, $x = 1 \Rightarrow y = \frac{10-1}{2} = \frac{9}{2} \notin A$.

This shows that 1 is not related to any element in A. Similarly we can observe. that 3, 5, 7, 9 and 10 are not related to any element of A under the defined relation

Further we find that :

For $x = 2,\ y = \frac{10-2}{2} = 4 \in A \quad \therefore (2, 4) \in R$

For $x = 4,\ y = \frac{10-4}{2} = 3 \in A \quad \therefore (4, 3) \in R$

For $x = 6,\ y = \frac{10-6}{2} = 2 \in A \quad \therefore (6, 2) \in R$

For $x = 8,\ y = \frac{10-8}{2} = 1 \in A \quad \therefore (8, 1) \in R$

Thus, $R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$

$\Rightarrow \quad R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$

Clearly, $Dom(R) = \{2, 4, 6, 8\} = Range(R^{-1})$ and, $Range(R) = \{4, 3, 2, 1\} = Dom(R^{-1})$

Ex. 15 Let $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$. Let R be the relation on A defined by $\{(x, y) : x \in A, y \in A \text{ and } x \text{ divides } y\}$. Find domain and range of R.

Sol. The relation R is

$R = \{(2, 2), (2, 4), (2, 6), (2, 8), (3, 3), (3, 6), (3, 9), (4, 4), (4, 8), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9)\}$

Domain of $R = \{2, 3, 4, 5, 6, 7, 8, 9\} = A$

Range of $R = \{2, 3, 4, 5, 6, 7, 8, 9\} = A$

Ex. 16 Which of the following are identity relations on set $A = \{1, 2, 3\}$.

$R_1 = \{(1, 1), (2, 2)\}, R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}, R_3 = \{(1, 1), (2, 2), (3, 3)\}$.

Sol. The relation R_3 is idenity relation on set A.

R_1 is not identity relation on set A as $(3, 3) \notin R_1$.

R_2 is not identity relation on set A as $(1, 3) \in R_2$

Ex. 17 Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1 \text{ is congruent to } T_2\}$. Show that R is an equivalence relation.

Sol. Since a relation R in T is said to be an equivalenece relation if R is reflexive, symmetric and transitive.

(i) Since every triangle is congruent to itself

$\therefore$ R is reflexive

(ii) $(T_1, T_2) \in R \Rightarrow T_1$ is congruent to T_2

$\Rightarrow T_2$ is congruent to $T_1 \Rightarrow (T_2, T_1) \in R$

Hence R is symmetric

(iii) Let $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$

$\Rightarrow T_1$ is congruent to T_2

and T_2 is congruent to T_3

$\Rightarrow T_1$ is congruent to $T_3 \Rightarrow (T_1, T_3) \in R$

$\therefore$ R is transitive

Hence R is an equivalence relation.

Ex. 18 Show that the relation R in R defined as $R = \{(a, b) : a \leq b\}$ is transitive.

Sol. Let $(a, b) \in R$ and $(b, c) \in R$

$\therefore$ $(a \leq b)$ and $b \leq c \Rightarrow a \leq c$

$\therefore$ $(a, c) \in R$

Hence R is transitive.

Ex. 19 Show that the relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ is symmetric.

Sol. Let $(a, b) \in R$ $[\because (1, 2) \in R]$

$\therefore$ $(b, a) \in R$ $[\because (2, 1) \in R]$

Hence R is symmetric.

Ex. 20 Prove that the relation R on the set Z of all integers defined by

$(x, y) \in R \Leftrightarrow x - y$ is divisible by n is an equivalence relation on Z.

Sol. We observe the following properties

Reflexivity : For any $a \in N$, we have $a - a = 0 = 0 \times n \Rightarrow a - a$ is divisible by n $\Rightarrow (a, a) \in R$

Thus, $(a, a) \in R$ for all $a \in Z$

So, R is reflexive on Z

symmetry : Let $(a, b) \in R$. Then,

$(a, b) \in R \Rightarrow (a - b)$ is divisible by n

$\Rightarrow$ $a - b = np$ for some $p \in Z$ $\Rightarrow$ $b - a = n(-p)$

$\Rightarrow$ $b - a$ is divisible by n $[\because p \in Z \Rightarrow -p \in Z]$

$\Rightarrow$ $(b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$ for all a, b, $\in Z$

So, R is symmetric on Z.

Transitivity : Let a, b, $c \in Z$ such that $(a, b) \in R$ and $(b, c) \in R$. Then,

$(a, b) \in R \Rightarrow (a - b)$ is divisible by n $\Rightarrow a - b = np$ for some $p \in Z$

$(b, c) \in R \Rightarrow (b - c)$ is divisible by n $\Rightarrow b - c = nq$ for some $q \in Z$ $\therefore (a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $a - b = np$ and $b - c - nq \Rightarrow (a - b) + (b - c) = np + nq$

$\Rightarrow$ $a - c = n(p + q)$ $\Rightarrow$ $a - c$ is divisible by n $[\because p, q \in Z \Rightarrow p + q = Z]$

$\Rightarrow$ $(a, c) \in R$

thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all a, b, $c \in Z$. so, R is transitive realtion in Z.

Exercise # 1

[Single Correct Choice Type Questions]

PART 1 SETS

1. If A and B are two sets, then $A \cup B = A \cap B$ iff-

(A) $A \subseteq B$ (B) $B \subseteq A$ (C) $A = B$ (D) none of these

2. Let A and B be two sets in the universal set. Then A – B equals-

(A) $A \cap B'$ (B) $A' \cap B$ (C) $A \cap B$ (D) none of these

3. Two sets A, B are disjoint iff-

(A) $A \cup B = \phi$ (B) $A \cap B \neq \phi$ (C) $A \cap B = \phi$ (D) $A - B = A$

4. Which of the following is a null set ?

(A) $\{0\}$ (B) $\{x : x > 0 \text{ or } x < 0\}$

(C) $\{x : x^2 = 4 \text{ or } x = 3\}$ (D) $\{x : x^2 + 1 = 0, x \in R\}$

5. If $A \subseteq B$, then $A \cap B$ is equal to-

(A) A (B) B (C) A' (D) B'

6. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to-

(A) A (B) B (C) ϕ (D) none of these

7. If A is any set, then-

(A) $A \cup A' = \phi$ (B) $A \cup A' = U$ (C) $A \cap A' = U$ (D) none of these

8. If A, B be any two sets, then $(A \cup B)'$ is equal to-

(A) $A' \cup B'$ (B) $A' \cap B'$ (C) $A \cap B$ (D) $A \cup B$

9. If A and B be any two sets, then $(A \cap B)'$ is equal to-

(A) $A' \cap B'$ (B) $A' \cup B'$ (C) $A \cap B$ (D) $A \cup B$

10. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 5\}$, $B = \{6, 7\}$ then $A \cap B'$ is-

(A) B' (B) A (C) A' (D) B.

11. The set of intelligent students in a class is-

(A) a null set (B) a singleton set

(C) a finite set (D) not a well defined collection

12. Which of the following is the empty set

(A) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$ (B) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$

(C) $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$ (D) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

13. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is

(A) Null set (B) Singleton set (C) Infinite set (D) None of these

14. If $A = \{x : |x| < 3, x \in Z\}$ then the number of subsets of A is -

(A) 120 **(B)** 30 **(C)** 31 **(D)** 32

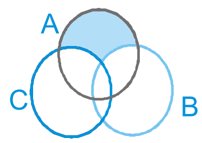
15. Which of the following are true ?

(A) $[3, 7] \subseteq (2, 10)$ **(B)** $(0, \infty) \subseteq (4, \infty)$ **(C)** $(5, 7] \subseteq [5, 7)$ **(D)** $[2, 7] \subseteq (2.9, 8)$

16. If $A = \{2, 3, 4, 8, 10\}$, $B = \{3, 4, 5, 10, 12\}$, $C = \{4, 5, 6, 12, 14\}$ then $(A \cap B) \cup (A \cap C)$ is equal to

(A) $\{3, 4, 10\}$ **(B)** $\{2, 8, 10\}$ **(C)** $\{4, 5, 6\}$ **(D)** $\{3, 5, 14\}$

17. The shaded region in the given figure is



(A) $A \cap (B \cup C)$ **(B)** $A \cup (B \cap C)$ **(C)** $A \cap (B - C)$ **(D)** $A - (B \cup C)$

18. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A' \cap B') =$

(A) 400 **(B)** 600 **(C)** 300 **(D)** 200

19. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 5\}$, $B = \{6, 7\}$, then $A \cap B'$ is

(A) B' **(B)** A **(C)** A' **(D)** B

20. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) ; n \in N\}$, then $X \cup Y$ is equal to

(A) X **(B)** Y **(C)** N **(D)** None of these

21. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to-

(A) A **(B)** B **(C)** A' **(D)** B'

22. If A and B are not disjoint, then $n(A \cup B)$ is equal to-

(A) $n(A) + n(B)$ **(B)** $n(A) + n(B) - n(A \cap B)$

(C) $n(A) + n(B) + n(A \cap B)$ **(D)** $n(A).n(B)$

23. If $A = \{2, 4, 5\}$, $B = \{7, 8, 9\}$ then $n(A \times B)$ is equal to-

(A) 6 **(B)** 9 **(C)** 3 **(D)** 0

24. Let A and B be two sets such that $n(A) = 70$, $n(B) = 60$ and $n(A \cup B) = 110$. Then $n(A \cap B)$ is equal to-

(A) 240 **(B)** 20 **(C)** 100 **(D)** 120

25. Which set is the subset of all given sets ?

(A) $\{1, 2, 3, 4, ...\}$ **(B)** $\{1\}$ **(C)** $\{0\}$ **(D)** $\{\ \}$

26. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 18

27. Given the sets $A = \{1, 2, 3\}, B = \{3, 4\}, C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is

(A) $\{3\}$ **(B)** $\{1, 2, 3, 4\}$ **(C)** $\{1, 2, 4, 5\}$ **(D)** $\{1, 2, 3, 4, 5, 6\}$

28. Let $A = \{x : x \in R, |x| < 1\}$, $B = \{x : x \in R, |x - 1| \geq 1\}$ and $A \cup B = R - D$, then the set D is

(A) $\{x : 1 < x \leq 2\}$ **(B)** $\{x : 1 \leq x < 2\}$ **(C)** $\{x : 1 \leq x \leq 2\}$ **(D)** None of these

29. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is

(A) $\{2, 3, 5\}$ **(B)** $\{3, 5, 9\}$ **(C)** $\{1, 2, 5, 9\}$ **(D)** None of these

30. Let A and B be two sets. Then

(A) $A \cup B \leq A \cap B$ **(B)** $A \cap B \leq A \cup B$ **(C)** $A \cap B = A \cup B$ **(D)** None of these

31. If $Q = \left\{x : x = \frac{1}{y},\ \text{where}\ y \in N\right\}$, then-

(A) $0 \in Q$ **(B)** $1 \in Q$ **(C)** $2 \in Q$ **(D)** $\frac{2}{3} \in Q$

32. $A = \{x : x \neq x\}$ represents-

(A) $\{0\}$ **(B)** $\{\ \}$ **(C)** $\{1\}$ **(D)** $\{x\}$

33. Which of the following statements is true ?

(A) $3 \subseteq \{1, 3, 5\}$ **(B)** $3 \in \{1, 3, 5\}$ **(C)** $\{3\} \in \{1, 3, 5\}$ **(D)** $\{3, 5\} \in \{1, 3, 5\}$

34. Which of the following is a null set ?

(A) $A = \{x : x > 1 \text{ and } x < 1]$ **(B)** $B = \{x : x + 3 = 3\}$

(C) $C = \{\phi\}$ **(D)** $D = \{x : x \geq 1 \text{ and } x \leq 1\}$

35. $P(A) = P(B) \Rightarrow$

(A) $A \subseteq B$ **(B)** $B \subseteq A$ **(C)** $A = B$ **(D)** none of these

PART 2 RELATION

36. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is-

(A) 2^{mn} **(B)** $2^{mn}-1$ **(C)** $2mn$ **(D)** m^n

37. In the set $A = \{1, 2, 3, 4, 5\}$, a relation R is defined by $R = \{(x, y) \mid x, y \in A \text{ and } x < y\}$. Then R is-

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** None of these

38. For real numbers x and y, we write $x\,R\,y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is-

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** None of these

39. Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is relations from X to Y-

(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$ **(B)** $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$

(C) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$ **(D)** $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

40. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then R is-

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** None of these

41 If A = {a, b}, B = {c, d}, C = {d, e}, then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} is equal to

(A) $A \cap (B \cup C)$ **(B)** $A \cup (B \cap C)$ **(C)** $A \times (B \cup C)$ **(D)** $A \times (B \cap C)$

42 If A = {2, 4, 5}, B = {7, 8, 9}, then $n(A \times B)$ is equal to

(A) 6 **(B)** 9 **(C)** 3 **(D)** 0

43 If $A = \{x : x^2 - 5x + 6 = 0\}$, B = {2, 4}, C = {4, 5} then $A \times (B \cap C)$ is-

(A) {(2, 4), (3, 4)} **(B)** {(4, 2), (4, 3)} **(C)** {(2, 4), (3, 4), (4, 4)} **(D)** {(2, 2), (3, 3), (4, 4), (5, 5)}

44 Let A = {a, b, c} and B = {1, 2}. Consider a relation R defined from set A to set B. Then R can equal to set

(A) A **(B)** B **(C)** $A \times B$ **(D)** $B \times A$

45. A and B are two sets having 3 and 4 elements respectively and having 2 elements in common. The number of relation which can be defined from A to B is

(A) 2^5 **(B)** $2^{10} - 1$ **(C)** $2^{12} - 1$ **(D)** none of these

46 Let R be relation from a set A to a set B, then

(A) $R = A \cup B$ **(B)** $R = A \cap B$ **(C)** $R \subseteq A \times B$ **(D)** $R \subseteq B \times A$

47. Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is not a relation from X to Y

(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$ **(B)** R_2 = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}

(C) R_3 = {(1, 1), (1, 3) (3, 5), (3, 7), (5, 7)} **(D)** R_4 = {(1, 3), (2, 5), (2, 4), (7, 9)}

48. Let R be a relation defined in the set of real numbers by $a R b \Leftrightarrow 1 + ab > 0$. Then R is-

(A) Equivalence relation **(B)** Transitive **(C)** Symmetric **(D)** Anti-symmetric

49. Which one of the following relations on R is equivalence relation-

(A) $x R_1 y \Leftrightarrow |x| = |y|$ **(B)** $x R_2 y \Leftrightarrow x \ge y$ **(C)** $x R_3 y \Leftrightarrow x \mid y$ **(D)** $x R_4 y \Leftrightarrow x < y$

50 Two points P and Q in a plane are related if OP = OQ, where O is a fixed point. This relation is-

(A) Reflexive but symmetric **(B)** Symmetric but not transitive

(C) An equivalence relation **(D)** none of these

51. The relation R defined in A = {1, 2, 3} by a R b if $|a^2 - b^2| \le 5$. Which of the following is false-

(A) R={(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2) **(B)** $R^{-1} = R$

(C) Domain of R = {1, 2, 3} **(D)** Range of R = {5}

52. Let a relation R is the set N of natural numbers be defined as $(x, y) \in R$ if and only if $x^2 - 4xy + 3y^2 = 0$ for all $x, y \in N$. The relation R is-

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** An equivalence relation

53. Let A = {2, 3, 4, 5} and let R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation in A. Then R is-

(A) Reflexive and transitive **(B)** Reflexive and symmetric

(C) Reflexive and antisymmetric **(D)** none of these

54. If $A = \{2, 3\}$ and $B = \{1, 2\}$, then $A \times B$ is equal to-

(A) $\{(2, 1), (2, 2), (3, 1), (3, 2)\}$ **(B)** $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$

(C) $\{(2, 1), (3, 2)\}$ **(D)** $\{(1, 2), (2, 3)\}$

55. Let R be a relation over the set $N \times N$ and it is defined by $(a, b)\ R\ (c, d) \Rightarrow a + d = b + c$. Then R is-

(A) Reflexive only **(B)** Symmetric only **(C)** Transitive only **(D)** An equivalence relation

56. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$ if $ad(b + c) = bc(a + d)$, then R is-

(A) Symmetric only **(B)** Reflexive only **(C)** Transitive only **(D)** An equivalence relation

57. If $A = \{1, 2, 3\}$, $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by 'x is greater than y'. Then range of R is-

(A) $\{1, 4, 6, 9\}$ **(B)** $\{4, 6, 9\}$ **(C)** $\{1\}$ **(D)** none of these

58. Let $A = \{1, 2, 3, 4\}$ and R be a relation in A given by $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)\}$, then relation R is

(A) Reflexive **(B)** Symmetric **(C)** Equivalence **(D)** Reflexive and Symmetric

59. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is

(A) Reflexive but not symmetric **(B)** Reflexive but not transitive

(C) Symmetric and Transitive **(D)** Neither symmetric nor transitive

60. The relation "less than" in the set of natural number is

(A) Only symmetric **(B)** Only transitive **(C)** Only reflexive **(D)** Equivalence relation

61. The relation R defined in N as $aRb \Leftrightarrow b$ is divisible by a is

(A) Reflexive but not symmetric **(B)** Symmetric but not transitive

(C) Symmetric and transitive **(D)** None of these

62. In the set $A = \{1, 2, 3, 4, 5\}$ a relation R is defined by $R = \{(x, y) | x, y \in A \text{ and } x < y\}$. Then R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** None of these

63. Let R_1 be a relation defined by $R_1 = \{(a, b) | \ a \geq b ; a, b \in R\}$. Then R_1 is

(A) An equivalence relation on R **(B)** Reflexive, transitive but not symmetric

(C) Symmetric, Transitive but not reflexive **(D)** Neither transitive nor reflexive but symmetric

64. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. The R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** None of these

65. Let S be the set of all real numbers. Then the relation $R = \{(a, b) : 1 + ab > 0\}$ on S is

(A) Reflexive and symmetric but not transitive **(B)** Reflexive, transitive but not symmetric

(C) Symmetric, transitive but not reflexive **(D)** Reflexive, transitive and symmetric

66. Let R be a relation on the set N be defined by $\{(x, y) | x, y \in N, 2x + y = 41\}$. Then R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** None of these

67. Let L be the set of all straight lines in the Euclidean plane. Two lines $\bullet_1$ and $\bullet_2$ are said to be related by the relation R if $\bullet_1$ is parallel to $\bullet_2$. Then the relation R is-
(A) Reflexive (B) Symmetric (C) Transitive (D) Equivalence

68. A and B are two sets having 3 and 4 elements respectively and having 2 elements in common. The number of relations which can be defined from A to B is-
(A) 2^5 (B) $2^{10} - 1$ (C) $2^{12} - 1$ (D) none of these

69. For n, m $\in$ N, n|m means that n is a factor of m, the relation | is-
(A) reflexive and symmetric (B) transitive and symmetric
(C) reflexive, transitive and symmetric (D) reflexive, transitive and not symmetric

70. Let $R = \{(x, y) : x, y \in A, x + y = 5\}$ where $A = \{1, 2, 3, 4, 5\}$ then
(A) R is not reflexive, symmetric and not transitive
(B) R is an equivalence relation
(C) R is reflexive, symmetric but not transitive
(D) R is not reflexive, not symmetric but transitive

71. Let R be a relation on a set A such that $R = R^{-1}$ then R is-
(A) reflexive (B) symmetric (C) transitive (D) none of these

72. For real numbers x and y, we write $x R y \Rightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is-
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

73. Which one of the following relations on R is equivalence relation-
(A) $x R_1 y \Leftrightarrow |x| = |y|$ (B) $x R_2 y \Leftrightarrow x \geq y$
(C) $x R_3 y \Leftrightarrow x \mid y$ (x divides y) (D) $x R_4 y \Leftrightarrow x < y$

74. The relation R defined in $A = \{1, 2, 3\}$ by a R b if $|a^2 - b^2| \leq 5$. Which of the following is false-
(A) $R = \{(1, 2), (2, 2), (3, 3), (2, 1), (2, 3), (3, 2)\}$ (B) Co-domain of $R = \{1, 2, 3\}$
(C) Domain of $R = \{1, 2, 3\}$ (D) Range of $R = \{1, 2, 3\}$

75. Let $P = \{(x, y) \mid x^2 + y^2 = 1, x, y \in R\}$, then P is
(A) reflexive (B) symmetric (C) transitive (D) equilance

76. Let $A = \{p, q, r\}$. Which of the following is an equivalence relation on A ?
(A) $R_1 = \{(p, q), (q, r), (p, r) (p, p)\}$ (B) $R_2 = \{(r, q), (r, p), (r, r) (q, q)\}$
(C) $R_3 = \{(p, p), (q, q), (r, r) (p, q)\}$ (D) none of these

77. Let x, y $\in$ I and suppose that a relation R on I is defined by x R y if and only if $x \leq y$ then
(A) R is partial order ralation (B) R is an equivalence relation
(C) R is reflexive and symmetric (D) R is symmetric and transitive

78. Let R be a relation from a set A to a set B, then-
(A) $R = A \cup B$ (B) $R = A \cap B$ (C) $R \subseteq A \times B$ (D) $R \subseteq B \times A$

79. Given the relation R = = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, the minimum number of ordered pairs which when added to R make it an equivalence relation is-

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8

80. Let P = {(x, y) | $x^2 + y^2 = 1$, x, y ∈ R} Then P is-

(A) reflexive **(B)** symmetric **(C)** transitive **(D)** anti-symmetric

81. Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then R is-

(A) reflexive **(B)** symmetric **(C)** anti-symmetric **(D)** transitive

82. In order that a relation R defined in a non-empty set A is an equivalence relation, it is sufficient that R

(A) is reflexive **(B)** is symmetric

(C) is transitive **(D)** possesses all the above three properties

83. If R be a relation '<' from A = {1, 2, 3, 4} to B = {1, 3, 5} i.e. (a, b) ∈ R iff a < b, then ROR^{-1} is-

(A) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)} **(B)** {(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}

(C) {(3, 3), (3, 5), (5, 3), (5, 5)} **(D)** {(3, 3), (3, 4), (4, 5)}

84. If R is an equivalence relation in a set A, then R^{-1} is-

(A) reflexive but not symmetric **(B)** symmetric but not transitive

(C) an equivalence relation **(D)** none of these

85. Let R and S be two equivalence relations in a set A. Then-

(A) R ∪ S is an equivalence relation in A **(B)** R ∩ S is an equivalence relation in A

(C) R – S is an equivalence relation in A **(D)** none of these

Exercise # 2 Part # I [Comprehension Type Questions]

Comprehension # 1

In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali.

1. Number of people who can speak Hindi only is
(A) 300 (B) 400 (C) 500 (D) 600

2. Number of people who can speak Bengali only is
(A) 150 (B) 250 (C) 50 (D) 100

3. Number of people who can speak both Hindi and Bengali is
(A) 50 (B) 100 (C) 150 (D) 200

Comprehension # 2

Let R be a relation defined as $R = \{ (x, y) : y = |x - 1|, x \in Z \text{ and } |x| \le 3\}$

4. Relation R is equal to :
(A) $\{(1, 0), (1, 2), (3, 2), (4, 3)\}$
(B) $\{(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)\}$
(C) $\{(4, -3), (3, -2), (2 -1), (1, 0), (2, 3)\}$
(D) None of these

5. Domain of R is :
(A) $\{0, 1, 2, 3, 4\}$ (B) $\{1, 3, 4\}$ (C) $\{-3, -2, -1, 0, 1, 2, 3\}$ (D) $\{0, 1, 2, 3, 4\}$

6. Range of R is
(A) $\{0, 1, 2, 3, 4\}$ (B) $\{-3, -2, -1, 0, 1, 2, 3\}$ (C) $\{-4, -3, -1, -2, 0\}$ (D) $\{-1, 0, 1, 2, 3, 4\}$

Exercise # 3 Part # I [Subjective Type Questions]

1. Write the set of all vowels in English alphabet which precede letter O.

2. Classify the following as a finite or infinite set :
(i) $A = \{x \in N : (x - 1)(x - 2) = 0\}$ **(ii)** $B = \{x \in N : x \text{ is odd}\}$

3. Write the following set by roster method : The set of all natural numbers 'x' such that $4x + 9 < 50$.

4. Describe the following set by set property method $\{0, 3, 8, 15, 24, 35\}$

5. Describe the following set by roster method the set of all letters in the word TRIGONOMETRY.

6. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

7. Which of the following are true ?
(i) If $A = \{1, 5, 5, 5\}$, $B = \{1, 3, 5\}$, then $A \not\subset B$.
(ii) If $A = \{x : x^3 - 1 = 0, x \in N\}$, $B = \{x : x^2 - 4x + 3 = 0, x \in N\}$ then $A \subseteq B$.

8. Assume that P(A) = P(B). Prove that A = B.

9. If $A = \{x : x = 4n + 1, n \le 5, n \in N\}$ and B $\{3n : n \le 8, n \in N\}$, then find $A - (A - B)$.

10. Prove that $A \cup B = A \cap B$ iff $A = B$.

11. Prove that : $A - (B \cup C) = (A - B) \cap (A - C)$ without using venn diagram.

12. Prove by using venn diagram
(i) $A - (B \cup C) = (A - B) \cap (A - C)$ **(ii)** $A \subseteq B \Rightarrow B' \subseteq A'$

13. A and B are two sets such that n(A) = 3 and n(B) = 6.
Find **(i)** minimum value of $n(A \cup B)$ **(ii)** maximum value of $n(A \cup B)$

14. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, and 12 play football and cricket. Eight play all the three games. Find the total number of members in the three athletic teams.

15. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. Find the number of students who have taken exactly one subject.

16. Determine the domain and the range of the relation R defined by $R = \{(x + 1, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

17. If $A = \{3, 4, 6\}$, $B = \{1, 3\}$ and $C = \{1, 2, 6\}$ then find $(A - B) \times (A - C)$.

18. Let n be a fixed positive integer. Define a relation R on the set of integers Z, aRb $\Leftrightarrow$ n|(a – b). Then prove that R is equivalence relation

19. Let R be a relation over the set N × N and it is defined by (a, b) R (c, d) $\Rightarrow$ a + d = b + c. Then prove that R is equivalence relation

20. Let L be the set of all straight lines in the Euclidean plane. Two lines $\bullet_1$ and $\bullet_2$ are said to be related by the relation R if $\bullet_1$ is parallel to $\bullet_2$. Then prove that R is equivalence relation.

21. For n, m $\in$ N, n | m means that n is a factor of m, then prove that relation | is reflexive, transitive but not symmetric.

22. Let $R = \{(x, y) : x, y \in A, x + y = 5\}$ where $A = \{1, 2, 3, 4, 5\}$ then prove that R is neither reflexive nor transitive but symmetric.

Exercise # 4

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is-

(A) transitive **(B)** not symmetric **(C)** reflexive **(D)** a function **[AIEEE-2004]**

2. Let R = {(3, 3), (6, 6), (9, 9), (12, 12) (6, 12), (3, 9), (3, 12), (3, 6)} be relation on the set A = {3, 6, 9, 12}. Then the relation R is **[AIEEE - 2005]**

(A) reflexive and transitive only **(B)** reflexive only

(C) an equilvalence relation **(D)** reflexive and symmetric only

3. Let W denote the words in the english dictionary. Define the relation R by : R = {(x, y) ∈ W × W | the words x and y have at least one letter in common}. Then R is- **[AIEEE - 2006]**

(A) reflexive, symmetric and not transitive **(B)** reflexive, symmetric and transitive

(C) reflexive, not symmetric and transitive **(D)** not reflexive, symmetric and transitive

4 The set S : {1, 2, 3, ..., 12} is to be partitioned into three sets A, B, C of equalsize.Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is- **[AIEEE - 2007]**

(A) $12!/3!(4!)^3$ **(B)** $12!/3!(3!)^4$ **(C)** $12!/(4!)^3$ **(D)** $12!/(3!)^4$

5. Let R be the real line. Consider the following subsets of the plane R × R **[AIEEE-2008]**

S = {(x, y) : y = x + 1 and 0 < x < 2}

T = {(x, y) : x – y is an integer}

Which one of the following is true ?

(A) T is an equivalence relation on R but S is not **(B)** Neither S nor T is an equivalence relation on R

(C) Both S and T are equivalence relations on R **(D)** S is an equivalence relation on R but T is not

6. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE-2009]**

(A) A = C **(B)** B = C **(C)** $A \cap B = \phi$ **(D)** A = B

7. Consider the following relations : **[AIEEE-2010]**

R : {(x, y)| x ,y are real numbers and x = wy for some rational number w}

$S = \{\left(\frac{m}{n}, \frac{p}{q}\right)$ | m, n, p and q are integers such that n, q ≠ 0 and qm = pn}

Then

(A) neither R nor S is an equivalence relation

(B) S is an equivalence relation but R is not an equivalence relation

(C) R and S both are equivalence relations

(D) R is an equivalence relation but S is not an equivalence relation

8. Let R be the set of real numbers. **[AIEEE-2011]**

Statement-1 : A = {(x, y) ∈ R × R : y – x is an integer} is an equivalence relation on R.

Statement-2 : B = {(x, y) ∈ R × R : x = αy for some rational number α} is an equivalence relation on R.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true;Statement-2 is **not** a correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false. **(D)** Statement-1 is false, Statement-2 is true.

9. Consider the following relation R on the set of real square matrices of order 3. **[AIEEE - 2011]**

$R = \{(A, B) | A = P^{-1} BP$ for some invertible matrix $P\}$.

Statement - 1 : R is equivalence relation.

Statement - 2 : For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.

(A) Statement-1 is true, statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

10. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE-2012]**

(A) 5^2 **(B)** 3^5 **(C)** 2^5 **(D)** 5^3

11. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is **[AIEEE - 2013]**

(A) 256 **(B)** 220 **(C)** 219 **(D)** 211

Part # II [Previous Year Questions][CBSE]

1. Show that the relation R defined by $(a, b) R(c, d) \Rightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation. **[CBSE - 2008]**

2. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even$\}$, is an equivalence relation. **[CBSE - 2009]**

3. Let Z be the set of all integers and R be the relation on Z defined as
$R = \{(a, b) : a, b \in Z$, and $(a - b)$ is divisible by $5\}$ Prove that R is an equivalence relation. **[CBSE - 2010]**

4. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b) : a, b \in R$ and $a \le b^3\}$ is neither reflexive, nor symmetric nor transitive. **[CBSE - 2010]**

5. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by $4\}$ is an equivalence relation. Find the set of all elements related to 1. **[CBSE - 2010]**

6. Show that the relation S defined on the set $N \times N$ by $(a, b) S(c, d) \Rightarrow a + d = b + c$ is an equivalence relation. **[CBSE - 2010]**

7. Let $f : X \to Y$ be a function. Define a relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$. Show that R is an equivalence relation on X . **[CBSE - 2010]**

8. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive. **[CBSE - 2011]**

ANSWER KEY

EXERCISE - 1

1. C	**2.** A	**3.** C	**4.** D	**5.** A	**6.** C	**7.** B	**8.** B	**9.** B	**10.** B	**11.** D	**12.** B	**13.** A
14. D	**15.** A	**16.** A	**17.** D	**18.** C	**19.** B	**20.** B	**21.** A	**22.** B	**23.** B	**24.** B	**25.** D	**26.** B
27. B	**28.** B	**29.** B	**30.** B	**31.** B	**32.** B	**33.** B	**34.** A	**35.** C	**36.** A	**37.** C	**38.** A	**39.** A
40. B	**41.** C	**42.** B	**43.** A	**44.** C	**45.** D	**46.** C	**47.** D	**48.** C	**49.** A	**50.** C	**51.** D	**52.** A
53. B	**54.** A	**55.** D	**56.** D	**57.** C	**58.** D	**59.** A	**60.** B	**61.** A	**62.** C	**63.** B	**64.** B	**65.** A
66. D	**67.** D	**68.** D	**69.** D	**70.** A	**71.** B	**72.** A	**73.** A	**74.** A	**75.** B	**76.** D	**77.** A	**78.** C
79. C	**80.** B	**81.** B	**82.** D	**83.** C	**84.** C	**85.** B						

EXERCISE - 2 : PART # I

Comprehension # 1 : **1.** D **2.** B **3.** C

Comprehension # 2 : **1.** B **2.** C **3.** A

EXERCISE - 4 : PART # I

1. B **2.** A **3.** A **4.** C **5.** A **6.** B **7.** B **8.** C **9.** B **10.** B **11.** C