LOG & MODULUS

EXERCISE # 1

Question based on **Logarithms**

Q.1 $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(A) 1/2 (B) 1 (C) 2 (D) 4

Sol.[B] $= \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab}$

$= \log_{abc} abc = 1$

Q.2 $\log_4 18$ is

(A) a prime number (B) a rational number

(C) an irrational number (D) None of these

Sol.[C] $\log_4 18 = \log_4 2 + \log_4 3^2 = \frac{1}{2}\log_2 2 + \frac{2}{2}\log_2 3$

$= \frac{1}{2} + \log_2 3$ an irrational no.

Q.3 If $a^2 + 4b^2 = 12ab$, then $\log(a + 2b) =$

(A) $\frac{1}{2}(\log a + \log b - \log 2)$

(B) $\log a/2 + \log b/2 + \log 2$

(C) $\frac{1}{2}(\log a + \log b + 4\log 2)$

(D) $\frac{1}{2}(\log a - \log b + 4\log 2)$

Sol. **[C]**

Θ $a^2 + 4b^2 = 12ab \Rightarrow (a + 2b)^2 = 16\,ab$

Taking log both sides

$\Rightarrow 2\log(a + 2b) = \log a + \log b + \log 16$

$\Rightarrow \log(a + 2b) = \frac{1}{2}(\log a + \log b + 4\log 2)$

Q.4 If $N = \frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - 125^{\log_{25} 6}\right)$

Then $\log_2 N$ has the value-

(A) 0 (B) 1

(C) –1 (D) None of these

Sol. **[A]**

$N = \frac{9^{2.\log_9 5} + 3^{3\log_3 6^{1/2}}}{409}\left(7^{\log_7 25} - 5^{3.\log_{5^2} 6}\right)$

$= \frac{5^2 + (6)^{3/2}}{409}(25 - (6)^{3/2}) = \frac{625 - 216}{409} = \frac{409}{409} = 1$

$\therefore \log_2 N = \log_2 1 = 0$

Q.5 The expression $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\ldots\sqrt[p]{p}}}}}_{\text{n radical sign}}$

where $p \geq 2$, $p \in N$; $n \in N$ when simplified is.

(A) independent of p

(B) independent of p and of n

(C) depend on both p & n

(D) positive

Sol. **[A]**

$\log_p \log_p p^{\frac{1}{p^n}} = \log_P \log_p p^{p^{-n}}$

$= \log_P p^{-n} \log_p p = -n\log_P p = -n$

Q.6 If $x_n > x_{n-1} > \ldots > x_2 > x_1 > 1$ then the value of

$\log_{x_1}\log_{x_2}\log_{x_3}\ldots\log_{x_n} x_n^{x_{n-1}^{N^{x_1}}}$ is equal to-

(A) 0 (B) 1

(C) 2 (D) None of these

Sol. **[B]**

$\log_{x_1}\log_{x_2}\log_{x_3}\ldots\log_{x_n} x_n^{x_{n-1}^{N^{x_1}}}$

$\Rightarrow \log_{x_1}\log_{x_2}\log_{x_3}\ldots\log_{x_{n-1}} x_{n-1}^{N^{x_1}} \log_{x_n} x_n$

$\Rightarrow \log_{x_1}\log_{x_2}\log_{x_3}\ldots\log_{x_{n-2}} x_{n-2}^{N^{x_1}}$

$\log_{x_{n-1}} x_{n-1}.1$

$\Rightarrow \log_{x_1} x_1 = 1$

Q.7 The ratio $\frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$

(C) $a^2 - a + 1$ (D) $a^2 + a + 1$

Sol.[D] $\Rightarrow \dfrac{2^{4\log_2 a} - 3^{\frac{3}{3}\log_3(a^2+1)} - 2a}{7^{\frac{4}{2}\log_7 - a - 1}} \Rightarrow \dfrac{a^4 - (a^2+1) - 2a}{a^2 - a - 1}$

$\Rightarrow \dfrac{a^4 - a^2 - 2a - 1}{a^2 - a - 1} \Rightarrow a^2 + a + 1$

Q.8 The value of $a^{\frac{\log_b \log_b N}{\log_b a}}$ is-

(A) $\log_b N$ (B) $-\log_b N$ (C) $\log_N b$ (D) $-\log_N b$

Sol. **[A]**

$a^{\log_a \log_b N} = \log_b N \qquad (\because a^{\log_a x} = x)$

Q.9 If $\left(a^{\log_b x}\right)^2 - 5\,x^{\log_b a} + 6 = 0$ where $a > 0$, $b > 0$ & $ab \neq 1$. Then the value of x is equal to

(A) $2^{\log_b a}$ (B) $3^{\log_a b}$

(C) $2^{\log_a 2}$ (D) $a^{\log_b 3}$

Sol. **[B]**

$\left(a^{\log_b x}\right)^2 - 5\,x^{\log_b a} + 6 = 0, a > 0, b > 0, \& ab \neq 1$

$\because a^{\log_b x} = x^{\log_b a}$

$\therefore \left(x^{\log_b a}\right)^2 - 5\,x^{\log_b a} + 6 = 0$

Let $x^{\log_b a} = t$

$t^2 - 5t + 6 = 0$

$\Rightarrow (t-2)(t-3) = 0 \qquad \Rightarrow t = 2, 3$

$\therefore x^{\log_b a} = 2$ or 3

$\Rightarrow x = 2^{\log_a b}$ or $3^{\log_a b}$

Question based on Inequalities

Q.10 The solution set of the inequation

$\log_{1/3}(x^2 + x + 1) + 1 > 0$ is

(A) $(-\infty, -2) \cup (1, +\infty)$

(B) $[-1, 2]$

(C) $(-2, 1)$

(D) $(-\infty, +\infty)$

Sol. **[C]**

$\log_{1/3}(x^2 + x + 1) > -1$

$\Rightarrow x^2 + x + 1 < \left(\frac{1}{3}\right)^{-1}$

$\Rightarrow x^2 + x + 1 < 3$

$\Rightarrow x^2 + x - 2 < 0$

$\Rightarrow (x-1)(x+2) < 0$

$\Rightarrow x \in (-2, 1)$

Q.11 Find the values of x,

$\left(\frac{1}{2}\right)^{\log_2 \log_{\frac{1}{5}}\left(x^2 - \frac{4}{5}\right)} < 1$

(A) $-1 < x < -\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}} < x < 1$

(B) $-1 < x < -0, \frac{2}{\sqrt{5}} < x < 1$

(C) $-1 < x < -\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}} < x < 3$

(D) None of these

Sol. **[A]**

$\left(\frac{1}{2}\right)^{\log_2 \log_{\frac{1}{5}}\left(x^2 - \frac{4}{5}\right)} < 1$

$\Rightarrow 2^{-1 \log_2 \log_{1/5}\left(x^2 - \frac{4}{5}\right)} < 1$

$\Rightarrow 2^{\log_2\left[\log_{1/5}\left(x^2 - \frac{4}{5}\right)\right]^{-1}} < 1$

$\Rightarrow \left[\log_{1/5}\left(x^2 - \frac{4}{5}\right)\right]^{-1} < 1$

$\Rightarrow \log_{1/5}\left(x^2 - \frac{4}{5}\right) > 1$

For log defined, $x^2 - \frac{4}{5} > 0 \qquad \Rightarrow x^2 > \frac{4}{5}$

$\Rightarrow x > \pm \frac{2}{\sqrt{5}} \qquad \ldots (1)$

and $x^2 - \frac{4}{5} < \left(\frac{1}{5}\right)^1, \qquad x^2 < \frac{4}{5} + \frac{1}{5}$

< 1

$x^2 < 1 \Rightarrow x < \pm 1 \qquad \ldots (2)$

(1) & (2) $\Rightarrow -1 < x < -\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}} < x < 1$

Q.12 $\log_4(2x^2 + x + 1) - \log_2(2x - 1) \leq -\tan\frac{7\pi}{4}$

(A) $x \geq -1$ (B) $x \geq 1$

(C) $x \leq -1$ (D) None of these

Sol. **[B]**

$\log_4 (2x^2 + x + 1) - \log_2 (2x - 1) \le -\tan \frac{7\pi}{4}$

$\Rightarrow \log_{2^2} (2x^2 + x + 1) - \log_2 (2x - 1) \le 1$

$(\because \tan \frac{7\pi}{4} = -1)$

$\Rightarrow \log_2 \frac{\sqrt{2x^2 + x + 1}}{2x - 1} \le 1$

$\Rightarrow \frac{\sqrt{2x^2 + x + 1}}{2x - 1} \le 2 \Rightarrow \sqrt{2x^2 + x + 1} \le 2(2x - 1)$

$\Rightarrow 2x^2 + x + 1 \le 16x^2 - 16x + 4$

$\Rightarrow 14x^2 - 17x + 3 \ge 0$

$\Rightarrow 14x^2 - 14x - 3x + 3 \ge 0$

$\Rightarrow (x - 1)(14x - 3) \ge 0$

$\Rightarrow x \ge 1$ or $x \le 3/14$

$= x \ge 1$

Q.13 $x^{\log_5 x} > 5$ implies -

(A) $x \in (0, \infty)$ (B) $x \in (0, 1/5) \cup (5, \infty)$

(C) $x \in (1, \infty)$ (D) $x \in (1, 2)$

Sol. **[B]**

$x^{\log_5 x} > 5$

taking $\log_5$ both sides

$\Rightarrow \log_5 x \log_5 x > 1$

$\Rightarrow (\log_5 x)^2 - 1 > 0$

$\Rightarrow (\log_5 x - 1)(\log_5 x + 1) > 0$

$\therefore \log_5 x < -1$ & $\log_5 x > 1$ $(\because x > 0)$

$\therefore x < 5^{-1}$ & $x > 5$

$\therefore x \in (0, 1/5) \cup (5, \infty)$

Q.14 Set of values of x satisfying the inequality $\frac{(x-3)^2 (2x+5)(x-7)}{(x^2 + x + 1)(3x+6)^2} \le 0$ is $[a, b) \cup (b, c]$ then $2a + b + c$ is equal to

(A) 0 (B) 2 (C) 5 (D) 7

Sol.[A] $x^2 + x + 1 > 0$; $3x + 6 \ne 0$

$x \ne -2$

Put $(x - 3) = 0 \Rightarrow x = 3$

so the inequality becomes $(2x + 5)(x - 7) \le 0$

$x \in \left[-\frac{5}{2}, 7\right]$

but $x \ne -2$ so $x \in \left[-\frac{5}{2}, 2\right) \cup (2, 7]$

$a = -\frac{5}{2}$, $b = 2$, $c = 7$

$2a + b + c = 0$

Q.15 Number of integral values of x satisfying the inequality $\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$ is

(A) 6 (B) 7 (C) 8 (D) Infinite

Sol.[B] $\left(\frac{3}{4}\right)^{6x+10-x^2} < \left(\frac{3}{4}\right)^3$

$\Rightarrow 6x + 10 - x^2 > 3 \Rightarrow x^2 - 6x - 7 < 0$

$\Rightarrow (x + 1)(x - 7) < 0 \Rightarrow x \in (-1, 7)$

$\Rightarrow$ 7 solution.

Question based on Characteristics and Mantissa

Q.16 Number of ciphers after decimal before a significant figure comes in $\left(\frac{5}{3}\right)^{-100}$ is-

(A) 21 (B) 22

(C) 23 (D) None of these

Sol.[B] $N = \left(\frac{5}{3}\right)^{-100}$ $\log_{10} N = -100(\log 5 - \log 3)$

$= -100(1 - 0.3010 - 0.4771)$

$= -100(0.2219) = -22.19 = -23 + 0.81$

characteristic $= -23$

$\therefore$ number of ciphers after decimal (22)

Q. 17 If $\log_{10} 3 = 0.477$, the no. of digits in 3^{40} is

(A) 18 (B) 19 (C) 20 (D) 21

Sol. **[C]**

Let $z = 3^{40}$

$\therefore \log_{10} z = \log_{10} 3^{40}$

$\Rightarrow \log_{10} z = 40 \log_{10} 3$

$= 40 \times 0.477$ $[\because \log_{10} 3 = 0.477$ given$]$

$= 19.08$

$\therefore$ number of digits in z is $= 19 + 1 = 20$

Question based on Modulus function

Q.18 If x_1 and x_2 are two solutions of the equation $\log_3 |2x - 7| = 1$ where $x_1 < x_2$, then the number of integer(s) between x_1 and x_2 is/are-

(A) 2 (B) 3 (C) 4 (D) 5

Sol.[A] $\log_3 |2x - 7| = 1$

$\Rightarrow |2x - 7| = 3$

$\Rightarrow 2x - 7 = \pm 3$
$\Rightarrow 2x = 4$ and $2x = 10$
$\Rightarrow x = 2$ and $x = 5$
$\Rightarrow x_1 = 2, x_2 = 5$
Integers between 2 and 5 are 2

Q.19 If $|x-1| + |x-2| + |x-3| \geq 6$ then.
(A) $0 \leq x \leq 4$ (B) $x \leq -2$ or $x \geq 4$
(C) $x \leq 0$ or $x \geq 4$ (D) None of these

Sol. **[C]**
$|x-1| + |x-2| + |x-3| \geq 6$
$\therefore$ if $x < 1$, the inequation becomes
$-(x-1) - (x-2) - (x-3) \geq 6$
or $-3x + 6 \geq 6$
$\therefore x \leq 0$
$\therefore$ x is such that $x < 1$ and $x \leq 0 \Rightarrow x \leq 0$
If $1 \leq x < 2$, the inequation becomes
$(x-1) + (x-2) - (x-3) \geq 6$
or $-x + 4 \geq 6$, $\therefore x \leq -2$
No such values satisfy $1 \leq x \leq 2$
If $2 \leq x \leq 3$, the inequation becomes
$(x-1) + (x-2) - (x-3) \geq 6$
or $x \geq 6$, no such values satisfy $2 \leq x < 3$
If $x \geq 3$, the inequation becomes
$(x-1) + (x-2) + (x-3) \geq 6$
or $3x - 6 \geq 6$ or $3x \geq 12$
$\therefore x \geq 4$ which satisfy $x \geq 3$
$\therefore$ The values of x satisfy $x \leq 0$ or $x \geq 4$
$\therefore$ required solution is $x \leq 0$ or $x \geq 4$

Q.20 The set of real values of x satisfying $||x-1| - 1| \leq 1$ is-
(A) $[-1, 3]$ (B) $[0, 2]$
(C) $[-1, 1]$ (D) None of these

Sol. **[A]**
$||x-1| - 1| \leq 1$
$\Rightarrow -1 \leq |x-1| - 1 \leq 1$
$\Rightarrow 0 \leq |x-1| \leq 2$
$\therefore |x-1| \leq 2$
$\Rightarrow -2 \leq x - 1 \leq 2$
$\Rightarrow x \in [-1, 3]$

Q.21 The solution of the inequation
$\log_{0.1}\left(\log_2 \frac{x^2+1}{|x-1|}\right) < 0$ lies in the interval -
(A) $(1, \infty)$ (B) $(-\infty, 1)$
(C) $[1, \infty)$ (D) None of these

Sol. **[A]**
$\log_2 \frac{x^2+1}{|x-1|} > 1$
$\Rightarrow \frac{x^2+1}{|x-1|} > 2$ for log defined here $x > 1$
$\Rightarrow \frac{x^2+1-2|x-1|}{|x-1|} > 0 \quad \therefore x > 1$
$\Rightarrow x^2 + 2x - 1 > 0$ $(1, \infty)$
$x > 0.41$

Question based on Miscellaneous points

Q.22 Which is greater -
(i) $\log_2 3$ or $\log_{1/2} 5$
(ii) $\log_7 11$ or $\log_8 5$
(iii) $\log_4 5$ or $\log_{1/16} 25$
(iv) $\log_2 3$ or $\log_3 11$
(v) $\log_{1/3} 1/2$ or $\log_{1/2} 1/3$
(vi) $\log_3 5$ or $\log_{17} 25$

Sol. (i) $\log_2 3 > 1$; $\log_{1/2} 5 = -\log_2 5 < 0$
(ii) $\log_7 11 > 1$; $\log_8 5 < 1$
(iii) $\log_4 5 > 1$; $\log_{1/16} 25 = -\log_4 5 < 0$
(iv) $2 > \log_2 3 > 1$; $\log_3 11 > 2$
(v) $\log_{1/3}\frac{1}{2} = \log_3 2 < 1; \log_{1/2}\frac{1}{3} = \log_2 3 > 1$
(vi) $\log_3 5$, $\log_{17} 25 = \log_{(\sqrt{17})^2}(5)^2 = \log_{\sqrt{17}} 5$
$\frac{\log 5}{\log 3} > \frac{\log 5}{\log \sqrt{17}}$

Q.23 $\log_2 7$ is
(A) an integer
(B) a rational number
(C) an irrational number
(D) a prime number

Sol.[C] $\log_2 7$ is an irrational number.

Q.24 Which is the correct order for a given number α in increasing order:
(A) $\log_2\alpha, \log_3\alpha, \log_e\alpha, \log_{10}\alpha$
(B) $\log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$
(C) $\log_{10}\alpha, \log_2\alpha, \log_e\alpha, \log_3\alpha$
(D) $\log_3\alpha, \log_e\alpha, \log_2\alpha, \log_{10}\alpha$

Sol.[B] Clearly increating order is
$\log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$

EXERCISE # 2

Part-A **Only single correct answer type questions**

Q.1 If $\log_{16}x = \frac{3}{4}$ and $\log_y 0.125 = -3$, then the value of $\log_{0.25}\left(\frac{x}{y}\right)$ is-

(A) 1 (B) –1

(C) 2 (D) 4

Sol.[B] $\log_{16}x = \frac{3}{4}$, $\log_y(0.125) = -3$

$\Rightarrow x = 8, y = 2$

$\Rightarrow \log_{0.25}\left(\frac{x}{y}\right) = \log_{0.25}\left(\frac{8}{2}\right) = \log_{0.25}(4)$

$= -\log_{0.25}(0.25) = -1$

Q.2 Let $x_1 = 97$, $x_2 = \frac{2}{x_1}$, $x_3 = \frac{3}{x_2}$, $x_4 = \frac{4}{x_3}$, ……., $x_8 = \frac{8}{x_7}$ then $\log_{3\sqrt{2}}\left(\prod_{i=1}^{8} x_i - 60\right) =$

(A) $\frac{3}{2}$ (B) 4 (C) 6 (D) $\frac{5}{2}$

Sol.[B] $x_1 = 97$, $x_2 = \frac{2}{x_1}$, $x_3 = \frac{3}{x_2}$, ……. $x_8 = \frac{8}{x_7}$

$\Rightarrow \prod_{i=1}^{8} x_i = x_1.x_2.x_3.x_4.x_5.x_6.x_7.x_8$

$= x_1.\frac{2}{x_1}.x_3.\frac{4}{x_3}.x_5.\frac{6}{x_5}.x_7.\frac{8}{x_7}$

$= 2.4.6.8 = 384$

$\Rightarrow \log_{3\sqrt{2}}\left(\prod_{i=1}^{8} x_i - 60\right)$

$= \log_{18^{\frac{1}{2}}}(384 - 60) = 2\log_{18}324$

$= 2\log_{18}18^2 = 4$

Q.3 If $2\log_a x = \log_b x + \log_c x$ where a, b, c > 0 and ≠ 1 then which of the following holds-

(A) $c^2 = (bc)^{\log_c a}$ (B) $a = bc(\log_c b)$

(C) $b^2 = (bc)^{\log_c a}$ (D) $a^2 = bc$

Sol.[C] $2\log_a x = \log_b x + \log_c x$

$\Rightarrow \frac{2}{\log a} = \frac{1}{\log b} + \frac{1}{\log c} \Rightarrow \frac{2}{\log a} = \frac{\log bc}{\log b \log c}$

$\Rightarrow 2\log b = \frac{\log a}{\log c}.\log bc \Rightarrow \log b^2 = \log_c a \log bc$

$\Rightarrow b^2 = (bc)^{\log_c a}$

Q.4 If $\log_\alpha 8 = \gamma$, $\log_\beta \alpha = -1$ and $\log_{1/4}\beta = -1$ then $\left(\frac{1}{\alpha}+1\right)^{\log_{\sqrt{5}}(\beta^2+4\gamma^2)}$ is equal to-

(A) $\sqrt{5}$ (B) 5

(C) 25 (D) 625

Sol.[D] $\log_\alpha 8 = \gamma$, $\log_\beta \alpha = 1$, $\log_{1/4}\beta = -1$

$\Rightarrow \beta = 4$, $\alpha = \frac{1}{4}$, $\gamma = -\frac{3}{2}$

$\Rightarrow \left(\frac{1}{\alpha}+1\right)^{\log_{\sqrt{5}}(\beta^2+4y^2)}$

$= 5^{\log_{\sqrt{5}}(16+9)} = 5^{\log_{5^{1/2}}5^2} = 5^4 = 625$

Q.5 The equation, $\log_2(2x^2) + \log_2 x.\ x^{\log_x(\log_2 x+1)} + \frac{1}{2}\log_4^2(x^4) + 2^{-3\log_{1/2}(\log_2 x)} = 1$ has

(A) exactly one real solution

(B) two real solutions

(C) 3 real solutions

(D) no solution

Sol.[D] Given equation can be written as

$\log_2 x(\log_2^2 x + 3\log_2 x + 3) = 0$

$\Rightarrow \log_2 x = 0 \Rightarrow x = 1$

and $\log_2^2 x + 3\log_2 x + 3 = 0 \Rightarrow$ imaginary sol.

But x = 1 does not satisfy the given equation so equation has no solution.

Part-B **One or more than one correct answer type questions**

Q.6 Which of the following when simplified, vanishes?

(A) $\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8}$

(B) $\log_2\left(\frac{2}{3}\right) + \log_4\left(\frac{9}{4}\right)$

(C) $-\log_8 \log_4 \log_2 16$

(D) $\log_{10}\cot 1° + \log_{10}\cot 2° + \log_{10}\cot 3° + \ldots\ldots + \log_{10}\cot 89°$

Sol.[A,B,C,D]

(A) $\log_2 3 + 2\log_{2^2} 3^2 - 3\log_{2^3} 3^3$

$\Rightarrow \log_2 3 + 2\log_2 3 - 3\log_2 3 = 0$ correct

(B) $\log_2\left(\frac{2}{3}\right) - \log_{2^2}\left(\frac{2}{3}\right)^2 = 0$ correct

(C) $-\log_8 \log_4 \log_2 2^4 - \log_8 \log_4 4$

$= -\log_8 1 = 0$ correct.

(D) $\log_{10}\cot 1^o + \log_{10}\cot 2^o + \ldots + \log_{10}\cot 89^o$
$+ \ldots\ldots + \log_{10}$
$\tan 1^o$

$\Rightarrow \log_{10}\cot 1^o + \log_{10}\cot 2^o + \ldots\ldots + \log_{10}$ $\cot 45^o + \log_{10}\tan 44^o + \ldots\ldots\ldots + \log_{10}\tan 1^o$

$\Rightarrow \log_{10}(\cot 1^o . \tan 1^o) + \ldots\ldots\ldots + \log_{10}\cot 45^o$

$= \log_{10} 1 = 0$ correct

$\Rightarrow$ option A, B, C, D are correct.

Q.7 The number $N = \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \log_6^2 2$ when simplified reduces to-

(A) a prime number

(B) an irrational number

(C) a real which is less then $\log_3\pi$

(D) a real which is greater than $\log_7 6$

Sol.[C,D]

$N = \frac{1+2\log_3 2}{(1+\log_3 2)^2} + \log_6^2 2$

$= \frac{1}{1+\log_3 2} + \frac{\log_3 2}{(1+\log_3 2)} + \log_6^2 2$

$= \frac{1}{\log_3 6} + \frac{\log_3 2}{\log_3^2 6} + \log_6^2 2$

$= \log_6 3 + \log_6 2\,(\log_6 3 + \log_6 2)$

$= \log_6 3 + \log_6 2 = \log_6 6 = 1$

Clearly $\log_7 6 < N < \log_3\pi$

$\Rightarrow$ option C, D are correct.

Q.8 Which of the following numbers are positive?

(A) $\log_{\log_3^2}\left(\frac{1}{2}\right)$ (B) $\log_2\left(\frac{2}{3}\right)^{-2/3}$

(C) $\log_{10}\log_{10} 9$ (D) $\log_{10}\sin 125^3$

Sol.[A,B]

(A) $\log_{\log_3 2}\left(\frac{1}{2}\right)$ is +ve Θ $\log_3 2 < 1$

(B) $\log_2\left(\frac{3}{2}\right)^{2/3} = \frac{2}{3}\log_2\frac{3}{2}$ is +ve

(C) $\log_{10}\log_{10} 9$ is – ve

(D) $\log_{10}\sin 125^3$ is –ve

$\Rightarrow$ option A, B are correct.

Q.9 Which of the following are correct?

(A) $\log_3 19 . \log_{1/7} 3 . \log_4 1/7 > 2$

(B) $\log_5(1/23)$ lies between –2 and –1

(C) if $m = 4^{\log_4 7}$ and $n = \left(\frac{1}{9}\right)^{-2\log_3 7}$ then $n = m^4$

(D) $\log_{\sqrt{5}}\sin\left(\frac{\pi}{5}\right) . \log_{\sqrt{\sin\frac{\pi}{5}}} 5$ simplifies to an irrational number

Sol.[A,B,C]

(A) $\log_3 19 . \log_{1/7} 3 . \log_4 1/7$

$= \frac{\log 19}{\log 3} \frac{\log 3}{\log\frac{1}{7}} . \frac{\log\frac{1}{7}}{\log 4}$

$= \log_4 19 > 2$

(B) $\log_5 (23)^{-1} = -\log_5 23$

Θ $\log_5 23$ lies between 1 and 2

$\Rightarrow -2 < -\log_5 23 < -1$

(C) $m = 4^{\log_4 7}$, $n = 9^{2\log_3 7}$

$\Rightarrow m = 7,\ n = 7^4$

$\Rightarrow n = m^4$

(D) $2\log_5\sin\frac{\pi}{5} . 2\log_{\sin\frac{\pi}{5}} 5$

$\Rightarrow 4\frac{\log\sin\frac{\pi}{5}}{\log 5} . \frac{\log 5}{\log\sin\frac{\pi}{5}} = 4$ rational

$\Rightarrow$ A, B, C are correct.

Q.10 The solution set of the system of equations,

$\log_{12}x \left(\frac{1}{\log_x 2} + \log_2 y\right) = \log_2 x$ and

$\log_2 x (\log_3(x + y)) = 3 \log_3 x$ is

(A) $x = 6; y = 2$ (B) $x = 4, y = 3$

(C) $x = 2; y = 6$ (D) $x = 3; y = 4$

Sol.[A,C]

Given equations can be written as

$\log_2 xy = \log_2 12 \Rightarrow xy = 12$

and $\log_3(x + y) = \log_3 2^3 \Rightarrow x + y = 8$

Solvign we get

$x = 2, y = 6$ and $x= 6, y = 2$

$\Rightarrow$ option A, C are correct.

Q.11 If x_1 and x_2 are the solution of the equation

$x^{3\log^3 x - \frac{2}{3}\log x} = 100\sqrt[3]{10}$ then

(A) $x_1x_2 = 1$ (B) $x_1 . x_2 = x_1 + x_2$

(C) $\log_{x_2} . x_1 = -1$ (D) $\log (x_1 . x_2) = 0$

Sol.[A,C,D]

$x^{3\log_{10}^3 x - \frac{2}{3}\log_{10} x} = 10^{\frac{7}{3}}$

$\Rightarrow \left(3\log_{10}^3 x - \frac{2}{3}\log_{10} x\right) \log_{10}x = \frac{7}{3}$

$\Rightarrow \left(9\log_{10}^2 x - 2\right) \log_{10}^2 x = 7$

Solving we get

$\log_{10}^2 x = 1$

$\Rightarrow x_1 = 10, x_2 = \frac{1}{10}$

$\Rightarrow x_1 . x_2 = 1, \log_{x_2} x_1 = -1, \log (x_1 . x_2) = 0$

$\Rightarrow$ A, C, D are correct.

Q.12 The solutions of the equation

$|x - 1|^{(\log x)^2 - \log x^2} = |x - 1|^3$, where base of logarithm is 10 are-

(A) $x = 2$ (B) $x = \frac{1}{10}$

(C) $x = 1000$ (D) $x = 100$

Sol.[A,B,C]

$(\log x)^2 - 2 \log x = 3$

$\Rightarrow (\log x)^2 - 2 \log x - 3 = 0$

$\Rightarrow (\log x - 3) (\log x + 1) = 0$

$\Rightarrow \log x = 3, \log x = -1$

$\Rightarrow x = 1000, x = \frac{1}{10}$

and $x = 2$ equation is

Satisfy so $x = 2$ is correct $\Rightarrow$ Option A, B , C are correct.

Q.13 The in-equation $(\log_x 2)(\log_{2x} 2) (\log_2 4x) > 1$

(A) has a meaning for all x

(B) has a meaning if $x > 2$

(C) is satisfied in $\left(2^{-\sqrt{2}}, \frac{1}{2}\right)$

(D) is satisfied in $(1, 2^{\sqrt{2}})$

Sol. **[B,C,D]**

$(\log_x 2)(\log_{2x} 2) (\log_2 4x) > 1$

$\log_x 2$ is defined for $x > 0$ and $x \neq 3$

$\log_{2x} 2$ is defined for $x > 0$ and $x \neq \frac{1}{2}$

$\log_2 4x$ is defined for $x > 0$

so domain of $(\log_x 2)(\log_{2x} 2) (\log_2 4x)$ is $x > 0$

and $x \neq 1$ also $x \neq \frac{1}{2}$

so choice (A) is ruled out.

Since $x > 2$ is subset of domain of the inequation is meaningful if $x > 2$

The given expressions is

$\frac{2 + \log_2 x}{\log_2 x(1 + \log_2 x)} > 1$

put $\log_2 x = t$

$\Rightarrow \frac{2 + t}{t(1 + t)} > 1$

If numerator & denominator > 0

$\Rightarrow t^2 + t - t - 2 < 0$

$\Rightarrow -\sqrt{2} < t < \sqrt{2}$

$\Rightarrow 2^{-\sqrt{2}} < x < 2^{\sqrt{2}}$

Q.14 The solution set of

$\left|\frac{x + 1}{x}\right| + |x + 1| = \frac{(x + 1)^2}{|x|}$ is

(A) $\{x \mid x \geq 0\}$

(B) $\{x \mid x > 0\} \cup \{-1\}$

(C) $\{-1, 1\}$

(D) $\{x \mid x \geq 1 \text{ or } x \leq -1\}$

Sol. **[B]**

$$\left|\frac{x+1}{x}\right| + |x+1| = \frac{(x+1)^2}{|x|}$$

$$\Rightarrow \left|\frac{x+1}{x}\right| + |x+1| = \frac{(x+1)^2}{|x|}$$

$$\Rightarrow |x+1|\left\{\frac{1}{|x|} + 1 - \frac{|x+1|^2}{|x|}\right\} = 0$$

$\therefore$ $|x + 1| = 0$ or $1 + |x| - |x + 1| = 0$

$\Rightarrow x = -1$: If $x < -1$, $1 + |x| - |x + 1| = 0$

$\Rightarrow 1 - x + x + 1 = 0 \Rightarrow 2 = 0$

(Not possible, Rejected)

if $-1 \le x < 0$, $1 + |x| - |x + 1| = 0$

$\Rightarrow 1 - x - (x + 1) = 0$

$\Rightarrow x = 0$ (not possible)

if $x \ge 0$, $1 + x - (x + 1) = 0 \Rightarrow 0 = 0$

$\Rightarrow$ x can have any value in the interval

$\therefore$ $x = -1$, $x > 0$ $\quad$ ($\because x \ne 0$)

$\{x \mid x > 0\} \cup \{-1\}$

Q.15 $\log_{|\sin x|}(x^2 - 8x + 23) > \dfrac{3}{\log_2 |\sin x|}$ -

(A) $3 < x < \pi$, $\pi < x < \frac{3\pi}{2}$, $\frac{3\pi}{2} < x < 5$

(B) $3 < x < \pi$, $\pi < x < \frac{\pi}{2}$, $\frac{\pi}{2} < x < 5$

(C) $3 < x < \frac{5\pi}{2}$, $\frac{5\pi}{2} < x < \frac{\pi}{2}$, $\frac{\pi}{2} < x < 5$

(D) None of these

Sol. **[A]**

$\log_{|\sin x|}(x^2 - 8x + 23) > \dfrac{3}{\log_2 |\sin x|}$

$$\Rightarrow \frac{\log_2(x^2 - 8x + 23)}{\log_2 |\sin x|} > \frac{3}{\log_2 |\sin x|}$$

$\Rightarrow \log_2 (x^2 - 8x + 23) < 3$ $\quad$ ($\because \log_2 |\sin x| < 0$)

$\Rightarrow x^2 - 8x + 23 < 2^3\ (= 8)$

$\Rightarrow x^2 - 8x + 15 < 0$

$\Rightarrow (x - 5)(x - 3) < 0$

$\Rightarrow 3 < x < 5$

For log defined $|\sin x| \ne 0$

$\Rightarrow x \notin \{n\pi, n \in I\}$

and $\quad x \notin \{(2k + 1)\frac{\pi}{2}, k \in I\}$

for $x \in (3, 5)$;

$x \ne \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

Hence $x \in (3, \pi) \cup (\pi, \frac{3\pi}{2}) \cup (\frac{3\pi}{2}, 5)$

i.e. $3 < x < \pi$, $\pi < x < \frac{3\pi}{2}$, $\frac{3\pi}{2} < x < 5$.

Part-C Assertion-Reason type questions

The following questions 16 to 19 consists of two statements each, printed as Statement (1) and Statement (2). While answering these questions you are to choose any one of the following four responses.

(A) If both Statement (1) and Statement (2) are true and the Statement (2) is correct explanation of the Statement (1).

(B) If both Statement (1) and Statement (2) are true but Statement (2) is not correct explanation of the Statement (1).

(C) If Statement (1) is true but the Statement (2) is false.

(D) If Statement (1) is false but Statement (2) is true

Q.16 **Statement (1) :** The equation $(\log x)^2 - \log x^3 + 2 = 0$ has only one solution.

Statement (2) : $\log x^2 = 2 \log x$ if $x > 0$.

Sol. **[D]**

$\because$ log x occurs in the equation $\therefore$ $x > 0$

$(\log x)^2 - 3 \log x + 2 = 0$

$\Rightarrow (\log x - 1)(\log x - 2) = 0$

$\Rightarrow \log x = 1, \log x = 2$

$\Rightarrow x = 10, 100$

$\Rightarrow$ The equation has two solutions

Q.17 **Statement (1) :** The equation $t.\ 2^x + 2^{-x} = 5$ has a unique solution for two values of t.

Statement (2) : Sum of a positive number and its reciprocal is not less than 2.

Sol. **[B]**

$t.\ 2^x + 2^{-x} = 5$

If $t = 0$ then $2^{-x} = 5$

$\Rightarrow x = -\log_2 5$

$\Rightarrow$ equation has a unique solution.

If $t \ne 0$ then put $2^x = y$

$t\,y + \frac{1}{y} = 5$

$\Rightarrow y = \frac{5 \pm \sqrt{25-4t}}{2}$

This will have unique solution if $t = \frac{25}{4}$ and the solution will be 5/2 which is > 0 ($\because y > 0$)

Hence for $t = 0$ and $t = \frac{25}{4}$ the system has a unique solution.

Q.18 **Statement- (1) :**

If $\log_{(\log_5 x)} 5 = 2$, then $x = 5^{\sqrt{5}}$

Statement (2) : $\log_x a = b$, if $a > 0$, then $x = a^{1/b}$.

Sol.[A] $\log_{\log_5 x} 5 = 2 \Rightarrow (\log_5 x)^2 = 5$

$\log_5 x = \sqrt{5} \quad \Rightarrow x = 5^{\sqrt{5}}$

Q.19 **Statement- (1) :**

The equation

$\log_{\frac{1}{2+|x|}}(5+x^2) = \log_{(3+x^2)}(15+\sqrt{x})$ has no solution.

Statement (2) : $\log_{1/b} a = -\log_b a$ and if number and base both are greater then unity then the number is positive

Sol.[B] $-\log_{2+|x|}(5+x^2) = \log_{3+x^2}(15+\sqrt{x})$

If a number and base are both greater than unity then the logarithm value is > 0)

$2 + |x| > 1, \; 3 + x^2 > 1, \; 5 + x^2 > 1, \; 15 + \sqrt{x} > 1$

hence $\log_{2+|x|}(5 + x^2) > 0$ & $\log_{3+x^2}(15+\sqrt{x}) > 0$

$-\log_{2+|x|}(5 + x^2) < 0$

LHS < 0

RHS > 0 hence no solution

Part-D Column Matching type questions

Q.20 Match the column for values of x which satisfy the equation in Column 1

	Column-1	Column-2
(A)	$\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \frac{1}{2}$	(P) 5
(B)	$x^{\log x + 4} = 32$, where base of logarithm is 2	(Q) 100
(C)	$5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$ where the base of logarithm is 10	(R) 2
(D)	$9^{1+\log x} - 3^{1+\log x} - 210 = 0$; where base of log is 3	(S) 1/32

Sol. **A → P ; B → R, S ; C → Q ; D → P**

(A) $\because (x-3)^2 = x^2 - 21$

$\Rightarrow -6x + 9 = -21 \Rightarrow x = 5$

(B) $(\log_2 x + 4) \log_2 x = 5$

$\Rightarrow (\log_2 x)^2 + 4\log_2 x - 5 = 0$

$\Rightarrow (\log_2 x + 5)(\log_2 x - 1) = 0$

$\Rightarrow \log_2 x = -5, \log_2 x = 1 \Rightarrow x = \frac{1}{32}, x = 2$

(C) Solving we get

$5^{\log_{10} x} = \frac{50}{18} \; 3^{\log_{10} x}$

$\Rightarrow \log_{10} x \log_{10} 5 = \log_{10} \frac{50}{18} + \log_{10} x \log_{10} 3$

$\Rightarrow \log_{10} x \left(\log_{10} \frac{5}{3}\right) = \log_{10} \frac{25}{9}$

$\Rightarrow \log_{10} x = \log_{5/3} \left(\frac{5}{3}\right)^2 \Rightarrow x = 100$

(D) $\because 9.3^{2\log x} - 3.3^{\log x} - 210 = 0$

$\Rightarrow 3x^2 - x - 70 = 0$ (wehre $3^{\log_3 x} = x$)

$\Rightarrow (3x + 14)(x - 5) = 0 \Rightarrow x = 5, x = -\frac{14}{3}$

$\Rightarrow x = 5$ ($x = \frac{-14}{3}$ not possible)

Q.21

	Column-1	Column-2
(A)	If 3^x and $7^{1/x}$ are two distinct prime number $\forall x \in R$ then number of x so that $3^x + 7^{1/x} = 10$ is/are	(P) 4
(B)	If N be the number of solution and S be the sum of all roots of the equation $\lvert x^2 - x - 6\rvert = x + 2$, then N + S equals	(Q) 2

(C) The value of (R) 1

$4\left[\frac{2\log_6 3}{\log_2 6}+(\log_6 2)^2+\frac{1}{(\log_3 6)^2}\right]$ is

(D) The number of values (S) 7

of x satisfying

$3^{\log_{27}(x-3)+\log_{\sqrt{3}}5} = 50$ is/are

Sol. A → Q, B → S, C → P, D → R

(A)

(B) $|x^2 - x - 6| = x + 2$

$|(x+2)(x-3)| = x + 2$

case 1 : $x \geq 3$

$(x + 2)(x - 3) = (x + 2)$

$\Rightarrow \quad x = 4$

case 2 : $-2 \leq x < 3$

$6 + x - x^2 = x + 2$

$x^2 = 4 \Rightarrow x = \pm 2$

case 3 : $x < -2$

$(x + 2)(x - 3) = x + 2; \quad x = 4$ (wrong)

$x = 4, -2, 2$

S = 4; N = 3; S + N = 7

(C) $4[2\log_6 3\log_6 2 + (\log_6 2)^2 + (\log_6 3)^2]$

$= 4[((\log_6 2) + (\log_6 3))^2]$

$= 4(\log_6 6)^2 = 4$

(D) $3^{\log_{27}(x-3)} \times 3^{\log_{\sqrt{3}}5} = 50$

$3^{\frac{1}{3}\log_3(x-3)} \times 3^{2\log_3 5} = 50$

$(x-3)^{1/3} \times 25 = 50$

$(x-3)^{1/3} = 2$

$x - 3 = 8$

➢ Fill in the blanks type questions

Q.22 If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$

Then answer the following

(i) xyz = ...

(ii) $x^a y^b z^c$ = ...

Sol. (i) $x = e^{k(b-c)}$, $y = e^{k(c-a)}$, $z = e^{k(a-b)}$

$\Rightarrow xyz = e^{\circ} = 1$

(ii) $x^a y^b z^c = e^{\circ} = 1$

Q.23 If $x^{[(\log_2 x)^2 - 6\log_2 x + 11]} = 64$ then

x equal to ....

Sol. $[(\log_2 x)^2 - 6\log_2 x + 11]\log_2 x = 6$

Put $\log_2 x = t$ we get

$t^3 - 6t^2 + 11t - 6 = 0 \Rightarrow (t-1)(t-2)(t-3) = 0$

$\Rightarrow t = 1, 2, 3 \Rightarrow \log_2 x = 1, 2, 3 \Rightarrow x = 2, 4, 8$

Q.24 If $x + \log_{10}(2^x + 1) = \log_{10}6 + x\log_{10}5$

then the value of x is..................

Sol. $(2^x + 1)\,10^x = 6.5^x$

$(2^x)^2 + 2^x - 6 = 0$ put $2^x = t$

$t^2 + t - 6 = 0 \Rightarrow t = 2, -3$ (rejected)

$\therefore 2^x = 2 \Rightarrow x = 1$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Compute the following :

(i) $\sqrt[3]{5^{\frac{1}{\log_7 5}} + \dfrac{1}{\sqrt{-\log_{10} 0.1}}}$

(ii) $\log_{0.75} \log_2 \sqrt{\sqrt[-2]{0.125}}$

(iii) $\left(\dfrac{1}{49}\right)^{1+\log_7 2} + 5^{-\log_{1/5} 7}$

Sol. (i) $\left(5^{\log_5 7} + \dfrac{1}{1}\right)^{1/3} \Rightarrow \left(2^3\right)^{1/3} = 2$

(ii) $\log_{0.75} \log_2 \sqrt{(0.125)^{-1/2}}$

$\Rightarrow \log_{0.75} \log_2 (0.125)^{-1/4}$

$\Rightarrow \log_{0.75} \log_2 (0.5)^{-3/4}$

$\Rightarrow \log_{0.75} (-3/4)(-1)$

$\Rightarrow \log_{0.75} 0.75 = 1$

(iii) $7^{-2}.7^{-2\log_7 2} + 5^{\log_5 7}$

$\Rightarrow 49^{-1} \times 2^{-2} + 7$

$\Rightarrow \dfrac{1}{196} + 7 \Rightarrow 7 + \dfrac{1}{196}$

Q.2 Simplify the following:

(i) $5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \dfrac{4}{\sqrt{7}+\sqrt{3}} + \log_{\frac{1}{2}} \dfrac{1}{10+2\sqrt{21}}$

(ii) $\log_{1/3} \sqrt[4]{\sqrt{729}.\sqrt[3]{9^{-1}.27^{-4/3}}}$

(iii) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

(iv) $4^{5\log_{4\sqrt{2}}(3-\sqrt{6})-6\log_8(\sqrt{3}-\sqrt{2})}$

Sol. (i) $5^{\log_{5^{-1}} 2^{-1}} + \log_{2^{1/2}} \dfrac{4}{(\sqrt{7}+\sqrt{3})} + \log_{2^{-1}} \dfrac{1}{(10+2\sqrt{2})}$

$= 5^{\log_5 2} + 4^{\log_2 2} - 2\log_2(\sqrt{7}+\sqrt{3}) + \log_2(10+2\sqrt{21})$

$= 2+4-\log_2(\sqrt{7}+\sqrt{3})^2 + \log_2(\sqrt{7}+\sqrt{3})^2 = 6$ Ans.

(ii) $\log_{1/3}\left[(729)^{1/2}\left(\dfrac{1}{9}\times\dfrac{1}{(27)^{4/3}}\right)^{1/3}\right]^{1/4}$

$\Rightarrow \log_{1/3}\left[(27)^{2.\frac{1}{2}}\left(\dfrac{1}{9}\times\dfrac{1}{3^4}\right)\right]^{1/4}$

$\Rightarrow \log_{1/3}\left[27\times\dfrac{1}{9\times 3^4}\right]^{1/4}$

$= \log_{1/3}\left[27\times 3^{-2/3}.3^{-4/3}\right]^{1/4}$

$= \log_{3^{-1}} 3^{1/4} \qquad = -\dfrac{1}{4}$

(iii) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

$= 5^{\log_3 7} + 7^{\log_5 3} - 5^{\log_3 7} - 7^{\log_5 3}$

$= 0$

(iv) $4^{5\log_{2^{5/2}}(3-\sqrt{6})-6\log_{2^3}(\sqrt{3}-\sqrt{2})}$

$= 4^{2\log_2(3-\sqrt{6})-2\log_2(\sqrt{3}-\sqrt{2})}$

$= 4^{\log_2\frac{(3-\sqrt{6})^2}{(\sqrt{3}-\sqrt{2})^2}} = 4^{\log_2 3}$

$= 2^{2\log_2 3} = 2^{\log_2 3^2} = 9$

Q.3 Find the value of $49^A + 5^B$ where $A = 1-\log_7 2$ & $B = -\log_5 4$

Sol. $49^A + 5^B$

$49^{1-\log_7 2} + 5^{-\log_5 4}$

$\Rightarrow 49.\ 49^{-\log_7 2} + 5^{\log_5 4^{-1}} \Rightarrow 49.\ 7^{\log_7 2^{-2}} + 4^{-1}$

$\Rightarrow 49.\ \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{25}{2}$ Ans.

Q.4 Solve for x:

(i) $\log_{10}(x^2 - 12x + 36) = 2$

(ii) $9^{1+\log x} - 3^{1+\log x} - 210 = 0$; where base of log is 3.

Sol. (i) $\log_{10}(x^2 - 12x + 36) = 2$

$\Rightarrow x^2 - 12x + 36 = 100$

$\Rightarrow x^2 - 16x + 4x - 64 = 0$

$\Rightarrow (x-16)(x+4) = 0$

$\Rightarrow x = -4, 16$ Ans.

(ii) $9.9^{\log_3 x} - 3.3^{\log_3 x} - 210 = 0$

$= 9.3^{2\log_3 x} - 3x - 210 = 0$

$\Rightarrow 9x^2 - 3x - 210 = 0$

$\Rightarrow 3x^2 - x - 70 = 0$

$\Rightarrow (x - 5)(3x - 14) = 0$

$\Rightarrow x = 5$ Ans.

Q.5 Solve for x :

$\log_{x+1} (x^2 + x - 6)^2 = 4$

Sol. $(x + 1)^4 = (x^2 + x - 6)^2$

$(x + 1)^2 = \pm (x^2 + x - 6)$

$x^2 + 2x + 1 = x^2 + x - 6$

$\Rightarrow x = -7$

or

$x^2 + 2x + 1 = -x^2 - x + 6$

$\Rightarrow 2x^2 + 3x - 5 = 0$

$\Rightarrow 2x^2 + 5x - 2x - 5 = 0$

$\Rightarrow 2x(x - 1) + 5(x - 1) = 0$

$\Rightarrow x = -\frac{5}{2}, 1$

$\Rightarrow x = 1 \quad$ (Θ $x + 1 > 0$)

Q.6 Solve $\frac{6}{5} a^{\log_a x . \log_{10} a . \log_a 5} - 3^{\log_{10}\left(\frac{x}{10}\right)}$

$= 9^{\log_{100} x + \log_4 2}$

Sol. Θ $\log_a x \log_{10} a = \log_{10} x$

$\Rightarrow \log_{10}\left(\frac{x}{10}\right) = \log_{10} x - 1$

$\Rightarrow \log_{100} x = \frac{1}{2}\log_{10} x$

and $\log_4 2 = \frac{1}{2}\log_2 2 = \frac{1}{2}$

None the equation can be written as –

$\frac{6}{5} a^{\log_{10} x \log_a 5} - 3^{\log_{10} x - 1} = 9^{\frac{1}{2}\log_{10} x + \frac{1}{2}}$

or $\frac{6}{5}\left(a^{\log_a 5}\right)^{\log_{10} x} = 3^{\log_{10} x}.3^{-1} + 3^{\log_{10} x}.3$

or $\frac{6}{5}.5^{\log_{10} x} = 3^{\log_{10} x}\left(\frac{1}{3}+3\right) = \frac{10}{3}.3^{\log_{10} x}$

or $\frac{1}{5^2} 5^{\log_{10} x} = \frac{1}{3^2} 3^{\log_{10} x}$

or $5^{(\log_{10} x - 2)} = 3^{(\log_{10} x - 2)}$

since base are different so it is true

$\log_{10} x - 2 = 0$

$\Rightarrow \log_{10} x = 2, \Rightarrow x = 10^2 = 100$ Ans.

Q.7 Find x satisfying the equation

$\log^2\left(1+\frac{4}{x}\right) + \log^2\left(1-\frac{4}{x+4}\right)$

$= 2\log^2\left(\frac{2}{x-1} - 1\right)$

Sol. L.H.S.

$\log^2\left(\frac{x+4}{x}\right) + \log^2\left(\frac{x}{x+4}\right)$

$= \log^2\left(\frac{x+4}{x}\right) + \log^2\left(\frac{x+4}{x}\right)$

$= 2\log^2\left(\frac{x+4}{x}\right) \qquad \ldots(1)$

R.H.S.

$2\log^2 \frac{3-x}{x-1} \qquad \ldots(2)$

from (1) & (2)

$\frac{x+4}{x} = \frac{3-x}{x-1}$ or $\frac{x-1}{3-x}$

$\Rightarrow x^2 = 2$ or $x^2 = 6$

$x = \pm\sqrt{2}, \pm\sqrt{6}$

for log defined $x \neq -\sqrt{2}, -\sqrt{6}$

$\therefore x = \sqrt{2}, \sqrt{6}$ Ans.

Q.8 If $4^A + 9^B = 10^C$, where $A = \log_{16} 4$, $B = \log_3 9$ and $C = \log_x 83$ then find x.

Sol. $4^A + 9^B = 10^C$ where $A = \log_{16} 4$, $B = \log_3 9$, $C = \log_x 83$.

$\Rightarrow 4^{\log_{16} 4} + 9^{\log_3 9} = 10^{\log_x 83}$

$= 4^{1/2} + 9^2 = 83^{\log_x 10}$

$= 83^1 = 83^{\log_x 10}$

$= \log_x 10 = 1$

$\Rightarrow x = 10$

Q.9 Solve the following inequalities

(i) $\frac{1}{1+\log x}+\frac{1}{1-\log x}>2.$

(ii) $\frac{1}{\log_4[(x+1)/(x+2)]}<\frac{1}{\log_4(x+3)}$

Sol. (i) $\frac{1}{1+\log x}+\frac{1}{1-\log x}>2.$

$\Rightarrow \frac{1-\log x+1+\log x}{1-\log^2 x}>2$

$\Rightarrow \frac{1}{1-\log^2 x}>1 \Rightarrow 1-\log^2 x<1$

$\Rightarrow \ -\log^2 x<0 \Rightarrow \ x<1$

(1) $1+\log x \neq 0 \Rightarrow \ \log x \neq -1$

$\Rightarrow \ x \neq 10^{-1} \Rightarrow \ \ x \neq 1/10 \neq 0.1$

(2) $1+\log x>0 \Rightarrow \ x>0.1$

(3) $1-\log x \neq 0 \Rightarrow \ \log x \neq 1 \Rightarrow \ \ x \neq 10$

(4) $1-\log x>0 \Rightarrow \ \log x<1 \Rightarrow \ \ x<10$

$x \in (0.1, 1) \cup (1, 10)$ Ans.

Q.10 Solve the inequality

$\log_{\frac{1}{2}} \frac{x^2+6x+9}{2(x+1)}<-\log_2(x+1)$

Sol. $\log_{2^{-1}} \frac{(x+3)^2}{2(x+1)}+\log_2(x+1)<0$

$\Rightarrow \log_2^{-1}(x+3)^2-\log_{2^{-1}} 2-\log_{2^{-1}}(x+1)+\log_2(x+1)<0$

$\Rightarrow -2\log_2(x+3)+\log_2 2+\log_2(x+1)+\log_2(x+1)<0$

$\Rightarrow 2\log_2(x+1)-2\log_2(x+3)+1<0$

$\Rightarrow \log_2(x+1)^2-\log_2(x+3)^2<-1$

$\Rightarrow \log_2 \frac{(x+1)^2}{(x+3)^2}<-1 \qquad \Rightarrow \frac{x^2+2x+1}{x^2+6x+9}<\frac{1}{2}$

$\Rightarrow x^2-2x-7<0$

$\Rightarrow \frac{2 \pm \sqrt{4+28}}{2}=2 \pm \sqrt{32}=(1 \pm \sqrt{8})$

$x>-1, \ x>-3$

$x>-1$ & $x<1+2\sqrt{2}$

$(-1, 1+2\sqrt{2})$

Q.11 Solve the inequality $x^{\frac{1}{\log_{10} x}}.\log_{10} x<1$

Sol. $x^{\log_x 10}.\log_{10} x<1 \qquad \Rightarrow 10.\ \log_{10} x<1$

$\Rightarrow \log_{10} x<\frac{1}{10}$ for log defined $x>0$

$\Rightarrow x<10^{1/10} \qquad \Rightarrow 0<x<10^{1/10}$

$0<x<\sqrt[10]{10}$ Ans.

Q.12 $\log_{\frac{1}{3}}(x-1)+\log_{\frac{1}{3}}(x+1)$

$+\log_{\sqrt{3}}(5-x)<1$ solve for x.

Sol. $\log_{3^{-1}}(x-1)+\log_{3^{-1}}(x+1)+\log_{3^{1/2}}(5-x)<1$

$\Rightarrow -\log_3(x-1)-\log_3(x+1)+2\log_3(5-x)<1$

$\Rightarrow -[\log_3(x-1)+\log_3(x+1)]+\log_3(5-x)^2<$
1

$\Rightarrow -\log_3(x^2-1)+\log_3(5-x)^2<1$

$\Rightarrow \log_3 \frac{(5-x)^2}{x^2-1}<1 \qquad \Rightarrow \frac{(5-x)^2}{x^2-1}<3$

$\Rightarrow x^2+5x-14>0 \quad \Rightarrow x(x+7)-2(x+7)>0$

$\Rightarrow (x+7)(x-2)>0 \quad \Rightarrow x>2$ & $x<5$

$\Rightarrow \ x \in (2, 5)$

Q.13 Solve $x^{(\log_{10} x)^2-\log_{10} x^3+1}>1000$

Sol. $x^{(\log_{10} x)^2-\log_{10} x^3}+1>1000$

put $\log_{10} x=t$ and take log of both sides

$t^2-3t+1>\log_x 10^3=\frac{3\log 10}{\log x}=\frac{3}{t}$

or $t^3-3t^2+t-3>0$

or $(t^2+1)(t-3)>0 \qquad \therefore t>3$

or $\log_{10} x>3$

or $x>10^3=1000$

$\therefore \ x \in (1000, \infty)$ Ans.

Q.14 If $\log_{10}2=0.3010$, $\log_{10}3=0.4771$. Find the number of integers in

(i) 5^{200}

(ii) 6^{15} &

(iii) the number of zeros after the decimal in 3^{-100}

Sol. (i) 5^{200}

$=200\log_{10}5=200\log_{10}\frac{10}{2}=200[1-\log_{10}2]$

$= 200\,[1 - 0.3010]$

$= 139.8$

$\therefore$ No. of integers = 139 + 1 = 140

(ii) 6^{15}

= 15 log 6

= 15 [log 2 + log 3]

= 15 [0.3010 + 0.4771]

= 11.8065

$\therefore$ No. of integers = 11 + 1 = 12

Q.15 Solve the equations

(i) $|x-4| - |x+4| = 8$

(ii) $|x-3| + |x+2| - |x-4| = 3$

(iii) $8x^2 + |-x| + 1 > 0$

Sol. (i) $(-\infty, -4]$ (ii) $-6, 2$

(iii) $(-\infty, \infty)$

Q.16 Solve $2^{|x+1|} - 2^x = |2^x - 1| + 1$

Sol. Case I : $x \le -1$

$2^{-(x+1)} - 2^x = -(2^x - 1) + 1$

$\Rightarrow 2^{-(x+1)} = 2^1 \therefore -(x+1) = 1$

$\Rightarrow x = -2$

Case II : $-1 < x < 0$

$2^{x+1} - 2^x = 1 - 2^x + 1$

$\Rightarrow 2^{x+1} = 2^1 \Rightarrow x + 1 = 1 \Rightarrow x = 0$

$\therefore$ x = 0 does not satisfy the condition,

$\therefore$ x = 0 is not a root

Case III: $x \ge 0$

$2^{x+1} - 2^x = 2^x - 1 + 1 \Rightarrow 2^{x+1} = 2.2^x$

$\Rightarrow 2^{x+1} = 2^{x+1}$ true for $x \ge 0$

$\therefore x = -2$ & $x \ge 0$ Ans.

Q.17 Solve the following systems of equations:

(i) $2^{\log_{1/2}(x+y)} = 5^{\log_5(x-y)}$

$\log_2 x + \log_2 y = \frac{1}{2}$

(ii) $\log_2 xy \,.\, \log_2 \frac{x}{y} = -3$

$\log_2^2 x + \log_2^2 y = 5$

Sol. (i) $2^{\log_{2^{-1}}(x+y)} = 5^{\log_5(x-y)}$

$\Rightarrow \frac{1}{x+y} = x - y$

$\Rightarrow x^2 - y^2 = 0$

$\Rightarrow x = \pm y$ …(1)

$\log_2 x + \log_2 y = \frac{1}{2}$

$\Rightarrow \log_2 xy = \frac{1}{2}$

$\Rightarrow xy = \sqrt{2}$

$\Rightarrow y = \frac{\sqrt{2}}{x}$ …(2)

from (1) & (2) $x = \pm \frac{\sqrt{2}}{x}$

$\Rightarrow x^2 = \pm\sqrt{2}$, $\Rightarrow x = \pm\sqrt{2}$

(Θ x > y) $\therefore x = \sqrt{2}$, y = 1 Ans.

(ii) $\log_2 xy \,.\, \log_2 \frac{x}{y} = -3$

$\Rightarrow (\log_2 x + \log_2 y)(\log_2 x - \log_2 y) = 3$

$\Rightarrow \log_2^2 x - \log_2^2 y = -3$ …(1)

$\log_2^2 x + \log_2^2 y = 5$ …(2)

(1) + (2) $\Rightarrow$

$2\log_2^2 x = 2$

$\Rightarrow \log_2^2 x = 1$, $\Rightarrow \log_2 x = \pm 1$

$$\left.\begin{array}{l} \Rightarrow x = 2, \frac{1}{2} \\ \Rightarrow y = 4, \frac{1}{4} \end{array}\right\} \Rightarrow (2,4), \left(\frac{1}{2}, \frac{1}{4}\right) \text{ Ans.}$$

Part-B Passage based objective questions

Passage-1 (Q.18 to Q.20)

In comparison of two numbers, logarithm of smaller number is smaller, if base of the logarithm is greater than one. Logarithm of smaller number is larger, if base of logarithm is in between zero and one. For example $\log_2 4$ is smaller than $\log_2 8$ and $\log_{1/2} 4$ is larger than $\log_{1/2} 8$.

On the basis of the above information, answer the following questions:

Q.18 Identify the correct order:

(A) $\log_2 6 < \log_3 8 < \log_3 6 < \log_4 6$

(B) $\log_2 6 > \log_3 8 > \log_3 6 > \log_4 6$

(C) $\log_3 8 > \log_2 6 > \log_3 6 > \log_4 6$

(D) $\log_3 8 > \log_4 6 > \log_3 6 > \log_2 6$

Sol. [B] $\log_2 6 = \dfrac{\log 6}{\log 2}$; $\log_3 6 = \dfrac{\log 6}{\log 3}$; $\log_4 6 = \dfrac{\log 6}{\log 4}$

$\log 2 < \log 3 < \log 4$

Hence, $\log_2 6 > \log_3 6 > \log_4 6$

also $\log_3 8 > \log_3 6$

also $\log_2 6 > 2$ and $\log_3 8 < 2$

so $\log_2 6 > \log_3 8$

Q.19 $\log_{1/20} 40$ is

(A) greater than one

(B) smaller than one

(C) greater than zero and smaller than one

(D) None of these

Sol.[B] $\log_{1/20} 40 = -\log_{20} 40 < 0$

Q.20 $\log_{1/4}(x-1) < \log_{1/4}(3-x)$ is satisfied when

(A) x is greater than one

(B) x is greater than two and smaller than 3

(C) x is smaller than three

(D) insufficient information

Sol.[B] $x - 1 > 0 \Rightarrow \quad x > 1$

$3 - x > 0 \Rightarrow \quad x < 3$

$x - 1 > 3 - x \Rightarrow 2x > 4 \Rightarrow x > 2$

Hence, $x \in (2, 3)$

Passage-2 (Q.21 to Q.23)

Let $f(x) = \log_{\left(\frac{25-x^2}{16}\right)}\left(\dfrac{24-2x-x^2}{14}\right)$

On the basis of above information, answer the following:

Q.21 The values of x for which the

$\sqrt{f(x)} \times \log_{\sec^2(8.5)} \dfrac{25-x^2}{16} > 0$

(A) $(-3, 3)$

(B) $(-1-\sqrt{11}, 3)$

(C) $(-3, -1+\sqrt{11})$

(D) $(-5-\sqrt{11}, -1+\sqrt{11})$

Sol.[C] $\sqrt{\log_{\left(\frac{25-x^2}{16}\right)}\left(\dfrac{24-2x-x^2}{14}\right)}$

$\times \log_{\sec^2(8.5)}\left(\dfrac{25-x^2}{16}\right) > 0$

$\Rightarrow \log_{\sec^2(8.5)}\left(\dfrac{25-x^2}{16}\right) > 0$

$\Rightarrow \sec^2 8.5 > 1$

hence $\dfrac{25-x^2}{16} > 1$

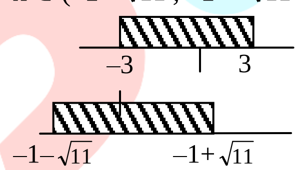
$x^2 < 9 \Rightarrow -3 < x < 3$

also $\log_{\left(\frac{25-x^2}{16}\right)}\left(\dfrac{24-2x-x^2}{14}\right) > 0$

$\dfrac{24-2x-x^2}{14} > 1$

$x^2 + 2x - 10 < 0$

$x \in (-1-\sqrt{11}, -1+\sqrt{11})$



$(-3, -1+\sqrt{11})$

Q.22 If $\left(\dfrac{25-x^2}{16}\right) \in (0, 1)$ then values of x for which

f(x) > 1 will be

(A) (3, 4) (B) (–5, –3)

(C) (3, 5) (D) $(-6, -3) \cup (3, 4)$

Sol. [A] $\log_{\frac{25-x^2}{16}} \dfrac{(24-2x-x^2)}{14} > 1$

$0 < \dfrac{25-x^2}{16} < 1$

$\dfrac{24-2x-x^2}{14} < \dfrac{25-x^2}{16}$

$192 - 16x - 8x^2 < 175 - 7x^2$

$x^2 + 16x - 17 > 0$

$0 < \dfrac{25-x^2}{16} < 1 \quad x > 1, x < -17 \quad ....(1)$

$-1 < \frac{x^2-25}{16} < 0$ $\qquad -16 < x^2 - 25 < 0$

$9 < x^2 < 25$

$\frac{24-2x-x^2}{14} > 0$ $\qquad x \in (-5, -3) \cup (3, 5)$

...(2)

$x^2 + 2x - 24 < 0$

$(x + 6)(x - 4) < 0$

$-6 < x < 4$ ....(3)

Solving (1), (2) & (3); $x \in (3, 4)$

Q.23 If $\left(\frac{25-x^2}{16}\right) > 1$ then values of x for which f(x) > 1 will be-

(A) (–3, 0) (B) (–3, 1)

(C) (–3, 2) (D) (–3, 3)

Sol.[B] $\frac{25-x^2}{16} > 1 \Rightarrow -3 < x < 3$

$\frac{24-2x-x^2}{14} > \frac{25-x^2}{16} \Rightarrow x^2 + 16x - 17 < 0$

$-17 < x < 1$

solving, x : (–3, 1)

Passage-3 (Q.24 to Q.26)

Given that $N = 7^{\log_{49} 900}$,

$A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3}$

$D = (\log_5 49)\ (\log_7 125)$

Then answer the following questions .

(using the values of N, A, D)

Q.24 If $\log_A D = a$, then the value of $\log_6 12$ is (in terms of a)

(A) $\frac{1+3a}{3a}$ (B) $\frac{1+2a}{3a}$

(C) $\frac{1+2a}{2a}$ (D) $\frac{1+3a}{2a}$

Sol.[A] $N = 7^{\log_{49} 900} = 7^{\frac{1}{2}\log_7 900} = 30$

$A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3}$

$= 4 + 3^{\log_2 4} + 4 - 3^{\log_2 4} = 8$

$D = (\log_5 49)(\log_7 125) = \frac{2\log 7}{\log 5} \times \frac{3\log 5}{\log 7} = 6$

$\log_A D = a \Rightarrow \log_8 6 = a \Rightarrow \frac{\log 6}{\log 8} = a$

$\frac{\log 2 + \log 3}{3\log 2} = a \Rightarrow \frac{\log 3}{\log 2} = 3a - 1$

$\log_6 12 = \frac{\log 12}{\log 6} = \frac{2\log 2 + \log 3}{\log 2 + \log 3} = \frac{2 + \frac{\log 3}{\log 2}}{1 + \frac{\log 3}{\log 2}}$

$\frac{2+3a-1}{1+3a-1} = \frac{3a+1}{3a}$

Q.25 If the value obtained in the previous question is $\frac{1+ma}{na}$ then choose the correct option.

(A) $\log_N m < \log_m N = \log_n N$

(B) $\log_N m < \log_n N < \log_m N$

(C) $\log_m N < \log_N m < \log_n N$

(D) $\log_m N < \log_N m = \log_n N$

Sol.[A] m = n = 3

$\log_N m = \log_{30} 3$

$\log_m N = \log_3 30 = \log_n N$

Q.26 The value of $\log_{\left(A-\frac{N}{10}\right)} |N + A + D + m + n| - \log_5 2$ is -

(A) 1 (B) 2

(C) 3 (D) 4

Sol.[B] $\log_{\left(A-\frac{N}{10}\right)} |N + A + D + m + n| - \log_5 2$

$\log_5(50) - \log_5 2 = \log_5\left(\frac{50}{2}\right) = 2$

EXERCISE # 4

Old IIT-JEE questions

Q.1 If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval- **[IIT 1985]**

(A) $(2, \infty)$ (B) $(1, 2)$

(C) $(-2, -1)$ (D) None of these

Sol. **[A]**

$\log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$

for log to be defined

$x - 1 > 0 \Rightarrow x > 1$

$\Rightarrow \log_{0.3}(x-1) < \frac{1}{2}\log_{0.3}(x-1)$

$\Rightarrow \log_{0.3}(x-1) < \log_{0.3}(x-1)^{1/2}$

$\Rightarrow (x-1) > (x-1)^{1/2}$

$\Rightarrow (x-1)^2 > (x-1)$

$\Rightarrow (x-1)^2 - (x-1) > 0$

$\Rightarrow (x-1)(x-1-1) > 0$

$\Rightarrow (x-1)(x-2) > 0$

$\Rightarrow x < 1$ or $x > 2$

$\Rightarrow \therefore x > 2$

$\Rightarrow x \in (2, \infty)$

Q.2 Solve for x the following equation:

$\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$ **[IIT 1987]**

Sol. The given equation is

$\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{3x+7}(4x^2 + 12x + 9)$

$\Rightarrow \log_{(2x+3)}(6x^2 + 23x + 21) + \log_{(3x+7)}(4x^2 + 12x + 9) = 4$

$\Rightarrow \log_{(2x+3)}(2x+3)(3x+7) + \log_{(3x+7)}(2x+3)^2 = 4$

$\Rightarrow 1 + \log_{(2x+3)}(3x+7) + 2\log_{(3x+7)}(2x+3) = 4$

[Using $\log ab = \log a + \log b$ and $\log a^n = n \log a$]

$\Rightarrow \log_{(2x+3)}(3x+7) + \frac{2}{\log_{(2x+3)}(3x+7)} = 3$

[Using $\log_a^b = \frac{1}{\log_b^a}$]

Let $\log_{(2x+3)}(3x+7) = y$ ..(1)

$\Rightarrow y + \frac{2}{y} = 3$

$\Rightarrow y^2 - 3y + 2 = 0$

$\Rightarrow (y-1)(y-2) = 0 \Rightarrow y = 1, 2$

Substituting the values of y in (1), we get

$\log_{(2x+3)}(3x+7) = 1$ and $\log_{(2x+3)}(3x+7) = 2$

$\Rightarrow 3x + 7 = 2x + 3$ and $3x + 7 = (2x+3)^2$

$\Rightarrow x = -4$ and $4x^2 + 9x + 2 = 0$

$\Rightarrow x = -4$ and $(x+2)(4x+1) = 0$

$\Rightarrow x = -4$ and $x = -2, x = -\frac{1}{4}$

As $\log_a x$ is defined for $x > 0$ and $a > 0$ $(a \neq 1)$, the possible value of x should satisfy all of the following inequalities :

$\Rightarrow$ $2x + 3 > 0$ and $3x + 7 > 0$

Also $2x + 3 \neq 1$ and $3x + 7 \neq 1$

Out of $x = -4$, $x = -2$ & $x = -\frac{1}{4}$ only $x = -\frac{1}{4}$ satisfies the above inequalities.

So only solution is $x = -\frac{1}{4}$.

Q.3 Solve the following equations for x and y:

$\log_{100}|x+y| = 1/2$, $\log_{10}y - \log_{10}|x| = \log_{100}4$ **[REE 1996]**

Sol. $\log_{100}|x+y| = \frac{1}{2}$

$\Rightarrow |x+y| = 100^{1/2} = 10$

and $\log_{10} y - \log_{10}|x| = \log_{100}4$

$\Rightarrow \log_{10}\frac{y}{|x|} = \frac{1}{2}\log_{10}4 = \log_{10}2$

$\Rightarrow \frac{y}{|x|} = 2 \Rightarrow y = 2|x|$

$\therefore |x + 2|x|| = 10$ $(x > 0)$

$\Rightarrow 3x = 10 \Rightarrow x = 10/3$

$x < 0$ $|x - 2x| = 10$

$\Rightarrow |-x| = 10$

$\Rightarrow x = -10$

$\Rightarrow x = \frac{10}{3}, y = \frac{20}{3}$

& $x = -10$, $y = 20$ Ans.

Q.4 Find the set of all solution of the equation

$2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$ **[IIT 1997]**

Sol. Case I :

$y < 0$ $2^{-y} - (-(2^{y-1} - 1)) = 2^{y-1} + 1$

$\Rightarrow 2^{-y} + 2^{y-1} - 1 = 2^{y-1} + 1$

$\therefore 2^{-y} = 2 \Rightarrow y = -1$

$\Rightarrow y = -1$ which is true.

Case II :

$0 \le y < 1,\ 2^{y} - (-(2^{y-1} - 1)) = 2^{y-1} + 1$

$2^y = 2 \Rightarrow y = 1$ which is not true. Θ $0 \le y < 1$.

Case III

$y \ge 1$

$2^y - (2^{y-1} - 1) = 2^{y-1} + 1$

$\therefore\ 2^y = 2.2^{y-1} = 2^y$ for all $y \ge 1$

$\therefore\ y \in \{-1\} \cup [1, \infty)$

Q.5 Find the no. of solution of

$\log_4 (x-1) = \log_2 (x-3)$ **[IIT-2001]**

(A) 3 (B) 1

(C) 2 (D) 0

Sol. **[B]**

$\log_4 (x-1) = \log_2 (x-3)$

$\Rightarrow \log_{2^2} (x-1) = \log_2 (x-3)$

$\Rightarrow \frac{1}{2} \log_2 (x-1) = \log_2 (x-3)$

$\Rightarrow \log_2 \sqrt{(x-1)} = \log_2 (x-3)$

For log defined $x - 1 > 0$ & $x - 3 > 0$

$\Rightarrow x > 1$ & $x > 3$

$\Rightarrow x > 3$

$\sqrt{x-1} = x - 3$

$\Rightarrow x - 1 = (x-3)^2$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x-2)(x-5)\ 0$

$\Rightarrow x = 2$ & $x = 5$

Θ $x > 3$ $\therefore$ x = 2 (Rejected)

$\therefore$ x = 5 is the reqd. solu.

$\therefore$ Number of solution is one.

Q.6 The set of all real numbers x for which $x^2 - |x+2| + x > 0$, is **[IITSc.-2002]**

(A) $(-\infty, -2) \cup (2, \infty)$

(B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(C) $(-\infty, -1) \cup (1, \infty)$

(D) $(\sqrt{2}, \infty)$

Sol. **[B]**

$x^2 - |x+2| + x > 0$

if $x + 2 > 0$

$x^2 - (x+2) + x > 0$

$\Rightarrow x^2 - x - 2 + x > 0$

$\Rightarrow x^2 - 2 > 0$

$\Rightarrow x^2 > 2$

$\Rightarrow x > \pm \sqrt{2}$

If $x + 2 < 0$

$x^2 + x + 2 + x > 0$

$\Rightarrow x^2 + 2x + 2 > 0$

$\Rightarrow D < 0$, no real roots

$\therefore\ x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Q.7 Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\lambda n 2} = (3y)^{\lambda n 3}$$
$$3^{\lambda n x} = 2^{\lambda n y}$$

Then x_0 is **[IIT 2011]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

Sol.[C] $(2x)^{\lambda n 2} = (3y)^{\lambda n 3}$

$\lambda n\, 2\, (\lambda n\, 2 + \lambda n\, x) = \lambda n\, 3\, (\lambda n\, 3 + \lambda n\, y)$

$\lambda n\, 2 \,.\, \lambda n\, x - \lambda n\, 3\ \lambda n\, y = (\lambda n\, 3)^2 - (\lambda n\, 2)^2$ .....(1)

$3^{\lambda n x} = 2^{\lambda n y}$

$\lambda n\, x \,.\, \lambda n\, 3 = \lambda n y \,.\, \lambda n\, 2$

$\lambda n\, y = \lambda n\, x\ \frac{\lambda n 3}{\lambda n 2}$ .....(2)

Solving (1) & (2)

$\lambda n\, x = -\lambda n\, 2 \Rightarrow$ $x = \frac{1}{2}$

Q.8 The value of

$$6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}....}}}\right)$$ is

[IIT 2012]

Sol. Let $x = \sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}...}}}$

$x^2 = 4 - \frac{1}{3\sqrt{2}}\ x$

$3\sqrt{2}\, x^2 + x - 12\sqrt{2} = 0$

$x = \frac{-1 + \sqrt{1 + 4.3\sqrt{2}.12\sqrt{2}}}{6\sqrt{2}}$

$x = \frac{-1+17}{6\sqrt{2}} = \frac{8}{3\sqrt{2}}$

$6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3./2}\left(\frac{3}{2}\right)^{-2} = 6 - 2 = 4$

EXERCISE # 5

Q.1 Solve $\log_2 (x-1) - \log_{\sqrt{2}} \sqrt{x+3} = \log_8 (x-a)^3 + \log_{1/2} (x-3)$

Sol. $\log_2 (x-1) - \log_{2^{1/2}} (x+3)^{1/2} = \log_{2^3} (x-a)^3 + \log_{2^{-1}} (x-3)$

$\Rightarrow \log_2(x-1) - \log_2(x+3) = \log_2(x-a) - \log_2 (x-3)$

$\Rightarrow \log_2 \frac{x-1}{x+3} = \log_2 \frac{x-a}{x-3}$

$\Rightarrow \frac{x-1}{x+3} = \frac{x-a}{x-3}$

$\Rightarrow x^2 - 4x + 3 = x^2 + (3-a)x - 3a$

$\Rightarrow \{(3-a) + 4\} x = 3(a+1)$

$\Rightarrow x = \frac{3(a+1)}{3-a+4} = \frac{3(a+1)}{7-a}$

$\Rightarrow x = \frac{3a+3}{7-a}$ for $a \in (3, 7)$ Ans.

and $x = \phi$ for $a \notin (3, 7)$ Ans.

Q.2 Solve for x,
$\log_{3/4} \log_8 (x^2+7) + \log_{1/2} \log_{1/4} (x^2+7)^{-1} = -2$

Sol. $= \log_{3/4} [(\frac{1}{3} \log_2 (x^2+7)] +$

$\log_{1/2} [\frac{1}{2} \log_2 (x^2+7)] = -2$

$(\Theta \ \log_{b^p} a^n = \frac{n}{p} \log_b a)$

$\log_{2^{-2}} (x^2+7)^{-1} = \frac{-1}{-2} \log_2 (x^2+7)$

Let $\log_2 (x^2+7) = t$

$\Rightarrow \quad x^2 + 7 = 2^t$

$= \log_{3/4} (\frac{1}{3} t) + \log_{1/2} (\frac{1}{2} t) = -2$

$= \frac{\log\left(\frac{1}{3}t\right)}{\log\frac{3}{4}} + \log_{1/2} \frac{1}{2} + \log_{1/2} t = -2$

$= \frac{\log t - \log 3}{\log 3 - \log 4} + \log_{1/2} \frac{1}{2} + \log_{1/2} t = -2$

$= \frac{\log t}{\log 3 - 2\log 2} - \frac{\log 3}{\log 3 - 2\log 2} + 1 - \frac{\log t}{\log 2} = -2$

$\therefore \log t \frac{\log 2 - \log 3 + 2\log 2}{(\log 3 - 2\log 2)\log 2}$

$= -3 + \frac{\log 3}{\log 3 - 2\log 2}$

$\frac{\log t(3\log 2 - \log 3)}{(\log 3 - 2\log 2)\log 2} = \frac{6\log 2 - 2\log 3}{\log 3 - 2\log 2}$

$\therefore \log t = 2\log 2 = \log 2^2 = \log 4$

$\therefore t = 4,\ x^2 + 7 = 2^t = 2^4 = 16 \Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$ Ans.

Q. 3 For $a \le 0$ determine all real roots of the equation $x^2 - 2a|x-a| - 3a^2 = 0$.

Sol. $x^2 - 2a|x-a| - 3a^2 = 0$

Case I :

When $x > a \Rightarrow (x-a) > 0$

$x^2 - 2a(x-a) - 3a^2 = 0 \Rightarrow x^2 - 2ax - a^2 = 0$

$\therefore x = a(1 \pm \sqrt{2})$

$\therefore x - a = \pm a\sqrt{2}$

$\Theta\ a \le 0$ given,

$\therefore x - a = -a\sqrt{2} = +ve$

Since $x = a(1-\sqrt{2})$ satisfies the condition

Case II :

When $x < a \therefore$ i.e. $x - a < 0$

$x^2 + 2a(x-a) - 3a^2 = 0 \Rightarrow x^2 + 2ax - 5a^2 = 0$

$\therefore x = a(-1 \pm \sqrt{6})$

$\therefore x - a = a(-2 \pm \sqrt{6}) = -ve$

$\Theta\ a \le 0,\ \therefore x - a = (-2 + \sqrt{6})$

$\therefore$ roots are $x = a(1-\sqrt{2}),\ a(-1+\sqrt{6})$ Ans.

Q.4 Solve the following inequalities

(i) $\frac{x-1}{\log_3(9-3^x)-3} \le 1$.

(ii) $\log_{1/6} (x^2 - 3x + 2) + 1 < 0$.

(iii) $\log_{\frac{3x}{x^2+1}} (x^2 - 2.5x + 1) \ge 0$

Sol. (i) $\frac{x-1}{\log_3(9-3^x)-3} \le 1$.

$\Rightarrow \frac{x-1}{\log_3(9-3^x)-3} - 1 \le 0$

$\Rightarrow \frac{x-1-\log_3(9-3^x)+3}{\log_3(9-3^x)-3} \le 0$

$\Rightarrow x + 2 - \log_3 (9-3^x)$

(ii) $\log_{1/6} (x^2 - 3x + 2) + 1 < 0$.

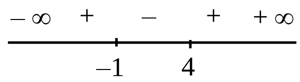
$\Rightarrow \log_{1/6} (x^2 - 3x + 2) < -1$

$\Rightarrow x^2 - 3x + 2 > (1/6)^{-1}$

$\Rightarrow x^2 - 3x + 2 > 6$

$\Rightarrow x^2 - 3x - 4 > 0$

$\Rightarrow (x-4)(x+1) > 0$



$x \in (-\infty, -1) \cup (4, \infty)$

Q.5 Solve the equations

(i) $|x-3|^{3x^2-10x+3} = 1$

(ii) $\log_3 \dfrac{|x^2-4x|+3}{x^2+|x-5|} \geq 0$

Sol. (i) x = 2, 4, 1/3

(ii) $\left(-\infty, -\dfrac{2}{3}\right] \cup \left[\dfrac{1}{2}, 2\right]$

Q.6 Let $E = \dfrac{a^2+b^2+c^2}{ab+bc+ac}$

(where a, b, c ∈ R, ab + bc + ac ≠ 0), then -

(A) E ≥ 1 for all a, b, c

(B) E ≥ 1 if ab + bc + ac > 0

(C) E ≤ –2 for all a, b, c

(D) E ≤ –2 if ab + bc + ac < 0

Sol. **[B,D]**

$E = \dfrac{a^2+b^2+c^2}{ab+bc+ac}$

Θ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2\,(ab + bc + ca)$

$a^2 + b^2 + c^2 + 2\,(ab + bc + ca) \geq 0$

$E = \dfrac{a^2+b^2+c^2}{ab+bc+ac} \geq -2$

$\therefore E \leq -2 \text{ if } ab+bc+ca<0$

Θ $a^2 + b^2 + c^2 \geq ab + bc + ca$

$\Rightarrow \dfrac{a^2+b^2+c^2}{ab+bc+ac} \geq 1$

Q.7 The equation || x – 1 | + a | = 4 can have real solutions for x if a belongs to the interval

(A) (–∞, 4) (B) (–∞, –4)

(C) (4, +∞) (D) [–4, 4]

Sol. **[A, B, D]**

$||x-1|+a| = 4$

$\Rightarrow |x-1| + a = \pm 4$

$\Rightarrow |x-1| = \pm 4 - a \Rightarrow |x-1| = 4 - a \quad a \leq 4$

$\Rightarrow |x-1| = 4 - a$ or $|x-1| = -4 - a$

$a \in (-\infty, 4)$

$|x-1| = 4 - a$ or $|x-1| = -4 - a$

4 – a & – 4 – a should be positive

4 – a > 0   – 4 – a > 0

a < 4 ; a ∈ (– ∞, 4) & 4 + a < 0, a < – 4

a ∈ (– ∞, –4)

a ∈ (– ∞, 4)

Q.8 The system of equations |x| + |y| = 1, $x^2 + y^2 = a^2$ will have-

(A) four solutions if a = 1

(B) two solutions if a = 1/2

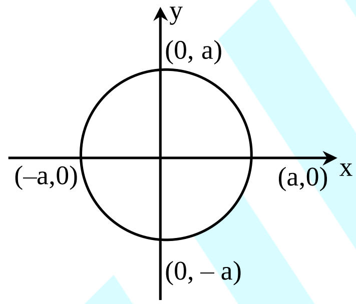
(C) eight solutions if $\dfrac{1}{\sqrt{2}} < a < 1$

(D) All above

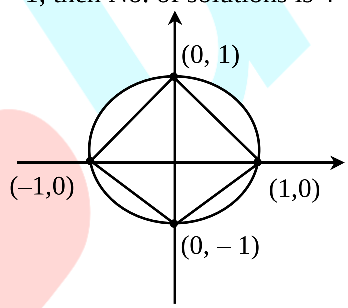
Sol. **[A,C]**

|x| + |y| = 1 represents a square

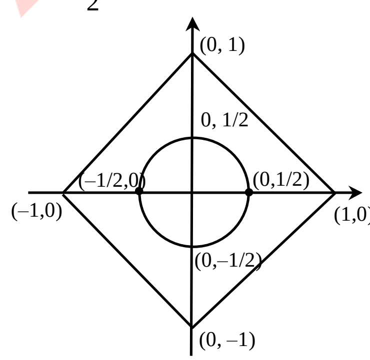
$x^2 + y^2 = a^2$ represents a circle



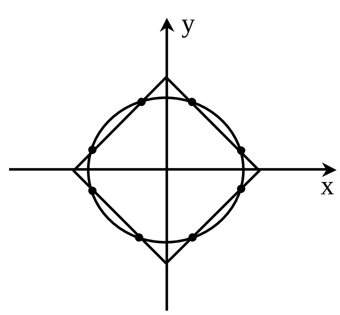
if a = 1, then No. of solutions is 4



If $a = \dfrac{1}{2}$, then there is no solution



if $\dfrac{1}{2} < a < 1$. then there is 8 solutions



ANSWER KEY

EXERCISE # 1

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	C	A	A	B	D	A	B	C	A	B	B	A	B	B	C	A	C	A
Qus.	21																			
Ans.	A																			

22. (i) $\log_2 3$ (ii) $\log_7 11$ (iii) $\log_4 5$ (iv) $\log_3 11$ (v) $\log_{1/2} 1/3$ (vi) $\log_3 5$ **23.** (C) **24.** (B)

EXERCISE # 2

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	B	B	C	D	D	A,B,C,D	C, D	A,B	A,B,C	A,C	A,C,D	A,B,C	B,C,D	B	A	D	B	A	B

20. $A \to P$; $B \to R, S$; $C \to Q$; $D \to P$

21. $A \to Q$; $B \to S$; $C \to P$; $D \to R$

22. **(i)** 1 **(ii)** 1

23. x = 2, 4, 8

24. 1

EXERCISE # 3

1. **(i)** 2 **(ii)** 1 **(iii)** $7 + \frac{1}{196}$

2. **(i)** 6 **(ii)** – 1/4 **(iii)** 0 **(iv)** 9

3. 25/2

4. **(i)** 16 or – 4 **(ii)** 5

5. x = 1

6. x = 100

7. $x = \sqrt{2}$ or $\sqrt{6}$

8. x = 10

9. **(i)** $(0.1, 1) \cup (1, 10)$ **(ii)** $(-1, \infty)$

10. $(-1, 1 + 2\sqrt{2})$

11. $0 < x < \sqrt[10]{10}$

12. (2, 5)

13. $(1000, \infty)$

14. **(i)** 140 **(ii)** 12 **(iii)** 47

15. **(i)** $(-\infty, -4]$ **(ii)** $(-\infty, \infty)$ **(iii)** –6, 2

16. –2, $[0, \infty)$

17. **(i)** $x = \sqrt{2}$, y = 1 **(ii)** x = 2, y = 4; x = 1/2, y = 1/4

18. (B)

19. (B)

20. (B)

21. (C)

22. (A)

23. (B)

24. (A)

25. (A)

26. (B)

EXERCISE # 4

1. (A) **2.** $x = -\frac{1}{4}$ **3.** $(x, y) = \left(\frac{10}{3}, \frac{20}{3}\right)$, (–10, 20) **4.** $y \in \{-1\} \cup [1, \infty)$

5. (B) **6.** (B) **7.** (C) **8.** 4

EXERCISE # 5

1. $\left(\frac{3a+3}{7-a}\right)$ for $a \in (3, 7)$, φ for $a \notin (3, 7)$ **2.** $x = \pm 3$

3. $a(1-\sqrt{2})$, $a(-1+\sqrt{6})$

4. **(i)** $[\log_3 0.9, 2)$ **(ii)** $(-\infty, -1) \cup (4, +\infty)$ **(iii)** $\left(0, \frac{3-\sqrt{5}}{2}\right) \cup \left[\frac{5}{2}, \frac{3+\sqrt{5}}{2}\right)$

5. **(i)** x = 2, 4, 1/3 **(ii)** $\left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{1}{2}, 2\right]$ **6.** (B, D) **7.** (A, B, D)

8. (A, C)