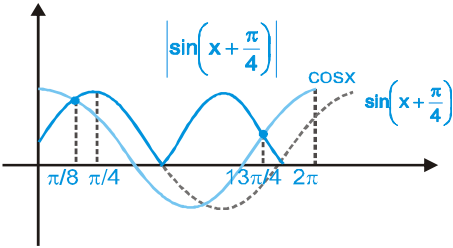


HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

- $2 \cos x = \sqrt{2 + 2 \sin 2x}$
 $\Rightarrow \sqrt{2} \cos x = \sqrt{1 + \sin 2x} = |\sin x + \cos x|$
 $\Rightarrow \cos x = \left| \frac{1}{\sqrt{2}} (\sin x + \cos x) \right|$

 $\Rightarrow \cos x = \left| \sin \left(x + \frac{\pi}{4} \right) \right|$
 $\Rightarrow$ see from graph or we can put values given in options to verify.
- $4 \sin \theta \cdot \cos \theta - 2 \cos \theta - 2 \sqrt{3} \sin \theta + \sqrt{3} = 0$
 $\Rightarrow 2 \cos \theta (2 \sin \theta - 1) - \sqrt{3} (2 \sin \theta - 1) = 0$
 $\Rightarrow (2 \sin \theta - 1) (2 \cos \theta - \sqrt{3}) = 0$
 $\Rightarrow \sin \theta = \frac{1}{2}, \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$
- $\tan^2 x - \sec^{10} x + 1 = 0$
 $\Rightarrow \sec^2 x - 1 - \sec^{10} x + 1 = 0$
 $\Rightarrow \sec^2 x (1 - \sec^8 x) = 0 \Rightarrow \sec^8 x = 1$
 $\Rightarrow \cos x = \pm 1 \Rightarrow x = \pi, 2\pi, 3\pi$
- Since $\tan \theta < 0$ and $\cos \theta > 0$, θ lies in the fourth quadrant.
 Then $\theta = 7\pi/4$
 Hence, the general value of θ is $2n\pi + 7\pi/4, n \in \mathbb{Z}$
- For the given relation
 $\cos \theta = (2 \sin \theta \cos \theta) \sin \theta, \sin \theta \neq 0$
 or $\sin \theta = \pm \frac{1}{\sqrt{2}}$ or $\cos \theta = 0$
 or $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}$
 Then the sum of roots is $3\pi/2$

- $\sin x + 3 \sin 2x + \sin 3x = \cos x + 3 \cos 2x + \cos 3x$
 $\Rightarrow 2 \sin 2x \cos x + 3 \sin 2x = 2 \cos 2x \cos x + 3 \cos 2x$
 $\Rightarrow \sin 2x [2 \cos x + 3] = \cos 2x [2 \cos x + 3]$
 $\Rightarrow \tan 2x = 1$
 $\Rightarrow 2x = n\pi + \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$
 $\therefore x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{13\pi}{8}, \frac{9\pi}{8}$
- $\frac{\cos 3\theta}{2 \cos 2\theta - 1} = \frac{1}{2}$
 $\Rightarrow 2(4 \cos^3 \theta - 3 \cos \theta) = 2(2 \cos^2 \theta - 1) - 1$
 $\Rightarrow 8 \cos^3 \theta - 4 \cos^2 \theta - 5 \cos \theta + 3 = 0$
 $\Rightarrow (4 \cos^2 \theta - 3)(2 \cos \theta - 1) = 0$
 $\Rightarrow \cos \theta = \frac{1}{2}, \pm \frac{\sqrt{3}}{2}$
 But when $\cos \theta = \pm \frac{\sqrt{3}}{2}$ then $2 \cos 2\theta - 1 = 0$
 $\therefore$ rejecting this value,
 $\cos \theta = \frac{1}{2}$ is valid only
 $\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{I}$
- $\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 0$
 $2 \cos 3x \cos 2x + 2 \cos 3x \cos x + \cos 3x = 0$
 $\cos 3x [2 \cos 2x + 2 \cos x - 1] = 0$
 $x = (2n - 1) \frac{\pi}{6} \Rightarrow \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6} = 3$
 2nd equation gives $\cos x = \frac{1 \pm \sqrt{2}}{4} = 2$
- We know that
 $\tan x \cdot \tan 2x \cdot \tan 3x = \tan 3x - \tan 2x - \tan x$
 hence $\tan x + \tan 2x + \tan 3x = \tan 3x - \tan 2x - \tan x$
 $\Rightarrow \tan x + \tan 2x = 0$
 $\therefore \tan 2x = \tan(-x)$
 $2x = n\pi - x$
 $x = \frac{n\pi}{3}, n \in \mathbb{I}$

$$11. \frac{1}{\cos x} = \frac{1}{1 - \cos x}$$

$$\Rightarrow \cos x = \frac{1}{2} = \cos \frac{\pi}{3}; \quad x = 2n\pi \pm \frac{\pi}{3}$$

$$12. f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$$

$$= 1 + \cos^2 x + \cos^2 2x - \sin^2 3x = 1 + \cos^2 x + \cos 5x \cdot \cos x$$

$$= 1 + \cos x [\cos x + \cos 5x] = 1 + 2 \cos x \cdot \cos 2x \cdot \cos 3x$$

$$\Rightarrow \cos x \cdot \cos 2x \cdot \cos 3x = 0$$

$$\text{now } f(x) = 1 \text{ if } \cos x = 0 \text{ or } \cos 2x = 0$$

$$\text{or } \cos 3x = 0$$

$$x = (2n-1)\frac{\pi}{2} \quad \text{or} \quad x = (2n-1)\frac{\pi}{4}$$

$$\text{or } x = (2n-1)\frac{\pi}{6}$$

$$\Rightarrow x \in \left\{ \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$\Rightarrow \text{number of values of } x = 5$$

$$13. f(x) = \frac{(\operatorname{cosec}^2 x - 1)^2}{(\operatorname{cosec}^2 x - 1) + 1 - \cot x + \cot x},$$

$$\text{defined for } \mathbb{R} - n\pi, n \in \mathbb{I}$$

$$f(x) = \frac{(\cot x)^4}{1 + \cot^2 x}$$

$$f(x) = 0 \Rightarrow \cot x = 0 \Rightarrow x = (2n-1)\frac{\pi}{2}$$

$$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, \frac{199\pi}{2}$$

$$\text{sum} = \frac{\pi}{2} [1 + 3 + 5 + \dots + 199]$$

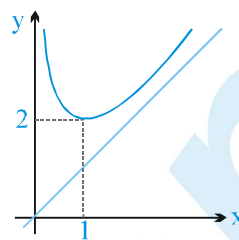
$$(100 \text{ solutions})$$

$$= \frac{\pi}{2} \cdot \frac{100}{2} \cdot 200 = 5000\pi$$

$$14. \cos x \cdot \sin \left(x + \frac{1}{x} \right) = 0$$

$$\cos x = 0 \Rightarrow x = \pi/2$$

$$\sin \left(x + \frac{1}{x} \right) = 0 \Rightarrow x + \frac{1}{x} = n\pi, n \in \mathbb{I}$$



if $x \in (0, 1)$ then $x + \frac{1}{x} \in (2, \infty)$ for $x > 0$

Hence there are infinite solution

$$15. 2 \sin x + 7 \cos px = 9 \text{ is possible only if } \sin x = 1$$

$$\text{and } \cos px = 1$$

$$x = (4n+1)\frac{\pi}{2} \quad \text{and} \quad px = 2m\pi$$

$$\Rightarrow x = \frac{2m\pi}{p} \quad (m, n \in \mathbb{I})$$

$$\therefore (4n+1)\frac{\pi}{2} = \frac{2m\pi}{p}$$

$$\Rightarrow p = \frac{4m}{4n+1}$$

$$\therefore p \in \text{rational}$$

$$16. 2^{2+4\sin^2 x} = 2^{6\sin x}$$

$$\Rightarrow 2 + 4\sin^2 x = 6 \sin x$$

$$\Rightarrow 2 \sin^2 x - 3 \sin x + 1 = 0$$

$$\Rightarrow (2 \sin x - 1)(\sin x - 1) = 0$$

$$\therefore \sin x = \frac{1}{2} \quad \text{or} \quad \sin x = 1$$

$$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$$

$$17. \sin^2 x + a \cos x + a^2 > 1 + \cos x$$

Putting $x = 0$, we get

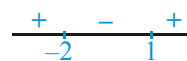
$$\text{or } a + a^2 > 2$$

$$\text{or } a^2 + a - 2 > 0$$

$$\text{or } (a+2)(a-1) > 0$$

$$\text{or } a < -2 \text{ or } a > 1$$

Therefore, we have the largest negative integral value of $a = -3$.



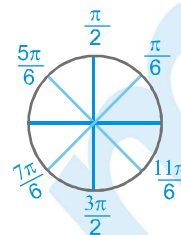
EXERCISE - 2

Part # 1 : Multiple Choice

- $\sin^6 x + \cos^6 x = a^2$
 $\Rightarrow (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) = a^2$
 $\Rightarrow (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x = a^2$
 $\Rightarrow 1 - 3\sin^2 x \cos^2 x = a^2$
 $\Rightarrow 1 - \frac{3}{4} \sin^2 2x = a^2 \Rightarrow \frac{4(1-a^2)}{3} = \sin^2 2x$
 $\Rightarrow 0 \leq \frac{4}{3} (1-a^2) \leq 1$
 $1-a^2 \geq 0$ and $4-4a^2 \leq 3$
 $a^2 \leq 1$ and $\frac{1}{4} \leq a^2$
 $-1 \leq a \leq 1$ and $a \geq \frac{1}{2}$ or $a \leq -\frac{1}{2}$
 $a \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$
- $\sin^2 x - \cos 2x = 2 - \sin 2x$
 $\Rightarrow \sin^2 x - (1 - 2\sin^2 x) = 2 - 2\sin x \cos x$
 $\Rightarrow 3\sin^2 x + 2\sin x \cos x = 3$
Case-I : $\cos x \neq 0$
 $\therefore 3\tan^2 x + 2\tan x = 3(1 + \tan^2 x) \Rightarrow \tan x = \frac{3}{2}$
Case-II : $\cos x = 0$
 $\therefore 3(1) + 2(\pm 1)(0) = 3$ which is true $\therefore x = (2n+1)\frac{\pi}{2}$
- $\sin^4 x - \cos^2 x \sin x + 2\sin^2 x + \sin x = 0$
 or $\sin x [\sin^3 x - \cos^2 x + 2\sin x + 1] = 0$
 or $\sin x [\sin^3 x - 1 + \sin^2 x + 2\sin x + 1] = 0$
 or $\sin x [\sin^3 x + \sin^2 x + 2\sin x] = 0$
 or $\sin^2 x = 0$ or $\sin^2 x + \sin x + 2 = 0$
 (not possible for real x)
 or $\sin x = 0$
 Hence, the solutions are $x = 0, \pi, 2\pi, 3\pi$.
- $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$
 $\Rightarrow \cos^2 x - \sin^2 2x + \cos^2 3x = 0$
 $\Rightarrow \cos x \cos 3x + \cos^2 3x = 0$
 $\Rightarrow \cos 3x (\cos x + \cos 3x) = 0$
 $\Rightarrow \cos 3x \cos 2x \cos x = 0$
 $\Rightarrow x = (2p+1)\frac{\pi}{6}$

$$x = (2q+1)\frac{\pi}{4},$$

$$x = (2r+1)\frac{\pi}{2}, n \in I$$



$$\Rightarrow x = n\pi \pm \frac{\pi}{6} \text{ also satisfy the equation.}$$

$$7. y + \frac{1}{y} \geq 2 \Rightarrow \sqrt{y + \frac{1}{y}} \geq 2$$

$$\text{But } \sin x + \cos x \leq \sqrt{2}$$

$$\Rightarrow y + \frac{1}{y} = 2 \text{ and } \sin x + \cos x = \sqrt{2}$$

$$y = 1 \text{ and } \sin\left(x + \frac{\pi}{4}\right) = 1 \text{ or } y = 1 \text{ and } x = \frac{\pi}{4}$$

$$8. 3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$$

$$\sin 2\theta - 2\cos\theta + 3 - 4\sin\theta - (1 - 2\sin^2\theta) = 0$$

$$2\cos\theta (\sin\theta - 1) + 2\sin^2\theta - 4\sin\theta + 2 = 0$$

$$\cos\theta (\sin\theta - 1) + (\sin\theta - 1)^2 = 0$$

$$(\sin\theta - 1)(\cos\theta + \sin\theta - 1) = 0$$

$$\sin\theta = 1 \text{ or } \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\theta = 2n\pi + \frac{\pi}{2} \text{ or } \theta - \frac{\pi}{4} = 2m\pi + \frac{\pi}{4}, 2m\pi - \frac{\pi}{4}$$

$$\theta = 2m\pi + \frac{\pi}{2}, 2m\pi$$

$$9. 3\theta = n\pi + (-1)^n(3\alpha)$$

$$\therefore 3\theta = 3\alpha; 3\theta = \pi - 3\alpha; 3\theta = -\pi - 3\alpha$$

$$\text{or } 3\theta = 2\pi + 3\alpha; 3\theta = -2\pi + 3\alpha$$

$$\text{Hence } \theta = \alpha; \theta = \frac{\pi}{3} - \alpha; \theta = -\left(\frac{\pi}{3} + \alpha\right);$$

$$\theta = \left(\frac{2\pi}{3} + \alpha\right); \theta = -\frac{2\pi}{3} + 3\alpha$$

$$\Rightarrow \cos\theta = \cos\left(\frac{\pi}{3} \pm \alpha\right) \text{ or } \cos\theta = \cos\left(\frac{2\pi}{3} \pm \alpha\right)$$

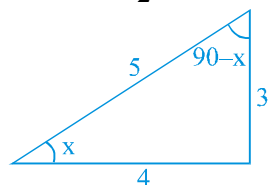
$$\Rightarrow \text{(A), (B), (C) and (D) all are correct}$$

10. Squaring and adding the given equations,
 $4 + 9 + 12 \sin(x+y) = 25$

$$\Rightarrow \sin(x+y) = 1 = \sin \frac{\pi}{2}$$

$$\therefore x+y = 2n\pi + \frac{\pi}{2} = (4n+1)\frac{\pi}{2} \quad n \in \mathbb{I} \Rightarrow \text{(A)}$$

$$\text{if } x+y = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} - x$$



$$5 \sin x = 3 \Rightarrow \sin x = \frac{3}{5} \quad \text{or} \quad \cos x = \frac{4}{5}$$

$$\text{also } \cos y = \frac{3}{5} \text{ and } \sin y = \frac{4}{5}; \text{ hence } y > x \Rightarrow \text{(D)}$$

11. only $x = \frac{\pi}{4}$ and $\frac{11\pi}{12}$ in $(0, \pi)$ satisfies

$$13. 2\cos^3 3\theta + 3\cos 3\theta + 4 = 3\sin^2 3\theta$$

$$\Rightarrow \left(\cos 3\theta + \frac{1}{2} \right) (2\cos^2 3\theta + 2\cos 3\theta + 2) = 0$$

$$\Rightarrow \cos 3\theta = -\frac{1}{2} \Rightarrow 3\theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{2n\pi}{3} \pm \frac{2\pi}{9}$$

$$14. \sin \alpha - \cos \alpha \cdot \tan \beta = \tan(\alpha - \beta)$$

$$\frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \beta} = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$$

$$\sin(\alpha - \beta) \cos(\alpha - \beta) = \sin(\alpha - \beta) \cos \beta$$

$$\sin(\alpha - \beta) [\cos(\alpha - \beta) - \cos \beta] = 0$$

$$\therefore \sin(\alpha - \beta) = 0 \quad \text{or} \quad \cos(\alpha - \beta) = \cos \beta$$

$$\therefore \alpha - \beta = n\pi \quad \alpha - \beta = 2m\pi \pm \beta$$

$$\therefore \alpha = n\pi + \beta \quad (+) \quad \alpha = 2m\pi + 2\beta$$

$$(-) \quad \alpha = 2m\pi$$

$$\therefore \text{A, B, C are correct}$$

$$15. \cos 3\theta = \cos 3\alpha \quad \text{put } n=0, 1$$

$$3\theta = 2n\pi \pm 3\alpha$$

$$\therefore 3\theta = 3\alpha \quad \text{or} \quad -3\alpha \quad \text{or} \quad 2\pi + 3\alpha \quad \text{or} \quad 2\pi - 3\alpha$$

$$\theta = \alpha \quad \text{or} \quad -\alpha \quad \text{or} \quad \frac{2\pi}{3} + \alpha \quad \text{or} \quad \frac{2\pi}{3} - \alpha$$

$$\Rightarrow \text{(A), (C), (D) are correct}$$

$$\text{if } n = -1$$

$$3\theta = -2\pi \pm 3\alpha$$

$$\theta = -\frac{2\pi}{3} \pm \alpha$$

$$\sin \theta = \sin \left(-\frac{2\pi}{3} \pm \alpha \right) = -\sin \left(\frac{2\pi}{3} \pm \alpha \right)$$

$$= -\sin \left(\pi - \frac{\pi}{3} \pm \alpha \right) = -\sin \left(\pi - \left(\frac{\pi}{3} \pm \alpha \right) \right)$$

$$= -\sin \left(\frac{\pi}{3} \pm \alpha \right)$$

hence (B) is not correct.

$$16. \cos 4x \cos 8x - \cos 5x \cos 9x = 0$$

$$\Rightarrow 2\cos 4x \cos 8x = 2\cos 5x \cos 9x$$

$$\Rightarrow \cos 12x + \cos 4x = \cos 14x + \cos 4x$$

$$\Rightarrow 14x = 2n\pi \pm (12x)$$

$$\Rightarrow 2x = 2n\pi \quad \text{or} \quad 26x = 2n\pi$$

$$\Rightarrow x = n\pi \quad \text{or} \quad \frac{n\pi}{13}$$

$$\therefore \sin x = 0 \quad \text{or} \quad \sin 13x = 0$$

$$17. \text{Let } E = \sin x - \cos^2 x - 1$$

$$\Rightarrow E = \sin x - 1 + \sin^2 x - 1 = \sin^2 x + \sin x - 2$$

$$= \left(\sin x + \frac{1}{2} \right)^2 - \frac{9}{4} \quad \text{assumes least value}$$

$$\text{when } \sin x = -\frac{1}{2} \Rightarrow x = n\pi + (-1)^n \left(-\frac{\pi}{6} \right).$$

$$18. \cos x \cdot \cos 6x = -1$$

$$\Rightarrow \text{Either } \cos x = 1 \quad \text{and} \quad \cos 6x = -1$$

$$\text{or } \cos x = -1 \quad \text{and} \quad \cos 6x = 1$$

$$\Rightarrow x = 2n\pi \quad \text{and} \quad \cos 6x = -1$$

$$\text{or } x = (2n+1)\pi \quad \text{and} \quad \cos 6x = 1$$

$$\text{If } x = 2n\pi \text{ then } \cos 6x \text{ cannot be } -1$$

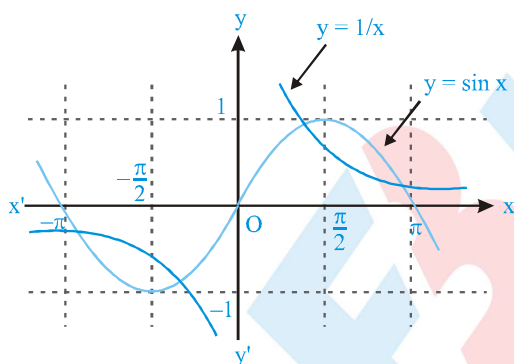
$$\text{However if } x = (2n+1)\pi \text{ then } \cos 6x = 1$$

$$\therefore x = (2n+1)\pi$$

$$x = (2n-1)\pi \text{ is also as above.}$$

Part # II : Assertion & Reason

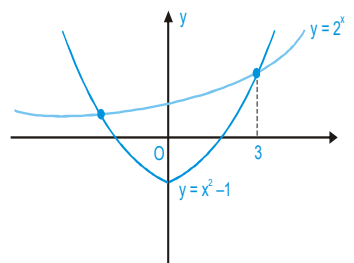
- $(\sin x + \cos x)^{1+\sin 2x} = 2$
 $\Rightarrow (\sin x + \cos x)^{(\sin x + \cos x)^2} = 2$
 Now we know that the maximum value of $\sin x + \cos x$ is $\sqrt{2}$ which occurs at $x = \pi/4$ for $0 \leq x \leq \pi/4$.
 Also, the given equation has roots only if $\sin x + \cos x = \sqrt{2}$
 Hence, there is only one solution for $0 \leq x \leq \pi/4$.
 Thus, the correct answer is (A).
- We know that $\sin^2 x \leq 1$ and $\cos^2 y \leq 1$, then $\sin^2 x + \cos^2 y \leq 2$
 Also, $\sec^2 z \geq 1$, then $2 \sec^2 z \geq 2$.
 Hence, the given equation is solvable only if $\sin^2 x + \cos^2 y = 2$ and $2 \sec^2 z = 2$, for which $\sin^2 x, \cos^2 y, \sec^2 z = 1$
 Then $\sin x, \cos y, \sec z = \pm 1$.
 Hence, statement 1 is false and statement 2 is true.
- Draw the graph of $y = \sin x$ and $y = 1/x$ and verify.



EXERCISE - 3

Part # I : Matrix Match Type

- $|\tan x| = \frac{m}{n} \Rightarrow \tan x = \frac{m}{n} \text{ \& \; } \tan x = -\frac{m}{n}$
 In $[0, 2\pi]$ it has 4 solutions
 - $\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 5$
 $\Rightarrow \cos x = \cos 2x = \cos 3x = \cos 4x = \cos 5x = 1$
 $\Rightarrow x = 2n_1\pi, x = n_2\pi,$
 $x = 2n_3\frac{\pi}{3}, x = \frac{n_4\pi}{2}, x = \frac{2n_5\pi}{5}$
 $\Rightarrow x = 0, 2\pi$ are common solutions.
 - $2^{\frac{1}{1-|\cos x|}} = 4$
 $\Rightarrow \frac{1}{1-|\cos x|} = 2 \Rightarrow 1 - |\cos x| = \frac{1}{2}$
 $\Rightarrow |\cos x| = \frac{1}{2} \Rightarrow \cos x = \pm \frac{1}{2}$
 $\therefore$ In $(-\pi, \pi)$ there are 4 solutions
 - $\tan \theta + \tan 2\theta + \tan 3\theta = \tan \theta \tan 2\theta \tan 3\theta$
 $\Rightarrow \theta + 2\theta + 3\theta = n\pi \Rightarrow \theta = n\pi/6$
 $\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$ satisfy equation only.
- $\sin^2 \theta + 3 \cos \theta = 3 \Rightarrow 1 - \cos^2 \theta + 3 \cos \theta = 3$
 $\Rightarrow \cos^2 \theta - 3 \cos \theta + 2 = 0 \Rightarrow \cos \theta = 1, 2$
 $\Rightarrow \cos \theta = 1 \text{ (} \rightarrow \cos \theta \neq 2 \text{)} \Rightarrow \theta = 0 \text{ in } [-\pi, \pi]$
 $\therefore$ No. of solution = 1
 - $\sin x \cdot \tan 4x = \cos x \Rightarrow \sin x \cdot \frac{\sin 4x}{\cos 4x} = \cos x$
 $\Rightarrow \sin 4x \sin x - \cos 4x \cos x = 0 \Rightarrow \cos 5x = 0$
 $\Rightarrow 5x = (2n+1)\pi/2 \Rightarrow x = (2n+1)\pi/10$
 $\Rightarrow x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10} \text{ in } (0, \pi)$



So there are five solutions.

(C) $(1 - \tan^2 \theta) \sec^2 \theta + 2^{\tan^2 \theta} = 0$

$\Rightarrow (1 - \tan^4 \theta) + 2^{\tan^2 \theta} = 0$

$\Rightarrow (1 - x^2) + 2^x = 0$ where $x = \tan^2 \theta$

$\Rightarrow 2^x = x^2 - 1 \Rightarrow x = 3$

$\Rightarrow \tan^2 \theta = 3 \Rightarrow \tan \theta = \pm \sqrt{3}$

$\Rightarrow \theta = \pm \frac{\pi}{3}$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Number of solutions = 2

(D) $[\sin x] + [\sqrt{2} \cos x] = -3$

$\Rightarrow [\sin x] = -1$ and $[\sqrt{2} \cos x] = -2$

$\Rightarrow \pi < x < 2\pi$ and $-2 \leq \sqrt{2} \cos x < -1$

$\Rightarrow -2 \leq \cos x < -\frac{1}{\sqrt{2}}$

$\Rightarrow -1 \leq \cos x < -\frac{1}{\sqrt{2}}$

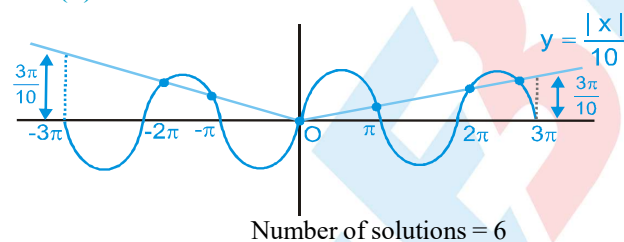
$\Rightarrow \pi \leq x < \frac{5\pi}{4}$ for $x \in [0, 2\pi]$

$\pi \leq x < \frac{5\pi}{4}, x \in [0, 2\pi]$

$\therefore \pi < x < \frac{5\pi}{4} \Rightarrow 2\pi < 2x < \frac{5\pi}{2}$

$\Rightarrow 0 < \sin 2x < 1 \Rightarrow [\sin 2x] = 0$

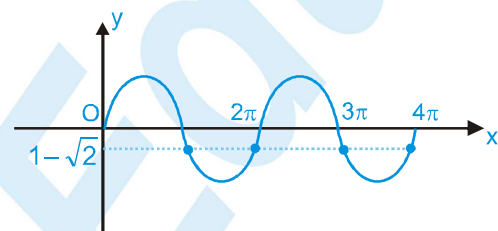
3. (A)



(B) $\sin x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

$\Rightarrow \sin x = 1 - \sqrt{2}$

As $\sin x$ takes at least four values



in $[0, n\pi] \therefore n \geq 4$

(C) $1 + \sin^4 x = \cos^2 3x$

L.H.S. ≥ 1 and R.H.S. ≤ 1

$\therefore$ L.H.S. = R.H.S. = 1

$\Rightarrow \sin^4 x = 0$ and $\cos^2 3x = 1 \Rightarrow x = n\pi$ and $3x = m\pi$

$\Rightarrow x = n\pi$ and $3x = m\pi \Rightarrow x = n\pi$ and $x = \frac{m\pi}{3}$

$\Rightarrow x = n\pi$

$\Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$ in $\left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$

$\therefore$ Number of solutions = 5.

(D) A, B, C are in A.P. $\Rightarrow B = 60^\circ$

As $\sin(2A + B) = \frac{1}{2} \Rightarrow 2A + B = 30^\circ$ or 150°

$\Rightarrow 2A = -30^\circ$ or $90^\circ \Rightarrow 2A = 90^\circ$

$\Rightarrow A = 45^\circ$

$\therefore C = 180^\circ - A - B = 75^\circ = \frac{5\pi}{12} \therefore p = 12.$

Part # II : Comprehension

Comprehension 1

1. $x^3 - (1 + \cos \theta + \sin \theta)x^2 + (\cos \theta \sin \theta + \cos \theta + \sin \theta)x - \sin \theta \cos \theta = 0$

Given cubic function is

$f(x) = (x - 1)(x - \cos \theta)(x - \sin \theta)$

Therefore, roots are 1, $\sin \theta$, and $\cos \theta$

Hence, $x_1^2 + x_2^2 + x_3^2 = 1 + \sin^2 \theta + \cos^2 \theta = 2$

2. Now if $1 = \sin \theta$, we get $\theta = \pi/2$

If $1 = \cos \theta$, then $\theta = 0, 2\pi$

and if $\sin \theta = \cos \theta$, we get $\tan \theta = 1$.

Therefore, $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

Therefore, the number of values of θ in $[0, 2\pi]$ is 5.

3. Again the maximum possible difference between the two roots is 2.

$1 - \sin \theta = 2$, when $\theta = 3\pi/2$

or $1 - \cos \theta = 2$, when $\theta = \pi$

Comprehension 2

1. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = (2\sin x + 1)(2\sin^2 x - 1) = 0$

$\therefore \sin x = -\frac{1}{2}, \pm \frac{1}{\sqrt{2}}$

$\therefore$ there are 6 solutions.

$$2. \quad 3 = \cos 4x + \frac{10 \tan x}{1 + \tan^2 x} = \cos 4x + 5 \sin 2x$$

$$\text{i.e. } 3 = 1 - 2 \sin^2 2x + 5 \sin 2x$$

$$\text{i.e. } \sin 2x = \frac{1}{2}$$

$$\therefore 2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Thus there are two solutions.

3. (i) when $\tan x \geq 0$, then the equation becomes

$$\tan x = \tan x + \frac{1}{\cos x} \quad \text{i.e. } \frac{1}{\cos x} = 0 \quad (\text{not possible})$$

- (ii) when $\tan x < 0$, then the equation becomes

$$-\tan x = \tan x + \frac{1}{\cos x} \quad \text{i.e. } \sin x = -\frac{1}{2}$$

$$\therefore x = \frac{11\pi}{6} \text{ is the only solution.}$$

Comprehension 3

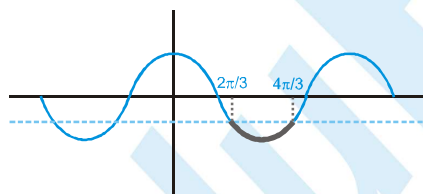
$$1. \quad \sin^6 x + \cos^6 x < \frac{7}{16} \Rightarrow 1 - 3 \sin^2 x \cos^2 x < \frac{7}{16}$$

$$\Rightarrow \sin^2 x \cos^2 x > \frac{3}{16} \Rightarrow \sin^2 2x > \frac{3}{4}$$

$$\Rightarrow \frac{1 - \cos 4x}{2} > \frac{3}{4} \Rightarrow 1 - \cos 4x > \frac{3}{2}$$

$$\Rightarrow \cos 4x < -\frac{1}{2}$$

$$\Rightarrow \text{Principal is value } 4x \in \left(\frac{2\pi}{3}, \frac{4\pi}{3} \right)$$



$$\Rightarrow \text{General value is } 4x \in \left(2n\pi + \frac{2\pi}{3}, 2n\pi + \frac{4\pi}{3} \right)$$

$$\Rightarrow x \in \left(\frac{n\pi}{2} + \frac{\pi}{6}, \frac{n\pi}{2} + \frac{\pi}{3} \right), n \in \mathbb{I}$$

$$2. \quad \cos 2x + 5 \cos x + 3 \geq 0$$

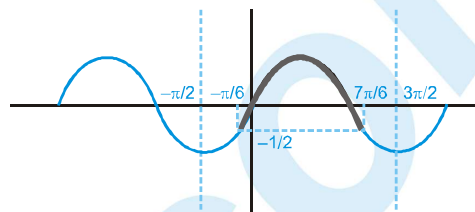
$$\Rightarrow 2 \cos^2 x + 5 \cos x + 2 \geq 0$$

$$\Rightarrow (\cos x + 2)(2 \cos x + 1) \geq 0$$

$$\Rightarrow 2 \cos x + 1 \geq 0 \quad (\rightarrow \cos x + 2 > 0)$$

$$\Rightarrow \cos x \geq -\frac{1}{2} \Rightarrow x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3} \right]$$

$$3. \quad 2 \sin^2 \left(x + \frac{\pi}{4} \right) + \sqrt{3} \cos 2x \geq 0$$



$$\Rightarrow 1 - \cos \left(2x + \frac{\pi}{2} \right) + \sqrt{3} \cos 2x \geq 0$$

$$\Rightarrow \sqrt{3} \cos 2x + \sin 2x \geq -1$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cos 2x + \frac{1}{2} \sin 2x \geq -\frac{1}{2}$$

$$\Rightarrow \sin \left(2x + \frac{\pi}{3} \right) \geq -\frac{1}{2}$$

$$\Rightarrow 2x + \frac{\pi}{3} \in \left[2n\pi - \frac{\pi}{6}, 2n\pi + \frac{7\pi}{6} \right]$$

$$\Rightarrow 2x \in \left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6} \right]$$

$$\Rightarrow x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{5\pi}{12} \right]$$

$$\Rightarrow x \in \left[-\pi, -\frac{7\pi}{12} \right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12} \right] \cup \left[\frac{3\pi}{4}, \pi \right] \text{ in } [-\pi, \pi]$$

EXERCISE - 4

Subjective Type

1. $\sin^4\left(\frac{x}{3}\right) + \cos^4\left(\frac{x}{3}\right) > \frac{1}{2} \quad (n \in \mathbb{Z})$

or $1 - 2\sin^2\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right) > \frac{1}{2}$

or $1 - \frac{1}{2}\sin^2\left(\frac{2x}{3}\right) > \frac{1}{2}$

or $\sin^2\left(\frac{2x}{3}\right) < 1$

which is always true except when $\sin^2(2x/3) = 1$.

This means $2x/3 = n\pi \pm (\pi/2)$

or $x = (3n\pi/2) \pm (3\pi/4), n \in \mathbb{Z}$

Hence, solution set of the inequality is

$\mathbb{R} - \{x : x = (3n\pi/2) \pm (3\pi/4), n \in \mathbb{Z}\}.$

2. $\sin x + \sin y = \sin(x + y)$

or $2\sin\frac{x+y}{2}\left[\cos\frac{x-y}{2} - \cos\frac{x+y}{2}\right] = 0$

or $4\sin\frac{x+y}{2}\sin\frac{x}{2}\sin\frac{y}{2} = 0$

(a) $\sin\frac{x+y}{2} = 0 \Rightarrow x + y = 2n\pi, n \in \mathbb{Z}$

$\Rightarrow x + y = 0$

($\rightarrow |x| + |y| = 1 \Rightarrow -1 \leq x, y \leq 1$)

(b) $\sin\frac{x}{2} = 0 \Rightarrow x = 2m\pi, m \in \mathbb{Z}$

$\Rightarrow x = 0$

(c) $\sin\frac{y}{2} = 0 \Rightarrow y = 2p\pi, p \in \mathbb{Z}$

From $|x| + |y| = 1$

If $x = 0$, then $|y| = 1 \Rightarrow y = \pm 1$

If $y = 0$, then $|x| = 1 \Rightarrow x = \pm 1$

If $y = -x$, then $|x| + |-x| = 2$

$\Rightarrow$ Hence, solutions are

$(0, 1), (0, -1), (1, 0), (-1, 0), \left(\frac{1}{2}, -\frac{1}{2}\right), \text{ and } \left(-\frac{1}{2}, \frac{1}{2}\right).$

3. $\tan\left(\frac{\pi}{2}\cos\theta\right) = \cot\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{2}\sin\theta\right)$

$\Rightarrow \frac{\pi}{2}\cos\theta = n\pi + \frac{\pi}{2} - \frac{\pi}{2}\sin\theta, n \in \mathbb{Z}$

or $\frac{\pi}{2}(\sin\theta + \cos\theta) = n\pi + \frac{\pi}{2} = \left(n + \frac{1}{2}\right)\pi$

or $\sin\theta + \cos\theta = (2n + 1)$

$\Rightarrow \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) = (2n + 1)$

Hence, $n = 0, -1$ are the only possibilities.

So, $\sin\left(\theta + \frac{\pi}{4}\right) = \pm \frac{1}{\sqrt{2}} = \sin\left(\pm \frac{\pi}{4}\right)$

or $\theta + \frac{\pi}{4} = m\frac{\pi}{2} + \frac{\pi}{4}, m \in \mathbb{Z}$

or $\theta = m\frac{\pi}{2}, m \in \mathbb{Z}$

However, for the values of $m = 2k, k \in \mathbb{Z}$, the equation is not defined.

Hence, $\theta = (2k + 1)\frac{\pi}{2}, \text{ where } k \in \mathbb{Z}$

4. $\sin^2 x + \frac{1}{4}\sin^2 3x = \sin x \sin^2 3x$

or $\sin^2 x - \sin x \sin^2 3x + \frac{1}{4}\sin^2 3x = 0$

or $\left(\sin x - \frac{1}{2}\sin^2 3x\right)^2 + \frac{1}{4}\sin^2 3x(1 - \sin^2 3x) = 0$

or $\left(\sin x - \frac{1}{2}\sin^2 3x\right)^2 + \frac{1}{4}\sin^2 3x \cos^2 3x = 0$

or $\left(\sin x - \frac{1}{2}\sin^2 3x\right)^2 + \frac{1}{16}\sin^2 6x = 0$

or $\sin x - \frac{1}{2}\sin^2 3x = 0 \text{ and } \sin 6x = 0$

or $2\sin x = \sin^2 3x \text{ and } \sin 6x = 0$

From $\sin 6x = 0, x = k\pi/6, k \in \mathbb{Z}$

From here, we choose those values which satisfy the equation $2\sin x = \sin^2 3x$. Now,

$\sin^2 3\left(\frac{k\pi}{6}\right) = \sin^2 \frac{k\pi}{2} = \begin{cases} 1, & \text{if } k \text{ is odd} \\ 0, & \text{if } k \text{ is even} \end{cases}$

$\Rightarrow \sin x = 0 \text{ or } \frac{1}{2}$

$x = n\pi \text{ or } x = n\pi + \frac{\pi}{6} (-1)^n, n \in \mathbb{Z}$

5. $\sin^4 x + \cos^4 x - 2 \sin^2 x + \frac{3}{4} \sin^2 2x = 0$

or $(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x - 2 \sin^2 x + \frac{3}{4} \cdot 4 \sin^2 x \cdot \cos^2 x = 0$

or $1 - 2 \sin^2 x + \sin^2 x \cdot \cos^2 x = 0$

or $\sin^4 x + \sin^2 x - 1 = 0$

or $\sin^2 x = \frac{\sqrt{5}-1}{2}$

$\therefore \cos 2x = 2 - \sqrt{5}$

$\Rightarrow x = n\pi \pm \frac{1}{2} \cos^{-1}(2 - \sqrt{5}), n \in \mathbb{Z}$

6. Given $\sin^3 x \cos 3x + \cos^3 x \sin 3x + \frac{3}{8} = 0$

$\Rightarrow \sin^3 x (4 \cos^3 x - 3 \cos x) + \cos^3 x (3 \sin x - 4 \sin^3 x) + \frac{3}{8} = 0$

or $3 \sin x \cos x (\cos^2 x - \sin^2 x) + \frac{3}{8} = 0$

or $8(\sin x \cos x) \cos 2x + 1 = 0$

or $2 \sin 4x = -1$

or $\sin 4x = -\frac{1}{2}$

$\therefore x = \frac{n\pi}{4} + (-1)^{n+1} \frac{\pi}{6}; n \in \mathbb{Z}$

7. $\sin^{10} x + \cos^{10} x = \frac{29}{16} \cos^4 2x$

$\Rightarrow \left(\frac{1-\cos 2x}{2}\right)^5 + \left(\frac{1+\cos 2x}{2}\right)^5 = \frac{29}{16} \cos^4 2x$

Let $\cos 2x = t$. Then,

$\left(\frac{1-t}{2}\right)^5 + \left(\frac{1+t}{2}\right)^5 = \frac{29}{16} t^4$

or $24t^4 - 10t^2 - 1 = 0$

or $(2t^2 - 1)(12t^2 + 1) = 0$

or $t^2 = \frac{1}{2}$

or $\cos^2 2x = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\cos \frac{\pi}{4}\right)^2$

or $2x = n\pi \pm \frac{\pi}{4}$

or $x = \frac{n\pi}{2} \pm \frac{\pi}{8}, n \in \mathbb{Z}$

8. $\sqrt{13 - 18 \tan x} = 6 \tan x - 3 \dots (i)$

$\Rightarrow 13 - 18 \tan x = 36 \tan^2 x + 9 - 36 \tan x$

$\Rightarrow \tan x = \frac{2}{3}, -\frac{1}{6}$

Put in (i) $\Rightarrow \tan x = \frac{2}{3}$ is correct

$\Rightarrow x = n\pi + \tan^{-1} \frac{2}{3}$

$= n\pi + \alpha = \alpha, \pi + \alpha, -\pi + \alpha, -2\pi + \alpha$ in $(-2\pi, 2\pi)$

9. $(\sin 2\theta + \sqrt{3} \cos 2\theta)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$

$\Rightarrow 4\left(\frac{1}{2} \sin 2\theta + \frac{\sqrt{3}}{2} \cos 2\theta\right)^2 - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0$

$\Rightarrow 4 \cos^2\left(\frac{\pi}{6} - 2\theta\right) - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0$

$\Rightarrow \cos\left(\frac{\pi}{6} - 2\theta\right) = \frac{5}{4}, -1$

$\Rightarrow \cos\left(\frac{\pi}{6} - 2\theta\right) = -1 = \cos \pi$

$\Rightarrow \frac{\pi}{6} - 2\theta = 2n\pi \pm \pi \Rightarrow 2\theta = \frac{\pi}{6} - 2n\pi \mu \pi$

$\Rightarrow \theta = \frac{2n\pi}{2} + \frac{\pi}{12} \pm \frac{\pi}{2} \Rightarrow \theta = \frac{7\pi}{12}, \frac{19\pi}{12}$

10. $1 + 2 \operatorname{cosec} x = \frac{-\sec^2\left(\frac{x}{2}\right)}{2}$

$\Rightarrow 1 + \frac{2}{\sin x} = \frac{-1}{1 + \cos x}$

$\Rightarrow (2 + \sin x)(1 + \cos x) = -\sin x$

$\Rightarrow 2 + 2 \cos x + \sin x + \sin x \cos x = -\sin x$

$\Rightarrow 2(\sin x + \cos x) + \sin x \cos x + 2 = 0$

Put $\sin x + \cos x = t$

$\Rightarrow 1 + 2 \sin x \cos x = t^2$

$\therefore 2t + \frac{t^2 - 1}{2} + 2 = 0$

$\Rightarrow t^2 + 4t + 3 = 0$

$\Rightarrow t = -1, -3$

$$\Rightarrow \sin x + \cos x = -1 \Rightarrow \cos \left(x - \frac{\pi}{4} \right) = \frac{-1}{\sqrt{2}} = \cos \frac{3\pi}{4}$$

$$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4} \Rightarrow x = 2n\pi + \pi, 2n\pi - \frac{\pi}{2}$$

$$\Rightarrow x = 2n\pi + \pi \text{ at which } \operatorname{cosec} x \text{ is not defined}$$

$$\therefore x = 2n\pi - \frac{\pi}{2}$$

11. $\sin^2 4x + \cos^2 x = 2 \sin 4x \cdot \cos^4 x$

$$\Rightarrow \sin^2 4x - 2 \sin 4x \cos^4 x + \cos^2 x = 0$$

$$\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x - \cos^8 x = 0$$

$$\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x (1 - \cos^6 x) = 0$$

$$\Rightarrow \sin 4x - \cos^4 x = 0 \quad \dots\dots(i)$$

$$\text{and } \cos^2 x (1 - \cos^6 x) = 0 \quad \dots\dots(ii)$$

$$\text{From (2) } \cos^2 x = 0, 1$$

Case-I $\cos^2 x = 0 \Rightarrow x = n\pi \pm \frac{\pi}{2}$

$$\Rightarrow 4x = 4n\pi \pm 2\pi$$

$$\therefore \sin 4x = 0$$

$$\Rightarrow \text{equation (1) is also true}$$

Case-II $\cos^2 x = 1 \Rightarrow \sin^2 x = 0$

$$\Rightarrow x = n\pi \quad \therefore \text{equation (1) becomes}$$

$$0 - 1 = 0 \text{ false} \quad \therefore \text{solution is } x = n\pi \pm \frac{\pi}{2}$$

EXERCISE - 5

Part # 1 : AIEEE/JEE-MAIN

1. Clearly, given equation is defined for $x \neq \pi/2, 3\pi/2$.

Now, $\tan x + \sec x = 2 \cos x$

$$\Rightarrow 1 + \sin x = 2 \cos^2 x$$

$$\Rightarrow 2 \sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow (2 \sin x - 1)(\sin x + 1) = 0$$

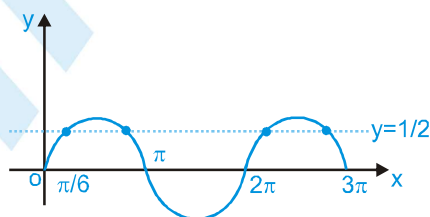
$$\Rightarrow \sin x = \frac{1}{2}, \sin x = -1$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

2. Given equation is $2 \sin^2 x + 5 \sin x - 3 = 0$

$$\Rightarrow (2 \sin x - 1)(\sin x + 3) = 0$$

$$\Rightarrow \sin x = \frac{1}{2} \quad (\rightarrow \sin x \neq -3)$$



It is clear from figure that the curve intersect the line at four points in the given interval.

Hence, number of solutions are 4.

3. We have,

$$\sin \theta = \sin 4\theta + \sin 7\theta = 0$$

$$\Rightarrow (\sin \theta + \sin 7\theta) + \sin 4\theta = 0$$

$$\Rightarrow 2 \sin 4\theta \cos 3\theta + \sin 4\theta = 0$$

$$\Rightarrow \sin 4\theta = 0 \quad \text{or} \quad \cos 3\theta = -\frac{1}{2}$$

$$\Rightarrow 4\theta = \pi, 2\pi, 3\pi \quad \text{or} \quad 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$$

$$\Rightarrow \theta = \frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{4\pi}{9}, \frac{3\pi}{4}, \frac{8\pi}{9}$$

$$\begin{aligned}
 4. \quad & \cos^2\left(x + \frac{\pi}{6}\right) + \cos^2 x - 2\cos\left(x + \frac{\pi}{6}\right)\cos\frac{\pi}{6} = \sin^2\frac{\pi}{6} \\
 \Rightarrow & \cos^2\left(x + \frac{\pi}{6}\right) + \cos^2 x - \sin^2\frac{\pi}{6} - 2\cos\left(x + \frac{\pi}{6}\right)\cos\frac{\pi}{6} = 0 \\
 \Rightarrow & \cos^2\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)\cos\left(x - \frac{\pi}{6}\right) \\
 \Rightarrow & -2\cos\left(x + \frac{\pi}{6}\right)\cos\frac{\pi}{6} = 0 \\
 \Rightarrow & \cos\left(x + \frac{\pi}{6}\right)\left\{\cos\left(x + \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{6}\right) - 2\cos\frac{\pi}{6}\right\} = 0 \\
 \Rightarrow & \cos\left(x + \frac{\pi}{6}\right)\left\{2\cos x \cos\frac{\pi}{6} - 2\cos\frac{\pi}{6}\right\} = 0 \\
 \Rightarrow & 2\cos\left(x + \frac{\pi}{6}\right)\cos\frac{\pi}{6}(\cos x - 1) = 0 \\
 \Rightarrow & \cos\left(x + \frac{\pi}{6}\right)(\cos x - 1) = 0 \\
 \Rightarrow & x + \frac{\pi}{6} = \pm\frac{\pi}{2} \text{ or } x = 0 \\
 \Rightarrow & x = \frac{\pi}{3}, -\frac{2\pi}{3}, 0 \\
 \Rightarrow & x = 0, \frac{\pi}{3} \quad \left[Q \ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \cos x + \cos 2x + \cos 3x + \cos 4x = 0 \\
 & 2\cos\frac{5x}{2} \cdot \cos\frac{3x}{2} + 2\cos\frac{5x}{2} \cdot \cos\frac{x}{2} = 0 \\
 & 2\cos\frac{5x}{2} \times 2\cos x \cdot \cos\frac{x}{2} = 0 \\
 & x = \frac{(2n+1)\pi}{5}, \frac{(2k+1)\pi}{2}, (2r+1)\pi, \\
 & \text{where } n, k \in \mathbb{Z} \text{ or } 0 \leq x \leq 2\pi \\
 & \text{Hence } x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{2\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}
 \end{aligned}$$

Part # II : IIT-JEE ADVANCED

1. To simplify the determinant, let $\sin x = a$; $\cos x = b$. Then the equation becomes

$$\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = 0$$

Operating $C_2 \rightarrow C_2 - C_1$; $C_3 \rightarrow C_3 - C_2$, we get

$$\begin{vmatrix} a & b-a & 0 \\ b & a-b & b-a \\ b & 0 & a-b \end{vmatrix} = 0$$

$$\begin{aligned}
 \text{or } & a(a-b)^2 - (b-a)[b(a-b) - b(b-a)] = 0 \\
 \text{or } & a(a-b)^2 - 2b(b-a)(a-b) = 0 \\
 \text{or } & (a-b)^2(a-2b) = 0 \\
 \text{or } & a = b \quad \text{or } a = 2b
 \end{aligned}$$

$$\text{or } \frac{a}{b} = 1 \quad \text{or } \frac{a}{b} = 2$$

$$\Rightarrow \tan x = 1 \quad \text{or } \tan x = 2$$

$$\text{But we have } -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

$$\Rightarrow \tan\left(\frac{\pi}{4}\right) \leq \tan x \leq \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow -1 \leq \tan x \leq 1$$

$$\therefore \tan x = 1 \Rightarrow x = \pi/4$$

Therefore, there is only one real root.

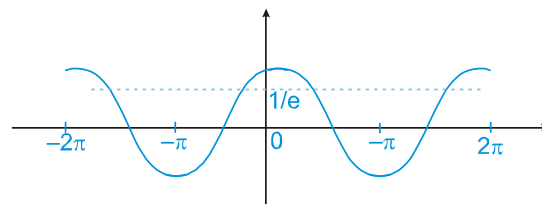
2. We know that

$$-\sqrt{a^2 + b^2} \leq a \cos \theta + b \sin \theta \leq \sqrt{a^2 + b^2}$$

$$\Rightarrow -\sqrt{74} \leq 2k + 1 \leq \sqrt{74}$$

$$\Rightarrow -8 \leq 2k + 1 \leq 8 \Rightarrow -4.5 \leq k \leq 3.5$$

Considering only integral values, which means k can take eight integral values.



$$\alpha - \beta = 0, -2\pi \text{ or } 2\pi$$

$$\alpha - \beta = 0 \Rightarrow \alpha = \beta \Rightarrow \cos 2\beta = \frac{1}{e}$$

This is true for '4' value of ' α ', ' β '

$$\text{If } \alpha - \beta = -2\pi$$

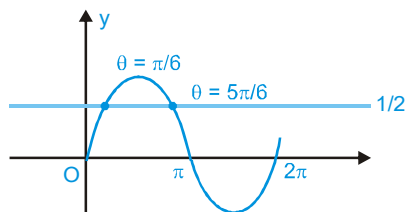
$$\Rightarrow \alpha = -\pi \text{ and } \beta = \pi \text{ and } \cos(\alpha + \beta) = 1$$

$$\Rightarrow (\text{No solution})$$

$$\text{similarly if } \alpha - \beta = 2\pi$$

$$\Rightarrow \alpha = \pi \text{ and } \beta = -\pi \text{ again no solution results}$$

4. $2\sin^2\theta - 5\sin\theta + 2 > 0$
 $\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$



$\Rightarrow \sin\theta < \frac{1}{2}$ [$\rightarrow -1 \leq \sin\theta \leq 1$]

From graph, we get $\theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

6. We have,

$2\sin^2\theta + \sin^2 2\theta = 2$

$\Rightarrow 1 - \cos 2\theta + 1 - \cos^2 2\theta = 2$

$\Rightarrow \cos 2\theta(1 + \cos 2\theta) = 0 \Rightarrow \cos 2\theta = 0$ or $\cos 2\theta = -1$

$\Rightarrow 2q = \frac{\pi}{2}$ or $\cos 2\pi = \pi \Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}$

7. We have

$(y+z)\cos 3\theta = xyz \sin 3\theta$ (i)

$x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z}$ (ii)

$xyz \sin 3\theta = (y+2z)\cos 3\theta + y \sin 3\theta$ (iii)

From (i) and (ii), we get

$y(\cos 3\theta - 2 \sin 3\theta) - z \cos 3\theta = 0$

From (ii) and (iii), we get

$y(\cos 3\theta - \sin 3\theta) = 0$

Since the given system of equations have a solution

(x_0, y_0, z_0) such that $y_0, z_0 \neq 0$

$\therefore \cos 3\theta - \sin 3\theta = 0 \Rightarrow \tan 3\theta = 1$

$\Rightarrow 3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

Hence, that are three values of θ .

5. $2\sin^2\theta - \cos 2\theta = 0$ (i)

$\Rightarrow \sin\theta = \pm \frac{1}{2} \Rightarrow 2\cos^2\theta - 3\sin\theta = 0$ (ii)

$-2\sin^2\theta - 3\sin\theta + 2 = 0 \Rightarrow \sin\theta = \frac{1}{2}, -2$

So $\sin\theta = \frac{1}{2}$ is the only solution at $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

8. $\tan\theta = \cot 5\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos 5\theta}{\sin 5\theta} \Rightarrow \cos 6\theta = 0$

$\Rightarrow 6\theta = (2n+1)\frac{\pi}{2} \Rightarrow \theta = (2n+1)\frac{\pi}{12}; n \in \mathbb{I}$

$\Rightarrow \theta = -\frac{5\pi}{12}, -\frac{\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$ (i)

$\sin 2\theta = \cos 4\theta$

$\Rightarrow \sin 2\theta = 1 - 2\sin^2 2\theta \Rightarrow 2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$\Rightarrow \sin 2\theta = -1, \frac{1}{2} \Rightarrow 2\theta = (4m-1)\frac{\pi}{2}, p\pi + (-1)^p$

$\frac{\pi}{6} \Rightarrow \theta = (4m-1)\frac{\pi}{4}, \frac{p\pi}{2} + (-1)^p \frac{\pi}{12}; m, p \in \mathbb{I}$

$\Rightarrow \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$ (ii)

From (i) & (ii)

$\theta \in \left\{-\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}\right\}$ Number of solution is 3.

9. As $\tan(2\pi - \theta) > 0, -1 < \sin\theta < -\frac{\sqrt{3}}{2}, \theta \in [0, 2\pi]$

$\Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$

Now $2\cos\theta(1 - \sin\phi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\phi - 1$

$\Rightarrow 2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$

$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \phi)$

As $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow 2\cos\theta + 1 \in (1, 2)$

$\Rightarrow 1 < 2\sin(\theta + \phi) < 2 \Rightarrow \frac{1}{2} < \sin(\theta + \phi) < 1$

As $\theta + \phi \in [0, 4\pi]$

$\Rightarrow \theta + \phi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ or $\theta + \phi \in \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

$\Rightarrow \frac{\pi}{6} - \theta < \phi < \frac{5\pi}{6} - \theta$ or $\frac{13\pi}{6} - \theta < \phi < \frac{17\pi}{6} - \theta$

$\Rightarrow \phi \in \left(-\frac{3\pi}{2}, -\frac{2\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{7\pi}{6}\right)$

$\left(Q \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)\right) \therefore$ correct option is (A, C, D)

11. $\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow \sqrt{3} \sin x + \cos x + 2(\sin^2 x - \cos^2 x) = 0$

$\Rightarrow \sqrt{3} \sin x + \cos x - 2\cos 2x = 0 \Rightarrow \sin\left(x + \frac{\pi}{3}\right) = \cos 2x$

$\cos(\pi/3 - x) = \cos 2x \Rightarrow 2x = 2n\pi \pm (\pi/3 - x)$

$x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = 2n\pi - \frac{\pi}{3}$

$-100^\circ - 60^\circ + 20^\circ + 140^\circ = 0$

MOCK TEST

$$1. \cot \theta = -\sqrt{3} \Rightarrow \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\operatorname{cosec} \theta = -2 \Rightarrow \sin \theta = -\frac{1}{2}$$

$$\therefore \theta \text{ lies in IV quadrant } \therefore \theta = 2n\pi - \frac{\pi}{6}$$

$$2. \frac{2.2 \cos \theta / 2 \cdot \cos \theta / 2}{2 \sin \theta / 2 \cdot \cos \theta / 2} = (1 + \cot \theta)^2$$

$$\text{or } \frac{2(1 + \cos \theta)}{\sin \theta} = \operatorname{cosec}^2 \theta + 2 \cot \theta$$

$$\text{or } 2 + 2 \cos \theta = \operatorname{cosec} \theta + 2 \cos \theta$$

$$\text{or } \sin \theta = 1/2 \Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$$

3. (C)

$$\text{We have } \tan \theta = -1 \text{ and } \cos \theta = 1/\sqrt{2}$$

The value of θ lying between $3\pi/2$ and 2π and satisfying these two is $7\pi/4$. Therefore the most general solution is $\theta = 2n\pi + 7\pi/4$ where $n \in \mathbb{Z}$.

$$4. 12 \cos^2 \theta - 6 + 1 + \cos \theta + 2 - 2 \cos^2 \theta = 0$$

$$\text{or } 10 \cos^2 \theta + \cos \theta - 3 = 0$$

$$\text{or } (5 \cos \theta + 3)(2 \cos \theta - 1) = 0$$

$$\Rightarrow \cos \theta = -\frac{3}{5}, \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right), -\frac{\pi}{3}$$

5. (D)

$$\text{Since } \sin \theta - \cos \theta \neq 0$$

$$\tan \theta \neq 1$$

$$\text{and also to define } \tan \theta, \theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\therefore \theta \neq \frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}$$

Now

$$\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta - |\sin \theta| |\cos \theta| - 2 \tan \theta \cot \theta = -1$$

$$\Rightarrow 1 + \cos \theta (\sin \theta - |\sin \theta|) - 2 = -1$$

$$\Rightarrow \cos \theta (\sin \theta - |\sin \theta|) = 0$$

$$\therefore \theta \in (0, \pi) - \left\{ \frac{\pi}{4}, \frac{\pi}{2} \right\}$$

$$6. \cos 6x (1 + \tan^2 x) = 1 - \tan^2 x$$

$$\text{or } \cos 6x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\text{or } \cos 6x = \cos 2x \quad \text{or } 6x = 2n\pi \pm 2x$$

$$\Rightarrow x = \frac{n\pi}{2}, \frac{n\pi}{4}$$

$$\Rightarrow x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$$

$$(\text{At } x = \frac{\pi}{2}, \frac{3\pi}{2}, \tan x \text{ does not exist})$$

7. (B)

$$\text{note : } \sin \theta \neq \cos \theta$$

$$\Rightarrow \theta \notin \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right); \theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi, 2\pi$$

$$\text{and equality holds if } \theta \in \left(\frac{\pi}{2}, \pi\right)$$

8. (B)

$$\therefore -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \Rightarrow -1 \leq \sin \theta \leq 1$$

$$\text{Here } 0 < \sin \theta < 1$$

$$\Rightarrow \cos 2\theta = \sin^2 \theta$$

$$\Rightarrow 3 \sin^2 \theta = 1$$

$$\therefore \sin \theta = \frac{1}{\sqrt{3}} \left\{ \because 0 < \sin \theta < 1 \right\} \text{ a unique solution.}$$

9.

$$\frac{\frac{1}{1 + \sin x}}{\frac{1}{1 - \sin x}} = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\Rightarrow \frac{1 - \sin x}{1 + \sin x} = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\Rightarrow \frac{1 - \sin x}{1 + \sin x} = \frac{\sin^2 x}{(1 - \sin^2 x)} \Rightarrow \sin x = \frac{1}{2}$$

$$(\Rightarrow \sin x \neq \pm 1)$$

10. (C)

$$\text{Given equation } \sec^2 (a + 2)x + a^2 - 1 = 0$$

$$\Rightarrow \tan^2 (a + 2)x + a^2 = 0$$

$$\Rightarrow \tan^2 (a + 2)x = 0 \text{ and } a = 0$$

$$\Rightarrow \tan^2 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$$

$$\therefore (0, 0), (0, \pi/2), (0, -\pi/2) \text{ are ordered pairs satisfying the equation.}$$

11. $4 \sin^4 x + (1 - \sin^2 x)^2 = 1$

$$5 \sin^4 x - 2 \sin^2 x = 0$$

$$\sin^2 x (5 \sin^2 x - 2) = 0$$

$$\Rightarrow \sin^2 x = 0; \quad \sin^2 x = \frac{2}{5}$$

$$\Rightarrow x = n\pi; n \in \mathbb{I} \text{ or } \cos 2x = 1 - 2 \sin^2 x = 1 - \frac{4}{5}$$

$$\therefore \cos 2x = \frac{1}{5} = \cos \alpha$$

$$\therefore 2x = 2n\pi \pm \alpha$$

$$\therefore x = n\pi \pm \frac{1}{2} \cos^{-1} \left(\frac{1}{5} \right); n \in \mathbb{I}$$

12. $\rightarrow 2 \sin 2x \cos x + \sin 2x = 2 \cos 2x \cos x + \cos 2x$

$$\Rightarrow \sin 2x (1 + 2 \cos x) = \cos 2x (2 \cos x + 1)$$

$$\Rightarrow (2 \cos x + 1) (\sin 2x - \cos 2x) = 0$$

$$\Rightarrow \cos x = -\frac{1}{2} \quad \text{or} \quad \tan 2x = 1$$

$$\Rightarrow x = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{I}$$

$$\text{or } x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in \mathbb{I}$$

15. $\sin^2 x - \cos 2x = 2 - \sin 2x$

$$\Rightarrow \sin^2 x - (1 - 2 \sin^2 x) = 2 - 2 \sin x \cos x$$

$$\Rightarrow 3 \sin^2 x + 2 \sin x \cos x = 3$$

Case-I : $\cos x \neq 0$

$$\therefore 3 \tan^2 x + 2 \tan x = 3 (1 + \tan^2 x) \Rightarrow \tan x = \frac{3}{2}$$

Case-II : $\cos x = 0$

$$\therefore 3(1) + 2(\pm 1)(0) = 3 \text{ which is true } \therefore x = (2n+1) \frac{\pi}{2}$$

16. $2^x + 2^{-x} \geq 2$

$$\Rightarrow \sin x \geq 2 \quad (\text{impossible})$$

$$\rightarrow |\sin x| \leq 1$$

17. $y + \frac{1}{y} \geq 2$ (AM $\geq$ GM)

$$\Rightarrow \sqrt{\left(y + \frac{1}{y}\right)} \geq \sqrt{2}$$

$$\text{or } \sin x + \cos x \geq \sqrt{2}$$

$$\text{and } |\sin x + \cos x| \leq \sqrt{2}$$

$$\text{Hence, } y + \frac{1}{y} = 2 \quad \text{and} \quad \sin x + \cos x = \sqrt{2}$$

$$\text{which is possible for } y = 1, x = \frac{\pi}{4}.$$

18. The given equation is equivalent to

$$\tan(\theta + 2\theta + 3\theta) = 0$$

$$\text{or } \tan 6\theta = 0$$

$$\text{Then, } 6\theta = n\pi$$

$$\therefore \theta = \frac{n\pi}{6}, n \in \mathbb{I}$$

$$\text{In } (0, \pi) \text{ we have } \theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}$$

$$\text{However } \tan \theta \text{ and } \tan 3\theta \text{ are not defined at } \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore \frac{\pi}{3}, \frac{2\pi}{3} \text{ are the only solutions.}$$

19. $\sin(\cos x) = \cos(\sin x)$

$$\Rightarrow \cos(\sin x) = \cos\left(\frac{\pi}{2} - \cos x\right)$$

$$\Rightarrow \sin x = 2n\pi \pm \left(\frac{\pi}{2} - \cos x\right), n \in \mathbb{I}$$

$$\Rightarrow \sin x \pm \cos x = \left(2n \pm \frac{1}{2}\right)\pi$$

Squaring

$$\Rightarrow 1 \pm \sin 2x = \left(2n \pm \frac{1}{2}\right)^2 \pi^2$$

$$\Rightarrow |\sin 2x| = \left(2n \pm \frac{1}{2}\right)^2 \pi^2 - 1$$

$$\text{But } \left(2n \pm \frac{1}{2}\right)^2 \pi^2 > 2 \text{ for all } n \in \mathbb{I}$$

$$\therefore |\sin 2x| > 1 \text{ which is inadmissible.}$$

$$\text{Hence, the given equation does not possess real roots.}$$

$$\text{and } \sin x > 0$$

$$(x \text{ lies in I and II quadrant})$$

$$\therefore 2n\pi < x < (2n+1)\pi, n \in \mathbb{I}$$

20. $\sin^2 A = \sin^2 B$ and $\cos^2 A = \cos^2 B$

$$\therefore \cos 2A = \cos 2B$$

$$\Rightarrow 2A = 2n\pi \pm B, n \in \mathbb{I}$$

or $A = n\pi \pm B$

or $A = n\pi + B, n \in \mathbb{I}$

($\rightarrow$ Both sides square given)

Now, $\sin A = \sin B$

$$\Rightarrow A = n\pi + (-1)^n B, n \in \mathbb{I}$$

If n is even, then

$$A = n\pi + B$$

and $\cos A = \cos B$

$$\Rightarrow A = 2n\pi \pm B, n \in \mathbb{I}$$

Hence, Assertion is true but Reason is false.

22. (A) $\sin x = -\frac{1}{2}$

$$= -\sin \frac{\pi}{6} \Rightarrow \sin \left(\pi + \frac{\pi}{6} \right), \sin \left(2\pi - \frac{\pi}{6} \right)$$

$$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \dots(i)$$

and $\cos x = -\frac{\sqrt{3}}{2} = -\cos \frac{\pi}{6}$

$$= \cos \left(\pi - \frac{\pi}{6} \right), \cos \left(\pi + \frac{\pi}{6} \right)$$

$$\therefore x = \frac{5\pi}{6}, \frac{7\pi}{6} \quad \dots(ii)$$

From eq. (i) and (ii). It is clear that

$$\alpha = \frac{7\pi}{6}, \beta = \frac{11\pi}{6}, \gamma = \frac{5\pi}{6}$$

$$\Rightarrow \alpha + \beta = 3\pi(S), \beta - \gamma = \pi(Q)$$

(B) $\cot x = -\sqrt{3}$

$$= -\cot \frac{\pi}{6}$$

$$= \cot \left(\pi - \frac{\pi}{6} \right), \cot \left(2\pi - \frac{\pi}{6} \right)$$

$$\therefore x = \frac{5\pi}{6}, \frac{11\pi}{6} \quad \dots(i)$$

and $\operatorname{cosec} x = -2 = -\operatorname{cosec} \frac{\pi}{6}$

$$= \operatorname{cosec} \left(\pi + \frac{\pi}{6} \right), \operatorname{cosec} \left(2\pi - \frac{\pi}{6} \right)$$

$$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \dots(ii)$$

from eq. (i) and (ii), It is clear that

$$\alpha = \frac{11\pi}{6}, \beta = \frac{5\pi}{6}, \gamma = \frac{7\pi}{6}$$

$$\Rightarrow \beta + \gamma = 2\pi(T), \alpha - \beta = \pi(P)$$

(C) $\sin x = -\frac{1}{2}$

$$= -\sin \frac{\pi}{6}$$

$$= \sin \left(\pi + \frac{\pi}{6} \right), \sin \left(2\pi - \frac{\pi}{6} \right)$$

$$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \dots(i)$$

and $\tan x = \frac{1}{\sqrt{3}}$

$$= \tan \frac{\pi}{6}$$

$$= \tan \frac{\pi}{6}, \tan \left(\pi + \frac{\pi}{6} \right)$$

$$\therefore x = \frac{\pi}{6}, \frac{7\pi}{6} \quad \dots(ii)$$

from eq. (i) and (ii), It is clear that

$$\alpha = \frac{7\pi}{6}, \beta = \frac{11\pi}{6}, \gamma = \frac{\pi}{6}$$

$$\Rightarrow \alpha + \beta = 3\pi(S), \beta + \gamma = 2\pi(T), \alpha - \gamma = \pi(R)$$

24.

 1. $AM \geq GM$

$$\therefore 3^x - 3^{-x} \geq 2$$

$$\Rightarrow 2\cos \left(\frac{x}{2} \right) \geq 2$$

or $\cos \left(\frac{x}{2} \right) \geq 1$

$$\therefore \cos \left(\frac{x}{2} \right) = 1 \quad \left(Q \cos \frac{x}{2} \text{ is never } > 1 \right)$$

$$\Rightarrow \frac{x}{2} = 2n\pi, n \in \mathbb{I}$$

$$\therefore x = 4n\pi$$

Hence (A) corresponding to $n = 0$,

because other values of n do not satisfy the equation.

2 Let $y = 2\cos^2\left(\frac{x}{2}\right) \sin^2 x = x^2 + x^{-2}$

$\therefore y = x^2 + x^{-2} \geq 2$ ($\rightarrow$ AM $\geq$ GM)

$\Rightarrow y \geq 2$ (i)

and $y = 2\cos^2\left(\frac{x}{2}\right) \sin^2 x$

$= (1 + \cos x) \cdot \sin^2 x$

$= (\text{a number} < 2) \cdot (\text{a number} \leq 1) < 2$

i.e., $y < 2$ (ii)

No value of y can be obtained satisfying eq. (i) and (ii) simultaneously.

$\Rightarrow$ No real solution of the equation exists.

3 AM $\geq$ GM

$\therefore 5^x + 5^{-x} \geq 2$ ($\rightarrow$ sin $e^x = 5^x + 5^{-x}$)

$\Rightarrow \sin e^x \geq 2$

But the value of $\sin(e^x)$ can never be > 1 .

Hence, the given equation has no solution.

25.

1. Let $\cos x - \sin x = t$

$\therefore 1 - 2 \sin x \cos x = t^2$

Then, the given equation can be written as

$t^2 = t$

$\Rightarrow t(t-1) = 0$

$\therefore t = 0, t = 1$

$\Rightarrow \cos x - \sin x = 0, \cos x - \sin x = 1$

$\therefore \tan x = 1, \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \tan x = 1, \cos\left(x + \frac{\pi}{4}\right) = \cos \frac{\pi}{4}$

$\therefore x = n\pi + \frac{\pi}{4}, x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

Hence, $x = 2n\pi, 2n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{4}, n \in I$

2. Let $\sin x + \cos x = t$

$\therefore 1 + 2 \sin x \cos x = t^2$

Then, the given equation can be written as

$\Rightarrow t = 1 + \frac{t^2 - 1}{2}$

$\Rightarrow 2t = 2 + t^2 - 1$

$\Rightarrow (t-1)^2 = 0$

or $t = 1$

$\Rightarrow \sin x + \cos x = 1$

$\Rightarrow \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos \frac{\pi}{4}$

or $x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in I$

$\therefore x = 2n\pi, 2n\pi + \frac{\pi}{2}$

3. $\sin^4 x + \cos^4 x = \sin x \cos x$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x = \sin x \cos x$

$\Rightarrow 1 - 2 \sin^2 x \cos^2 x = \sin x \cos x$

Let $\sin x \cos x = \lambda$, then

$1 - 2\lambda^2 = \lambda$

$\Rightarrow 2\lambda^2 + \lambda - 1 = 0$

$\Rightarrow (\lambda + 1)(2\lambda - 1) = 0$

$\therefore \lambda = -1, \frac{1}{2}$

$\Rightarrow \sin x \cos x = -1, \frac{1}{2}$

$\Rightarrow \sin 2x = -2, 1$

$\Rightarrow \sin 2x \neq -2$

$\Rightarrow \sin 2x = 1$

or $2x = 2n\pi + \frac{\pi}{2}, n \in I$

$\therefore x = (4n + 1) \frac{\pi}{4}, n \in I$

26. (0)

$|\sin x \cos x| + |\tan x + \cot x| = \sqrt{3}$

$\Rightarrow |\sin x \cos x| + \frac{1}{|\sin x \cos x|} = \sqrt{3}$

$|\sin x \cos x| + \frac{1}{|\sin x \cos x|} \geq 2$

Hence, there is no solution.

27. (6)

$$\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = 0$$

$$\text{or } \frac{2 \sin x \cos x}{2 \cos 3x \cos x} + \frac{2 \sin 3x \cos 3x}{2 \cos 9x \cos 3x} + \frac{2 \sin 9x \cos 9x}{2 \cos 27x \cos 9x} = 0$$

$$\text{or } \frac{\sin(3x - x)}{2 \cos 3x \cos x} + \frac{\sin(9x - 3x)}{2 \cos 9x \cos 3x} + \frac{\sin(27x - 9x)}{2 \cos 27x \cos 9x} = 0$$

$$\text{or } (\tan 3x - \tan x) + (\tan 9x - \tan 3x) + (\tan 27x - \tan 9x) = 0$$

$$\text{or } \tan 27x - \tan x = 0$$

$$\text{or } \tan x = \tan 27x$$

$$\Rightarrow 27x = n\pi + x, n \in I$$

$$\text{or } x = \frac{n\pi}{26}, n \in I$$

$$\text{or } x = \frac{\pi}{26}, \frac{2\pi}{26}, \frac{3\pi}{26}, \frac{4\pi}{26}, \frac{5\pi}{26}, \frac{6\pi}{26}$$

Hence, there are six solutions.

28. (1)

Adding given equations, we get

$$2 = \frac{3a}{2} + \frac{a^2}{2}$$

$$\text{or } a^2 + 3a - 4 = 0$$

$$\text{or } (a + 4)(a - 1) = 0$$

$$\text{or } a = 1 \quad (\text{as } a = -4 \text{ is rejected})$$

29. (6)

$$a \sin x + 1 - 2 \sin^2 x = 2a - 7$$

$$\text{or } 2 \sin^2 x - a \sin x + (2a - 8) = 0$$

$$\text{or } \sin x = \frac{a \pm \sqrt{a^2 - 8(2a - 8)}}{4} = \frac{a \pm (a - 8)}{4}$$

$$= 2 \quad \text{or} \quad \frac{a - 4}{2}$$

$$\text{For a solution } -1 \leq \frac{a - 4}{2} \leq 1, \text{ we have } 2 \leq a \leq 6.$$

30. (4)

$$\sin^4 x - \cos^2 x \sin x + 2 \sin^2 x + \sin x = 0$$

$$\text{or } \sin x [\sin^3 x - \cos^2 x + 2 \sin x + 1] = 0$$

$$\text{or } \sin x [\sin^3 x + \sin^2 x + 2 \sin x] = 0$$

$$\text{or } \sin^2 x [\sin^2 x + \sin x + 2] = 0$$

$$\text{or } \sin x = 0, \text{ where } x = 0, \pi, 2\pi, 3\pi$$

Hence, there are four solutions.