

SOLVED EXAMPLES

Ex. 1 Prove that

(i) $\sin(45^\circ + A) \cos(45^\circ - B) + \cos(45^\circ + A) \sin(45^\circ - B) = \cos(A - B)$

(ii) $\tan\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{3\pi}{4} + \theta\right) = -1$

Sol. (i) Clearly $\sin(45^\circ + A) \cos(45^\circ - B) + \cos(45^\circ + A) \sin(45^\circ - B)$
 $= \sin(45^\circ + A + 45^\circ - B) = \sin(90^\circ + A - B) = \cos(A - B)$

(ii) $\tan\left(\frac{\pi}{4} + \theta\right) \times \tan\left(\frac{3\pi}{4} + \theta\right) = \frac{1 + \tan \theta}{1 - \tan \theta} \times \frac{-1 + \tan \theta}{1 + \tan \theta} = -1$

Ex. 2 Prove that $\sin 5A + \sin 3A = 2 \sin 4A \cos A$

Sol. L.H.S. $\sin 5A + \sin 3A = 2 \sin 4A \cos A$ = R.H.S.

[→ $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

Ex. 3 Find the value of $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1$.

Sol. $2[(\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)] - 3[(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta] + 1$
 $= 2[1 - 3 \sin^2 \theta \cos^2 \theta] - 3[1 - 2 \sin^2 \theta \cos^2 \theta] + 1$
 $= 2 - 6 \sin^2 \theta \cos^2 \theta - 3 + 6 \sin^2 \theta \cos^2 \theta + 1 = 0$

Ex. 4 If the arcs of same length in two circles subtend angles of 60° and 75° at their centres. Find the ratio of their radii.

Sol. Let r_1 and r_2 be the radii of the given circles and let their arcs of same length s subtend angles of 60° and 75° at their centres.

Now, $60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$ and $75^\circ = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$

$\therefore \frac{\pi}{3} = \frac{s}{r_1}$ and $\frac{5\pi}{12} = \frac{s}{r_2}$

$\Rightarrow \frac{\pi}{3} r_1 = s$ and $\frac{5\pi}{12} r_2 = s \Rightarrow \frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2 \Rightarrow 4r_1 = 5r_2 \Rightarrow r_1 : r_2 = 5 : 4$

Ex. 5 Find the value of θ for $\sin \theta = -\frac{1}{2}$ and $\tan \theta = \frac{1}{\sqrt{3}}$.

Sol. Let us first find out θ lying between 0 and 360° .

Since $\sin \theta = -\frac{1}{2} \Rightarrow \theta = 210^\circ$ or 330° and $\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$ or 210°

Hence, $\theta = 210^\circ$ or $\frac{7\pi}{6}$ is the value satisfying both.



Ex. 6 Prove that

$$(i) \frac{\sin 8\theta \cos \theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos \theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$$

$$(ii) \frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta$$

Sol. (i)
$$\frac{2 \sin 8\theta \cos \theta - 2 \sin 6\theta \cos 3\theta}{2 \cos 2\theta \cos \theta - 2 \sin 3\theta \sin 4\theta}$$

$$= \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos \theta - \cos \theta + \cos 7\theta} = \frac{2 \sin 2\theta \cos 5\theta}{2 \cos 5\theta \cos 2\theta} = \tan 2\theta$$

$$(ii) \frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = \frac{\sin 5\theta \cos 3\theta + \sin 3\theta \cos 5\theta}{\sin 5\theta \cos 3\theta - \sin 3\theta \cos 5\theta} = \frac{\sin 8\theta}{\sin 2\theta} = 4 \cos 2\theta \cos 4\theta$$

Ex. 7 Prove that $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ = 4$.

Sol. L.H.S.
$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cdot \cos 20^\circ}$$

$$= \frac{4 \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ \right)}{2 \sin 20^\circ \cos 20^\circ} = \frac{4 (\sin 60^\circ \cdot \cos 20^\circ - \cos 60^\circ \cdot \sin 20^\circ)}{\sin 40^\circ}$$

$$= 4 \cdot \frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} = 4 \cdot \frac{\sin 40^\circ}{\sin 40^\circ} = 4 = \text{R.H.S.}$$

Ex. 8 Which of the following is greatest ?

(A) $\tan 1$

(B) $\tan 4$

(C) $\tan 7$

(D) $\tan 10$

Sol. (A) $\tan 4 = \tan(\pi + (4 - \pi)) = \tan(4 - \pi) = \tan(0.86)$

$$\tan 7 = \tan(2\pi + (7 - 2\pi)) = \tan(7 - 2\pi) = \tan(0.72)$$

$$\tan 10 = \tan(3\pi + (10 - 3\pi)) = \tan(10 - 3\pi) = \tan(0.58)$$

Now, $1 > 0.86 > 0.72 > 0.58$

$\Rightarrow \tan 1 > \tan(0.86) > \tan(0.72) > \tan(0.58)$ [as 1, 0.86, 0.72, 0.58 lie in the first quadrant and tangent functions increase in all the quadrant]

Hence, $\tan 1$ is greatest

Ex. 9 Find the value of $\cos^2 \frac{\pi}{16} + \cos^2 \frac{3\pi}{16} + \cos^2 \frac{5\pi}{16} + \cos^2 \frac{7\pi}{16}$.

Sol. LHS
$$= \cos^2 \frac{\pi}{16} + \cos^2 \frac{3\pi}{16} + \cos^2 \left(\frac{\pi}{2} - \frac{3\pi}{16} \right) + \cos^2 \left(\frac{\pi}{2} - \frac{\pi}{16} \right)$$

$$= \cos^2 \frac{\pi}{16} + \cos^2 \frac{3\pi}{16} + \sin^2 \frac{3\pi}{16} + \sin^2 \frac{\pi}{16}$$

$$= \left(\cos^2 \frac{\pi}{16} + \sin^2 \frac{\pi}{16} \right) + \left(\cos^2 \frac{3\pi}{16} + \sin^2 \frac{3\pi}{16} \right)$$

$$= 1 + 1 = 2$$

Ex. 10 Simplify $\frac{\sin 5\theta + \sin 2\theta - \sin \theta}{\cos 5\theta + 2 \cos 3\theta + 2 \cos^2 \theta + \cos \theta}$.

Sol. L.H.S. = $\frac{2 \sin 2\theta \cos 3\theta + \sin 2\theta}{2 \cos 3\theta \cdot \cos 2\theta + 2 \cos 3\theta + 2 \cos^2 \theta} = \frac{\sin 2\theta [2 \cos 3\theta + 1]}{2 [\cos 3\theta (\cos 2\theta + 1) + (\cos^2 \theta)]}$

$$= \frac{\sin 2\theta [2 \cos 3\theta + 1]}{2 [\cos 3\theta (2 \cos^2 \theta) + \cos^2 \theta]} = \frac{\sin 2\theta (2 \cos 3\theta + 1)}{2 \cos^2 \theta (2 \cos 3\theta + 1)} = \tan \theta$$

Ex. 11 If $\sin \alpha + \sin \beta = a$ and $\cos \alpha + \cos \beta = b$, prove that $\tan \frac{\alpha - \beta}{2} = \pm \sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$

Sol. Given, $\sin \alpha + \sin \beta = a$ (i)
 and $\cos \alpha + \cos \beta = b$ (ii)
 Now, $(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = b^2 + a^2$
 or $\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta + \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta = b^2 + a^2$
 or $(\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) = a^2 + b^2$
 or $2 + 2 \cos (\alpha - \beta) = a^2 + b^2$
 or $\cos (\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$
 use $\tan \theta = \pm \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}}$
 here $2\theta = \alpha - \beta \Rightarrow \theta = (\alpha - \beta)/2$
 $\tan \frac{\alpha - \beta}{2} = \pm \sqrt{\frac{1 - (a^2 + b^2 - 2)/2}{1 + (a^2 + b^2 - 2)/2}} \Rightarrow \tan \frac{\alpha - \beta}{2} = \pm \sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$

Ex. 12 Prove that : $\tan A + \tan(60^\circ + A) + \tan(120^\circ + A) = 3 \tan 3A$

Sol. L.H.S. = $\tan A + \tan(60^\circ + A) + \tan(120^\circ + A)$
 $= \tan A + \tan(60^\circ + A) + \tan\{180^\circ - (60^\circ - A)\}$
 $= \tan A + \tan(60^\circ + A) - \tan(60^\circ - A)$ [$\rightarrow \tan(180^\circ - \theta) = -\tan \theta$]
 $= \tan A + \frac{\tan 60^\circ + \tan A}{1 - \tan 60^\circ \tan A} - \frac{\tan 60^\circ - \tan A}{1 + \tan 60^\circ \tan A} = \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3} \tan A}$
 $= \tan A + \frac{\sqrt{3} + \tan A + 3 \tan A + \sqrt{3} \tan^2 A - \sqrt{3} + \tan A + 3 \tan A - \sqrt{3} \tan^2 A}{(1 - \sqrt{3} \tan A)(1 + \sqrt{3} \tan A)}$
 $= \tan A + \frac{8 \tan A}{1 - 3 \tan^2 A} = \frac{\tan A - 3 \tan^3 A + 8 \tan A}{1 - 3 \tan^2 A}$
 $= \frac{9 \tan A - 3 \tan^3 A}{1 - 3 \tan^2 A} = 3 \left(\frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \right) = 3 \tan 3A = \text{R.H.S.}$

Ex. 13 Find maximum and minimum values of following :

(i) $3\sin x + 4\cos x$ (ii) $1 + 2\sin x + 3\cos^2 x$

Sol. (i) We know

$$-\sqrt{3^2 + 4^2} \leq 3\sin x + 4\cos x \leq \sqrt{3^2 + 4^2}$$

$$-5 \leq 3\sin x + 4\cos x \leq 5$$

(ii) $1 + 2\sin x + 3\cos^2 x = -3\sin^2 x + 2\sin x + 4$

$$= -3 \left(\sin^2 x - \frac{2\sin x}{3} \right) + 4 = -3 \left(\sin x - \frac{1}{3} \right)^2 + \frac{13}{3}$$

Now $0 \leq \left(\sin x - \frac{1}{3} \right)^2 \leq \frac{16}{9}$

$$\Rightarrow -\frac{16}{3} \leq -3 \left(\sin x - \frac{1}{3} \right)^2 \leq 0$$

$$-1 \leq -3 \left(\sin x - \frac{1}{3} \right)^2 + \frac{13}{3} \leq \frac{13}{3}$$

Ex. 14 In any triangle ABC, $\sin A - \cos B = \cos C$, then angle B is

Sol. We have, $\sin A - \cos B = \cos C$

$$\sin A = \cos B + \cos C$$

$$\Rightarrow 2 \sin \frac{A}{2} \cos \frac{A}{2} = 2 \cos \left(\frac{B+C}{2} \right) \cos \left(\frac{B-C}{2} \right)$$

$$\Rightarrow 2 \sin \frac{A}{2} \cos \frac{A}{2} = 2 \cos \left(\frac{\pi - A}{2} \right) \cos \left(\frac{B-C}{2} \right) \quad \rightarrow A+B+C=\pi$$

$$\Rightarrow 2 \sin \frac{A}{2} \cos \frac{A}{2} = 2 \sin \frac{A}{2} \cos \left(\frac{B-C}{2} \right)$$

$$\Rightarrow \cos \frac{A}{2} = \cos \frac{B-C}{2} \quad \text{or} \quad A=B-C ; \quad \text{But} \quad A+B+C=\pi$$

$$\text{Therefore } 2B=\pi \quad \Rightarrow \quad B=\pi/2$$

Ex. 15 Find the summation of the following series

(i) $\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}$

(ii) $\cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{5\pi}{7} + \cos \frac{6\pi}{7}$

(iii) $\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$

Sol. (i) $\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} = \frac{\cos \left(\frac{2\pi}{7} + \frac{6\pi}{7} \right) \sin \frac{3\pi}{7}}{\sin \frac{\pi}{7}}$

$$= \frac{\cos \frac{4\pi}{7} \sin \frac{3\pi}{7}}{\sin \frac{\pi}{7}} = \frac{-\cos \frac{3\pi}{7} \sin \frac{3\pi}{7}}{\sin \frac{\pi}{7}} = -\frac{\sin \frac{6\pi}{7}}{2 \sin \frac{\pi}{7}} = -\frac{1}{2}$$

(ii) $\cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{5\pi}{7} + \cos \frac{6\pi}{7}$

$$= \frac{\cos \left(\frac{\pi}{7} + \frac{6\pi}{7} \right) \sin \frac{6\pi}{14}}{\sin \frac{\pi}{14}} = \frac{\cos \frac{\pi}{2} \sin \frac{6\pi}{14}}{\sin \frac{\pi}{14}} = 0$$

(iii) $\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$

$$= \frac{\cos \frac{10\pi}{22} \sin \frac{5\pi}{11}}{\sin \frac{\pi}{11}} = \frac{\sin \frac{10\pi}{11}}{2 \sin \frac{\pi}{11}} = \frac{1}{2}$$

Ex. 16 If $\tan \beta = \frac{\tan \alpha + \tan \gamma}{1 + \tan \alpha \tan \gamma}$, prove that $\sin 2\beta = \frac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \sin 2\gamma}$

Sol. $\tan \beta = \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \gamma}{\cos \gamma}}{1 + \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \gamma}{\cos \gamma}} = \frac{\sin(\alpha + \gamma)}{\cos(\alpha - \gamma)}$

$$\sin 2\beta = \frac{2 \tan \beta}{1 + \tan^2 \beta} = \frac{2 \frac{\sin(\alpha + \gamma)}{\cos(\alpha - \gamma)}}{1 + \frac{\sin^2(\alpha + \gamma)}{\cos^2(\alpha - \gamma)}}$$

$$= \frac{2 \sin(\alpha + \gamma) \cos(\alpha - \gamma)}{\cos^2(\alpha - \gamma) + \sin^2(\alpha + \gamma)}$$

$$= \frac{\sin 2\alpha + \sin 2\gamma}{\frac{1 + \cos 2(\alpha - \gamma)}{2} + \frac{1 - \cos 2(\alpha + \gamma)}{2}}$$

$$= \frac{\sin 2\alpha + \sin 2\gamma}{1 + \frac{1}{2} \times 2 \sin 2\alpha \sin 2\gamma} = \frac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \sin 2\gamma}.$$

Ex. 17 Prove that

$$(i) \quad \frac{\sin 2A}{1 + \cos 2A} = \tan A \quad (ii) \quad \tan A + \cot A = 2 \operatorname{cosec} 2A$$

$$(iii) \quad \frac{1 - \cos A + \cos B - \cos(A+B)}{1 + \cos A - \cos B - \cos(A+B)} = \tan \frac{A}{2} \cot \frac{B}{2}$$

Sol. (i) **L.H.S.** $\frac{\sin 2A}{1 + \cos 2A} = \frac{2 \sin A \cos A}{2 \cos^2 A} = \tan A$

(ii) **L.H.S.** $\tan A + \cot A = \frac{1 + \tan^2 A}{\tan A} = 2 \left(\frac{1 + \tan^2 A}{2 \tan A} \right) = \frac{2}{\sin 2A} = 2 \operatorname{cosec} 2A$

(iii) **L.H.S.**
$$\frac{1 - \cos A + \cos B - \cos(A+B)}{1 + \cos A - \cos B - \cos(A+B)} = \frac{2 \sin^2 \frac{A}{2} + 2 \sin \frac{A}{2} \sin \left(\frac{A}{2} + B \right)}{2 \cos^2 \frac{A}{2} - 2 \cos \frac{A}{2} \cos \left(\frac{A}{2} + B \right)}$$

$$= \tan \frac{A}{2} \left[\frac{\sin \frac{A}{2} + \sin \left(\frac{A}{2} + B \right)}{\cos \frac{A}{2} - \cos \left(\frac{A}{2} + B \right)} \right] = \tan \frac{A}{2} \left[\frac{2 \sin \frac{A+B}{2} \cos \left(\frac{B}{2} \right)}{2 \sin \frac{A+B}{2} \sin \left(\frac{B}{2} \right)} \right] = \tan \frac{A}{2} \cot \frac{B}{2}$$

Ex. 18 Evaluate $\cos a \cos 2a \cos 3a \dots \cos 999a$, where $a = \frac{2\pi}{1999}$.

Sol. **Let** $P = \cos a \cos 2a \cos 3a \dots \cos 999a$

$$Q = \sin a \sin 2a \sin 3a \dots \sin 999a.$$

Then, $2^{999} PQ = (2 \sin a \cos a)(2 \sin 2a \cos 2a) \dots (2 \sin 999a \cos 999a)$

$$= \sin 2a \sin 4a \dots \sin 1998a$$

$$= (\sin 2a \sin 4a \dots \sin 998a) [-\sin(2\pi - 1000a)] \cdot [-\sin(2\pi - 1002a)] \dots [-\sin(2\pi - 1998a)]$$

$$= \sin 2a \sin 4a \dots \sin 998a \sin 999a \sin 997a \dots \sin a = Q.$$

It is easy to see that $Q \neq 0$. Hence, the desired product is $P = \frac{1}{2^{999}}$.

Ex. 19 If $x + y + z = xyz$, Prove that $\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2x}{1-x^2} \cdot \frac{2y}{1-y^2} \cdot \frac{2z}{1-z^2}$.

Sol. **Put** $x = \tan A$, $y = \tan B$ and $z = \tan C$,

So that we have

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C \quad \Rightarrow \quad A + B + C = n\pi, \text{ where } n \in \mathbb{I}$$

Hence **L.H.S.**

$$\therefore \frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2 \tan A}{1 - \tan^2 A} + \frac{2 \tan B}{1 - \tan^2 B} + \frac{2 \tan C}{1 - \tan^2 C}$$

$$= \tan 2A + \tan 2B + \tan 2C \quad [\rightarrow A + B + C = n\pi]$$

$$= \tan 2A \tan 2B \tan 2C = \frac{2x}{1-x^2} \cdot \frac{2y}{1-y^2} \cdot \frac{2z}{1-z^2}$$

Ex. 20 Find the maximum value of $1 + \sin \left(\frac{\pi}{4} + \theta \right) + 2 \cos \left(\frac{\pi}{4} - \theta \right)$ -

Sol. We have $1 + \sin \left(\frac{\pi}{4} + \theta \right) + 2 \cos \left(\frac{\pi}{4} - \theta \right)$

$$= 1 + \frac{1}{\sqrt{2}} (\cos \theta + \sin \theta) + \sqrt{2} (\cos \theta + \sin \theta) = 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2} \right) (\cos \theta + \sin \theta)$$

$$= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2} \right) \cdot \sqrt{2} \cos \left(\theta - \frac{\pi}{4} \right)$$

$\therefore$ maximum value $= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2} \right) \cdot \sqrt{2} = 4$

Ex. 21 Evaluate $\sum_{r=1}^{n-1} \cos^2 \left(\frac{r\pi}{n} \right)$; $n \geq 2$

Sol. Sum $= \frac{1}{2} \sum_{r=1}^{n-1} \left(1 + \cos \frac{2r\pi}{n} \right) = \frac{1}{2} (n-1) + \frac{1}{2} \left\{ \cos \frac{2\pi}{n} + \cos \frac{4\pi}{n} + \dots + \cos \frac{(2n-2)\pi}{n} \right\}$

$$= \frac{1}{2} (n-1) + \frac{1}{2} \left\{ \frac{\sin (n-1) \frac{2\pi}{n}}{\sin \frac{2\pi}{n}} \cdot \cos \left[\frac{2 \left(\frac{2\pi}{n} \right) + (n-2) \frac{2\pi}{n}}{2} \right] \right\}$$

Using, $\cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots + \cos(\alpha + (n-1)\beta) = \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \cdot \cos \left\{ \frac{2\alpha + (n-1)\beta}{2} \right\}$

$$= \frac{1}{2} (n-1) + \frac{1}{2} \left\{ \frac{\sin \frac{(n-1)\pi}{n} \cdot \cos \pi}{\sin \left(\frac{\pi}{n} \right)} \right\} = \frac{1}{2} (n-1) - \frac{1}{2} = \frac{n}{2} - 1$$

$\therefore \sum_{r=1}^{n-1} \cos^2 \left(\frac{r\pi}{n} \right) = \frac{n-2}{2}$

Ex. 22 Prove that $\tan \alpha + 2 \tan 2\alpha + 2^2 \tan^2 \alpha + \dots + 2^{n-1} \tan 2^{n-1} \alpha + 2^n \cot 2^n \alpha = \cot \alpha$

Sol. We know $\tan \theta = \cot \theta - 2 \cot 2\theta$ (i)

Putting $\theta = \alpha, 2\alpha, 2^2\alpha, \dots$ in (i), we get

$$\tan \alpha = (\cot \alpha - 2 \cot 2\alpha)$$

$$2 (\tan 2\alpha) = 2(\cot 2\alpha - 2 \cot 2^2\alpha)$$

$$2^2 (\tan 2^2 \alpha) = 2^2 (\cot 2^2 \alpha - 2 \cot 2^3 \alpha)$$

$$\dots\dots\dots$$

$$2^{n-1} (\tan 2^{n-1} \alpha) = 2^{n-1} (\cot 2^{n-1} \alpha - 2 \cot 2^n \alpha)$$

Adding,

$$\tan \alpha + 2 \tan 2\alpha + 2^2 \tan^2 \alpha + \dots + 2^{n-1} \tan 2^{n-1} \alpha = \cot \alpha - 2^n \cot 2^n \alpha$$

$\therefore \tan \alpha + 2 \tan 2\alpha + 2^2 \tan^2 \alpha + \dots + 2^{n-1} \tan 2^{n-1} \alpha + 2^n \cot 2^n \alpha = \cot \alpha$

Exercise # 1

[Single Correct Choice Type Questions]

- The value of the expression $\left(1 + \cos \frac{\pi}{10}\right) \left(1 + \cos \frac{3\pi}{10}\right) \left(1 + \cos \frac{7\pi}{10}\right) \left(1 + \cos \frac{9\pi}{10}\right)$ is
 (A) $\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $\frac{1}{4}$ (D) 0
- Which of the following is correct ?
 (A) $\sin 1^\circ > \sin 1$ (B) $\sin 1^\circ < \sin 1$ (C) $\sin 1^\circ = \sin 1$ (D) $\sin 1^\circ = \frac{\pi}{180} \sin 1$
- If $x + y = 3 - \cos 4\theta$ and $x - y = 4 \sin 2\theta$ then
 (A) $x^4 + y^4 = 9$ (B) $\sqrt{x} + \sqrt{y} = 16$
 (C) $x^3 + y^3 = 2(x^2 + y^2)$ (D) $\sqrt{x} + \sqrt{y} = 2$
- If $\tan B = \frac{n \sin A \cos A}{1 - n \cos^2 A}$ then $\tan(A + B)$ equals
 (A) $\frac{\sin A}{(1 - n) \cos A}$ (B) $\frac{(n - 1) \cos A}{\sin A}$ (C) $\frac{\sin A}{(n - 1) \cos A}$ (D) $\frac{\sin A}{(n + 1) \cos A}$
- If $A = \sin \frac{2\pi}{7} + \sin \frac{4\pi}{7} + \sin \frac{8\pi}{7}$ and $B = \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{8\pi}{7}$ then $\sqrt{A^2 + B^2}$ is equal to
 (A) 1 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{3}$
- The expression $\frac{\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ + \cos 157^\circ \cos 97^\circ}$ when simplified reduces to -
 (A) 1 (B) -1 (C) 2 (D) none
- If $\tan \theta = \sqrt{\frac{a}{b}}$ where a, b are positive reals then the value of $\sin \theta \sec^7 \theta + \cos \theta \operatorname{cosec}^7 \theta$ is -
 (A) $\frac{(a + b)^3 (a^4 + b^4)}{(ab)^{7/2}}$ (B) $\frac{(a + b)^3 (a^4 - b^4)}{(ab)^{7/2}}$ (C) $\frac{(a + b)^3 (b^4 - a^4)}{(ab)^{7/2}}$ (D) $-\frac{(a + b)^3 (a^4 + b^4)}{(ab)^{7/2}}$
- If $\frac{\sin 2\alpha - \sin 3\alpha + \sin 4\alpha}{\cos 2\alpha - \cos 3\alpha + \cos 4\alpha} = \tan k\alpha$ is an identity then the value of k is equal to -
 (A) 2 (B) 3 (C) 4 (D) 6
- Exact value of $\cos 20^\circ + 2 \sin^2 55^\circ - \sqrt{2} \sin 65^\circ$ is -
 (A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) zero
- If $\cos \theta = \frac{1}{2} \left(a + \frac{1}{a}\right)$ then $\cos 3\theta$ in terms of 'a' =
 (A) $\frac{1}{4} \left(a^3 + \frac{1}{a^3}\right)$ (B) $4 \left(a^3 + \frac{1}{a^3}\right)$ (C) $\frac{1}{2} \left(a^3 + \frac{1}{a^3}\right)$ (D) none

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11. The product $\cot 123^\circ \cdot \cot 133^\circ \cdot \cot 137^\circ \cdot \cot 147^\circ$, when simplified is equal to -
 (A) -1 (B) $\tan 37^\circ$ (C) $\cot 33^\circ$ (D) 1
12. If $5 \tan \theta = 4$, then $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}$ is equal to
 (A) 0 (B) 1 (C) $1/6$ (D) 6
13. The roots of the equation $4x^2 - 2\sqrt{5}x + 1 = 0$ are
 (A) $\sin 36^\circ, \sin 18^\circ$ (B) $\sin 18^\circ, \cos 36^\circ$ (C) $\sin 36^\circ, \cos 18^\circ$ (D) $\cos 18^\circ, \cos 36^\circ$
14. If $\cos(A - B) = 3/5$ and $\tan A \tan B = 2$, then
 (A) $\cos A \cos B = 1/5$ (B) $\sin A \sin B = -2/5$ (C) $\cos A \cos B = -1/5$ (D) $\sin A \sin B = -1/5$
15. If $\cos \alpha = \frac{2 \cos \beta - 1}{2 - \cos \beta}$ then $\tan \frac{\alpha}{2} \cdot \cot \frac{\beta}{2}$ has the value equal to $\{ \text{where } \alpha, \beta \in (0, \pi) \}$
 (A) 2 (B) $\sqrt{2}$ (C) 3 (D) $\sqrt{3}$
16. The graphs of $y = \sin x$, $y = \cos x$, $y = \tan x$ and $y = \operatorname{cosec} x$ are drawn on the same axes from 0 to $\pi/2$. A vertical line is drawn through the point where the graphs of $y = \cos x$ and $y = \tan x$ cross, intersecting the other two graphs at points A and B. The length of the line segment AB is:
 (A) 1 (B) $\frac{\sqrt{5}-1}{2}$ (C) $\sqrt{2}$ (D) $\frac{\sqrt{5}+1}{2}$
17. If $\frac{5\pi}{2} < x < 3\pi$, then the value of the expression $\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}$ is
 (A) $-\cot \frac{x}{2}$ (B) $\cot \frac{x}{2}$ (C) $\tan \frac{x}{2}$ (D) $-\tan \frac{x}{2}$
18. The value of $\cot x + \cot(60^\circ + x) + \cot(120^\circ + x)$ is equal to :
 (A) $\cot 3x$ (B) $\tan 3x$ (C) $3 \tan 3x$ (D) $\frac{3 - 9 \tan^2 x}{3 \tan x - \tan^3 x}$
19. The value of $\sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8}$ is
 (A) 1 (B) 2 (C) $1 \frac{1}{8}$ (D) $2 \frac{1}{8}$
20. The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is
 (A) 2 (B) 3 (C) 4 (D) None of these

Exercise # 2

Part # I

[Multiple Correct Choice Type Questions]

- The value of $\cos \frac{\pi}{10} \cos \frac{2\pi}{10} \cos \frac{4\pi}{10} \cos \frac{8\pi}{10} \cos \frac{16\pi}{10}$ is :
 (A) $\frac{\sqrt{10 + 2\sqrt{5}}}{64}$ (B) $-\frac{\cos(\pi/10)}{16}$ (C) $\frac{\cos(\pi/10)}{16}$ (D) $-\frac{\sqrt{10 + 2\sqrt{5}}}{64}$
- If $x + y = z$, then $\cos^2 x + \cos^2 y + \cos^2 z - 2 \cos x \cos y \cos z$ is equal to
 (A) $\cos^2 z$ (B) $\sin^2 z$ (C) $\cos(x + y - z)$ (D) 1
- Let $m = \tan 3$ & $n = \sec 6$, then which of following statement(s) does/do not hold good ?
 (A) m & n both are positive (B) m & n both are negative
 (C) m is positive & n is negative (D) m is negative & n is positive
- In a triangle $\tan A + \tan B + \tan C = 6$ and $\tan A \tan B = 2$, then the values of $\tan A$, $\tan B$ and $\tan C$ are
 (A) 1, 2, 3 (B) 2, 1, 3 (C) 1, 2, 0 (D) none
- If $\sin \theta + \sqrt{\sin \theta + \sqrt{\sin \theta + \sqrt{\sin \theta + \dots \infty}}} = \sec^4 \alpha$, then $\sin \theta$ is equal to -
 (A) $\sec^2 \alpha \tan^2 \alpha$ (B) $2 \frac{(1 - \cos 2\alpha)}{(1 + \cos 2\alpha)^2}$ (C) $2 \frac{(1 + \cos 2\alpha)}{(1 - \cos 2\alpha)^2}$ (D) $\cot^2 \alpha \operatorname{cosec}^2 \alpha$
- Factors of $\cos 4\theta - \cos 4\phi$ are -
 (A) $(\cos \theta + \cos \phi)$ (B) $(\cos \theta - \cos \phi)$ (C) $(\cos \theta + \sin \phi)$ (D) $(\cos \theta - \sin \phi)$
- If $\cos(A - B) = \frac{3}{5}$ & $\tan A \tan B = 2$, then -
 (A) $\cos A \cos B = \frac{1}{5}$ (B) $\sin A \sin B = -\frac{2}{5}$ (C) $\cos(A + B) = -\frac{1}{5}$ (D) $\sin A \sin B = \frac{2}{5}$
- If $\sqrt{\frac{1 - \sin A}{1 + \sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$, for all permissible values of A , then A belongs to -
 (A) first quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant
- Which of the following when simplified reduces to unity ?
 (A) $\frac{1 - 2 \sin^2 \alpha}{2 \cot\left(\frac{\pi}{4} + \alpha\right) \cos^2\left(\frac{\pi}{4} - \alpha\right)}$ (B) $\frac{\sin(\pi - \alpha)}{\sin \alpha - \cos \alpha \tan \frac{\alpha}{2}} + \cos(\pi - \alpha)$
 (C) $\frac{1}{4 \sin^2 \alpha \cos^2 \alpha} + \frac{(1 - \tan^2 \alpha)^2}{4 \tan^2 \alpha}$ (D) $\frac{1 + \sin 2\alpha}{(\sin \alpha + \cos \alpha)^2}$
- If $A + B = \frac{\pi}{3}$ and $\cos A + \cos B = 1$, then -
 (A) $\cos(A - B) = 1/3$ (B) $|\cos A - \cos B| = \sqrt{\frac{2}{3}}$ (C) $\cos(A - B) = -\frac{1}{3}$ (D) $|\cos A - \cos B| = \frac{1}{2\sqrt{3}}$

11. $f(x) = \frac{\sin x}{\sqrt{1 + \tan^2 x}} + \frac{\cos x}{\sqrt{1 + \cot^2 x}}$ is constant in which of following interval -
 (A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(\frac{3\pi}{2}, 2\pi\right)$
12. For a positive integer n, let $f_n(\theta) = \left(\tan \frac{\theta}{2}\right)(1 + \sec \theta)(1 + \sec 2\theta)(1 + \sec 4\theta) \dots (1 + \sec 2^n \theta)$. Then
 (A) $f_2\left(\frac{\pi}{16}\right) = 1$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$ (C) $f_4\left(\frac{\pi}{64}\right) = 1$ (D) $f_5\left(\frac{\pi}{128}\right) = 1$
13. $(a + 2) \sin \alpha + (2a - 1) \cos \alpha = (2a + 1)$ if $\tan \alpha =$
 (A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{2a}{a^2 + 1}$ (D) $\frac{2a}{a^2 - 1}$
14. If $\tan x = \frac{2b}{a - c}$, ($a \neq c$)
 $y = a \cos^2 x + 2b \sin x \cos x + c \sin^2 x$
 $z = a \sin^2 x - 2b \sin x \cos x + c \cos^2 x$, then
 (A) $y = z$ (B) $y + z = a + c$ (C) $y - z = a - c$ (D) $y - z = (a - c)^2 + 4b^2$
15. The equation $\sin^6 x + \cos^6 x = a^2$ has real solution if
 (A) $a \in (-1, 1)$ (B) $a \in \left(-1, -\frac{1}{2}\right)$ (C) $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $a \in \left(\frac{1}{2}, 1\right)$

Part # II

[Assertion & Reason Type Questions]

Each question has four choices (A), (B), (C) and (D) out of which only one is correct. These questions contains, Statement I (assertion) and Statement II (reason).

- (A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
 (B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.
 (C) Statement-I is true, Statement-II is false.
 (D) Statement-I is false, Statement-II is true.

1. **Statement-I :** $\cos^3 \alpha + \cos^3 \left(\alpha + \frac{2\pi}{3}\right) + \cos^3 \left(\alpha + \frac{4\pi}{3}\right) = 3 \cos \alpha \cos \left(\alpha + \frac{2\pi}{3}\right) \cos \left(\alpha + \frac{4\pi}{3}\right)$
Statement-II : If $a + b + c = 0 \Leftrightarrow a^3 + b^3 + c^3 = 3abc$
2. **Statement-I :** If A is obtuse angle in $\triangle ABC$, then $\tan B \tan C < 1$
Statement-II : In $\triangle ABC$, $\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$
3. **Statement-I :** $\sec^2 \theta = \frac{4xy}{(x+y)^2}$ is positive for all real values of x and y only when $x = y$
Statement-II : $t^2 \geq 0 \forall t \in \mathbb{R}$

4. **Statement-I :** If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^n\theta + \operatorname{cosec}^n\theta = 2^n$.
Statement-II : If $a + b = 2$, $ab = 1$, then $a = b = 1$
5. **Statement-I :** $\tan 5\theta - \tan 3\theta - \tan 2\theta = \tan 5\theta \tan 3\theta \tan 2\theta$
Statement-II : $x = y + z \Rightarrow \tan x - \tan y - \tan z = \tan x \tan y \tan z$.
6. **Statement-I :** If $x + y + z = xyz$, then at most one of the numbers can be negative,
Statement-II : In a triangle ABC, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ and there can be at most one obtuse angle in a triangle.
7. **Statement-I :** $\cos 1 < \sin 1$.
Statement-II : In the first quadrant, cosine decreases but sine increases.
8. Let f be any one of the six trigonometric functions. Let $A, B \in \mathbb{R}$ satisfying $f(2A) = f(2B)$.
Statement-I : $A = n\pi + B$, for some $n \in \mathbb{Z}$.
Statement-II : 2π is one of the period of f .

Exercise # 3

Part # I

[Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one** statement in **Column-II**.

- | column-I | column-II |
|--|--------------------|
| (A) $\operatorname{cosec} 10^\circ - \sqrt{3} \sec 10^\circ =$ | (p) $-\frac{1}{2}$ |
| (B) $4 \cos 20^\circ - \sqrt{3} \cot 20^\circ =$ | (q) -1 |
| (C) $\frac{2 \cos 40^\circ - \cos 20^\circ}{\sin 20^\circ} =$ | (r) $\sqrt{3}$ |
| (D) $2\sqrt{2} \sin 10^\circ \left[\frac{\sec 5^\circ}{2} + \frac{\cos 40^\circ}{\sin 5^\circ} - 2 \sin 35^\circ \right] =$ | (s) 4 |
- | Column - I | Column - II |
|---|-------------|
| (A) If for some real x, the equation $x + \frac{1}{x} = 2 \cos \theta$ holds, then $\cos \theta$ is equal to | (p) 2 |
| (B) If $\sin \theta + \operatorname{cosec} \theta = 2$, then $\sin^{2008} \theta + \operatorname{cosec}^{2008} \theta$ is equal to | (q) 1 |
| (C) Maximum value of $\sin^4 \theta + \cos^4 \theta$ is | (r) 0 |
| (D) Least value of $2 \sin^2 \theta + 3 \cos^2 \theta$ is | (s) -1 |
- | Column - I | Column - II |
|--|-------------|
| (A) $\sin 420^\circ \cos 390^\circ + \cos(-660^\circ) \sin(-330^\circ)$ | (p) 0 |
| (B) $\tan 315^\circ \cot(-405^\circ) + \cot 495^\circ \tan(-585^\circ)$ | (q) 1 |
| (C) The value of $\frac{(1 + \tan 8^\circ)(1 + \tan 37^\circ)}{(1 + \tan 22^\circ)(1 + \tan 23^\circ)} =$ | (r) 2 |
| (D) Value of $\left[\frac{\pi}{4} \right] + \left[\frac{1}{3} \sin^2 x \right]$ is
(where $[.]$ represents greatest integer function) | (s) 5 |

Part # II

[Comprehension Type Questions]

Comprehension # 1

If $\sin \alpha = A \sin(\alpha + \beta)$, $A \neq 0$, then

1. The value of $\tan \alpha$ is

(A) $\frac{A \sin \beta}{1 - A \cos \beta}$ (B) $\frac{A \sin \beta}{1 + A \cos \beta}$ (C) $\frac{A \cos \beta}{1 - A \sin \beta}$ (D) $\frac{A \sin \beta}{1 + A \cos \beta}$

2. The value of $\tan \beta$ is

(A) $\frac{\sin \alpha(1 + A \cos \beta)}{A \cos \alpha \cos \beta}$ (B) $\frac{\sin \alpha \cos \alpha}{A \cos \beta - \sin^2 \alpha}$ (C) $\frac{\cos \alpha(1 - A \cos \beta)}{A \cos \alpha \cos \beta}$ (D) $\frac{\cos \alpha(1 + A \sin \beta)}{A \cos \alpha \cos \beta}$

3. Which of the following is not the value of $\tan(\alpha + \beta)$?

(A) $\frac{\sin \beta}{\cos \beta - A}$ (B) $\frac{\sin \alpha \cos \alpha}{A \cos \beta - \sin^2 \alpha}$ (C) $\frac{\sin \alpha \cos \alpha}{A \cos \beta + \sin^2 \alpha}$ (D) none of these

Comprehension # 2

The measure of an angle in degrees, grades and radians be D, G and C respectively, then the relation between them

$$\frac{D}{90} = \frac{G}{100} = \frac{2C}{\pi} \text{ but } 1^\circ = \left(\frac{180}{\pi}\right)^\circ$$

; $57^\circ, 17', 44.8''$

and sum of interior angles of a n-sided regular polygon is $(2n - 4)\pi/2$

On the basis of above information, answer the following questions :

1. Which of the following are correct -

(A) $\sin 1^\circ < \sin 1$ (B) $\cos 1^\circ > \cos 1$ (C) $\cos 1^\circ < \cos 1$ (D) $\sin 1^\circ < \frac{\pi}{180} \sin 1$

2. The angles between the hour hand and minute hand of a clock at half past three is -

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

3. The number of sides of two regular polygon are as 5 : 4 and the difference between their angles is $\frac{\pi}{20}$, then the number of sides in the polygons respectively are-

(A) 25, 20 (B) 20, 16 (C) 15, 12 (D) 10, 8

4. One angle of a triangle is $\frac{4x}{3}$ grades and another is $3x$ degrees, while the third is $\frac{2\pi x}{75}$ radians. Then the angles in degrees are-

(A) $20^\circ, 60^\circ, 100^\circ$ (B) $24^\circ, 60^\circ, 96^\circ$ (C) $36^\circ, 60^\circ, 84^\circ$ (D) $20^\circ, 40^\circ, 120^\circ$



Comprehension # 3

Continued product $\cos \alpha \cos 2\alpha \cos 2^2\alpha \dots \cos 2^{n-1}\alpha$

$$= \begin{cases} \frac{\sin 2^n \alpha}{2^n \sin \alpha}, & \text{if } \alpha \neq n\pi \\ \frac{1}{2^n}, & \text{if } \alpha = \frac{\pi}{2^n + 1} \quad \text{i.e. } 2^n \alpha = \pi - \alpha \\ -\frac{1}{2^n}, & \text{if } \alpha = \frac{\pi}{2^n - 1} \quad \text{i.e. } 2^n \alpha = \pi + \alpha \end{cases}$$

Where, $n \in \mathbb{I}$ (Integer)

On the basis of above information, answer the following questions :

- The value of $\cos \frac{2\pi}{7} \cos \frac{4\pi}{7} \cos \frac{6\pi}{7}$ is -
 (A) $-1/2$ (B) $1/2$ (C) $1/4$ (D) $1/8$
- If $\alpha = \frac{\pi}{15}$, then the value of $\prod_{r=1}^7 \cos r\alpha$ is -
 (A) $\frac{1}{128}$ (B) $-\frac{1}{128}$ (C) $\frac{1}{64}$ (D) $\frac{1}{32}$
- The value of $\sin\left(\frac{\pi}{14}\right) \sin\left(\frac{3\pi}{14}\right) \sin\left(\frac{5\pi}{14}\right) \sin\left(\frac{7\pi}{14}\right) \sin\left(\frac{9\pi}{14}\right) \sin\left(\frac{11\pi}{14}\right) \sin\left(\frac{13\pi}{14}\right)$ is -
 (A) 1 (B) $\frac{1}{8}$ (C) $\frac{1}{32}$ (D) $\frac{1}{64}$

Exercise # 4

[Subjective Type Questions]

1. If $m \tan(\theta - 30^\circ) = n \tan(\theta + 120^\circ)$, show that $\cos 2\theta = \frac{m+n}{2(m-n)}$.
2. If $\tan \beta = \frac{\tan \alpha + \tan \gamma}{1 + \tan \alpha \cdot \tan \gamma}$, prove that $\sin 2\beta = \frac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \cdot \sin 2\gamma}$.
3. If $\sin x + \sin y = a$ & $\cos x + \cos y = b$, show that,

$$\sin(x+y) = \frac{2ab}{a^2+b^2} \text{ and } \tan \frac{x-y}{2} = \pm \sqrt{\frac{4-a^2-b^2}{a^2+b^2}}.$$
4. If $\sin(\theta + \alpha) = a$ & $\sin(\theta + \beta) = b$ ($0 < \alpha, \beta, \theta < \pi/2$) then find the value of $\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta)$
5. Prove that $\sin 3x \cdot \sin^3 x + \cos 3x \cdot \cos^3 x = \cos^3 2x$.
6. If $\tan \alpha = \frac{p}{q}$ where $\alpha = 6\beta$, α being an acute angle, prove that ; $\frac{1}{2}(p \operatorname{cosec} 2\beta - q \sec 2\beta) = \sqrt{p^2 + q^2}$.
7. Show that:
 (i) $\cot 7\frac{1^\circ}{2} \text{ or } \tan 82\frac{1^\circ}{2} = (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1) \text{ or } \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$
 (ii) $\tan 142\frac{1^\circ}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$.
8. Prove that, $\tan \alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + 8 \cot 8\alpha = \cot \alpha$.
9. Calculate the following without using trigonometric tables:
 (i) $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ (ii) $\operatorname{cosec} 10^\circ - \sqrt{3} \sec 10^\circ$
 (iii) $2\sqrt{2} \sin 10^\circ \left[\frac{\sec 5^\circ}{2} + \frac{\cos 40^\circ}{\sin 5^\circ} - 2 \sin 35^\circ \right]$ (iv) $\cot 70^\circ + 4 \cos 70^\circ$
 (v) $\tan 10^\circ - \tan 50^\circ + \tan 70^\circ$
10. If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = \frac{-3}{2}$, prove that
 $\cos \alpha + \cos \beta + \cos \gamma = 0, \sin \alpha + \sin \beta + \sin \gamma = 0$.
11. Let $A_1, A_2, \dots, A_n$ be the vertices of an n -sided regular polygon such that;

$$\frac{1}{A_1 A_2} = \frac{1}{A_1 A_3} + \frac{1}{A_1 A_4}.$$
 Find the value of n .
12. If $\frac{ax}{\cos \theta} + \frac{by}{\sin \theta} = a^2 - b^2, \frac{ax \sin \theta}{\cos^2 \theta} - \frac{by \cos \theta}{\sin^2 \theta} = 0$.
 Show that $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

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13. If $P_n = \cos^n \theta + \sin^n \theta$ and $Q_n = \cos^n \theta - \sin^n \theta$, then show that
 $P_n - P_{n-2} = -\sin^2 \theta \cos^2 \theta P_{n-4}$ $Q_n - Q_{n-2} = -\sin^2 \theta \cos^2 \theta Q_{n-4}$
and hence show that
 $P_4 = 1 - 2 \sin^2 \theta \cos^2 \theta$ $Q_4 = \cos^2 \theta - \sin^2 \theta$
14. If $A + B + C = \pi$, prove that $\tan B \tan C + \tan C \tan A + \tan A \tan B = 1 + \sec A \cdot \sec B \cdot \sec C$.
15. If $\tan^2 \alpha + 2 \tan \alpha \cdot \tan 2\beta = \tan^2 \beta + 2 \tan \beta \cdot \tan 2\alpha$, then prove that each side is equal to 1 or $\tan \alpha = \pm \tan \beta$.
16. Find the general solution of $\sec 4\theta - \sec 2\theta = 2$.
17. Solve the equation $\cot x - 2 \sin 2x = 1$.
18. Solve the equation $\sin 5x = 16 \sin^5 x$.
19. Solve for x , the equation $\sqrt{13 - 18 \tan x} = 6 \tan x - 3$, where $-2\pi < x < 2\pi$.
20. If α & β are two distinct roots of the equation $a \tan \theta + b \sec \theta = c$, then prove that $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$.
21. Solve the equation for $0 \leq \theta \leq 2\pi$; $(\sin 2\theta + \sqrt{3} \cos 2\theta)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$.
22. If $\tan \theta + \sin \phi = \frac{3}{2}$ & $\tan^2 \theta + \cos^2 \phi = \frac{7}{4}$, then find the general value of θ & ϕ .
23. If α & β satisfy the equation $a \cos 2\theta + b \sin 2\theta = c$ then prove that: $\cos^2 \alpha + \cos^2 \beta = \frac{a^2 + ac + b^2}{a^2 + b^2}$.
 $\cos^2 \alpha + \cos^2 \beta = \frac{a^2 + ac + b^2}{a^2 + b^2}$.
24. Solve the equation $3 - 2\cos \theta - 4 \sin \theta - \cos 2\theta + \sin 2\theta = 0$
25. Solve the equation $1 + 2 \operatorname{cosec} x = -\frac{\sec^2 \frac{x}{2}}{2}$.
26. Solve the equation $\sin^2 4x + \cos^2 x = 2 \sin 4x \cdot \cos^4 x$
27. Solve the equation $2 \sin x = 3x^2 + 2x + 3$.

Exercise # 5

Part # I

[Previous Year Questions] [AIEEE/JEE-MAIN]

1. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to
 (1) $\frac{24}{25}$ (2) $-\frac{24}{25}$ (3) $\frac{13}{18}$ (4) $-\frac{13}{18}$ [AIEEE 2002]
2. The upper $\left(\frac{3}{4}\right)$ th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is-
 (1) 20 m (2) 40 m (3) 60 m (4) 80 m [AIEEE 2003]
3. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 m away from the tree, the angle of elevation becomes 30° . The breadth of the river is-
 (1) 20 m (2) 30 m (3) 40 m (4) 60 m [AIEEE 2004]
4. If $u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$, then the difference between the maximum and minimum values of u^2 is given by
 (1) $2(a^2 + b^2)$ (2) $2\sqrt{a^2 + b^2}$ (3) $(a + b)^2$ (4) $(a - b)^2$ [AIEEE 2004]
5. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$, then the value of $\cos\left(\frac{\alpha - \beta}{2}\right)$ is
 (1) $\frac{-3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$ (3) $\frac{6}{65}$ (4) $\frac{-6}{65}$ [AIEEE 2004]
6. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$; $a \neq 0$ then
 (1) $b = a + c$ (2) $b = c$ (3) $c = a + b$ (4) $a = b + c$ [AIEEE 2005]
7. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is
 (1) $\frac{4 - \sqrt{7}}{3}$ (2) $-\left(\frac{4 + \sqrt{7}}{3}\right)$ (3) $\frac{1 + \sqrt{7}}{4}$ (4) $\frac{1 - \sqrt{7}}{4}$ [AIEEE 2006]
8. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2 \sin^2 x + 5 \sin x - 3 = 0$ is
 (1) 6 (2) 1 (3) 2 (4) 4 [AIEEE 2006]

9. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is [AIEEE 2006]
 (1) $\frac{(4-\sqrt{7})}{3}$ (2) $-\frac{(4+\sqrt{7})}{3}$ (3) $\frac{(1+\sqrt{7})}{4}$ (4) $\frac{(1-\sqrt{7})}{4}$
10. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB (= a)$ subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30° . The height of the tower is- [AIEEE 2007]
 (1) $\frac{2a}{\sqrt{3}}$ (2) $2a\sqrt{3}$ (3) $\frac{a}{\sqrt{3}}$ (4) $\sqrt{3}$
11. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60° . He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45° . Then the height of the pole is- [AIEEE 2008]
 (1) $\frac{7\sqrt{3}}{2} \left(\frac{1}{\sqrt{3}+1} \right)$ m (2) $\frac{7\sqrt{3}}{2} \left(\frac{1}{\sqrt{3}-1} \right)$ m (3) $\frac{7\sqrt{3}}{2} (\sqrt{3}+1)$ m (4) $\frac{7\sqrt{3}}{2} (\sqrt{3}-1)$ m
12. Let A and B denote the statements
 A : $\cos \alpha + \cos \beta + \cos \gamma = 0$
 B : $\sin \alpha + \sin \beta + \sin \gamma = 0$
 If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then : [AIEEE 2009]
 (1) A is false and B is true (2) both A and B are true
 (3) both A and B are false (4) A is true and B is false
13. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha =$ [AIEEE 2010]
 (1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$
14. If $A = \sin^2 x + \cos^4 x$, then for all real x : [AIEEE 2011]
 (1) $\frac{3}{4} \leq A \leq 1$ (2) $\frac{13}{16} \leq A \leq 1$ (3) $1 \leq A \leq 2$ (4) $\frac{3}{4} \leq A \leq \frac{13}{16}$
15. In a ΔPQR , if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to : [AIEEE 2012]
 (1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$
16. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to : [AIEEE 2013]
 (1) $\frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$ (2) $\frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$ (3) $\frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$ (4) $\frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$

17. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as : [JEE MAIN 2013]
 (1) $\sin A \cos A + 1$ (2) $\sec A \operatorname{cosec} A + 1$ (3) $\tan A + \cot A$ (4) $\sec A + \operatorname{cosec} A$
18. If $f_k(x) = \frac{1}{k} (\sin^k x + \cos^k x)$, where $x \in \mathbb{R}, k \geq 1$, then $f_4(x) - f_6(x)$ is equal to [JEE MAIN 2014]
 (1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$
19. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45° . It flies off horizontally straight away from the point O. After 1 s, the elevation of the bird from O is reduced to 30° . Then, the speed (in m/s) of the bird is [JEE MAIN 2014]
 (1) $40(\sqrt{2} - 1)$ (2) $40(\sqrt{3} - \sqrt{2})$ (3) $20\sqrt{2}$ (4) $20(\sqrt{3} - 1)$
20. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are $30^\circ, 45^\circ$ and 60° respectively, then the ratio, AB : BC, is : [JEE MAIN 2015]
 (1) $1 : \sqrt{3}$ (2) $2 : 3$ (3) $\sqrt{3} : 1$ (4) $\sqrt{3} : \sqrt{2}$
21. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30° . After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60° . Then the time taken (in minutes) by him, from B to reach the pillar, is : [JEE MAIN 2016]
 (1) 10 (2) 20 (3) 5 (4) 6

Part # II

[Previous Year Questions][IIT-JEE ADVANCED]

1. The maximum value of $(\cos \alpha_1)(\cos \alpha_2) \dots (\cos \alpha_n)$ under the restrictions $0 \leq \alpha_1, \alpha_2, \dots, \alpha_n \leq \pi/2$ and $(\cot \alpha_1)(\cot \alpha_2) \dots (\cot \alpha_n) = 1$ is [IIT JEE 2001]
 (A) $1/2^{n/2}$ (B) $1/2^n$ (C) $1/2n$ (D) 1
2. If $\sin \alpha = 1/2$ and $\cos \theta = 1/3$, then the values of $\alpha + \theta$ (if θ, α are both acute) will lie in the interval [IIT JEE 2004]
 (A) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$ (C) $\left[\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left[\frac{5\pi}{6}, \pi\right]$
3. Find the range of values of 't' for which $2 \sin t = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. [IIT JEE 2005]
4. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan \theta)^{\tan \theta}$, $t_2 = (\tan \theta)^{\cot \theta}$, $t_3 = (\cot \theta)^{\tan \theta}$ and $t_4 = (\cot \theta)^{\cot \theta}$, then [IIT JEE 2006]
 (A) $t_1 > t_2 > t_3 > t_4$ (B) $t_2 < t_1 < t_3 < t_4$ (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

5. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then [IIT JEE 2009]
- (A) $\tan^2 x = \frac{2}{3}$ (B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
- (C) $\tan^2 x = \frac{1}{3}$ (D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$
6. The maximum value of the expression $\frac{1}{\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta}$ is [IIT JEE 2010]
7. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is [IIT JEE 2011]
8. Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and $Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then [IIT JEE 2011]
- (A) $P \subset Q$ and $Q - P \neq \emptyset$ (B) $Q \not\subset P$
- (C) $P \not\subset Q$ (D) $P = Q$
9. Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2} \right) \cos\phi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy [IIT JEE 2012]
- (A) $0 < \phi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \phi < 2\pi$
10. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$, then possible value of $\cos \frac{x-y}{2}$ is [JEE Ad. 2013]
11. If $\cos\left(\frac{\pi}{4} - x\right) \cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right) \cos 2x$, then possible value of $\sec x$ is [JEE Ad. 2013]
12. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals [JEE Ad. 2016]
- (A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0
13. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to [JEE Ad. 2016]
- (A) $3 - \sqrt{3}$ (B) $2(3 - \sqrt{3})$ (C) $2(\sqrt{3} - 1)$ (D) $2(2 + \sqrt{3})$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If $2 \cos x + \sin x = 1$, then value of $7 \cos x + 6 \sin x$ is equal to
 (A) 2 or 6 (B) 1 or 3 (C) 2 or 3 (D) None of these
2. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is equal to
 (A) $\frac{4}{3}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) 3
3. Value of $\frac{3 + \cot 80^\circ \cot 20^\circ}{\cot 80^\circ + \cot 20^\circ}$ is equal to
 (A) $\cot 20^\circ$ (B) $\tan 50^\circ$ (C) $\cot 50^\circ$ (D) $\cot \sqrt{20^\circ}$
4. The value of $\cos \frac{\pi}{19} + \cos \frac{3\pi}{19} + \cos \frac{5\pi}{19} + \dots + \cos \frac{17\pi}{19}$ is equal to:
 (A) $1/2$ (B) 0 (C) 1 (D) None
5. If $\sin x + \cos x = \sqrt{2} \cos x$, then $\cos x - \sin x$ is equal to
 (A) $\sqrt{2} \cos x$ (B) $-\sqrt{2} \cos x$ (C) $\sqrt{2} \sin x$ (D) $-\sqrt{2} \sin x$
6. If $f(\theta) = \sin^2 \theta + \sin^2 \left(\theta + \frac{2\pi}{3} \right) + \sin^2 \left(\theta + \frac{4\pi}{3} \right)$, then $f\left(\frac{\pi}{15}\right)$ is equal to
 (A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$
7. If $\sin 2\beta$ is the geometric mean between $\sin \alpha$ and $\cos \alpha$, then $\cos 4\beta$ is equal to
 (A) $2 \sin^2 \left(\frac{\pi}{4} - \alpha \right)$ (B) $2 \cos^2 \left(\frac{\pi}{4} - \alpha \right)$ (C) $2 \cos^2 \left(\frac{\pi}{2} + \alpha \right)$ (D) $2 \sin^2 \left(\frac{\pi}{4} + \alpha \right)$
8. If $0^\circ < x < 90^\circ$ & $\cos x = \frac{3}{\sqrt{10}}$, then the value of $\log_{10} \sin x + \log_{10} \cos x + \log_{10} \tan x$ is
 (A) 0 (B) 1 (C) -1 (D) None of these
9. If $\sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$, then the value of $1 + \cot \alpha \tan \beta$ is
 (A) 1 (B) -1 (C) 2 (D) none of these
10. If $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ and $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}$, $0 < A, B < \pi/2$, then $\tan A + \tan B$ is equal to
 (A) $\sqrt{3} / \sqrt{5}$ (B) $\sqrt{5} / \sqrt{3}$ (C) 1 (D) $(\sqrt{5} + \sqrt{3}) / \sqrt{5}$

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $2 \sec^2 \alpha - \sec^4 \alpha - 2 \operatorname{cosec}^2 \alpha + \operatorname{cosec}^4 \alpha = 15/4$, then $\tan \alpha$ is equal to
 (A) $1/\sqrt{2}$ (B) $1/2$ (C) $1/2\sqrt{2}$ (D) $-1/\sqrt{2}$
12. If $3 \sin \beta = \sin(2\alpha + \beta)$, then $\tan(\alpha + \beta) - 2 \tan \alpha$ is
 (A) independent of α (B) independent of β
 (C) dependent of both α and β (D) independent of α but dependent of β
13. Which of following functions have the maximum value unity ?
 (A) $\sin^2 x - \cos^2 x$ (B) $\sqrt{\frac{6}{5}} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{3}} \cos x \right)$
 (C) $\cos^6 x + \sin^6 x$ (D) $\cos^2 x + \sin^4 x$
14. If $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$, then
 (A) A, B, C may be angles of a triangle (B) $A + B + C$ is an integral multiple of π
 (C) sum of any two of A, B, C is equal to third (D) none of these
15. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if
 (A) $\cos 12x = \cos 14x$ (B) $\sin 13x = 0$ (C) $\sin x = 0$ (D) $\cos x = 0$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** $\sin 2 > \sin 3$
Statement-II : If $x, y \in \left(\frac{\pi}{2}, \pi\right)$, $x < y$, then $\sin x > \sin y$
 (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True
17. **Statement-I :** The number of integral values of λ , for which the equation $7 \cos x + 5 \sin x = 2\lambda + 1$ has a solution, is 8
Statement-II : $a \cos \theta + b \sin \theta = c$ has atleast one solution if $|c| > \sqrt{a^2 + b^2}$
 (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True
18. **Statement-I :** The maximum value of $\sin \theta + \cos \theta$ is 2
Statement-II : The maximum value of $\sin \theta$ is 1 and that of $\cos \theta$ is also 1.
 (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True

19. **Statement-I** : If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^n\theta + \operatorname{cosec}^n\theta = 2^n$.

Statement-II : If $a + b = 2$, $ab = 1$, then $a = b = 1$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. Let $\alpha, \beta, \gamma > 0$ and $\alpha + \beta + \gamma = \frac{\pi}{2}$.

Statement-I : $\left| \tan\alpha \tan\beta - \frac{a!}{6} \right| + \left| \tan\beta \tan\gamma - \frac{b!}{2} \right| + \left| \tan\gamma \tan\alpha - \frac{c!}{3} \right| \leq 0$, where $n! = 1.2 \dots n$, then $\tan\alpha \tan\beta, \tan\beta \tan\gamma, \tan\gamma \tan\alpha$ are in A.P.

Statement-II : $\tan\alpha \tan\beta + \tan\beta \tan\gamma + \tan\gamma \tan\alpha = 1$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column I

(A) The number of solutions of the equation

$$|\cot x| = \cot x + \frac{1}{\sin x} \quad (0 < x < \pi) \text{ is}$$

(B) If $\sin\theta + \sin\phi = \frac{1}{2}$ and $\cos\theta + \cos\phi = 2$, then value of

$$\cot\left(\frac{\theta + \phi}{2}\right) \text{ is}$$

(C) The value of $\sin^2\alpha + \sin\left(\frac{\pi}{3} - \alpha\right) \sin\left(\frac{\pi}{3} + \alpha\right)$ is

(D) If $\tan\theta = 3\tan\phi$, then maximum value of $\tan^2(\theta - \phi)$ is

Column II

(p) No solution

(q) $\frac{1}{3}$

(r) 1

(s) 2

(t) 4

22.

Column I

Column II

(A) If maximum and minimum values of $\frac{7 + 6 \tan \theta - \tan^2 \theta}{(1 + \tan^2 \theta)}$ for

(p) $\lambda + \mu = 2$

all real values of $\theta \sim \frac{\pi}{2}$ are λ and μ respectively, then

(B) If maximum and minimum values of

(q) $\lambda - \mu = 6$

$5 \cos \theta + 3 \cos \left(\theta + \frac{\pi}{3} \right) + 3$ for all real values of θ are

λ and μ respectively, then

(r) $\lambda + \mu = 6$

(C) If maximum and minimum values of

$1 + \sin \left(\frac{\pi}{4} + \theta \right) + 2 \cos \left(\frac{\pi}{4} - \theta \right)$ for all real values of θ are λ

(s) $\lambda - \mu = 10$

and μ respectively, then

(t) $\lambda - \mu = 14$

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

If $P_n = \sin^n \theta + \cos^n \theta$ where $n \in \mathbb{W}$ (whole number) and $\theta \in \mathbb{R}$ (real number)

1. If $P_1 = m$, then the value of $4(1 - P_6)$ is

(A) $3(m-1)^2$

(B) $3(m^2-1)^2$

(C) $3(m+1)^2$

(D) $3(m^2+1)^2$

2. The value of $2P_6 - 3P_4 + 10$ is

(A) 0

(B) 6

(C) 9

(D) 15

3. The value of $6P_{10} - 15P_8 + 10P_6 + 7$ is

(A) 8

(B) 6

(C) 4

(D) 2

24. Read the following comprehension carefully and answer the questions.

If θ is an angle which measured in radian and $\theta \in [0, 2\pi]$, then $r\theta$ is length of arc AB, of circle of radius r , subtending angle θ at the centre O, of the circle. Area of sector OAB is $\frac{1}{2} r^2 \theta$.

1. The angle between minute hand and hour hand of a clock at "half past 4" equals

(A) 42°

(B) 43°

(C) 44°

(D) none of these

2. The wheel of a train is 1 meter in diameter and it makes 5 revolutions per second. Then the speed of the train is approximately equal to

(A) 57 km/hr

(B) 66 km/hr

(C) 68 km/hr

(D) 42.6 km/hr.



3. Two lines drawn through a point on the circumference of a circle divide the circle into three regions of equal area. Then the angle θ between the lines is given by
 (A) $3\theta + 3 \sin \theta = \pi$ (B) $6\theta + 3 \sin \theta = \pi$ (C) $2\theta + \sin \theta = \pi$ (D) $\theta + \sin \theta = \pi/2$

25. Read the following comprehension carefully and answer the questions.

Given $\cos 2^m \theta \cos 2^{m+1} \theta \dots \cos 2^n \theta = \frac{\sin 2^{n+1} \theta}{2^{n-m+1} \sin 2^m \theta}$, where $2^m \theta \neq k\pi$, $n, m, k \in \mathbb{I}$

Solve the following :

1. $\sin \frac{9\pi}{14} \cdot \sin \frac{11\pi}{14} \sin \frac{13\pi}{14}$ is equal to
 (A) $\frac{1}{64}$ (B) $-\frac{1}{64}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$
2. $\cos 2^3 \frac{\pi}{10} \cos 2^4 \frac{\pi}{10} \cos 2^5 \frac{\pi}{10} \dots \cos 2^{10} \frac{\pi}{10}$ is equal to
 (A) $\frac{1}{128}$ (B) $\frac{1}{256}$ (C) $\frac{1}{512} \sin \frac{\pi}{10}$ (D) $\frac{\sqrt{5}-1}{512} \sin \frac{3\pi}{10}$
3. $\cos \frac{\pi}{11} \cos \frac{2\pi}{11} \cos \frac{3\pi}{11} \dots \cos \frac{11\pi}{11}$ is equal to
 (A) $-\frac{1}{32}$ (B) $\frac{1}{512}$ (C) $\frac{1}{1024}$ (D) $-\frac{1}{2048}$

SECTION - VI : INTEGER TYPE

26. In a triangle ABC, if $\sin 10A + \sin 10B + \sin 10C = \lambda \sin 5A \sin 5B \sin 5C$, then find the value of λ .
27. Find the absolute value of the expression $\tan \frac{\pi}{16} + \tan \frac{5\pi}{16} + \tan \frac{9\pi}{16} + \tan \frac{13\pi}{16}$.
28. If $\tan \beta = \frac{\tan \alpha + \tan \gamma}{1 + \tan \alpha \tan \gamma}$ and $\sin 2\beta = \frac{\sin 2\alpha + \sin 2\gamma}{\lambda + \sin 2\alpha \sin 2\gamma}$, then find the value of λ .
29. The value of $64\sqrt{3} \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6}$ is
30. Let $f(\theta) = \frac{1}{1 + (\cos \theta)^x}$ and $S = \sum_{\theta=1^\circ}^{89^\circ} f(\theta)$, then the value of $\sqrt{2S-8}$ is

ANSWER KEY

EXERCISE - 1

1. B 2. B 3. D 4. A 5. B 6. A 7. A 8. B 9. A 10. C 11. D 12. C 13. B
14. A 15. D 16. A 17. B 18. D 19. B 20. C

EXERCISE - 2 : PART # I

1. BD 2. CD 3. ABC 4. AB 5. AB 6. ABCD 7. ACD 8. AD 9. ABD 10. BC 11. BD
12. ABCD 13. BD 14. BD 15. BD

PART - II

1. C 2. A 3. B 4. D 5. A 6. D 7. B 8. A

EXERCISE - 3 : PART # I

1. $A \rightarrow s$ $B \rightarrow q$ $C \rightarrow r$ $D \rightarrow s$ 2. $A \rightarrow q, s$ $B \rightarrow p$ $C \rightarrow q$ $D \rightarrow p$ 3. $A \rightarrow q$ $B \rightarrow r$ $C \rightarrow q$ $D \rightarrow p$

PART - II

Comprehension # 1 : 1. A 2. B 3. C Comprehension # 2 : 1. A, B 2. C 3. D 4. B

Comprehension # 3 : 1. D 2. A 3. D

EXERCISE - 5 : PART # I

1. 2 2. 1 3. 4 4. 1 5. 3 6. 2 7. 4 8. 2 9. 3 10. 3 11. 2 12. 1 13. 1
14. 2 15. 1 16. 2 17. 2 18. 4 19. 4 20. 3 21. 3

PART - II

1. B 2. $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ 3. B 4. D 5. B 6. A 7. C 8. A 9. 2
10. $n=7$ 11. 3 12. D 13. A, C, D

MOCK TEST

1. A 2. C 3. B 4. A 5. C 6. B 7. A 8. C 9. D
10. D 11. A, D 12. A, B 13. A, B, C, D 14. AB 15. ABCD 16. A 17. C 18. D
19. D 20. D 21. $A \rightarrow r$ $B \rightarrow p$ $C \rightarrow p$ $D \rightarrow q$ 22. $A \rightarrow r, s$ $B \rightarrow p, t$ $C \rightarrow p, q$
23. 1. B 2. C 3. A 24. 1. D 2. A 3. A 25. 1. C 2. B 3. C
26. 4 27. 4 28. 29. 6 30. 9