

HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $u(x) = 7v(x) \Rightarrow u'(x) = 7v'(x) \Rightarrow p = 7$ (given)

again $\frac{u(x)}{v(x)} = 7 \Rightarrow \left(\frac{u(x)}{v(x)} \right)' = 0 \Rightarrow q = 0;$

now $\frac{p+q}{p-q} = \frac{7+0}{7-0} = 1$

4. multiply Numerator and Denominator by x^m, x^n and x^p respectively

5. Integrating both sides we gets $f^3(x) = x^2 + C ; f(2) = 1$
 $\Rightarrow C = -3$

$f^3(x) = x^2 - 3 \Rightarrow f^3(3) = 6 \Rightarrow f(3) = \sqrt[3]{6}$

6. $f[g(x)] = x \Rightarrow f'[g(x)] \cdot [g'(x)] = 1$
 $\Rightarrow f'(A) \cdot g'(2) = 1$ [putting $x = 2$]

given, $f'(A) = \frac{a^{10}}{1+a^2}$

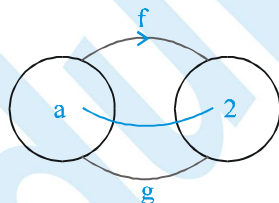
$\Rightarrow g'(2) = \frac{1+a^2}{a^{10}}$

Alternative $g[f(x)] = x$

$g'[f(x)] \cdot f'(x) = 1$

now $g(2) = a \Rightarrow f(A) = 2$

$\therefore g$ and f are inverse of each other



now $f(x) = 2 \Rightarrow g(2) = x = a$

$\therefore g'(2) \cdot f'(A) = 1$

$g'(2) = \frac{1}{f'(a)} = \frac{1+a^2}{a^{10}}$

8. $f'(x) \cdot f'(x) - f(x) \cdot f''(x) = 0$

or $\frac{(f'(x))^2 - f(x)f''(x)}{(f'(x))^2} = 0$

$\frac{d}{dx} \left[\frac{f(x)}{f'(x)} \right] = 0$

Integrating, $\frac{f(x)}{f'(x)} = C$ (i)

put $x = 0$, $\frac{f(0)}{f'(0)} = C \Rightarrow C = \frac{1}{2}$,

Hence $\frac{f(x)}{f'(x)} = \frac{1}{2}$

from (1) $2f(x) = f'(x)$

$\therefore \frac{f'(x)}{f(x)} = 2$

again Integrating $\ln[f(x)] = 2x + k$

put $x = 0$ to get $k = 0$

$f(x) = e^{2x}$

9. $g(x) = f(-x + f(f(x)))$; $f(0) = 0; f'(0) = 2$

$g'(x) = f'(-x + f(f(x))) \cdot [-1 + f'(f(x)) \cdot f'(x)]$

$g'(0) = f'(f(0)) \cdot [-1 + f'(0) \cdot f'(0)]$

$= f'(0) [-1 + (2)(2)]$

$= (2)(3) = 6$

10. $f(x) = (x-1)^{100} \cdot (x-2)^{2.99} \cdot (x-3)^{3.98} \dots (x-100)^{100.1}$

Take log & then differentiate we get

Now $\frac{f'(x)}{f(x)} = \frac{1.100}{x-1} + \frac{2.99}{x-2} + \frac{3.98}{x-3} + \dots + \frac{100.1}{x-100}$

$\frac{f'(101)}{f(101)} = 1 + 2 + 3 + \dots + 100 = 5050$

$\therefore \frac{f'(101)}{f(101)} = \frac{1}{5050}$

11. $f'(0) = \lim_{h \rightarrow 0} \frac{g(0+h)\cos(1/h) - 0}{h} = \lim_{h \rightarrow 0} \frac{g(h)\cos(1/h)}{h}$

$= \lim_{h \rightarrow 0} \frac{g(h)}{h} \cdot \lim_{h \rightarrow 0} \cos \frac{1}{h} = g'(0) \lim_{h \rightarrow 0} \cos \frac{1}{h} = 0$

$g'(x) = -g'(x) \Rightarrow g'(0) = 0$



12. $h''(x) = 0$

13. $x^{\left(\frac{1+m}{(m-n)(n-1)} + \frac{m+n}{(n-1)(1-m)} + \frac{n+1}{(1-m)(m-n)}\right)} = x^{\left(\frac{1^2-m^2+m^2-n^2+n^2-1^2}{(m-n)(n-1)(1-m)}\right)}$
 $= x^0 = 1 \quad \therefore \frac{d}{dx}(1) = 0$

14. $f'(x) = \frac{1}{x} + \pi \cos(\pi x) + C$

$f'(2) = \frac{1}{2} + \pi + C = \frac{1}{2} + \pi \Rightarrow C = 0$

$f(x) = \ln|x| + \sin(\pi x) + C'$

$f(1) = C' = 0$

$f(x) = \ln|x| + \sin(\pi x)$

15. Put $\cos \phi = \frac{2}{\sqrt{13}}$; $\sin \phi = \frac{3}{\sqrt{13}}$; $\tan \phi = \frac{3}{2}$

$y = \cos^{-1}\{\cos(x+\phi)\} + \sin^{-1}\{\cos(x-\phi)\}$

$= \cos^{-1}\{\cos(x+\phi)\} + \frac{\pi}{2} - \cos^{-1}\{\cos(\phi-x)\}$ (think !)

$= x + \phi + \frac{\pi}{2} - \phi - x$

$y = 2x + \frac{\pi}{2}$; $z = \sqrt{1+x^2}$

now compute $\frac{dy}{dz}$

18. $D^*f(x) = 2f(x).f'(x)$

$D^*(x \ln x) = 2x \ln x (1 + \ln x)$

19. between two consecutive roots of $f'(x) = 0$ the curve can cut the axis of x at most once
 i.e. may cut or may not cut

21. $f(x) = \sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}$

$\therefore f(x) = \sqrt{(\sqrt{x-2} + \sqrt{2})^2} + \sqrt{(\sqrt{x-2} - \sqrt{2})^2}$

$= |\sqrt{x-2} + \sqrt{2}| + |\sqrt{x-2} - \sqrt{2}|$

for $\sqrt{x-2}$ to exist $x \geq 2$

Also, $\sqrt{x-2} + \sqrt{2} > 0$ (always true, think ! why ?)

but $\sqrt{x-2} - \sqrt{2} \geq 0$ only if $x \geq 4$
 < 0 only if $x < 4$

$\therefore$ now $f(x)$ becomes

$f(x) = \sqrt{x-2} + \sqrt{2} - \sqrt{x-2} + \sqrt{2}$

for $2 \leq x < 4$

$= \sqrt{x-2} + \sqrt{2} + \sqrt{x-2} - \sqrt{2}$

for $x \geq 4$

$\therefore f(x) = 2\sqrt{2}$, for $2 \leq x < 4$

$= 2\sqrt{x-2}$, for $4 \leq x < \infty$

→ f is continuous $[2, 4) \cup [4, \infty)$ (verify)

$\therefore f'(x) = 0, 2 \leq x < 4$

$= \frac{1}{\sqrt{x-2}}$, $4 \leq x < \infty$

$\therefore f'(102_+) = \frac{1}{\sqrt{102-2}} = \frac{1}{10}$

$\therefore 10 f'(102_+) = 1$

22. Let $f(x) = px^2 + qx + r$

$f(1) = f(-1)$ gives $p + q + r = p - q + r$

hence $q = 0$

Hence $f(x) = px^2 + r$

$f'(x) = 2px$ (i)

Given a, b, c are in A.P.

hence $2pa, 2pb, 2pc$ will also be in A.P.

or $f'(A), f'(B), f'(C)$ will also be in A.P. $\Rightarrow$ (D)

23. $2x + 2yy' = 0$

$x + yy' = 0 \Rightarrow y' = -\frac{x}{y}$ (i)

$1 + yy'' + (y')^2 = 0$

$y'' = -\frac{1 + (y')^2}{y}$

now $k = \frac{y''}{(1 + (y')^2)^{3/2}} = -\frac{1 + (y')^2}{y(1 + (y')^2)^{3/2}}$
 $= -\frac{1}{y\sqrt{1 + (y')^2}} = -\frac{1}{y\sqrt{1 + \frac{x^2}{y^2}}} = -\frac{1}{\sqrt{y^2 + x^2}} = -\frac{1}{R}$

24. $y = \frac{x}{a + \frac{x}{b+y}} \Rightarrow y = \frac{x(b+y)}{ab + ay + x}$

$\Rightarrow aby + ay^2 + xy = xb + xy$

$\Rightarrow ab \frac{dy}{dx} + 2ay \frac{dy}{dx} = b$

$\Rightarrow \frac{dy}{dx} = \frac{b}{ab + 2ay}$



25. $n = 2$ or 0 only

26. Degree of $f(x) = n$; degree of $f'(x) = n - 1$
degree of $f''(x) = (n - 2)$

hence $n = (n - 1) + (n - 2) = 2n - 3$

$\therefore n = 3$

hence $f(x) = ax^3 + bx^2 + cx + d, (a \neq 0)$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$\therefore ax^3 + bx^2 + cx + d = (3ax^2 + 2bx + c)(6ax + 2b)$

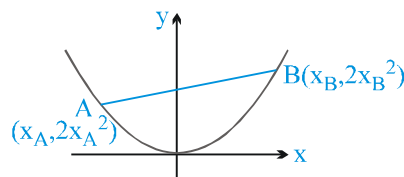
$\therefore 18a^2 = a \Rightarrow a = \frac{1}{18}$

28. We have $y = 2x^2$

$(AB)^2 = (x_B - x_A)^2 + (2x_B^2 - 2x_A^2)^2 = 5$

or $(x_B - x_A)^2 + 4(x_B^2 - x_A^2)^2 = 5$

differentiating w.r.t. x_A and denoting $\frac{dx_B}{dx_A} = D$



$2(x_B - x_A)(D - 1) + 8(x_B^2 - x_A^2)(2x_B D - 2x_A) = 0$

put $x_A = 0$; $x_B = 1$

$2(1 - 0)(D - 1) + 8(1 - 0)(2D - 0) = 0$

$2D - 2 + 16D = 0 \Rightarrow D = 1/9$

30. for objective note that in y highest degree of x is 4 and

therefore $\frac{d^3y}{dx^3}$ is a linear function of x .

Which satisfies only in (D)

31. $y = f(x) - f(2x)$

$y' = f'(x) - 2f'(2x)$

$y'(1) = f'(1) - 2f'(2) = 5$ (i)

and $y'(2) = f'(2) - 2f'(4) = 7$ (ii)

now let $y = f(x) - f(4x)$

$y' = f'(x) - 4f'(4x)$

$y'(1) = f'(1) - 4f'(4)$ (iii)

Substituting the value of $f'(2) = 7 + 2f'(4)$ in (i)

$f'(1) - 2[7 + 2f'(4)] = 5$

$f'(1) - 4f'(4) = 19 \Rightarrow$ (A)

34. $y = 3e^2 - x$

let $x^y = X^{3e^2 - x}$

$f(x) = X^{3e^2 - x}$

$\ln(f(x)) = (3e^2 - x) \ln x$

$\frac{1}{f(x)} \cdot f'(x) = \frac{3e^2 - x}{x} - \ln x$

$\therefore f'(x) = 0$

$\Rightarrow 3e^2 - x = x \ln x$

$\Rightarrow 3e^2 = x(1 + \ln x)$

$\Rightarrow x = e^2$ (by verification)

35. $f(x) = x^n$

$\therefore f'(x) = nx^{n-1}, f''(x) = n(n-1)x^{n-2}$

$f^{(n)}(x) = n!$

Now $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \dots - \frac{f^{(n)}(1)}{n!}$

$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \dots - \frac{n!}{n!} = (1-1)^n = 0$

EXERCISE - 2

Part # I : Multiple Choice

1. $y = e^{-x}$ & $y = e^{-x} \sin x$
 $y' = -e^{-x}$... (i) & $y' = -e^{-x}(\sin x - \cos x)$... (ii)
 equating (i) & (ii)

$$e^{-x}(1 - \sin x + \cos x) = 0$$

$$e^{-x} \neq 0$$

$$\Rightarrow 1 - \sin x + \cos x = 0$$

$$\Rightarrow 2\cos^2 \frac{x}{2} = 2\sin \frac{x}{2} \cos \frac{x}{2}$$

$$\Rightarrow 2\cos \frac{x}{2} \left(\sin \frac{x}{2} - \cos \frac{x}{2} \right) = 0$$

$$\Rightarrow x = \frac{\pi}{2}, \pi$$

$$\text{slope can be } -e^{-\pi/2} \text{ \& } -e^{-\pi}.$$

2. $x = \cos t, y = \sin t$

$$\frac{dy}{dx} = \frac{1}{t} \cdot \frac{1}{-\sin t}$$

$$\text{at } t = \frac{\pi}{2} \quad \left| \quad \text{at } t = \frac{\pi}{6} \right.$$

$$\frac{dy}{dx} = \frac{-2}{\pi} \quad \frac{dy}{dx} = -\frac{12}{\pi}$$

3. square both sides, differentiate and rationalise

5. $\rightarrow t = \frac{1}{2} \sin(x^2 + y^2)$

$$\Rightarrow \frac{1}{2} \sin(x^2 + y^2) = \sin^{-1} \left(\frac{y}{\sqrt{x^2 + y^2}} \right)$$

Case-I : When $x \geq 0$

$$\Rightarrow \sin(x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x} \right)$$

$$\Rightarrow \frac{1}{x^2 + y^2} (2x + 2yy') = \frac{2}{1 + \frac{y^2}{x^2}} \left(\frac{xy' - y}{x^2} \right)$$

$$\Rightarrow xy' - yy' = x + y \Rightarrow y' = \frac{x + y}{x - y}$$

let $y = x \tan \theta$; $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$

$$\Rightarrow \sin^{-1} \left(\frac{y}{\sqrt{x^2 + y^2}} \right) = \sin^{-1} \left(\frac{x \tan \theta}{x \sec \theta} \right) = \sin^{-1}$$

$$\left(\frac{x}{|x|} \sin \theta \right) = \begin{cases} \theta = \tan^{-1} \left(\frac{y}{x} \right), & x \geq 0 \\ -\theta = -\tan^{-1} \left(\frac{y}{x} \right), & x < 0 \end{cases}$$

Case-II : When $x < 0$

$$\bullet \sin(x^2 + y^2) = -2 \tan^{-1} \left(\frac{y}{x} \right)$$

$$\Rightarrow \frac{(2x + 2yy')}{x^2 + y^2} = \frac{-2}{1 + \frac{y^2}{x^2}} \left(\frac{xy' - y}{x^2} \right)$$

$$y'(x + y) = y - x$$

$$y' = \frac{y - x}{y + x}$$

9. $f^{-1}(x) = g(x) \Rightarrow x = f(g(x))$

Differentiating both sides,

$$1 = f'(g(x)) g'(x) \Rightarrow g'(x) = \frac{1}{f'(g(x))}$$

Now $f(x) = 2x + 3$

$$\text{So } g'(x) = \frac{1}{2g(x) + 3} \Rightarrow g'(1) = \frac{1}{2(g(1)) + 3}$$

$$g \circ f(x) = x \Rightarrow g'(f(x)) f'(x) = 1$$

$$f(x) = 1 \text{ at } x = 1 \quad \& \quad f(1) = 5$$

$$g'(1)f(1) = 1 \Rightarrow g'(1) = 1/5$$

11. $u = e^x \sin x, v = e^x \cos x$

$$\begin{aligned} v \frac{du}{dx} - u \frac{dv}{dx} &= v(e^x \cos x + e^x \sin x) - u(e^x \cos x - e^x \sin x) \\ &= e^x \sin x(v + u) + e^x \cos x(v - u) \\ &= u(v + u) + v(v - u) \\ &= v^2 + u^2 \end{aligned}$$

again $\frac{du}{dx} = e^x \sin x + e^x \cos x$

$$\frac{d^2u}{dx^2} = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x$$

$$\frac{d^2u}{dx^2} = 2v$$

Similarly other options can be checked.

12. $f(x) = (x^2 + bx + c)e^x$

$\therefore f'(x) = (x^2 + (b+2)x + (b+c))e^x$

$f(x) > 0$ iff $D = b^2 - 4c < 0$

now $f'(x) > 0$ iff $D' = (b+2)^2 - 4(b+c) = D + 4 < 0$

Thus for $f'(x) > 0$ $D + 4 < 0$ holds. $\Rightarrow D < 0$

$\Rightarrow f(x) > 0$

Note that the converse need not be true, e.g. $b = c = 1$,

$f(x) > 0$ but $f'(-1) = 0$

14. $f'(x) = -f(x)$ (i)

$f'(x) = g(x)$ (ii)

$h'(x) = (f(x))^2 + (g(x))^2$ (iii)

$h(0) = 2, h(1) = 4$

Differentiating equation (ii) w.r.t. x

$f'(x) = g'(x) = -f(x)$

Differentiating equation (iii) w.r.t. x

$h''(x) = 2f(x) \cdot f'(x) + 2g(x) \cdot g'(x)$

$= 2f(x) \cdot f'(x) - 2f(x) \cdot f(x) = 0 \quad \{ \rightarrow g'(x) = -f(x) \}$

$\Rightarrow h'(x)$ is constant

$\Rightarrow h(x)$ is linear function

$\rightarrow h(0) = 2 \Rightarrow h(x)$ not passing through $(0, 0)$

Let $y = h(x) = ax + b$

at $x = 0$

$y = 2 = b \Rightarrow y = ax + 2$

at $x = 1$

$a + 2 = 4$

$a = 2$

$\Rightarrow$ curve is $y = 2x + 2$

15. $f(x) = \frac{\sqrt{(\sqrt{x-1})^2 + 1} - 2\sqrt{x-1}}{\sqrt{x-1} - 1} \cdot x$

$= \frac{|\sqrt{x-1} - 1|}{\sqrt{x-1} - 1} \cdot x = \begin{cases} -x & \text{if } x \in [1, 2) \\ x & \text{if } x \in (2, \infty) \end{cases}$

17. $f(x) = \begin{cases} x^3 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

18. $y = \cos^{-1} \sqrt{\frac{1+x^2+1}{2\sqrt{1+x^2}}}$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \left(\frac{1}{2} + \frac{1}{2\sqrt{1+x^2}}\right)}} \cdot \frac{1}{2\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{1+x^2}}}} \cdot \frac{1}{2}$

$\frac{2x}{(-2)(1+x^2)^{3/2}}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\frac{1}{4} - \frac{1}{4(1+x^2)}}} \cdot \frac{x}{4(1+x^2)^{3/2}}$

$\Rightarrow \frac{dy}{dx} = \frac{2\sqrt{1+x^2}}{\sqrt{1+x^2}-1} \cdot \frac{x}{4\sqrt{1+x^2}(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{x}{2|x|(1+x^2)}$

when $x < 0$

$\frac{dy}{dx} = \frac{-1}{2(1+x^2)}$

when $x > 0$

$\frac{dy}{dx} = \frac{1}{2(1+x^2)}$

19. $y = x^{(\ln x)^{\lambda \ln(\lambda \ln x)}}$

$\lambda \ln y = (\lambda \ln x)^{\lambda \ln(\lambda \ln x)} \cdot \lambda \ln x$ (1)

$\lambda \ln(\lambda \ln y) = \lambda \ln(\lambda \ln x) \cdot \lambda \ln(\lambda \ln x) + \lambda \ln(\lambda \ln x)$

$\frac{1}{\lambda \ln y} \cdot \frac{1}{y} \frac{dy}{dx} = \frac{2 \lambda \ln(\lambda \ln x)}{\lambda \ln x} \cdot \frac{1}{x} + \frac{1}{x \lambda \ln x}$

$= \frac{2 \lambda \ln(\lambda \ln x) + 1}{x \lambda \ln x}$

$\therefore \frac{dy}{dx} = \frac{y}{x} \cdot \frac{\lambda \ln y}{\lambda \ln x} (2 \lambda \ln(\lambda \ln x) + 1) \Rightarrow D$

Substituting the value of $\ln y$ from (1)

$\frac{dy}{dx} = \frac{y}{x} (\lambda \ln x)^{\lambda \ln(\lambda \ln x)} (2 \lambda \ln(\lambda \ln x) + 1) \Rightarrow B$

Part # II : Assertion & Reason

1. **Hint :** Statement I : $f(x)$ is constant function

Statement II : It is true

2. $y = x^2$

$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2$$

again $2x \frac{dx}{dy} = 1$

$$2 \left(\frac{dx}{dy} \right)^2 + 2x \frac{d^2x}{dy^2} = 0$$

$$\Rightarrow x \frac{d^2x}{dy^2} = - \left(\frac{dx}{dy} \right)^2 \Rightarrow \frac{d^2x}{dy^2} = - \frac{1}{4x^2}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2} \right) \left(\frac{d^2x}{dy^2} \right) \neq 1$$

Statement-2 :

$$\rightarrow \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = - \frac{1}{\left(\frac{dx}{dy} \right)^2} \cdot \frac{d^2x}{dy^2} \cdot \frac{dy}{dx} = - \left(\frac{dy}{dx} \right)^3 \cdot \frac{d^2x}{dy^2}$$

4. Consider $f(x) = x + \sin x$ which is aperiodic
but $f'(x) = 1 + \cos x$ which is periodic with period 2π .

EXERCISE - 3

Part # I : Matrix Match Type

3. (A) $f(x) = 3x^2 + 1 \Rightarrow f(x^2 + 1) = 3(x^2 + 1)^2 + 1$
 $f(x^2 + 1)$ at $x = 0$ is 4

(B) $f(x) = \log_{x^2} \log(x) = \frac{1}{2} \log_x(\log x) = \frac{1}{2} \frac{\log(\log x)}{\log x}$

$$f(x) = \frac{1}{2} \left(\frac{\log x \cdot \frac{1}{\log x} \cdot \frac{1}{x} - \frac{\log(\log(x))}{x}}{(\log x)^2} \right)$$

$$= \frac{1}{2} \left(\frac{1 - \log(\log(x))}{x(\log x)^2} \right) \Rightarrow f(e^e) = 0$$

(C) $y = \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$

$$\frac{dy}{dx} = \frac{1}{2} \frac{\sec^2 \left(\frac{\pi}{4} + \frac{x}{2} \right)}{\tan \left(\frac{\pi}{4} + \frac{x}{2} \right)} = \frac{1}{\sin \left(\frac{\pi}{2} + x \right)} = \frac{1}{\cos x}$$

$$= \sec x$$

Hence $p = 0$

(D) $f(x) = |x^3 - x^2 + x - 1| \sin x$
 $f(x) = |(x^2 + 1)(x - 1)| \sin x$
 $= (x^2 + 1)(x - 1) \sin x$ when $x \geq 1$
 $= -(x^2 + 1)(x - 1) \sin x$ when $x < 1$

Now $28f(\pi) = 0$

$\therefore$ At $x = 0$

$$f(x) = -[2x(x-1) \sin x + (x^2 + 1) \cos x]$$

$$4f(0) = 4$$

4. (A) $y = f(x^3)$

$$\therefore \frac{dy}{dx} = f'(x^3) \cdot 3x^2$$

$$\therefore \left. \frac{dy}{dx} \right|_{x=1} = f'(1) \cdot 3 = 9$$

(B) $f(xy) = f(x) + f(y)$

$$f(1) = f(1) + f(1)$$

$$\therefore f(1) = 0$$

$$\rightarrow f(1) = f(e) + f\left(\frac{1}{e}\right)$$

$$\therefore f(e) + f\left(\frac{1}{e}\right) = 0$$

(C) $f''(x) = -f(x), f'(x) = g(x)$

$\therefore g'(x) = f''(x) = -f(x)$

$h(x) = (f(x))^2 + (g(x))^2$

$\therefore h'(x) = 2f(x) \cdot f'(x) + 2g(x) \cdot g'(x)$
 $= 2f(x) \cdot g(x) + 2g(x) \cdot (-f(x)) = 0$

$\therefore h(x) = c, x \in \mathbb{R}$

$\therefore h(10) = h(5) = 9.$

(D) $y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x), \frac{\pi}{2} < x < \pi$

$$\frac{dy}{dx} = \frac{-\operatorname{cosec}^2 x}{1 + \cot^2 x} + \frac{-1}{1 + \tan^2 x} \cdot \sec^2 x$$

$$= -1 - 1 = -2$$

Part # II : Comprehension

Comprehension # 1

$f(x+y) - f(x) = f(y) - 1 + 2xy$

$\Rightarrow f(0+0) - f(0) = f(0) - 1 + 2(0)(0) \Rightarrow f(0) = 1$

and $f'(0) = 1$ (given)

Also $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \left[\frac{f(h) - 1}{h} + \frac{2xh}{h} \right]$

$= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} + 2x = f'(0) + 2x$

$f'(x) = 1 + 2x$

Integrate it

$f(x) = x^2 + x + c$

$f(x) = x^2 + x + 1 \quad [f(0) = 1 \Rightarrow c = 1]$

1. $\bullet n(x^2 + x + 1) \rightarrow \text{Domain } \mathbb{R}$

2. $y = \log_{3/4}(x^2 + x + 1)$

Now $x^2 + x + 1 \geq \frac{3}{4}$

hence range is $(-\infty, 1]$

3. $g(0) = \frac{g(0) + g(0)}{k} \Rightarrow 2g(0) = kg(0)$

$\Rightarrow g(0) = 0$ (as $k \neq 2$)

$g'(x) = \lim_{h \rightarrow 0} \frac{g\left(\frac{x+h}{1}\right) - g(x)}{h}$

$= \lim_{h \rightarrow 0} \frac{\frac{g(x) + g(h)}{1} - g(x)}{h}$

$= \lim_{h \rightarrow 0} \frac{g(h) - g(0)}{h} = g'(0) = \lambda$

$g(x) = \lambda x + c$

$\Rightarrow g(x) = \lambda x \quad [g(0) = 0]$

Now $x^2 + x + 1 = \lambda x \Rightarrow x^2 + (1 - \lambda)x + 1 = 0$

For coincident pt. $D = 0$

$(1 - \lambda)^2 - 4 = 0$

$\Rightarrow \lambda = 3, -1$

Comprehension # 3

1. $\text{LHD} = \lim_{h \rightarrow 0^-} \frac{f(-a+h) - f(-a)}{h}$

$= \lim_{h \rightarrow 0^-} \frac{-f(a-h) + f(a)}{h} = \lim_{h \rightarrow 0^-} \frac{f(a-h) - f(a)}{-h}$

2. $\text{RHD} = \lim_{h \rightarrow 0^+} \frac{f'(a) - f'(a-h)}{h} = \frac{f'(a) + f'(h-a)}{h}$

Since derivative of even function is odd & vice versa.

3. $\lim_{h \rightarrow 0} \frac{f(-x) - f(-x-h)}{h} = \lim_{h \rightarrow 0} \frac{f(-x-h) - f(-x)}{-h}$

$= f'(-x) \quad \dots \text{(i)}$

and $\lim_{h \rightarrow 0} \frac{f(x) - f(x-h)}{-h} = -f'(x) \quad \dots \text{(ii)}$

from (i) and (ii) $f'(x)$ is odd function and hence $f(x)$ is even function.

Comprehension # 4

(i) $y = e^t \sin t \Rightarrow \frac{dy}{dt} = e^t[\cos t + \sin t]$

$x = e^t \cos t \Rightarrow \frac{dx}{dt} = e^t[\cos t - \sin t]$

$\therefore \frac{dy}{dx} = \frac{\cos t + \sin t}{\cos t - \sin t} = \tan \alpha$

$\therefore \tan\left(\frac{\pi}{4} + t\right) = \tan \alpha$

$\left(\frac{\pi}{4} + t\right) = \alpha$

$t = \alpha - \frac{\pi}{4}$



$$(ii) \frac{d^2y}{dx^2} = \frac{\sec^2\left(\frac{\pi}{4} + t\right)}{e^t(\cos t - \sin t)}$$

$$\left. \frac{d^2y}{dx^2} \right|_{t=0} = 2$$

$$(iii) F(t) = \int e^t(\cos t + \sin t) dt = e^t \sin t + C$$

$$F\left(\frac{\pi}{2}\right) - F(0) = (e^{\pi/2} + C) - 0 = e^{\pi/2}$$

Comprehension # 5

$$\text{Sol. } \frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{2x+5y-2}{5x+2y+1} = \frac{-5}{8} \text{ at } (1, 1)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{(5x+2y+1)(2+5\frac{dy}{dx}) - (2x+5y-2)(5+2\frac{dy}{dx})}{(5x+2y+1)^2}$$

$$\Rightarrow \left. \frac{d^2y}{dx^2} \right|_{(1,1)} = \frac{111}{256}$$

For question 6

$$\text{Slope of normal at } (1, 1) = -\frac{dx}{dy} = \frac{8}{5}$$

Equation of normal

$$y - 1 = \frac{8}{5}(x - 1) \Rightarrow 5y - 5 = 8x - 8$$

$$\Rightarrow 8x - 5y - 3 = 0$$

EXERCISE - 4

Subjective Type

$$1. \sqrt{5}$$

$$2. \frac{(n+1)x^{n+2} - (n+2)x^{n+1} + 1}{(1-x)^2}$$

$$3. 2$$

$$5. f_1(x) = e^{f_0(x)} = e^x$$

$$f_2(x) = e^{f_1(x)} = e^{e^x}$$

$$f_3(x) = e^{e^{e^x}}$$

$$\text{similarly } f_n(x) = e^{e^{e^{\dots (n-1) \text{ times } (x)}}}$$

$$\text{Now } \frac{d}{dx}[f_n(x)] = e^{f_{n-1}(x)} \cdot \frac{d}{dx}(e^{f_{n-1}(x)})$$

On differentiating it completely we get

$$\begin{aligned} \frac{d}{dx}[f_n(x)] &= e^{f_{n-1}(x)} \cdot e^{f_{n-2}(x)} \cdot e^{f_{n-3}(x)} \dots e^{f_1(x)} \\ &= f_n(x) \cdot f_{n-1}(x) \dots f_1(x) \end{aligned}$$

$$7. x = \frac{1}{z} \Rightarrow \frac{dx}{dz} = -\frac{1}{z^2}$$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{dy}{dz}(-z^2)$$

$$\frac{d}{dx}(y') = \frac{dy'}{dz} \cdot \frac{dz}{dx} + \frac{dy}{dz} \cdot \frac{d^2z}{dx^2}$$

$$\frac{d^2y}{dx^2} = 2z^3 \cdot \frac{dy}{dz} + z^4 \frac{d^2y}{dz^2}$$

$$\Rightarrow \frac{d^2f}{dx^2} = 2z^3 \frac{dy}{dz} + z^4 \frac{d^2y}{dz^2}$$

$$10. 2^n$$

$$11. y = 1 + \frac{c_1}{x - c_1} + \frac{c_2 x}{(x - c_1)(x - c_2)}$$

$$+ \frac{c_3 x^2}{(x - c_1)(x - c_2)(x - c_3)}$$

$$= \frac{x}{x - c_1} + \frac{c_2 x}{(x - c_1)(x - c_2)} + \frac{c_3 x^2}{(x - c_1)(x - c_2)(x - c_3)}$$

$$= \frac{x^2}{(x - c_1)(x - c_2)} + \frac{c_3 x^2}{(x - c_1)(x - c_2)(x - c_3)}$$



$$= \frac{x^3}{(x-c_1)(x-c_2)(x-c_3)} = \frac{x}{x-c_1} \cdot \frac{x}{x-c_2} \cdot \frac{x}{x-c_3}$$

$$\Rightarrow \bullet ny = \bullet nx - \bullet n(x-c_1) + \bullet nx - \bullet n(x-c_2) + \bullet nx - \bullet n(x-c_3)$$

Differentiating both sides w.r.t. x , we get

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \left(\frac{1}{x} - \frac{1}{x-c_1} \right) + \left(\frac{1}{x} - \frac{1}{x-c_2} \right) + \left(\frac{1}{x} - \frac{1}{x-c_3} \right)$$

$$= -\frac{c_1}{x(x-c_1)} - \frac{c_2}{x(x-c_2)} - \frac{c_3}{x(x-c_3)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \left[\frac{c_1}{c_1-x} + \frac{c_2}{c_2-x} + \frac{c_3}{c_3-x} \right]$$

12. 100

13. $f(2x) = f'(x)f''(x)$

Let the degree of 'f' be n .

Comparing highest power on both sides

$$n = n-1 + n-2 \Rightarrow n = 3$$

Let $f(x) = a_0x^3 + a_1x^2 + a_2x + a_3$

$$f(2x) = f'(x)f''(x)$$

$$\therefore (8a_0x^3 + 4a_1x^2 + 2a_2x + a_3) = (3a_0x^2 + 2a_1x + a_2)(6a_0x + 2a_1)$$

Comparing coefficient of x^3

$$8a_0 = 18a_0^2 \Rightarrow a_0 = \frac{4}{9}$$

Rest all are zero

$$\therefore f(x) = \frac{4}{9}x^3$$

14. $\frac{b}{ab+2ay}$

15. $f(x) = x^2 - 4x - 3$ & $f(x) = 9$

For $x = 6, -2$

$$\Rightarrow x = 6 \quad (x > 2)$$

$$\text{Now } y = f(x) \Rightarrow f^{-1}(y) = x$$

$$\Rightarrow g(y) = x$$

$$\Rightarrow g'(y) = \frac{dx}{dy} = \frac{1}{2x-4} = \frac{1}{8}$$

16. $\operatorname{cosec}^2 x - (1/x^2)$

17. $z = \bullet n \left(\tan \frac{x}{2} \right)$

$$\frac{dz}{dx} = \frac{1}{\sin x}$$

Now $\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz} = \frac{dy}{dx} \sin x$

$$\frac{d^2y}{dz^2} = \frac{d}{dx} \left(\frac{dy}{dx} \sin x \right) \cdot \frac{dx}{dz}$$

$$\frac{d^2y}{dz^2} = \left(\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} \right) \sin x \quad \dots (i)$$

$$\frac{d^2y}{dz^2} = \sin^2 x \frac{d^2y}{dx^2} + \sin x \cos x \frac{dy}{dx}$$

$$\operatorname{cosec}^2 x \frac{d^2y}{dz^2} = \frac{d^2y}{dx^2} + \cot x \frac{dy}{dx}$$

Now put value from given equation

$$\operatorname{cosec}^2 x \frac{d^2y}{dz^2} + 4y \operatorname{cosec}^2 x = 0$$

$$\frac{d^2y}{dz^2} + 4y = 0$$

18. $\lim_{x \rightarrow 0} \frac{\log |\tan 2x|}{\log |\tan x|} = \lim_{x \rightarrow 0} 2 \left(\frac{\sec^2 2x}{\sec^2 x} \cdot \frac{\tan x}{\tan 2x} \right) = 1$

19. $H'(1) = 1, g'(1) = 2, H(1) = 1, g(1) = 2$

$$\lim_{x \rightarrow 1} \frac{H(x) \cdot g(1) - g(x) \cdot H(1)}{\sin(x-1)} \quad \left(\frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow 1} \frac{g(1) \cdot H'(x) - H(1) \cdot g'(x)}{\cos(x-1)} \quad \text{By L.H. Rule}$$

$$= \frac{2 \times 1 - 1 \times 2}{1} = 0$$

20.
$$\begin{vmatrix} X & sX & tX \\ X_1 & sX_1 + s_1X & t_1X + tX_1 \\ X_2 & sX_2 + 2s_1X_1 + Xs_2 & tX_2 + 2X_1t_1 + Xt_2 \end{vmatrix}$$

$$[C_2 \rightarrow C_2 - sC_1, C_3 \rightarrow C_3 - tC_1]$$

$$= \begin{vmatrix} X & 0 & 0 \\ X_1 & Xs_1 & Xt_1 \\ X_2 & Xs_2 + 2s_1X_1 & Xt_2 + 2t_1X_1 \end{vmatrix}$$

$$= X \begin{vmatrix} Xs_1 & Xt_1 \\ Xs_2 + 2s_1X_1 & Xt_2 + 2t_1X_1 \end{vmatrix}$$



$$= X^2 \begin{vmatrix} s_1 & t_1 \\ Xs_2 + 2s_1X_1 & Xt_2 + 2t_1X_1 \end{vmatrix}$$

$$(R_2 \rightarrow R_2 - 2X_1R_1)$$

$$= X^2 \begin{vmatrix} s_1 & t_1 \\ Xs_2 & Xt_2 \end{vmatrix} = X^3 \begin{vmatrix} s_1 & t_1 \\ s_2 & t_2 \end{vmatrix}$$

21. $x = at^3$ and $y = bt^2$

$$\frac{dx}{dt} = 3at^2, \frac{dy}{dt} = 2bt$$

$$\frac{dy}{dx} = \frac{2b}{3a} \cdot \frac{1}{t}$$

$$\frac{d^2y}{dx^2} = \frac{2b}{3a} \left(-\frac{1}{t^2} \right) \cdot \frac{dt}{dx} = -\frac{2b}{3a} \cdot \frac{1}{t^2} \cdot \frac{1}{3at^2}$$

$$\begin{aligned} \frac{d^3y}{dx^3} &= -\frac{2b}{9a^2} \cdot \left(-\frac{4}{t^3} \right) \cdot \frac{dt}{dx} = \frac{8b}{9a^2} \cdot \frac{1}{t^3} \cdot \left(\frac{1}{3at^2} \right) \\ &= \frac{8b}{27a^3} \cdot \frac{1}{t^7} \end{aligned}$$

22. (i) $y = \tan^{-1} \left(\frac{1+2x}{1-2x} \right), z = \sqrt{1+4x^2}$

$$y = \tan^{-1}(1) + \tan^{-1}(2x)$$

$$\frac{dy}{dx} = \frac{2}{1+4x^2}$$

$$\frac{dz}{dx} = \frac{1}{2\sqrt{1+4x^2}} \cdot 8x$$

$$\Rightarrow \frac{dy}{dz} = \frac{2}{1+4x^2} \cdot \frac{2\sqrt{1+4x^2}}{8x} = \frac{1}{2x\sqrt{1+4x^2}}$$

(ii) $y = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right), z = \tan^{-1}x,$

$$z \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \Rightarrow x = \tan z$$

$$\Rightarrow y = \tan^{-1} \left(\frac{\sec z - 1}{\sec z} \right)$$

$$y = \tan^{-1}(\tan z/2)$$

$$y = \frac{z}{2} \Rightarrow -\frac{\pi}{4} < \frac{z}{2} < \frac{\pi}{4}$$

$$\frac{dy}{dz} = \frac{1}{2}$$

26. $\frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$

27. (A) Not differentiable at $x=0$

(B) Not derivable at $x=\pm 1/2$

30. 3

33. $\rightarrow F(x) = f(x) \cdot g(x)$

Differentiating both sides w.r.t. x

$$F'(x) = f'(x)g(x) + g'(x)f(x)$$

Again differentiating both sides w.r.t. x

$$F''(x) = f''(x)g(x) + 2f'(x)g'(x) + g''(x)f(x)$$

dividing both sides by $f(x) \cdot g(x)$

$$\frac{F''(x)}{f(x)g(x)} = \frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2f'(x)g'(x)}{f(x)g(x)}$$

$$\Rightarrow \frac{F''(x)}{f(x)g(x)} = \frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2c}{f(x)g(x)}$$

$$\rightarrow F''(x) = f''(x)g(x) + g''(x)f(x) + 2c$$

Differentiating both sides w.r.t. x

$$F'''(x) = f'''(x)g(x) + f''(x)g'(x) + g'''(x)f(x) + g''(x)f'(x) \dots (i)$$

$$\rightarrow f(x) \cdot g'(x) = c \Rightarrow f''(x)g'(x) + g''(x)f'(x) = 0$$

$$\therefore \text{from (i), we get } \frac{F'''(x)}{F(x)} = \frac{f'''(x)}{f(x)} + \frac{g'''(x)}{g(x)}$$

34. $f(x) = \begin{cases} -\frac{2}{3} \left[\frac{1}{6} + \ln \frac{3}{2} \right] x & \text{if } x \leq 0 \\ \left(\frac{1+x}{2+x} \right)^{1/x} & \text{if } x > 0 \end{cases}$

35. $a=6, b=6, c=0; 3/40$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

3. $y = \log_y x$ $y = \frac{\log x}{\log y}$

$y \times \log y = \log x$

$\frac{dy}{dx} \log y + y \times \frac{1}{y} \times \frac{dy}{dx} = \frac{1}{x}$

$\frac{dy}{dx} (\log y + 1) = \frac{1}{x}$ $\frac{dy}{dx} = \frac{1}{x(1 + \log y)}$

4. $x = 3\cos\theta - 2\cos^3\theta$ $y = 3\sin\theta - 2\sin^3\theta$

$\frac{dx}{d\theta} = -3\sin\theta + 6\cos^2\theta \times \sin\theta$

$\frac{dy}{d\theta} = 3\cos\theta - 6\sin^2\theta \cos\theta$

$\frac{dy}{dx} = \frac{3\cos\theta - 6\sin^2\theta \cos\theta}{-3\sin\theta + 6\cos^2\theta \sin\theta}$

$= \frac{\cos\theta - 2\sin^2\theta \cos\theta}{-\sin\theta + 2\cos^2\theta \sin\theta} = \cot\theta$

6. $f(x) = x^n$

$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} \dots \frac{(-1)^x f^n(1)}{n!}$

put $n = 1$

$f(x) = x$ for series $f(1) - \frac{f'(1)}{1} = 0$

$f(1) = 1$ put $n = 2$

$f(x) = x^2$ $f'(x) = 2x$ $f''(x) = 2$

$f(1) = 1$

so series $= f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} = 1 - 2 + \frac{2}{2} = 0$

Put $x = 3$ $f(x) = x^3$

$f(x) = 3x^2$ $f''(x) = 6x$

$f(1) = 1$ $f'''(x) = 6$

series $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!}$

$= 1 - 3 + \frac{6}{2} - \frac{6}{6} = 0$

7. $f(x)$ is a polynomial function

$f(x) = ax^2 + bx + c$ $f(1) = f(-1)$

$a + b + c = a - b + c$ $b = 0$

a, b, c in A.P. $b = \frac{a+c}{2}$ $a = -c$

$f(x) = ax^2 + bx + c$

$f(a) = 2a^2 + b$

$f(b) = 2ab + b$

$f(b) = 0$ $f(a) = 2a^2$

$f(x) = 2ax + b$

$f(c) = 2ac + b$

then $f(a), f(b), f(c)$

$f(c) = -2a^2$

so that $f(a), f(b), f(c)$ are in A.P.

8. $x = e^{y+e^{y+\dots\infty}}$ $x > 0$ $\frac{dy}{dx} = ?$

$x = e^{y+x}$ $1 = e^{y+x} \left(1 + \frac{dy}{dx}\right)$

$\frac{1}{x} = 1 + \frac{dy}{dx}$ $\frac{1-x}{x} = \frac{dy}{dx}$

10. $(x^x)^2 - 2 \cot y x^x - 1 = 0$

$x^x = \frac{2 \cot y \pm \sqrt{4 \cot^2 y + 4}}{2}$ $\begin{cases} \text{at } x = 1, \\ 1 = \cot y + \operatorname{cosec} y \\ \Rightarrow y = \frac{\pi}{2} \end{cases}$

$= \cot y \pm \operatorname{cosec} y$

$x^x = \cot y + \operatorname{cosec} y$

diff. w.r. to x

$x^x (1 + \log x) = [-\operatorname{cosec}^2 y - \operatorname{cosec} y \cot y] \frac{dy}{dx}$

$1 = -\operatorname{cosec} y [\operatorname{cosec} y + \cot y] \frac{dy}{dx}$

$\frac{dy}{dx} = -1$

11. $g(x) = [f(2f(x) + 2)]^2$

$g'(x) = 2f(2f(x) + 2) f'(2f(x) + 2) 2f'(x)$

Put $x = 0$

$g'(0) = 2f(2f(0) + 2) f'(2f(0) + 2) 2f'(0)$

$= 2f(2(-1) + 2) f'(2(-1) + 2) 2f'(0)$

$= 2f(0) f'(0) 2f'(0)$

$= 4(-1)(1)(1) = -4$

12. $\frac{d}{dy} \left(\left(\frac{dy}{dx} \right)^{-1} \right) = \frac{d}{dy} \left(\left(\frac{dy}{dx} \right)^{-1} \right) \cdot \frac{dx}{dy}$

$= -\left(\frac{dy}{dx} \right)^{-2} \cdot \frac{d^2 x}{dx^2} \cdot \frac{dx}{dy} = -\left(\frac{d^2 y}{dx^2} \right) \left(\frac{dy}{dx} \right)^{-3}$

13. $y = \sec(\tan^{-1} x) = \sqrt{1 + x^2}$

$\frac{dy}{dx} = \frac{x}{\sqrt{1 + x^2}} \Rightarrow \frac{dy}{dx} \Big|_{x=1} = \frac{1}{\sqrt{2}}$



Part # II : IIT-JEE ADVANCED

2. (B) Let $P(x) = ax^2 + bx + c$

$$P(0) = 0 \Rightarrow c = 0$$

$$P(1) = 1 \Rightarrow a + b = 1$$

$$\therefore P(x) = (1-a)x^2 + ax$$

$$P'(x) = 2(1-a)x + a > 0$$

$$\text{put } x=0, \quad a > 0$$

$$x=1, \quad a < 2$$

$$S = \{(1-a)x^2 + ax; 0 < a < 2\}.$$

5. (A) $g(x+1) = \log(f(x+1)) = \log x + \log f(x)$

$$\Rightarrow g(x+1) = \log x + g(x) \Rightarrow g(x+1) - g(x) = \log x$$

$$\Rightarrow g'(x+1) - g'(x) = \frac{1}{x} \Rightarrow g''(x+1) - g''(x) = -\frac{1}{x^2}$$

$$\Rightarrow g''\left(1 + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4$$

$$\Rightarrow g''\left(2 + \frac{1}{2}\right) - g''\left(1 + \frac{1}{2}\right) = -\frac{4}{9}$$

$$g''\left(N + \frac{1}{2}\right) - g''\left(N - \frac{1}{2}\right) = \frac{-4}{(2N-1)^2}$$

By adding

$$\text{Hence } g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$$

$$= -4\left(1 + \frac{1}{9} + \dots + \frac{1}{(2N-1)^2}\right)$$

$$(B) \lim_{x \rightarrow 0} \frac{[g(x) \cos x - g(0)]}{\sin x}$$

$$= \lim_{x \rightarrow 0} \frac{g'(x) \cos x - g(x) \sin x}{\cos x} = 0$$

Now $f(x) = g(x) \sin x$

$$f(x) = g'(x) \sin x + g(x) \cos x$$

$$\therefore f(0) = 0$$

$$f'(x) = g''(x) \sin x + g'(x) \cos x - g(x) \sin x + g'(x) \cos x$$

$$f'(0) = 0$$

$$\therefore \text{Given limit} = f'(0) \text{ \& also } f(0) = g(0)$$

So S(I) & S(II) both are correct but S(III) is not correct
explanation of S(I)

6. $f(x) = x^3 + e^{x/2}, g(x) = f^{-1}(x)$

$$\Rightarrow g'(f(x)) \cdot f'(x) = 1$$

$$\text{Put } f(x) = 1 \Rightarrow x^3 + e^{x/2} = 1$$

$$\Rightarrow x = 0$$

$$\Rightarrow g'(1) \cdot f'(0) = 1, f'(x) = 3x^2 + e^{x/2} \cdot \frac{1}{2}$$

$$\Rightarrow g'(1) = 2$$

7. Let $f(\theta) = \sin \alpha$ where $\alpha = \tan^{-1}\left(\frac{\sin \theta}{\sqrt{\cos 2\theta}}\right)$

$$\Rightarrow \tan \alpha = \frac{\sin \theta}{\sqrt{\cos 2\theta}}$$

$$\Rightarrow \sin \alpha = \frac{\sin \theta}{\cos \theta} = \tan \theta \quad \left(Q \theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)\right)$$

$$\Rightarrow f(\theta) = \tan \theta$$

$$\Rightarrow \frac{d(f(\theta))}{d(\tan \theta)} = 1$$

MOCK TEST

1. $y = x^{(\ln x)^{\ln \ln x}}$

... (i)

taking $\log_e$ both sides, we get $\bullet ny = (\bullet nx) \bullet n(\bullet nx) \bullet nx$

Again taking $\log_e$, we get $\bullet n(\bullet ny) = \bullet n(\bullet nx) \bullet n(\bullet nx)$

$$+ \bullet n(\bullet nx) = \{\bullet n(\bullet nx)\} [\bullet n(\bullet nx) + 1]$$

Diff. w.r.t. x,

$$\left(\frac{1}{\ln y}\right) \left(\frac{1}{y} \cdot \frac{dy}{dx}\right) = \left(\frac{1}{x \ln x}\right) [\bullet n(\bullet nx) + 1] + \{\bullet n(\bullet nx)\} \left[\frac{1}{x \ln x}\right]$$

$$\Rightarrow \frac{dy}{dx} = y \bullet ny \left[\frac{\ln(\ln x)}{x \ln x} + \frac{1}{x \ln x} + \frac{\ln(\ln x)}{x \ln x} \right]$$

$$\Rightarrow \frac{dy}{dx} = y \bullet ny \left[\frac{2 \ln(\ln x) + 1}{x \ln x} \right]$$

3. $\therefore x = \operatorname{cosec} \theta - \sin \theta$

$$\Rightarrow x^2 + 4 = (\operatorname{cosec} \theta + \sin \theta)^2$$

and $y^2 + 4 = (\operatorname{cosec} \theta + \sin \theta)^2$

Now $\frac{dy}{dx} = \frac{(dy/d\theta)}{(dx/d\theta)} = \frac{n \cot \theta (\operatorname{cosec} \theta + \sin \theta)}{\cot \theta (\operatorname{cosec} \theta + \sin \theta)}$

$$= \frac{n \sqrt{y^2 + 4}}{\sqrt{x^2 + 4}}$$

Squaring both sides, we get $\left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2 + 4)}{x^2 + 4}$

or $(x^2 + 4) \left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$

4. (C)

$$y^2 = P(x)$$

$$\Rightarrow 2y \frac{dy}{dx} = P'(x)$$

or $2 \left(\frac{dy}{dx}\right)^2 + 2y \frac{d^2y}{dx^2} = P''(x)$

or $2y \frac{d^2y}{dx^2} = P'' - 2 \left(\frac{dy}{dx}\right)^2 = P'' - \frac{P'^2}{2y^2}$

$$\rightarrow 2y^3 \frac{d^2y}{dx^2} = y^2 P'' - \frac{1}{2} P'^2 = PP'' - \frac{1}{2} P'^2$$

$$\therefore 2 \frac{d}{dx} \left(y^3 \frac{d^2y}{dx^2} \right) = P' P'' + PP''' - P' P'' = PP'''$$

5. Given that $g^{-1}(x) = f(x)$

$$\Rightarrow x = g(f(x)) \quad \text{or} \quad g'(f(x)) f'(x) = 1$$

$$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow g''(f(x)) \cdot f'(x) = \frac{-f''(x)}{(f'(x))^2}$$

$$\Rightarrow g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$$

6. (A)

$$F'(x) = \left[f\left(\frac{x}{2}\right) \cdot f'\left(\frac{x}{2}\right) + g\left(\frac{x}{2}\right) g'\left(\frac{x}{2}\right) \right]$$

here $g(x) = f'(x)$ & $g'(x) = f''(x) = -f(x)$

so $F'(x) = f\left(\frac{x}{2}\right) g\left(\frac{x}{2}\right) - f\left(\frac{x}{2}\right) g\left(\frac{x}{2}\right) = 0$

$$\Rightarrow F(x) \text{ is constant function}$$

so $F(10) = 5$

7. $x^y \cdot y^x = 1 \Rightarrow y \bullet nx + x \bullet ny = 0$

Diff. w.r.t. x, we get

$$\left(\frac{dy}{dx}\right) \bullet nx + \frac{y}{x} + \bullet ny + \frac{x}{y} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\left(\frac{y}{x} + \ln y\right)}{\frac{x}{y} + \ln x} = -\frac{y}{x} \frac{(x \ln y + y)}{(y \ln x + x)}$$

8. (D)

$$f(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$f'(x) = \begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$f'''(0) = \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = 0$$



9. $y = x(\log nx - \log n(a + bx))$ (i)

$\Rightarrow \frac{dy}{dx} = \log nx - \log n(a + bx) + \frac{a}{a + bx}$ (ii)

$\Rightarrow \frac{d^2y}{dx^2} = \frac{a}{x(a + bx)} - \frac{ab}{(a + bx)^2}$

$\Rightarrow x^3 \frac{d^2y}{dx^2} = \frac{a^2 x^2}{(a + bx)^2}$ (iii)

By (i) and (ii), $x \frac{dy}{dx} - y = \frac{ax}{a + bx}$

$\Rightarrow x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^2$

10. (B)

$S_1: y = \sin 2x$

$\frac{dy}{dx} = 2 \cos 2x$

$\frac{d^2y}{dx^2} = -4 \sin 2x$

$\frac{d^3y}{dx^3} = -8 \cos 2x$

$\frac{d^4y}{dx^4} = 16 \sin 2x$

$\frac{d^6y}{dx^6} = -64 \sin 2x$

$\therefore \frac{d^6y}{dx^6} \text{ at } x = \frac{\pi}{2} \text{ is } 0$ $\therefore$ Statement is false.

$S_2: x = e^{y+e^{y+\dots\infty}} = e^{y+x}$

$\therefore 1 = e^{y+x} \left(1 + \frac{dy}{dx} \right) = x \left(1 + \frac{dy}{dx} \right)$

$\therefore \frac{dy}{dx} = \frac{1}{x} - 1$

$\therefore \frac{dy}{dx} \Big|_{\text{at } x=1} = 0$ $\therefore$ Statement is true.

$S_3: y = 2t^2 \quad x = 4t$

$\therefore y = 2 \left(\frac{x}{4} \right)^2 = \frac{x^2}{8}$

$\therefore \frac{dy}{dx} = \frac{x}{4} \quad \therefore \frac{d^2y}{dx^2} = \frac{1}{4}$

$\therefore$ Statement is false.

$S_4: x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$

$\therefore \frac{dx}{dt} = 2t + 3; \frac{dy}{dt} = 4t - 2$

$\therefore \frac{dy}{dx} = \frac{4t - 2}{2t + 3}$

$\therefore \frac{d^2y}{dx^2} = \frac{(2t + 3)4 - (4t - 2)2}{(2t + 3)^2} \cdot \frac{dt}{dx}$
 $= \frac{16}{(2t + 3)^2} \times \frac{1}{(2t + 3)}$

$\therefore \frac{d^2y}{dx^2} = \frac{16}{(2t + 3)^3}$

When $x = 2$ and $y = -1$ then $t = 2$

$\therefore \frac{d^2y}{dx^2} \text{ at } (2, -1) \text{ is } = \frac{16}{7^3}$ $\therefore$ Statement is true.

11. (A, B)

$y = \sec(\tan^{-1} x)$

$\therefore \frac{dy}{dx} = \frac{\sec(\tan^{-1} x) \cdot \tan(\tan^{-1} x)}{1 + x^2}$

$\therefore \frac{dy}{dx} \Big|_{x=1} = \frac{1}{\sqrt{2}} = \sin^{-1} \left(\sin \frac{1}{\sqrt{2}} \right)$

12. (D)

$y = \tan^{-1} \left(\frac{2^{x+1} - 2^x}{1 + 2^x \cdot 2^{x+1}} \right) = \tan^{-1} 2^{(x+1)} - \tan^{-1} 2^x$

$\Rightarrow y' = \frac{2^{x+1} \ln 2}{1 + (2^{x+1})^2} - \frac{2^x \ln 2}{1 + (2^x)^2}$

$\Rightarrow y'(0) = -\frac{1}{10} \ln 2$

13. (B, C)

$y = 10^{10^x}$

$\therefore \frac{dy}{dx} = 10^{10^x} \log 10 \cdot 10^x \log 10 = y 10^x (\log 10)^2$

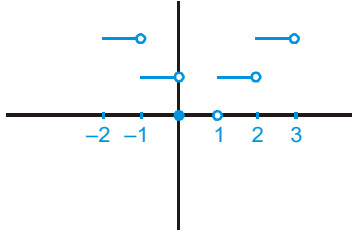
$\therefore \frac{1}{y} \frac{dy}{dx} = 10^x (\log 10)^2$

$\therefore \lambda = (\log 10)^2 = e^{\ln(10)^2}$

14. (B)

S_1 : the graph of function $f(|f(x)|) = [|f(x)|]$

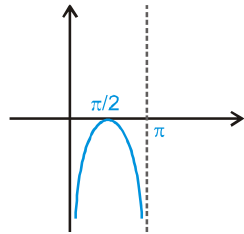




it is clear from the graph $f\left(f\left(\frac{1}{2}\right)\right) = 0$

S₂ : Function is not defined at $x = n\pi$

S₃ : Graph of $f(x) = \log |\sin x|$



clearly $f'(x) < 0 \forall x \in \left(\frac{\pi}{2}, \pi\right)$

$$S_4 : f'\left(\frac{\pi}{2}\right) = \lim_{h \rightarrow 0} \frac{e^{\cosh h} - e}{|h|}$$

$\Rightarrow$ does not exist

15. (A, D)

$$f(x) = -\frac{x^3}{3} + x^2 \sin 1.5a - x \sin a \cdot \sin 2a - 5 \sin^{-1}(a^2 -$$

$$8a + 17)$$

$$f(x) \text{ is defined when } -1 \leq a^2 - 8a + 17 \leq 1$$

$$-1 \leq (a-4)^2 + 1 \leq 1$$

$$\Rightarrow a = 4$$

$$\therefore f(x) = -\frac{x^3}{3} + x^2 \sin 6 - x \sin 4 \sin 8 - \frac{5\pi}{2}$$

$$\therefore f'(x) = -x^2 + 2x \sin 6 - \sin 4 \sin 8$$

$$f'(\sin 8) = -\sin^2 8 + 2 \sin 8 \sin 6 - \sin 4 \sin 8 = \sin 8 [2 \sin 6 - (\sin 8 + \sin 4)]$$

$$= \sin 8 [2 \sin 6 - 2 \sin 6 \cos 2] = 2 \sin 6 \sin 8 (1 - \cos 2)$$

$$\sin 6 < 0, \sin 8 > 0, 1 - \cos 2 > 0$$

$$\therefore f'(\sin 8) < 0$$

16. Statement I : $f(x)$ is constant function

Statement II : It is true

17. (A)

$$\frac{d}{dx} \ln |x| = \frac{d}{dx} \ln(-x) = \frac{1}{(-x)}(-1) = \frac{1}{x}$$

19. (B)

$$\frac{d}{dx} \left\{ \tan^{-1}(\sec x + \tan x) \right\} = \frac{d}{dx} \left\{ \tan^{-1} \left(\frac{1 + \sin x}{\cos x} \right) \right\}$$

$$= \frac{d}{dx} \left\{ \tan^{-1} \left(\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right) \right\} = \frac{d}{dx} \left(\frac{\pi}{4} + \frac{x}{2} \right) = \frac{1}{2}$$

$$\frac{d}{dx} \left\{ \cot^{-1}(\operatorname{cosec} x + \cot x) \right\} = \frac{d}{dx} \left\{ \cot^{-1} \left(\frac{1 + \cos x}{\sin x} \right) \right\}$$

$$= \frac{d}{dx} \left\{ \cot^{-1} \left(\cot \frac{x}{2} \right) \right\} = \frac{d}{dx} \left(\frac{x}{2} \right) = \frac{1}{2}$$

20. $y = x^2$

$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2$$

$$\text{again } 2x \frac{dx}{dy} = 1$$

$$2 \left(\frac{dx}{dy} \right)^2 + 2x \frac{d^2x}{dy^2} = 0 \Rightarrow x \frac{d^2x}{dy^2} = - \left(\frac{dx}{dy} \right)^2$$

$$\Rightarrow \frac{d^2x}{dy^2} = - \frac{1}{4x^2} \Rightarrow \left(\frac{d^2y}{dx^2} \right) \left(\frac{d^2x}{dy^2} \right) \neq 1$$

Statement-II :

$$\rightarrow \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = - \frac{1}{\left(\frac{dx}{dy} \right)^2} \cdot \frac{d^2x}{dy^2} \cdot \frac{dy}{dx} = - \left(\frac{dy}{dx} \right)^3 \cdot \frac{d^2x}{dy^2}$$

21. (A) $\rightarrow$ (q), (B) $\rightarrow$ (p), (C) $\rightarrow$ (t), (D) $\rightarrow$ (p)

$$(A) \lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$$

$$= \lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{x - \sqrt{1-x^2}}{1 - \frac{x}{\sqrt{1-x^2}}} = \lim_{x \rightarrow \frac{1}{\sqrt{2}}} \left(-\sqrt{1-x^2} \right) = -\frac{1}{\sqrt{2}}$$

(B) $x = \frac{1}{2}$ is not in the domain

$\therefore f\left(\frac{1}{2}\right)$ does not exist

(C) $y = f(x) = \sec \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$

$$\therefore \frac{dy}{dx} = \frac{\sec^2 \left(\frac{\pi}{4} + \frac{x}{2} \right)}{\tan \left(\frac{\pi}{4} + \frac{x}{2} \right)} \cdot \frac{1}{2}$$

$$= \frac{1}{2} \cdot \frac{1}{\sin \left(\frac{\pi}{4} + \frac{x}{2} \right) \cos \left(\frac{\pi}{4} + \frac{x}{2} \right)} = \frac{1}{\sin \left(\frac{\pi}{2} + x \right)} = \sec x$$

(D) $\lim_{x \rightarrow 0} \frac{1}{x} \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \lim_{x \rightarrow 0} \frac{|\tan x|}{x} = \text{does not exist}$

22. (A) $\rightarrow$ (t), (B) $\rightarrow$ (q), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)

(A) $y = \cos^{-1}(\cos x)$

$$y' = \frac{-1}{\sqrt{1 - \cos^2 x}} \cdot (-\sin x) = \frac{\sin x}{|\sin x|}$$

$\therefore y'$ at $x = 5$ is -1

(B) $y = f(x) = \sec |\tan x|$

$$\therefore f'(x) = (1/\tan x) (\sec^2 x) \cdot \left(\frac{|\tan x|}{\tan x} \right)$$

$$f'\left(-\frac{\pi}{4}\right) = 2$$

(C) $\frac{d}{dx} \tan^{-1} \left(\frac{1+x}{1-x} \right) = \frac{1}{1 + \left(\frac{1+x}{1-x} \right)^2} \cdot \frac{d}{dx} \left(\frac{1+x}{1-x} \right)$

$$= \frac{(1-x)^2}{2(1+x^2)} \cdot \frac{2}{(1-x)^2} = \frac{1}{1+x^2}$$

at $x = -1$

$$\frac{d}{dx} \tan^{-1} \left(\frac{1+x}{1-x} \right) = \frac{1}{2}$$

(D) $\frac{d}{dx} \frac{\ln |x|}{x} = \frac{x \cdot \frac{1}{x} - \ln |x|}{x^2} = \frac{1 - \ln |x|}{x^2}$

$$\Rightarrow \frac{d}{dx} \frac{\ln |x|}{x} = 1 \quad \text{at } x = -1$$

24.

1. (C)

$$f(x) g(x) = x^3 g(x)$$

$$3x^2 \cdot g'(x) = 3x^2 g(x) + x^3 g'(x)$$

$$3g'(x) = 3g(x) + xg'(x)$$

$$(3-x)g'(x) = 3g(x)$$

$$\int \frac{g'(x)}{g(x)} dx = \int \frac{3}{3-x} dx$$

$$\ln g(x) = -3 \ln |3-x| + \ln c$$

$$\therefore g(x) = \frac{c}{|3-x|^3}$$

$$g(0) = \frac{c}{27} = \frac{1}{3} \quad \therefore c = 9$$

$$\therefore g(x) = \frac{9}{|3-x|^3}$$

2. (A)

$$f(x-3) \cdot g(x) = (x-3)^3 \cdot g(x) = 9$$

$\therefore$ derivative of $f(x-3) \cdot g(x)$ is 0

3. (A)

$$\lim_{x \rightarrow 0} \frac{f(x) \cdot g(x)}{x(1+g(x))} = \lim_{x \rightarrow 0} \frac{x^3 \cdot \frac{9}{|3-x|^3}}{x \left(1 + \frac{9}{|3-x|^3} \right)} = 0$$

25.

1. (D)

$$f(x) = -\frac{2x}{(1+x^2)^2}$$

$$f'(x) = \frac{-2+6x^2}{(1+x^2)^3} \quad f'(x) = 0 \text{ if } x = \pm \frac{1}{\sqrt{3}}$$

$\therefore f(x)$ is greatest at $x = -\frac{1}{\sqrt{3}}$

2. (A)

Equation of tangent at $x = \alpha$ is

$$y - \frac{1}{1+\alpha^2} = \frac{-2\alpha}{(1+\alpha^2)^2} (x - \alpha)$$

$$\therefore b = \frac{1}{1+\alpha^2} + \frac{2\alpha^2}{(1+\alpha^2)^2} = \frac{1+3\alpha^2}{(1+\alpha^2)^2}$$

$$\therefore \frac{db}{d\alpha} = \frac{(1+\alpha^2)^2 \cdot 6\alpha - 2(1+3\alpha^2)(1+\alpha^2) \cdot 2\alpha}{(1+\alpha^2)^4}$$

$$= \frac{2\alpha(1-3\alpha^2)}{(1+\alpha^2)^3}$$

$$\frac{db}{d\alpha} = 0 \quad \text{if} \quad \alpha = 0, \pm \frac{1}{\sqrt{3}}$$

$$\text{at } \alpha = \pm \frac{1}{\sqrt{3}}, \quad b = \frac{9}{8}$$

3. (A)

$$a = \frac{1+3\alpha^2}{2\alpha}$$

$$\therefore \frac{1}{a} = \frac{2\alpha}{1+3\alpha^2} \text{ its greatest value is } \frac{1}{\sqrt{3}}$$

26. (5)

$$y = \tan^{-1} \frac{4x}{1+5x^2} + \tan^{-1} \frac{2+3x}{3-2x}$$

$$= \tan^{-1} \frac{5x-x}{1+5x \cdot x} + \tan^{-1} \frac{\frac{2}{3}+x}{1-\frac{2}{3} \cdot x}$$

$$= \tan^{-1} 5x - \tan^{-1} x + \tan^{-1} \frac{2}{3} + \tan^{-1} x$$

$$= \tan^{-1} 5x + \tan^{-1} \frac{2}{3}$$

$$\therefore \frac{dy}{dx} = \frac{5}{1+25x^2}$$

27. $\rightarrow y = \left[\ln \left(\frac{x + \sqrt{x^2 - a^2}}{a} \right) \right]^2 + k \bullet n (x + \sqrt{x^2 - a^2})$

Differentiating both sides w. r. t. x , we get

$$\frac{dy}{dx} = 2 \left[\ln \left(\frac{x + \sqrt{x^2 - a^2}}{a} \right) \right] \cdot \frac{a}{(x + \sqrt{x^2 - a^2})}$$

$$\frac{(x + \sqrt{x^2 - a^2})}{a\sqrt{x^2 - a^2}} + \frac{k}{(x + \sqrt{x^2 - a^2})} \cdot \frac{(x + \sqrt{x^2 - a^2})}{\sqrt{(x^2 - a^2)}}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{x^2 - a^2}} \bullet n \left[\frac{x + \sqrt{x^2 - a^2}}{a} \right] + \frac{k}{\sqrt{x^2 - a^2}}$$

$$\Rightarrow \sqrt{(x^2 - a^2)} \frac{dy}{dx} = 2 \bullet n (x + \sqrt{x^2 - a^2}) - 2 \bullet n a + k$$

Differentiating both sides w.r.t. x , we get

$$\sqrt{(x^2 - a^2)} \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{x}{\sqrt{x^2 - a^2}}$$

$$= \frac{2}{(x + \sqrt{x^2 - a^2})} \cdot \frac{(x + \sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}}$$

$$\text{or } \sqrt{x^2 - a^2} \frac{d^2y}{dx^2} + \frac{x}{\sqrt{x^2 - a^2}} \frac{dy}{dx} = \frac{2}{\sqrt{x^2 - a^2}}$$

$$\text{or } (x^2 - a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 2$$

$$\text{hence value of } (x^2 - a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} \text{ is } 2$$

28. Let $y_1 = \tan^{-1} \left(\frac{\ln(e/x^2)}{\ln(ex^2)} \right)$

$$\text{and } y_2 = \tan^{-1} \left(\frac{3+21nx}{1-61nx} \right)$$

$$\text{Let } a = 2 \bullet n x,$$

$$\text{then } y_1 = \tan^{-1} \left(\frac{\ln e - \ln x^2}{\ln e + \ln x^2} \right) = \tan^{-1} \left(\frac{1-a}{1+a} \right) = \tan^{-1}$$

$$\left(\tan \left(\frac{\pi}{4} - \alpha \right) \right) = \frac{\pi}{4} - \alpha, \text{ where } a = \tan \alpha$$

$$\text{Similarly } y_2 = \tan^{-1} \left(\frac{3+a}{1-3a} \right) = \beta + \alpha, \text{ where } \tan \beta = 3$$

$$\Rightarrow y = \left(\frac{\pi}{4} - \alpha \right) + (\beta + \alpha) = \frac{\pi}{4} + \tan^{-1} 3 = \text{constant}$$

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$$



$$29. y = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{\sqrt{a-b}}{\sqrt{a+b}} \tan \frac{x}{2} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{a^2 - b^2}} \cdot \frac{\sec^2 \left(\frac{x}{2} \right) \cdot \left(\frac{a-b}{a+b} \right)^{1/2}}{1 + \left(\frac{a-b}{a+b} \right) \tan^2 \frac{x}{2}} \cdot \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(a+b) \cos^2 \frac{x}{2} + (a-b) \sin^2 \frac{x}{2}} = \frac{1}{a + b \cos x}$$

$$\text{at } a=1, b=1, x=0$$

$$\frac{dy}{dx} = 1/2$$

30. (2)

$$x = e^t \sin t \text{ and } y = e^t \cos t$$

$$\Rightarrow x^2 + y^2 = e^{2t} \Rightarrow e^t = \sqrt{x^2 + y^2} \quad \dots\dots(i)$$

$$\text{and } \tan t = \frac{x}{y} \Rightarrow t = \tan^{-1} \left(\frac{x}{y} \right) \text{ put in (i)}$$

$$\therefore e^{\tan^{-1} \left(\frac{x}{y} \right)} = \sqrt{x^2 + y^2} \quad \dots\dots(ii)$$

taking ●n of both sides.

$$\tan^{-1} \left(\frac{x}{y} \right) = \frac{1}{2} \bullet n (x^2 + y^2)$$

differentiate both sides w.r.t. 'x'

$$\Rightarrow \left(\frac{1}{1 + \frac{x^2}{y^2}} \right) \left(\frac{y \cdot 1 - x \cdot y'}{y^2} \right) = \frac{1}{2} \cdot \frac{(2x + 2yy')}{(x^2 + y^2)}$$

$$\Rightarrow y' = \frac{y-x}{x+y} \quad \dots\dots(iii)$$

again differentiate equation (iii) w.r.t. 'x'

$$\Rightarrow y'' = \frac{(x+y)(y'-1) - (y-x)(1+y')}{(x+y)^2}$$

$$\Rightarrow y''(x+y)^2 = y'(2x) - 2y$$

$$\Rightarrow y''(x+y)^2 = 2(xy' - y). \text{ Hence proved.}$$