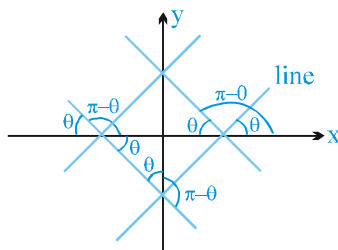


HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

2. Reflecting a graph over the x-axis results in the line M whose equation is $ax - by = c$, while a reflection through the y-axis results in the line N whose equation is $-ax + by = c$. Both clearly have slope equal to a/b (from, say, the slope-intercept form of the equation.)



6. $AP = \sqrt{x^2 + (y-4)^2}$

$BP = \sqrt{x^2 + (y+4)^2}$

$\rightarrow |AP - BP| = 6$

$AP - BP = \pm 6$

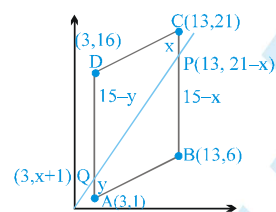
$\sqrt{x^2 + (y-4)^2} - \sqrt{x^2 + (y+4)^2} = \pm 6$

On squaring we get the locus of P is

$9x^2 - 7y^2 + 63 = 0$

9. as shown $= \frac{21-x}{13} = \frac{x+1}{3}$

$63 - 3x = 13x + 13$



$16x = 50$

$x = \frac{25}{8}$; Hence $m = \left(\frac{25}{8} + 1\right) \cdot \frac{1}{3} = \frac{33}{24} = \frac{11}{8}$

11. Let (h, k) be the centroid of triangle

$3h = \cos\alpha + \sin\alpha + 1$

$\Rightarrow (3h - 1) = \cos\alpha + \sin\alpha$ (i)

$3k = \sin\alpha - \cos\alpha + 2$

$\Rightarrow (3k - 2) = \sin\alpha - \cos\alpha$ (ii)

square & add (i) & (ii)

$9(x^2 + y^2) + 6(x - 2y) = -3$

12. $D = 0$

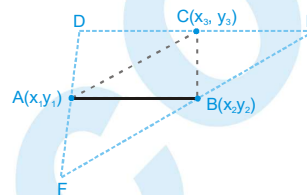
$x^2 = 4(x-y)^2$

$x = 2(x-y)$ or $x = -2(x-y)$

$x = 2y$ or $3x = 2y$

$\Rightarrow$ line pair with slope $3/2$ and $1/2$ $\Rightarrow D$

- 13.



ABCD, ABEC, ACBF are three possible parallelograms.

14. $\Delta = \frac{1}{2} \begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0$

$\Rightarrow (2a - c)(2b - c) - (3a - c)(3b - c) = 0$

$\Rightarrow 4ab - 2ac - 2bc + c^2 - (9ab - 3ac - 3bc + c^2) = 0$

$\Rightarrow ac + bc - 5ab = 0$

$\frac{1}{a} + \frac{1}{b} = \frac{5}{c} \Rightarrow \frac{1}{a} + \frac{1}{b} = 2\left(\frac{5}{2c}\right)$

$\therefore a, \frac{2c}{5}, b$ are in H.P.

17. $(2y - x)(y - mx) = mx^2 - xy(2m + 1) + 2y^2 = 0$

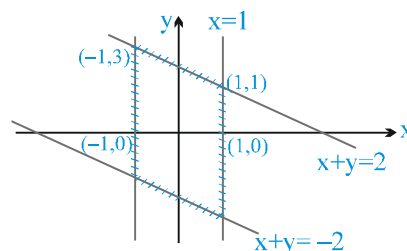
$\Rightarrow$ the equation to the pair of bisectors are :

$\frac{x^2 - y^2}{m - 2} = \frac{-2xy}{2m + 1} \Rightarrow 12x^2 - 7xy - 12y^2$

$\Rightarrow \frac{2m+1}{12} = \frac{2(m-2)}{-7}$ or $38m = 41 \Rightarrow m = \frac{41}{38}$

19. Figure is a parallelogram

Area $= 2\left(\frac{1+3}{2} \cdot 2\right) = 8$ Ans.



24. $x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$

$\Rightarrow (x + \sqrt{2}y + p)(x + \sqrt{2}y + q) = 0$

$p + q = 4$

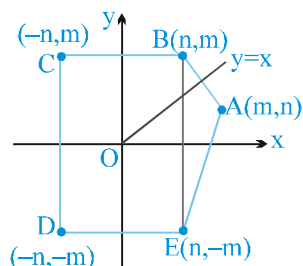
$\Rightarrow pq = 1$

Distance between parallel lines is $\left| \frac{p-q}{\sqrt{3}} \right| =$

$\frac{\sqrt{(p+q)^2 - 4pq}}{\sqrt{3}} = \frac{\sqrt{16-4}}{\sqrt{3}} = 2$

26. Area of rectangle BCDE = 4mn

Area of $\Delta ABC = \frac{2m(m-n)}{2}$



$= m^2 - mn$

$\therefore$ area of pentagon = $4mn + m^2 - mn$
 $= m^2 + 3mn$

30. Here, $x + 2y - 3 = 0$ and $3x + 4y - 7 = 0$ intersect $(1, 1)$, which does not satisfy $2x + 3y - 4 = 0$ and $4x + 5y - 6 = 0$. Also, $3x + 4y - 7 = 0$ and $2x + 3y - 4 = 0$ intersect at $(5, -2)$ which does not satisfy $x + 2y - 3 = 0$ and $4x + 5y - 6 = 0$. Intersection point of $x + 2y - 3 = 0$ and $2x + 3y - 4 = 0$ is $(-1, 2)$ which satisfy $4x + 5y - 6 = 0$.

Hence, only three lines are concurrent.

32. $m_1 + m_2 = -10 \Rightarrow m_1 m_2 = \frac{a}{1}$
 given $m_1 = 4m_2 \Rightarrow m_2 = -2, m_1 = -8, a = 16$

34. Homogenizing the curve with the help of the straight line.

$5x^2 + 12xy - 6y^2 + 4x(x+ky) - 2y(x+ky) + 3(x+ky)^2 = 0$

$12x^2 + (10 + 4k + 6k)xy + (3k^2 - 2k - 6)y^2 = 0$

Lines are equally inclined to the coordinate axes

$\therefore$ coefficient of $xy = 0$

$\Rightarrow 10k + 10 = 0 \Rightarrow k = -1$

35. Curve passing through points of intersection of $S_1 = 0$ & $S_2 = 0$ is

$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right) + \lambda(x^2 + y^2 + 2gx + 2fy + c) = 0$

above equation represents a pair of straight lines. They

are parallel to the lines $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \lambda(x^2 + y^2) = 0$ which

represents a pair of lines equally inclined to axis as the term containing xy is absent

36. Let the third PH reading is x

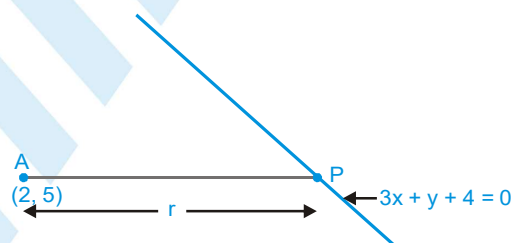
$7.4 < \frac{7.48 + 8.42 + x}{3} < 8.2$

$22.2 < 15.90 + x < 24.6$

$6.3 < x < 8.7$

PH range should be in between 6.3 to 8.7

37. Let distance be 'r'.



Co-ordinates of 'P' are

$(2 + r \cos \theta, 5 + r \sin \theta)$ where $\tan \theta = \frac{3}{4}$

which lies on the line $3x + y + 4 = 0$

$3(2 + r \cos \theta) + 5 + r \sin \theta + 4 = 0$

$r \left(3 \cdot \frac{4}{5} + \frac{3}{5} \right) + 15 = 0$

$\Rightarrow r = -\frac{15}{3} = -5$

but distance can not be negative

$\therefore r = 5$

EXERCISE - 2

Part # I : Multiple Choice

8. Use the condition of concurrency for three lines
13. The lines will pass through (4, 5) & parallel to the bisectors between them

$$\frac{3x - 4y - 7}{5} = \pm \frac{12x - 5y + 6}{13}$$

by taking + sign, we get $21x + 27y + 121 = 0$

Now by taking - sign, we get $99x - 77y - 61 = 0$

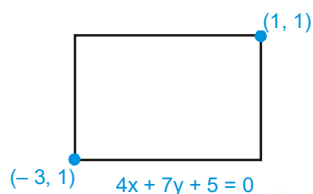
so slopes of bisectors are $-\frac{7}{9}, \frac{9}{7}$

Equation of lines are

$$y - 5 = -\frac{7}{9}(x - 4) \quad \text{and} \quad y - 5 = \frac{9}{7}(x - 4)$$

$$\Rightarrow 7x + 9y = 73 \quad \text{and} \quad 9x - 7y = 1$$

18. Line $\perp$ to $4x + 7y + 5 = 0$ is



$$7x - 4y + \lambda = 0$$

It passes through $(-3, 1)$ and $(1, 1)$

$$-11 - 4 + \lambda = 0 \Rightarrow \lambda = 15$$

$$7 - 4 + \lambda = 0 \Rightarrow \lambda = -3$$

Hence lines are $7x - 4y + 15 = 0, 7x - 4y - 3 = 0$

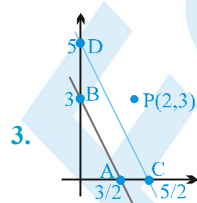
line $\parallel$ to $4x + 7y + 5 = 0$ passing through

$(1, 1)$ is $4x + 7y + 1 = 0$

$$\Rightarrow 1 = -1$$

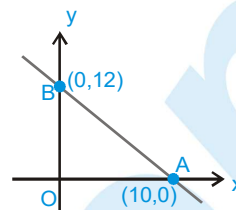
$$\Rightarrow 4x + 7y - 11 = 0$$

Part # II : Assertion & Reason



3. ; P lies outside the quadrilateral

4. S_1 : equation of such line is $\frac{x}{5} + \frac{y}{6} = 2$



$$\Rightarrow \text{Area of } \triangle OAB = \frac{1}{2} \times 10 \times 12 = 60$$

S_2 : In this situation area obtained is least infact.

6. $AB = \sqrt{(8)^2 + (19)^2} = \sqrt{425}$; $AC = \sqrt{(16)^2 + (13)^2}$
 $\therefore \Delta$ is isosceles

8. $ax^3 + bx^2y + cxy^2 + dy^3 = 0$

since this is homogeneous pair represent there straight lines passing through origin

$$ax^3 + bx^2y + cxy^2 + dy^3 = (y - m_1x)(y - m_2x)(y - m_3x)$$

or put $y = mx$ in given equation we get

$$m^3d + cm^2 + bm + a = 0$$

$$m_1 + m_2 + m_3 = -\frac{c}{d}$$

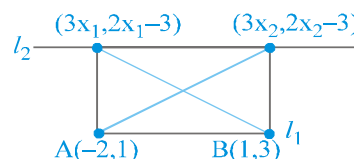
$$m_1m_2 + m_2m_3 + m_3m_1 = \frac{+b}{d}$$

$$m_1m_2m_3 = \frac{+a}{d}$$

given two lines + hence $m_1m_2 = -1 \Rightarrow m_3 = a/d$
 eliminate m_3 from remaining equation

$$10. m_1 = \frac{2}{3}$$

$$m_2 = \frac{2(x_2 - x_1)}{3(x_2 - x_1)} = \frac{2}{3}$$



$$A = \frac{1}{2} \begin{vmatrix} -2 & 1 & 1 \\ 1 & 3 & 1 \\ 3x & (2x-3) & 1 \end{vmatrix} = 8$$

EXERCISE - 3

Part # I : Matrix Match Type

1. (A) Let the lines $4x + 5y = 0$ and $7x + 2y = 0$ represents the sides AB & AD of the parallelogram ABCD, then the vertices of

A, B, D are $(0,0)$, $\left(\frac{5}{3}, -\frac{4}{3}\right)$ and $\left(-\frac{2}{3}, \frac{7}{3}\right)$ respectively

the mid point of BD is $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ the equation of the line passing through $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $(0, 0)$ will be $x - y = 0$ which is the required equation of the other diagonal

So $a = 1, b = -1, c = 0$

$\therefore a + b + c = 0$

- (B) Joint equation of lines OA & OB, O being the origin will be

$$2x^2 - by^2 + (2b - 1)xy - (x + by)(-2x + by) = 0$$

$$\Rightarrow 4x^2 - (b + b^2)y^2 + (3b - 1)xy = 0$$

If these lines are perpendicular then

$$4 - b - b^2 = 0 \Rightarrow b + b^2 = 4$$

- (C) Equation of line passing through intersection of $4x + 3y = 12$ and $3x + 4y = 12$ will be

$$(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$$

$$\text{If passes through } (3, 4) \Rightarrow (12 + \lambda(13)) = 0$$

$$\Rightarrow \lambda = -\frac{12}{13}$$

$\therefore$ Equation of the required line

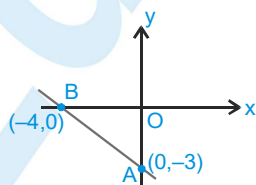
$$16x - 9y - 12 = 0$$

length of intercepts on x and y axes are $\frac{3}{4}$ and $\frac{4}{3}$

So $ab = 1$

2. (A) Slope of such line is ± 1

(B) area of $\Delta OAB = \frac{1}{2} \times 3 \times 4 = 6$ sq. units



(C) To represent pair of straight lines $\begin{vmatrix} 2 & -1 & -3 \\ -1 & -1 & 3 \\ -3 & 3 & c \end{vmatrix} = 0$

$$\Rightarrow c = 3$$

- (D) Lines represented by given equation are $x + y + a = 0$ and $x + y - 9a = 0$

$\therefore$ distance between these parallel lines is

$$= \frac{10a}{\sqrt{2}} = 5\sqrt{2}a$$

Part # II : Comprehension

Comprehension # 5

1. $d(OR) = d(AR)$

$$|x - 0| + |y - 0| = |x - 1| + |y - 2|$$

$$x + y = |x - 1| + |y - 2| \quad (\rightarrow x > 0, y > 0)$$

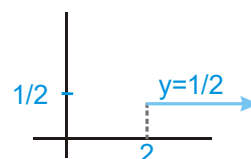
$$x + y = -x + 1 - y + 2$$

$$2x + 2y = 3. \quad (\rightarrow 0 \leq x < 1 \text{ \& } 0 \leq y < 2)$$

2. $d(OS) = d(BS)$

$$|x - 0| + |y - 0| = |x - 2| + |y - 3|$$

$$x + y = x - 2 + 3 - y \quad (\rightarrow x \geq 2 \text{ \& } 0 \leq y < 3).$$



$$y = 1/2.$$

which is an infinite ray

3. $d(TO) = d(TC)$

$$|x - 0| + |y - 0| = |x - 4| + |y - 3|$$

$$x + y = |x - 4| + |y - 3|$$

Case : I $0 \leq x < 4$ & $0 \leq y < 3$.

$$x + y = -x + 4 - y + 3$$

$$x + y = 7/2.$$

Case : II $0 \leq x < 4$ & $y \geq 3$.

$$x + y = -x + 4 + y - 3$$

$$x = 1/2.$$

Case : III $x \geq 4$ & $0 \leq y < 3$.

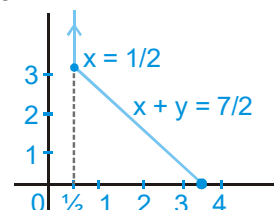
$$x + y = x - 4 - y + 3$$

$$y = -1/2.$$

Case : IV $x \geq 4$ & $y \geq 3$.

$$x + y = x - 4 + y - 3$$

$$0 = -7 \text{ (so rejected)}$$



Comprehension # 6

Slopes of the lines

$$3x + 4y = 5 \text{ is } m_1 = -\frac{3}{4}$$

$$\text{and } 4x - 3y = 15 \text{ is } m_2 = \frac{4}{3}$$

$$\rightarrow m_1 m_2 = -1$$

$\therefore$ given lines are perpendicular and $\angle A = \frac{\pi}{2}$

Now required equation of BC is

$$(y-2) = \frac{m \pm \tan(\pi/4)}{1 \mp m \tan(\pi/4)} (x-1) \dots\dots(i)$$

$$\text{where } m = \text{slope of AB} = -\frac{3}{4}$$

$\therefore$ equation of BC is (on solving (1))

$$x - 7y + 13 = 0 \text{ and } 7x + y - 9 = 0$$

$$L_1 \equiv x - 7y + 13 = 0$$

$$L_2 \equiv 7x + y - 9 = 0$$

1. $c + f = 4$

2. Equation of a straight line

through (2, 3) and inclined at an angle of $(\pi/3)$ with y-axis
($(\pi/6)$ with x-axis) is

$$\frac{x-2}{\cos(\pi/6)} = \frac{y-3}{\sin(\pi/6)} \Rightarrow x - \sqrt{3}y = 2 - 3\sqrt{3}$$

Points at a distance $c + f = 4$ units from point P are

$$(2 + 4 \cos(\pi/6), 3 + 4 \sin(\pi/6)) \equiv (2 + 2\sqrt{3}, 5)$$

$$\text{and } (2 - 4 \cos(\pi/6), 3 - 4 \sin(\pi/6)) \equiv (2 - 2\sqrt{3}, 1)$$

only (A) is true out of given options

3. Let required line be $x + y = a$

which is at $|b - 2a - 1| = |5 - 4 - 4\sqrt{3} - 1| = 4\sqrt{3}$ units from origin

$\therefore$ required line is $x + y - 4\sqrt{6} = 0$ (since intercepts are on positive axes only)

EXERCISE - 4

Subjective Type

3. $ax^2 + 2hxy + by^2 = (y - m_1x)(y - m_2x)$

given that $m_2 = m_1^n$

$$\text{Hence } m_1 + m_2 = -\frac{2h}{b} \Rightarrow m_1 + m_1^n = -\frac{2h}{b}$$

$$\Rightarrow m_1 \cdot m_1^n = \frac{a}{b} \Rightarrow m_1 = \left(\frac{a}{b}\right)^{\frac{1}{1+n}}$$

Eliminate m_1 from both.

4. The combined equation of AB and AD is

$$S_1 \equiv ax^2 + 2hxy + by^2 = 0$$

Now equation of lines through (p,q) and parallel to $S_1 = 0$ is

$$S_2 \equiv a(x-p)^2 + 2h(x-p)(y-q) + b(y-q)^2 = 0$$

Hence equation of diagonal BD is $S_1 - S_2 = 0$

$$\Rightarrow (2x-p)(ap+hq) + (2y-q)(hp+bq) = 0$$

5. Consider a line $\bullet x + my + n = 0$

point $\left(\frac{r^3}{r-1}, \frac{r^2-3}{r-1}\right)$ lies on the above line

$$\therefore \bullet \left(\frac{r^3}{r-1}\right) + m\left(\frac{r^2-3}{r-1}\right) + n = 0$$

$$\bullet r^3 + mr^2 + nr - (3m+n) = 0$$

a, b, c are the roots of the equation.

$$a+b+c = \frac{-m}{1}, ab+bc+ca = \frac{n}{1}, abc = \frac{3m+n}{1}$$

Now taking LHS

$$3(a+b+c) = \frac{-3m}{1}$$

RHS

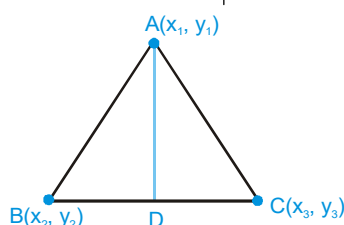
$$ab+bc+ca-abc = \frac{n}{1} - \left(\frac{3m+n}{1}\right) = -\frac{3m}{1}$$

10. (i) D is mid point of BC Hence co-ordinates of D are

$$\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

Therefore, equation of the median AD is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2 + x_3}{2} & \frac{y_2 + y_3}{2} & 1 \end{vmatrix} = 0$$



Applying $R_3 \rightarrow 2R_3$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 + x_3 & y_2 + y_3 & 2 \end{vmatrix} = 0$$

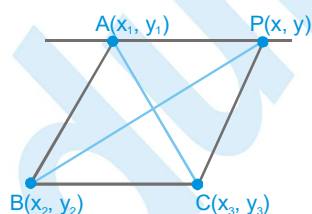
$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

(using the addition property of determinants)

- (ii) Let $P(x, y)$ be any point on the line parallel to BC

Area of $\triangle ABP$ = Area of $\triangle ACP$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

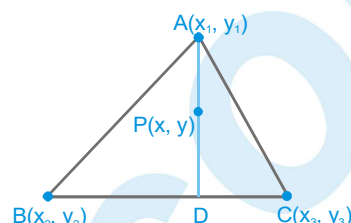
This gives the equation of line AP.

- (iii) Let AD be the internal bisector of angle A,

$$\therefore \frac{BD}{DC} = \frac{BA}{CA} = \frac{c}{b}$$

$$\therefore D \equiv \left(\frac{cx_3 + bx_2}{c+b}, \frac{cy_3 + by_2}{c+b} \right)$$

Let $P(x, y)$ be any point on AD then P, A, D are collinear



$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{cx_3 + bx_2}{b+c} & \frac{cy_3 + by_2}{b+c} & 1 \end{vmatrix} = 0$$

$R_3 \rightarrow (b+c) R_3$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3 + bx_2 & cy_3 + by_2 & b+c \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3 & cy_3 & c \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ bx_2 & by_2 & b \end{vmatrix} = 0$$

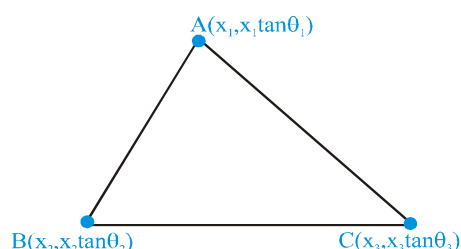
(Addition property)

$$\Rightarrow c \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + b \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

This is the equation of AD.

11. Circumcentre is origin

$$\therefore OA^2 = OB^2 = OC^2$$



$$x_1^2 + x_1^2 \tan^2 \theta_1 = x_2^2 + x_2^2 \tan^2 \theta_2$$

$$= x_3^2 + x_3^2 \tan^2 \theta_3 = r^2$$

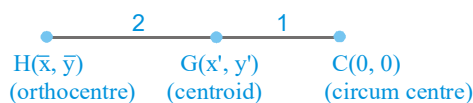
$$x_1 = r \cos \theta_1, x_2 = r \cos \theta_2, x_3 = r \cos \theta_3$$

∴ co-ordinate of vertices of the triangle become -

$$A(r \cos \theta_1, r \sin \theta_1), B(r \cos \theta_2, r \sin \theta_2),$$

$$C(r \cos \theta_3, r \sin \theta_3)$$

$$x' = \frac{\sum r \cos \theta_i}{3}, \quad y' = \frac{\sum r \sin \theta_i}{3}$$



$$\text{Now, } x' = \frac{0 + \bar{x}}{3}$$

$$\bar{x} = r(\cos \theta_1 + \cos \theta_2 + \cos \theta_3)$$

$$\bar{y} = r(\sin \theta_1 + \sin \theta_2 + \sin \theta_3)$$

$$\therefore \frac{\bar{x}}{\bar{y}} = \frac{\cos \theta_1 + \cos \theta_2 + \cos \theta_3}{\sin \theta_1 + \sin \theta_2 + \sin \theta_3}$$

13. Let point of intersection of lines is (x, y) using parametric form of line

$$\frac{x-2}{\cos \theta} = \frac{y-1}{\sin \theta} = 3$$

$$x = 3 \cos \theta + 2, \quad y = 3 \sin \theta + 1$$

This point satisfy equation of line

$$4y - 4x + 4 + 3\sqrt{2} + 3\sqrt{10} = 0$$

$$12(\sin \theta - \cos \theta) = -3\sqrt{2}(1 + \sqrt{5})$$

$$\sin \theta - \cos \theta = -\frac{(1 + \sqrt{5})}{2\sqrt{2}}$$

$$\Rightarrow \cos(\theta + 45^\circ) = -\frac{(1 + \sqrt{5})}{4} \quad \dots (i)$$

$$\Rightarrow \cos(\theta + 45^\circ) = \cos(180^\circ - 36^\circ)$$

$$\Rightarrow \cos(\theta + 45^\circ) = \cos 144^\circ \Rightarrow \theta = 99^\circ$$

Now from (i)

$$\cos(\theta + 45^\circ) = \cos(180^\circ + 36^\circ) \Rightarrow \theta = 171^\circ$$

14. $y + 2at = tx - at^3$

slope = t.

Let it passes through P(h, k)

$$\therefore k + 2at = th - at^3$$

$$at^3 + t(2a - h) + k = 0 \quad \dots (1)$$

$$t_1 t_2 t_3 = -\frac{k}{a} \quad \{t_1 t_2 = -1\}$$

$$t_3 = \frac{k}{a}$$

Substituting t_3 in (1) we can get the desired locus

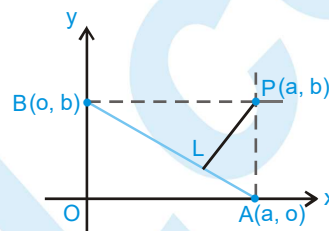
16. $a^2 + b^2 = c^2 \quad \dots (i)$

Let L is (x_1, y_1)

L is foot of perpendicular from point P(a, b) on line AB

equation of AB is $bx + ay - ab = 0$

$$\Rightarrow \frac{x_1 - a}{b} = \frac{y_1 - b}{a} = \frac{-(ab + ab - ab)}{a^2 + b^2}$$



$$\frac{x_1 - a}{b} = \frac{y_1 - b}{a} = \frac{-ab}{c^2}$$

$$\Rightarrow x_1 - a = \frac{ab^2}{c^2} = \frac{a(c^2 - b^2)}{c^2} = a^3/c^2 \Rightarrow a^3 = c^2 x_1 \quad \dots (ii)$$

$$\text{similarly } b^3 = c^2 y_1 \quad \dots (iii)$$

using these relations (ii) & (iii) in equation (i), we get required locus.

20. Since A(4, 2) and B(2, 4) both lie on same side of $3x + 2y + 10 = 0$

- (i) $PA + PB \geq AB$

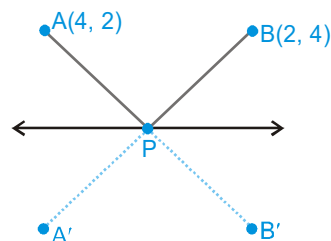
$$PA + PB' \geq AB \Rightarrow PA + PB = PA + PB' (\text{min.}) = AB$$

Hence A, P, B' are collinear.

Image of B(2, 4) in $3x + 2y + 10 = 0$ is ... (i)

$$\frac{x-2}{3} = \frac{y-4}{2} = -2 \left(\frac{6+8+10}{3^2+2^2} \right)$$

$$\Rightarrow B'(x, y) \left(-\frac{118}{13}, -\frac{44}{13} \right)$$



EXERCISE - 5

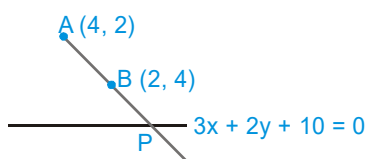
Part # I : AIEEE/JEE-MAIN

now equation of AB' is $y - 2 = \frac{2 + \frac{49}{13}}{4 + \frac{118}{13}} (x - 4)$

$\Rightarrow 7x - 17y + 6 = 0 \dots\dots(ii)$

solving (i) and (ii) we get $\left(-\frac{14}{5}, -\frac{4}{5}\right)$

(ii) in any triangle.



$|PA - PB| \leq AB$

Hence $|PA - PB| = AB$ when P, A, B are collinear

Hence equation of AB is

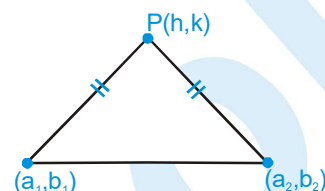
$y - 2 = -1(x - 4)$

$x + y - 6 = 0 \dots\dots(i)$

solving (i) with $3x + 2y + 10 = 0$

we get $(-22, 28)$

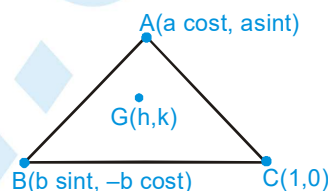
1. $(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$
 $2h(a_1 - a_2) + 2k(b_1 - b_2) + (a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$



compare with $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$

$c = \frac{(a_2^2 + b_2^2 - a_1^2 - b_1^2)}{2}$

2. $3h - 1 = a \cos t + b \sin t$
 $3k = a \sin t - b \cos t$
 squaring and add. (Locus)



$(3x - 1)^2 + 9y^2 = a^2 + b^2$

3. $x^2 - 2pxy - y^2 = 0$

pair of angle bisector of this pair $\frac{x^2 - y^2}{1 - (-1)} = \frac{xy}{-p}$

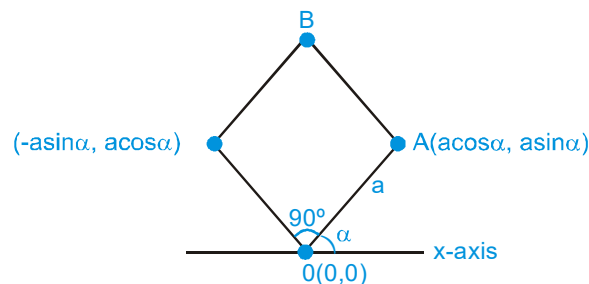
$\Rightarrow x^2 - y^2 + \frac{2}{p}xy = 0$

compare this bisector pair with $x^2 - 2qxy - y^2 = 0$

$\frac{2}{p} = -2q \Rightarrow pq = -1$

4. Equation of AC

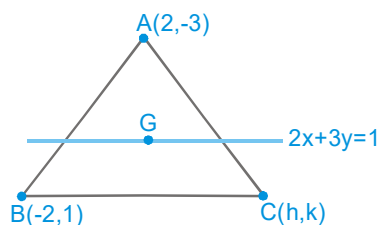
$y - a \sin \alpha = \frac{\sin \alpha - \cos \alpha}{\cos \alpha + \sin \alpha} (x - a \cos \alpha)$



$y(\cos \alpha + \sin \alpha) + x(\cos \alpha - \sin \alpha)$
 $= a(\sin \alpha \cos \alpha + \sin^2 \alpha - \sin \alpha \cos \alpha + \cos^2 \alpha)$
 $y(\cos \alpha + \sin \alpha) + x(\cos \alpha - \sin \alpha) = a$

5. $G\left(\frac{h}{3}, \frac{k-2}{3}\right)$

$$\Rightarrow \frac{2h}{3} + (k-2) = 1 \Rightarrow 2h + 3k = 9$$



Locus $2x + 3y = 9$.

6. Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

it passes through (4, 3) $\frac{4}{a} + \frac{3}{b} = 1$

sum of intercepts is -1

$$\Rightarrow a + b = -1 \Rightarrow a = -1 - b$$

$$\Rightarrow \frac{4}{-1-b} + \frac{3}{b} = 1 \Rightarrow 4b - 3 - 3b = -b - b^2$$

$$\Rightarrow b^2 + 2b - 3 = 0 \Rightarrow b = -3, 1$$

$b = 1, a = -2$ $\frac{x}{-2} + \frac{y}{1} = 1$

$b = -3, a = 2$ $\frac{x}{2} + \frac{y}{-3} = 1$.

7. $x^2 - 2cxy - 7y^2 = 0$

sum of the slopes $m_1 + m_2 = \frac{2c}{-7}$

Product of slopes $m_1 m_2 = \frac{-1}{7}$

given $m_1 + m_2 = 4m_1 m_2 \Rightarrow \frac{2c}{-7} = \frac{-4}{7} \Rightarrow c = 2$.

8. Pair $6x^2 - xy + 4cy^2 = 0$ has its one line $3x + 4y = 0$

$$\Rightarrow y = \frac{-3x}{4}$$

$$6x^2 + \frac{3x^2}{4} + 4c \frac{9x^2}{16} = 0 \Rightarrow 24x^2 + 3x^2 + 9cx^2 = 0$$

$$\Rightarrow c = -3.$$

9. $ax + 2by + 3b = 0$

$$bx - 2ay - 3a = 0$$

$$\frac{x}{-6ab + 6ab} = \frac{y}{3b^2 + 3a^2} = \frac{1}{-2a^2 - 2b^2}$$

Hence point of intersection $(0, -3/2)$

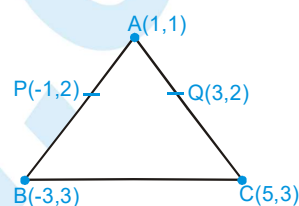
Line parallel to x-axis $y = -3/2$.

10. $\rightarrow$ a, b, c are in H.P. $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

given line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$

Clearly line passes through $(1, -2)$.

11. Centroid is $\left(1, \frac{7}{3}\right)$



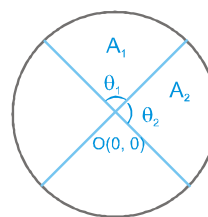
12. Pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$

Area of sector $A_1 = \frac{1}{2}r^2\theta_1$

$$A_2 = \frac{1}{2}r^2\theta_2$$

$$\theta_1 + \theta_2 = 180^\circ$$

given $A_1 = 3A_2 \Rightarrow \theta_1 = 3\theta_2$
 $\Rightarrow \theta_2 = 45^\circ, \theta_1 = 135^\circ$



Angle between lines is $= \left| \frac{2\sqrt{(a+b)^2 - ab}}{a+b} \right| = 1$

$$\Rightarrow 4(a^2 + b^2 + ab) = a^2 + b^2 + 2ab$$

$$\Rightarrow 3a^2 + 3b^2 + 2ab = 0.$$

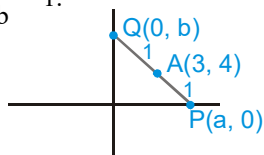
13. Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$.

By section formula

$$\frac{a}{2} = 3 \Rightarrow a = 6$$

$$\frac{b}{2} = 4 \Rightarrow b = 8$$

$$\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24.$$

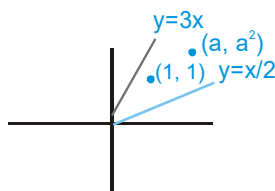


14. Since (1, 1) and (a, a²) Both lies same side with respect to both lines

$$a - 2a^2 < 0 \Rightarrow 2a^2 - a > 0$$

$$\Rightarrow a(2a - 1) > 0$$

$$a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$$



$$3a - a^2 > 0 \Rightarrow a^2 - 3a < 0 \Rightarrow a \in (0, 3)$$

$$\text{Hence after taking intersection } a \in \left(\frac{1}{2}, 3\right).$$

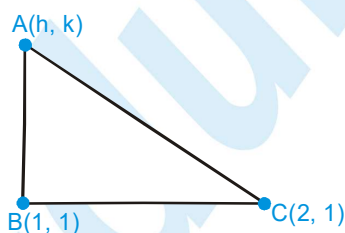
15. $AB = \sqrt{(h-1)^2 + (k-1)^2}$

$$BC = 1$$

$$AC = \sqrt{(h-2)^2 + (k-1)^2}$$

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow (h-1)^2 + (k-1)^2 + 1 = (h-2)^2 + (k-1)^2$$



$$\Rightarrow 2h = 2 \Rightarrow h = 1$$

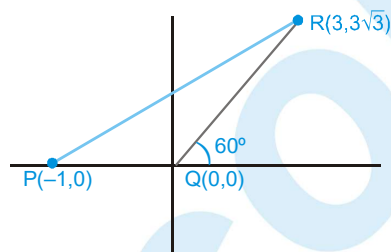
$$\text{Area of } \triangle ABC = \frac{1}{2} \sqrt{(h-1)^2 + (k-1)^2} \times 1 = 1$$

$$(K-1)^2 = 4 \Rightarrow k-1 = \pm 2 \Rightarrow k = 3, -1.$$

16. The line segment QR makes an angle 60° with the positive direction of x-axis.
hence bisector of angle PQR will make 120° with +ve direction of x-axis.

∴ Its equation

$$y - 0 = \tan 120^\circ (x - 0)$$



$$y = -\sqrt{3}x$$

$$x\sqrt{3} + y = 0$$

17. Bisector of $x = 0$ and $y = 0$ is either $y = x$ or $y = -x$

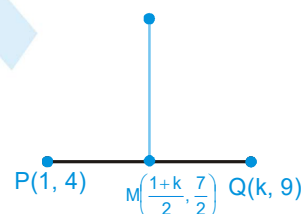
If $y = x$ is Bisector, then

$$mx^2 + (1 - m^2)x^2 - mx^2 = 0 \Rightarrow m + 1 - m^2 - m = 0$$

$$\Rightarrow m^2 = 1 \Rightarrow m = \pm 1.$$

18. Slope of PQ = $\frac{1}{1-k}$

Hence equation of $\perp$ to line PQ



$$y - \frac{7}{2} = (k-1) \left(x - \frac{(1+k)}{2} \right)$$

Put $x = 0$

$$y = \frac{7}{2} + \frac{(1-k)(1+k)}{2} = -4$$

$$7 + (1 - k^2) = -8$$

$$\Rightarrow k^2 = 16 \Rightarrow k = \pm 4.$$

Hence possible answer = -4.

19. $p(p^2 + 1)x - y + q = 0$

$(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular for a common line

∴ lines are parallel ∴ slopes are equal

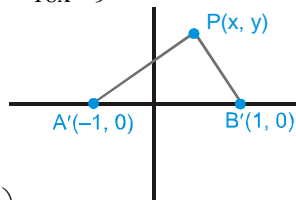
$$\therefore \frac{p(p^2 + 1)}{1} = -\frac{(p^2 + 1)^2}{(p^2 + 1)} \Rightarrow p = -1$$

20. $\therefore \frac{PA'}{PB'} = \frac{3}{1}$

$$\begin{aligned} \therefore (x+1)^2 + y^2 &= 9((x-1)^2 + y^2) \\ x^2 + 2x + 1 + y^2 &= 9x^2 + 9y^2 - 18x + 9 \\ 8x^2 + 8y^2 - 20x + 8 &= 0 \end{aligned}$$

$$x^2 + y^2 - \frac{10}{4}x + 1 = 0$$

$$\therefore \text{circumcentre} \left(\frac{5}{4}, 0 \right).$$



21. $\frac{x}{5} + \frac{y}{b} = 1$

$$\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5} \Rightarrow b = -20$$

$$\frac{x}{5} - \frac{y}{20} = 1 \Rightarrow 4x - y = 20$$

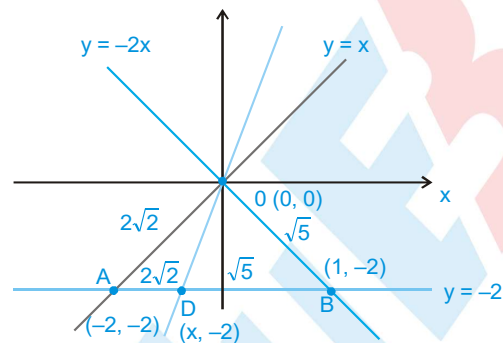
Line K has same slope $\Rightarrow -\frac{3}{c} = 4$

$$c = -\frac{3}{4} \Rightarrow 4x - y = -3$$

$$\text{distance} = \frac{23}{\sqrt{17}}$$

Hence correct option is (3)

22.



$$\therefore AD : DB = 2\sqrt{2} : \sqrt{5}$$

→ OD is angle bisector of angle AOB

$\therefore$ St : 1 true

St. 2 false (obvious)

23. $x + y = |a|$
 $ax - y = 1$

If $a > 0$

$$x + y = a$$

$$ax - y = 1$$

$$x(1+a) = 1+a \text{ as } x = 1$$

$$y = a - 1$$

It is in the first quadrant

So $a - 1 \geq 0$

$$a \geq 1$$

$$a \in [1, \infty)$$

If $a < 0$

$$x + y = -a$$

$$ax - y = 1$$

+

$$x(1+a) = 1-a$$

$$x = \frac{1-a}{1+a} > 0 \Rightarrow \frac{a-1}{a+1} < 0$$



..... (i)

$$\begin{aligned} y &= -a - \frac{1-a}{1+a} \\ &= \frac{-a - a^2 - 1 + a}{1+a} > 0 \end{aligned}$$

$$-\left(\frac{a^2+1}{a+1}\right) > 0 \Rightarrow \frac{a^2+1}{a+1} < 0$$



..... (ii)

from (1) and (2) $a \in \{\phi\}$

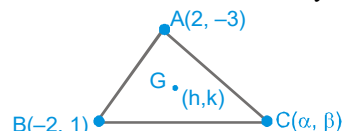
correct answer is $a \in [1, \infty)$

24. $\alpha = 3h$

$$\beta - 2 = 3k$$

$$\beta = 3k + 2$$

third vertex on the line $2x + 3y = 9$



$$2\alpha + 3\beta = 9$$

$$2(3h) + 3(3k + 2) = 9$$

$$2h + 3k = 1$$

$$2x + 3y - 1 = 0$$

25. $\therefore C\left(\frac{8}{5}, \frac{14}{5}\right)$



Line $2x + y = k$ passes $C\left(\frac{8}{5}, \frac{14}{5}\right)$

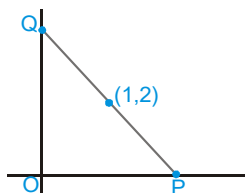
$$\frac{2 \times 8}{5} + \frac{14}{5} = k$$

$$k = 6$$

26. $(y-2) = m(x-1)$

$$OP = 1 - \frac{2}{m}$$

$$OQ = 2 - m$$



$$\text{Area of } \Delta POQ = \frac{1}{2} (OP)(OQ) = \frac{1}{2} \left(1 - \frac{2}{m}\right) (2 - m)$$

$$= \frac{1}{2} \left[2 - m - \frac{4}{m} + 2\right]$$

$$= \frac{1}{2} \left[4 - \left(m + \frac{4}{m}\right)\right]$$

$$m = -2$$

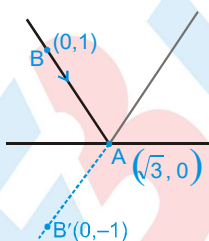
27. Take any point $B(0, 1)$ on given line
Equation of AB'

$$y - 0 = \frac{-1 - 0}{0 - \sqrt{3}} (x - \sqrt{3})$$

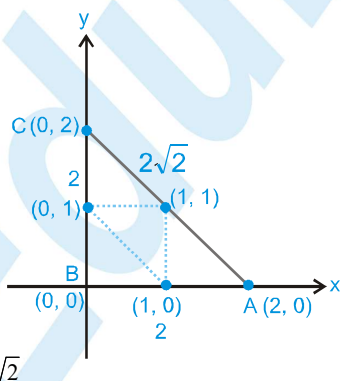
$$-\sqrt{3}y = -x + \sqrt{3}$$

$$x - \sqrt{3}y = \sqrt{3}$$

$$\Rightarrow \sqrt{3}y = x - \sqrt{3}$$

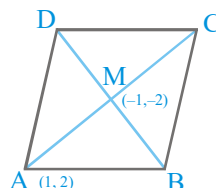


28. x - coordinate of incentre $= \frac{2 \times 0 + 2\sqrt{2} \cdot 0 + 2 \cdot 2}{2 + 2 + 2\sqrt{2}} = \frac{2}{2 + \sqrt{2}}$



$$= 2 - \sqrt{2}$$

31. Point of intersection of sides



$$x - y + 1 = 0 \quad \text{and} \quad 7x - y - 5 = 0$$

$$\therefore x = 1, y = 2$$

$$\text{Slope of } AM = \frac{4}{2} = 2$$

$$\therefore \text{Equation of } BD: y + 2 = -\frac{1}{2}(x + 1)$$

$$\Rightarrow x + 2y + 5 = 0$$

$$\text{Solving } x + 2y + 5 = 0 \quad \text{and} \quad 7x - y - 5 = 0$$

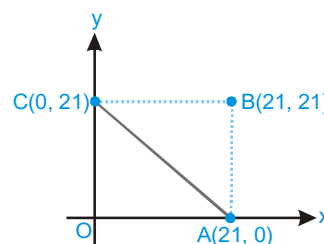
$$x = \frac{1}{3}, y = -\frac{8}{3} \Rightarrow \left(\frac{1}{3}, -\frac{8}{3}\right)$$

Part # II : IIT-JEE ADVANCED

1. The number of integral points that lie in the interior of square OABC is 20×20 . These points are (x, y) where $x, y = 1, 2, \dots, 20$. Out of these 400 points 20 lie on the line AC. Out of the remaining exactly half lie in ΔABC .

$\therefore$ number of integral point in the triangle OAC

$$= \frac{1}{2} [20 \times 20 - 20] = 190$$

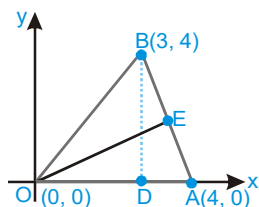


Alternative Solution

There are 19 points that lie in the interior of ΔABC and on the line $x = 1$, 18 point that lie on the line $x = 2$ and so on. Thus, the number of desired points is

$$19 + 18 + 17 + \dots + 2 + 1 = \frac{20 \times 19}{2} = 190.$$

2. Refer Figure



Equation of altitude BD is $x = 3$.

$$\text{slope of AB is } \frac{4-0}{3-0} = \frac{4}{3}$$

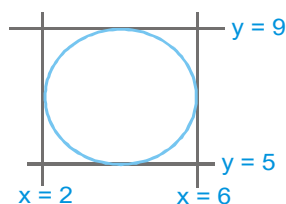
$\therefore$ slope of OE is $\frac{1}{4}$

Equation of OE is

$$y = \frac{1}{4}x$$

Lines BD and OE meet at $(3, 3/4)$

3. The lines given by $x^2 - 8x + 12 = 0$ are $x = 2$ and $x = 6$.



The lines given by $y^2 - 14y + 45 = 0$ are $y = 5$ and $y = 9$
Centre of the required circle is the centre of the square.

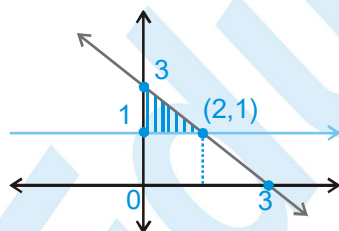
$\therefore$ Required centre is

$$\left(\frac{2+6}{2}, \frac{5+9}{2} \right) = (4, 7)$$

4. $x^2 - y^2 + 2y = 1$

$$x = \pm(y-1)$$

Bisector of above lines are $x = 0, y = 1$



so Area between $x = 0, y = 1$ and $x + y = 3$

$$= \frac{1}{2} \times 2 \times 2 = 2 \text{ sq. units}$$

5. A line passing through $P(h, k)$ and parallel to x-axis is $y = k$.
.....(i)

The other lines given are $y = x$ (ii)

and

$$y + x = 2 \text{(iii)}$$

Let ABC be the Δ formed by the points of intersection of the lines (i), (ii) and (iii).

$$\therefore A(k, k), B(1, 1), C(2-k, k)$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$$

$$C_1 \rightarrow C_1 - C_2 \quad \frac{1}{2} \begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$$

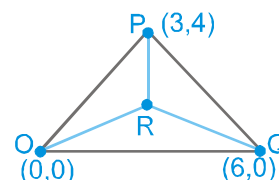
$$\Rightarrow \frac{1}{2} |(2-2k)(k-1)| = 4h^2$$

$$\Rightarrow (k-1)^2 = 4h^2 \Rightarrow k-1 = 2h, k-1 = -2h$$

$$\Rightarrow k = 2h+1 \quad k = -2h+1$$

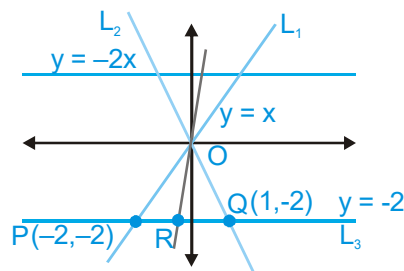
$$\therefore \text{locus of } (h, k) \text{ is } y = 2x+1 \text{ and } y = -2x+1$$

6.



$$R \text{ is centroid hence } R = \left(3, \frac{4}{3} \right)$$

$$7. \frac{PR}{RQ} = \frac{OP}{OQ}$$



$$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$$

but statement - 2 is false

$\therefore$ Ans. (C)

8. $P \equiv (-\sin(\beta - \alpha), -\cos \beta)$
 $Q \equiv (\cos(\beta - \alpha), \sin \beta)$
 $R \equiv (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$

$$0 < \alpha, \beta, \theta < \frac{\pi}{4}$$

$$x_R = \cos(\beta - \alpha) \cos \theta - \sin(\beta - \alpha) \sin \theta$$

$$\Rightarrow x_R = x_Q \cdot \cos \theta + x_P \cdot \sin \theta$$

$$y_R = \sin \beta \cos \theta - \cos \beta \sin \theta$$

$$\Rightarrow y_R = y_Q \cdot \cos \theta + y_P \cdot \sin \theta$$

For P, Q, R to be collinear

$$\sin \theta + \cos \theta = 1$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \text{not possible for the given interval } \theta \in \left(0, \frac{\pi}{4}\right)$$

$\Rightarrow$ non collinear

9. $(1 + p)x - py + p(1 + p) = 0$ (i)

$(1 + q)x - qy + q(1 + q) = 0$ (ii)

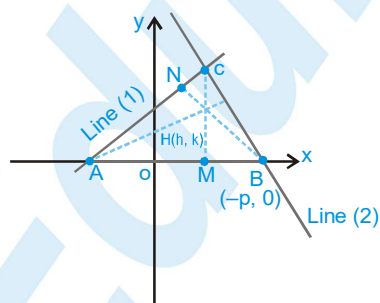
on solving (1) and (2), we get $C(pq, (1 + p)(1 + q))$

$\therefore$ equation of altitude CM passing through C and perpendicular to AB is $x = pq$ (iii)

$\rightarrow$ slope of line (2) is $= \left(\frac{1+q}{q}\right)$

$\therefore$ slope of altitude BN (as shown in figure) is $= \frac{-q}{1+q}$

$\therefore$ equation of BN is $y - 0 = \frac{-q}{1+q}(x + p)$



$$\Rightarrow y = \frac{-q}{(1+q)}(x + p) \text{ (4)}$$

Let orthocentre of triangle be $H(h, k)$ which is the point of intersection of (3) and (4)

$\therefore$ on solving (3) and (4), we get

$$x = pq \text{ and } y = -pq \Rightarrow h = pq \text{ and } k = -pq$$

$$\therefore h + k = 0$$

$\therefore$ locus of $H(h, k)$ is $x + y = 0$

10. Let slope of line $L = m$

$$\therefore \left| \frac{m - (-\sqrt{3})}{1 + m(-\sqrt{3})} \right| = \tan 60^\circ = \sqrt{3} \Rightarrow \left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right| = \sqrt{3}$$

taking positive sign, $m + \sqrt{3} = \sqrt{3} - 3m$

$$m = 0$$

taking negative sign

$$m + \sqrt{3} + \sqrt{3} - 3m = 0$$

$$m = \sqrt{3}$$

As L cuts x -axis $\Rightarrow m = \sqrt{3}$

So L is $y + 2 = \sqrt{3}(x - 3)$

11. (A) or (C) or Bonus

As $a > b > c > 0$

$$\Rightarrow a - c > 0 \text{ and } b > 0$$

$$\Rightarrow a - c > 0 \text{ and } b > 0$$

$$\Rightarrow a + b - c > 0$$

$$\Rightarrow \text{option (A) is correct}$$

Further $a > b$ and $c > 0$

$$\Rightarrow a - b > 0 \text{ and } c > 0$$

$$\Rightarrow a - b > 0 \text{ and } c > 0$$

$$\Rightarrow a - b + c > 0 \Rightarrow \text{option (C) is correct}$$

Aliter

$$(a - b)x + (b - a)y = 0 \Rightarrow x = y$$

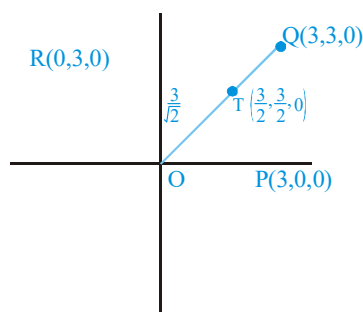
$$\Rightarrow \text{Point of intersection } \left(\frac{-c}{a+b}, \frac{-c}{a+b} \right)$$

Now $\sqrt{\left(1 + \frac{c}{a+b}\right)^2 + \left(1 + \frac{c}{a+b}\right)^2} < 2\sqrt{2}$

$$\Rightarrow \sqrt{2} \left(\frac{a+b+c}{a+b} \right) < 2\sqrt{2}$$

$$\Rightarrow a + b - c > 0$$

12.



$$S \equiv \left(\frac{3}{2}, \frac{3}{2}, 3 \right)$$

$$\overrightarrow{OQ} = 3\hat{i} + 3\hat{j} \Rightarrow \overrightarrow{OS} = \frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 3\hat{k}$$

$$\cos \theta = \frac{\frac{1}{2} + \frac{1}{2}}{\sqrt{2} \sqrt{\frac{1}{2} + \frac{1}{4} + 1}} = \frac{1}{\sqrt{2} \sqrt{\frac{3}{2}}} = \frac{1}{\sqrt{3}}$$

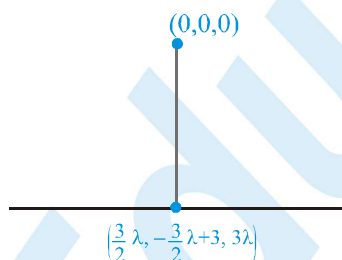
$$\begin{aligned} \vec{r}_n &= \overrightarrow{OQ} \times \overrightarrow{OS} = (\hat{i} + \hat{j}) \times (\hat{i} + \hat{j} + 2\hat{k}) \\ &= \hat{k} - 2\hat{j} - \hat{k} + 2\hat{i} \Rightarrow 2\hat{i} - 2\hat{j} \end{aligned}$$

$$x - y = \lambda \Rightarrow x = y \Rightarrow \perp (3, 0, 0) \Rightarrow \frac{3}{\sqrt{2}}$$

$$RS \rightarrow \frac{x-0}{\frac{3}{2}} = \frac{y-3}{-\frac{3}{2}} = \frac{z-0}{3} = \lambda$$

$$\Rightarrow x = \frac{3}{2}\lambda, y = -\frac{3}{2}\lambda + 3, z = 3\lambda$$

$$T \text{ distance} \Rightarrow \sqrt{\frac{3}{2} - 3 + 9} \Rightarrow \sqrt{\frac{15}{2}}$$



$$D = \frac{9}{4}\lambda^2 + \left(3 - \frac{3}{2}\lambda\right)^2 + 9\lambda^2 = \frac{27}{2}\lambda^2 - 9\lambda + 9$$

$$\Rightarrow \lambda = \frac{9}{27} = \frac{1}{3}$$

MOCK TEST

1. Condition for concurrency $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$$3bc - 4bc - 2a(c - b) + a(4c - 3b) = 0$$

$$-bc + 2ac - ab = 0 \Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

So a, b, c are in H.P.

2. (A)

point of intersection is A (-2, 0). The required line will be one which passes through (-2, 0) and is perpendicular to the line joining (-2, 0) and (2, 3)

3. $x^2(\sec^2 \theta - \sin^2 \theta) - 2xy \tan \theta + y^2 \sin^2 \theta = 0$

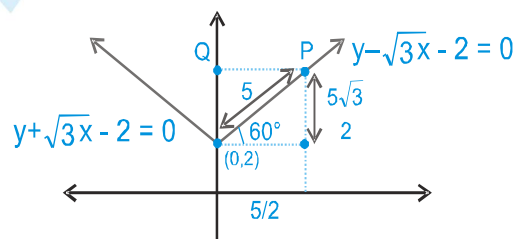
$$|m_1 - m_2| = \sqrt{(m_1 + m_2)^2 - 4m_1 m_2}$$

$$\sqrt{\left(\frac{2 \tan \theta}{\sin^2 \theta}\right)^2 - 4\left(\frac{\sec^2 \theta - \sin^2 \theta}{\sin^2 \theta}\right)} = 2$$

4. (B)

for $x > 0$ $y - \sqrt{3}x - 2 = 0$

$x < 0$ $y + \sqrt{3}x - 2 = 0$



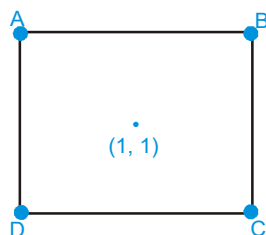
$$P = \left(\frac{5}{2}, \frac{4+5\sqrt{3}}{2} \right) \text{ or } \left(-\frac{5}{2}, \frac{4+5\sqrt{3}}{2} \right)$$

distance of p on its angle bisector i.e.

$$y\text{-axis is } \left(0, \frac{4+5\sqrt{3}}{2} \right)$$

5. To find equations of AB and CD

→ AB and CD are parallel to $3x - 4y = 0$ and at a distance of 2 units from (1, 1)



$$\therefore 3x - 4y + k = 0 \text{ and } \left| \frac{3 - 4 + k}{5} \right| = 2$$

$$\Rightarrow k - 1 = \pm 10 \Rightarrow k = 11, -9$$

$\therefore$ equations of two sides of the square which are parallel to $3x - 4y = 0$ are

$$3x - 4y + 11 = 0 \text{ and } 3x - 4y - 9 = 0$$

Now the remaining two sides will be perpendicular to $3x - 4y = 0$ and at a distance of 2 unit from $(1, 1)$

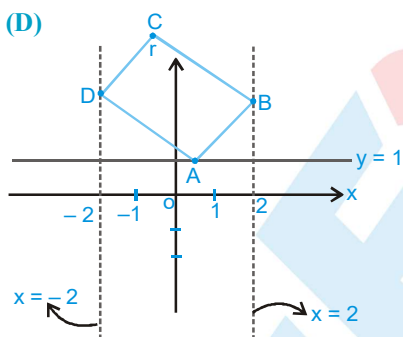
$$\therefore 4x + 3y + k = 0$$

$$\text{and } \left| \frac{4 + 3 + k}{5} \right| = 2 \Rightarrow k + 7 = \pm 10 \Rightarrow k = 3, -17$$

$\therefore$ remaining two sides are

$$4x + 3y + 3 = 0 \text{ and } 4x + 3y - 17 = 0$$

6. (D)



Let equation of AB be $y = x + a$

$$\therefore A(1 - a, 1) \text{ and } B(2, 2 + a)$$

$\therefore$ equation of AD is

$$y - 1 = -1(x - 1 + a)$$

$$\therefore D(-2, 4 - a)$$

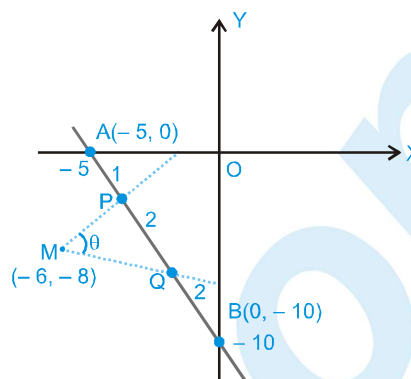
Let $C(h, k)$ mid point of AC = mid point of BD

$$\Rightarrow h + 1 - a = 2 - 2 \Rightarrow h = a - 1$$

$$\text{and } k + 1 = 2 + a + 4 - a \Rightarrow k = 5$$

$\therefore$ Locus of $C(h, k)$ is $y = 5$

7.



$$\therefore P \equiv (-4, -2) \text{ and } Q \equiv (-2, -6)$$

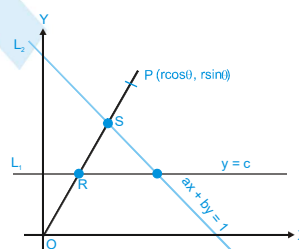
$\therefore$ Let slopes of PM and QM be m_1 and m_2 respectively.

$$\therefore m_1 = 3 \text{ and } m_2 = \frac{1}{2}$$

Let ' θ ' be the acute angle between PM and QM

8. (A)

Let O be taken as the origin and a line through O parallel to L_1 as the x-axis and the line through O perpendicular to x-axis as y-axis (figure).



Let equations of L_1 and L_2 in this system of coordinates be $y = c$ and $ax + by = 1$ respectively, where a, b, c are fixed constants.

Let equation of the variable line through O be

$$\frac{x}{\cos \theta} = \frac{y}{\sin \theta} = r$$

Then $(r \cos \theta, r \sin \theta)$ are the coordinates of a point on this line at a distance r from the origin O.

Let $OP = r$, $OR = r_1$ and $OS = r_2$ so that coordinates of P, R and S are respectively

$$(r \cos \theta, r \sin \theta), (r_1 \cos \theta, r_1 \sin \theta) \text{ and } (r_2 \cos \theta, r_2 \sin \theta).$$

Since R lies on L_1 , $r_1 \sin \theta = c$ and S lies on L_2 , $a \cdot r_2 \cos \theta + b \cdot r_2 \sin \theta = 1$.

$$\text{so that } r_1 = \frac{c}{\sin \theta} \text{ and } r_2 = \frac{1}{a \cos \theta + b \sin \theta} \dots (1)$$

Now we are given $\frac{m+n}{OP} = \frac{m}{OR} + \frac{n}{OS}$

$$\Rightarrow \frac{m+n}{r} = \frac{m}{r_1} + \frac{n}{r_2}$$

$$\Rightarrow \frac{m+n}{r} = \frac{m \sin \theta}{c} + n (\frac{a \cos \theta}{c} + \frac{b \sin \theta}{c})$$

[from (1)]

$$\Rightarrow (m+n)c = m r \sin \theta + n (a \cos \theta + b \sin \theta)$$

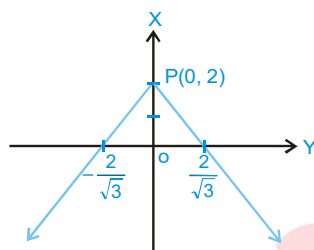
Therefore locus of P (r cos θ, r sin θ) is

$$cn(ax + by - 1) + m(y - c) = 0$$

which is a straight line passing through the intersection of $L_1 : y - c = 0$ and $L_2 : ax + by = 1$

9. ➔ point of intersection of the two rays is P(0, 2)

$$\therefore \text{Point A is } \left(\frac{2}{\sqrt{3}}, 0\right) \text{ or } \left(-\frac{2}{\sqrt{3}}, 0\right)$$



and PO is bisector of the angle between two rays

∴ required point is (0, 0)

$$\therefore \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \Rightarrow \tan \theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

10. (D)

S_1 : Image of (2, 1) in the line $x + 1 = 0$ is (-4, 1)

∴ S_1 is false

S_2 : $\bullet + m = 4$

$$\therefore \frac{1+m}{2} = 2 \quad \therefore S_2 \text{ is true}$$

S_3 : A (10, 20), B(22, 25), C(10, 25)

$$AB^2 = (22-10)^2 + (25-20)^2 = 169, BC^2 = 12^2 + 0 = 144,$$

$$CA^2 = 0^2 + 5^2 = 25$$

ABC is right angled triangle

Hence (10, 25) is orthocentre ∴ S_3 is true

S_4 : Equation of pair of bisectors of angles between lines $ax^2 - 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{-h}$$

$$\Rightarrow -h(x^2 - y^2) = (a - b)xy$$

but $y = mx$ is one of these lines, then it will satisfy it.

Substituting $y = mx$ in (1)

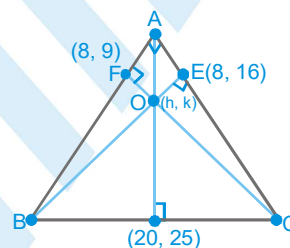
$$-h(x^2 - m^2 x^2) = (a - b)x \cdot mx$$

$$\text{Dividing by } x^2, h(1 - m^2) + m(a - b) = 0$$

11. Orthocentre O of the ΔABC is the incentre of the pedal ΔDEF .

$$ED = \sqrt{(20-8)^2 + (25-16)^2} = 1$$

$$FD = 20, EF = 7$$



$$H = \frac{7 \times 20 + 20 \times 8 + 15 \times 8}{7 + 20 + 15} = 10$$

$$K = \frac{7 \times 25 + 20 \times 16 + 15 \times 9}{7 + 20 + 15} = 15$$

$$O(10, 15)$$

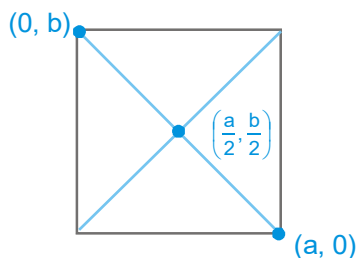
$$AC \equiv y - 2x = 0$$

$$AB \equiv 3y + x - 35 = 0$$

$$BC \equiv x + y - 45 = 0 \Rightarrow A(5, 10), B(50, -5), C(15, 30)$$

12. (A,C)

$$\frac{\frac{x - \frac{a}{2}}{b}}{\sqrt{a^2 + b^2}} = \frac{\frac{y - \frac{b}{2}}{a}}{\sqrt{a^2 + b^2}} = \pm \frac{\sqrt{a^2 + b^2}}{2}$$



$$x = \frac{a}{2} + \frac{b}{2}, y = \frac{b}{2} + \frac{a}{2}$$

$$\text{and } x = \frac{a}{2} - \frac{b}{2}, y = \frac{b}{2} - \frac{a}{2}$$

$$\therefore \text{ the required points are } \left(\frac{a+b}{2}, \frac{a+b}{2} \right)$$

$$\text{and } \left(\frac{a-b}{2}, \frac{b-a}{2} \right)$$

13. Take $A(0, 0)$, $B(a, 0)$, $C(a, a)$ and $D(0, a)$ then $M(a, a/2)$ and $P(a/2, a)$

$$\Delta AMP = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8}$$

$$\Delta MAP = \frac{a^2}{8} \Rightarrow \Delta ABM = \Delta ARP = \frac{a^2}{4}$$

$$\text{Area of quad. AMCP} = \frac{3a^2}{8} + \frac{a^2}{8} = \frac{a^2}{2}$$

14. (A,C)

$$\tan \alpha \tan \beta = -1$$

$$\Rightarrow \cos(\alpha - \beta) = 0$$

$$\Rightarrow \alpha - \beta = \frac{\pi}{2}$$

16. (B)

$$\text{Put } 2h = -(a+b) \text{ in } ax^2 + 2hxy + by^2 = 0$$

$$\Rightarrow ax^2 - (a+b)xy + by^2 = 0$$

$$\Rightarrow (x-y)(ax-by) = 0$$

$\Rightarrow$ one of the line bisects the angle between co-ordinate axes in positive quadrant.

$$\text{Also put } b = -2h - a \text{ in } ax - by \text{ we have } ax - by = ax - (-2h - a)y = ax + (2h + a)y$$

$$\text{Hence } ax + (2h + a)y \text{ is a factor of } ax^2 + 2hxy + by^2 = 0$$

17. (D)

Statement-II is true (standard result from high school classes)

Statement-I :

Since AB may not be equal to AC,

$\therefore$ perpendicular drawn from A to BC may not bisect BC

$\therefore$ statement is false

18. (B)

$$ax^3 + bx^2y + cxy^2 + dy^3 = 0$$

$$\Rightarrow d\left(\frac{y}{x}\right)^3 + c\left(\frac{y}{x}\right)^2 + b\left(\frac{y}{x}\right) + a = 0$$

$$\Rightarrow dm^3 + cm^2 + bm + a = 0 \quad \dots (i)$$

$$m_1 m_2 m_3 = -a/d$$

$$\Rightarrow m_3 = a/d$$

as two lines are perpendicular, put $m_3 = a/d$ in $\dots (i)$

$$7 \Rightarrow a^2 + ac + bd + d^2 = 0$$

19. (A)

ABC is a right angled triangle, right angled at C as (m_{AC})

$$(m_{BC}) = \left(\frac{-4+2}{5+5} \right) \left(\frac{-4-6}{5-7} \right) = -1$$

Hence circumcentre is mid pt. of AB $\equiv (1, 2)$

20. (B)

$$\text{Bisector at C } \frac{|3x+2y|}{\sqrt{13}} = \frac{|2x+3y+6|}{\sqrt{13}}$$

$$\Rightarrow x - y - 6 = 0 \text{ and } 5x + 5y + 6 = 0$$

according to given equations of sides, internal angle bisector at C will have negative slope.

Image of A will lie on BC with respect to both bisectors.

21. (A) $\rightarrow$ (t), (B) $\rightarrow$ (s), (C) $\rightarrow$ (p), (D) $\rightarrow$ (q)

(A) For point (a, a^2) to lie inside the triangle must satisfy

$$a > 0 \quad \dots (i)$$

$$a^2 > 0 \quad \dots (ii)$$

$$\text{and } a + 2a^2 - 3 < 0 \quad \dots (iii)$$

$$(2a+3)(a-1) < 0$$

$$\Rightarrow a < 1$$

$$\Rightarrow a \in (0, 1)$$

Hence correct answer is t

(B) Since $\angle BCA = 90^\circ$

Points A, O, B, C are concyclic

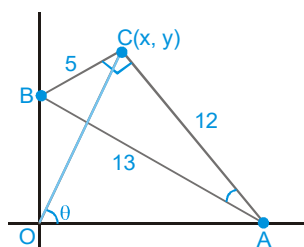
Let $\angle AOC = \theta$

$$\angle BOC = \angle BAC$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{5}{12}$$

$$\frac{x}{y} = \frac{5}{12}$$

$$\Rightarrow 12x - 5y = 0$$



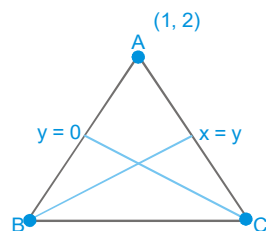
(C) Slope of line joining the point $(t-1, 2t+2)$ and its image

$$(2t+1, t) \text{ is } \frac{(2t+2) - t}{t-1-2t-1} = \frac{t+2}{-(t+2)} = -1.$$

So slope of line is 1

(D) Image of point A(1, 2) in bisector of angles B and C lie on the line BC.

Image of A in $x = y$ is (2, 1) and image of A in $y = 0$ is (1, -2).



So equation of line BC is $y = 3x - 5$

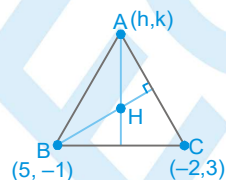
$$\text{So } d(A, BC) = \frac{4}{\sqrt{10}} \text{ So } \sqrt{10} d(A, BC) = 4.$$

22. (A) $\rightarrow$ (p), (B) $\rightarrow$ (q), (C) $\rightarrow$ (s), (D) $\rightarrow$ (s)

$$(A) AH \perp BC. \Rightarrow \left(\frac{k}{h}\right) \left(\frac{3+1}{-2-5}\right) = -1$$

$$4k = 7h \quad \dots (i)$$

$$BH \perp AC. \Rightarrow \left(\frac{0+1}{0-5}\right) \left(\frac{k-3}{h+2}\right) = -1$$



$$k - 3 = 5(h + 2) \quad \dots (ii)$$

$$\Rightarrow 7h - 12 = 20h + 40$$

$$13h = -52$$

$$h = -4$$

$$\therefore k = -7$$

$$\therefore A(-4, -7)$$

$$(B) x + y - 4 = 0 \quad \dots (i)$$

$$4x + 3y - 10 = 0 \quad \dots (ii)$$

Let $(h, 4-h)$ be the point on (i),

$$\text{then } \left| \frac{4h + 3(4-h) - 10}{5} \right| = 1 \quad \text{i.e. } h + 2 = \pm 5$$

$$\text{i.e. } h = 3; h = -7$$

$$\therefore \text{required point is either } (3, 1) \text{ or } (-7, 11)$$

(C) Orthocentre of the triangle is the point of intersection of the lines

$$x + y - 1 = 0 \text{ and } x - y + 3 = 0 \quad \text{i.e. } (-1, 2)$$

(D) Since a, b, c are in A.P.

$$\therefore b = \frac{a+c}{2}$$

$$\therefore \text{the family of lines is } ax + \frac{a+c}{2}y = c$$

$$\text{i.e. } a \left(x + \frac{y}{2} \right) + c \left(\frac{y}{2} - 1 \right) = 0$$

$$\therefore \text{point of concurrency is } (-1, 2)$$

23.

1. (B)

$$\omega = 60^\circ, m = 2$$

$$\tan \theta = \frac{m \sin \omega}{1 + m \cos \omega} = \frac{2 \sin 60^\circ}{1 + 2 \cos 60^\circ} = \frac{2 \times \sqrt{3}/2}{1 + 2 \times 1/2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

2. (D) $\omega = 60^\circ, m_1 = 2, m_2 = -\frac{1}{2}$

$$\tan \theta_1 = \frac{m_1 \sin \omega}{1 + m_1 \cos \omega} = \frac{2 \times \sqrt{3}/2}{1 + 2 \times 1/2} = \frac{\sqrt{3}}{2}$$

$$\tan \theta_2 = \frac{-1/2 \times \sqrt{3}/2}{1 - 1/2 \times 1/2} = \frac{-\sqrt{3}/4}{3/4} = -\frac{1}{\sqrt{3}}$$

Let angle between the lines be ϕ then

$$\tan \phi = \left| \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2} \right| = \left| \frac{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}}{1 - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}} \right|$$

$$\Rightarrow \phi = \tan^{-1} \left(\frac{5}{\sqrt{3}} \right)$$

3. (C) $m = \frac{\sin 60^\circ}{\sin(30^\circ - 60^\circ)} = -\sqrt{3}$
 $\therefore$ equation of the line is $y - 0 = -\sqrt{3}(x - 2)$

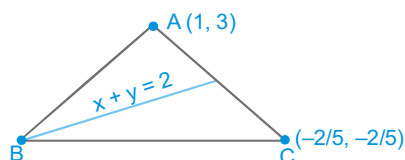
i.e. $\sqrt{3}x + y = 2\sqrt{3}$

24.

1. (B)

Image of A(1, 3) in line $x + y = 2$ is

$$\left(1 - \frac{2(2)}{2}, 3 - \frac{2(2)}{2} \right) \equiv (-1, 1)$$



So line BC passes through $(-1, 1)$ and $\left(-\frac{2}{5}, -\frac{2}{5}\right)$.

Equation of line BC is $y - 1 = \frac{-2/5 - 1}{-2/5 + 1}(x + 1)$

$$\Rightarrow 7x + 3y + 4 = 0$$

2. (C) Vertex B is point of intersection of $7x + 3y + 4 = 0$ and $x + y = 2$ i.e. $B = (-5/2, 9/2)$

3. (A) Line AB is $y - 3 = \frac{3 - 9/2}{1 + 5/2}(x - 1)$
 $\Rightarrow 3x + 7y = 24$

25.

1. $\frac{x-2}{3} = \frac{y-3}{-4} = -15 \cdot \frac{6-12+1}{25} = 3$

$$\therefore x = 11, y = -9$$

$$\therefore \alpha = 2$$

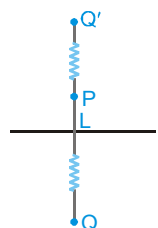
2. $\frac{x-1}{-5} = \frac{y-1}{12} = 26 \cdot \frac{-5+12+6}{169} = 2$

$$x = -9, y = 25$$

$$\therefore \beta = 16$$

3. since $PQ = 16PL$, therefore, $LQ = 15PL$ and so $PQ' = 14PL$.

Thus $n = 14$ for the point Q' .



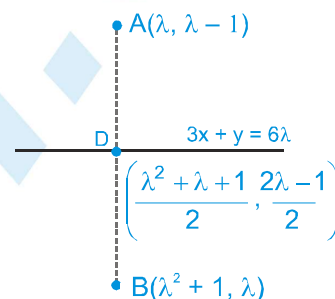
Since L and Q' are on opposite sides of P

$$\therefore \frac{x-2}{1} = \frac{y+1}{-1} = 14 \cdot \frac{2+1+1}{2} = 28 \quad \therefore Q'(30, -29)$$

26. D is mid point of AB and lies on the line $3x + y = 6\lambda$

$$\Rightarrow 3 \cdot \frac{\lambda^2 + \lambda + 1}{2} + \frac{2\lambda - 1}{2} = 6\lambda$$

$$3\lambda^2 - 7\lambda + 2 = 0 \quad \dots\dots\dots(1)$$



$$\lambda = \frac{1}{3}, 2$$

multiplication of slope of AB & line = -1

$$\frac{-1}{\lambda - \lambda^2 - 1}(-3) = -1$$

$$\lambda^2 - \lambda - 2 = 0 \quad \dots\dots\dots(2)$$

$$\lambda = -1, 2$$

$\lambda = 2$ satisfies both (1) & (2)

27. Let the line (L) through the origin is

$$x = r \cos \theta$$

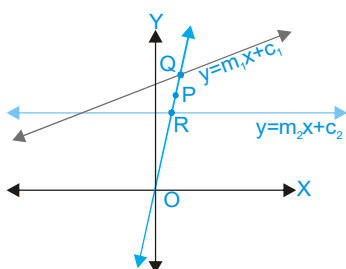
$$y = r \sin \theta$$

as L intersects L_1 at Q and $OQ = r_1$

$$\therefore r_1 \sin \theta = m_1 r_1 \cos \theta + c_1 \quad \dots\dots\dots(1)$$

similarly, L intersects L_2 at R and $OR = r_2$

$$r_2 \sin \theta = m_2 r_2 \cos \theta + c_2 \quad \dots\dots\dots(2)$$



Let $P \equiv (h, k)$ & $OP = r$

$$\therefore r^2 = r_1 r_2 \quad \dots\dots\dots(3)$$

$$\& h = r \cos \theta \quad \dots\dots\dots(4)$$

$$k = r \sin \theta \quad \dots\dots\dots(5)$$

putting the values of r_1 and r_2 from (1) and (2) in (3)

$$\therefore r^2 = \frac{c_1}{(\sin \theta - m_1 \cos \theta)} \cdot \frac{c_2}{(\sin \theta - m_2 \cos \theta)} \quad \dots\dots\dots(6)$$

putting the value of $\cos \theta$ and $\sin \theta$ from (4) and (5) in (6), we get

$$\Rightarrow r^2 = \frac{c_1 c_2}{\left(\frac{k}{r} - m_1 \frac{h}{r}\right) \left(\frac{k}{r} - m_2 \frac{h}{r}\right)}$$

$$\Rightarrow (k - m_1 h)(k - m_2 h) = c_1 c_2$$

replacing (h, k) by (x, y) we get the desired locus

$$\text{as } (y - m_1 x)(y - m_2 x) = c_1 c_2$$

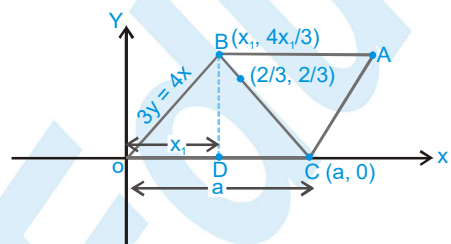
28. (6)

Let $OC = a$

$$\therefore OC = CA = AB = BO = a$$

$$\text{Let } \left(x_1, \frac{4x_1}{3}\right) \therefore A\left(a + x_1, \frac{4x_1}{3}\right)$$

$$\rightarrow x_1^2 + \frac{16x_1^2}{9} = a^2 \quad (\rightarrow ODB \text{ is a right angle triangle})$$



$$\therefore a = \frac{5x_1}{3}$$

$\rightarrow$ equation of BC is

$$y - 0 = \frac{\frac{4x_1}{3} - 0}{x_1 - a} (x - a) \rightarrow a = \frac{5x_1}{3}$$

$$\therefore y = -2x + \frac{10x_1}{3}$$

$$\therefore BC \text{ passes through } \left(\frac{2}{3}, \frac{2}{3}\right)$$

$$\therefore x_1 = 3/5 \quad \therefore a = 1$$

$$\therefore A\left(1 + \frac{3}{5}, \frac{4}{3} \times \frac{3}{5}\right)$$

$$\therefore A\left(\frac{8}{5}, \frac{4}{5}\right) \quad \therefore \frac{5}{2}(\alpha + \beta) = 6.$$

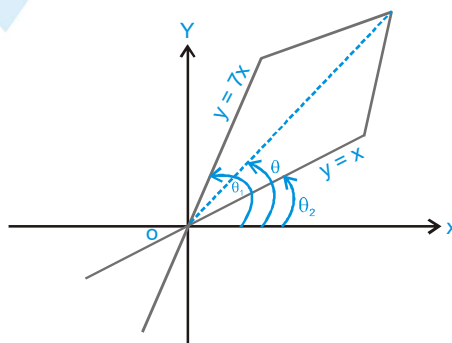
29. (2)

$$\rightarrow \theta_1 - \theta = \theta - \theta_2 \Rightarrow 2\theta = \theta_1 + \theta_2$$

$$\therefore \tan 2\theta = \frac{\tan \theta_1 + \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2}$$

$$\Rightarrow \tan 2\theta = \frac{7+1}{1-7} \Rightarrow \tan 2\theta = \frac{-4}{3}$$

$$\Rightarrow \frac{2 \tan \theta}{1 - \tan^2 \theta} = -\frac{4}{3}$$



$$\Rightarrow \tan \theta = 2 \text{ or } -\frac{1}{2}$$

$\therefore$ slope of longer diagonal is = 2

30. Consider region OABC, x-coordinate downward very from 0 to -10

$$\left. \begin{array}{lll} (0, 0) & (0, 1) & \dots\dots\dots (0, 10) \rightarrow 11 \\ (-1, 0) & (-1, 1) & \dots\dots\dots (-1, 10) \rightarrow 11 \\ (-10, 0) & (-10, 1) & (-10, 2), (-10, 10) \rightarrow 11 \end{array} \right\} 11^2$$

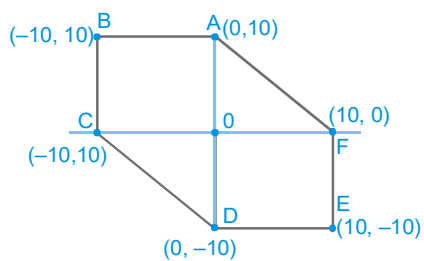
Similarly $ODEF = 11^2$

origin is common to both $\Rightarrow$ integral point in
region $OABC$ and $ODEF = 11^2 + 11^2 - 1 = 241$ (1)

consider region OAF excluding OA & OF

$(1, 9), (1, 8) \dots (1, 1) \rightarrow 9$

$(2, 8), (2, 7) \dots (2, 1) \rightarrow 8$



$(9, 1) \rightarrow 1$

= total points $1 + 2 + \dots + 8 + 9 = \frac{9 \times 10}{2} = 45$ points
..... (2)

similarly region $OCD = 45$ points

..... (3)

total integral points = $241 + 45 + 45 = 331$