SOLVED EXAMPLES

Ex. 1 Find the distance between the point $P(a\cos\alpha,\ a\sin\alpha)$ and $Q(a\cos\beta,\ a\sin\beta)$.

Sol. $d^2 = (a\cos\alpha - a\cos\beta)^2 + (a\sin\alpha - a\sin\beta)^2 = a^2(\cos\alpha - \cos\beta)^2 + a^2(\sin\alpha - \sin\beta)^2$

$$= a^2\left\{2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right\}^2 + a^2\left\{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right\}^2$$

$$= 4a^2\sin^2\frac{\alpha-\beta}{2}\left\{\sin^2\frac{\alpha+\beta}{2} + \cos^2\frac{\alpha+\beta}{2}\right\} = 4a^2\sin^2\frac{\alpha-\beta}{2} \Rightarrow d = 2a\sin\frac{\alpha-\beta}{2}$$

Ex. 2 Find the equation of the straight line which passes through the origin and making angle 60º with the line $x + \sqrt{3}\,y + 3\sqrt{3} = 0$.

Sol. Given line is $x + \sqrt{3}\,y + 3\sqrt{3} = 0$.

$\Rightarrow \quad y = \left(-\frac{1}{\sqrt{3}}\right)x - 3 \qquad \therefore \qquad$ Slope of (1) $= -\frac{1}{\sqrt{3}}$.

Let slope of the required line be m. Also between these lines is given to be 60º.

$$\Rightarrow \tan 60^\circ = \left|\frac{m - (-1/\sqrt{3})}{1 + m(-1/\sqrt{3})}\right| \Rightarrow \sqrt{3} = \left|\frac{\sqrt{3}m + 1}{\sqrt{3} - m}\right| \Rightarrow \frac{\sqrt{3}m+1}{\sqrt{3}-m} = \pm\sqrt{3}$$

$$\frac{\sqrt{3}m+1}{\sqrt{3}-m} = \sqrt{3} \Rightarrow \sqrt{3}\,m + 1 = 3 - \sqrt{3}\,m \Rightarrow m = \frac{1}{\sqrt{3}}$$

Using $y = mx + c$, the equation of the required line is $y = \frac{1}{\sqrt{3}}x + 0$

i.e. $\quad x - \sqrt{3}\,y = 0. \qquad$ (∵ This passes through origin, so c = 0)

$$\frac{\sqrt{3}m+1}{\sqrt{3}-m} = -\sqrt{3} \Rightarrow \sqrt{3}\,m + 1 = -3 + \sqrt{3}\,m$$

$\Rightarrow$ m is not defined

$\therefore$ The slope of the required line is not defined. Thus, the required line is a vertical line. This line is to pass through the origin.

$\therefore$ The equation of the required line is $x = 0$

Ex. 3 The vertices of a triangle are A(0, –6), B(–6, 0) and C(1,1) respectively, then find coordinates of the ex-centre opposite to vertex A.

Sol. $a = BC = \sqrt{(-6-1)^2 + (0-1)^2} = \sqrt{50} = 5\sqrt{2}$

$b = CA = \sqrt{(1-0)^2 + (1+6)^2} = \sqrt{50} = 5\sqrt{2}$

$c = AB = \sqrt{(0+6)^2 + (-6-0)^2} = \sqrt{72} = 6\sqrt{2}$

coordinates of ex-centre opposite to vertex A will be :

$$x = \frac{-ax_1 + bx_2 + cx_3}{-a+b+c} = \frac{-5\sqrt{2}.0 + 5\sqrt{2}(-6) + 6\sqrt{2}(1)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{-24\sqrt{2}}{6\sqrt{2}} = -4$$

$$y = \frac{-ay_1 + by_2 + cy_3}{-a+b+c} = \frac{-5\sqrt{2}(-6) + 5\sqrt{2}.0 + 6\sqrt{2}(1)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{36\sqrt{2}}{6\sqrt{2}} = 6$$

Hence coordinates of ex-centre is (–4, 6)

Ex.4 Obtain the equations of the lines passing through the intersection of lines $4x - 3y - 1 = 0$ and $2x - 5y + 3 = 0$ and equally inclined to the axes.

Sol. The equation of any line through the intersection of the given lines is $(4x - 3y - 1) + \lambda(2x - 5y + 3) = 0$

or $x(2\lambda + 4) - y(5\lambda + 3) + 3\lambda - 1 = 0$ ...... **(i)**

Let m be the slope of this line. Then $m = \dfrac{2\lambda+4}{5\lambda+3}$

As the line is equally inclined with the axes, therefore

$m = \tan 45^\circ$ or $m = \tan 135^\circ$

$\Rightarrow$ $m = \pm 1$, $\dfrac{2\lambda+4}{5\lambda+3} = \pm 1$

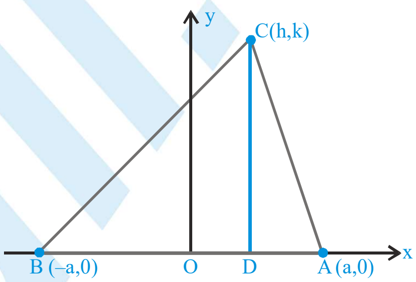
$\Rightarrow$ $\lambda = -1$

or $\dfrac{1}{3}$, putting the values of λ in **(i)**, we get $2x + 2y - 4 = 0$ and $14x - 14y = 0$

i.e. $x + y - 2 = 0$ and $x = y$ as the equations of the required lines.

Ex. 5 A(a, 0) and B(–a, 0) are two fixed points of ΔABC. If its vertex C moves in such a way that $\cot A + \cot B = \lambda$, where λ is a constant, then find the locus of the point C.

Sol. Given that coordinates of two fixed points A and B are (a, 0) and (–a, 0) respectively. Let variable point C is (h, k). From the adjoining figure



$$\cot A = \frac{DA}{CD} = \frac{a-h}{k}$$

$$\cot B = \frac{BD}{CD} = \frac{a+h}{k}$$

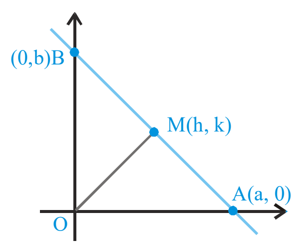
But $\cot A + \cot B = \lambda$, so we have

$$\frac{a-h}{k} + \frac{a+h}{k} = \lambda \quad \Rightarrow \quad \frac{2a}{k} = \lambda$$

Hence locus of C is $y\lambda = 2a$

Ex. 6 Two points A and B move on the positive direction of x-axis and y-axis respectively, such that OA + OB = K. Show that the locus of the foot of the perpendicular from the origin O on the line AB is $(x + y)(x^2 + y^2) = Kxy$.

Sol. Let the equation of AB be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ...... **(i)**



given, $a + b = K$ ...... **(ii)**

now, $m_{AB} \times m_{OM} = -1 \Rightarrow ah = bk$ ...... **(iii)**

from **(ii)** and **(iii)**,

$$a = \frac{kK}{h+k} \quad \text{and} \quad b = \frac{hK}{h+k}$$

$\therefore$ from **(i)** $\dfrac{x(h+k)}{k.K} + \dfrac{y(h+k)}{h.K} = 1$

as it passes through (h, k)

$$\frac{h(h+k)}{k.K} + \frac{k(h+k)}{h.K} = 1 \quad \Rightarrow \quad (h+k)(h^2+k^2) = Khk$$

$\therefore$ locus of (h, k) is $(x + y)(x^2 + y^2) = Kxy$.

Ex. 7 Find the equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy + 4y^2 = 0$

Sol. Given equation is $3x^2 - 5xy + 4y^2 = 0$ ...... **(i)**

comparing it with the equation $ax^2 + 2hxy + by^2 = 0$ ...... **(ii)**

we have $a = 3$, $2h = -5$; and $b = 4$

Now the equation of the bisectors of the angle between the pair of lines (i) is $\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$

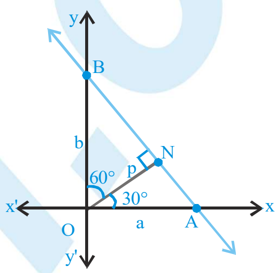
or $\dfrac{x^2 - y^2}{3 - 4} = \dfrac{xy}{-\frac{5}{2}}$; or $\dfrac{x^2 - y^2}{-1} = \dfrac{2xy}{-5}$

or $5x^2 - 2xy - 5y^2 = 0$

Ex. 8 Find the equation of the straight line on which the perpendicular from origin makes an angle 30° with positive x-axis and which forms a triangle of area $\left(\dfrac{50}{\sqrt{3}}\right)$ sq. units with the co-ordinates axes.

Sol. $\angle NOA = 30°$

Let $ON = p > 0$, $OA = a$, $OB = b$



In ΔONA, $\cos 30° = \dfrac{ON}{OA} = \dfrac{p}{a} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{p}{a}$

or $a = \dfrac{2p}{\sqrt{3}}$

and in ΔONB, $\cos 60° = \dfrac{ON}{OB} = \dfrac{p}{b} \Rightarrow \dfrac{1}{2} = \dfrac{p}{b}$

or $b = 2p$

$\therefore$ Area of $\Delta OAB = \dfrac{1}{2}\,ab = \dfrac{1}{2}\left(\dfrac{2p}{\sqrt{3}}\right)(2p) = \dfrac{2p^2}{\sqrt{3}}$

$\therefore$ $\dfrac{2p^2}{\sqrt{3}} = \dfrac{50}{\sqrt{3}} \Rightarrow p^2 = 25$

or $p = 5$

$\therefore$ Using $x\cos\alpha + y\sin\alpha = p$, the equation of the line AB is $x\cos 30° + y\sin 30° = 5$

or $x\sqrt{3} + y = 10$

Ex.9 Prove that the equation $2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$ represents a pair of straight lines. Find the co-ordinates of their point of intersection.

Sol. Given equation is $2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$

Writing the equation (1) as a quadratic equation in x we have

$2x^2 + (5y + 6)x + 3y^2 + 7y + 4 = 0$

$$\therefore \quad x = \frac{-(5y+6) \pm \sqrt{(5y+6)^2 - 4.2(3y^2 + 7y + 4)}}{4}$$

$$= \frac{-(5y+6) \pm \sqrt{25y^2 + 60y + 36 - 24y^2 - 56y - 32}}{4}$$

$$= \frac{-(5y+6) \pm \sqrt{y^2 + 4y + 4}}{4} = \frac{-(5y+6) \pm (y+2)}{4}$$

$\therefore \quad x = \dfrac{-5y-6+y+2}{4}, \dfrac{-5y-6-y-2}{4}$

or $\quad 4x+4y+4=0 \quad$ and $\quad 4x+6y+8=0$

or $\quad x+y+1=0 \quad$ and $\quad 2x+3y+4=0$

Hence equation (1) represents a pair of straight lines whose equation are

$x+y+1=0$ .....**(1)**

and $\quad 2x+3y+4=0$ .....**(2)**

Solving these two equations, the required point of intersection is $(1, -2)$.

Ex. 10 A variable line is drawn through O, to cut two fixed straight lines L_1 and L_2 in A_1 and A_2, respectively. A point A is taken on the variable line such that $\dfrac{m+n}{OA} = \dfrac{m}{OA_1} + \dfrac{n}{OA_2}$. Show that the locus of A is a straight line passing through the point of intersection of L_1 and L_2 where O is being the origin.

Sol. Let the variable line passing through the origin is $\dfrac{x}{\cos\theta} = \dfrac{y}{\sin\theta} = r_i$ ...... **(i)**

Let the equation of the line L_1 is $p_1x + q_1y = 1$ ...... **(ii)**

Equation of the line L_2 is $p_2x + q_2y = 1$ ...... **(iii)**

the variable line intersects the line **(ii)** at A_1 and **(iii)** at A_2.

Let $OA_1 = r_1$.

Then $A_1 = (r_1\cos\theta, r_1\sin\theta) \quad \Rightarrow \quad A_1$ lies on L_1

$\Rightarrow \quad r_1 = OA_1 = \dfrac{1}{p_1\cos\theta + q_1\sin\theta}$

Similarly, $r_2 = OA_2 = \dfrac{1}{p_2\cos\theta + q_2\sin\theta}$

Let $OA = r$

Let co-ordinate of A are (h, k) $\Rightarrow$ $(h, k) \equiv (r\cos\theta, r\sin\theta)$

Now $\dfrac{m+n}{r} = \dfrac{m}{OA_1} + \dfrac{n}{OA_2} \quad \Rightarrow \quad \dfrac{m+n}{r} = \dfrac{m}{r_1} + \dfrac{n}{r_2}$

$\Rightarrow \quad m+n = m(p_1r\cos\theta + q_1r\sin\theta) + n(p_2r\cos\theta + q_2r\sin\theta)$

$\Rightarrow \quad (p_1h + q_1k - 1) + \dfrac{n}{m}(p_2h + q_2k - 1) = 0$

Therefore, locus of A is $(p_1x+q_1y-1) + \dfrac{n}{m}(p_2x + q_2y - 1) = 0$

$\Rightarrow \quad L_1 + \lambda L_2 = 0$ where $\lambda = \dfrac{n}{m}$.

This is the equation of the line passing through the intersection of L_1 and L_2.

Ex. 11 A straight line through P(–2, –3) cuts the pair of straight lines $x^2 + 3y^2 + 4xy - 8x - 6y - 9 = 0$ in Q and R. Find the equation of the line if PQ. PR = 20.

Sol. Let line be $\dfrac{x+2}{\cos\theta} = \dfrac{y+3}{\sin\theta} = r$

$\Rightarrow$ $x = r\cos\theta - 2,\ y = r\sin\theta - 3$ ...... **(i)**

Now, $x^2 + 3y^2 + 4xy - 8x - 6y - 9 = 0$ ...... **(ii)**

Taking intersection of **(i)** with **(ii)** and considering terms of r^2 and constant (as we need PQ . PR = $r_1 . r_2$ = product of the roots)

$r^2(\cos^2\theta + 3\sin^2\theta + 4\sin\theta\cos\theta) + (\text{some terms})r + 80 = 0$

$\therefore$ $r_1.r_2 = PQ.\ PR = \dfrac{80}{\cos^2\theta + 4\sin\theta\cos\theta + 3\sin^2\theta}$

$\therefore$ $\cos^2\theta + 4\sin\theta\cos\theta + 3\sin^2\theta = 4$ ($\because$ PQ . PR = 20)

$\therefore$ $\sin^2\theta - 4\sin\theta\cos\theta + 3\cos^2\theta = 0$

$\Rightarrow$ $(\sin\theta - \cos\theta)(\sin\theta - 3\cos\theta) = 0$

$\therefore$ $\tan\theta = 1,\ \tan\theta = 3$

hence equation of the line is $y + 3 = 1(x + 2)$ $\Rightarrow$ $x - y = 1$ and $y + 3 = 3(x + 2)$ $\Rightarrow$ $3x - y + 3 = 0$.

Ex. 12 Prove that no line can be drawn through the point (4, –5) so that its distance from (–2, 3) will be equal to 12.

Sol. Suppose, if possible.

Equation of line through (4, –5) with slope of m is

$y + 5 = m(x - 4)$

$\Rightarrow$ $mx - y - 4m - 5 = 0$

Then $\dfrac{|m(-2) - 3 - 4m - 5|}{\sqrt{m^2+1}} = 12$

$\Rightarrow$ $|-6m - 8| = 12\sqrt{(m^2+1)}$

On squaring, $(6m + 8)^2 = 144(m^2 + 1)$

$\Rightarrow$ $4(3m + 4)^2 = 144(m^2 + 1)$

$\Rightarrow$ $(3m + 4)^2 = 36(m^2 + 1)$

$\Rightarrow$ $27m^2 - 24m + 20 = 0$ ...... **(i)**

Since the discriminant of **(i)** is $(-24)^2 - 4.27.20 = -1584$ which is negative, there is no real value of m. Hence no such line is possible.

Ex. 13 Prove that the angle between the lines joining the origin to the points of intersection of the straight line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is $\tan^{-1}\dfrac{2\sqrt{2}}{3}$.

Sol. Equation of the given curve is $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$

and equation of the given straight line is $y - 3x = 2$; $\therefore$ $\dfrac{y-3x}{2} = 1$

Making equation (1) homogeneous equation of the second degree in x any y with the help of (1), we have

$$x^2 + 2xy + 3y^2 + 4x\left(\frac{y-3x}{2}\right) + 8y\left(\frac{y-3x}{2}\right) - 11\left(\frac{y-3x}{2}\right)^2 = 0$$

or $\quad x^2 + 2xy + 3y^2 + \frac{1}{2}(4xy + 8y^2 - 12x^2 - 24xy) - \frac{11}{4}(y^2 - 6xy + 9x^2) = 0$

or $\quad 4x^2 + 8xy + 12y^2 + 2(8y^2 - 12x^2 - 20xy) - 11(y^2 - 6xy + 9x^2) = 0$

or $\quad -119x^2 + 34xy + 17y^2 = 0$ or $119x^2 - 34xy - 17y^2 = 0$

or $\quad 7x^2 - 2xy - y^2 = 0$

This is the equation of the lines joining the origin to the points of intersection of (1) and (2).

Comparing equation (3) with the equation $ax^2 + 2hxy + by^2 = 0$

we have $a = 7$, $b = -1$ and $2h = -2$ i.e. $h = -1$

If θ be the acute angle between pair of lines (3), then

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{1+7}}{7-1}\right| = \frac{2\sqrt{8}}{6} = \frac{2\sqrt{2}}{3}$$

$\therefore \quad \theta = \tan^{-1}\frac{2\sqrt{2}}{3}$.

Ex. 14 A ray of light is sent along the line $x - 2y - 3 = 0$. Upon reaching the line mirror $3x - 2y - 5 = 0$, the ray is reflected from it. Find the equation of the line containing the reflected ray.

Sol. Let Q be the point of intersection of the incident ray and the line mirror,

Then, $\quad x_1 - 2y_1 - 3 = 0$ & $3x_1 - 2y_1 - 5 = 0$

On solving these equations,

We get, $x_1 = 1$ & $y_1 = -1$

Since P(–1, –2) be a point lies on the incident ray, so we can find the image of the point P on the reflected ray about the line mirror (by property of reflection).

Let P'(h, k) be the image of point P about line mirror, then

$$\frac{h+1}{3} = \frac{k+2}{-2} = \frac{-2(-3+4-5)}{13} \Rightarrow \quad h = \frac{11}{13} \text{ and } k = \frac{-42}{13}.$$

So $\quad P'\left(\frac{11}{13}, \frac{-42}{13}\right)$

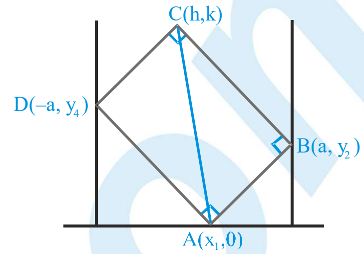
Then equation of reflected ray will be

$$(y+1) = \frac{\left(\frac{-42}{13}+1\right)(x-1)}{\left(\frac{11}{13}-1\right)}$$

$\Rightarrow \quad 2y - 29x + 31 = 0$ is the required equation of reflected ray.

Ex. 15 ABCD is a variable rectangle having its sides parallel to fixed directions (say m). The vertices B and D lie on x = a and x = –a and A lies on the line y = 0. Find the locus of C.

Sol. Let A be $(x_1, 0)$, $B(a, y_2)$ and D be $(-a, y_4)$. We are given AB and AD have fixed directions and hence their slopes are constants. i.e. m & m_1 (say)



$\therefore \quad \frac{y_2}{a - x_1} = m$ and $\frac{y_4}{-a - x_1} = m_1$.

Further, $mm_1 = -1$. Since ABCD is a rectangle.

$$\frac{y_2}{a - x_1} = m \quad \text{and} \quad \frac{y_4}{-a - x_1} = -\frac{1}{m}$$

The mid point of BD is $\left(0, \frac{y_2 + y_4}{2}\right)$ and mid point of AC is $\left(\frac{x_1 + h}{2}, \frac{k}{2}\right)$, where C is taken to be (h, k). This gives $h = -x_1$ and $k = y_2 + y_4$. So C is $(-x_1, y_2 + y_4)$.

Also, $\frac{y_2}{a - x_1} = m$ and $\frac{y_4}{a + x_1} = \frac{1}{m}$ gives $m(k - y_2) = a + x_1 = m(k - m(a - x_1)) = a + x_1$

$\Rightarrow \quad mk - m^2(a - x_1) = a + x_1 \qquad \Rightarrow \quad m^2(a + h) - mk + a - h = 0$

$\Rightarrow \quad (m^2 - 1)h - mk = -(m^2 + 1)a \qquad \Rightarrow \quad (1 - m^2)h + mk = (m^2 + 1)a$

$\Rightarrow \quad (1 - m^2)x + my = (m^2 + 1)a$

The locus of C is $(1 - m^2)x + my = (m^2 + 1)a$.

Ex. 16 If the lines $ax + by + p = 0$, $x\cos\alpha + y\sin\alpha - p = 0$ $(p \neq 0)$ and $x\sin\alpha - y\cos\alpha = 0$ are concurrent and the first two lines include an angle $\frac{\pi}{4}$, then $a^2 + b^2$ is equal to -

Sol. Since the given lines are concurrent,

$$\begin{vmatrix} a & b & p \\ \cos\alpha & \sin\alpha & -p \\ \sin\alpha & -\cos\alpha & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad a\cos\alpha + b\sin\alpha + 1 = 0$ .... **(i)**

As $ax + by + p = 0$ and $x\cos\alpha + y\sin\alpha - p = 0$ include an angle $\frac{\pi}{4}$.

$$\pm\tan\frac{\pi}{4} = \frac{-\frac{a}{b} + \frac{\cos\alpha}{\sin\alpha}}{1 + \frac{a}{b}\frac{\cos\alpha}{\sin\alpha}}$$

$\Rightarrow \quad -a\sin\alpha + b\cos\alpha = \pm(b\sin\alpha + a\cos\alpha)$

$\Rightarrow \quad -a\sin\alpha + b\cos\alpha = \pm 1$ [from **(i)**] ...... **(ii)**

Squaring and adding **(i)** & **(ii)**, we get

$a^2 + b^2 = 2$.

Exercise # 1 [Single Correct Choice Type Questions]

1. The circumcentre of the triangle with vertices (0, 0), (3, 0) and (0, 4) is -

(A) (1, 1) **(B)** (2, 3/2) **(C)** (3/2, 2) **(D)** none of these

2. If L is the line whose equation is $ax + by = c$. Let M be the reflection of L through the y-axis, and let N be the reflection of L through the x-axis. Which of the following must be true about M and N for all choices of a, b and c?

(A) The x-intercepts of M and N are equal. **(B)** The y-intercepts of M and N are equal.
(C) The slopes of M and N are equal. **(D)** The slopes of M and N are reciprocal.

3. If (3, –4) and (–6, 5) are the extremities of a diagonal of a parallelogram and (2, 1) is its third vertex, then its fourth vertex is -

(A) (–1, 0) **(B)** (–1, 1) **(C)** (0, –1) **(D)** (–5, 0)

4. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$, are concurrent at the point :

(A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ **(B)** (1, 3) **(C)** (3, 1) **(D)** $\left(\frac{3}{4}, \frac{1}{2}\right)$

5. The point A divides the join of the points (–5,1) and (3,5) in the ratio k : 1 and coordinates of points B and C are (1,5) and (7,–2) respectively. If the area of ΔABC be 2 units, then k equals -

(A) 7,9 **(B)** 6,7 **(C)** 7,31/9 **(D)** 9,31/9

6. Given the points A (0, 4) and B (0, –4), the equation of the locus of the point P (x, y) such that $|AP - BP| = 6$ is :

(A) $9x^2 - 7y^2 + 63 = 0$ **(B)** $9x^2 - 7y^2 - 63 = 0$ **(C)** $7x^2 - 9y^2 + 63 = 0$ **(D)** $7x^2 - 9y^2 - 63 = 0$

7. Area of a triangle whose vertices are $(a \cos \theta, b \sin\theta)$, $(-a \sin \theta, b \cos \theta)$ and $(-a \cos \theta, -b \sin \theta)$ is -

(A) $a\,b \sin \theta \cos \theta$ **(B)** $a \cos \theta \sin \theta$ **(C)** $\frac{1}{2}ab$ **(D)** ab

8. Find all pair of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less then 34

(A) (11, 9), (13, 15) **(B)** (9, 11), (15, 17) **(C)** (11, 13), (13, 15) **(D)** None of these

9. Vertices of a parallelogram ABCD are A(3, 1), B(13, 6), C(13, 21) and D(3, 16). If a line passing through the origin divides the parallelogram into two congruent parts then the slope of the line is

(A) $\frac{11}{12}$ **(B)** $\frac{11}{8}$ **(C)** $\frac{25}{8}$ **(D)** $\frac{13}{8}$

10. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + aby + 1 = 0\ \forall\ a \in R, b > 0$ are :

(A) $b \in (2, 4)$ **(B)** $b \in (0, 2)$ **(C)** $b \in [0, 2]$ **(D)** $(2, \infty)$

11. If $A(\cos\alpha, \sin\alpha)$, $B(\sin\alpha, -\cos\alpha)$, C (1,2) are the vertices of a ΔABC, then as α varies, the locus of its centroid is -

(A) $x^2 + y^2 - 2x - 4y + 3 = 0$ **(B)** $x^2 + y^2 - 2x - 4y + 1 = 0$
(C) $3(x^2 + y^2) - 2x - 4y + 1 = 0$ **(D)** none of these

12. Consider a quadratic equation in Z with parameters x and y as $Z^2 - xZ + (x - y)^2 = 0$
The parameters x and y are the co-ordinates of a variable point P w.r.t. an orthonormal co-ordinate system in a plane. If the quadratic equation has equal roots then the locus of P is
(A) a circle
(B) a line pair through the origin of co-ordinates with slope 1/2 and 2/3
(C) a line pair through the origin of co-ordinates with slope 3/2 and 2
(D) a line pair through the origin of co-ordinates with slope 3/2 and 1/2

13. $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are three non-collinear points in Cartesian plane. Number of parallelograms that can be drawn with these three points as vertices is -
(A) one **(B)** two **(C)** three **(D)** four

14. The points with the co-ordinates (2a, 3a), (3b, 2b) & (c, c) are collinear-
(A) for no value of a, b, c **(B)** for all values of a, b, c
(C) if a, $\frac{c}{5}$, b are in H.P. **(D)** if a, $\frac{2}{5}$ c, b are in H.P.

15. In a ΔABC, side AB has the equation $2x + 3y = 29$ and the side AC has the equation $x + 2y = 16$. If the mid point of BC is (5, 6), then the equation of BC is
(A) $2x + y = 7$ **(B)** $x + y = 11$ **(C)** $2x - y = 17$ **(D)** none of these

16. If the axes are rotated through an angle of 30° in the anti-clockwise direction, the coordinates of point $(4, -2\sqrt{3})$ with respect to new axes are-
(A) $(2, \sqrt{3})$ **(B)** $(\sqrt{3}, -5)$ **(C)** $(2, 3)$ **(D)** $(\sqrt{3}, 2)$

17. The equation of the pair of bisectors of the angles between two straight lines is, $12x^2 - 7xy - 12y^2 = 0$. If the equation of one line is $2y - x = 0$ then the equation of the other line is :
(A) $41x - 38y = 0$ **(B)** $11x + 2y = 0$ **(C)** $38x + 41y = 0$ **(D)** $11x - 2y = 0$

18. A stick of length 10 units rests against the floor and a wall of a room. If the stick begins to slide on the floor then the locus of its middle point is -
(A) $x^2 + y^2 = 2.5$ **(B)** $x^2 + y^2 = 25$ **(C)** $x^2 + y^2 = 100$ **(D)** none

19. The area enclosed by the graphs of $|x + y| = 2$ and $|x| = 1$ is
(A) 2 **(B)** 4 **(C)** 6 **(D)** 8

20. If the point (a, 2) lies between the lines $x - y - 1 = 0$ and $2(x - y) - 5 = 0$, then the set of values of a is -
(A) $(-\infty, 3) \cup (9/2, \infty)$ **(B)** $(3, 9/2)$ **(C)** $(-\infty, 3)$ **(D)** $(9/2, \infty)$

21. The equation of perpendicular bisector of the line segment joining the points (1, 2) and (–2, 0) is -
(A) $5x + 2y = 1$ **(B)** $4x + 6y = 1$ **(C)** $6x + 4y = 1$ **(D)** none of these

22. The combined equation of the bisectors of the angle between the lines represented by $(x^2 + y^2)\sqrt{3} = 4xy$ is
(A) $y^2 - x^2 = 0$ **(B)** $xy = 0$ **(C)** $x^2 + y^2 = 2xy$ **(D)** $\frac{x^2 - y^2}{\sqrt{3}} = \frac{xy}{2}$

23. The equation $2x^2 + 4xy - py^2 + 4x + qy + 1 = 0$ will represent two mutually perpendicular straight lines, if -
(A) p=1 and q = 2 or 6 **(B)** p = – 2 and q = – 2 or 8
(C) p = 2 and q = 0 or 8 **(D)** p = 2 and q = 0 or 6

24. The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is
(A) 4 **(B)** $\frac{4}{\sqrt{3}}$ **(C)** 2 **(D)** $2\sqrt{3}$

25. The equation of a straight line which passes through the point (–3, 5) such that the portion of it between the axes is divided by the point in the ratio 5 : 3, internally (reckoning from x-axis) will be -

(A) $x + y - 2 = 0$ (B) $2x + y + 1 = 0$ (C) $x + 2y - 7 = 0$ (D) $x - y + 8 = 0$

26. m, n are integer with $0 < n < m$. A is the point (m, n) on the Cartesian plane. B is the reflection of A in the line $y = x$. C is the reflection of B in the y-axis, D is the reflection of C in the x-axis and E is the reflection of D in the y-axis. The area of the pentagon ABCDE is

(A) $2m(m+n)$ (B) $m(m+3n)$ (C) $m(2m+3n)$ (D) $2m(m+3n)$

27. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is-

(A) square (B) circle (C) straight line (D) two intersecting lines

28. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is -

(A) one (B) two (C) three (D) four

29. The straight lines joining the origin to the points of intersection of the line $2x + y = 1$ and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

30. Given the four lines with the equations $x + 2y - 3 = 0$, $3x + 4y - 7 = 0$, $2x + 3y - 4 = 0$, $4x + 5y - 6 = 0$ then,

(A) they are all concurrent (B) they are the sides of a quadrilateral
(C) only three lines are concurrent (D) none of the above

31. The equation of the line passing through the point (c, d) and parallel to the line $ax + by + c = 0$ is -

(A) $a(x + c) + b(y + d) = 0$ (B) $a(x + c) - b(y + d) = 0$
(C) $a(x - c) + b(y - d) = 0$ (D) none of these

32. If the slope of one line of the pair of lines represented by $ax^2 + 10xy + y^2 = 0$ is four times the slope of the other line, then a =

(A) 1 (B) 2 (C) 4 (D) 16

33. Equation of the pair of straight lines through origin and perpendicular to the pair of straight lines $5x^2 - 7xy - 3y^2 = 0$ is -

(A) $3x^2 - 7xy - 5y^2 = 0$ (B) $3x^2 + 7xy + 5y^2 = 0$
(C) $3x^2 - 7xy + 5y^2 = 0$ (D) $3x^2 + 7xy - 5y^2 = 0$

34. If the straight lines joining the origin and the points of intersection of the curve $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the co-ordinate axis, then the value of k -

(A) is equal to 1 (B) is equal to –1
(C) is equal to 2 (D) does not exist in the set of real numbers

35. The curve passing through the points of intersection of $S_1 \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ and $S_2 \equiv x^2 + y^2 + 2gx + 2fyx + C = 0$ represents a pair of straight lines which are

(A) equally inclined to the x - axis (B) perpendicular to each other
(C) pass through a fixed point (D) None of above

36. The water acidity in a pool is considered normal when the average ph reading of three daily measurement is between 7.4 and 8.2 . If the first two ph reading are 7.48 and 8.42. Find the range of ph value for the third reading that will result in the acidity level being normal

(A) (6.3, 8.7) (B) (6.3, 9.2) (C) (5.4, 10.3) (D) None of these

37. Distance of the point (2, 5) from the line $3x + y + 4 = 0$ measured parallel to the line $3x - 4y + 8 = 0$ is -

(A) 15/2 (B) 9/2 (C) 5 (D) none

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. All the points lying inside the triangle formed by the points (1, 3), (5, 6) and (– 1, 2) satisfy

(A) $3x+2y\geq 0$ **(B)** $2x+y+1\geq 0$ **(C)** $2x+3y-12\geq 0$ **(D)** $-2x+11\geq 0$

2. The angle between the lines $y - x + 5 = 0$ and $\sqrt{3}\,x - y + 7 = 0$ is/are -

(A) 15° **(B)** 60° **(C)** 165° **(D)** 75°

3. The points A(–4, –1), B (–2, –4), C (4, 0) and D(2, 3) are the vertices of

(A) parallelogram **(B)** rectangle **(C)** rhombus **(D)** none of these

4. Two vertices of the Δ ABC are at the points A(– 1, – 1) and B(4, 5) and the third vertex lines on the straight line $y = 5(x-3)$. If the area of the Δ is 19/2 then the possible co–ordinates of the vertex C are:

(A) (5, 10) **(B)** (3, 0) **(C)** (2, –5) **(D)** (5, 4)

5. If one vertex of an equilateral triangle of side 'a' lies at the origin and the other lies on the line $x-\sqrt{3}y=0$, then the co-ordinates of the third vertex are -

(A) (0, a) **(B)** $\left(\frac{\sqrt{3}\,a}{2},-\frac{a}{2}\right)$ **(C)** (0, –a) **(D)** $\left(-\frac{\sqrt{3}\,a}{2},\frac{a}{2}\right)$

6. If the points (k, 2 – 2k), (1 – k, 2k) and (–k –4, 6 – 2k) be collinear, the possible values of k are

(A) $-\frac{1}{2}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** –1

7. If the equation $ax^2 - 6xy + y^2 + bx + cx + d = 0$ represents a pair of lines whose slopes are m and m^2, then value(s) of a is/are -

(A) a = –8 **(B)** a = 8 **(C)** a = 27 **(D)** a = –27

8. If $\frac{x}{c}+\frac{y}{d}=1$ is a line through the intersection of $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ and the lengths of the perpendiculars drawn from the origin to these lines are equal in lengths then :

(A) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}+\frac{1}{d^2}$ **(B)** $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{c^2}-\frac{1}{d^2}$

(C) $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}+\frac{1}{d}$ **(D)** none

9. Three lines px + qy + r = 0, qx + ry + p = 0 and rx + py + q = 0 are concurrent if -

(A) p + q + r = 0 **(B)** $p^2 + q^2 + r^2 = pr + qr + pq$

(C) $p^3 + q^3 + r^3 = 3pqr$ **(D)** none of these

10. The points which trisect the line segment joining the points (0, 0) and (9, 12) are

(A) (3, 4) **(B)** (8, 6) **(C)** (6, 8) **(D)** (4, 0)

11. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/are always rational point (s) ?

(A) centriod **(B)** incentre **(C)** circumcentre **(D)** orthocentre

12. A and B are two fixed points whose co-ordinates are (3, 2) and (5, 4) respectively . The co-ordinates of a point P if ABP is an equilateral triangle, is/are :

(A) $\left(4-\sqrt{3}\,,3+\sqrt{3}\right)$ **(B)** $\left(4+\sqrt{3}\,,3-\sqrt{3}\right)$ **(C)** $\left(3-\sqrt{3}\,,4+\sqrt{3}\right)$ **(D)** $\left(3+\sqrt{3}\,,4-\sqrt{3}\right)$

13. Equation of a straight line passing through the point (4, 5) and equally inclined to the lines $3x = 4y + 7$ and $5y = 12x + 6$ is

(A) $9x - 7y = 1$ **(B)** $9x + 7y = 71$ **(C)** $7x + 9y = 73$ **(D)** $7x - 9y + 17 = 0$

14. The sides of a triangle are the straight line $x + y = 1$, $7y = x$ and $\sqrt{3}\,y + x = 0$. Then which of the following is an interior points of triangle ?

(A) circumcentre **(B)** centroid **(C)** incentre **(D)** orthocentre

15. Lines, $L_i : x + \sqrt{3}y = 2$, and $L_2 : ax + by = 1$, meet at P and enclose an angle of 45° between them. Line $L_3 : y = \sqrt{3}x$, also passes through P then -

(A) $a^2 + b^2 = 1$ **(B)** $a^2 + b^2 = 2$ **(C)** $a^2 + b^2 = 3$ **(D)** $a^2 + b^2 = 4$

16. The straight lines $x + y = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form a triangle which is

(A) isosceles **(B)** right angled **(C)** obtuse angled **(D)** equilateral

17. The diagonals of a square are along the pair of lines whose equation is $2x^2 - 3xy - 2y^2 = 0$. If (2, 1) is a vertex of the square, then the vertex of the square adjacent to it may be -

(A) (1, 4) **(B)** (–1, –4) **(C)** (–1, 2) **(D)** (1, –2)

18. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Then the equations of other sides are :

(A) $7x - 4y + 25 = 0$ **(B)** $7x + 4y + 25 = 0$ **(C)** $7x - 4y - 3 = 0$ **(D)** $4x + 7y = 11$

19. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of a triangle, then the equation

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \text{ represents}$$

(A) the median through A
(B) the altitude through A
(C) the perpendicular bisector of BC
(D) the line joining the centroid with a vertex

20. The x-coordinates of the vertices of a square of unit area are the roots of the equation $x^2 - 3|x| + 2 = 0$ and the y-coordinates of the vertices are the roots of the equation $y^2 - 3y + 2 = 0$ then the possible vertices of the square is/are :

(A) (1, 1), (2, 1), (2, 2), (1, 2) **(B)** (–1, 1), (–2, 1), (–2, 2), (–1, 2)
(C) (2, 1), (1, –1), (1, 2), (2, 2) **(D)** (–2, 1), (–1, –1), (–1, 2), (–2, 2)

Part # II [Assertion & Reason Type Questions]

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true and Statement-II is correct explanation for statement-I.

(B) Statement-I is true, Statement-II is true and Statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. Consider the lines, $L_1: \frac{x}{3}+\frac{y}{4}=1$; $L_2 = \frac{x}{4}+\frac{y}{3}=1$; $L_3: \frac{x}{3}+\frac{y}{4}=2$ and $L_4: \frac{x}{4}+\frac{y}{3}=2$

Statement - I The quadrilateral formed by these four lines is a rhombus.

Statement - II If diagonals of a quadrilateral formed by any four lines are unequal and intersect at right angle then it is a rhombus.

2. **Statement - I** The diagonals of the parallelogram whose sides are ●x + my + n = 0, ●x + my + n' = 0, mx + ●y + n = 0, mx + ●y + n' = 0 are perpendicular.

Statement - II If the perpedicular distances between parallel sides of a parallelogram are equal, then it is a rhombus.

3. Given the lines $y + 2x = 3$ and $y + 2x = 5$ cut the axes at A, B and C, D respectively.

Statement- I ABDC forms quadrilateral and point (2, 3) lies inside the quadrilateral

Statement- II Point lies on same side of the lines.

4. **Statement - I** Area of triangle formed by the line which is passing through the point (5, 6) such that segment of the line between axes is bisected at the point, with coordinate axes is 60 sq. units

Statement - II Area of triangle formed by line passing through point (α, β), with axes is maximum when point (α, β) is mid point of segment of line between axes.

5. Let $L_1 : a_1x + b_1y + c_1 = 0$, $L_2 : a_2x + b_2y + c_2 = 0$ and $L_3 : a_3x + b_3y + c_3 = 0$.

Statement - I If L_1, L_2 and L_3 are three concurrent lines, then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Statement - II If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the lines L_1, L_2 and L_3 must be concurrent.

6. **Statement - I** Centroid of the triangle whose vertices are A(–1, 11); B(– 9, – 8) and C(15, – 2) lies on the internal angle bisector of the vertex A.

Statement - II Triangle ABC is isosceles with B and C as base angles.

7. **Statement-I :** The joint equation of lines $2y = x+1$ and $2y = -(x+1)$ is $4y^2 = -(x + 1)^2$.

Statement-II : The joint equation of two lines satisfy every point lying on any one of the line.

8. **Statement - I** Two of the straight lines represented by the equation $ax^3 + bx^2y + cxy^2 + dy^3 = 0$ will be at right angled if $a^2 + ac + bd + d^2 = 0$

Statement - II If roots of equation $px^3 + qx^2 + rx + s = 0$ are α, β and γ, then $\alpha\beta\gamma = -s/p$.

9. **Statement - I** The equation $2x^2 + 3xy - 2y^2 + 5x - 5y + 3 = 0$ represents a pair of perpendicular straight lines.

Statement - II A pair of lines given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are perpendicular, if $a + b = 0$

10. Consider a triangle whose vertices are $A(-2, 1)$, $B(1, 3)$ and $C(3x, 2x - 3)$ where x is a real number.

Statement - I The area of the triangle ABC is independent of x

Statement - II The vertex C of the triangle ABC always moves on a line parallel to the base AB.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1.

	Column-I		Column-II
(A)	Two adjacent sides of a parallogram are $4x + 5y = 0$ and $7x + 2y = 0$ and one diagonal is $ax + by + c = 0$, then $a + b + c$ is equal to	(p)	1
(B)	If line $2x - by + 1 = 0$ intersects the curve $2x^2 - by^2 + (2b - 1)xy - x - by = 0$ at points A & B and AB subtends a right angle at origin, then value of $b + b^2$ is equal to	(q)	0
(C)	A line passes through point (3, 4) and the point of intersection of the lines $4x + 3y = 12$ and $3x + 4y = 12$ and length of intercepts on the co-ordinate axes are a and b, then ab is equal to	(r)	5
(D)	A light ray emerging from the point source placed at P(2, 3) is reflected at a point 'Q' on the y-axis and then passes through the point R(5, 10). If co-ordinates of Q are (a, b), then a + b is	(s)	4

2.

	Column – I		Column – II
(A)	Slope of line bisecting the angle between co-ordinate axes, is	(p)	3
(B)	Area of Δ formed by line $3x + 4y + 12 = 0$ with co-ordinate axis is	(q)	1
(C)	If the equation $2x^2 - 2xy - y^2 - 6x + 6y + c = 0$ represents a pair of lines, then 'c' is	(r)	6
(D)	If distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is $25\sqrt{2}$, then 'a/5' is equal to	(s)	– 1

3.

	Column-I		Column-II
(A)	Let 'P' be a point inside the triangle ABC and is equidistant from its sides. DEF is a triangle obtained by the intersection of the external angle bisectors of the angles of the ΔABC. With respect to the triangle DEF point P is its	(p)	centroid
(B)	Let 'Q' be a point inside the triangle ABC If $(AQ)\sin\frac{A}{2} = (BQ)\sin\frac{B}{2} = (CQ)\sin\frac{C}{2}$ then with respect to the triangle ABC, Q is its	(q)	orthocentre
(C)	Let 'S' be a point in the plane of the triangle ABC. If the point is such that infinite normals can be drawn from it on the circle passing through A, B and C then with respect to the triangle ABC, S is its	(r)	incentre
(D)	Let ABC be a triangle. D is some point on the side BC such that the line segments parallel to BC with their extremities on AB and AC get bisected by AD. Point E and F are similarly obtained on CA and AB. If segments AD, BE and CF are concurrent at a point R then with respect to the triangle ABC, R is its	(s)	circumcentre

4.

	Column-I		Column-II
(A)	If $3a - 2b + 5c = 0$, then family of straight lines $ax + by + c = 0$ are always concurrent at a point whose co-ordinates is (a, b), then the values of $a - 5b$	(p)	$3\sqrt{2}$
(B)	Number of integral values of b for which the origin and the point $(1, 1)$ lie on the same side of the straight line $a^2x + aby + 1 = 0$ for all $a \in R - \{0\}$ is	(q)	5
(C)	Vetices of a right angled triangle lie on a circle and extrimites of whose hypotenuse are $(6, 0)$ and $(0, 6)$, then radius of circle is	(r)	12
(D)	If the slope of one of the lines represented by $ax^2 - 6xy + y^2 = 0$ is square of the other, then a is	(s)	3
		(t)	8

Part # II [Comprehension Type Questions]

Comprehension # 1

A locus is the curve traced out by a point which moves under certain geometrical conditions:

To find the locus of a point first we assume the co-ordinates of the moving point as (h,k) and then try to find a relation between h and k with the help of the given conditions of the problem. If there is any variable involved in the process then we eliminate them. At last we replace h by x and k by y and get the locus of the point which will be an equation in x and y.

On the basis of above information, answer the following questions :

1. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$ where t is a parameter is -

(A) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (B) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
(C) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ (D) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

2. A variable line cuts x-axis at A, y-axis at B where OA = a, OB = b (O as origin) such that $a^2 + b^2 = 1$ then the locus of circumcentre of Δ OAB is -

(A) $x^2 + y^2 = 4$ (B) $x^2 + y^2 = 1/4$ (C) $x^2 - y^2 = 4$ (D) $x^2 - y^2 = 1/4$

3. The locus of the point of intersection of the lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ where α is variable is -

(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 + y^2 = a^2 - b^2$ (C) $x^2 - y^2 = a^2 - b^2$ (D) $x^2 - y^2 = a^2 + b^2$

Comprehension # 2

Consider a general equation of degree 2, as $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

1. The value of 'λ' for which the line pair represents a pair of straight lines is

(A) 1 (B) 2 (C) 3/2 (D) 3

2. For the value of λ obtained in above question, if $L_1 = 0$ and $L_2 = 0$ are the lines denoted by the given line pair then the product of the abscissa and ordinate of their point of intersection is

(A) 18 (B) 28 (C) 35 (D) 25

3. If θ is the acute angle between $L_1 = 0$ and $L_2 = 0$ then θ lies in the interval

(A) $(45°, 60°)$ (B) $(30°, 45°)$ (C) $(15°, 30°)$ (D) $(0, 15°)$

Comprehension # 3

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points (3, 4) and (1, 2) respectively and centroid of Δ ABC is G(3, 2), then answer the following questions :

1. The equation of side AB is

(A) $2x + y = 4$ **(B)** $x + y - 3 = 0$ **(C)** $4x - 2y = 0$ **(D)** none of these

2. Co-ordinates of D are

(A) $(7, -4)$ **(B)** $(5, 0)$ **(C)** $(7, 4)$ **(D)** $(-3, 0)$

3. Height of altitude drawn from point A is (in units)

(A) $4\sqrt{2}$ **(B)** $3\sqrt{2}$ **(C)** $6\sqrt{2}$ **(D)** $2\sqrt{3}$

Comprehension # 4

Consider a line pair $ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0$ representing perpendicular lines intersecting each other at C and forming a triangle ABC with the x-axis.

1. If x_1 and x_2 are intercepts on the x-axis and y_1 and y_2 are the intercepts on the y-axis then the sum $(x_1 + x_2 + y_1 + y_2)$ is equal to

(A) 6 **(B)** 5 **(C)** 4 **(D)** 3

2. Distance between the orthocentre and circumcentre of the triangle ABC is

(A) 2 **(B)** 3 **(C)** 7/4 **(D)** 9/4

3. If the circle $x^2 + y^2 - 4y + k = 0$ is orthogonal with the circumcircle of the triangle ABC then 'k' equals

(A) 1/2 **(B)** 1 **(C)** 2 **(D)** 3/2

Comprehension # 5

For points $P \equiv (x_1, y_1)$ and $Q \equiv (x_2, y_2)$ of the coordinate plane, a new distance d(P, Q) is defined by

$$d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$$

Let $O \equiv (0, 0)$, $A \equiv (1, 2)$, $B \equiv (2, 3)$ and $C \equiv (4, 3)$ are four fixed points on x – y plane.

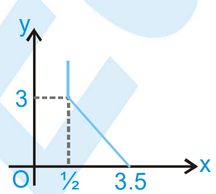
1. Let R(x, y), such that R is equidistant from the points O and A with respect to new distance and if $0 \le x < 1$ and $0 \le y < 2$, then R lies on a line segment whose equation is -

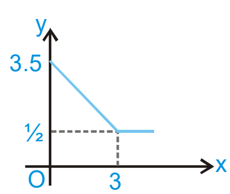
(A) $x + y = 3$ **(B)** $x + 2y = 3$ **(C)** $2x + y = 3$ **(D)** $2x + 2y = 3$

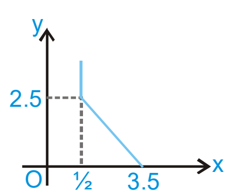
2. Let S(x, y), such that S is equidistant from points O and B with respect to new distance and if $x \ge 2$ and $0 \le y < 3$, then locus of S is -

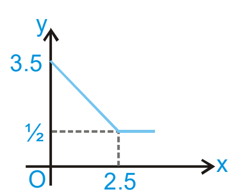
(A) a line segment **(B)** a line **(C)** a vertical ray **(D)** a horizontal ray

3. Let T(x, y), such that T is equidistant from point O and C with respect to new distance and if T lies in first quadrant, then T consists of the union of a line segment of finite length and an infinite ray whose labelled diagram is -

(A)


(B)


(C)


(D)


Comprehension # 6

Given two straight lines AB and AC whose equations are $3x + 4y = 5$ and $4x - 3y = 15$ respectively. Then the possible equation of line BC through (1, 2), such that ΔABC is isosceles, is $L_1 = 0$ or $L_2 = 0$, then answer the following questions

1. If $L_1 \equiv ax + by + c = 0$ & $L_2 \equiv dx + ey + f = 0$ where a, b, c, d, e, f $\in$ I, and a, d > 0, then c + f =

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

2. A straight line through P(2, c + f – 1), inclined at an angle of 60° with positive Y-axis in clockwise direction. The co-ordinates of one of the points on it at a distance (c + f) units from point P is (c, f obtained from previous question)

(A) $(2+2\sqrt{3}, 5)$ **(B)** $(3+2\sqrt{3}, 3)$ **(C)** $(2+3\sqrt{2}, 4)$ **(D)** $(2+3\sqrt{2}, 3)$

3. If (a, b) is the co-ordinates of the point obtained in previous question, then the equation of line which is at the distance |b – 2a – 1| units from origin and make equal intercept on co-ordinate axes in first quadrant, is

(A) $x+y+4\sqrt{6}=0$ **(B)** $x+y+2\sqrt{6}=0$ **(C)** $x+y-4\sqrt{6}=0$ **(D)** $x+y-2\sqrt{6}=0$

Exercise # 4 [Subjective Type Questions]

1. The line $3x + 2y = 24$ meets the y-axis at A & the x-axis at B. The perpendicular bisector of AB meets the line through $(0,-1)$ parallel to x-axis at C. Find the area of the triangle ABC.

2. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a + b) = ab(x + y)$.

3. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the n^{th} power of the other, then prove that

$$(ab^n)^{\frac{1}{n+1}} + (a^n b)^{\frac{1}{n+1}} + 2h = 0.$$

4. A parallelogram is formed by the lines $ax^2 + 2hxy + by^2 = 0$ and the lines through (p, q) parallel to them. Show that the equation of the diagonal of the parallelogram which doesn't pass through origin is
$(2x - p)(ap + hq) + (2y - q) + (2y - q)\ (hp + bq) = 0$

5. If a, b, c are all different and the points $\left(\frac{r^3}{r-1}, \frac{r^2-3}{r-1}\right)$ where $r = a, b, c$ are collinear,

then prove that $3(a + b + c) = ab + bc + ca - abc$.

6. Find the co-ordinates of the orthocentre of the triangle, the equations of whose sides are $x + y = 1$, $2x + 3y = 6$, $4x - y + 4 = 0$, without finding the co-ordinates of its vertices.

7. Let ABC be a triangle with AB = AC. If D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC and F the mid-point of DE, prove that AF is perpendicular to BE.

8. Find λ if $(\lambda, \lambda + 1)$ is an interior points of ΔABC, where $A \equiv (0, 3)$, $B \equiv (-2, 0)$ and $C \equiv (6, 1)$.

9. A line cuts the x-axis at A(7, 0) and the y-axis at B(0, –5). A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R.

10. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then show that :

(i) The median through A can be written in the form $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

(ii) the line through A & parallel to BC can be written in the form ; $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

(iii) equation to the angle bisector through A is $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

where b = AC & c = AB.

11. The vertices of a triangle are $A(x_1, x_1 \tan\theta_1)$, $B(x_2, x_2 \tan\theta_2)$ & $C(x_3, x_3 \tan\theta_3)$. If the circumcentre O of the triangle ABC is at the origin & $H(\bar{x}, \bar{y})$ be its orthocentre, then show that $\frac{\bar{x}}{\bar{y}} = \frac{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}$.

12. Reduce $x + \sqrt{3}y + 4 = 0$ to the :

(i) Slope intercepts form and find its slope and y-intercept.

(ii) Intercepts form and find its intercepts on the axes.

(iii) Normal form and find values of P and α.

13. Find the direction in which a straight line may be drawn through the point (2, 1) so that its point of intersection with the line $4y - 4x + 4 + 3\sqrt{2} + 3\sqrt{10} = 0$ is at a distance of 3 unit from (2, 1).

14. Equation of a line is given by $y + 2at = t(x - at^2)$, t being the parameter. Find the locus of the point intersection of the lines which are at right angles.

15. Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv \bullet x + my + n = 0$ intersect at the point P and makes an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1

16. Two ends A & B of a straight line segment of constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB is completed show that the locus of the foot of the perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$.

17. Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that AB = AC. Determine the possible equations of the line BC passing through the point (1, 2)

18. Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrence & if not, give reasons.

19. Find the locus of the centroid of a triangle whose vertices are (acost, asint), (bsin t, –bcost) and (1, 0), where 't' is the parameter.

20. Find a point P on the line $3x + 2y + 10 = 0$ such that

(i) PA + PB minimum

(ii) |PA – PB| maximum where $A \equiv (4, 2)$ and $B \equiv (2, 4)$.

21. Find the equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of 'c' is : **[AIEEE - 2003]**

(A) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ **(B)** $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(C) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ **(D)** $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

2. Locus of centroid of the triangle whose vertices are (acost, asint), (bsint, – bcost) and (1, 0), where t is a parameter is : **[AIEEE - 2003]**

(A) $(3x-1)^2 + (3y)^2 = a^2 - b^2$ **(B)** $(3x-1)^2 + (3y)^2 = a^2 + b^2$

(C) $(3x+1)^2 + (3y)^2 = a^2 + b^2$ **(D)** $(3x+1)^2 + (3y)^2 = a^2 - b^2$

3. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then : **[AIEEE - 2003]**

(A) $pq = -1$ **(B)** $p = q$ **(C)** $p = -q$ **(D)** $pq = 1$

4. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is : **[AIEEE - 2003]**

(A) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ **(B)** $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$ **(D)** $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

5. Let A(2,–3) and B(–2,1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line : **[AIEEE - 2004]**

(A) $2x + 3y = 9$ **(B)** $2x - 3y = 7$ **(C)** $3x + 2y = 5$ **(D)** $3x - 2y = 3$

6. The equation of the straight line passing through the point (4,3) and making intercepts on the co–ordinate axes whose sum is –1, is : **[AIEEE - 2004]**

(A) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$ **(B)** $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(C) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$ **(D)** $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

7. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value: **[AIEEE - 2004]**

(A) 1 **(B)** –1 **(C)** 2 **(D)** –2

8. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals : **[AIEEE - 2004]**

(A) 1 **(B)** –1 **(C)** 3 **(D)** –3

9. The line parallel to the x–axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a,b) \neq (0,0)$ is : **[AIEEE - 2005]**

(A) above the x–axis at a distance of (2/3) from it

(B) above the x–axis at a distance of (3/2) from it

(C) below the x–axis at a distance of (2/3) from it

(D) below the x–axis at a distance of (3/2) from it

10. If non–zero numbers a,b,c are in HP, then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is : **[AIEEE - 2005]**

(A) $\left(1, -\frac{1}{2}\right)$ **(B)** $(1, -2)$ **(C)** $(-1, -2)$ **(D)** $(-1, 2)$

11. If a vertex of a triangle is (1,1) and the mid–points of two sides through this vertex are (–1,2) and (3,2), then the centroid of the triangle is : **[AIEEE - 2005]**

(A) $\left(\frac{1}{3},\frac{7}{3}\right)$ **(B)** $\left(1, \frac{7}{3}\right)$ **(C)** $\left(-\frac{1}{3}, \frac{7}{3}\right)$ **(D)** $\left(-1\ , \frac{7}{3}\right)$

12. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameter of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector, then : **[AIEEE - 2005]**

(A) $3a^2+2ab+3b^2=0$ **(B)** $3a^2+10ab+3b^2=0$ **(C)** $3a^2-2ab+3b^2=0$ **(D)** $3a^2-10ab+3b^2=0$

13. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is : **[AIEEE - 2006]**

(A) $3x-4y+7=0$ **(B)** $4x+3y=24$ **(C)** $3x+4y=25$ **(D)** $x+y=7$

14. If (a, a^2) falls inside the angle made by the lines $y=\frac{x}{2}$, $x>0$ and $y=3x$, $x>0$, then 'a' belongs to : **[AIEEE - 2006]**

(A) $(3, \infty)$ **(B)** $\left(\frac{1}{2},3\right)$ **(C)** $\left(-3,-\frac{1}{2}\right)$ **(D)** $\left(0, \frac{1}{2}\right)$

15. Let A(h, k), B(1, 1) and C(2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of triangle is 1, then the set of values which 'k' can take is given by **[AIEEE - 2007]**

(A) $\{1, 3\}$ **(B)** $\{0, 2\}$ **(C)** $\{-1, 3\}$ **(D)** $\{-3, -2\}$

16. Let $P=(-1, 0)$ $Q=(0, 0)$ and $R=(3, 3\sqrt{3})$ be three points. The equation of the bisector of the $\angle PQR$ is **[AIEEE - 2007]**

(A) $\sqrt{3}\ x+y=0$ **(B)** $x+\frac{\sqrt{3}}{2}\ y=0$ **(C)** $\frac{\sqrt{3}}{2}\ x+y=0$ **(D)** $x+\sqrt{3}\ y=0$

17. If one of the lines of $my^2+(1-m^2)xy-mx^2=0$ is a bisector of the angle between the lines $xy=0$, then m is **[AIEEE - 2007]**

(A) $-\frac{1}{2}$ **(B)** -2 **(C)** ± 1 **(D)** 2

18. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept – 4. Then a possible value of k is **[AIEEE - 2008]**

(A) -4 **(B)** 1 **(C)** 2 **(D)** -2

19. The lines $p(p^2 + 1) x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1) y + 2q = 0$ are perpendicular to a common line for:

(A) exactly one value of p (B) exactly two values of p **[AIEEE - 2009]**

(C) more than two values of p (D) no value of p

20. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point : **[AIEEE - 2009]**

(A) $\left(\frac{5}{4}, 0\right)$ (B) $\left(\frac{5}{2}, 0\right)$ (C) $\left(\frac{5}{3}, 0\right)$ (D) 0, 0

21. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is **[AIEEE - 2010]**

(A) $\sqrt{17}$ (B) $\frac{17}{\sqrt{15}}$ (C) $\frac{23}{\sqrt{17}}$ (D) $\frac{23}{\sqrt{15}}$

22. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE - 2011]**

Statement - 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement - 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1

(B) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

23. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE - 2011]**

(A) $(0, \infty)$ (B) $[1, \infty)$ (C) $(-1, \infty)$ (D) $(-1, 1]$

24. If A(2, –3) and B(–2, 1) are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is : **[AIEEE - 2011]**

(A) $x - y = 1$ (B) $2x + 3y = 1$ (C) $2x + 3y = 3$ (D) $2x - 3y = 1$

25. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE - 2012]**

(A) $\frac{29}{5}$ (B) 5 (C) 6 (D) $\frac{11}{5}$

26. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE - 2012]**

(A) $-\frac{1}{4}$ (B) -4 (C) -2 (D) $-\frac{1}{2}$

27. A ray of light along $x + \sqrt{3}\,y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[AIEEE - 2013]**

(A) $y = x + \sqrt{3}$ **(B)** $\sqrt{3}y = x - \sqrt{3}$ **(C)** $y = \sqrt{3}x - \sqrt{3}$ **(D)** $\sqrt{3}y = x - 1$

28. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[AIEEE - 2013]**

(A) $2 + \sqrt{2}$ **(B)** $2 - \sqrt{2}$ **(C)** $1 + \sqrt{2}$ **(D)** $1 - \sqrt{2}$

29. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d =$ lies in the fourth quadrant and is equidistant from the two axes then **[JEE Main 2014]**

(A) $2bc - 3ad = 0$ **(B)** $2bc + 3ad = 0$ **(C)** $3bc - 2ad = 0$ **(D)** $3bc + 2ad = 0$

30. Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1, –1) parallel to PS is : **[JEE Main 2014]**

(A) $4x - 7y - 11 = 0$ **(B)** $2x + 9y + 7 = 0$ **(C)** $4x + 7y + 3 = 0$ **(D)** $2x - 9y - 11 = 0$

31. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus ? **[JEE Main 2016]**

(A) (–3, –8) **(B)** $\left(\frac{1}{3}, -\frac{8}{3}\right)$ **(C)** $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ **(D)** (–3, –9)

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is **[IIT-JEE - 2003]**

(A) 133 **(B)** 190 **(C)** 233 **(D)** 105

2. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is **[IIT-JEE - 2003]**

(A) $\left(3, \frac{5}{4}\right)$ **(B)** (3, 12) **(C)** $\left(3, \frac{3}{4}\right)$ **(D)** (3, 9)

3. The centre of circle inscribed in a square formed by lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is **[IIT-JEE - 2003]**

(A) (4, 7) **(B)** (7, 4) **(C)** (9, 4) **(D)** (4, 9)

4. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is **[IIT-JEE - 2004]**

(A) 2 sq units **(B)** 4 sq. units **(C)** 6 sq. units **(D)** 8 sq. units

5. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. **[IIT-JEE - 2005]**

6. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are **[IIT-JEE - 2007]**

(A) $\left(\frac{4}{3}, 3\right)$ **(B)** $\left(3, \frac{2}{3}\right)$ **(C)** $\left(3, \frac{4}{3}\right)$ **(D)** $\left(\frac{4}{3}, \frac{2}{3}\right)$

7. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[IIT-JEE - 2007]**

Statement - I : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement - II : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement - I is True, Statement - II is True; Statement - II **is** a correct explanation for Statement - I

(B) Statement - I is True, Statement - II is True; Statement - II is **NOT** a correct explanation for Statement - I

(C) Statement - I is True, Statement - II is False

(D) Statement - I is False, Statement - II is True

8. Consider three points

$P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where

$0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then, **[IIT-JEE - 2008]**

(A) P lies on the line segment RQ **(B)** Q lies on the line segment PR

(C) R lies on the line segment QP **(D)** P, Q, R are non-collinear

9. The locus of the orthocentre of the triangle formed by the lines **[IIT-JEE - 2009]**

$(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is

(A) a hyperbola **(B)** a parabola **(C)** an ellipse **(D)** a straight line

10. A straight line **L** through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If **L** also intersects the x-axis, then the equation of **L** is **[IIT-JEE 2011]**

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ **(B)** $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ **(D)** $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

11. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then **[JEE Ad. 2013]**

(A) $a + b - c > 0$ **(B)** $a - b + c < 0$ **(C)** $a - b + c > 0$ **(D)** $a + b - c < 0$

12. Consider a pyramid OPQRS located in the first octant ($x \geq 0, y \geq 0\ z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then **[JEE Ad. 2016]**

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$ & $x + 4cy + c = 0$ are concurrent, then a, b, c are in :

(A) A.P. **(B)** G.P. **(C)** H.P. **(D)** none

2. Given the family of lines, $a(3x + 4y + 6) + b(x + y + 2) = 0$. The line of the family situated at the greatest distance from the point P (2, 3) has equation

(A) $4x + 3y + 8 = 0$ **(B)** $5x + 3y + 10 = 0$ **(C)** $15x + 8y + 30 = 0$ **(D)** none

3. The absolute value of difference of the slopes of the lines $x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ is

(A) -2 **(B)** $1/2$ **(C)** 2 **(D)** 1

4. P is point on either of the two lines $y - \sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. The co-ordinates of the foot of the perpendicular from P on the bisector of the angle between them are :-

(A) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$ or $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ depending on which the points p is taken

(B) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$

(C) $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ **(D)** $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

5. The equations of the sides of a square whose each side is of length 4 units and centre is (1, 1). Given that one pair of sides is parallel to $3x - 4y = 0$.

(A) $3x - 4y + 11 = 0$, $3x - 4y - 9 = 0$, $4x + 3y + 3 = 0$, $4x + 3y - 17 = 0$

(B) $3x - 4y - 15 = 0$, $3x - 4y + 5 = 0$, $4x + 3y + 3 = 0$, $4x + 3y - 17 = 0$

(C) $3x - 4y + 11 = 0$, $3x - 4y - 9 = 0$, $4x + 3y + 2 = 0$, $4x + 3y - 18 = 0$

(D) None

6. A rectangle ABCD has its side AB parallel to the line $y = x$ and vertices A, B and D lie on $y = 1$, $x = 2$ and $x = -2$ respectively. Locus of vertex C is

(A) $x - y = 5$ **(B)** $x = 5$ **(C)** $x + y = 5$ **(D)** $y = 5$

7. The acute angle between two straight lines passing through the point M(−6, −8) and the points in which the line segment $2x + y + 10 = 0$ enclosed between the co−ordinate axes is divided in the ratio 1 : 2 : 2 in the direction from the point of its intersection with the x−axis to the point of intersection with the y−axis is :

(A) $\pi/3$ **(B)** $\pi/4$ **(C)** $\pi/6$ **(D)** $\pi/12$

8. A variable line is drawn through O to cut two fixed straight lines L_1 and L_2 in R and S. A point P is chosen on the variable line such that $\frac{m+n}{OP} = \frac{m}{OR} + \frac{n}{OS}$. Find the locus of P which is a straight line passing through the point of intersection of L_1 and L_2.

(A) $cn(ax + by - 1) + m(y - c) = 0$ **(B)** $n(ax + by - 1) + m(y - c) = 0$

(C) $cn(ax + by - 1) + (y - c) = 0$ **(D)** $n(ax + by - 1) + (y - c) = 0$

9. A is a point on either of two rays $y + \sqrt{3}\,|x| = 2$ at a distance of $\frac{4}{\sqrt{3}}$ units from their point of intersection. The co–ordinates of the foot of perpendicular from A on the bisector of the angle between them are

(A) $\left(-\frac{2}{\sqrt{3}}, 2\right)$ **(B)** $(0, 0)$ **(C)** $\left(\frac{2}{\sqrt{3}}, 2\right)$ **(D)** $(0, 4)$

10. Consider the following statements :

S_1 : The image of the point (2, 1) with respect to the line $x + 1 = 0$ is $(-2, 1)$.

S_2 : If $(\bullet, m)$ is a point on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$, then $\frac{1+m}{2}$ is a prime number.

S_3 : Orthocentre of the triangle with vertices (10, 20), (22, 25) and (10, 25) is (10, 25).

S_4 : The line $y = mx$ bisect the angle between the lines $ax^2 - 2hxy + by^2 = 0$ if $h(1 - m^2) + m(a - b) = 0$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) FTTF **(B)** FFTT **(C)** TTFF **(D)** FTTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. Let $u \equiv ax + by + a\sqrt[3]{b} = 0$, $v \equiv bx - ay + b\sqrt[3]{a} = 0$, where $a, b \in R$ be two straight lines. The equations of the bisectors of the angles formed by $k_1u - k_2v = 0$ & $k_1u + k_2v = 0$ for non zero real k_1 & k_2 are :

(A) $u = 0$ **(B)** $k_2u + k_1v = 0$ **(C)** $k_2u - k_1v = 0$ **(D)** $v = 0$

12. If one diagonal of a square is the portion of the line $\frac{x}{a} + \frac{y}{b} = 1$ intercepted by the axes, then the extremities of the other diagonal of the square are

(A) $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$ **(B)** $\left(\frac{a-b}{2}, \frac{a+b}{2}\right)$ **(C)** $\left(\frac{a-b}{2}, \frac{b-a}{2}\right)$ **(D)** $\left(\frac{a+b}{2}, \frac{b-a}{2}\right)$

13. The coordinates of the feet of $\perp$ from the vertices of a Δ on the opposite sides are (20, 25), (8, 16) and (8, 9). The coordinates of a vertex of the Δ are

(A) (5, 10) **(B)** (50, – 5) **(C)** (15, 30) **(D)** (10, 15)

14. The points A (0 , 0), B (cos α , sin α) and C (cos β, sin β) are the vertices of a right angled triangle if

(A) $\sin\frac{\alpha-\beta}{2} = \frac{1}{\sqrt{2}}$ **(B)** $\cos\frac{\alpha-\beta}{2} = -\frac{1}{\sqrt{2}}$ **(C)** $\cos\frac{\alpha-\beta}{2} = \frac{1}{\sqrt{2}}$ **(D)** $\sin\frac{\alpha-\beta}{2} = -\frac{1}{\sqrt{2}}$

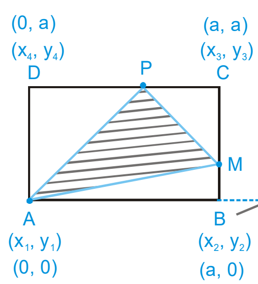
15. Let $D(x_4, y_4)$ be a point such that ABCD is a square & M & P are the midpoints of the sides BC & CD respectively, then

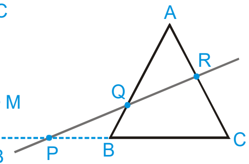
(A) Ratio of the areas of ΔAMP and the square is 3 : 8

(B) Ratio of the areas of ΔMCP & ΔAMD is 1 : 1

(C) Ratio of the areas of ΔABM & ΔADP is 1 : 3

(D) Ratio of the areas of the quandrilateral AMCP and the square is 1 : 3





SECTION - III : ASSERTION AND REASON TYPE

16. **Statement -I :** If $-2h = a + b$, then one line of the pair of lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between co-ordinate axes in positive quadrant.

Statement -II : If $ax + y(2h + a) = 0$ is a factor of $ax^2 + 2hxy + by^2 = 0$, then $b + 2h + a = 0$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

17. **Statement-I :** Perpendicular from point A (1, 1) to the line joining the points B $(c\cos\alpha, c\sin\alpha)$ and C $(c\cos\beta, c\sin\beta)$ bisects BC for all values of α and β.

Statement-II : Perpendicular drawn from the vertex to the base of an isosceles triangle bisects the base.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

18. **Statement-I :** Two of the straight lines represented by the equation $ax^3 + bx^2y + cxy^2 + dy^3 = 0$ will be right angled if $a^2 + ac + bc + d^2 = 0$.

Statement -II : Product of the slopes of two perpendicular lines is – 1.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

19. **Statement -I :** Let the vertices of a ΔABC are $A(-5, -2)$, $B(7, 6)$ and $C(5, -4)$, then co-ordinates of circumcentre is (1, 2).

Statement -II : In a right angle triangle, mid-point of hypotenuous is the circumcentre of the triangle.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

20. **Statement - I :** The internal angle bisector of angle C of a triangle ABC with sides AB, AC and BC are $y = 0$, $3x + 2y = 0$ and $2x + 3y + 6 = 0$ respectively, is $5x + 5y + 6 = 0$.

Statement -II : Image of point A with respect to $5x + 5y + 6 = 0$ lies on side BC of the triangle.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

SECTION - IV : MATRIX - MATCH TYPE

21. **Match the following**

Column - I	Column - II
(A) The number of integral values of 'a' for which the point $P(a, a^2)$ lies completely inside the triangle formed by the lines $x = 0$, $y = 0$ and $x + 2y = 3$	**(p)** 1
(B) Triangle ABC with AB = 13, BC = 5 and AC = 12 slides on the coordinate axis with A and B on the positive x-axis and positive y-axis respectively, the locus of vertex C is a line $12x - ky = 0$, then the value of k is	**(q)** 4
(C) The reflection of the point $(t - 1, 2t + 2)$ in a line is $(2t + 1, t)$, then the line has slope equals to	**(r)** 3
(D) In a triangle ABC the bisector of angles B and C lie along the lines $x = y$ and $y = 0$. If A is (1, 2) then $\sqrt{10}$ d(A,BC) where d(A, BC) represents distance of point A from side BC	**(s)** 5
	(t) 0

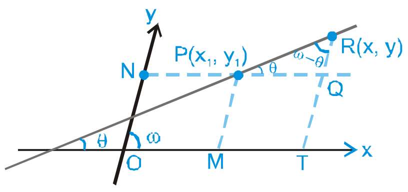
22.

Column - I	Column - II
(A) Two vertices of a triangle are $(5, -1)$ and $(-2, 3)$. If orthocentre is the origin, then coordinates of the third vertex are	**(p)** $(-4, -7)$
(B) A point on the line $x + y = 4$ which lies at a unit distance from the line $4x + 3y = 10$, is	**(q)** $(-7, 11)$
(C) Orthocentre of the triangle made by the lines $x + y - 1 = 0$, $x - y + 3 = 0$, $2x + y = 7$ is	**(r)** $(1, -2)$
(D) If a, b, c are in A.P., then lines $ax + by = c$ are concurrent at	**(s)** $(-1, 2)$
	(t) $(4, -7)$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Let us consider the situation when axes are inclined at an angle 'ω'. If coordinates of a point P are (x_1, y_1) then $PN = x_1$, $PM = y_1$. Where PM is parallel to y-axis and PN is parallel x-axis.



Now $RQ = y - y_1$, $PQ = x - x_1$

From ΔPQR, we have

$$\frac{PQ}{\sin(\omega - \theta)} = \frac{RQ}{\sin\theta}$$

$\therefore$ Equation of straight line through P and makes an angle θ with x-axis is

$$y - y_1 = \frac{\sin\theta}{\sin(\omega - \theta)}(x - x_1)$$

written in the form of

$y - y_1 = m(x - x_1)$ where $m = \dfrac{\sin\theta}{\sin(\omega - \theta)}$. (m is called of slope of line)

$\therefore$ Angle of inclination of line with x-axis is given by $\tan\theta = \left(\dfrac{m\sin\omega}{1 + m\cos\omega}\right)$

Read the above comprehension and answer the following questions.

1. The axes being inclined at an angle of 60°, then the inclination of the straight line $y = 2x + 5$ with the axis of x is

(A) 30° **(B)** $\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ **(C)** $\tan^{-1}2$ **(D)** 60°

2. The axes being inclined a t an angle of 60°, then angle between the two straight lines $y = 2x + 5$ and $2y + x + 7 = 0$ is

(A) 90° **(B)** $\tan^{-1}\left(\dfrac{5}{3}\right)$ **(C)** $\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ **(D)** $\tan^{-1}\left(\dfrac{5}{\sqrt{3}}\right)$

3. The axes being inclined at an angle of 30°, then equation of straight line which makes an angle of 60º with the positive direction of x-axis and x-intercept equal to 2, is

(A) $y - \sqrt{3}x = 0$ **(B)** $\sqrt{3}y = x$ **(C)** $y + \sqrt{3}x = 2\sqrt{3}$ **(D)** $y + 2x = 0$

24. Read the following comprehension carefully and answer the questions.

$A(1, 3)$ and $C\left(-\dfrac{2}{5}, -\dfrac{2}{5}\right)$ are the vertices of a triangle ABC and the equation of the angle bisector of $\angle ABC$ is $x + y = 2$.

Answer the following questions

1. Equation of side BC is

(A) $7x + 3y - 4 = 0$ **(B)** $7x + 3y + 4 = 0$ **(C)** $7x - 3y + 4 = 0$ **(D)** $7x - 3y - 4 = 0$

2. Coordinates of vertex B are

(A) $\left(\dfrac{3}{10}, \dfrac{17}{10}\right)$ **(B)** $\left(\dfrac{17}{10}, \dfrac{3}{10}\right)$ **(C)** $\left(-\dfrac{5}{2}, \dfrac{9}{2}\right)$ **(D)** $(1, 1)$

3. Equation of side AB is

(A) $3x + 7y = 24$ **(B)** $3x + 7y + 24 = 0$ **(C)** $13x + 7y + 8 = 0$ **(D)** $13x - 7y + 8 = 0$

25. Read the following comprehensions carefully and answer the questions.

Let $P(x_1, y_1)$ be a point not lying on the line ● : $ax + by + c = 0$. Let L be a point on line ● such that PL is perpendicular to the line ● .

Let Q(x, y) be a point on the line passing through P and L. Let absolute distance between P and Q is n times ($n \in R^+$) the absolute distance between P and L. If L and Q lie on the same side of P, then coordinates of Q are given by the formula $\frac{x-x_1}{a} = \frac{y-y_1}{b} = -n\frac{ax_1+by_1+c}{a^2+b^2}$ and if L and Q lie on the opposite sides of P, then the coordinates of Q are given by the formula $\frac{x-x_1}{a} = \frac{y-y_1}{b} = n\frac{ax_1+by_1+c}{a^2+b^2}$

1. Let (2, 3) be the point P and $3x - 4y + 1 = 0$ be the straight line ●, if the sum of the coordinates of a point Q lying on PL, where L and Q lie on the same side of P and n = 15 is α, then α =

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

2. Let (1, 1) be the point P and $-5x + 12y + 6 = 0$ be the straight line ●, if the sum of the coordinates of a point Q lying on PL, where L and Q are on opposite sides of P and $n = 13\alpha$ is β, then β =
(α is as obtained in the above question)

(A) –9 **(B)** 25 **(C)** 12 **(D)** 16

3. Let (2, –1) be the point P and $x - y + 1 = 0$ be the straight line ●, if a point Q lies on PL where L and Q are on the same side of P for which $n = \beta$, then the coordinates of the image Q′ of the point Q in the line ● are
(β is as obtained in the above question)

(A) (14, 28) **(B)** (30, –29) **(C)** (26, –27) **(D)** (–26, 27)

SECTION - VI : INTEGER TYPE

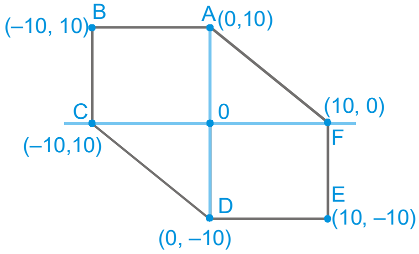
26. Is there a real value of λ for which the image of the point $(\lambda, \lambda - 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so, find λ.

27. Through the origin O a straight line is drawn to cut the lines $y = m_1x + C_1$ and $y = m_2x + C_2$ at Q and R. respectively. Find the locus of the point P on this variable line, such that OP is the geometric mean of OQ and OR.

28. The vertices B and C of a triangle ABC lie on the lines $3y = 4x$ and $y = 0$ respectively and the side BC passes through the point $\left(\frac{2}{3}, \frac{2}{3}\right)$. If ABOC is a rhombus, O being the origin. If co-ordinates of vertex A is (α, β), then find the value of $\frac{5}{2}(\alpha + \beta)$.

29. The equations of two adjacent sides of a rhombus formed in first quadrant are represented by $7x^2 - 8xy + y^2 = 0$, then slope of its longer diagonal is :

30. How many integral points are there on and inside the region bounded by straight lines as shown



ANSWER KEY

EXERCISE - 1

1. C **2.** C **3.** D **4.** D **5.** C **6.** A **7.** D **8.** C **9.** B **10.** B **11.** C **12.** D **13.** C
14. D **15.** B **16.** B **17.** A **18.** B **19.** D **20.** B **21.** C **22.** A **23.** C **24.** C **25.** D **26.** B
27. A **28.** C **29.** A **30.** C **31.** C **32.** D **33.** A **34.** B **35.** A **36.** A **37.** C

EXERCISE - 2 : PART # I

1. ABD **2.** AC **3.** AB **4.** AB **5.** ABCD **6.** BD **7.** BD **8.** AC **9.** ABC
10. AC **11.** ACD **12.** AB **13.** AC **14.** BC **15.** B **16.** AC **17.** CD **18.** ACD
19. AD **20.** AB

PART - II

1. C **2.** A **3.** D **4.** C **5.** C **6.** A **7.** D **8.** A **9.** D **10.** A

EXERCISE - 3 : PART # I

1. $A \rightarrow q, B \rightarrow s, C \rightarrow p, D \rightarrow r$ **2.** $A \rightarrow q\,s, B \rightarrow r, C \rightarrow p, D \rightarrow q\,s$
3. $A \rightarrow q, B \rightarrow r, C \rightarrow s, D \rightarrow p$ **4.** $A \rightarrow q, B \rightarrow s, C \rightarrow p, D \rightarrow t$

PART - II

Comprehension # 1 : **1.** B **2.** B **3.** A **Comprehension # 2 :** **1.** B **2.** C **3.** D
Comprehension # 3 : **1.** A **2.** B **3.** C **Comprehension # 4 :** **1.** B **2.** C **3.** D
Comprehension # 5 : **1.** D **2.** D **3.** A **Comprehension # 6 :** **1.** D **2.** A **3.** C

EXERCISE - 5 : PART # I

1. A **2.** B **3.** D **4.** D **5.** A **6.** D **7.** C **8.** D **9.** D **10.** B **11.** B **12.** A **13.** B
14. B **15.** C **16.** A **17.** C **18.** A **19.** A **20.** A **21.** C **22.** C **23.** B **24.** B **25.** C **26.** C
27. B **28.** B **29.** C **30.** B **31.** B

PART - II

1. B **2.** C **3.** A **4.** A **5.** $y = 2x + 1$ or $y = -2x + 1$ **6.** C **7.** C **8.** D **9.** D **10.** B
11. A or C **12.** BCD

MOCK TEST

1. C **2.** A **3.** C **4.** B **5.** A **6.** D **7.** B **8.** A **9.** B
10. D **11.** AD **12.** AC **13.** ABC **14.** AC **15.** A **16.** B **17.** D **18.** B
19. A **20.** B
21. $A \rightarrow t, B \rightarrow s, C \rightarrow p, D \rightarrow q$ **22.** $A \rightarrow p, B \rightarrow q, C \rightarrow s, D \rightarrow s$ **23.** **1.** B **2.** D **3.** C
24. **1.** B **2.** C **3.** A **25.** **1.** C **2.** D **3.** B **26.** 2
27. $(y - m_1x)(y - m_2x) = c_1c_2$ **28.** 6 **29.** 2 **30.** 331