

HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $60 = 2^2 \times 3^1 \times 5^1$

Number of divisor = 12

3. $A_\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

$$\Rightarrow A_\alpha A_\beta = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) \\ -\sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} = A_{\alpha+\beta}$$

4. $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$$abc = 120 = 2^3 \times 3^1 \times 5^1$$

Case-I $= {}^5C_2 \times {}^3C_2 \times {}^3C_2 = 90$

Case-II $= 3 \times ({}^5C_2 \times {}^3C_2 \times {}^3C_2) = 270$

$$= 90 + 270 = 360$$

5. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2+3 \\ 0 & 1 \end{bmatrix}$$

on multiplying the matrix we get

$$\begin{bmatrix} 1 & 1+2+\dots+n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow n(n+1) = 378 \times 2 \Rightarrow n = 27$$

6. Hint : $x = 11 - y$ & $x + 5 = y$

9. $A^2 - 2A + I = 0$

$$\Rightarrow (A - I)^2 = 0$$

$$A^n = (A - I + I)^n$$

$$= {}^nC_0 (A - I)^n + \dots + {}^nC_{n-2} (A - I)^2 \cdot I^{n-2} + {}^nC_{n-1} I$$

$$(A - I) \cdot I^{n-1} + {}^nC_n I^n$$

$$= 0 + 0 + \dots + 0 + n(A - I) + I = nA - (n - 1)I$$

11. $A^T = -A$ & $A^T A = I$

$$\Rightarrow A^2 = -I \Rightarrow A^{4n} = I$$

$$A^{4n-1} = A^{-1} \Rightarrow A^{4n-1} = A^T \text{ (A is orthogonal)}$$

17. A is involutory

$$\Rightarrow A^2 = I \Rightarrow A = A^{-1}$$

$$\text{now } A^2 = \left(\frac{A}{2}\right)(2A) = I \Rightarrow 2A = \left(\frac{A}{2}\right)^{-1}$$

18. $A = 3 \times 4$; $A' = 4 \times 3$

$$\text{As } A' B \text{ is defined} \Rightarrow \text{let order of } B = 3 \times n$$

$$\text{now } BA' = (3 \times n) \times (4 \times 3) \Rightarrow n = 4$$

$$\therefore \text{order of } B \text{ is } 3 \times 4$$

$$\therefore \text{order of } B' = 4 \times 3$$

$$\text{order of } B' A = (4 \times 3) \times (3 \times 4) = 4 \times 4$$

22. $AB = \begin{pmatrix} \cos^2 \alpha & \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha & \sin^2 \alpha \end{pmatrix}$

$$\begin{pmatrix} \cos^2 \beta & \sin \beta \cos \beta \\ \sin \beta \cos \beta & \sin^2 \beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2 \alpha \cos^2 \beta + \sin \alpha \cos \alpha \sin \beta \cos \beta \\ \cos^2 \beta \sin \alpha \cos \alpha + \sin^2 \alpha \sin \beta \cos \beta \end{pmatrix}$$

$$\begin{pmatrix} \cos^2 \alpha \sin \beta \cos \beta + \sin \alpha \cos \alpha \sin^2 \beta \\ \sin \alpha \cos \alpha \sin \beta \cos \beta + \sin^2 \alpha \sin^2 \beta \end{pmatrix}$$

$$\begin{pmatrix} \cos \alpha \cos \beta \cos(\alpha - \beta) & \cos \alpha \sin \beta \cos(\alpha - \beta) \\ \sin \alpha \cos \beta \cos(\alpha - \beta) & \sin \alpha \sin \beta \cos(\alpha - \beta) \end{pmatrix}$$

$$\Rightarrow \alpha - \beta \text{ must be an odd integral multiple of } \pi/2$$

$$\Rightarrow \text{(C)}$$

23. $t_r(A) + 2 t_r(B) = -1$ (from the given matrix)

$$\text{and } 2 t_r(A) - t_r(B) = 3 \text{ (from the given matrix)}$$

$$\text{Let } t_r(A) = x \text{ and } t_r(B) = y$$

$$x + 2y = -1$$

$$2x - y = 3$$

$$\text{solving } x = 1 \text{ and } y = -1$$

$$\text{Hence } t_r(A) - t_r(B) = x - y = 2$$



24. Obv. A is orthogonal as $a_{11}^2 + a_{12}^2 = 1 = a_{21}^2 + a_{22}^2$

for skew symmetric matrix $a_{ii} = 0 \Rightarrow \theta = (2n+1) \frac{\pi}{2}$

for symmetric matrix, $A = A^T \Rightarrow \sin \theta = 0 \Rightarrow \theta = n\pi$

Also $\text{adj} A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ and $|A| = 1$

hence $A = A^{-1}$ is possible if $\sin \theta = 0$

25. $A \cdot \text{adj} A = |A| I$

$|A| = xyz - 8x - 3(z-8) + 2(2-2y)$

$|A| = xyz - (87x + 3z + 4y) + 28 \Rightarrow 60 - 20 + 28 = 68$

$\Rightarrow$ (C)

26. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

hence $\det. A = \sec^2 x$

$\therefore \det A^T = \sec^2 x$

now $f(x) = \det. (A^T A^{-1})$

$= (\det. A^T) (\det. A^{-1})$

$= (\det. A^T) (\det. A)^{-1}$

$= \frac{\det. (A^T)}{\det. (A)} = 1$

hence $f(x) = 1$

27. As $A(\theta) = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$

$A(\theta)$ is certainly neither symmetric nor skew symmetric

Further, $A(\pi - \theta) = \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

and $A(\theta) \cdot A(\pi - \theta) = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix} \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{-1}(\theta) = A(\pi - \theta)$

$A^2(\theta) = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix} \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$

$= \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{bmatrix} \neq A\left(\frac{\pi}{2} - 2\theta\right)$

28. $\begin{vmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 4 & k \end{vmatrix} = 0$

$1(3k - 16) - 2(k - 12) + 2(4 - 9) = 0$

$3k - 16 - 2k + 24 - 10 = 0$

$k = 2$

29. $A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (i)

and $A^2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (ii)

Let A be given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$;

Hence $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$; $\begin{bmatrix} a-b \\ c-d \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

The first equation gives

$a - b = -1$ (iii)

and $c - d = 2$ (iv)

For second equation, $A^2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = A \left(A \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$
 $= A \left(\begin{bmatrix} -1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

This gives $-a + 2b = 1$ (v)

and $-c + 2d = 0$ (vi)

(iii) + (v) $\Rightarrow b = 0$ and $a = -1$

(iv) + (vi) $\Rightarrow d = 2$ and $c = 4$

so the sum $a + b + c + d = 5$

30. Given $A^2 = A$

$I = (I - 0.4A)(I - \alpha A)$

$= I - I\alpha A - 0.4AI + 0.4\alpha A^2$

$= I - A\alpha - 0.4A + 0.4\alpha A$

$= I - A(0.4 + \alpha) + 0.4\alpha A$

hence $0.4\alpha = 0.4 + \alpha \Rightarrow \alpha = -2/3$



EXERCISE - 2

Part # I : Multiple Choice

1. $A^2 = A \Rightarrow |A|^2 = |A| \Rightarrow |A| = 1$

$$A(\text{adj } A) = |A| I$$

$$\text{adj } A = A^{-1}$$

$$\text{also } A^2 = A$$

$$A = I \Rightarrow \text{adj } A = I$$

$$(\text{adj } A)^2 = I \Rightarrow (\text{adj } A)^2 = \text{adj } A$$

4. $A^T = BCD$

$$AA^T = ABCD \Rightarrow AA^T = S$$

$$\Rightarrow AA^T = S^T \Rightarrow S = S^T$$

$$D^T C^T B^T A^T = ABC \cdot DAB \cdot CDA \cdot BCD$$

$$(ABCD)^T = (ABCD)(ABCD)(ABCD)$$

$$S^T = S^3 \Rightarrow S = S^3$$

$$\Rightarrow S^2 = S^4$$

11. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$C = A - B \begin{bmatrix} 1-\alpha & 0 \\ 0 & -2 \end{bmatrix} \text{ is diagonal matrix, } \forall \alpha \in \mathbb{R}$$

12. $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$$\Rightarrow \lambda^2 - \lambda(a+d) + ad - bc = 0$$

This is characteristic equation. Comparing with given equation we get

$$k = ad - bc = |A|, \quad a + d = 0$$

14. For $|AB| = 0 \Rightarrow |A| \cdot |B| = 0$

$$\Rightarrow |A| = 0 \text{ or } |B| = 0$$

$$AA^{-1} = I \Rightarrow |A| \cdot |A|^{-1} = |I| = 1$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A|} = |A|^{-1}$$

15. We have $A^2B = A(AB) = AA = A^2$, $B^2A = B(BA) = BB = B^2$,

$$ABA = A(BA) = AB = A, \text{ and } BAB = B(AB) = BA = B$$

16. Note that A is non singular but B is singular hence only A^{-1} exists

$$\text{Now } XA = B \Rightarrow X = BA^{-1} \dots (i)$$

$$\text{and } AY = B \Rightarrow Y = A^{-1}B \dots (ii)$$

$$\text{Also } A^{-1} = \frac{1}{3} \begin{bmatrix} -1 & 1 \\ -4 & 1 \end{bmatrix} \text{ now verify}$$

17. $AB = O$

$$\therefore |AB| = 0 \Rightarrow |A| \cdot |B| = 0$$

$$\therefore \det A \neq 0$$

$$\therefore A^{-1} \text{ exist}$$

$$\therefore A^{-1}(AB) = A^{-1}(O) = 0$$

$$IB = 0$$

$$B = 0 \Rightarrow B \text{ must be null matrix.}$$

18. $PQ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow B, C, D$

19. $A = \begin{bmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{bmatrix}$; $x(x^2 - 1) - 1(x) = 6 - 2 = 4$

20. $|A^T A^{-1}| = |A^T| |A^{-1}| = |A^T| \frac{1}{|A|} = 1$

$$\Rightarrow f(x) = 1$$

Part # II : Assertion & Reason

3. Given $AB + A + B = O$

$$AB + A + B + I = I$$

$$A(B + I) + (B + I) = I$$

$$(A + I)(B + I) = I$$

$$\Rightarrow (A + I) \text{ and } (B + I) \text{ are inverse of each other}$$

$$\Rightarrow (A + I)(B + I) = (B + I)(A + I)$$

$$\Rightarrow AB = BA$$



4. Let $A = \begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix}$ $A^T = \begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$

$$AA^T = \begin{bmatrix} a^2 + p^2 & ab + pq & ac + pr \\ ab + pq & b^2 + q^2 & bc + qr \\ ac + pr & bc + qr & c^2 + r^2 \end{bmatrix}$$

$$|AA^T| = \begin{vmatrix} a & p & 0 \\ b & q & 0 \\ c & r & 0 \end{vmatrix} = \begin{vmatrix} a & b & c \\ p & q & r \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow AA^T$ is singular.

6. Statement-I :

$$\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} a+2c & b+2d \\ -a-c & -b-d \end{bmatrix} = \begin{bmatrix} a-b & 2a-b \\ c-d & 2c-d \end{bmatrix}$$

$\Rightarrow 2c = -b$ & $b = a - d$

$\therefore$ infinite matrix are there.

EXERCISE - 3

Part # II : Comprehension

1. $\det. A = \begin{vmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{vmatrix}$

$$= 2(-16 + 15) + 1(12 - 15) + 1(-15 + 20)$$

$$= -2 - 3 + 5 = 0 \Rightarrow A \text{ is singular}$$

$\det. B = \begin{vmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{vmatrix}$

$$= 1(-12 + 12) + 1(12 - 12) + 1(-12 + 12)$$

$$= 0$$

$\Rightarrow B$ is also singular

$\det. C = \begin{vmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{vmatrix} = -1(16 - 12) - 1(-12 + 9)$

$$= -4 + 3 = -1$$

$\Rightarrow C$ is non singular again

$A^2 = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = A \Rightarrow A \text{ is idempotent}$$

$B^2 = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow (B)$ is nilpotent

$C^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow C$ is involutory

(i) obvious (B) as B is nilpotent

(ii) $P = A^3 C^2 = A^3 = A \Rightarrow P^2 = A^2 = A$

$\therefore P^2 = P$

ly in B and D

hence BC^2 is not Idempotent.

(iii) Let $X = BC^2 \Rightarrow \det. X = 0$

$Y = A^2 C^2 \Rightarrow \det. Y = 0$

$Z = A^2 B \Rightarrow \det. Z = 0$

but $W = C^3 \Rightarrow \det. W \neq 0$

hence C^3 has an inverse $\Rightarrow$ (D)

2.

(i) $(A+B)C = (A+B)(A+B)^{-1}(A-B)$

$\Rightarrow (A+B)C = A-B \dots(1)$

$C^T = (A-B)^T \left((A+B)^{-1} \right)^T$

$= (A+B) \left((A+B)^T \right)^{-1} \{ \text{as } |A+B| \neq 0 \}$

$\Rightarrow |(A+B)^T| \neq 0 \Rightarrow |A-B| \neq 0$

$= (A+B)(A-B)^{-1} \dots(2)$

(1) & (2) $C^T(A+B)C = (A+B)(A-B)^{-1}(A-B)$

$= (A+B) \dots(3) \text{ Ans.}$

(ii) taking transpose in (3)

$C^T(A+B)^T(C^T)^T = (A+B)^T$

$C^T(A-B)C = A-B \dots(4) \text{ Ans.}$

(ii) adding (3) and (4)

$C^T[A+B+A-B]C = 2A$

$C^TAC = A \text{ Ans.}$

EXERCISE - 4

Subjective Type

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & -8 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 4+14 & 5-16 & 6+4 \\ -12+28 & 15+32 & 18-10 \\ 20-62 & 25-48 & 30+12 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 18 & -11 & 10 \\ -16 & 47 & 10 \\ 62 & -23 & 42 \end{bmatrix}$

$BA = \begin{bmatrix} 4+15+30 & 8-20+36 \\ 7-24+10 & 14+32+12 \end{bmatrix}$

$\Rightarrow BA = \begin{bmatrix} 49 & 24 \\ -7 & 58 \end{bmatrix}$

2. $(I+A) = (I-A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

L.H.S. $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}$

R.H.S.

$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} \right) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & +\tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos \alpha + \sin \alpha \tan \frac{\alpha}{2} & -\sin \alpha + \tan \frac{\alpha}{2} \cos \alpha \\ -\tan \frac{\alpha}{2} \cos \alpha + \sin \alpha & \sin \alpha \tan \frac{\alpha}{2} + \cos \alpha \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \cos \alpha + 2 \sin \frac{\alpha}{2} \sin \alpha & \sin \frac{\alpha}{2} \left(-2 \cos \frac{\alpha}{2} + \frac{\cos \alpha}{\cos \frac{\alpha}{2}} \right) \\ \sin \alpha \left(-\frac{\cos \alpha}{\cos \frac{\alpha}{2}} + 2 \cos \frac{\alpha}{2} \right) & \cos \alpha + 2 \sin^2 \frac{\alpha}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

4. $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}; A^3 = 5A^2 - 6AI + I^2$

$$A^2 = \begin{bmatrix} 3 & 3 & 7 \\ 1 & 4 & 4 \\ 3 & 1 & 6 \end{bmatrix} \quad A^3 = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix}$$

$$5A^2 + 6A + I = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix} = A^3$$

5. We have, $A - \lambda I$ and $B - \lambda I$ commute

$$\Leftrightarrow (A - \lambda I)(B - \lambda I) = (B - \lambda I)(A - \lambda I)$$

$$\Leftrightarrow AB - \lambda IA - \lambda IB + \lambda^2 I^2 = BA - \lambda BI - \lambda IA + \lambda^2 I^2$$

$$\Leftrightarrow AB - \lambda A - \lambda B + \lambda^2 I = BA - \lambda B - \lambda A + \lambda^2 I$$

$$\Leftrightarrow AB = BA$$

A and B commute

6. We have,

$$AB = A \text{ and } BA = B$$

Now

$$AB = A$$

$$\Rightarrow (AB)A = AA$$

[Multiplying both sides on right by A]

$$\Rightarrow A(BA) = A^2 \quad [\text{By associ. of matrix multip}]$$

$$\Rightarrow AB = A^2 \quad [\rightarrow BA = B]$$

$$\Rightarrow A = A^2 \quad [\rightarrow AB = A]$$

Similarly, $B^2 = B$.

7. $\left\{ \frac{1}{2} (A - A' + I) \right\}^{-1}$ for $A = \begin{bmatrix} -2 & 3 & 4 \\ 5 & -4 & -3 \\ 7 & 2 & 9 \end{bmatrix}$

$$\frac{1}{2} (A - A^T + I)^{-1} = \left[\frac{1}{2} \begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & -5 \\ 3 & 5 & 1 \end{bmatrix} \right]^{-1} = \left(\frac{1}{2} B \right)^{-1}$$

$$[B] = \frac{-39}{8}$$

$$\text{Adj } B = \frac{1}{4} \begin{bmatrix} 26 & -17 & 7 \\ -13 & 10 & -11 \\ 13 & -1 & 5 \end{bmatrix}^T = \frac{1}{4} \begin{bmatrix} 26 & -13 & 13 \\ -17 & 10 & -1 \\ 7 & -11 & 5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{2}{39} \begin{bmatrix} 26 & -13 & 13 \\ -17 & 10 & -1 \\ 7 & -11 & 5 \end{bmatrix}$$

8. $a^2 + b^2 + c^2 = 1$

$$\Delta = \begin{vmatrix} a^2 + (b^2 + c^2) \cos \phi & ab(1 - \cos \phi) & ac(1 - \cos \phi) \\ ba(1 - \cos \phi) & b^2 + (c^2 + a^2) \cos \phi & bc(1 - \cos \phi) \\ ca(1 - \cos \phi) & cb(1 - \cos \phi) & c^2 + (a^2 + b^2) \cos \phi \end{vmatrix}$$

$$\Delta = \frac{1}{abc}$$

$$\begin{vmatrix} a^3 + a(b^2 + c^2) \cos \phi & b^2 a(1 - \cos \phi) & ac^2(1 - \cos \phi) \\ ba^2(1 - \cos \phi) & b^3 + b(c^2 + a^2) \cos \phi & bc^2(1 - \cos \phi) \\ ca^2(1 - \cos \phi) & cb(1 - \cos \phi) & c^3 + c(a^2 + b^2) \cos \phi \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\frac{abc}{abc}$$

$$\begin{vmatrix} a^2 + (b^2 + c^2) \cos \phi & b^2(1 - \cos \phi) & c^2(1 - \cos \phi) \\ a^2(1 - \cos \phi) & b^2 + (c^2 + a^2) \cos \phi & c^2(1 - \cos \phi) \\ a^2(1 - \cos \phi) & b^2(1 - \cos \phi) & c^2 + (a^2 + b^2) \cos \phi \end{vmatrix}$$

$$= (a^2 + b^2 + c^2)$$

$$\begin{vmatrix} 1 & b^2(1 - \cos \phi) & c^2(1 - \cos \phi) \\ 1 & b^2 + (c^2 + a^2) \cos \phi & c^2(1 - \cos \phi) \\ 1 & b^2(1 - \cos \phi) & c^2 + (a^2 + b^2) \cos \phi \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1 \text{ or } R_3 \rightarrow R_3 - R_1$$



$$9. \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = \lambda - 3$$

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 2\lambda + \mu - 16$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 4(\lambda - \mu + 7)$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = (\mu - 10)$$

(i) $\Delta \neq 0 \therefore \lambda \neq 3$

(ii) infinite solutions $\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$.

$\therefore \lambda = 3, \mu = 10$

(iii) $\Delta = 0, \Delta_x, \Delta_y, \Delta_z$

$\therefore \lambda = 3, \mu \neq 10$

10. x, y, z

$x + y + z = 5000$ (i)

$6x + 7y + 8z = 35800$ (ii)

$6x + 7y - 8z = 7000$ (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$X = A^{-1}B$

$|A| = -16$

$$\text{adj} = \begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B = -\frac{1}{16} \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}, x = 1000, y = 2200, z = 1800$$

$$14. A = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix} \neq 0$$

$6\alpha - 5\alpha^2 \neq 0$

$\alpha(6 - 5\alpha) \neq 0$

$\alpha = 0, 6/5 \therefore \alpha \in \mathbb{R} - \{0, 6/5\}$

$\alpha = 1$

$$A = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} \Rightarrow |A| = 6 - 5 = 1$$

$$\Rightarrow \text{Adj } A = \begin{vmatrix} 3 & -1 & -1 \\ -5 & 6 & 5 \\ 3 & -2 & 2 \end{vmatrix}$$

$$\therefore A^{-1} = \begin{vmatrix} 3 & -1 & -1 \\ -5 & 6 & 5 \\ 5 & -2 & 2 \end{vmatrix}$$

$|A - xI| = 0$

$$|A - xI| = 0 \Rightarrow \begin{vmatrix} 2-x & 0 & -1 \\ 5 & 1-x & 0 \\ 0 & 1 & 3-x \end{vmatrix} = 0$$

$\Rightarrow x^3 - 6x^2 + 11x - 1 = 0$

$A^3 - 6A^2 + 11A = I$

$\Rightarrow A^{-1} = A^2 - 6A + 11I$

$(a^2 + b^2 + c^2)$

$$\begin{vmatrix} 1 & b^2(1 - \cos \phi) & c^2(1 - \cos \phi) \\ 0 & (a^2 + b^2 + c^2) \cos \phi & 0 \\ 0 & 0 & (a^2 + b^2 + c^2) \cos \phi \end{vmatrix}$$

$= (a^2 + b^2 + c^2) \cos^2 \phi = \cos^2 \phi$

a, b, c

EXERCISE - 5

Part # 1 : AIEEE/JEE-MAIN

3. $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$$10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

$$B = A^{-1}$$

$$AB = AA^{-1} = I$$

$$10AB = 10I$$

$$(A)(10B) = 10I$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 0 & 5 - \alpha \\ 0 & 10 & \alpha - 5 \\ 0 & 0 & 5 + \alpha \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$5 - \alpha = 0$$

$$\boxed{\alpha = 5}$$

4. $A^2 - A + I = 0$

multiplying by A^{-1}

$$A^{-1}AA - A^{-1}A + A^{-1}I = 0$$

$$IA - I + A^{-1} = 0$$

$$\boxed{A^{-1} = I - A}$$

7. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \quad a, b \in \mathbb{N}$$

$$AB = \begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$$

$$BA = \begin{pmatrix} a & 2b \\ 3b & 4b \end{pmatrix}$$

$$\text{For } AB = BA$$

$$b = a \rightarrow \text{their are infinite}$$

$$\text{Natural number for which } a = 6$$

$$\text{so Infinite matrix B possible}$$

8. $|A^2| = 25$

$$|A|^2 = 25$$

$$|A| = \pm 5$$

$$\begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix} = \pm 5$$

$$25\alpha = \pm 5$$

$$\alpha = \pm \frac{1}{5}$$

9. $A^2 = I$

$$|A^2| = |I|$$

$$|A^2| = 1$$

$$|A| = \pm 1$$

statement-1 :

$$\text{If } A \neq I, \quad A \neq -I$$

$$\text{but } |A| = \pm 1$$

$$\text{so this statement is true}$$

statement-2 :

$$\text{Let } A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|A| = -1 \quad \text{tr}(A) = 0$$

$$\text{but } A \neq I, \quad A \neq -I$$

$$\text{so statement-2 is false}$$

14. $A^T = A$

$$B^T = B$$

Statement-1 :

$$(A(BA))^T = (BA)^T A^T$$

$$= A^T B^T A^T = A(BA) \rightarrow \text{symetric}$$

$$((AB)A)^T = A^T B^T A^T = (AB)A \rightarrow \text{symetric}$$

$$\text{Statement - 1 is true}$$



Statement-2 :

$$(AB)^T = B^T A^T = BA$$

if $AB = BA$ then

$$(AB)^T = BA = AB$$

Statement.- 2 is true

but Not a correct expalnation.

15. **Statement-1 :** The value of det. of skew sym. matrix of odd order is always zero. So Statement-I. is true.

Statement-II : This st. is not always true depends on the order of matrix.

$|-A| = -|A|$ if order is odd, so Statement-II is wrong.

Statement-I is true and Statement-II is false.

16. Since H is a diagonal matrix.

We know that product of two diagonal matrix is always a diagonal matrix.

$$\text{So } H^{70} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \cdots \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \quad 70 \text{ times}$$

$$= \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$$

17. $(P^2 + Q^2)P = P^3 + Q^2P$ (i)

$$(P^2 + Q^2)Q = P^2Q + Q^3 \quad \text{.....(ii)}$$

Equation (i) – Equation (ii)

$$(P^2 + Q^2)(P - Q) = P^3 - Q^3 + Q^2P - P^2Q$$

$$(P^2 + Q^2)(P - Q) = 0 \quad \rightarrow (P \neq Q)$$

$$P^2 + Q^2 = 0$$

$$\text{So } |P^2 + Q^2| = 0$$

$$18. A^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$$

$$\text{and } A^{-1} A U_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad \text{.....(i)}$$

$$A^{-1} A U_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \quad \text{.....(ii)}$$

Eq. (i) + (ii)

$$U_1 + U_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

21.

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$(1\lambda + 1) - 1(-\lambda^2 + 1) - (\lambda + 1) = 0$$

$$(\lambda + 1)(\lambda + 1)(\lambda - 1) = 0$$

$$\lambda = -1 \quad \text{or} \quad 0 \quad \text{or} \quad 1$$

22. $A(\text{adj } A) = |A| I_n = AA^T$ [Given]

$$|A| = 10a + 3b$$

$$A^T = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$AA^T \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$$\Rightarrow 15a - 2b = 0 \quad \Rightarrow a = \frac{2b}{15} \quad \& \quad 10a + 3b = 13$$

$$\Rightarrow a = \frac{13 - 3b}{10}$$

$$\Rightarrow \frac{2b}{15} = \frac{13 - 3b}{10} \quad \Rightarrow 4b = 39 - 9b$$

$$\Rightarrow 13b = 39 \quad \Rightarrow b = 3$$

$$\Rightarrow a \frac{2}{15} \times 3 = \frac{6}{15} = \frac{2}{5} \Rightarrow 5a = 2$$

$$\therefore 5a + b = 2 + 3 = 5$$



Part # II : IIT-JEE ADVANCED

3. $|M - I| = |M - M M^T|$

$$|M - I| = |M| |I - M|$$

$$\Rightarrow |M - I| = |I - M|$$

$$\Rightarrow |M - I| = (-1)^3 |M - I|$$

$$\Rightarrow |M - I| = 0$$

4. $AX = U$

$$\Rightarrow \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$$

has infinitely many solutions.

$$\Rightarrow |A| = 0$$

$$\Rightarrow (c - d)(ab - 1) = 0$$

$$\& (\text{adj } A) U = 0$$

$$\begin{bmatrix} bc - bd & -c & d \\ d - c & ac & -ad \\ 0 & 1 - ab & ab - 1 \end{bmatrix} \begin{bmatrix} f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} fbc - fbd - gc + dh \\ fd - fc + agc - adh \\ g - abg + adh - h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow fd - fc + agc - agh = 0 \quad \dots(i)$$

$$BX = V$$

$$\Rightarrow \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

$$|B| = a(dh - gc) + fc - fd = 0 \text{ (from (i))}$$

$\therefore$ system can't have unique solution

Now $X = (\text{adj } B)V$

$$= \begin{bmatrix} dh - gc & g - h & c - d \\ fc & ah - f & -ac \\ -fd & -g + af & ad \end{bmatrix} \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\text{if } afd \neq 0 \Rightarrow (\text{adj } B) V \neq 0$$

$\therefore$ $af d \neq 0$ then $BX = V$ is inconsistent

5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \Rightarrow |A| = 6$

$$A^{-1} \Rightarrow \frac{\text{adj } A}{|A|} = \frac{1}{6} \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6} [A^2 + cA + dI]$$

$$\frac{1}{6} \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \frac{1}{6} \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix} + \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix} \right\}$$

on comparing we get

$$-1 = 5 + c \Rightarrow c = -6$$

$$1 = 14 + 4c + d \Rightarrow 1 = 14 - 24 + d$$

$$d = 11$$

6. $PP^T = I$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \& \text{ so on}$$

$$Q = PAP^T$$

$$Q^2 = (PAP^T)(PAP^T) = PA^2P^T$$

$$Q^{2005} = PA^{2005}P^T$$

$$x = P^T (PA^{2005}P^T)P$$

$$\Rightarrow x = A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

10. (C) (i) If A is symmetric, $A^T = A$

$$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} a & c \\ b & a \end{bmatrix}$$

$$\Rightarrow b = c$$

If A is skew symmetric, $A^T = -A$

$$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} -a & -c \\ -b & -a \end{bmatrix}$$

$$\Rightarrow a = 0, b + c = 0$$

$$\rightarrow b, c \geq 0$$

$$\Rightarrow a = 0, b = 0, c = 0$$

$$\text{Now, } \det(A) = a^2 - bc \\ = a^2 - b^2$$

($\rightarrow b = c$ for A being symmetric or skew symmetric or both)

$$= (a - b)(a + b) \text{ is divisible by } p.$$

Let $(a - b)(a + b) = \lambda p, \lambda \in I$

Range of $(a + b)$ is 0 to $2p - 2$ which includes only one multiple of p i.e. p

$$\therefore a + b = p \text{ \& } a - b \in I$$

$\Rightarrow$ possible number of pairs of a & b will be $p - 1$.

Also, range of $(a - b)$ is $1 - p$ to $p - 1$ which includes only one multiple of p i.e. 0

$$\therefore a - b = 0 \text{ \& } a + b \in I$$

$\Rightarrow$ Possible number of pairs of a & b will be p .

Hence total number of A in T_p will be

$$p + p - 1 = 2p - 1$$

(iii) Total number of A in $T_p = p^3$

when $a \neq 0$ & $\det(A)$ is divisible by p , then number of A will be $(p - 1)^2$

When $a = 0$ & $\det(A)$ is divisible by p , then number of A will be $2p - 1$.

$$\text{So, total number of A for which } \det(A) \text{ is divisible by } p \\ = (p - 1)^2 + 2p - 1 \\ = p^2$$

$$\text{So number of A for which } \det(A) \text{ is not divisible by } p \\ = p^3 - p^2$$

11. (Comment : Although 3×3 skew symmetric matrices can never be non-singular. Therefore the information given in question is wrong. Now if we consider only non singular skew symmetric matrices M & N, then the solution is-)

$$\text{Given } M^T = -M$$

$$N^T = -N$$

$$MN = NM$$

$$\text{according to question } M^2 N^2 (M^T N)^{-1} (MN^{-1})^T \\ = M^2 N^2 N^{-1} (M^T)^{-1} (N^{-1})^T M^T \\ = M^2 N^2 N^{-1} (-M)^{-1} (N^T)^{-1} (-M) \\ = M^2 N^2 N^{-1} (-M)^{-1} (N^T)^{-1} (-M)$$

$$\begin{cases} MN = NM \\ (MN)^{-1} = (NM)^{-1} \\ N^{-1} M^{-1} = M^{-1} N^{-1} \end{cases} \\ = -M^2 N M^{-1} N^{-1} M \\ = -M^2 N N^{-1} M^{-1} M = -M^2$$

$$12. \begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \\ = 1 - c\omega - a(\omega - \omega^2 c) = (1 - c\omega) - a\omega(1 - c\omega) \\ = (1 - c\omega)(1 - a\omega)$$

for non singular matrix

$$c \neq \frac{1}{\omega} \text{ \& } a \neq \frac{1}{\omega}$$

$$\Rightarrow c \neq \omega^2, a \neq \omega^2$$

$$\Rightarrow a \text{ \& } c \text{ must be } \omega \text{ \& } b \text{ can be } \omega \text{ or } \omega^2$$

$$\therefore \text{ total matrices} = 2$$

$$14. |Q| = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$\Rightarrow |Q| = 2^2 \cdot 2^3 \cdot 2^4 \cdot \begin{vmatrix} a_{11} & 2a_{12} & 2^2 a_{13} \\ a_{21} & 2a_{22} & 2^2 a_{23} \\ a_{31} & 2a_{32} & 2^2 a_{33} \end{vmatrix} \\ = 2^2 \cdot 2^3 \cdot 2^4 |P| \cdot 2^3 \\ = 2^2 \cdot 2^3 \cdot 2^4 \cdot 2 \cdot 2^3 = 2^{13}$$



15. $P^T = 2P + I$

$$\Rightarrow P = 2P^T + I$$

$$\Rightarrow P = 2(2P + I) + I$$

$$\Rightarrow P = 4P + 3I$$

$$\Rightarrow P = -I$$

$$\Rightarrow PX = -X$$

16. $|\text{adj}P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$

$$\Rightarrow |P|^2 = 4$$

$$\Rightarrow |P| = \pm 2$$

19. $\begin{pmatrix} P \\ K \end{pmatrix} \cdot Q = 1$

$$\therefore Q = \begin{pmatrix} P \\ K \end{pmatrix}^{-1}$$

Comparing q_{23} , we get

$$\frac{-K}{8} = \frac{-K(3a+4)}{(12a+20)}$$

$$\alpha = -1$$

Also, $|P| \cdot |Q| = K^3$

$$\therefore (12a+20) \frac{K^2}{2} = K^3$$

$$K = 6a + 10 = 4$$

Hence (B), (C) are correct.

20. $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix}$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{bmatrix}$$

$$\therefore P^n = \begin{bmatrix} 1 & 0 & 0 \\ 4n & 1 & 0 \\ 8(n^2+n) & 4n & 1 \end{bmatrix}$$

$$\therefore P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 8 \times 50(51) & 200 & 1 \end{bmatrix}$$

$$P^{50} - Q = I$$

$\therefore$ Equate we get

$$200 - q_{21} = 0 \Rightarrow q_{21} = 200$$

$$400 \times 51 - q_{31} = 0$$

$$q_{31} = 400 \times 51$$

$$200 - q_{32} = 0 \Rightarrow q_{32} = 200$$

$$\frac{q_{31} - q_{32}}{q_{21}} = \frac{400 \times 51 + 200}{200} = 2(51) + 1 = 103$$

MOCK TEST

1. (D)

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

if $AB = BA$, then $a + 2c = a + 3b$

$\Rightarrow 2c = 3b \Rightarrow b \neq 0$

$b + 2d = 2a + 4b$

$\Rightarrow 2a - 2d = -3b$

$$\frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b-\frac{3}{2}b} = -1$$

2. $AA' = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}$

$AA' = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}$ symmetric

3. (A)

$(B^T AB)^T = B^T A^T (B^T)^T = B^T A^T B$
 $= B^T AB$ iff A is symmetric

$\therefore B^T AB$ is symmetric iff A is symmetric

Also $(B^T AB)^T = B^T A^T B = (-B) A^T B$

$\therefore B^T AB$ is not skew symmetric if B is skew symmetric

4. (C)

$AB = A, BA = B \Rightarrow A^2 = A$ and $B^2 = B$

$(A+B)^2 = A^2 + B^2 + AB + BA$
 $= A + B + A + B = 2(A+B)$

$(A+B)^3 = (A+B)^2 (A+B)$
 $= 2(A+B)^2 = 2^2(A+B)$

$\therefore (A+B)^7 = 2^6(A+B) = 64(A+B)$

5. (B)

$A^3 = 0$

$(I + A + A^2)(I - A) = I - A^3 = I$

$\therefore I + A + A^2 = (I - A)^{-1}$

6. $A = \begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$

$|A| = (3x+\lambda) \begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$

by $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1 = (3x+\lambda) \begin{vmatrix} 1 & x & x \\ 1 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{vmatrix}$

$= (3x+\lambda)(\lambda^2)$

$|A| \neq 0$ for non-singular matrix

$\therefore 3x+\lambda \neq 0, \lambda \neq 0$

7. Let $X = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \quad x^n = 0 \quad \forall n \geq 2$

$\Rightarrow A = X + I$

$A^n = (X + I)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} I + \dots + {}^nC_{n-2} x^2 +$

${}^nC_{n-1} + {}^nC_n I = n x + I = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

$\lim_{n \rightarrow \infty} \frac{1}{n} \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$

8. (A)

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} (-1)$

$= \frac{1}{2} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$



10. (C)

$S_1 : A^T = A$

$((A^{-1})^{-1})^{-1} = A^{-1}$

$(A^{-1})^T = (A^T)^{-1} = A^{-1}$

So $((A^{-1})^{-1})^{-1}$ is also symmetric

S_2 : Obvious

S_3 : Obvious

$S_4 : |A| \neq 0, |B| \neq 0 \Rightarrow |AB| \neq 0$

So AB is invertible

12. (A, B, D)

$$A^2 - 4A - 5I_3 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$-4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore A^2 - 4A - 5I_3 = 0$

or $A^{-1}A^2 - 4A^{-1}A - 5A^{-1}I_3 = 0$

or $(A^{-1}A)A - 4I_3 - 5A^{-1} = 0$

or $IA - 4I_3 - 5A^{-1} = 0$

$\therefore A^{-1} = \frac{1}{5} (A - 4I_3)$

Also, $|A^2| = \begin{vmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{vmatrix} = 9(81 - 64) - 8(72 - 64) + 8(64 - 72)$
 $= 9 \times 17 - 8 \times 8 + 8 \times (-8) = 133 - 128 = 5 \neq 0$

$\therefore A^2$ is invertible

and $A^3 = A \cdot A^2 = A \cdot (4A - 5I_3) = 4A^2 - 5A$

$$= \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} + \begin{bmatrix} -5 & -10 & -10 \\ -10 & -5 & -10 \\ -10 & -10 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 22 & 22 \\ 22 & 31 & 22 \\ 22 & 22 & 31 \end{bmatrix}$$

$\therefore |A^3| \neq 0$

$\therefore A^3$ is invertible.

13. Taking $C_3 \rightarrow C_3 - (C_1\alpha - C_2)$

we get $|A| = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha + 1 \end{vmatrix} = (1 - 2\alpha)(ac - b^2)$

$\therefore$ non-invertible if $\alpha = \frac{1}{2}$ and if a, b, c are in G.P.

14. (A, B, C)

$\rightarrow |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 3(-3 + 4) - 2(-3 + 4) + 0 = 1$

$\rightarrow \text{adj}(\text{adj } A) = |A|^{3-2} A = A$ and $|\text{adj}(\text{adj } A)| = |A| = 1$

Also, $|\text{adj } A| = |A|^{3-1} = |A|^2 = 1^2 = 1$

15. (A, D)

$$A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$$

$A^3 = A^2A = 3A \cdot A = 3A^2 = 3 \cdot (3A) = 9A$ and $|A| = 0$

$\therefore A^{-1}$ does not exist

16. (D)

Statement-1 is false

$\rightarrow A = [a_{ij}]_{n \times n}$ where $a_{ij} = 0, i \geq j$

$\therefore |A| = 0$ hence A is singular inverse of A is not defined

Statement-2 $|A| = 0$

$\therefore$ inverse of A is not defined.

17. $\text{adj}(\text{adj } A) = |A|^{n-2} A$

Here, $n = 3$

$\therefore (\text{adj})(\text{adj } A) = |A| A \quad \dots\dots(i)$

Now, $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$= 3(-3+4) + 3(2) + 4(-2) = 1$

From eq. (i),

$\text{adj}(\text{adj } A) = A$

18. (A)

$A(\alpha) = \begin{bmatrix} f_1(\alpha) & f_2(\alpha) & f_3(\alpha) \\ f_4(\alpha) & f_5(\alpha) & f_6(\alpha) \\ f_7(\alpha) & f_8(\alpha) & f_9(\alpha) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x - \alpha$ is a factor of $f_1(x), f_2(x) \dots\dots\dots f_9(x)$

$f(x) = (x - \alpha) \phi(x)$

$f(\alpha) = 0 \Rightarrow x - \alpha$ is a factor of $f(x)$

19. In a unit matrix of order $n \times n$

Number of non-zero rows = n

$\therefore$ Rank is n . The rank of a non singular matrix of order $n \times n$ is n

$\therefore$ For non singular $|A| \neq 0$.

20. $A = \begin{pmatrix} 0 & -c & b \\ c & 0 & a \\ -b & -a & 0 \end{pmatrix}$

$\rightarrow A = -A' \quad (\rightarrow A \text{ is skew symmetric})$

$\therefore \det(A) = \det(-A')$

$= -\det(A')$

$= -\det A$

$\therefore \det A = 0$

$\rightarrow \det A' = \det(-A')$ is not true

$\therefore \det(-A') = (-1)^3 \det(A') = -\det A'$

21. (A) $\rightarrow (q), (B) \rightarrow (s, t), (C) \rightarrow (r), (D) \rightarrow (p)$

(A) $A^2 = -I \quad \therefore A$ is of even order

(B) $(I + A)^n = C_0 I^n + C_1 I A + C_2 I A^2 + \dots\dots\dots + C_n I A^n$

$= C_0 I + C_1 A + C_2 A + \dots\dots\dots + C_n A$

$= I + (2^n - 1) A$

$\therefore \lambda = 2^n - 1$

(C) $A^2 = A$ and $B = I - A$

$AB + BA + I - (I + A^2 - 2A)$

$= AB + BA - A + 2A = AB + BA + A$

$= A(I - A) + (I - A)A + A$

$= A - A + A - A + A = A$

(D) $A^* = A, B^* = B$

$(AB - BA)^* = B^* A^* - A^* B^* = BA - AB$

22. (A) $A^* B = \frac{1}{2}(AB + BA) = \frac{1}{2}(BA + AB) = B^* A$

$A^* A = \frac{1}{2}(A \cdot A + A \cdot A) = A^2$

$A^* I = \frac{1}{2}(AI + IA) = \frac{1}{2}(A + A) = A$

Now, $A^*(B + C) = \frac{1}{2}\{A(B + C) + (B + C)A\}$

(B) $A^* B = \frac{1}{2}(AB' + A'B) = \frac{1}{2}(A'B + AB')$

$= \frac{1}{2}(BA' + B'A) = B^* A(P)$

$A^* A = \frac{1}{2}(AA' + A'A) \neq A^2$

$A^*(B + C) = \frac{1}{2}(A(B + C)' + A'(B + C))$

$= \frac{1}{2}(A(B' + C') + A'(B + C))$

$= \frac{1}{2}(AB' + A'B) + \frac{1}{2}(AC' + A'C)$

$= A^* B + A^* C (Q)$

$A^* I = \frac{1}{2}(AI' + A'I) = \frac{1}{2}(A + A') \neq A$ or O



$$(C) A * B = \frac{1}{2}(AB - BA) - \frac{1}{2}(BA - AB) = -(B * A)$$

$$\begin{aligned} A * (B + C) &= \frac{1}{2} \{A(B + C) - (B + C)A\} \\ &= \frac{1}{2}(AB - BA) + \frac{1}{2}(AC - CA) \\ &= A * B + A * C(Q) \end{aligned}$$

$$A * A = \frac{1}{2}(A^2 - A^2) = O \neq A^2$$

$$A * I = \frac{1}{2}(AI - IA) = \frac{1}{2}(A - A) = O(T)$$

23.

1. A is singular

$$\therefore |A| = 0 \Rightarrow \begin{vmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$$

$$\Rightarrow 1(40 - 40) - 3(20 - 24) + (\lambda + 2)(10 - 12) = 0$$

$$\Rightarrow 12 - 2\lambda - 4 = 0$$

$$\Rightarrow \lambda = 4$$

$$\therefore A + B = \begin{pmatrix} 4 & 5 & 10 \\ 5 & 6 & 13 \\ 5 & 6 & 14 \end{pmatrix}$$

$$\Rightarrow \text{tr}(A + B) = 4 + 6 + 14 = 24$$

$$2. 2A + 3B = \begin{pmatrix} 11 & 12 & 2\lambda + 16 \\ 13 & 14 & 31 \\ 12 & 13 & 32 \end{pmatrix}$$

$$|2A + 3B| = 0$$

$$\text{we get } \lambda = \frac{17}{2}$$

$$\Rightarrow 2\lambda = 17$$

3. For $\lambda = 3$

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{pmatrix}$$

$$\rightarrow \text{tr}(AB) + \text{tr}(BA) = 2\text{tr}(AB)$$

$$\rightarrow \text{tr}(AB) = C_{11} + C_{22} + C_{33} = 119$$

$$\therefore \frac{1}{7}(\text{tr}(AB) + \text{tr}(BA)) = \frac{1}{7}(238) = 34$$

24.

$$1. \text{adj}(\text{adj } A) = |A|^{n-2} A = |A| A = 2A$$

$$2. |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = |A|^4 = 2^4 = 16$$

$$3. |\text{adj } B| = |B|^{n-1} = |B|^2 = 24^2 = 576$$

25.

1. (A)

Possible combinations are (1, 2, 6), (1, 3, 5), (1, 8, 9), (2, 3, 4), (2, 7, 9), (3, 6, 9), (3, 7, 8), (4, 5, 9), (4, 6, 8), (5, 6, 7)

Hence total 10 combinations are possible.

2. (B)

In a matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ a_{22} must be a multiple of 3

(3, 6 or 9) because from the above possible combinations only 3, 6 & 9 are repeated four times in row or column or diagonal.

3. (A)

Clearly maximum value of sum of the diagonal elements is 18 which is called the trace or the matrix A.

26. (4)

$$A'A = I$$

$$\therefore |A'A| = |I| \Rightarrow |A| = \pm 1$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \pm 1$$

$$\Rightarrow 3abc - a^3 - b^3 - c^3 = \pm 1$$

$$\Rightarrow a^3 + b^3 + c^3 = 2 \text{ or } 4$$



27. (1)

$$I + A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}$$

and $I - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ -1 & 0 \end{pmatrix}$

Now, $|I - A| = \begin{vmatrix} 0 & -2 \\ -1 & 0 \end{vmatrix} = 0 - 2 = -2$

$$\text{adj}(I - A) = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$$

$$(I - A)^{-1} = \begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}$$

$$\therefore \phi(A) = (I + A)(I - A)^{-1} = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix} = -A$$

28. (1)

$$\frac{|\text{adj}B|}{|C|} = \frac{|\text{adj}(\text{adj}A)|}{|5A|} = \frac{|A|^{(3-1)^2}}{5^3 |A|} = \frac{|A|^3}{125}$$

Now $|A| = 5$

$$\therefore \frac{|\text{adj}B|}{|C|} = 1$$

29. $P^2 = \begin{bmatrix} \cos \frac{\pi}{9} & \sin \frac{\pi}{9} \\ -\sin \frac{\pi}{9} & \cos \frac{\pi}{9} \end{bmatrix} = \begin{bmatrix} \cos \frac{\pi}{9} & \sin \frac{\pi}{9} \\ -\sin \frac{\pi}{9} & \cos \frac{\pi}{9} \end{bmatrix}$

$$= \begin{bmatrix} \cos^2 \frac{\pi}{9} - \sin^2 \frac{\pi}{9} & \cos \frac{\pi}{9} \sin \frac{\pi}{9} + \sin \frac{\pi}{9} \cos \frac{\pi}{9} \\ -\sin \frac{\pi}{9} \cos \frac{\pi}{9} - \cos \frac{\pi}{9} \sin \frac{\pi}{9} & -\sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{9} \end{bmatrix}$$

$$= \begin{bmatrix} \cos \frac{2\pi}{9} & \sin \frac{2\pi}{9} \\ -\sin \frac{2\pi}{9} & \cos \frac{2\pi}{9} \end{bmatrix}$$

$$P^3 = \begin{bmatrix} \cos \frac{2\pi}{9} & \sin \frac{2\pi}{9} \\ -\sin \frac{2\pi}{9} & \cos \frac{2\pi}{9} \end{bmatrix} \begin{bmatrix} \cos \frac{\pi}{9} & \sin \frac{\pi}{9} \\ -\sin \frac{\pi}{9} & \cos \frac{\pi}{9} \end{bmatrix}$$

$$P^3 = \begin{bmatrix} \cos \frac{3\pi}{9} & \sin \frac{3\pi}{9} \\ -\sin \frac{3\pi}{9} & \cos \frac{3\pi}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^6 = P^3 \cdot P^3 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\alpha P^6 + \beta P^3 + \alpha I = 0$$

i.e. $\begin{bmatrix} -\frac{\alpha}{2} + \frac{\beta}{2} + \gamma & \frac{\sqrt{3}}{2}\alpha + \frac{\sqrt{3}}{2}\beta \\ -\frac{\sqrt{3}}{2}\alpha - \frac{\sqrt{3}}{2}\beta & -\frac{\alpha}{2} + \frac{\beta}{2} + \gamma \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\alpha = -\beta \quad \beta = -\gamma \Rightarrow \alpha = \gamma$$

$$\Rightarrow \alpha + \beta = 0 ; \beta + \gamma = 0 ; \alpha - \gamma = 0$$

$$\Rightarrow (\alpha^2 + \beta^2 + \gamma^2)^0 = 1$$

30. Given $\sum_{k=1}^4 a_{ik} = 1 \quad \forall i \in \{1, 2, 3, 4\}$

Let's consider $B = A^2 \quad B = [b_{ij}]_{4 \times 4}$

$$b_{ij} = \sum_{k=1}^4 a_{ik} \cdot a_{kj}$$

Sum of elements of i^{th} row in A^2 is

$$\sum_{j=1}^4 b_{ij} = \sum_{j=1}^4 \sum_{k=1}^4 a_{ik} \cdot a_{kj} = \sum_{K=1}^4 \sum_{j=1}^4 a_{ik} \cdot a_{kj} = \sum_{K=1}^4 a_{ik} \sum_{j=1}^4 a_{kj}$$

$$\rightarrow \sum_{j=1}^4 a_{kj} = 1 = \sum_{K=1}^4 a_{ik} = 1$$

$$\Rightarrow \text{Sum of elements in a row of } A^2 \text{ is } 1.$$

$$\Rightarrow \text{Similarly sum of elements in a row of } A^{10} \text{ is } 1$$

$$\Rightarrow \text{Sum of all elements} = 4$$

